CMSC740
Advanced Computer Graphics

Fall 2025

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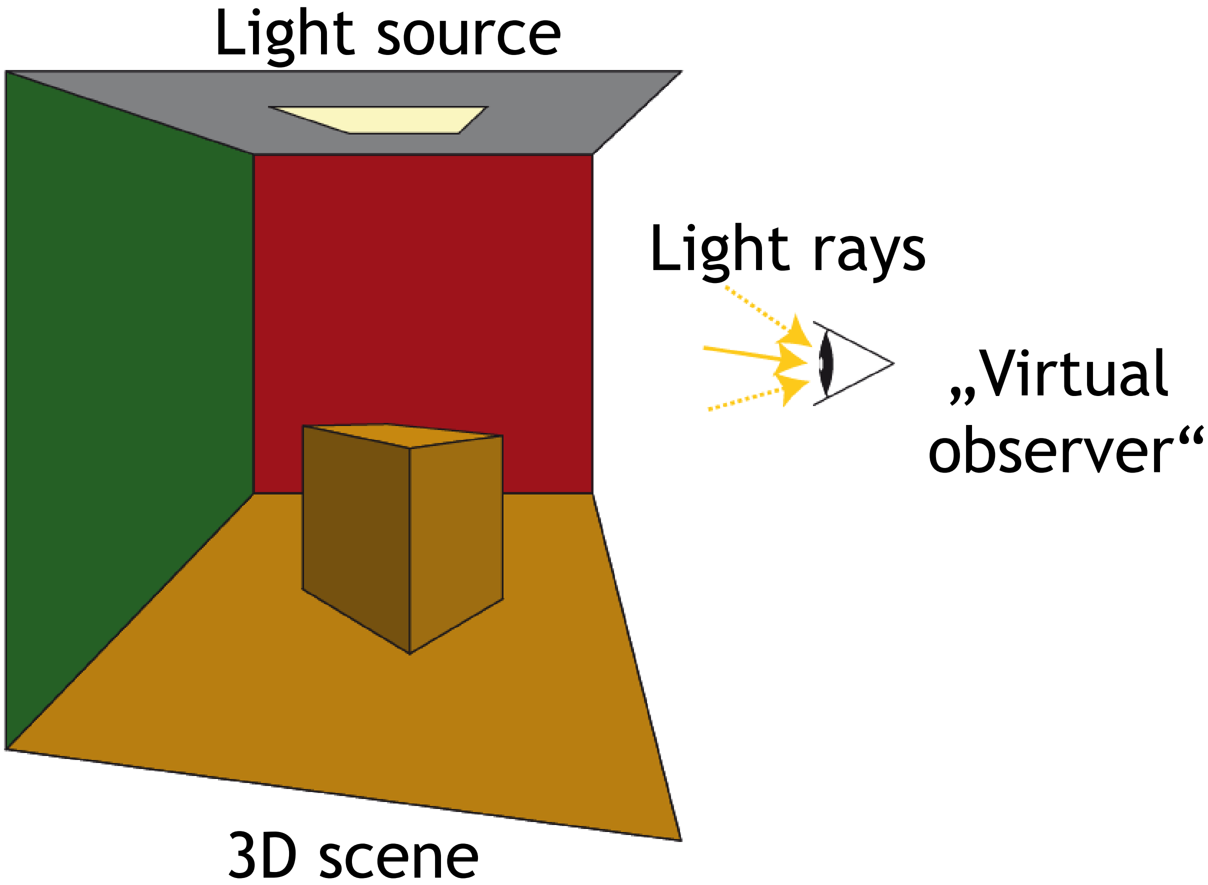
Today

Simulating light transport

- Introduction
- Solid angle, integration over the sphere
- Radiometry

Introduction

- Realistic rendering = simulating light transport in a scene



Approach

- Find suitable physical model that describes important properties of light transport
  - Model of light (today)
  - Model of light transport (later)
- Develop algorithms to simulate light transport
  - Based on ray tracing machinery, Monte Carlo integration

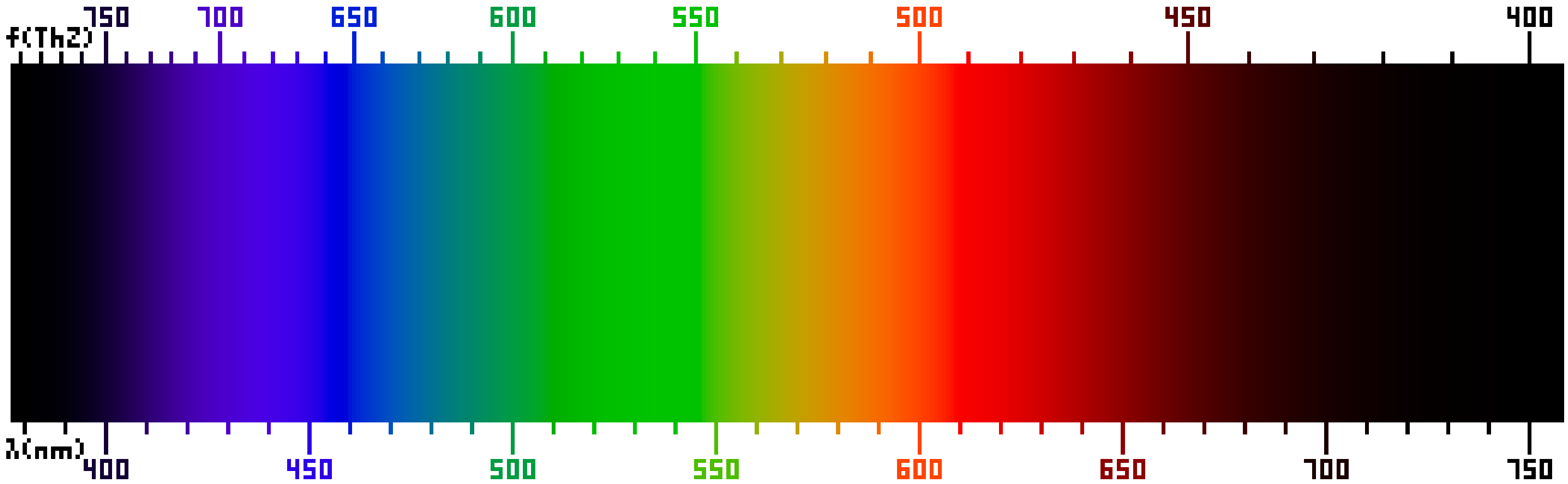
What is a suitable model for light transport simulation in computer graphics?

A. Light particles (photons)
B. Light rays
C. Light waves (electromagnetic waves)

Physical model of light

- Visible light is certain spectrum of electromagnetic radiation

Frequency

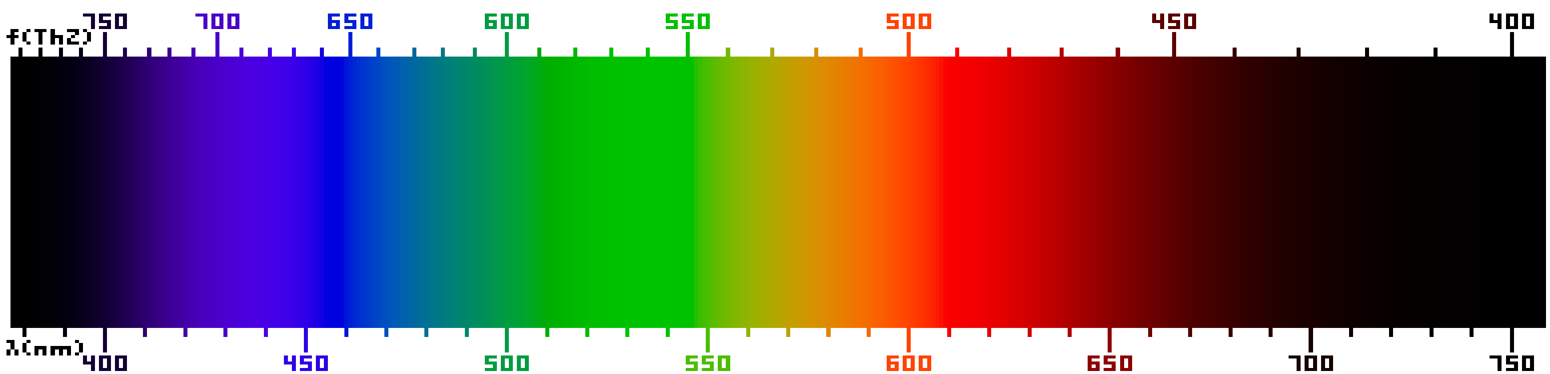


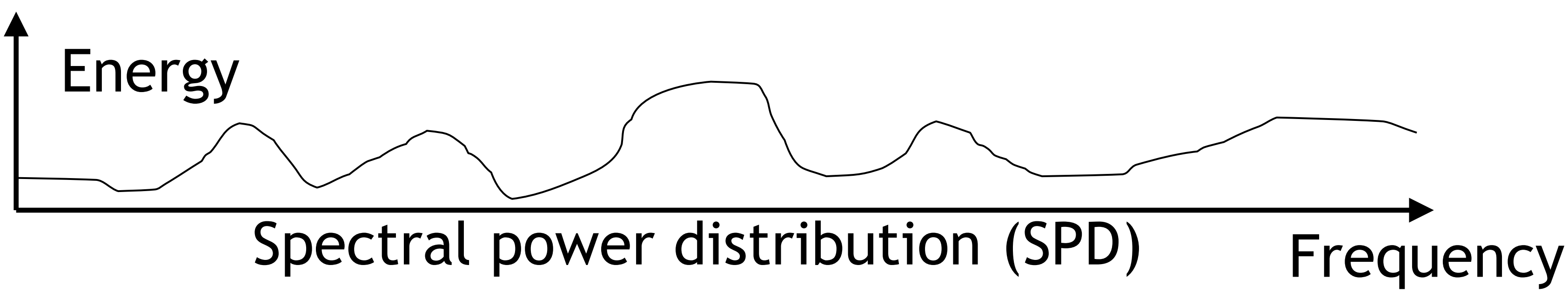
Wavelength

1nm=10^-9 meters, one-billionth of a meter

Geometrical optics (http://en.wikipedia.org/wiki/Geometrical_optics)

- Simple model, but explains most light transport effects visible to the naked eye
- Main assumption: light consists of rays
  - Idealized narrow beams of light
- Rays carry "spectrum of light", spectral power distribution (SPD)



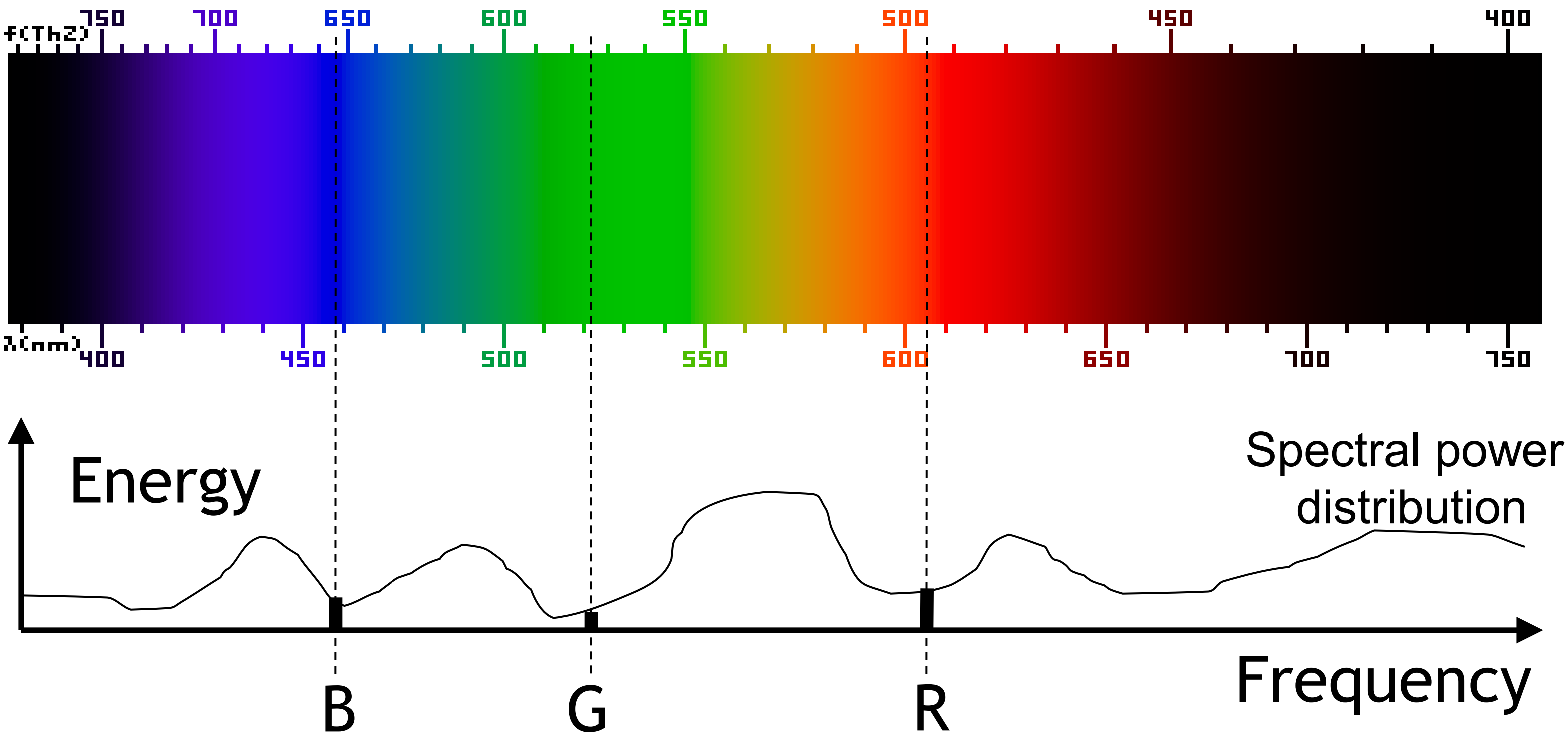


Geometrical optics (http://en.wikipedia.org/wiki/Geometrical_optics)

- „Rays reflect, refract, and scatter when they hit boundary (interface) between different materials (media)“
- As long as ray travels in homogeneous material
  - Ray travels along straight lines
  - In vacuum, power (SPD) along ray is constant
- Assumptions are valid when wavelength of light is small compared to objects it interacts with
  - Otherwise, wave effects appear (diffraction)

Color in computer graphics

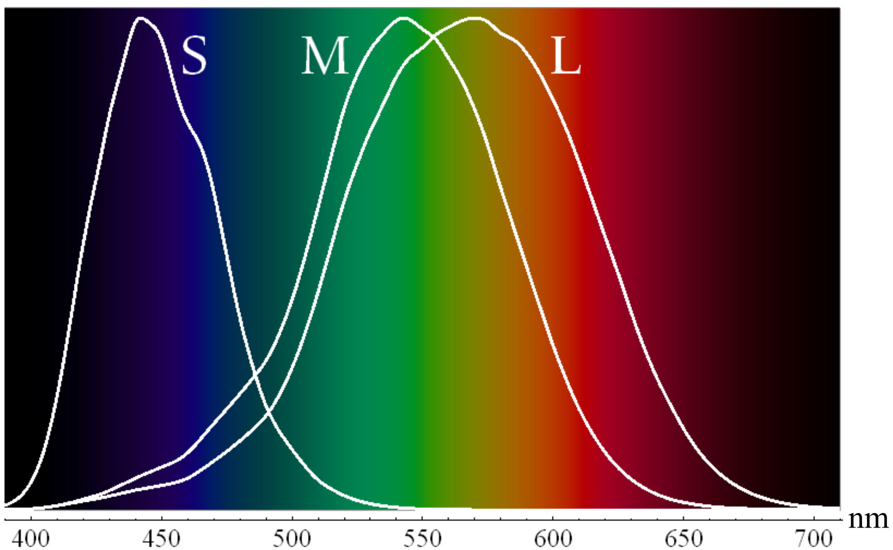
- Store only three samples (values) of spectral power distribution (SPD): red, green, blue (RGB)



Color in computer graphics

- Why is RGB enough?
  - Trichromatic color vision

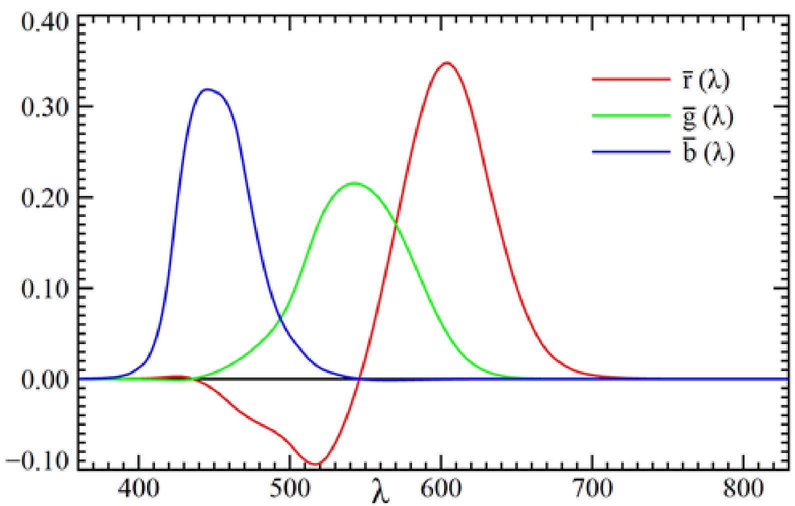
    http://en.wikipedia.org/wiki/Color_vision
    http://en.wikipedia.org/wiki/Trichromatic_color_vision



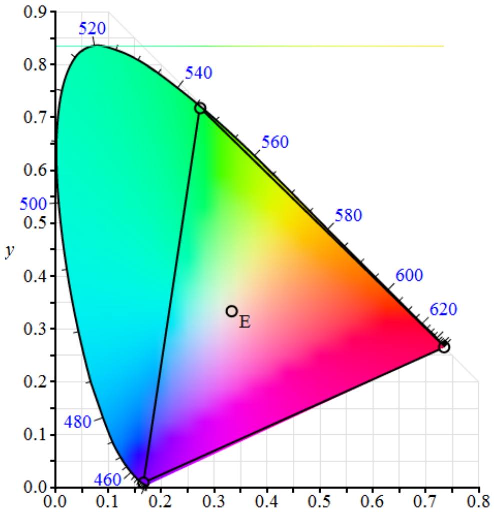
Absorption curves of
S,M,L photoreceptor cells

Color in computer graphics

- Color spaces: quantifying color
  http://en.wikipedia.org/wiki/Color_space
  http://en.wikipedia.org/wiki/CIE_1931_color_space
- Many color spaces derived from CIERGB color matching curves using three primary colors
  - Determined using tristimulus experiment
  - Gamut of 3 primaries doesn't cover all distinct colors



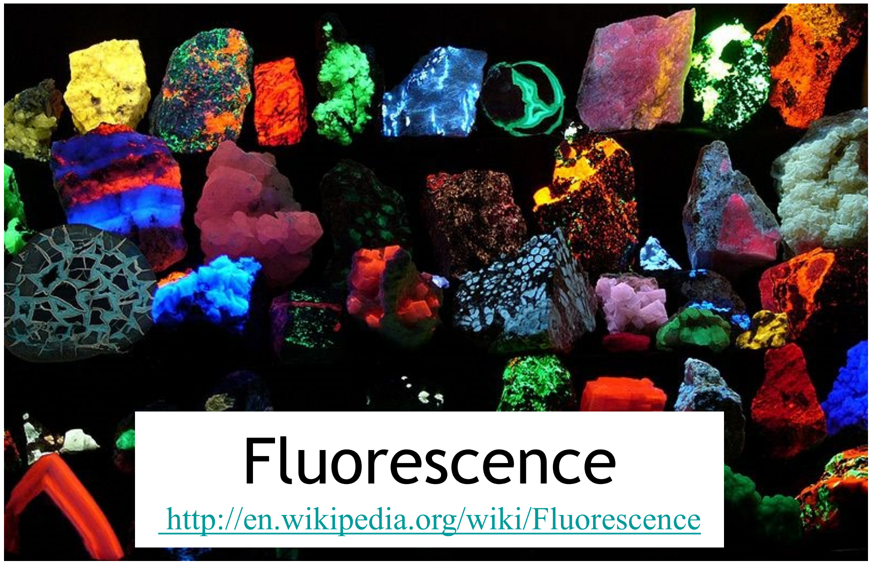
CIERGB color matching curves



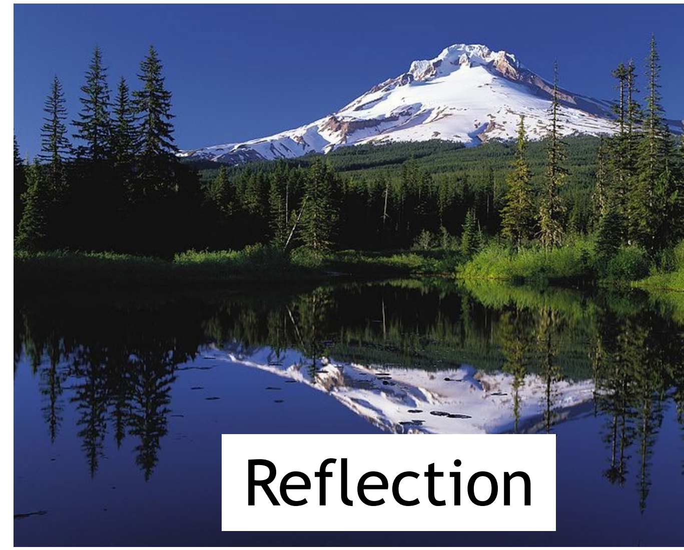
Gamut of CIERGB primaries

Light transport effects

Which ones can be represented with geometrical optics?



Fluorescence
http://en.wikipedia.org/wiki/Fluorescence



Reflection

Thin film interference
http://en.wikipedia.org/wiki/Thin-film_optics
http://en.wikipedia.org/wiki/Interference_(wave_propagation)

Diffraction
http://en.wikipedia.org/wiki/Diffraction

Light transport effects

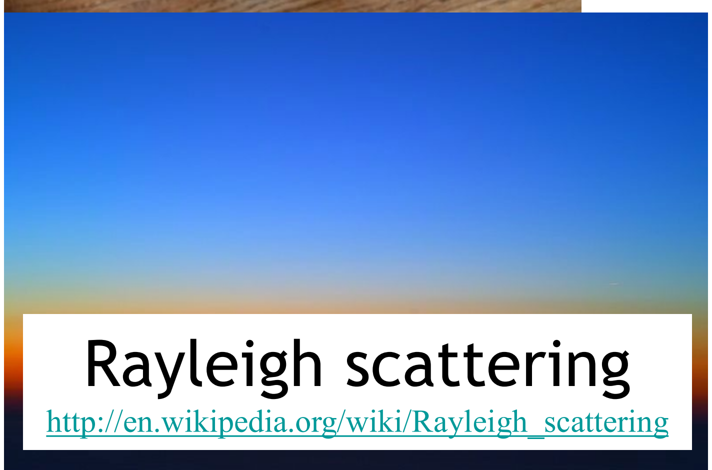
Which ones can be represented with geometrical optics?

Refraction

http://en.wikipedia.org/wiki/Refraction

Refraction, dispersion

http://en.wikipedia.org/wiki/Dispersion_(optics)



Rayleigh scattering

http://en.wikipedia.org/wiki/Rayleigh_scattering

Polarization

http://en.wikipedia.org/wiki/Polarization_(waves)

Light transport effects

Which ones can be represented with geometrical optics?

Scattering in volumes

Subsurface scattering

http://en.wikipedia.org/wiki/Subsurface_scattering

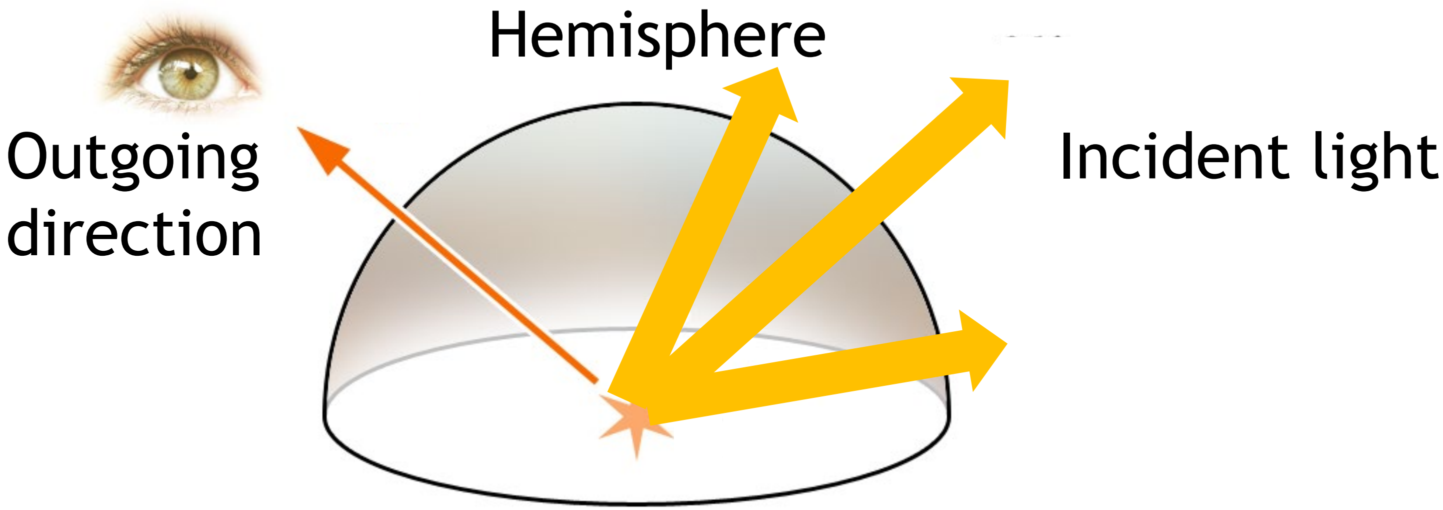
Today

Simulating light transport

- Introduction
- Solid angle, integration over the sphere
- Radiometry

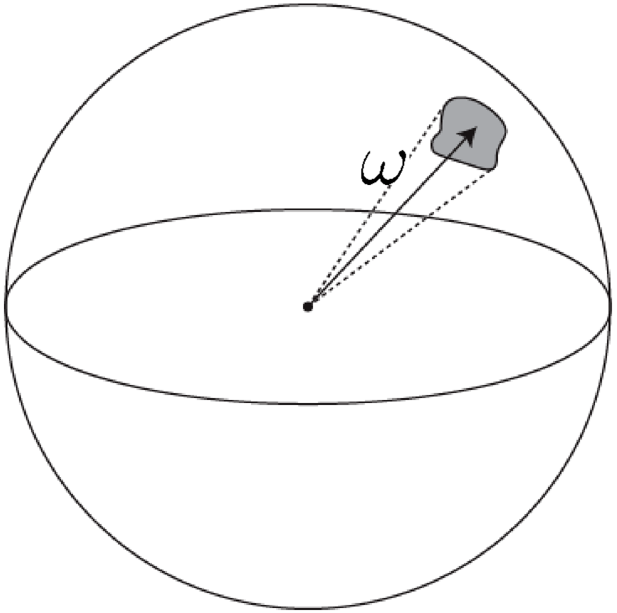
Math challenge

- Integrate functions over the sphere (or hemisphere)
- Application: integrate ("sum up") incident light over the hemisphere at a point on surface to calculate reflected light in some outgoing direction



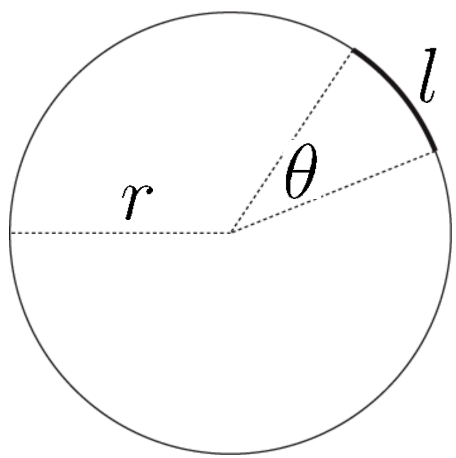
Solid angle

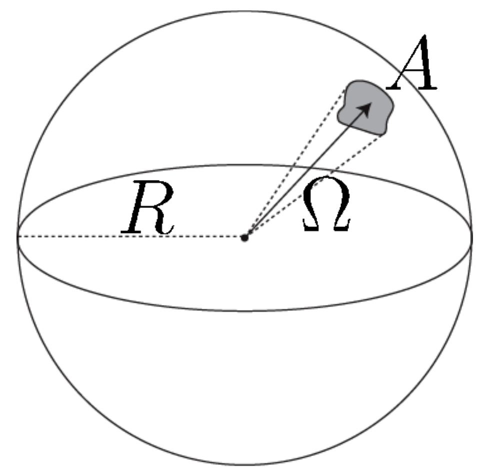
- Area on the unit sphere that is spanned by a set of directions
- Unit: steradian sr
- Directions usually denoted by ω



Angles and solid angles

Solid angle generalizes angles (in radians) from 2D to 3D



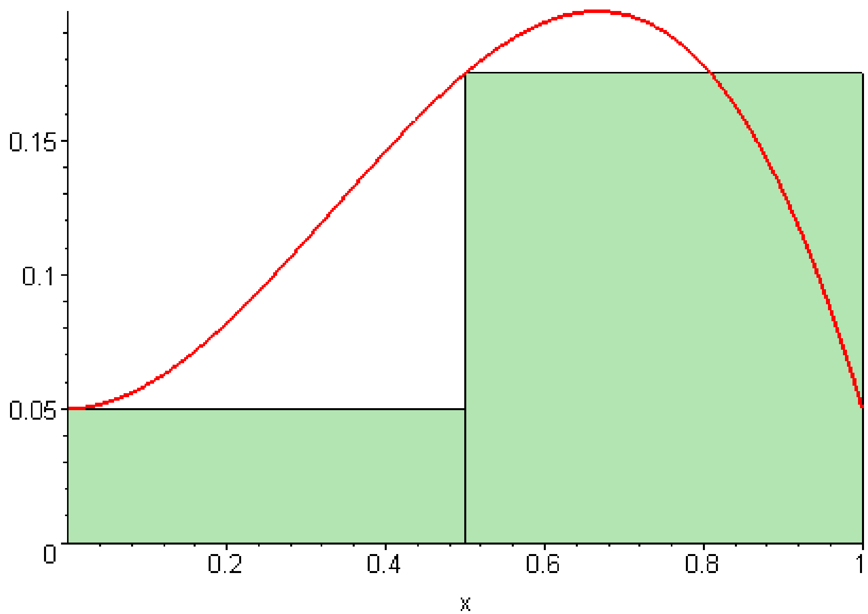


- Angle $\theta = l/r$
- Segment length l, radius r
- Unit circle has radians 2π

- Solid angle $\Omega = A/R^2$
- Area A, radius R
- Unit sphere has 4π steradians

Integrating over the sphere

- Given function of directions $f(\omega)$ defined over sphere S^2
- Want to integrate over sphere, i.e., $\int_{S^2} f(\omega)d\omega$
- Think of integral as a Riemann sum
(http://en.wikipedia.org/wiki/Riemann_sum)
  - Partition (hemi)sphere into small parts



1D Riemann sum

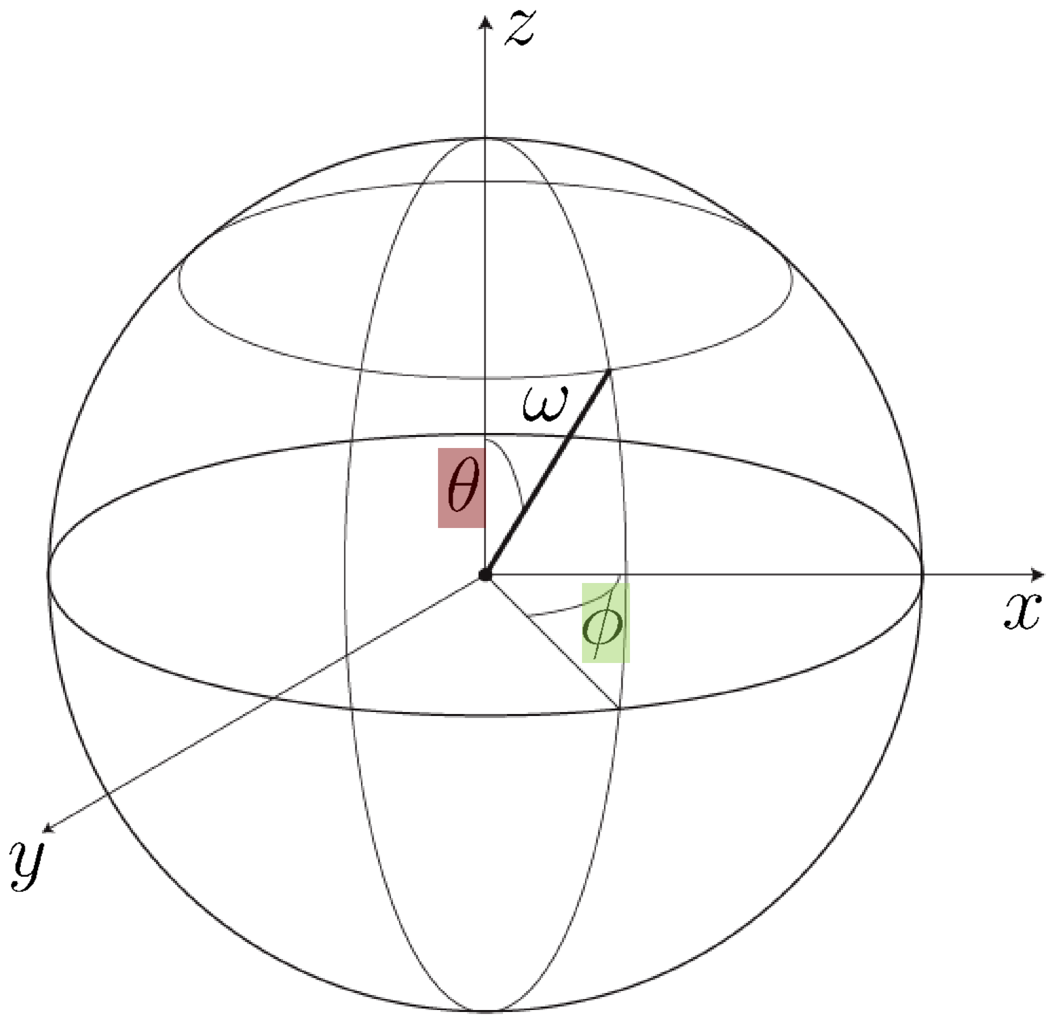
Integrating over the sphere

- Given function of directions $f(\omega)$ defined over sphere S^2
- Want to integrate over sphere, i.e., $\int_{S^2} f(\omega) d\omega$
- Think of integral as a Riemann sum (http://en.wikipedia.org/wiki/Riemann_sum)
  - Partition (hemi)sphere into small parts
  - For each part, multiply f evaluated at some direction ω inside the part by the „differential“ (i.e., small) solid angle (area) $d\omega$ spanned by the part
  - Sum over all parts
  - (Riemann) integral is defined as limit of this sum as parts get infinitely small

Integrating over the sphere

- Analytic integration in the form $\int_{S^2} f(\omega) d\omega$ not directly possible
- Will most often implement numerical approximations
  - Monte Carlo integration, next time
- For analytic integration, need parameterization of function over sphere using two variables
  - E.g., spherical coordinates

Spherical coordinates

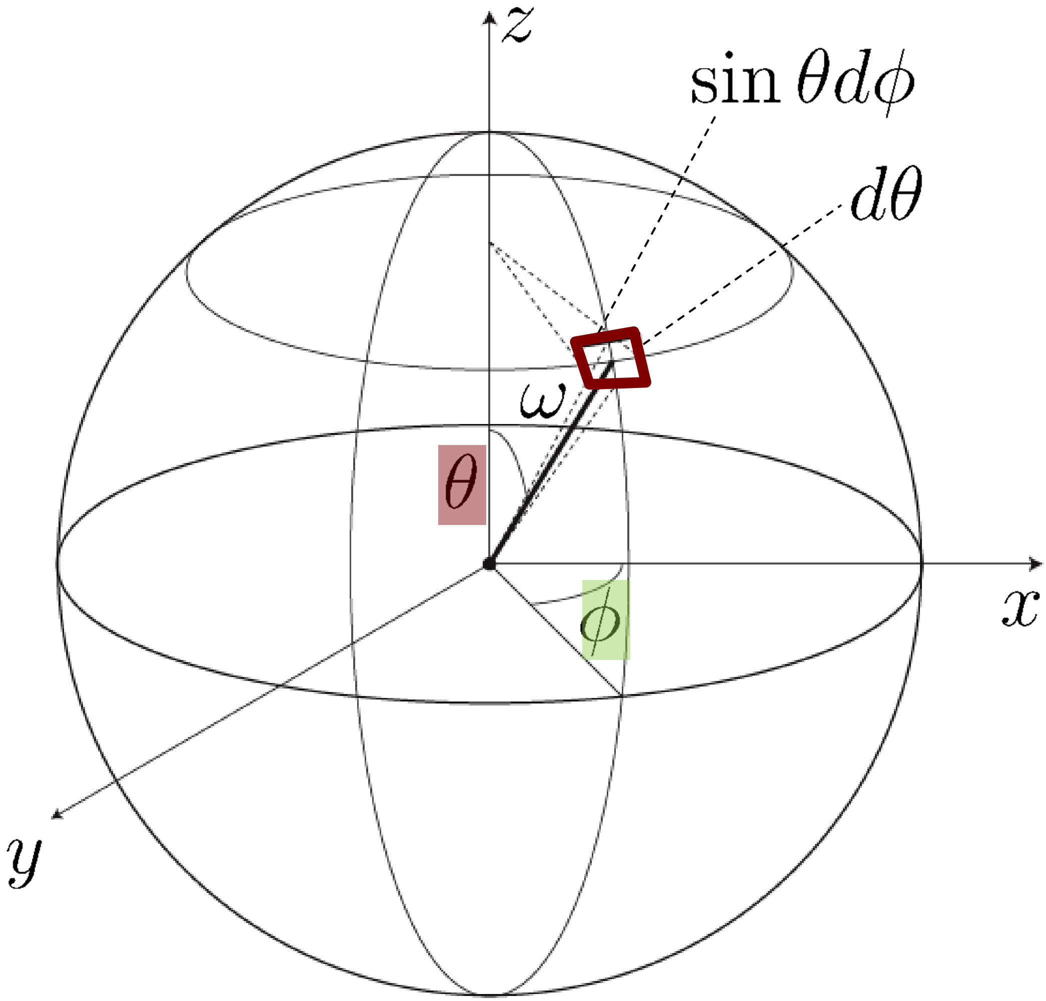


Elevation θ
Azimuth ϕ

Unit direction ω as vector in spherical co-ordinates

$$
\begin{aligned}
\omega_x &= \sin\theta\cos\phi \\
\omega_y &= \sin\theta\sin\phi \\
\omega_z &= \cos\theta
\end{aligned}
$$

Conversion of differential solid angle



Differential solid angle $d\omega$:
"solid angle of infinitely small patch around ω"

Conversion to spherical coords.:

$$d\omega = \sin\theta d\theta d\phi$$

Integrating over the sphere

- Given function over directions $f(\omega)$
- Denote $f(\theta,\phi)$ its reparameterization in spherical coordinates
- Integration

$$\int_{S^2} f(\omega)d\omega = \int_0^{2\pi}\int_0^{\pi} f(\theta,\phi)\sin\theta d\theta d\phi$$

- Mathematically, $\sin\theta$ is the determinant of the Jacobian $\mathrm{D}\rho(\theta,\phi)$ of the mapping $\omega=\rho(\theta,\phi)$ http://en.wikipedia.org/wiki/Integration_by_substitution

Example

- Integral of function $f(\omega)=1$ over the unit sphere?

Ingegral of $f(\omega)=1$ over sphere?

A. 1

B. π

C. 4π

Example

- Integral of function $f(\omega)=1$ over the unit sphere?
- Yields the surface area of the unit sphere

$$\int_{\mathcal{S}^2} d\omega = \int_0^{2\pi} \int_0^{\pi} \sin\theta d\theta d\phi = 4\pi$$

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- Radiometry

Radiometry (http://en.wikipedia.org/wiki/Radiometry)

- Quantify spatial energy distribution of light (in a way that is compatible with the geometrical optics model based on rays)
- Physical measurement of electromagnetic energy
- Units: Watts

Side note: photometry

http://en.wikipedia.org/wiki/Photometry_%28optics%29

- Perceptual measurement of perceived brightness
- Units: Lumen

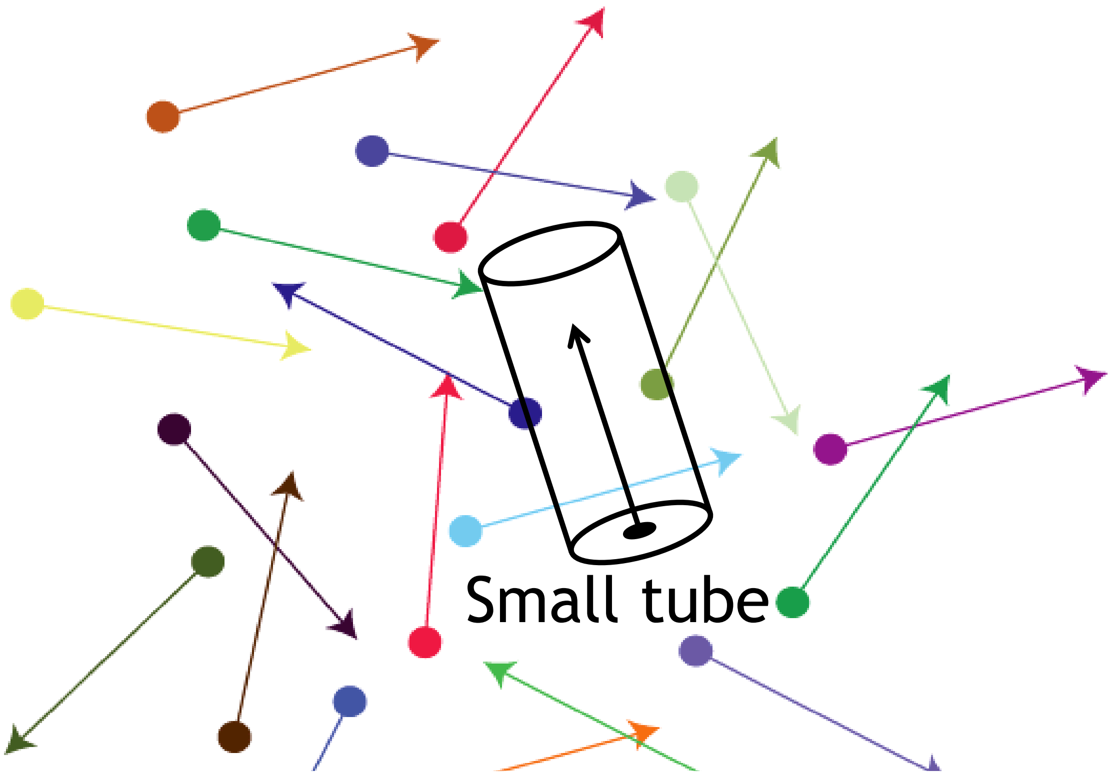
Radiometry

For intuition:

- Assume light consists of photons with
  - Position $\mathbf{x}$
  - Direction of motion ω
  - Wavelength λ
- Each photon has an energy of $h\upsilon$
  h Planck's constant
  $\upsilon = 1/\lambda$ frequency
- Measuring energy means "counting photons"

Radiometry

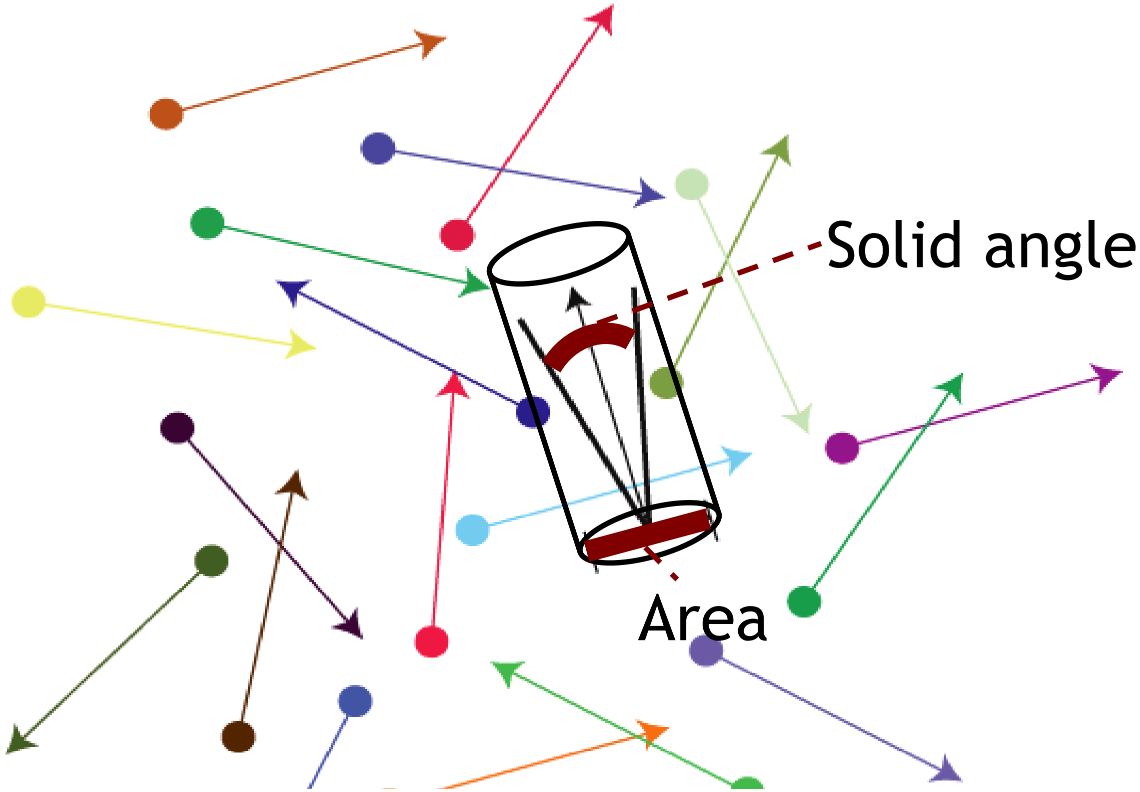
How do we measure the energy flow?



Energy density at some point in some direction:
number of photons traveling through infinitesimally small tube placed at that point/direction

Radiometry

How do we measure the energy flow?



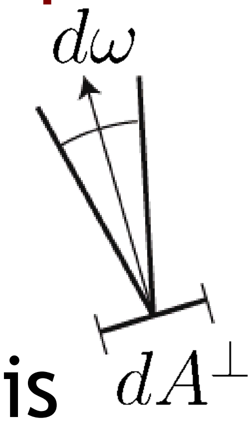
Energy density at some point in some direction:
photons per area per solid angle

Spectral radiance

http://en.wikipedia.org/wiki/Radiance

Definition: Spectral radiance is energy per time, per wavelength, per solid angle, per area

$$L(t, \lambda, \omega, \mathbf{x}) = \frac{d^4 Q(t,\lambda,\omega,\mathbf{x})}{dt\, d\lambda\, d\omega\, dA^{\perp}}$$



where energy ("number of photons") is $Q(t, \lambda, \omega, \mathbf{x})$

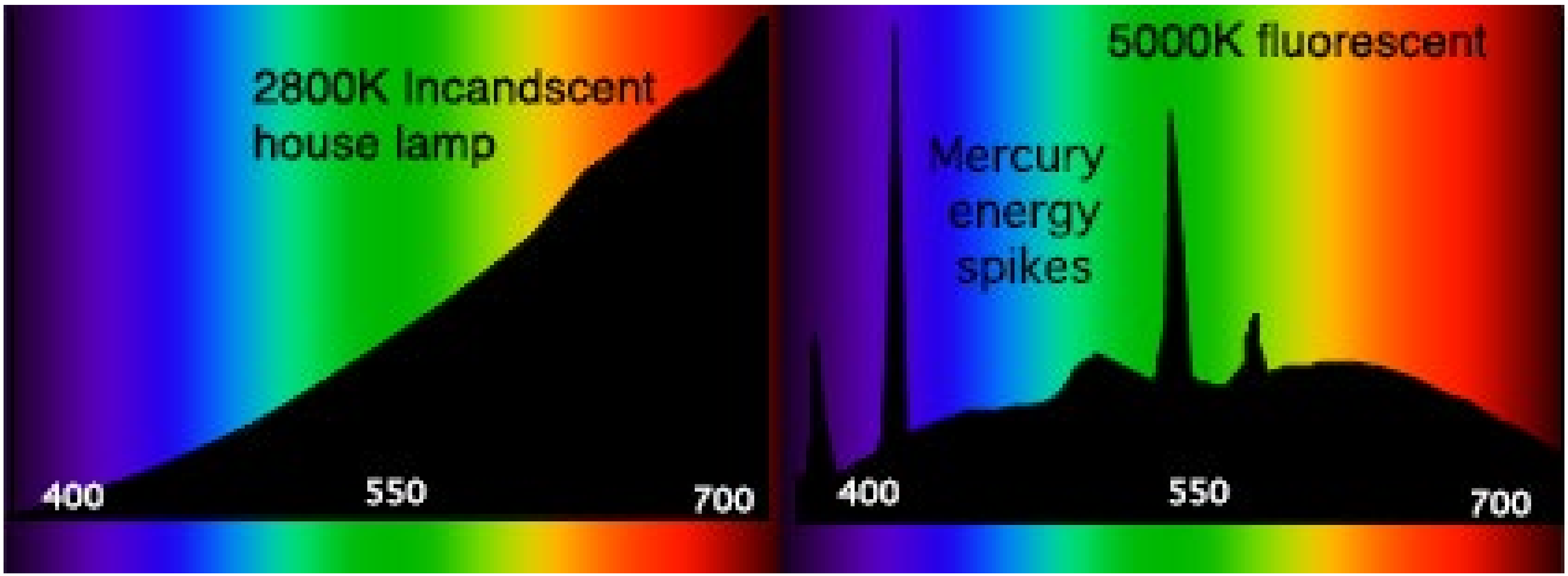
Note: $dA^{\perp}$ means differential area perpendicular to ω

Power (SPD) carried along a ray is measured in radiance
Radiance along ray is constant (in vacuum)

Spectral radiance

http://en.wikipedia.org/wiki/Radiance

- Measurement: spectroradiometer
- Examples:



https://en.wikipedia.org/wiki/Spectroradiometer

Radiance

http://en.wikipedia.org/wiki/Radiance

- In practice: assume steady state (no change over time), measure at discrete wavelengths R,G,B
  - R,G,B not explicit in our notation
- Introduce power, or radiant flux Q

$$\Phi(\omega, \mathbf{x}) = \frac{dQ(\omega,\mathbf{x})}{dt}, \left[W = J \cdot s^{-1}\right]$$

- Our definition for radiance L: power per solid angle per area; vector of 3 values for R,G,B; function L of position $\mathbf{x}$, direction ω

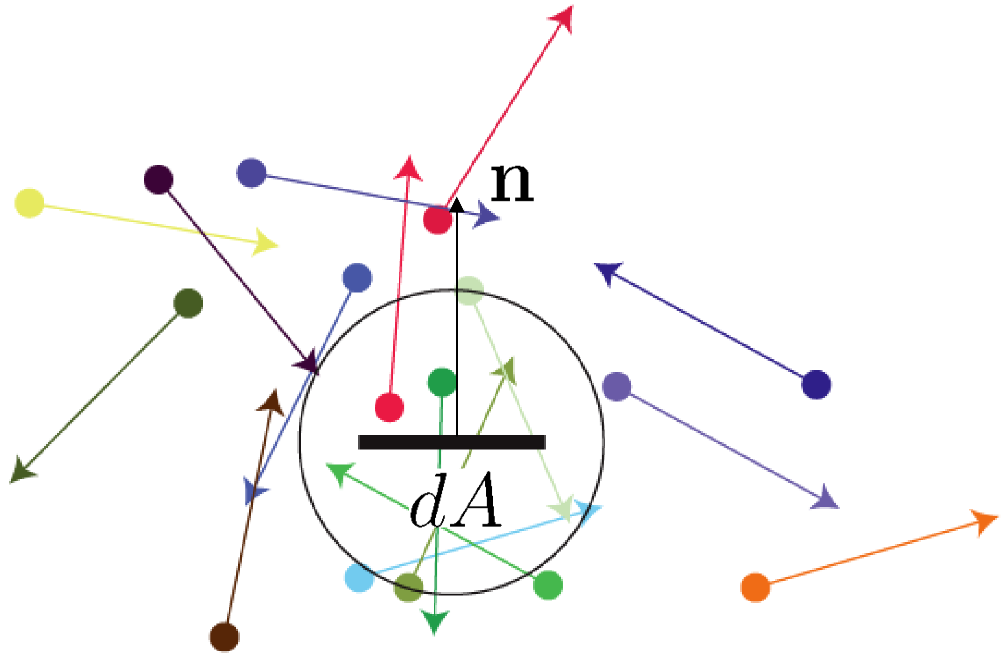
$$L(\omega, \mathbf{x}) = \frac{d^2\Phi(\omega,\mathbf{x})}{d\omega dA^{\perp}}, \left[W \cdot sr^{-1} \cdot m^{-2}\right]$$

Irradiance

http://en.wikipedia.org/wiki/Irradiance

Definition: Power per unit area

$$E(\mathbf{x}, \mathbf{n}) = \frac{d\Phi(\mathbf{x})}{dA}, \left[W \cdot m^{-2}\right]$$



"Count photons going through area dA, irrespective of direction"

Irradiance

http://en.wikipedia.org/wiki/Irradiance

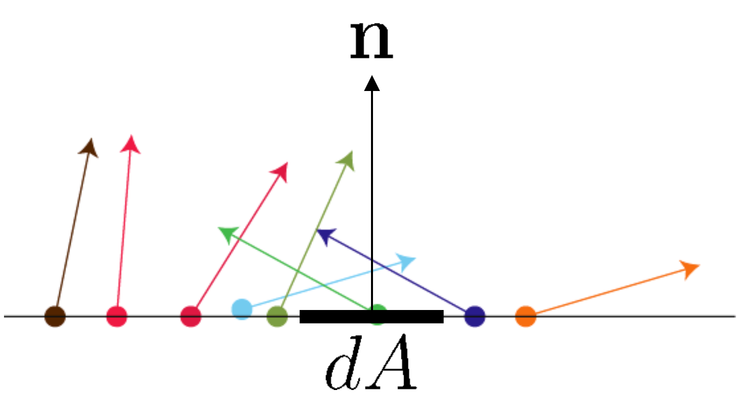
Definition: Power per unit area

$$E(\mathbf{x}, \mathbf{n}) = \frac{d\Phi(\mathbf{x})}{dA}, \left[W \cdot m^{-2}\right]$$

Radiant exitance

from reflected radiance

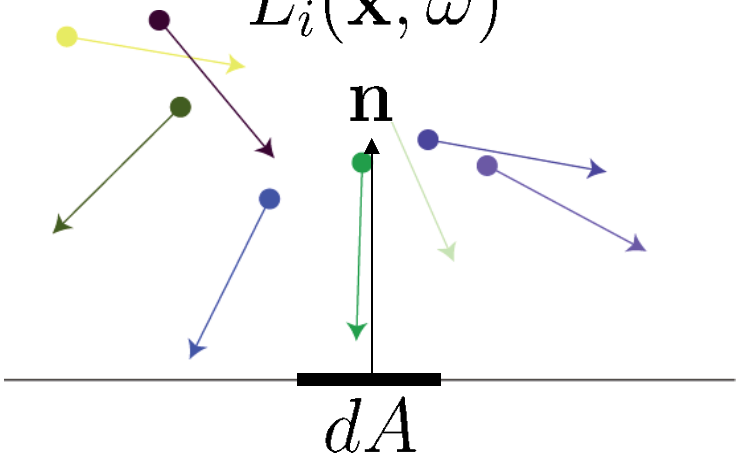
$$L_o(\mathbf{x}, \omega)$$



Irradiance

from incident radiance

$$L_i(\mathbf{x}, \omega)$$



Irradiance

http://en.wikipedia.org/wiki/Irradiance

- Irradiance is integration ("sum") of radiance $L(\mathbf{x},\omega)$ over hemisphere

$$E(\mathbf{x}, \mathbf{n}) = \int_{\mathcal{H}^2(\mathbf{n})} L_i(\mathbf{x}, \omega) \boxed{\cos\theta} d\omega$$

CAUTION: cosine term

Irradiance E: power per area perpendicular to $\mathbf{n}$

Radiance L: power per area perpendicular to ω

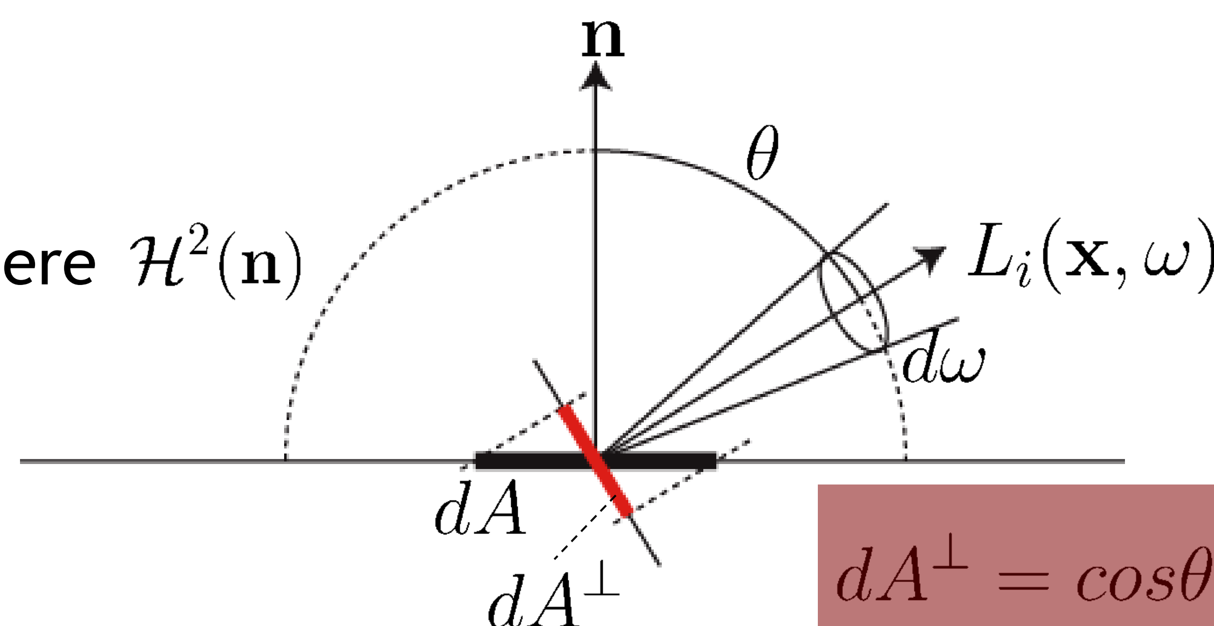
Irradiance

http://en.wikipedia.org/wiki/Irradiance

- Integration ("sum") of radiance over hemisphere

$$E(\mathbf{x}, \mathbf{n}) = \int_{\mathcal{H}^2(\mathbf{n})} L_i(\mathbf{x}, \omega) \cos\theta d\omega$$

Hemisphere $\mathcal{H}^2(\mathbf{n})$



$$dA^{\perp} = cos\theta dA$$

Relation of area perpendicular to ray, to area perpendicular to surface normal

Irradiance

http://en.wikipedia.org/wiki/Irradiance

Integration in spherical coordinates

- Before

$$E(\mathbf{x},\mathbf{n}) = \int_{\mathcal{H}^2(\mathbf{n})} L_i(\mathbf{x},\omega)\cos\theta d\omega$$

- Remember $d\omega = \sin\theta d\theta d\phi$
- In spherical coordinates

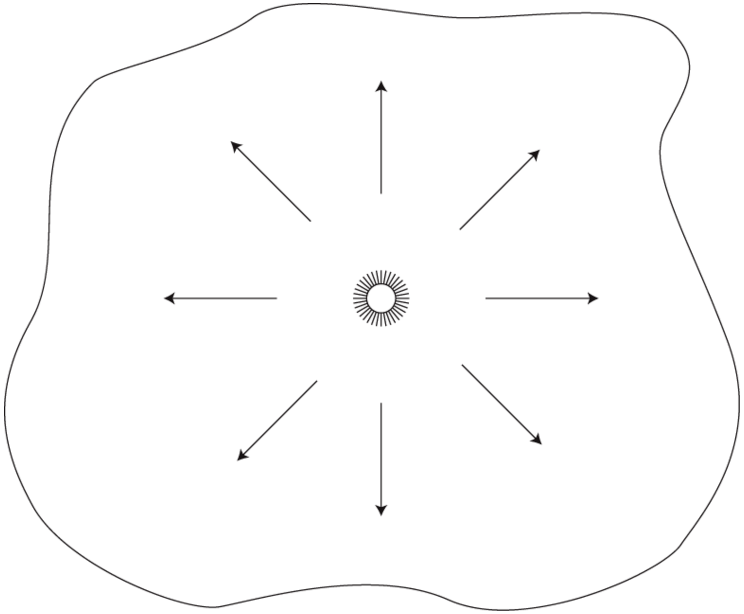
$$E(\mathbf{x},\mathbf{n}) = \int_0^{2\pi}\int_0^{\pi/2} L_i(\mathbf{x},\theta,\phi)\cos\theta\sin\theta d\theta d\phi$$

(Radiant) intensity

http://en.wikipedia.org/wiki/Radiant_intensity

Definition: Power (flux) per solid angle

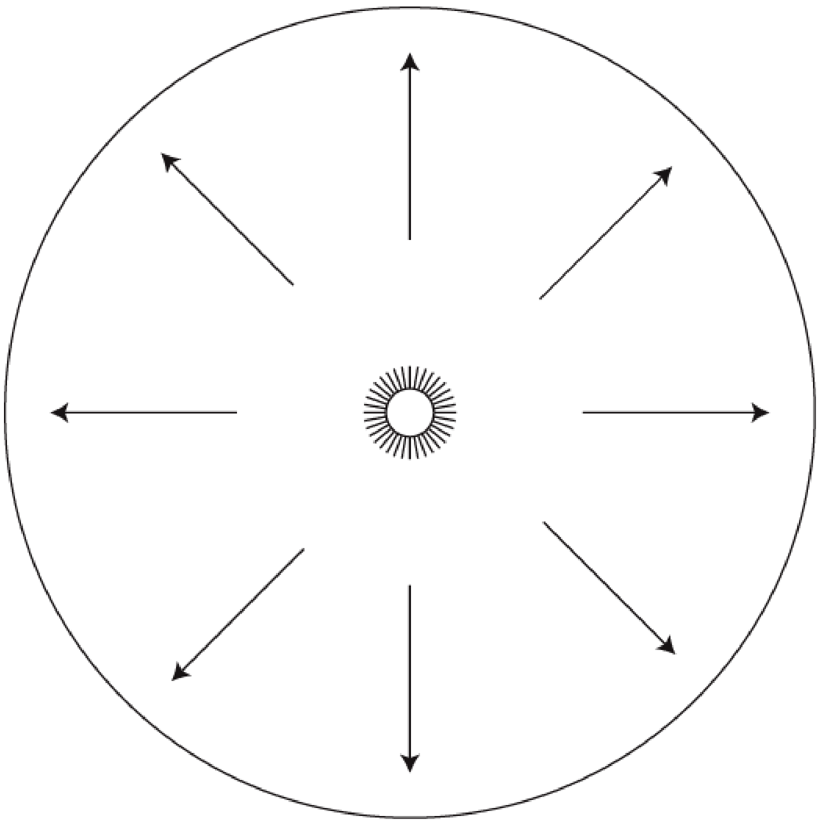
$$I(\omega) = \frac{d\Phi(\omega)}{d\omega}, \left[W \cdot sr^{-1}\right]$$



(Radiant) intensity

http://en.wikipedia.org/wiki/Radiant_intensity

Isotropic point source



- Intensity I constant in all directions
- Total power Φ

$$\Phi = \int_{S^2} I \, d\omega = 4\pi I$$

$$I = \frac{\Phi}{4\pi}$$

Next time

- How to describe reflection of light on surfaces: BRDFs, reflection equation

Amount of reflected light from a surface is always proportional to amount of incident light

A. True

B. False