CMSC740
Advanced Computer Graphics

Fall 2025

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Today

Participating media

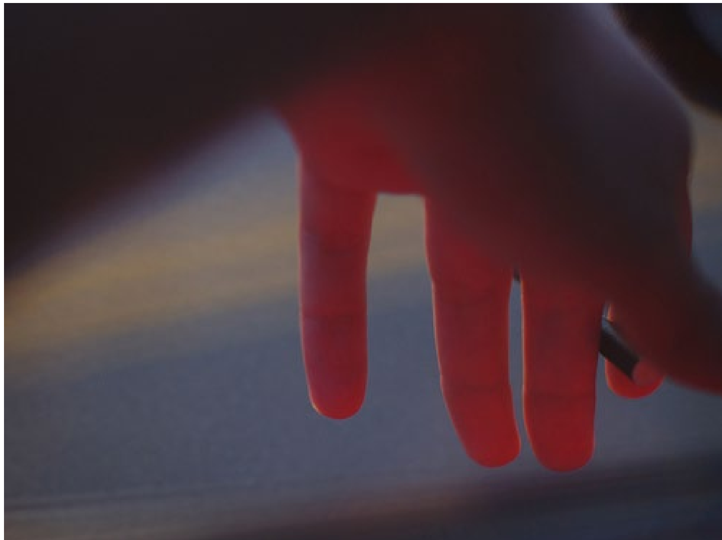
- Introduction
- Light interaction with participating media
- Volume rendering equation
- Approximations & rendering algorithms
- Subsurface scattering

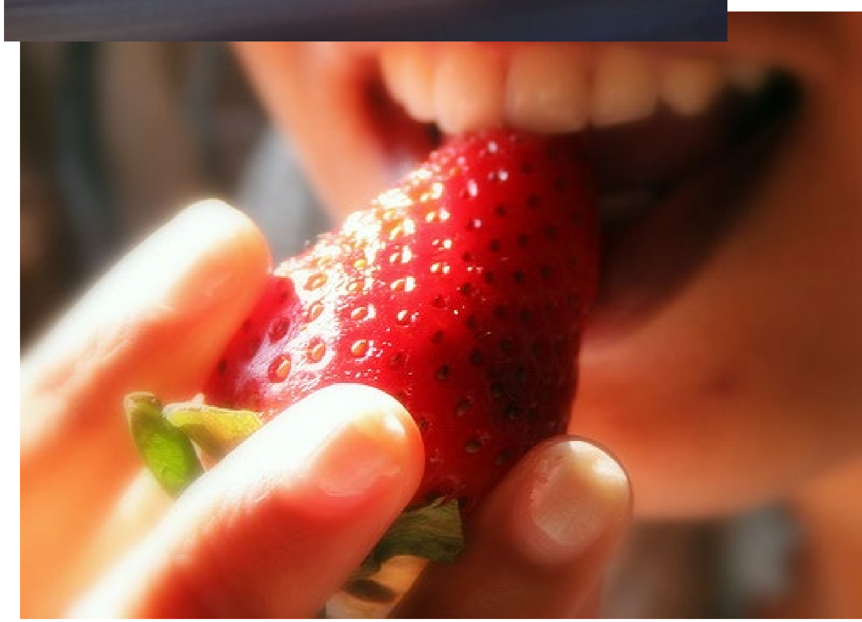
Participating media

- So far: light reflection on surfaces, BRDFs
- What about interaction of light with volumetric media?
- Also called “scattering media”

Participating media

Subsurface scattering





Participating media

Applications

- Clouds, smoke, water, fire, …
- Paint, skin,…
- Scientific/medical volume visualization (CT, MRI, etc.)

Participating media

- Before: „Radiance along straight rays through volume is constant“
- Participating media: volumetric media influence radiance along rays
  – What are types of interaction between media and light rays?

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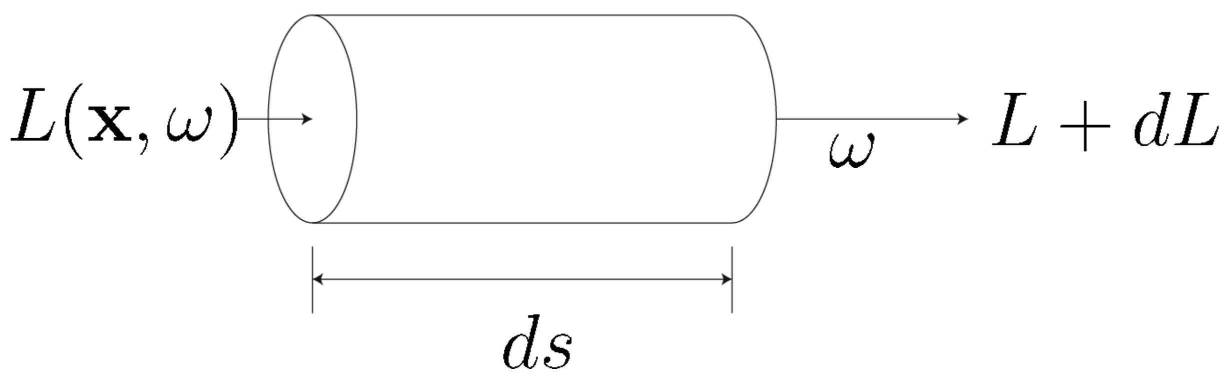
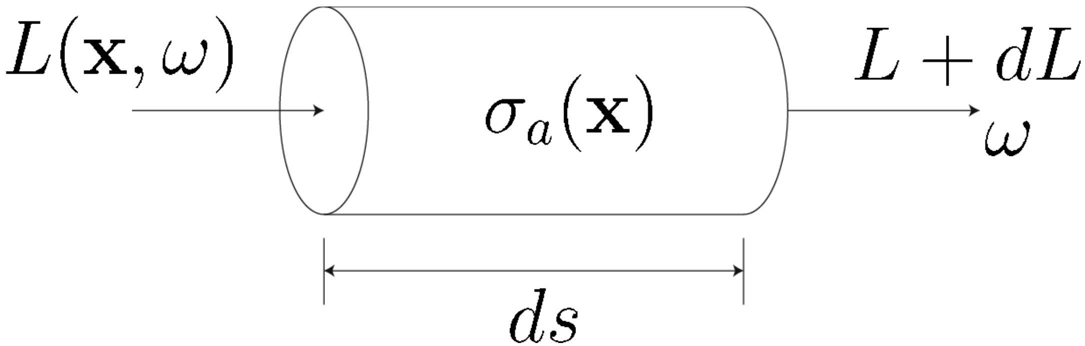
Change of radiance along ray?

- What are the causes for radiance change dL along ray $\mathbf{x}, \omega$?



- Note: all phenomena in following slides are wavelength dependent; omitted in notation

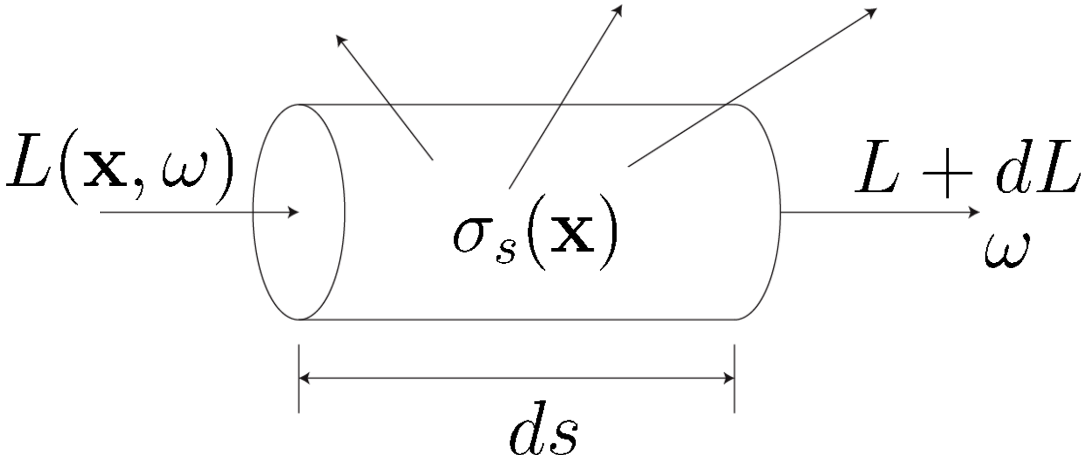
Absorption



$$dL(\mathbf{x},\omega) = -\sigma_a(\mathbf{x})L(\mathbf{x},\omega)ds$$

- Absorption coefficient $\sigma_a(\mathbf{x})$
  - Material property
  - Related to absorption cross section
    http://en.wikipedia.org/wiki/Absorption_cross_section
  - Rate of absorption, units are $1/\text{meter}$
  - "Probability per unit length that photon is absorbed"

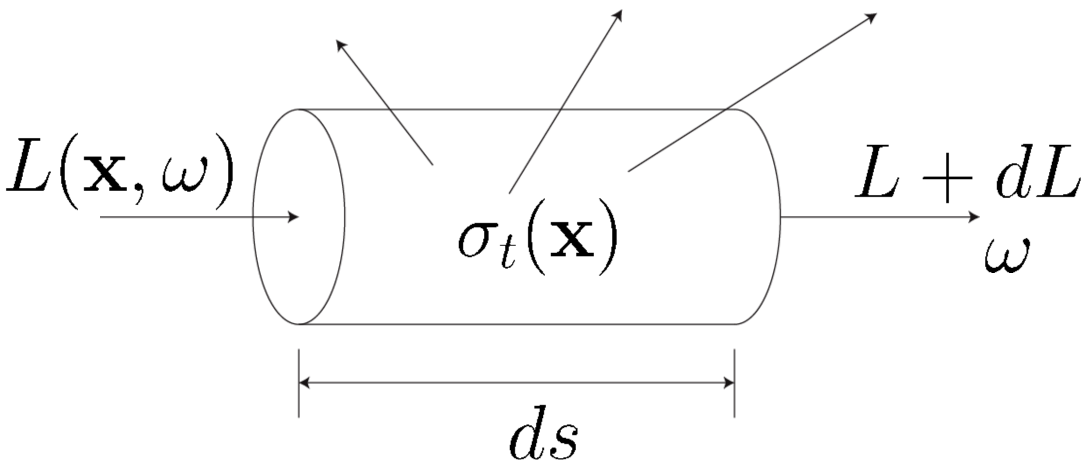
Out-scattering



$$dL(\mathbf{x}, \omega) = -\sigma_s(\mathbf{x}) L(\mathbf{x}, \omega) ds$$

- Scattering coefficient $\sigma_s(\mathbf{x})$
  - Rate of being scattered, units are $1/\text{meter}$
  - Related to scattering cross section
    http://en.wikipedia.org/wiki/Scattering_cross-section
  - "Probability per unit length that photon is scattered into different direction"

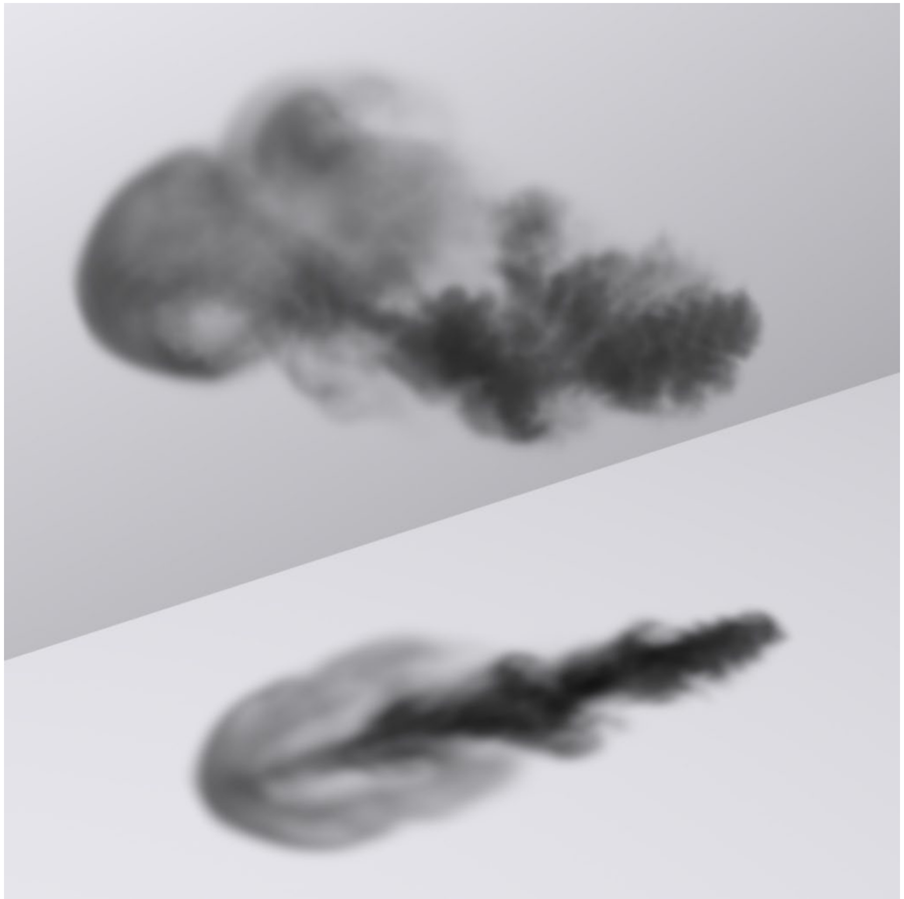
Extinction



$$dL(\mathbf{x},\omega) = -\sigma_t(\mathbf{x})L(\mathbf{x},\omega)ds$$

- Extinction: sum of absorption and out-scattering
  - Extinction coefficient $\sigma_t = \sigma_a + \sigma_s$

Extinction



[Pharr, Humphreys]

Scattering coefficient of milk is larger than the one of the atmosphere

A. True

B. False

Transmittance

- "Fraction of light transported over a certain distance along a ray"

$$dL(\mathbf{x} + s\omega, \omega) = -\sigma_t(\mathbf{x}) L(\mathbf{x} + s\omega, \omega) ds$$

$$\frac{dL(\mathbf{x} + s\omega, \omega)}{ds} = -\sigma_t(\mathbf{x}) L(\mathbf{x} + s\omega, \omega)$$

$$L(\mathbf{x} + s\omega, \omega) = e^{-\tau(s)} L(\mathbf{x}, \omega) = T(s) L(\mathbf{x}, \omega)$$

$$\tau(s) = \int_0^s \sigma_t(\mathbf{x} + s'\omega) ds'$$

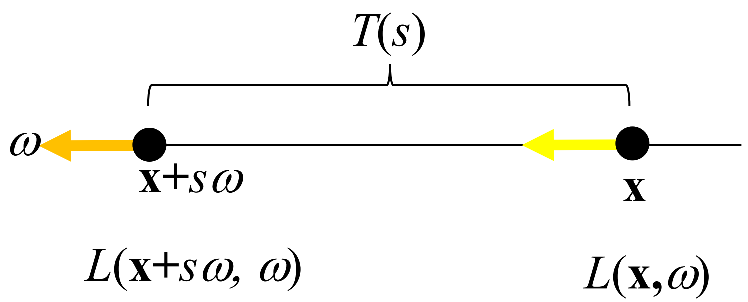
- Optical thickness $\tau(s)$
- Transmittance $T(s) = e^{-\tau(s)}$

Transmittance $T(s)$

$$L(\mathbf{x} + s\omega, \omega) = T(s)L(\mathbf{x}, \omega)$$

$$T(s) = e^{-\tau(s)}$$

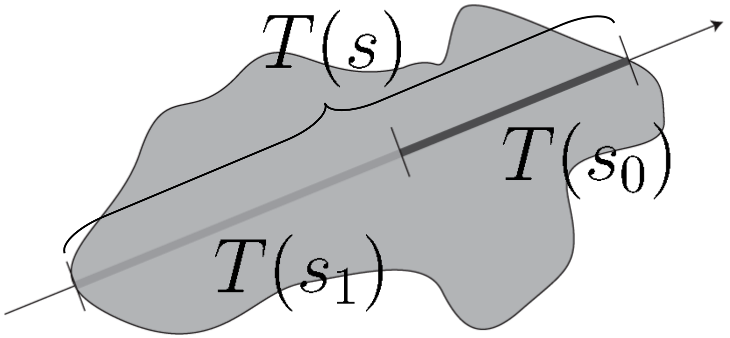
$$\tau(s) = \int_0^s \sigma_t(\mathbf{x} + s'\omega)ds'$$



Transmittance

- Multiplicative property

$$T(s) = T(s_0) \cdot T(s_1)$$



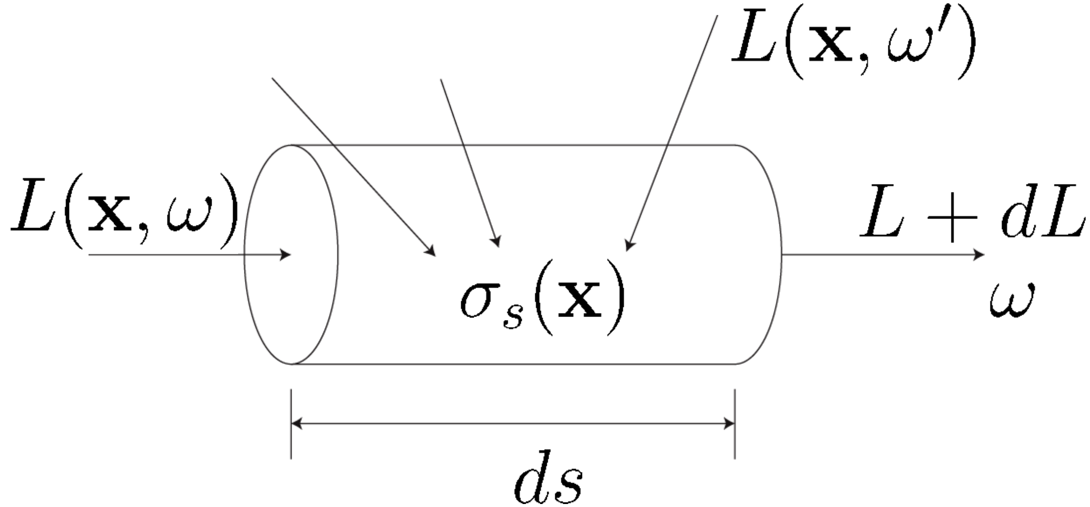
- **Homogeneous media** $\sigma_t(\mathbf{x}) \equiv \sigma_t$

Beer's law $T(s) = e^{-\sigma_t s}$

http://en.wikipedia.org/wiki/Beer%E2%80%93Lambert_law

In-scattering

- “Incoming light from all directions that is scattered into one given direction”

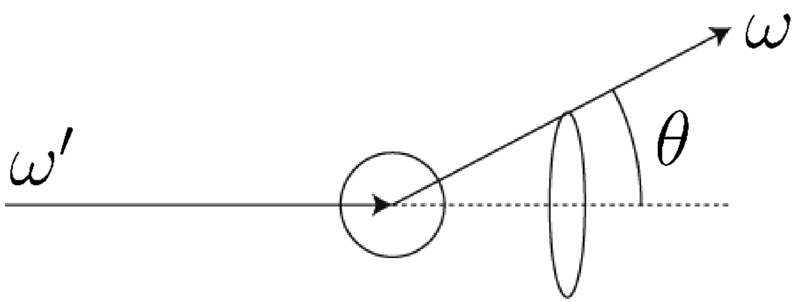


$$S(\mathbf{x}, \omega) = \frac{dL}{ds} = \sigma_s(\mathbf{x}) \int_{S^2} p(\omega' \rightarrow \omega) L(\mathbf{x}, \omega') d\omega'$$

- Phase function $p(\omega' \rightarrow \omega)$

Phase functions

- Phase function $p(\omega' \to \omega)$ is probability that incoming light from direction ω' is scattered into direction ω



- Expressed using phase angle $\cos\theta = \omega \cdot \omega'$
- Isotropic $p(\cos\theta) = \frac{1}{4\pi}$

Phase function

- Similar to BRDF, phase function is aggregate model of scattering
- Abstracts away more detailed physical model

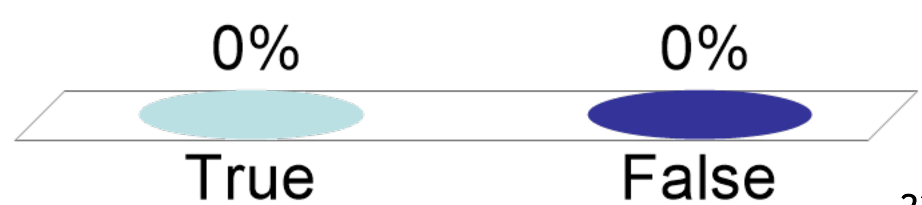
Phase functions: examples

- Rayleigh scattering: scattering from particles smaller than wavelength of light
  - For example, molecules in atmosphere
  - Details see, e.g., wikipedia
    http://en.wikipedia.org/wiki/Rayleigh_scattering
- Mie (Lorenz-Mie) theory: scattering by spherical particles (Mie scattering)
  http://en.wikipedia.org/wiki/Mie_theory

Sky would be brighter if there were no scattering in atmosphere

A. True

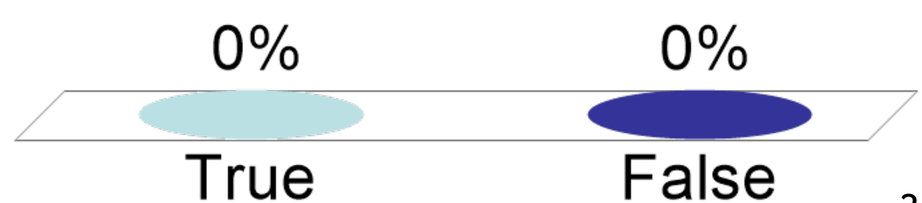
B. False



Sky is blue because blue light scatters less in atmosphere (lower scattering coefficient) than other wavelengths

A. True

B. False



Blue sky, red sunset

[Greenler]

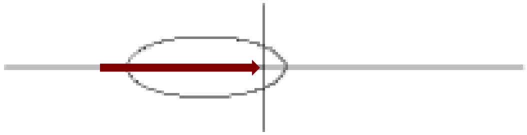
Henyey-Greenstein phase function

- Empirical phase function
- Useful for many phenomena (water, clouds, skin, stone)

$$p(\cos\theta) = \frac{1-g^2}{4\pi(1+g^2-2g\cos\theta)^{1.5}}$$

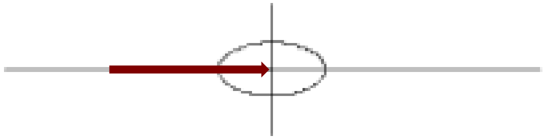
- Pareameter g: average phase angle, asymmetry parameter

$g=-0.3$

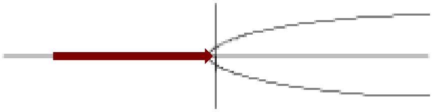


Backward scattering

$g=0.0$



$g=0.6$



Forward scattering

Phase functions: properties

- Unitless
- Reciprocity

$$p(\omega' \to \omega) = p(\omega \to \omega')$$

- Energy conservation

$$\int_{S^2} p(\omega' \to \omega) d\omega' = 1$$

- Amount (rate) of scattering controlled by σ_s

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The volume rendering equation

- Integro-differential form
  - Describes change of raradiance along a ray

$$\frac{dL(\mathbf{x},\omega)}{ds} = \underbrace{-\sigma_t L(\mathbf{x},\omega)}_{\text{Extinction}} + \underbrace{S(\mathbf{x},\omega)}_{\text{In-scattering}}$$

- Integro-integral form
  - Describes radiance arriving at a point along a ray

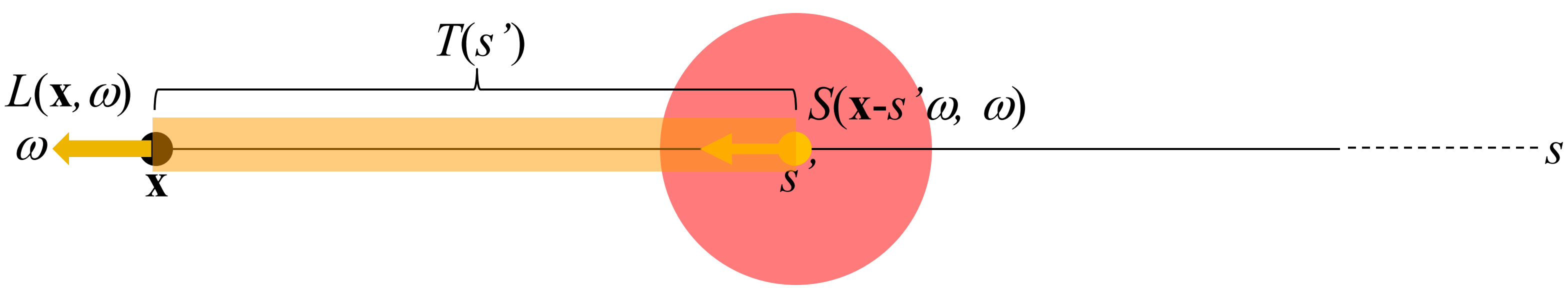
$$L(\mathbf{x},\omega) = \int_0^\infty \underbrace{e^{-\int_0^{s'} \sigma_t(\mathbf{x}-s''\omega)ds''}}_{\text{Transmittance } T(s') \text{ due to extinction (absorption, out-scattering)}} \underbrace{S(\mathbf{x}-s'\omega,\omega)}_{\text{Source (emission, in-scattering)}} ds$$

Integro-integral form

$$L(\mathbf{x},\omega) = \int_0^\infty e^{-\int_0^{s'} \sigma_t(\mathbf{x}-s''\omega)ds''} S(\mathbf{x}-s'\omega,\omega)ds$$

Transmittance $T(s')$ due to extinction (absorption, out-scattering)

Source (emission, in-scattering)



The volume rendering equation

- If there is scattering in volume and on surfaces, both contributions simply add up
- Surface reflection attenuated with transmittance

The volume rendering equation

- Full solution via Monte Carlo path tracing expensive to compute
- Rendering with simplified models
- Most common approximations
  - Emission/absorption only
  - Single scattering

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Absorption/emission only

- Simplified model

$$L(\mathbf{x},\omega) = T(s)L(s) + \int_0^s T(s')L_{ve}(s')ds'$$

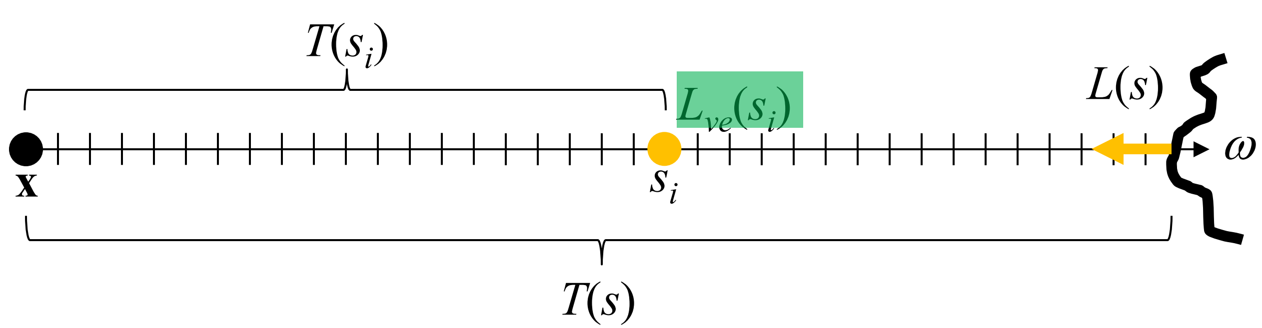
  - Integration is along ray $\mathbf{x}$, ω
  - Volume emission L_{ve}
    - No in-scattering (use given volume emission instead of in-scattering)
  - Contribution from surface (optional)
    - Assumption: ray hits opaque surface at distance s
    - Radiance reflected by surface at distance s is $L(s)$
    - Attenuated surface reflection at $\mathbf{x}$ is $T(s)L(s)$
- Remember transmittance $T(s) = e^{-\int_0^s \sigma_t(s')ds'}$

Ray marching

- Approximation of integral $\int_0^s T(s')L_{ve}(s')ds'$

$$L(\mathbf{x},\omega) \approx T(s)L(s) + \frac{s_{out} - s_{in}}{N}\sum_{i=1}^{N} T(s_i)L_{ve}(s_i)$$

  - Number of samples along ray N
  - Uniformly distributed locations along ray s_i



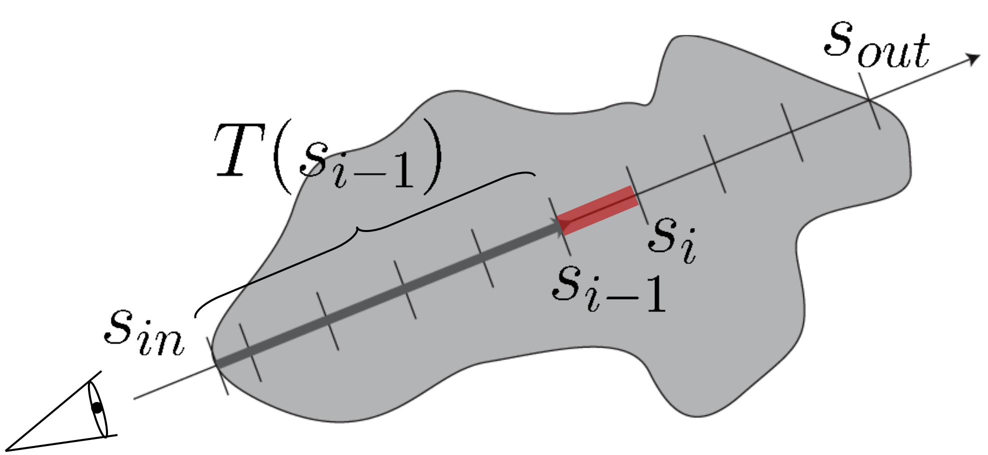
Ray marching

- Incremental computation of transmittance

$$T(s_i) = T(s_{i-1})T(s_{i-1} \to s_i)$$

$$\boxed{T(s_{i-1} \to s_i)} = e^{-\int_{s_{i-1}}^{s_i} \sigma_t(s')ds'} \approx \boxed{(1 - \sigma_t(s_i) \cdot \underbrace{(s_i - s_{i-1})}_{ds})}$$

Linear approximation for small ds



Ray marching

```
T = 1
L = 0
ds = (s_out - s_in) / N   // Number of steps N

for( s_i = s_in; s_i <= s_out; s_i += ds ) {
  L = L + T * L_ve
  T = T * ( 1 - sigma_t(s_i) * ds)
}

L = L * ds

L = L + T*L_s // Add attenuated surface
              // reflection T*L_s
```

Ray marching

- Deterministic step sizes leads to biased estimate of integral
- Could also sample locations s_i randomly
- However, resulting Monte Carlo estimate of transmittance is biased

$$T(s) = e^{-\int_0^s \sigma_t(s')ds'}$$

MC estimate of transmittance is biased

Optical thickness using MC sampling of locations s_i unbiased

$$E[\mathrm{e}^{-X}] \neq \mathrm{e}^{-E[X]}$$

Problem: unbiased MC estimator for transmittance?

Null scattering, delta tracking

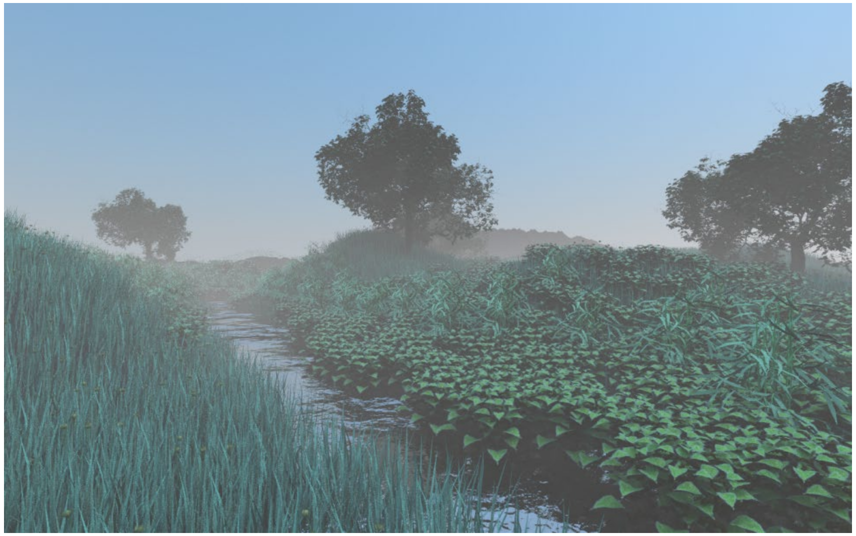
PBRT, 4th edition, Section 11.2.1

- Reformulate mathematical expression of transmittance using null scattering
- Null scattering: hypothetical type of scattering that doesn't affect radiance along ray
- Delta tracking: unbiased method to calculate transmittance using null scattering and Monte Carlo sampling
  - Place samples along ray step by step, with random step size
  - In each step, randomly decide if true scattering or null scattering occurs
  - If true scattering, stop; if null, scattering continue

Absorption/emission only

No absorption

With absorption



[Pharr, Humphreys]

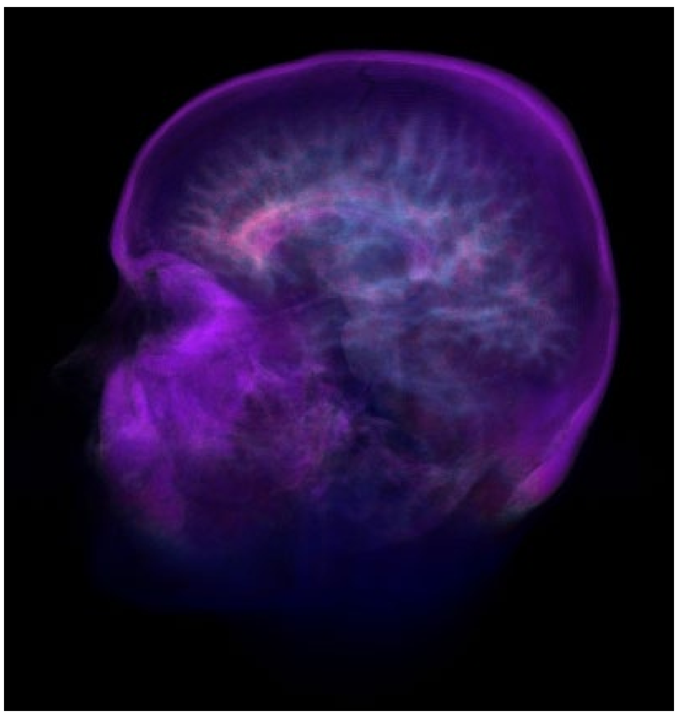
Applications: Volume visualization

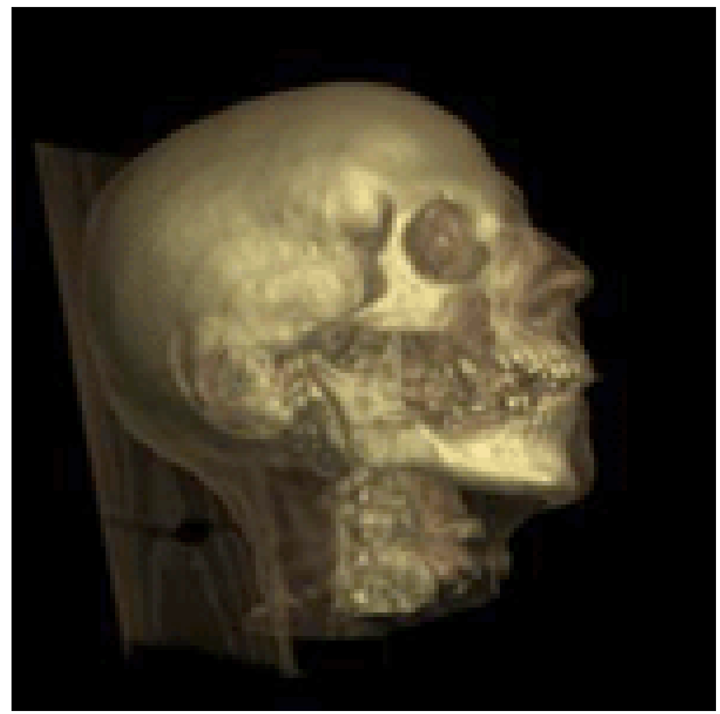
http://en.wikipedia.org/wiki/Volume_rendering

- Visualize scalar data on volumetric grid
- CT, MRI scans, …
- Scalar data represents some physical property of a material
- Medical/industrial applications
- Transfer function assigns color (volume emission) and opacity value (extinction coefficient) to each scalar value

Volume visualization

Volume rendering: ray marching/ray casting as described before





Applications: NeRF (Neural radiance fields)

- Context: novel view synthesis and 3D reconstruction from images
- Formulated as inverse rendering problem: find extinction function σ_t, volumetric "emission" L_{ve}, such that rendered images match given input photos
- Represent σ_t, L_{ve} as neural networks
- More later https://www.matthewtancik.com/nerf

Single scattering

- Include in-scattering of incident radiance due to direct illumination

$$\int_0^s T(s') \left(L_{ve}(s') + \sigma_s(s') \int_{S^2} p(s', \omega' \to \omega) L_d(s', \omega') d\omega' \right) ds'$$

Integral along ray

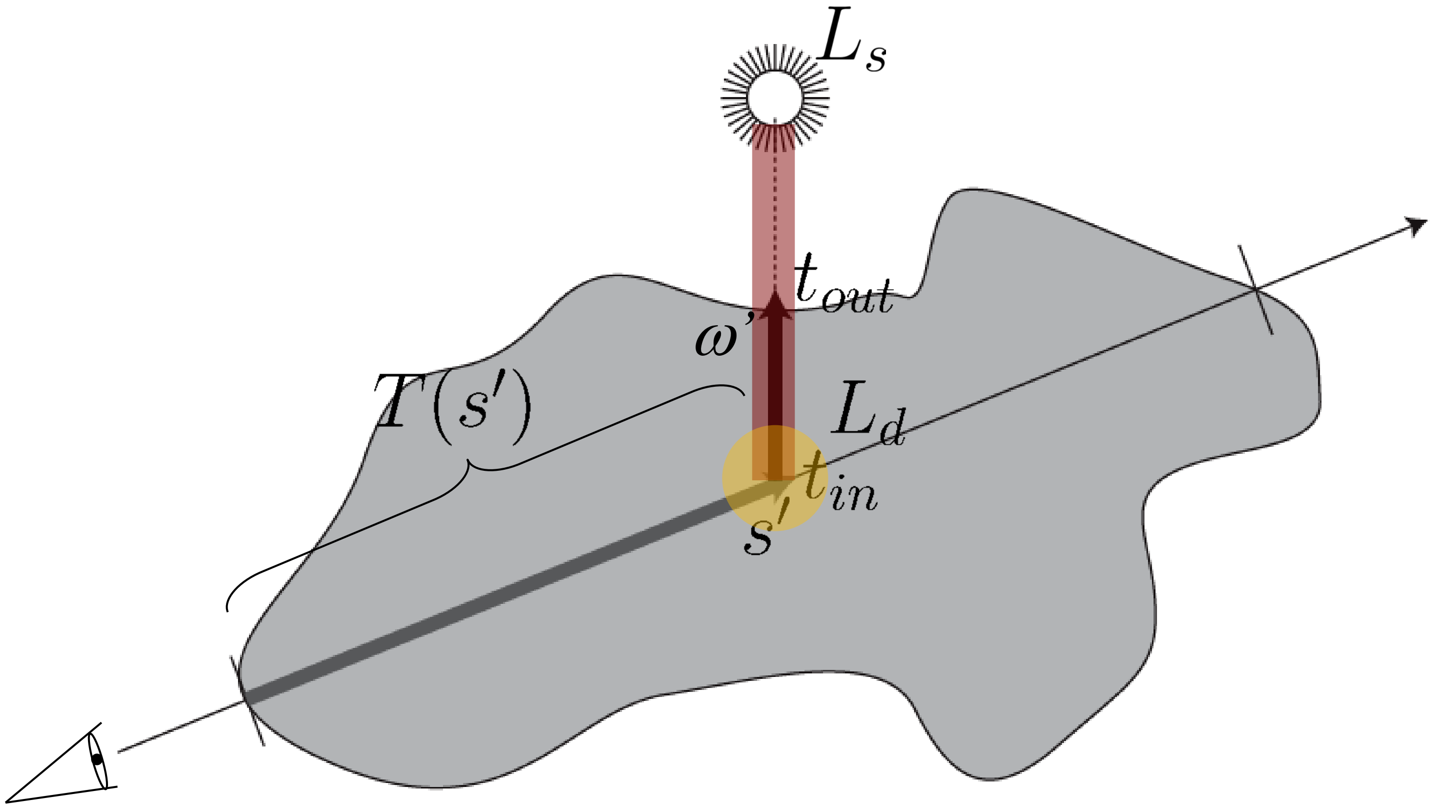
Optional

In-scattering (spherical integral), $S(s')$

- Direct illumination L_d
- Phase function $p(s', \omega' \to \omega)$
- Scattering coefficient σ_s
- Emission $L_{ve} = 0$, except for volumetric sources such as fire

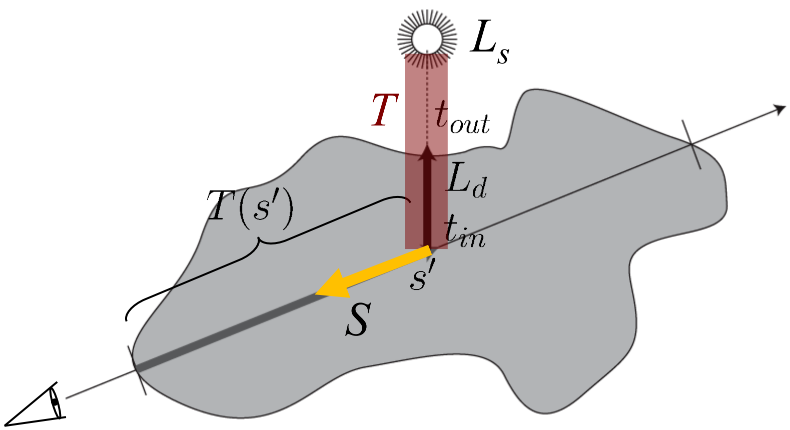
Single scattering

- Emission L_s needs to be attenuated by transmittance from t_{in} to t_{out} to calculate incident radiance L_d on camera ray
- Shoot shadow rays towards light sources at each step during ray marching



Shadow rays

- Ray march towards light
- Leads to two nested ray marching loops




```
T = 1
dt = t_out - t_in / N

for( t = t_in; t <= t_out; t += dt ) {
  T = T * ( 1 - sigma_t(t) * dt )
}

L_d = T * L_s  // L_s: light source
S = sigma_s * p * L_d  // p: phase function
```

Shadow rays

- Homogeneous media $\sigma_t(x) = \sigma_t$ (constant)
  - No ray marching necessary along shadow rays
  - Compute transmittance directly via Beer's law $T(s) = e^{-\sigma_t s}$

Single scattering

[Pharr, Humphreys]

Beams of light

[Minneart]

[Greenler]

Simulating multiple scattering

- Path tracing, PBRT, 4th Edition, Chapter 14
- Photon mapping
  "Efficient Simulation of Light Transport in Scenes with Participating Media using Photon Maps", Jensen, Christensen
  http://portal.acm.org/citation.cfm?id=280925
  "Realistic Image Synthesis using Photon Mapping", Jensen
  http://www.amazon.com/Realistic-Image-Synthesis-Photon-Mapping/dp/1568811470
- Many more recent improvements
  https://cs.dartmouth.edu/wjarosz/publications/bitterli17beyond.html

Photon mapping

- Volume caustics

[Jensen]

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Subsurface scattering

- Same physical phenomon as before
- Different approximation for specific materials
  - Highly scattering (large scattering coefficient)
- Different rendering algorithms

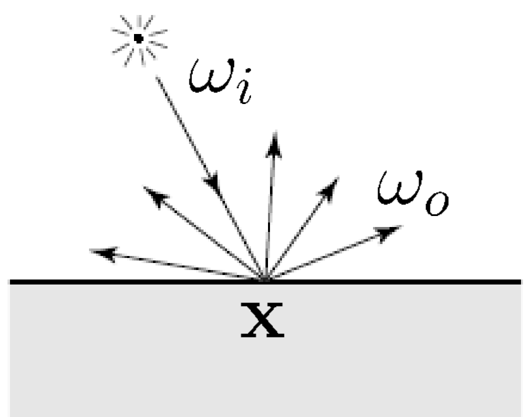
Subsurface scattering

- Could render using brute force Monte Carlo path tracing
  - Very expensive
- Idea: describe scattering properties of material using an extension of BRDFs
  - So far: BRDFs, light scatters exactly at the same point where it hits the surface
  - Subsurface scattering: light enters the material, bounces around, leaves at a different place

Subsurface scattering

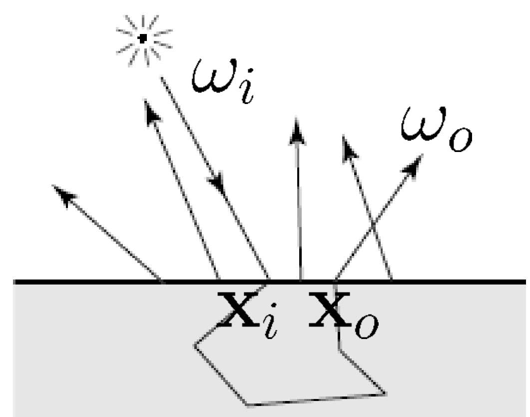
- BSSRDF: bidirectional surface scattering reflectance distribution function

BRDF



$f(\mathbf{x}, \omega_i, \omega_o)$

BSSRDF



$S(\mathbf{x}_i, \omega_i, \mathbf{x}_o, \omega_o)$

Subsurface scattering

- BSSRDF has 8 degrees of freedom (2 positions, 2 orientations)
  - Abstract model that describes aggregate effect of scattering inside material
- Hard to capture physically in the general case, requires specialized equipment
  http://omilab.naist.jp/~mukaigawa/papers/TCVA2013-BSSRDF.pdf
- Impractical to store in a table

Subsurface scattering

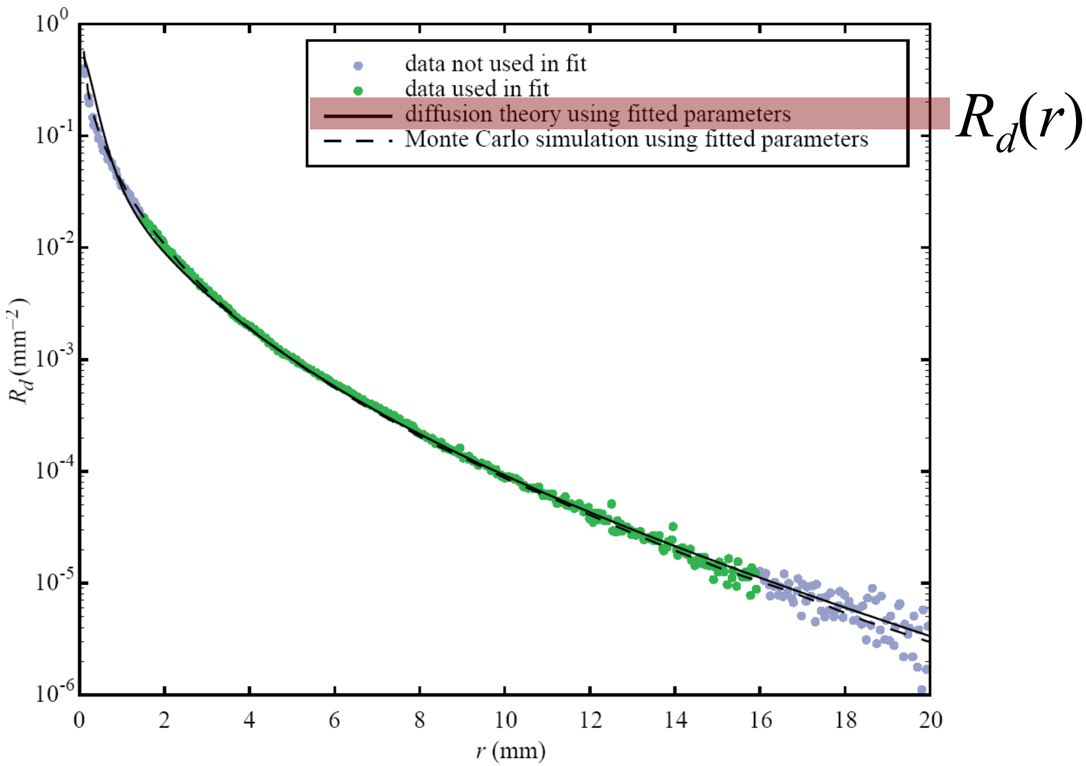
Diffusion approximation

http://www.graphics.stanford.edu/papers/bssrdf/

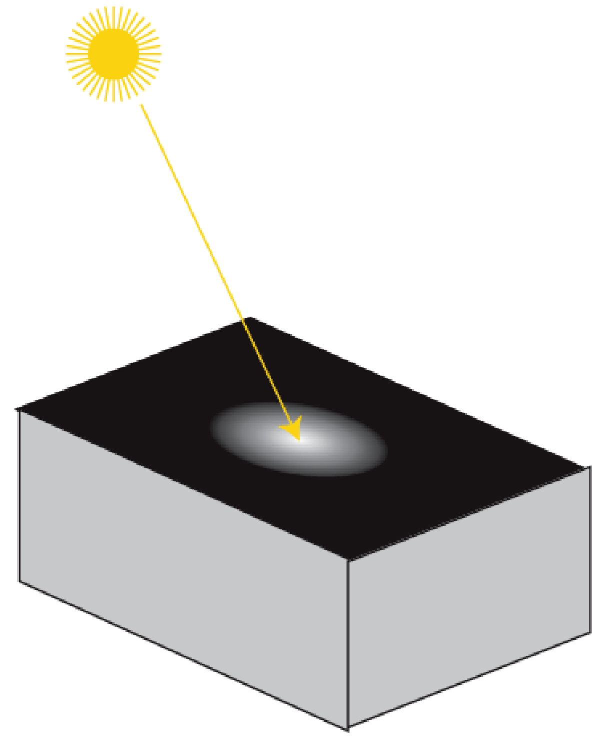
- Light distribution in highly scattering media tends to become isotropic
- Define a simple, analytic diffuse BSSRDF that approximates subsurface scattering
  - Express ratio of scattered over incident radiance as function $R_d(r)$ of distance $r=||\mathbf{x}_i-\mathrm{x}_o||$ between entry and exit points
- Approximation known as "dipole model"

Subsurface scattering

Diffusion approximation



Diffusion profile as function of radius r
[Jensen et al.]



Diffusion profile

Subsurface scattering

BSSRDF

BRDF

[Jensen et al.]

Subsurface scattering

BRDF	BSSRDF	Monte Carlo simulation
		

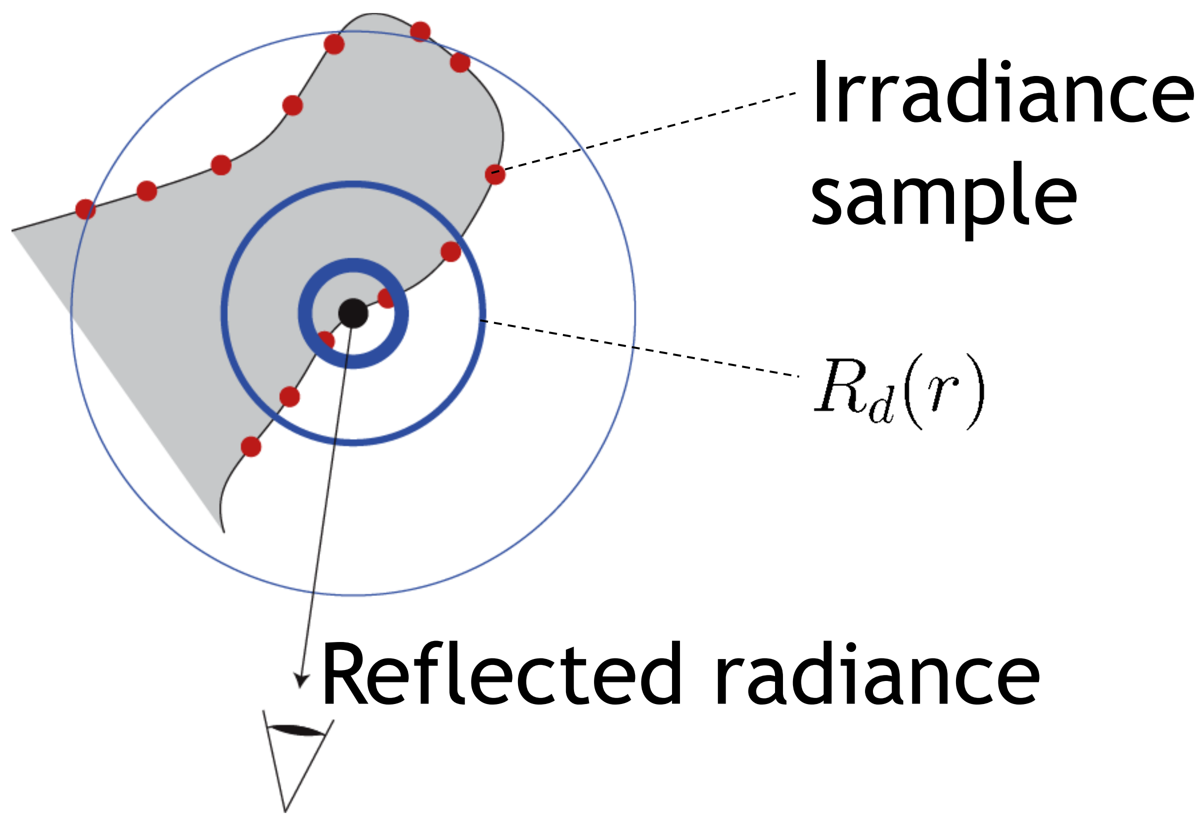
[Jensen et al.]

Efficient rendering

- "A Rapid Hierarchical Rendering Technique for Translucent Materials", Jensen, Buhler
  http://graphics.ucsd.edu/~henrik/papers/fast_bssrdf/fast_bssrdf.pdf
- Two pass approach
  - First, compute irradiance at a set of surface points
  - Second, evaluate diffuse BSSRDF by gathering irradiance samples
- Use efficient hierarchical scheme

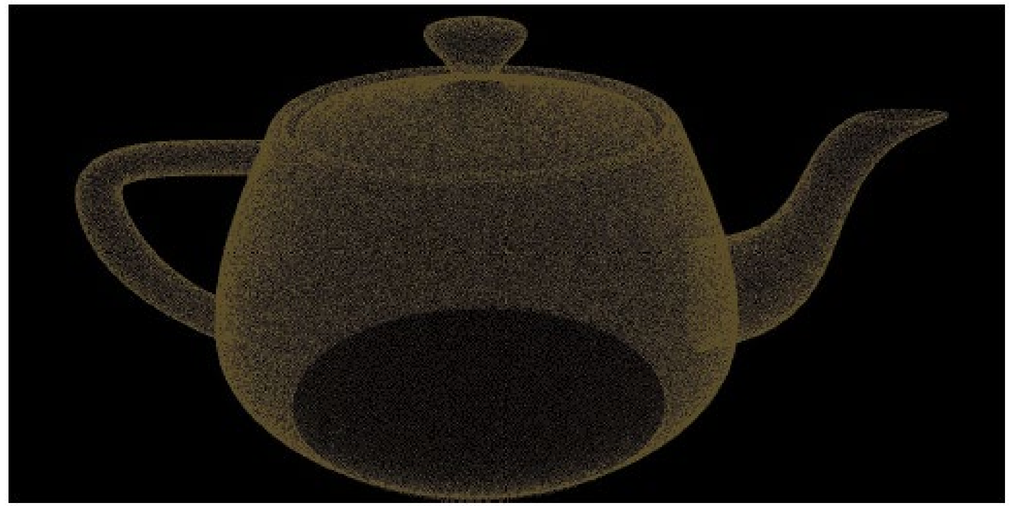
Efficient rendering

- Irradiance gathering using the diffuse BSSRDF



Efficient rendering

Irradiance samples

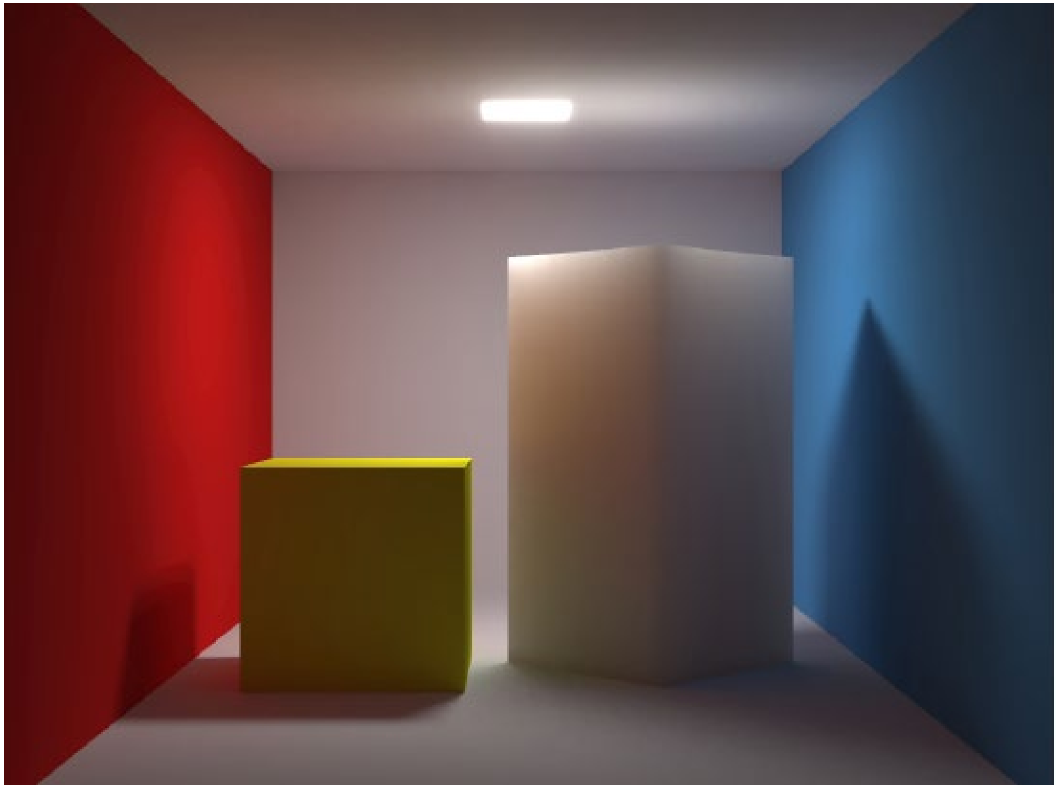


Final image

[Jensen, Buhler]

Efficient rendering

- Global illumination and subsurface scattering



[Jensen, Buhler]