CMSC740
Advanced Computer Graphics

Fall 2025

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Today

Simulating light transport

- BRDF & reflection integral
- BRDF examples

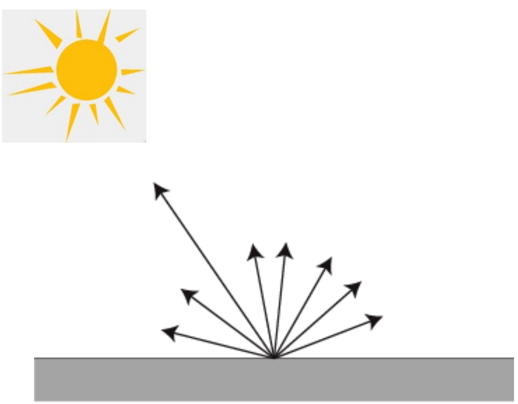
Surface appearance

- How is light reflected by a
  - Mirror
  - White sheet of paper
  - Blue sheet of paper
  - Glossy metal

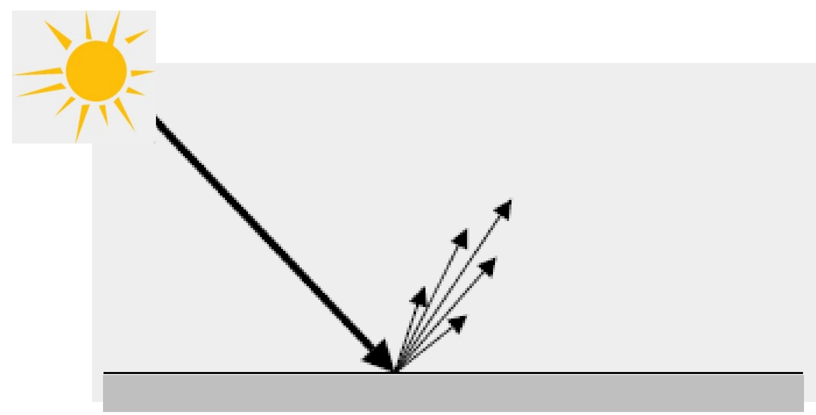
The BRDF (bidirectional reflectance distribution function)

http://en.wikipedia.org/wiki/Bidirectional_reflectance_distribution_function

- Describes quantitatively how light is reflected off a surface
- "For every pair of light and viewing directions, BRDF gives fraction of transmitted light"
- Captures appearance of surface



Diffuse reflection



Glossy reflection

The BRDF (bidirectional reflectance distribution function)

http://en.wikipedia.org/wiki/Bidirectional_reflectance_distribution_function

Relation of BRDF to physics

- BRDF is quantitative description of overall result of light scattering for a given type of material
- BRDF can be measured, or derived from (often simplified) physical assumptions

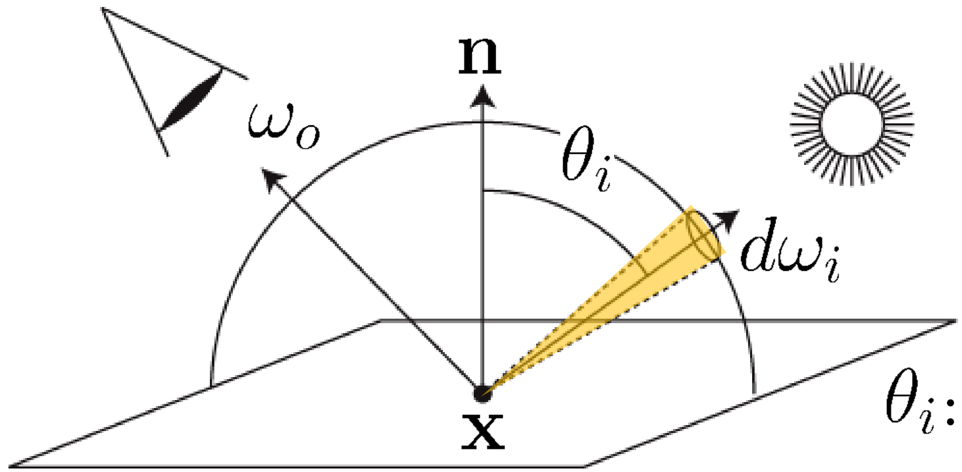
Types of reflection

- Diffuse
  - Matte paint
- Glossy
  - Plastic, high-gloss paint
- Perfect specular
  - Mirror
- Retro-reflective
  - Surface of the moon
    http://en.wikipedia.org/wiki/Retroreflector
- Natural surfaces are often combinations



Mathematical formulation

- Preliminaries: differential irradiance dE on surface due to incident radiance L_i within a small cone of around direction ω_i

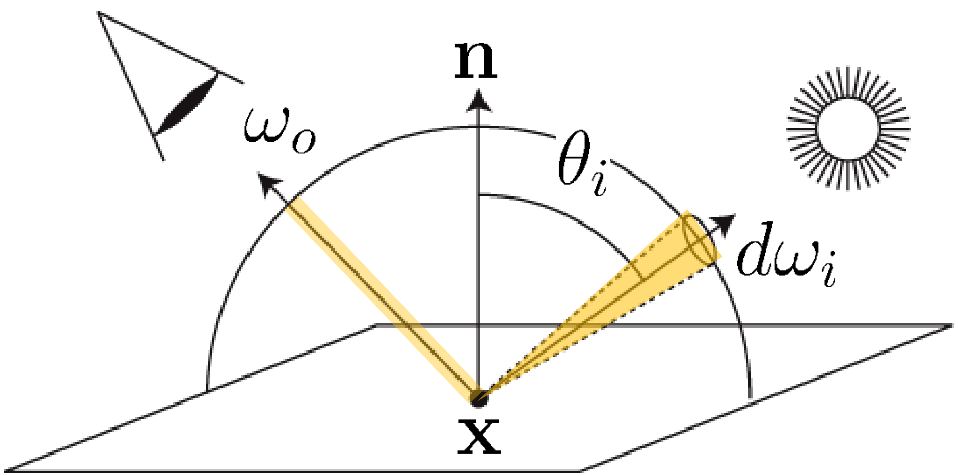


θ_i: angle between normal and ω_i

$$dE(\mathbf{x}, \omega_i) = L_i(\mathbf{x}, \omega_i) \cos\theta_i d\omega_i$$

Mathematical formulation

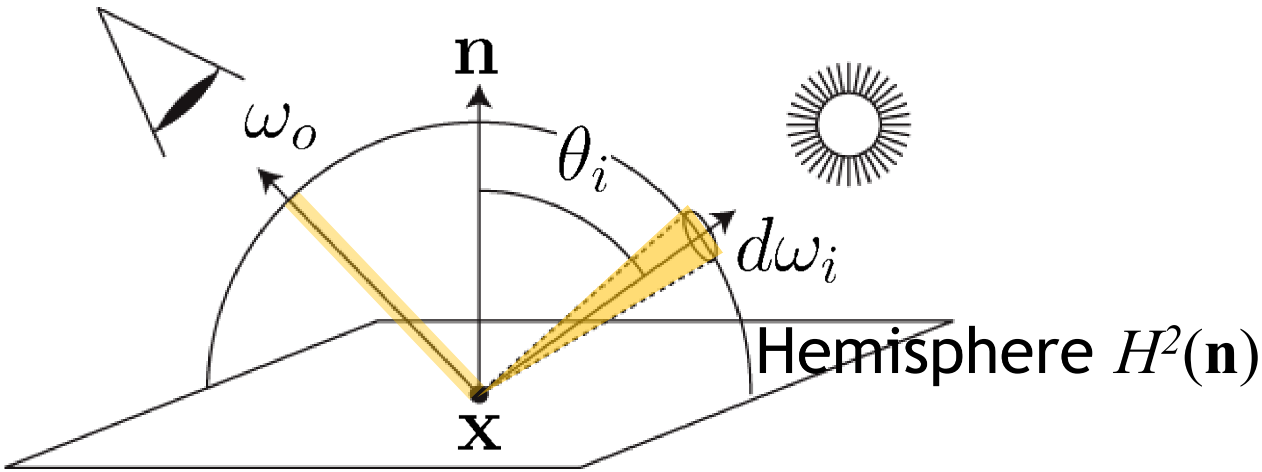
- BRDF: given surface location $\mathbf{x}$ with normal $\mathbf{n}$, BRDF is fraction of reflected radiance L_o in outgoing direction ω_o over differential irradiance dE from direction ω_i



$$f(\mathbf{x}, \omega_o, \omega_i) = \frac{dL_o(\mathbf{x},\omega_o)}{dE(\mathbf{x},\omega_i)} = \frac{dL_o(\mathbf{x},\omega_o)}{L_i(\mathbf{x},\omega_i)\cos\theta_i d\omega_i}, \left[sr^{-1}\right]$$

Reflection equation

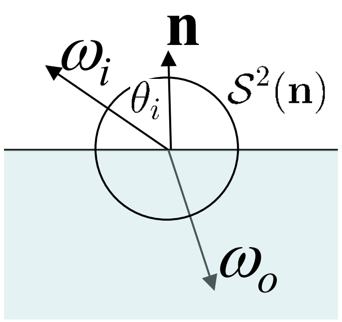
- Outgoing radiance L_o due to incident illumination L_i from all directions ω_i over hemisphere $H^2(\mathbf{n})$



$$L_o(\mathbf{x}, \omega_o) = \int_{\mathcal{H}^2(\mathbf{n})} f(\mathbf{x}, \omega_o, \omega_i) L_i(\mathbf{x}, \omega_i) \cos\theta_i d\omega_i$$

Scattering equation

- Generalization of reflection equation to materials that include refraction
  - Light changing direction as it enters material



- Integral over whole sphere of directions $\mathcal{S}^2(\mathbf{n})$
- BRDF is now also called bidirectional scattering distribution function (BSDF)

$$L_o(\mathbf{x}, \omega_o) = \int_{\mathcal{S}^2(\mathbf{n})} f(\mathbf{x}, \omega_o, \omega_i) L_i(\mathbf{x}, \omega_i) |\cos\theta_i| d\omega_i$$

What can we model with BRDFs?

- Semi-transparent objects like glass?
- Properties such as diffuse, glossy, specular reflection and combinations thereof?
- Surface color?
- Spatially varying surface color?
- Dispersion (e.g., rainbows)?
- Subsurface and volume scattering?
- Polarization?
- Diffraction?

Spatially varying surface color?

A. Yes
B. No

Semi-transparent objects like glass?

A. Yes

B. No

Scattering in 3D volumes (clouds, orange juice, etc.)?

A. Yes

B. No

Dispersion (e.g., rainbows)?

A. Yes

B. No

Properties of the BRDF

- BRDFs are wavelength dependent
  - Not explicit in our notation (similar as radiance)
  - In practice, separate for RGB channels
- BRDFs are four-dimensional functions
  - 2 degrees of freedom for both incident and outgoing directions
- Spatially varying BRDFs
  - BRDF is different at each surface point
  - Common in natural surfaces

Properties of BRDFs

- Energy conservation: total energy of reflected light is less than or equal to energy of incident light

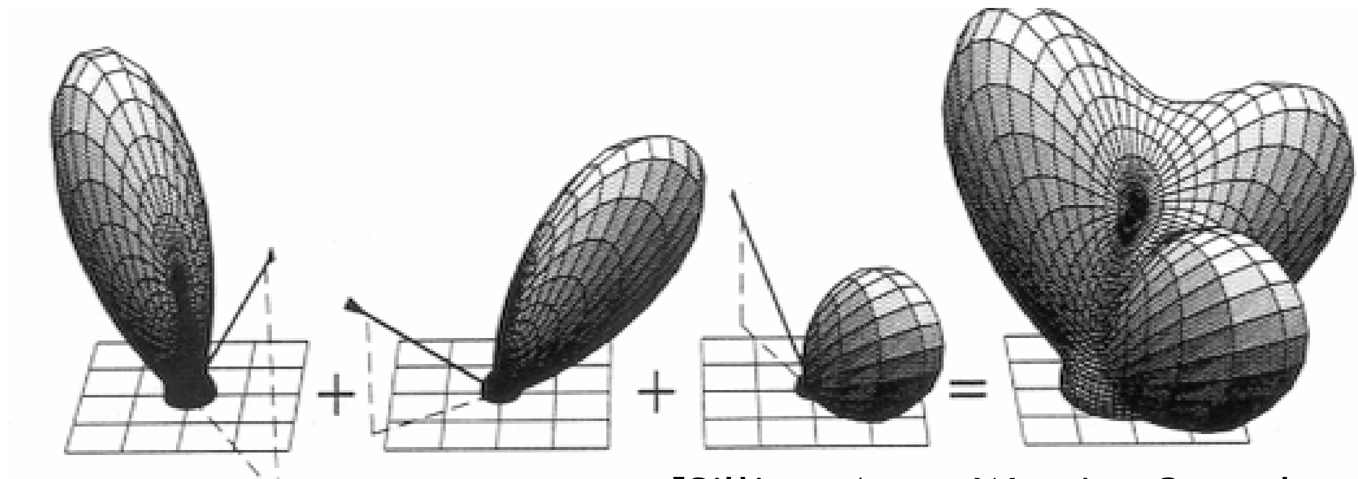
$$\int_{\mathcal{H}^2(\mathbf{n})} f(\mathbf{x}, \omega_o, \omega_i) \cos\theta_i d\omega_i \leq 1$$

- But BRDF values for some ω_o, ω_i can be larger than one!
  - BRDF can involve Dirac delta function

    https://en.wikipedia.org/wiki/Dirac_delta_function

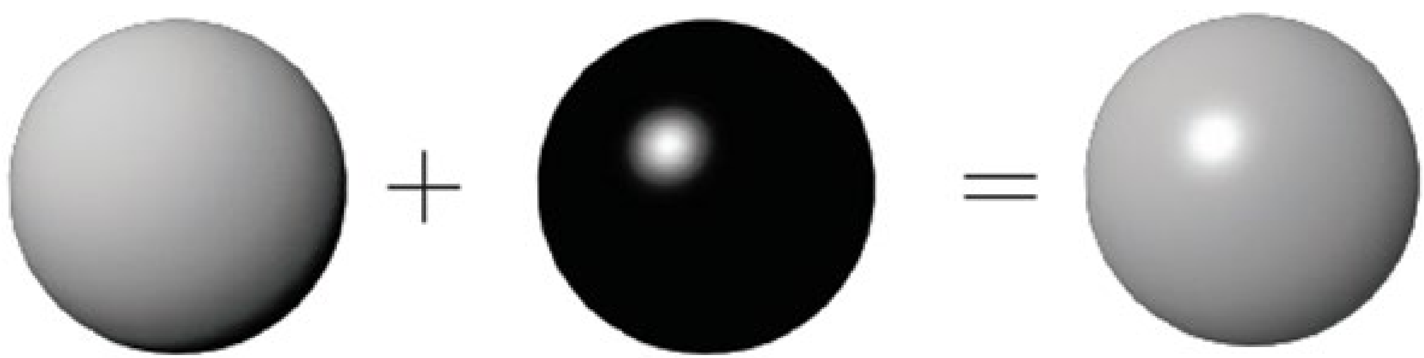
Properties of BRDFs

- Linear



[Sillion, Arvo, Westin, Greenberg]

- BRDF usually sum of different components

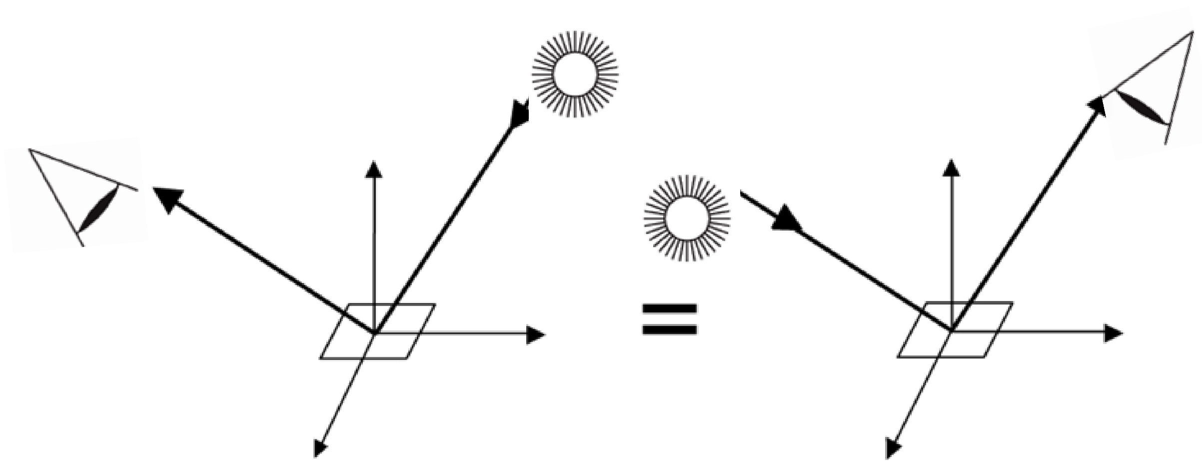


diffuse specular

Properties of BRDFs

- Helmoltz reciprocity
  http://en.wikipedia.org/wiki/Helmholtz_reciprocity

$$f(\mathbf{x}, \omega_o, \omega_i) = f(\mathbf{x}, \omega_i, \omega_o)$$



[Hanrahan]

- Can exchange role of light source and observer

Today

Simulating light transport

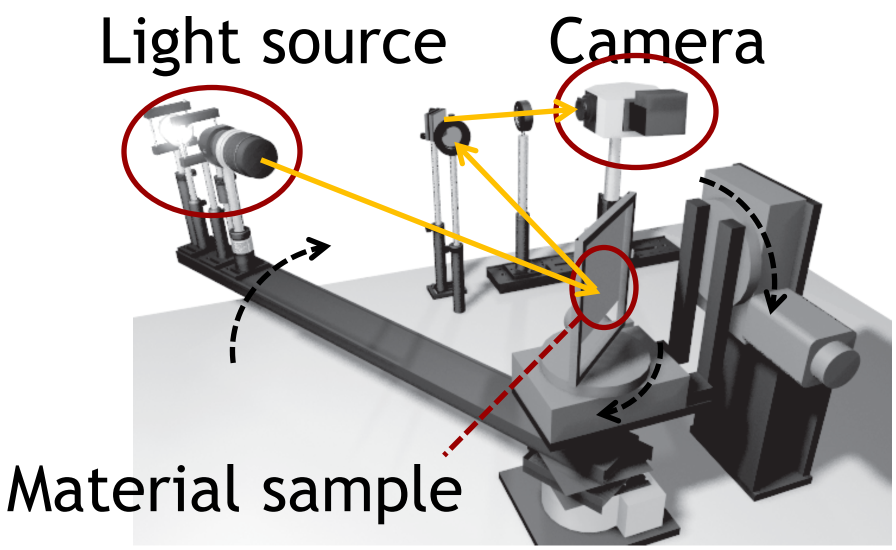
- BRDF & reflection integral
- BRDF examples

BRDF representation

1. As a table of values over incident and outgoing directions
   - General, based on measurements
   - Lots of data (4D table for each surface location)
2. Analytic BRDF models
   - Represent BRDFs as analytic functions
   - Specific functions to model physical behavior of different materials
   - Compact, material properties given by just a few parameters

Gonioreflectometer

- Device to measure and tabulate BRDFs
  - Robot with light source and camera
  - Measures reflection for each light/camera direction
  - Store measurements in table
- Public databases, e.g. http://www.merl.com/brdf/



Cornell University Gonioreflectometer

Analytic BRDFs

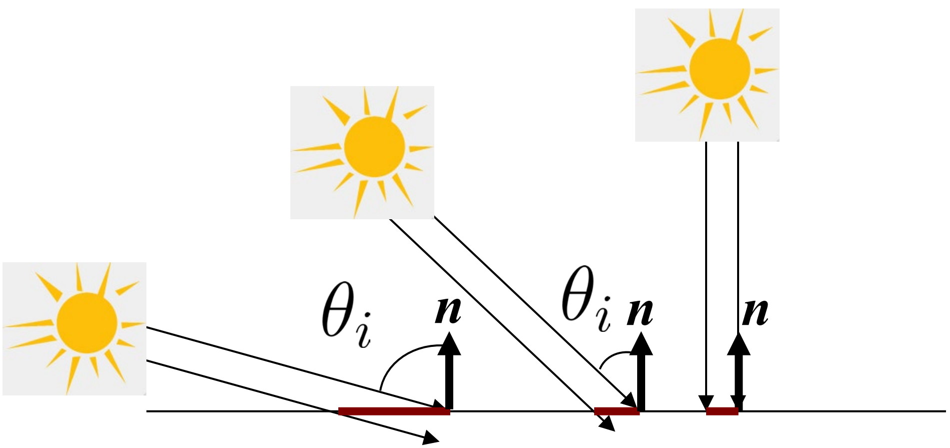
- Three examples next
  - Diffuse
  - Perfect specular reflection (mirror) and refraction
  - Torrance-Sparrow
  - Many details in PBRT book, Sections 8-8.4
    http://www.pbr-book.org/3ed-2018/Reflection_Models.html
- More examples on
  http://en.wikipedia.org/wiki/Brdf

- Phong http://en.wikipedia.org/wiki/Phong_reflection_model
  - Not physically plausible, may violate energy conservation property

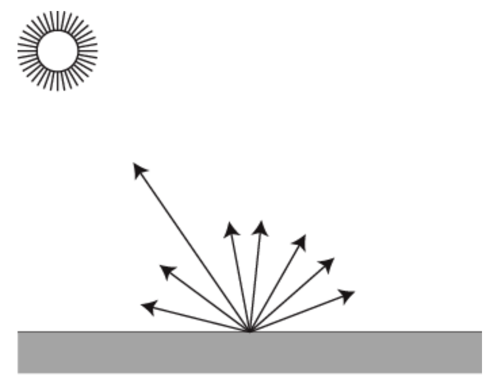
Diffuse reflection

http://en.wikipedia.org/wiki/Diffuse_reflection

- Ideal diffuse reflection: reflected radiance is equal over all directions
- Physical model: Lambert's cosine law
  http://en.wikipedia.org/wiki/Lambert's_cosine_law



Differential irradiance: Proportional to cosine of angle θ_i between incident light and normal



Reflected radiance equal in all outgoing directions

Diffuse reflection

- Diffuse BRDF is a constant ρ

$$L_o(\mathbf{x}, \omega_o) = \int_{\mathcal{H}^2(\mathbf{n})} \rho L_i(\mathbf{x}, \omega_i) \cos\theta_i d\omega_i$$

Reflection equation for diffuse surface

- Usually defined in terms of diffuse reflectance ρ_d, i.e., fraction of reflected over incoming light over whole hemisphere
  - $0 < \rho_d < 1$
  - $\rho = \rho_d / \pi$
  - Factor $1/\pi$ because of energy conservation
  - $\rho_d = 1$: all light reflected, none absorbed

Specular reflection & refraction

http://en.wikipedia.org/wiki/Specular_reflection

- Perfectly flat, homogeneous materials
  - Mirror like reflection, BSDF includes Dirac delta function
- Distinguish between dielectrics and conductors
- Dielectrics (insulators)
  - Glass, plastic
  - Refraction: light enters material and is transmitted in it
- Conductors
  - Metals
  - No light is transmitted in material
- Snell's law determines angle of refraction
- Fresnel equations determine amount of reflected and refracted light

Specular refraction

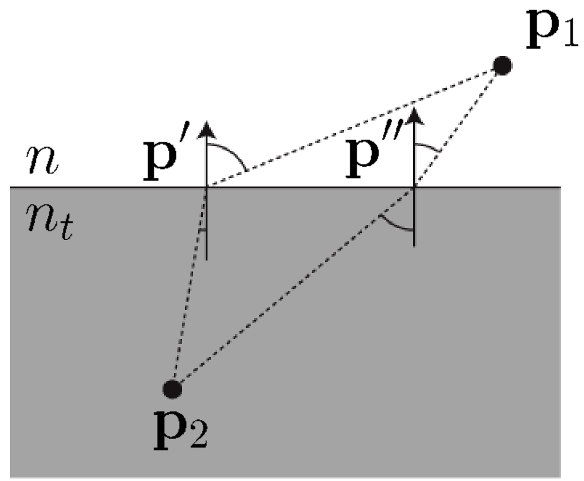
- Light travels at different speeds in different media
- Light is bent when it goes from one medium to another
- Dielectric materials
  - Diamond, glass, water, air

Index of refraction

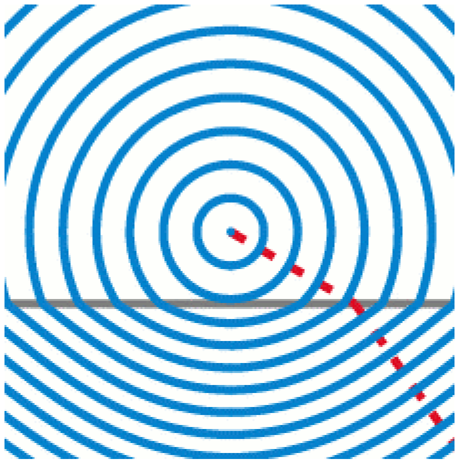
- Speed of light depends on medium
  - Speed of light in vacuum c
  - Speed of light in medium v
- Index of refraction $n = c/v$
  - Air 1.00029
  - Water 1.33
  - Acrylic glass 1.49

Explanations for bending of light rays

- Fermat's principle
  "*The actual path between two points taken by a beam of light is the one which is traversed in the least time.*"
  http://en.wikipedia.org/wiki/Specular_reflection



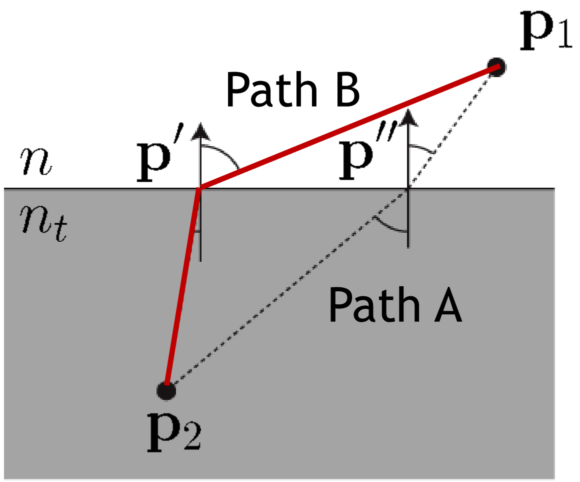
- "Change in phase velocity leads to bending of light rays"
  http://en.wikipedia.org/wiki/Snell%27s_law



Which path will light take if index of refraction n_t is larger than n?

A. Path A (black dotted)

B. Path B (red)



Snell's law

http://en.wikipedia.org/wiki/Snell%27s_law

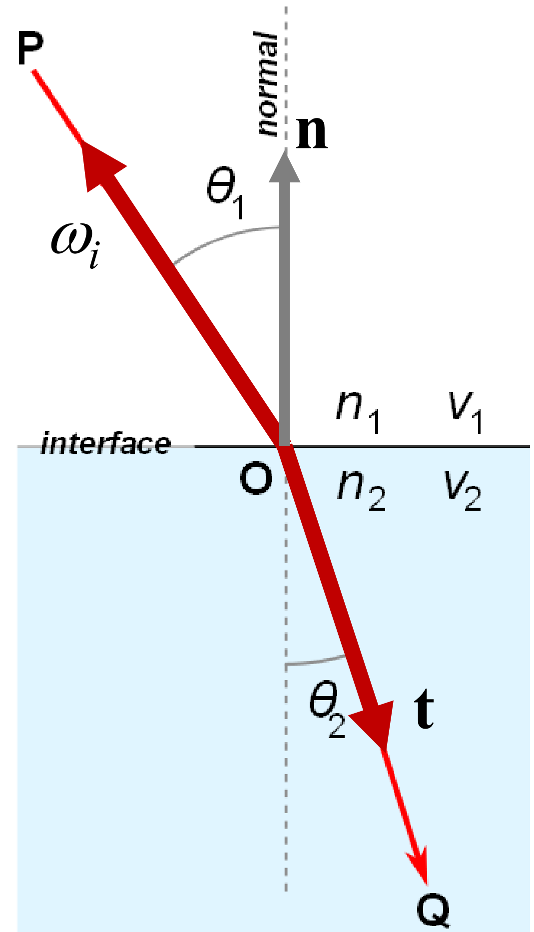
- Ratio of sines of angles of incidence and refraction is equal to opposite ratio of indices of refraction n_1, n_2

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}$$

- Vector form to obtain refracted (transmitted) 3D direction $\mathbf{t}$

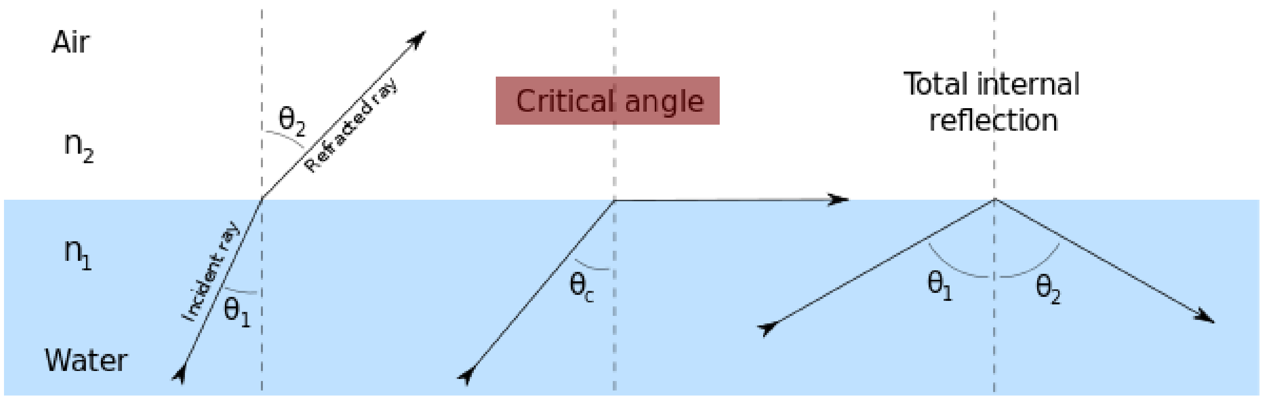
$$\mathbf{t}(\omega_i) = \frac{n_1}{n_2}(-\omega_i) + \left(\frac{n_1}{n_2}\cos\theta_1 - \cos\theta_2\right)\mathbf{n}$$

  - Incident, refracted 3D direction ω_i, $\mathbf{t}$
  - Normal vector $\mathbf{n}$



Total internal reflection

- Snell's law has no solutions for angles θ_1 larger than critical angle $\theta_c = \arcsin(\frac{\eta_2}{\eta_1})$
- Means all light is reflected, none refracted



https://en.wikipedia.org/wiki/Total_internal_reflection

Only reflection, no refraction

Fresnel equations

http://en.wikipedia.org/wiki/Fresnel_equations

- Fresnel equations describe fraction of intensity of light that is reflected and refracted
- Derived by solving Maxwell equations
  - Takes into account polarization of light
- Different versions for dielectrics and conductors
- See pbrt book for exact equations
  http://www.pbr-book.org/3ed-2018/Reflection_Models/Specular_Reflection_and_Transmission.html#FresnelReflectance

BRDF for refraction (specular transmission)

- BRDF only non-zero if ω_o is refracted direction given ω_i
- Dirac delta function δ

https://en.wikipedia.org/wiki/Dirac_delta_function

Refracted direction

$$f(\mathbf{x}, \omega_o, \omega_i) = \frac{\eta_2^2}{\eta_1^2}(1 - \underbrace{F_r(\omega_i)}_{\text{Fresnel term}}) \frac{\delta(\omega_o - \mathbf{t}(\omega_i))}{|\cos\theta_i|}$$

Fresnel term

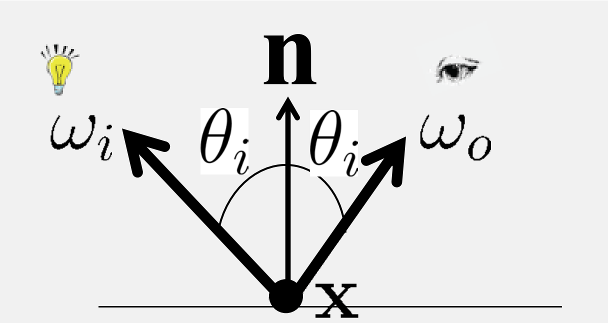
Cancels with cosine term in scattering equation

See PBRT book for detailed derivation

https://www.pbr-book.org/3ed-2018/Reflection_Models/Specular_Reflection_and_Transmission

BRDF for specular reflection

- Reflection only if incident direction ω_i is mirror reflection $\mathbf{r} = -\omega_o + 2(\mathrm{dot}(\mathbf{n},\omega_o))\mathbf{n}$ of outgoing direction
- Specular BRDF



$$f(\mathbf{x}, \omega_o, \omega_i) = F_r(\omega_o)\frac{\delta(\omega_i - \mathbf{r}(\omega_o, \mathbf{n}))}{|\cos\theta_i|}$$

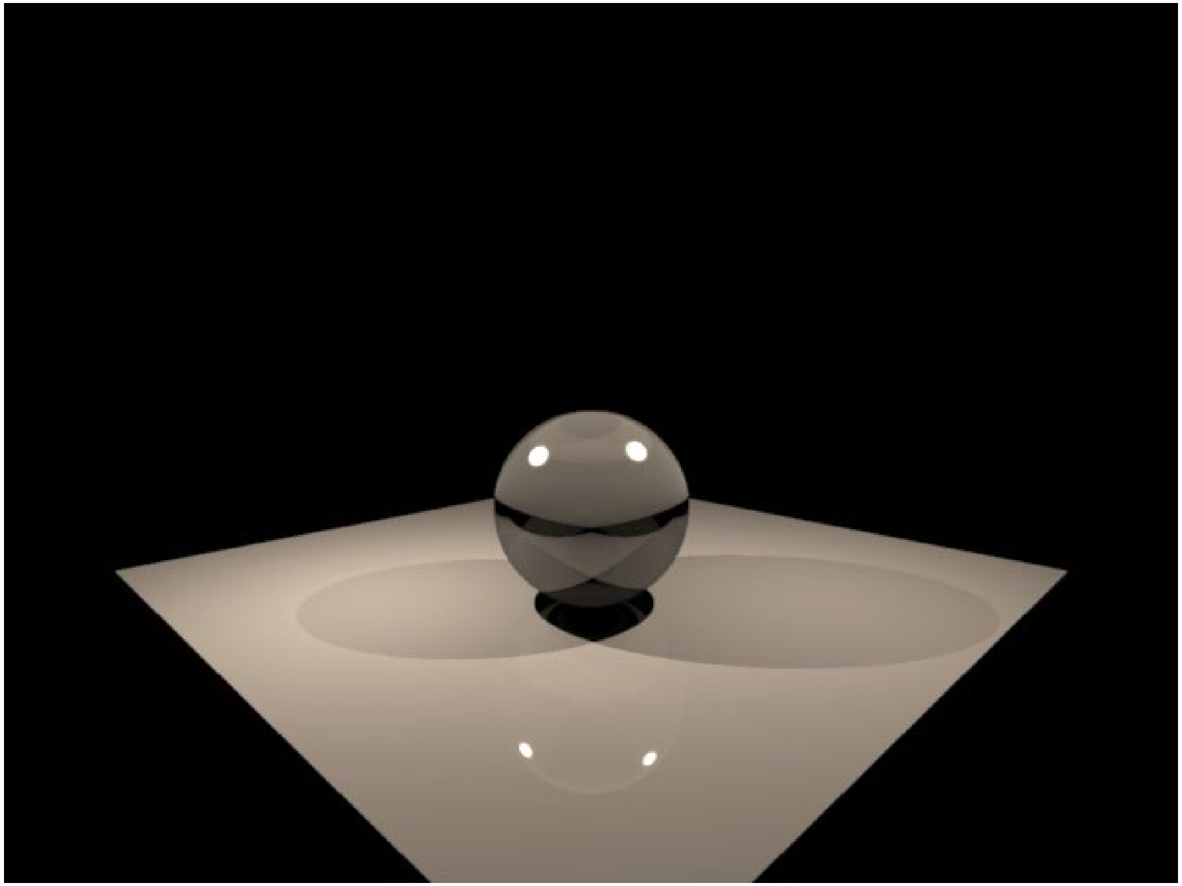
- Reflected light (result of plugging specular BRDF into reflection equation, $\cos\theta_i$ cancels out)

$$L_o(\mathbf{x}, \omega_o) = F_r(\omega_o) L_i(\mathbf{x}, \mathbf{r}(\omega_o, \mathbf{n}))$$

- Fraction of reflected light $F_r(\omega_o)$ given by Fresnel equations

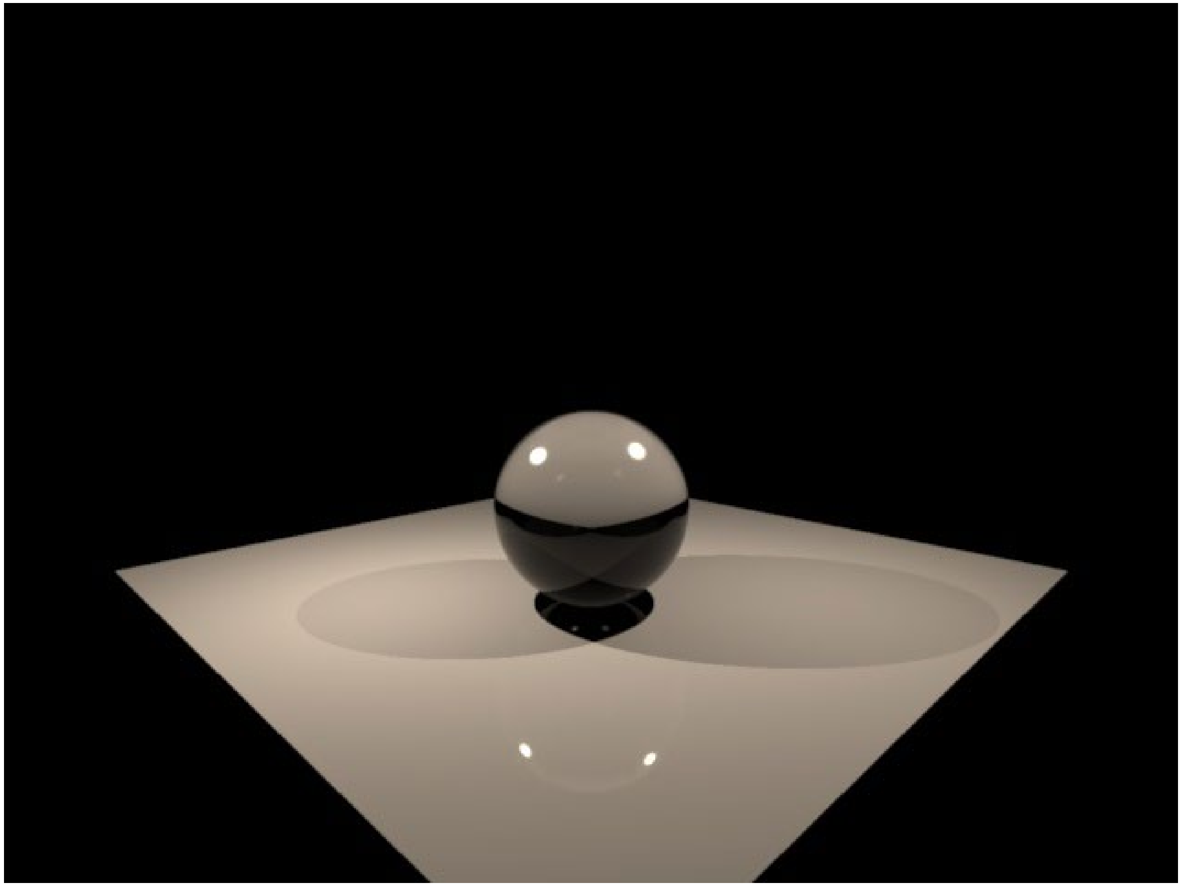
Glass sphere

- Fresnel term omitted



Glass sphere

- With Fresnel term

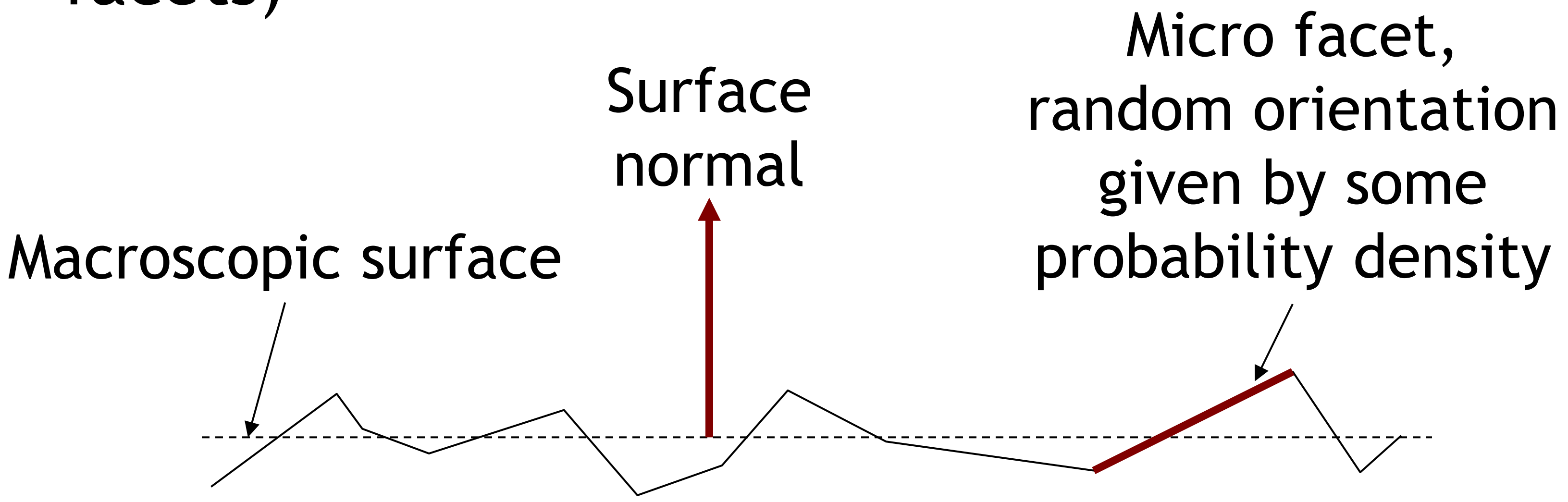


Micro facet models for glossy reflection

- Model assumption: surface consists of randomly oriented small mirrors (micro facets)
- Statistical distribution of micro facet orientations determines appearance, glossiness/shininess of reflection

Torrance-Sparrow model

- Physically-based model for specular highlight due to micro facet distribution
  http://en.wikipedia.org/wiki/Specular_highlight
  - Surfaces modeled as collections of randomly oriented small mirrors (micro mirrors, micro facets)

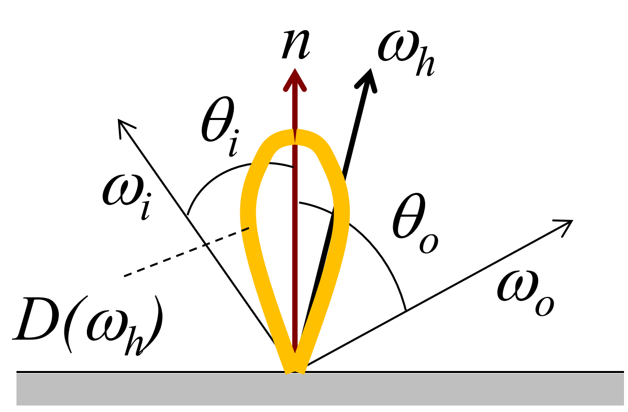


Torrance-Sparrow model

- Distribution of micro facet orientations $D(\omega_h)$
  - Half-vector ω_h between ω_i, ω_o
  - D usually function of dot product of ω_h and n
  - Note: denoting spherical coordinates of ω_h as θ, ϕ, then $D(\omega_h) = \mathrm{D}(\theta, \phi)/\sin(\theta)$, for any density D
- Distribution of microfacet normals determines surface roughness
  - Sharp or smooth specular highlight

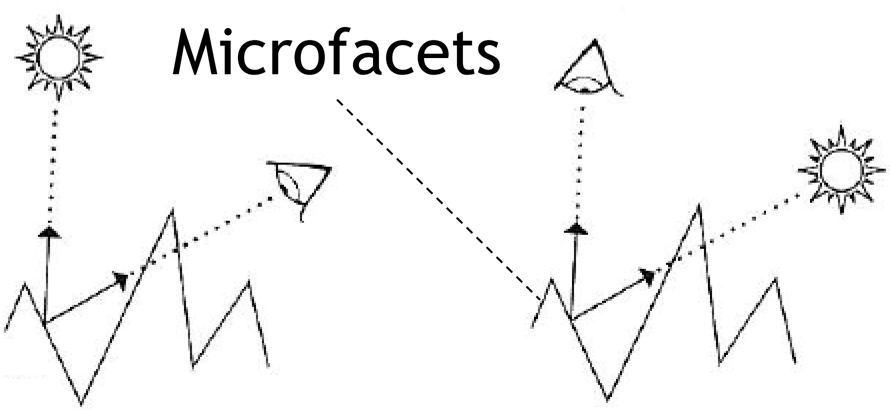
$$\omega_h = (\omega_i + \omega_o) / |\omega_i + \omega_o|$$

Half-vector

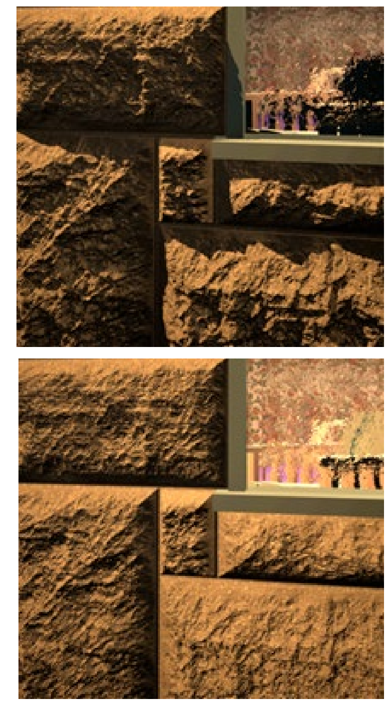


Masking and shadowing

- Rough surface consisting of microfacets leads to blocking of reflected light (masking) and shadowing of incident light
- Masking and shadowing depends on incident/outgoing angles



Masking Shadowing



Shadowing on a rough surface

Torrance-Sparrow model

$$f(\mathbf{x}, \omega_o, \omega_i) = \frac{D(\omega_h) G(\omega_o, \omega_i) F_r(\omega_o)}{4 \cos\theta_o \cos\theta_i}$$

- Geometry term G (masking and shadowing)
  - Derived from simplifying assumptions about micro facet geometry

$$G(\omega_o, \omega_i) = \min\left(1, \min\left(\frac{2(\mathbf{n}\cdot\omega_h)(\mathbf{n}\cdot\omega_o)}{\omega_o\cdot\omega_h}, \frac{2(\mathbf{n}\cdot\omega_h)(\mathbf{n}\cdot\omega_i)}{\omega_o\cdot\omega_h}\right)\right)$$

- Blinn microfacet distribution D

$$D(\omega_h) = \frac{e+2}{2\pi}(\omega_h \cdot \mathbf{n})^e$$

  - Smoothness parameter e: The higher e the more specular (mirror-like)
  - Other distributions D can be used

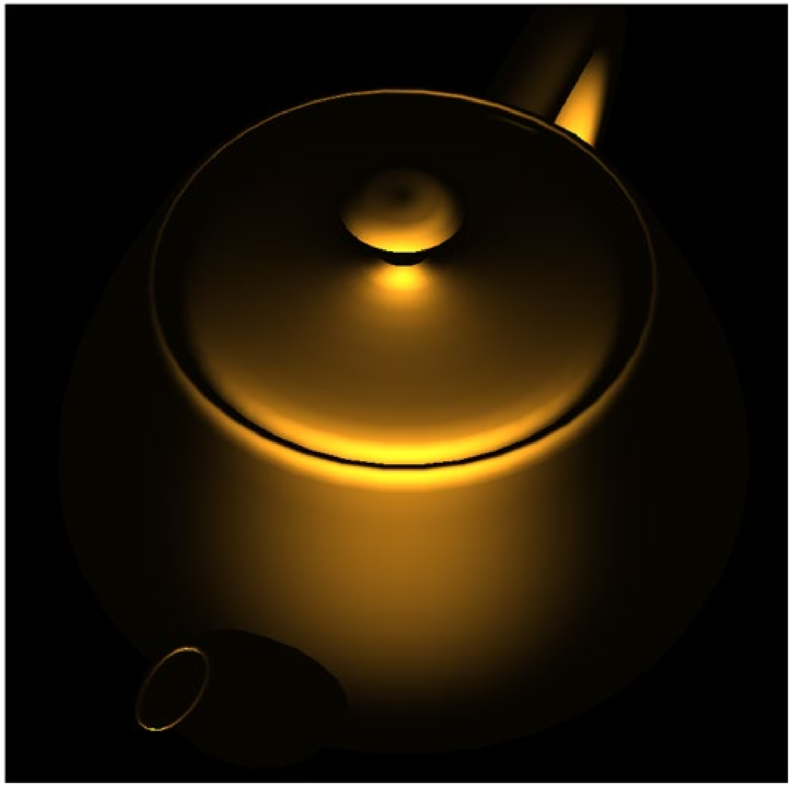
Torrance-Sparrow model

$$f(\mathbf{x}, \omega_o, \omega_i) = \frac{D(\omega_h) G(\omega_o, \omega_i) F_r(\omega_o)}{4 \cos\theta_o \cos\theta_i}$$

- Fresnell term F_r
  - Because each micro facet is modeled as a small mirror
- Normalization factors $4 \cos \theta_o \cos \theta_i$
- Derivation of complete model in PBRT book
  http://www.pbr-book.org/3ed-2018/Reflection_Models/Microfacet_Models.html

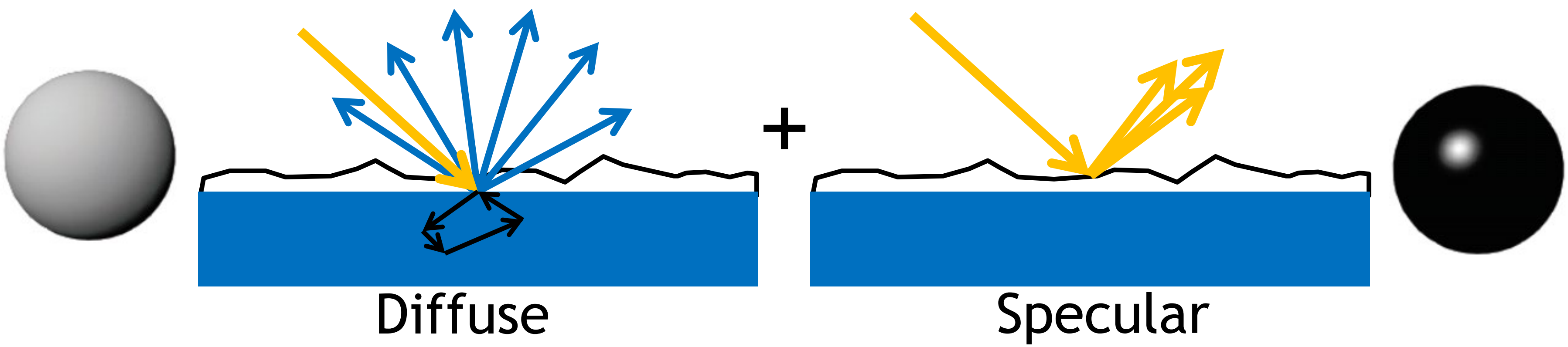
Torrance-Sparrow model

- Looks qualitatively similar to simpler models such as Phong-Blinn
  https://en.wikipedia.org/wiki/Blinn%E2%80%93Phong_shading_model



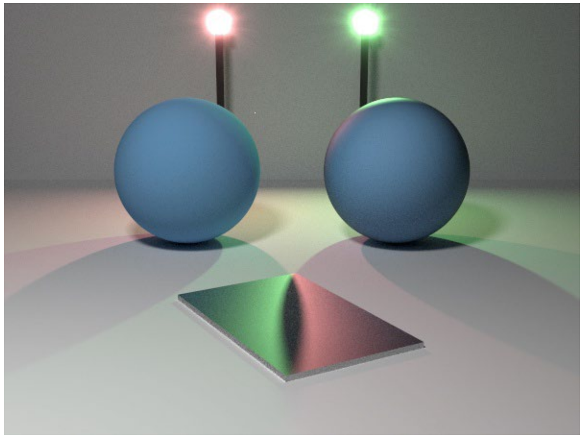
Layered surfaces

- Combine microfacet model (e.g. Torrance-Sparrow) with diffuse component to model layered surfaces
  - Glossy layer on top of diffuse



- Physically plausible model (reciprocity, energy conservation) requires adjustment of diffuse term
  - See work by Ashikmin and Shirley
    http://www.cs.utah.edu/~shirley/papers/jgtbrdf.pdf
- Comprehensive framework for layered materials
  https://rgl.epfl.ch/publications/Jakob2014Comprehensive

Layered materials



[Ashikmin & Shirley]

[Jakob et al. 2014, https://rgl.epfl.ch/publications/Jakob2014Comprehensive]

Relevant sections in PBRT book

- 8 Reflection Models
  - 8.1 Basic Interface
  - 8.2 Specular Reflection and Transmission
  - 8.3 Lambertian Reflection
  - 8.4 Microfacet Models

Next time

- Modeling light transport using the rendering equation

Evaluating the reflection equation for a diffuse surface requires more computation then for a mirror surface

A. True

B. False