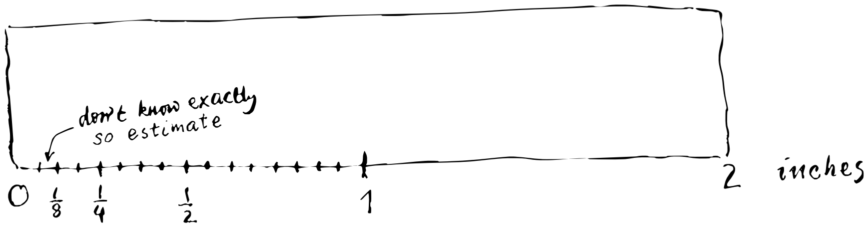


You cannot write the value of the ratio between a circle's perimeter and its diameter, π; it is necessarily inexact.

$\frac{23}{24} = \underbrace{\cdot 9583}_{\text{certain}}3+$ lies bet^n $\cdot 9583$ and $\cdot 9584$

If we express $\frac{23}{24}$ as $\cdot 9583$ we undershoot by $\cdot 00003+$

If we express $\frac{23}{24}$ as $\cdot 9584$ we overshoot by $\cdot 00007+$

Better choice is $\cdot 9583$

If we are discarding d_i after decimal point :

(a) $d_i < 5 \Rightarrow d_{i-1}$ unchanged

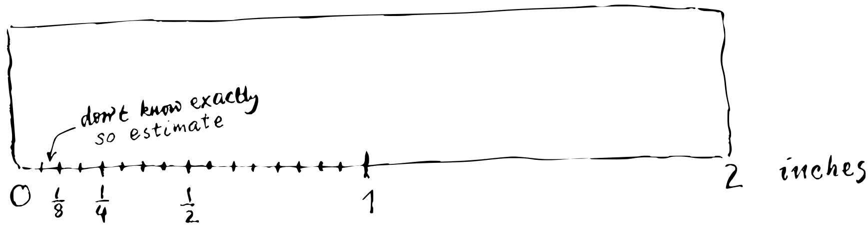
(b) $d_i \geq 5 \Rightarrow d_{i-1}$ bumped up by 1 ($\cdot 96$ is a better choice)

What if we did not know that true value is $\cdot 95833$?

True value could be $\cdot 95830, \cdot 95831, \ldots, \cdot 958399\ldots$

So max error we make is $\cdot 0001$ or $\frac{1}{10^4}$ or 10^{-4}

We may write $\cdot 9583 + [0, 10^{-4}]$



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$f: n \to a_n$, series terms are defined by a function.

- Arithmetical Progression

$$a_1 + a_2 + a_3 + \cdots + a_n, \quad a_n = a_{n-1} + \underbrace{b}_{\text{constant}}$$

$$a_1 + (a_1+b) + (a_2+b) + \cdots + a_{n-1} + b$$

$$= a_1 + (a_1+b) + (a_1+2b) + \cdots + a_1 + (n-1)b$$

$$= na_1 + \frac{n(n-1)}{2} b$$

$$= \frac{1}{2} n[2a_1 + (n-1)b]$$

$$= \frac{1}{2} n[a_1 + a_1 + (n-1)b]$$

$$= \frac{1}{2} n(a_1 + a_n)$$

So, the sum is a multiple of the average of first and last terms.

- Geometrical Progression

why this name?

$$a_n = \underset{\substack{\uparrow \\ \text{constant}}}{b} \cdot a_{n-1}$$

$$a_1 + a_2 + a_3 + \cdots + a_n$$

$$= a_1 + b \cdot a_1 + b^2 a_1 + \cdots + b^{n-1} a_1$$

$$= a_1 (1 + b + b^2 + \cdots + b^{n-1})$$

$$= a_1 \frac{b^n - 1}{b - 1} \quad (b \neq 1)$$

$$S = 1 + \cancel{b} + \cancel{b^2} + \cancel{\cdots} + \cancel{b^{n-1}}$$

$$bS = \cancel{b} + \cancel{b^2} + \cancel{b^3} + \cdots \cancel{+} b^n$$

$$S - bS = 1 - b^n$$

$$S = \frac{1 - b^n}{1 - b}$$

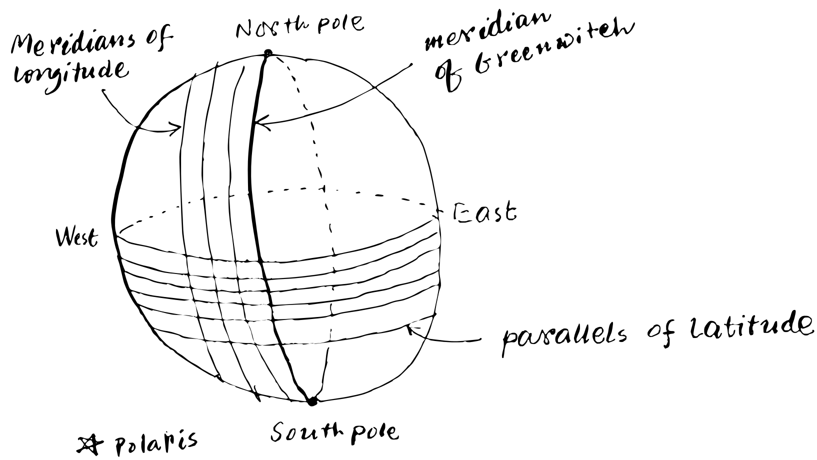
$$.666\ldots$$

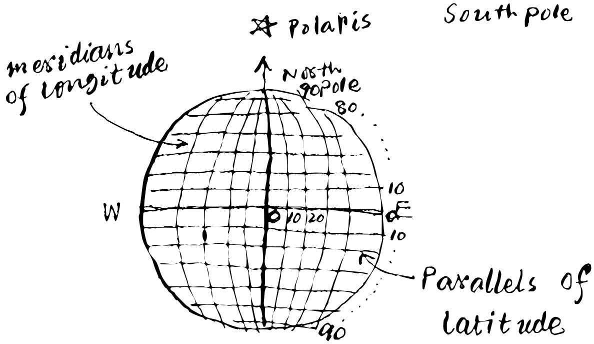
$$= .6 + .06 + .006 + \cdots$$

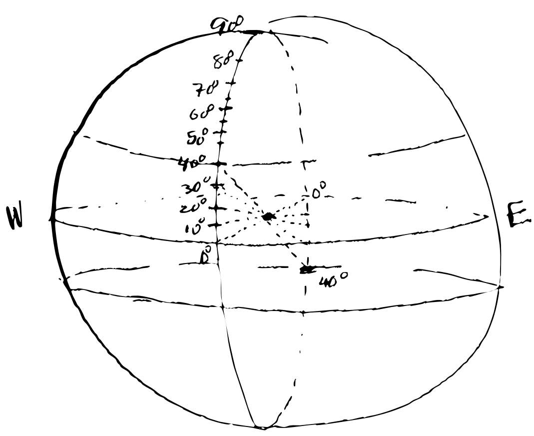
$$= \frac{6}{10} + \frac{6}{10^2} + \frac{6}{10^3} + \cdots$$

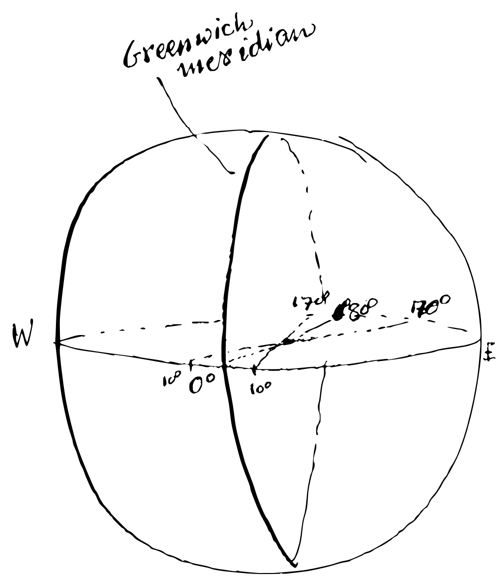
$$= \frac{6}{10}\left(1 + \frac{1}{10} + \frac{1}{10^2} + \cdots\right)$$

$$= \frac{6}{10}\left(\frac{1}{1 - \frac{1}{10}}\right) = \frac{6}{10} \cdot \frac{10}{9} = \frac{2}{3}$$





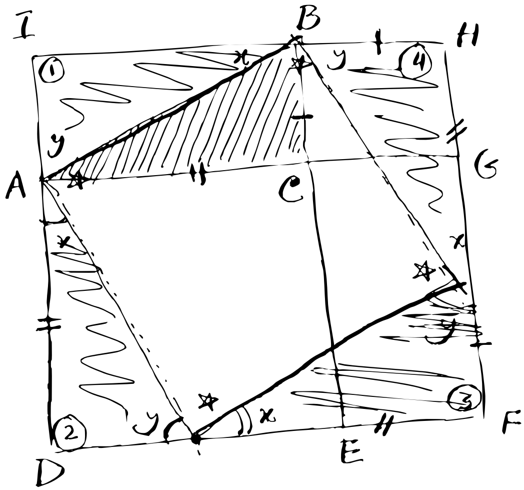


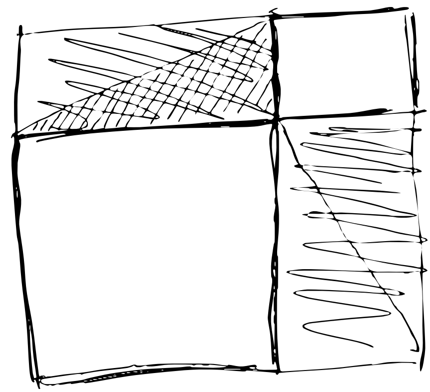


From equator ↕ lat ↔ long

$\overbrace{\text{lat, long, + N/S of equator, + W/E}}^{+}$ of prime m.

$\underbrace{\text{N/s of equator, + W/E}}_{-1}$



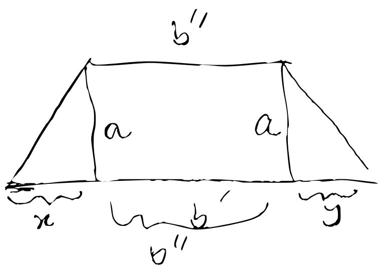


Basic idea is :

If we remove 4 triangles 2 different ways

- in one case we are left with square on hypotenuse
- in other case we are left with BCGH + ADEC

So Pythagoras: $AB^2 = AC^2 + BC^2$



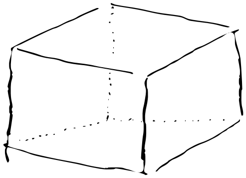
$\frac{1}{2} ax + \frac{1}{2} ay + b'' \cdot a$

$\frac{1}{2} a(x+y) + a \cdot b''$

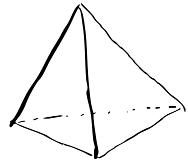
$\frac{1}{2} a \left(x+y+2b''\right)$

$= \frac{1}{2} a(x+y+b'' + b'')$

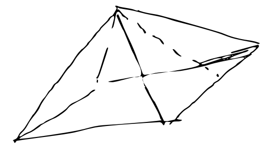
$= \frac{1}{2} a(b'+b'')$ □

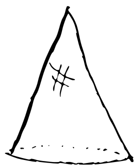


Cube

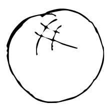


pyramid





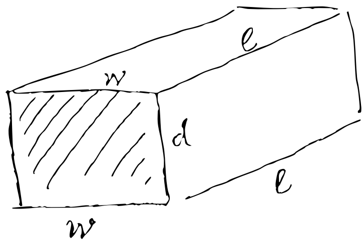
cone



sphere

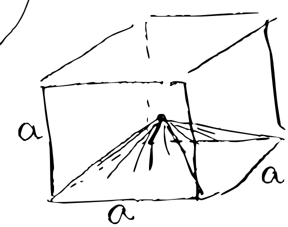
cylinder

- dimension, surface area, volume



Area = 2dw + 2dl + 2wl

Volume = dlw



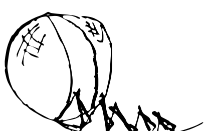
say h = pyramid's height $\frac{a^3}{6}$ = pyramid volume

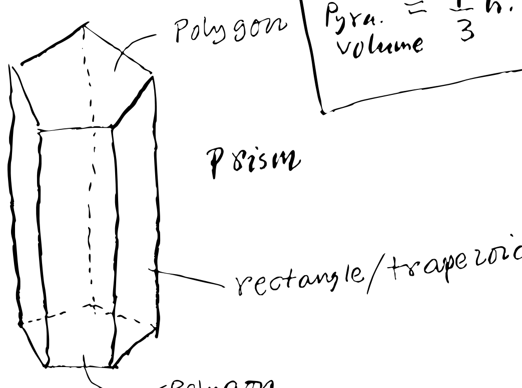
$a^3 = a \cdot \text{baseArea}$

$= 2 \cdot h \cdot \text{baseArea}$

$\frac{1}{6} a^3 = \frac{2}{6} h \cdot \text{baseArea}$

Pyra. volume $= \frac{1}{3} h \cdot \text{baseArea}$





Prism

$\frac{a^3}{6} = V_p$

$4\pi r^2 \times \frac{1}{3} r$

$\frac{4}{3}\pi r^3$

$\frac{1}{6}\ \overset{a^3}{h \cdot b}$

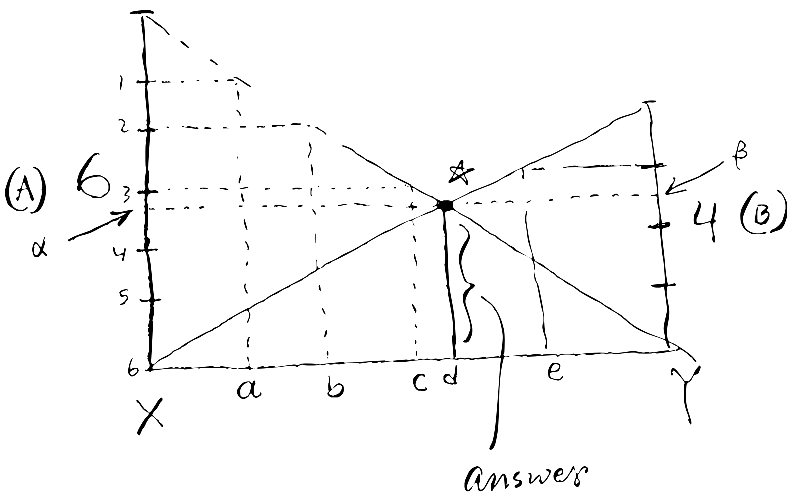
$\frac{1}{6}\ 2h\,b$

$\frac{1}{3} h\,b$

A finishes a job in 6 days

B finishes a job in 4 days

If A and B work together (assuming no contention for shared resources), how long it will take to finish?



We may think $\overline{XY}$ as the whole job.

In 1 day, A does $\overline{Xa}$ amount, in 2 days A does $\overline{Xb}$, ...

In 1 day, B does $\overline{Ye}$ amount, etc.

☆ is the point that determines exactly how long A needs to work to finish $\overline{Xd}$ and B needs to work to finish $\overline{Yd}$. Since $\overline{Xd} + \overline{Yd}$ is the whole work, ☆ is the number of required days.