

Why $-3 \times -5 = 15$?

0.1 Assuming negative numbers follow certain basic rules

To show $-3 \times -5 = 15$, we need the following assumptions:

1. Distributive property holds even for negative integers.
2. For integers a and b , if $a + b = 0$, then for a given a there is exactly one choice for b . For example, if $4 + x = 0$ then x can only be -4 .
3. $x \times 0 = 0$ holds even when x is a negative integer.

We first need to prove $-3 \times 5 = -15$. Consider the below:

$$\begin{aligned} -3 \times 5 + 15 &= -3 \times 5 + 3 \times 5 \\ &= (-3 + 3) \times 5 \quad /* \text{Using first assumption} */ \\ &= 0 \times 5 \\ &= 0 \end{aligned}$$

So, -3×5 is a number when added to 15 gives 0. From the second assumption we can thus say $-3 \times 5 = -15$.

Now consider the below:

$$\begin{aligned} 0 &= -3 \times 0 \quad /* \text{Using third assumption} */ \\ &= -3 \times (-5 + 5) \\ &= (-3 \times -5) + (-3 \times 5) \quad /* \text{Using first assumption} */ \\ &= (-3 \times -5) + (-15) \end{aligned}$$

So, -3×-5 is a number when added to -15 gives 0. From the second assumption, we can thus say $-3 \times -5 = 15$. ■

0.2 From pattern of series

Consider the below series:

$$\dots \dots 1, 2, 3, 4, 5, \dots \dots \quad (1)$$

Lets multiply the series by 3. Between each consecutive pair, the left one is 3 less than the right one.

$$\dots \dots 3, 6, 9, 12, 15, \dots \dots \quad (2)$$

Lets reveal the second series more to the left.

$$\dots \dots -15, -12, -9, -6, -3, 0, 3, 6, 9, 12, 15, \dots \dots \quad (3)$$

We can now show a correspondence between the first and the third series which essentially is a correspondence between x and $3x$.

...	...	-5	-4	-3	-2	-1	0	1	2	3	4	5
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
...	...	-15	-12	-9	-6	-3	0	3	6	9	12	15

We see that 3×-5 corresponds to -15 , thus we can say $3 \times -5 = -15$.

Now we do the same routine but this time we multiply the first series by -3 and get the fourth series below. Between every pair of consecutive integers, the left one is 3 greater than the right one.

$$\dots \dots -3, -6, -9, -12, -15, \dots \dots \quad (4)$$

If we reveal the above series more towards left, we find below:

$$\dots \dots 15, 12, 9, 6, 3, 0, -3, -6, -9, -12, -15 \dots \dots \quad (5)$$

Now we establish a correspondence between the first and the fifth series below, which actually is the correspondence between x and $-3x$.

...	...	-5	-4	-3	-2	-1	0	1	2	3	4	5
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
...	...	15	12	9	6	3	0	-3	-6	-9	-12	-15

Above, -5 corresponds to 15 , so we can say $-3 \times -5 = 15$. ■