

Problem-225

Problem Statement

If in the equation

$$ax^2 + bx + c = 0$$

the coefficient a is equal to zero then the equation takes the form

$$bx + c = 0$$

and has the unique solution

$$x = -\frac{c}{b}$$

Strictly speaking, the last sentence is wrong; when $b = 0$ the quotient $\frac{c}{b}$ is undefined. How are we to correct this error?

Solution

We should require that in the equation $ax^2 + bx + c = 0$, both a and b cannot be zero.

Problem-227

Problem Statement

Prove that $\sqrt{3}$ is irrational.

Solution

The statement ' $\sqrt{3}$ is irrational' is equivalent to the statement that $\sqrt{3} \neq \frac{a}{b}$ for any integer a and b . We shall make use of **proof by contradiction**, therefore, we shall assume that $\sqrt{3} = \frac{a}{b}$ and that should lead to a contradiction, completing the proof.

We shall consider four different cases for the pair (a, b) and we shall derive a contradiction for each.

1. a and b both are odd integer. Say $a = 2m + 1$ and $b = 2n + 1$, where m and n are integers. According to our assumption

$$\begin{aligned}\sqrt{3} &= \frac{2m+1}{2n+1} \\ 3 &= \frac{(2m+1)^2}{(2n+1)^2} \\ 3(2n+1)^2 &= (2m+1)^2 \\ 3(4n^2+4n+1) &= 4m^2+4m+1 \\ 12n^2+12n+3 &= 4m^2+4m+1 \\ 12n^2+12n+2 &= 4m^2+4m \\ 6n^2+6n+1 &= 2m^2+2m\end{aligned}$$

On the left side we have an odd integer and on the right side we have an even integer—a contradiction. ✖

2. a is an odd integer and b is an even integer. Say $a = 2m + 1$ and $b = 2n$, where m and n are integers. According to our assumption

$$\begin{aligned}\sqrt{3} &= \frac{2m+1}{2n} \\ 3 &= \frac{(2m+1)^2}{4n^2} \\ 12n^2 &= (2m+1)^2\end{aligned}$$

On the left side we have an even integer and on the right side we have an odd integer—a contradiction. ✖

3. a is an even integer and b is an odd integer. Say $a = 2m$ and $b = 2n + 1$, where m and n are integers. According to our assumption

$$\begin{aligned}\sqrt{3} &= \frac{2m}{2n+1} \\ 3 &= \frac{4m^2}{(2n+1)^2} \\ 3(2n+1)^2 &= 4m^2\end{aligned}$$

On the left side we have an odd integer and on the right side we have an even integer—a contradiction. ✖

4. a and b both are even integer. In this case, a and b must have 2 as a common factor. We can divide both a and b by 2. We can keep dividing by 2 as long as both remain even integer. At the end, we are in one of the previous three cases.