A degree 2 polynomial seems to have three distinct roots, how?

Consider the below identity:

$$\frac{(x-a)(x-b)}{(c-a)(c-b)} + \frac{(x-a)(x-c)}{(b-a)(b-c)} + \frac{(x-b)(x-c)}{(a-b)(a-c)} = 1$$

We can rearrange:

$$\frac{(x-a)(x-b)}{(c-a)(c-b)} + \frac{(x-a)(x-c)}{(b-a)(b-c)} + \frac{(x-b)(x-c)}{(a-b)(a-c)} - 1 = 0$$

Now the left side of the above can be considered as a polynomial $P(x)$ with degree at most 2.

$$P(x) = \frac{(x-a)(x-b)}{(c-a)(c-b)} + \frac{(x-a)(x-c)}{(b-a)(b-c)} + \frac{(x-b)(x-c)}{(a-b)(a-c)} - 1$$

However, a, b, c look like three distinct roots of $P(x)$. **How?**

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Assume that $x_1, \ldots, x_{10}$ are different numbers, and $y_1, \ldots, y_{10}$ are arbitrary numbers. Prove that there is one and only one polynomial $P(x)$ of degree not exceeding 9 such that $P(x_1) = y_1,\ P(x_2) = y_2,\ \ldots,\ P(x_{10}) = y_{10}$.

We get a set of 10 equations in 10 unknowns. As long as there is a solution, we have one polynomial. Uniqueness can be proved by reasoning about $\boldsymbol{R(x) = P(x) - Q(x)}$.