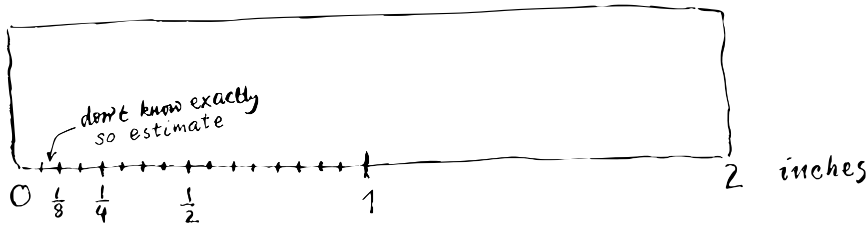


You cannot write the value of the ratio between a circle's perimeter and its diameter, π; it is necessarily inexact.

$\frac{23}{24} = \underbrace{\cdot 9583}_{\text{certain}}3+$ lies bet^n $\cdot 9583$ and $\cdot 9584$

If we express $\frac{23}{24}$ as $\cdot 9583$ we undershoot by $\cdot 00003+$

If we express $\frac{23}{24}$ as $\cdot 9584$ we overshoot by $\cdot 00007+$

Better choice is $\cdot 9583$

If we are discarding d_i after decimal point :

(a) $d_i < 5 \Rightarrow d_{i-1}$ unchanged

(b) $d_i \geq 5 \Rightarrow d_{i-1}$ bumped up by 1 ($\cdot 96$ is a better choice)

What if we did not know that true value is $\cdot 95833$?

True value could be $\cdot 95830, \cdot 95831, \ldots, \cdot 958399\ldots$

So max error we make is $\cdot 0001$ or $\frac{1}{10^4}$ or 10^{-4}

We may write $\cdot 9583 + [0, 10^{-4}]$

Approximation in Addition

$$
\begin{array}{r}
\cdot 234673 \\
\cdot 322135 \\
\cdot 114342 \\
+\ \cdot 563217 \\
\hline
1\cdot 234367
\end{array}
\quad \left.\right\} \text{ we know upto six digits (rounded perhaps)}
$$

$1\cdot234367 \leftarrow$ can we say the sum is this ?

When we possibly discarded 7 th digit of $\cdot 234673$ we may have incurred an error of $\cdot 0000005$. For example, say true value was $\cdot 2346725$ so difference is $\cdot 234673 - \cdot 2346725$ or $\cdot 0000005$, This is true for other numbers.

so, we may have $4 \times 0\cdot0000005 = 0\cdot000002$ error in the sum.

Thus the sum $1\cdot234367$ can be anywhere between $1\cdot234365$ and $1\cdot234369$; either case rounding rule gives $1\cdot23437$. Even though we knew 6 digits for each numbers, for the sum though we know 5 digits after decimal point.

If we added more numbers, things could be worse; in that case we may go back and see if we could measure the individual numbers more accurately.

$$
\begin{array}{r}
^{2}\ \ ^{1\,2\,1\,1\,2}\ \\
5\cdot 866314 \\
3\cdot 715918 \\
0\cdot 568286 \\
+\ 4\cdot 342233 \\
\hline
14\cdot 492751
\end{array}
$$

$14\cdot492751 \leftarrow$ How much are we certain about this ?

$(\pm\cdot 000002)$

we can say $14\cdot49275$ with rounding

Book says sthing else P27

Approximation in subtraction

$$
\begin{array}{rl}
 & .329528 \text{ (possible error } .0000005\text{)} \\
- & .238647 \\
\hline
 & .090881 \text{ (possible error } 2 \times 0.0000005 = 0.000001\text{)}
\end{array}
$$

we can say (with rounding) .09088

Approximation in multiplication

$$
\begin{array}{r}
3.14159 \\
\times\ 3.14159 \\
\hline
2827431 \\
1570795 \\
314159 \\
1256636 \\
314159 \\
942477 \\
\hline
9.869587281
\end{array}
$$

~ 9.8696

shortcut if we wanted only 4 digits in the product : left → right, we need 2 extra digits

$$
\begin{array}{r|r}
3.141 & 59 \\
3.141 & 59 \\
\hline
9.4247 & 7 \\
.3141 & 59 \\
.1256 & 64 \\
.0031 & 42 \\
.0015 & 71 \\
.0002 & 83 \\
\hline
9.8695 & 89
\end{array}
$$

~ 9.8696

we only need this much extra to round up to 4 digits

$$
\begin{array}{r}
9.42477 \\
.31416 \\
.12566 \\
.00314 \\
.00157 \\
.00028 \\
\hline
9.86938
\end{array}
$$

$\sim 9.8\cancel{69}4$

How does error in multiplier or multiplicand affect the product?

Exercise 19

1.

23·45|61|7
937·34|21|2
42·31|75|9
532·23|34|6
141·42|37|98

$$
\begin{array}{r}
{\scriptstyle 1\,1\,1\ \ 1\,2\,2} \\
23.4562 \\
937.3421 \\
42.3176 \\
532.2335 \\
+\ 141.4238 \\
\hline
\underbrace{1676.77}32
\end{array}
$$

2.

$$
\begin{array}{r}
993.624 \\
-\ 987.642 \\
\hline
\underbrace{5.9}82
\end{array}
$$

$$\downarrow$$

$$\boxed{6.0}$$

3.

$$
\begin{array}{r}
32.47\overset{3}{3}6 \\
\times\ 24.7955 \\
\hline
680
\end{array}
$$

least common multiple of a, b, c

$$a = \prod_{i=1}^{m} p_i^{a_i} \quad \text{where } a_i \geq 0 \text{ and } p_i \text{ are primes}$$

$$b = \prod_{i=1}^{n} p_i^{b_i} \quad , \quad c = \prod_{i=1}^{q} p_i^{c_i}$$

say $m \leq n \leq q$

$$\text{lcm}(a,b,c) = \prod_{i=1}^{q} p_i^{\max\{a_i, b_i, c_i\}}$$

$$\text{lcm}(2,3,4,5) = 2^{\max\{0,1,2\}} \cdot 3^{\max\{0,1\}} \cdot 5^{\max\{0,1\}}$$

$2 = 2^1 \cdot 3^0 \cdot 5^0$
$3 = 2^0 \cdot 3^1 \cdot 5^0$
$4 = 2^2 \cdot 3^0 \cdot 5^0$
$5 = 2^0 \cdot 3^0 \cdot 5^1$

$= 2^2 \cdot 3^1 \cdot 5^1$
$= 4 \cdot 15$
$= 60$

greatest common divisor of a, b, c

$$\text{gcd}(a,b,c) = \prod_{i=1}^{m} p_i^{\min\{a_i, b_i, c_i\}}$$

$$\text{gcd}(9, 18, 36, 54) = 2^0 \cdot 3^2 = 9$$

$9 = 2^0 \cdot 3^2$
$18 = 2^1 \cdot 3^2$
$36 = 2^2 \cdot 3^2$
$54 = 2^1 \cdot 3^3$

(1.) $(d_n d_{n-1} \cdots d_1 d_0)_{10}$ if $2 \mid d_0$, then $2 \mid (d_n d_{n-1} \cdots d_1 d_0)_{10}$

$(d_n d_{n-1} \cdots d_1 d_0)_{10} = \sum_{i=0}^{n} d_i \cdot 10^i$, for $i > 0$, $2 \mid 10^i$, so...

(2.) Say $x = (d_n d_{n-1} \cdots d_1 d_0)_{10}$. If $d_0 = 0$ or $d_0 = 5$, then $5 \mid x$.

– Similar logic as in (1)

(3) Say $x = (d_n d_{n-1} \cdots d_1 d_0)_{10}$. If $4 \mid (d_1 d_0)_{10}$, then $4 \mid x$.

If $25 \mid (d_1 d_0)_{10}$, then $25 \mid x$.

(4) Say $x = (d_n d_{n-1} \cdots d_2 d_1 d_0)_{10}$. If $8 \mid (d_2 d_1 d_0)_{10}$, then $8 \mid x$.

$$x = d_n \cdot 10^n + d_{n-1} \cdot 10^{n-1} + \cdots + d_3 \cdot \underbrace{10^3}_{} + (d_2 d_1 d_0)_{10}$$

$$\underbrace{2^3}_{=8} \cdot 5^3$$

(5) If $9 \mid \sum_{i=0}^{n} d_i$, then $9 \mid x$.

$$x = 10^n \cdot d_n + 10^{n-1} \cdot d_{n-1} + \cdots + 10^3 \cdot d_3 + 10^2 \cdot d_2 + 10^1 d_1 + 10^0 \cdot d_0$$

$$= (10^n - 1) d_n + d_n + \cdots + (10-1) d_1 + d_1 + d_0$$

$$= \underbrace{\sum_{i=1}^{n} d_i (10^i - 1)}_{\blacksquare} + \underbrace{\sum_{i=0}^{n} d_i}_{\star}$$

$$10^n = (9+1)^n = 9^n + \binom{n}{1} 9^{n-1} + \binom{n}{2} 9^{n-2} + \cdots + \binom{n}{n-1} 9 + 1$$

So, $9 \mid (10^n - 1)$. Therefore $9 \mid \blacksquare$ and $9 \mid \star$

– $(7362)_{10}$ has digit sum $(18)_{10}$

$9 \mid (18)_{10}$, so $9 \mid (7362)_{10}$

(6) Say $x = (d_n d_{n-1} \cdots d_1 d_0)_{10}$

If $3 \mid \sum d_i$, then $3 \mid x$. Since $3 \mid 9$, by (5) ...

(7) If $2 \mid x$ and $3 \mid x$, then $6 \mid x$.

- 2 and 3 are primes, so if 2 and 3 divides x; then x must have both 2 and 3 in its prime factorization. Thus 6 must be a factor of x.

(8) If $\sum_{i : 2 \mid i} d_i = \sum_{j : 2 \nmid j} d_j$, then $11 \mid x$.

$$\underset{3}{1}\,\underset{2}{1}\,\underset{1}{1}\,\underset{0}{1} = 1\cdot 10^3 + 1\cdot 10^2 + 1\cdot 10^1 + 1$$

$$= (1001-1)\,d_3 + (99+1)\,d_2 + (11-1)\,d_1 + 1\cdot d_0$$

$$= 1001\cdot d_3 + 99 d_2 + 11\cdot d_1 + (d_2 - d_3) + (d_0 - d_1)$$

$$\boxed{\begin{aligned} 10^3 &= 1000 = (1001-1) \\ 10^2 &= 100 = (99+1) \end{aligned}}$$

11) 1001 (91
99
11

Lemma: If $2 \nmid n$, then $11 \mid (10^n + 1)$. Say $n = 2k+1$, $k \geq 0$

$$10^{2k+1} + 1 = 10^{2k}\cdot 10 + 1$$
$$= (100)^k \cdot 10 + 1$$
$$= (99+1)^k \cdot 10 + 1$$
$$= \Big[\underbrace{99^k + \binom{k}{1} 99^{k-1} + \binom{k}{2} 99^{k-2} + \cdots + \binom{k}{k-1} 99}_{\square} + 1\Big]\cdot 10 + 1$$

$\square \rightarrow 11 \mid \square$

$$= \square \cdot 10 + 10 + 1$$
$$= \square \cdot 10 + 11$$

If $2 \mid n$, then $11 \mid (10^n - 1)$. Say $n = 2k$, $k \geq 0$.

$$
\begin{aligned}
10^{2k} - 1 &= 100^k - 1 \\
&= (99+1)^k - 1 \\
&= \left[99^k + \binom{k}{1} 99^{k-1} + \cdots + \binom{k}{k-1} 99 + 1\right] - 1 \\
&= 99 \cdot \square
\end{aligned}
$$

So, $11 \mid (10^{2k} - 1)$.

- Corollary

If $\displaystyle\sum_{i: 2 \nmid i} d_i - \sum_{i: 2 \mid i} d_i = m \cdot 11$, then $11 \mid x$.

$$
\begin{aligned}
x &= (d_n d_{n-1} \cdots d_1 d_0)_{10} \\
&= \sum_{i: 2 \nmid i} d_i \cdot 10^i + \sum_{i: 2 \mid i} d_i \cdot 10^i \\
&= \sum_{i: 2 \nmid i} d_i \cdot 10^{2k_i + 1} + \sum_{i: 2 \mid i} d_i \cdot 10^{2k_i} \\
&= \sum_{i: 2 \nmid i} d_i \cdot \left[(10^{2k_i+1} + 1) - 1\right] + \sum_{i: 2 \mid i} d_i \cdot \left[(10^{2k_i} - 1) + 1\right] \\
&= \underbrace{\sum_{i: 2 \nmid i} d_i (10^{2k_i+1} + 1) + \sum_{i: 2 \mid i} d_i \cdot (10^{2k_i} - 1)}_{\square} + \underbrace{\sum_{i: 2 \mid i} d_i - \sum_{i: 2 \nmid i} d_i}_{\star}
\end{aligned}
$$

Both $\square$ an $\star$ are divisible by 11.

$$\begin{array}{cccccccc} 3 & 4 & 2 & 6 & 7 & 8 & 9 & 3 \\ 7 & 6 & 5 & 4 & 3 & 2 & 1 & 0 \end{array}$$

Sum of even place digits $= 4+6+8+3$
$= 21$

Sum of odd place digits $= 3+2+7+9$
$= 21$

Since diff is 0 and divisible by 11, $11 \mid 34267893$.

$$
\begin{array}{l}
11 \overline{)\,34267893\,} \left(3115263 \right. \\
\quad 33 \\
\quad \overline{\ \ 12} \\
\quad \ \ 11 \\
\quad \overline{\ \ \ 16} \\
\quad \ \ \ 11 \\
\quad \overline{\ \ \ \ 57} \\
\quad \ \ \ \ 55 \\
\quad \overline{\ \ \ \ \ 28} \\
\quad \ \ \ \ \ 22 \\
\quad \overline{\ \ \ \ \ \ 69} \\
\quad \ \ \ \ \ \ 66 \\
\quad \overline{\ \ \ \ \ \ \ 33} \\
\quad \ \ \ \ \ \ \ 33 \\
\quad \overline{\ \ \ \ \ \ \ \ \ \ }
\end{array}
$$

Problem 1

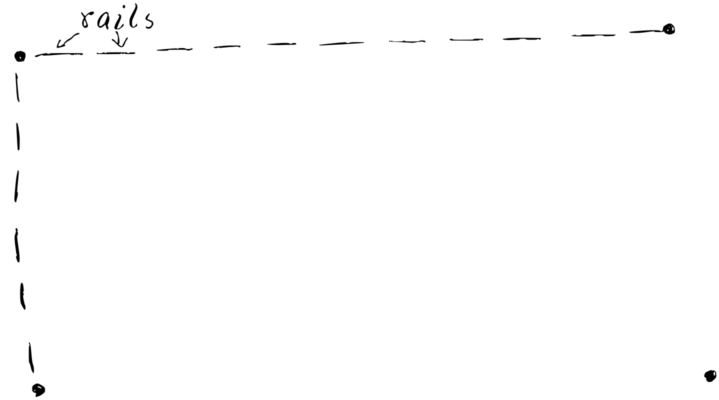
364 corn $\qquad 364 = 2 \cdot 182 = 2^2 \cdot 91$

455 oats $\qquad 455 = 5 \cdot 91$

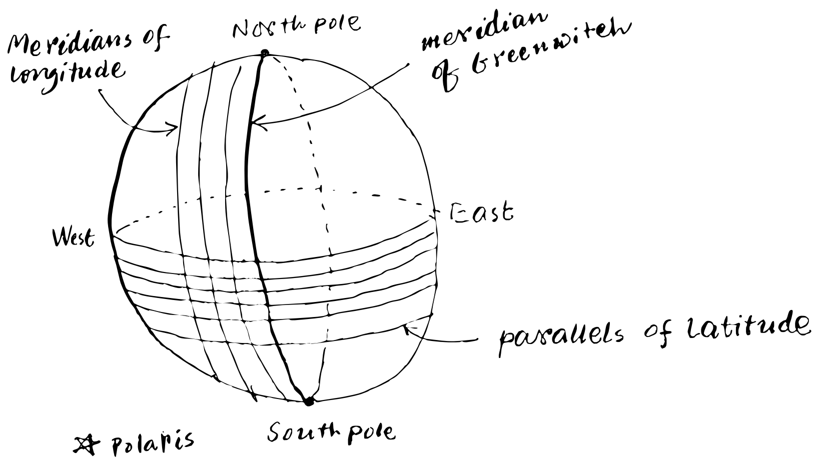
546 wheat $\qquad 546 = 2 \cdot 273 = 2 \cdot 3 \cdot 91$

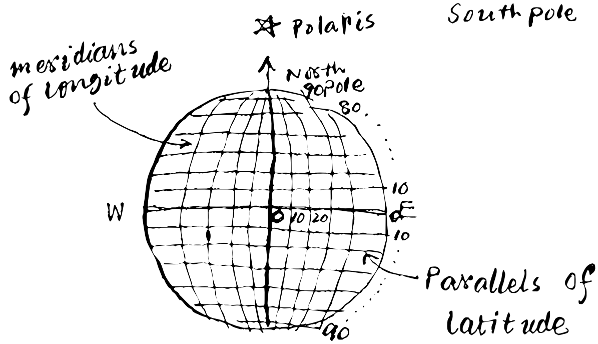
$\gcd(364, 455, 546) = 91$

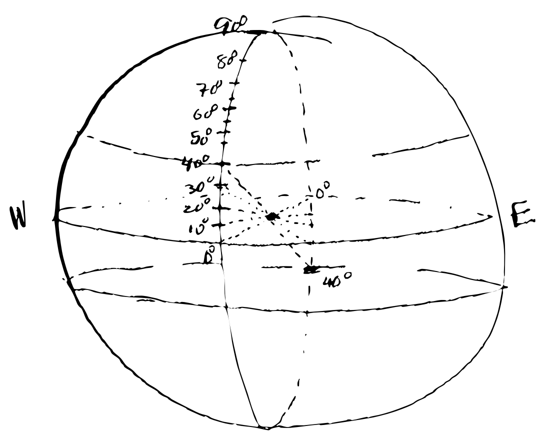
Problem 2

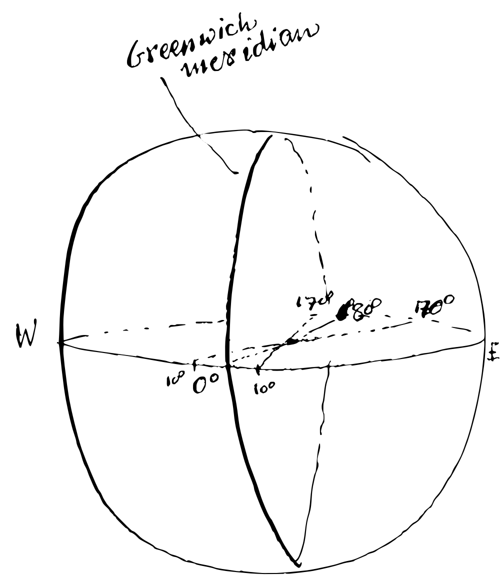


GCD.







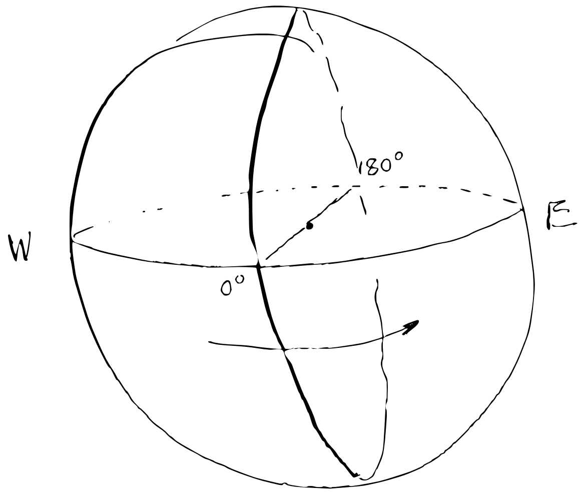


From equator lat long

+

lat, long, + N/S of equator, + W/E of prime m.

−1

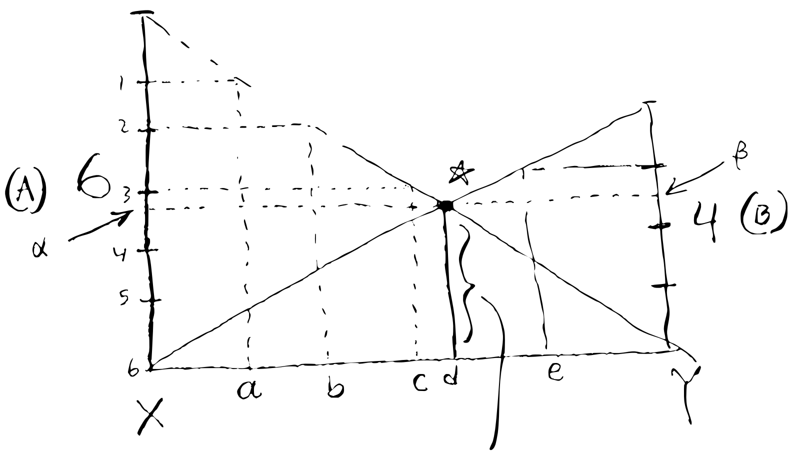


$$\frac{\overset{60}{\cancel{360}}°}{\underset{4}{\cancel{24}}\,hr} \sim 15°/hr$$

A finishes a job in 6 days

B finishes a job in 4 days

If A and B work together (assuming no contention for shared resources), how long it will take to finish?

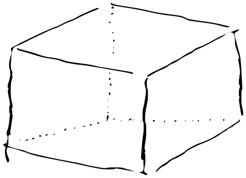


We may think $\overline{XY}$ as the whole job.

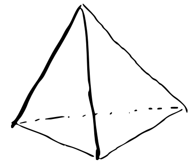
In 1 day, A does $\overline{Xa}$ amount, in 2 days A does $\overline{Xb}$, ...

In 1 day, B does $\overline{Ye}$ amount, etc.

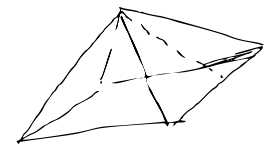
☆ is the point that determines exactly how long A needs to work to finish $\overline{Xd}$ and B needs to work to finish $\overline{Yd}$. Since $\overline{Xd} + \overline{Yd}$ is the whole work, ☆ is the number of required days.

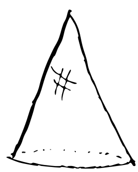


Cube

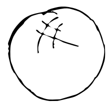


pyramid

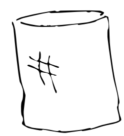




cone

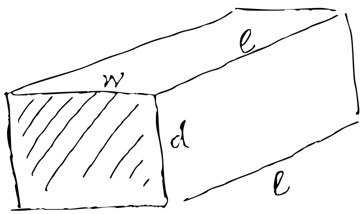


sphere



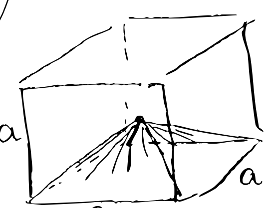
cylinder

- dimension, surface area, volume



Area = $2dw + 2dl + 2wl$

Volume = dlw



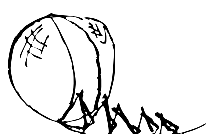
Say h = pyramid's height $\frac{a^3}{6}$ = pyramid volume

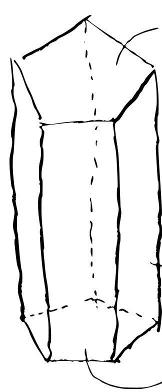
$a^3 = a \cdot \text{baseArea}$

$= 2 \cdot h \cdot \text{baseArea}$

$\frac{1}{6} a^3 = \frac{2}{6} h \cdot \text{baseArea}$

Pyra. volume $= \frac{1}{3} h \cdot \text{baseArea}$





Prism

$\frac{a^3}{6} = V_p$

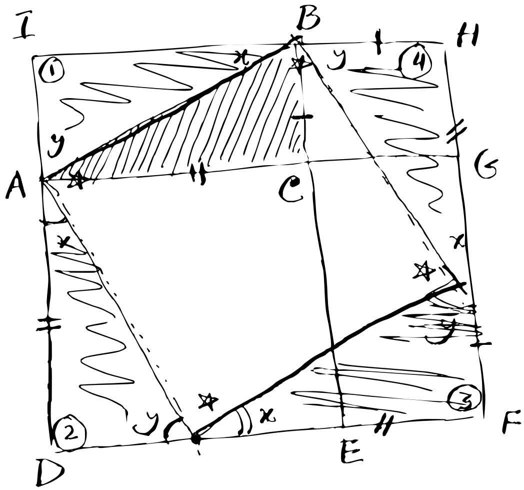
$4\pi r^2 \times \frac{1}{3} r$

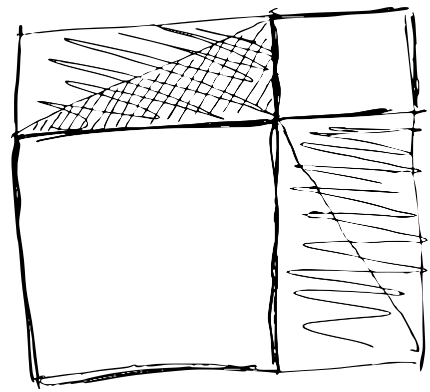
$\frac{4}{3} \pi r^3$

$\frac{1}{6} \overset{a^3}{h \cdot b}$

$\frac{1}{6} 2h b$

$\frac{1}{3} h b$



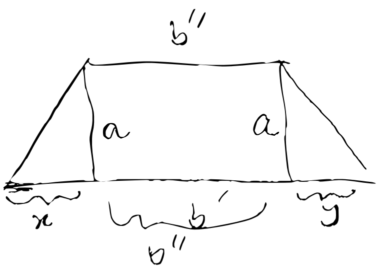


Basic idea is :

If we remove 4 triangles 2 different ways

- in one case we are left with square on hypotenuse
- in other case we are left with BCGH + ADEC

So Pythagoras: $AB^2 = AC^2 + BC^2$



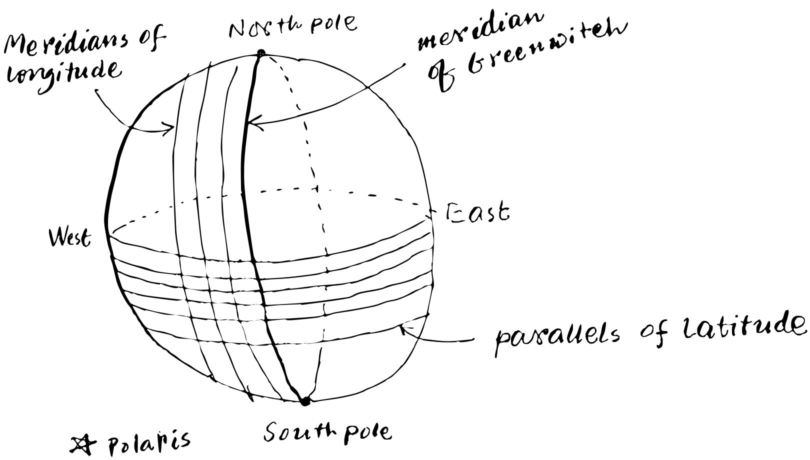
$$\frac{1}{2} ax + \frac{1}{2} ay + b'' \cdot a$$

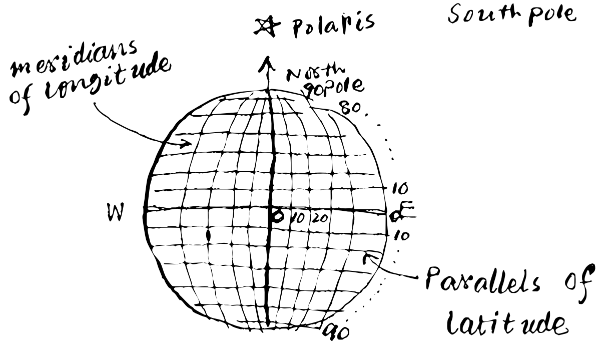
$$\frac{1}{2} a(x+y) + a \cdot b''$$

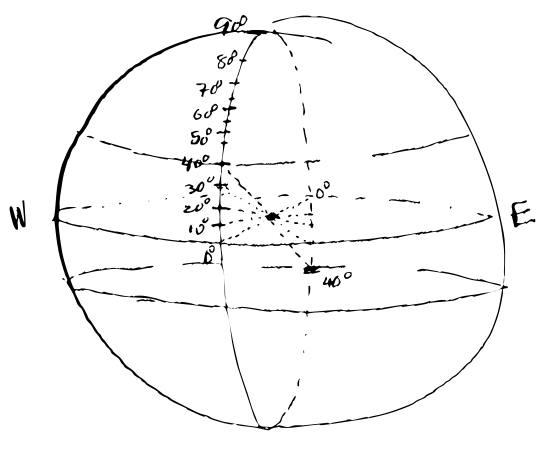
$$\frac{1}{2} a\,(x+y+2b'')$$

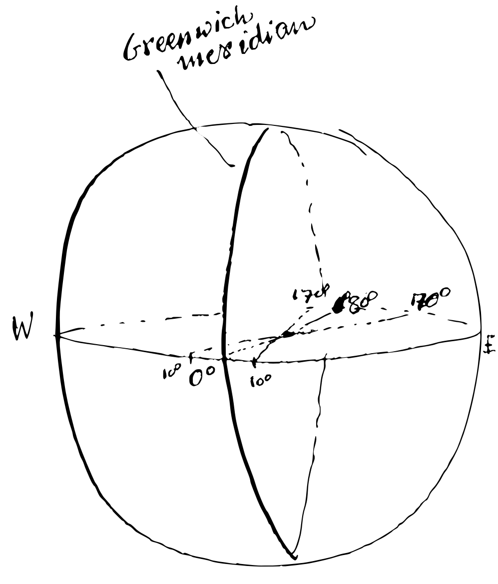
$$= \frac{1}{2} a(x+y+b''+b'')$$

$$= \frac{1}{2} a(b'+b'') \quad \square$$









From equator ↕ lat ↔ long

+

lat, long, + N/S of equator, + W/E of prime m.

−1