1 A degree 2 polynomial seems to have three distinct roots, how?

Consider the below identity:

$$\frac{(x-a)(x-b)}{(c-a)(c-b)}+\frac{(x-a)(x-c)}{(b-a)(b-c)}+\frac{(x-b)(x-c)}{(a-b)(a-c)}=1$$

We can rearrange:

$$\frac{(x-a)(x-b)}{(c-a)(c-b)}+\frac{(x-a)(x-c)}{(b-a)(b-c)}+\frac{(x-b)(x-c)}{(a-b)(a-c)}-1=0$$

Now the left side of the above can be considered as a polynomial $P(x)$ with degree at most 2.

$$P(x)=\frac{(x-a)(x-b)}{(c-a)(c-b)}+\frac{(x-a)(x-c)}{(b-a)(b-c)}+\frac{(x-b)(x-c)}{(a-b)(a-c)}-1$$

However, a, b, c look like three distinct roots of $P(x)$. **How?**