Problem-225

Problem Statement

If in the equation

$$ax^2 + bx + c = 0$$

the coefficient a is equal to zero then the equation takes the form

$$bx + c = 0$$

and has the unique solution

$$x = -\frac{c}{b}$$

Strictly speaking, the last sentence is wrong; when b=0 the quotient $\frac{c}{b}$ is undefined. How are we to correct this error?

Solution

We should require that in the equation $ax^2 + bx + c = 0$, both a and b cannot be zero.

Problem-227

Problem Statement

Prove that $\sqrt{3}$ is irrational.

Solution

The statement ' $\sqrt{3}$ is irrational' is equivalent to the statement that $\sqrt{3} \neq \frac{a}{b}$ for any integer a and b. We shall make use of **proof by contradiction**, therefore, we shall assume that $\sqrt{3} = \frac{a}{b}$ and that should lead to a contradiction, completing the proof.

We shall consider four different cases for the pair (a,b) and we shall derive a contradiction for each.

1. a and b both are odd integer. Say a = 2m + 1 and b = 2n + 1, where m and n are integers. According to our assumption

$$\sqrt{3} = \frac{2m+1}{2n+1}$$

$$3 = \frac{(2m+1)^2}{(2n+1)^2}$$

$$3 (2n+1)^2 = (2m+1)^2$$

$$3 (4n^2+4n+1) = 4m^2+4m+1$$

$$12n^2+12n+3 = 4m^2+4m+1$$

$$12n^2+12n+2 = 4m^2+4m$$

$$6n^2+6n+1 = 2m^2+2m$$

On the left side we have an odd number and on the right side we have an even number—a contradiction. *

2. a is an odd integer and b is an even integer. Say a = 2m + 1 and b = 2n, where m and n are integers. According to our assumption

$$\sqrt{3} = \frac{2m+1}{2n}$$
$$3 = \frac{(2m+1)^2}{4n^2}$$
$$2n^2 = (2m+1)^2$$

On the left side we have an even integer and on the right side we have an odd integer—a contradiction. *

3. a is an even integer and b is an odd integer. Say a = 2m and b = 2n + 1, where m and n are integers. According to our assumption

$$\sqrt{3} = \frac{2m}{2n+1}$$
$$3 = \frac{4m^2}{(2n+1)^2}$$
$$3 (2n+1)^2 = 4m^2$$

On the left side we have an odd integer and on the right side we have an even integer—a contradiction. *

4. a and b both are even integer. In this case, a and b must have 2 as a common factor. We can divide both a and b by 2. We can keep dividing by 2 as long as both remain even integer. At the end, we are in one of the previous three cases.