Problem 1

(a) Programming Assignment.

(b) Programming Assignment.

Problem 2

(a) Programming Assignment.

(b) Programming Assignment.

(c) Both the Likelihood weighting and particle filtering algorithms were run repeatedly for 100 times for each given *numsamples/numparticles* value and for each evidence sequence. The mean and the variance over the 100 different outputs of the probability estimate $P(R_{10}|u_{1:10})$ were calculated. The results were as follows:

Likelihood weighting :

For number of samples = 100 :

Evidence Sequence 1 : T,T,T,T,T,F,F,F,F,F
Filtering probability of $P(r10|u1:10)$ is : 0.056163
Mean LW estimate of $P(r10|u1:10)$ after 100 repeated runs: 0.064745
Variance after 100 repeated runs of likelihood weighting : 0.003829

Evidence Sequence 2 : F,F,F,F,F,F,F,T,T,T
Filtering probability of $P(r10|u1:10)$ is : 0.890216
Mean LW estimate of $P(r10|u1:10)$ after 100 repeated runs: 0.844464
Variance after 100 repeated runs of likelihood weighting : 0.023124

Evidence Sequence 3 : F,T,F,T,F,T,F,T,F,T
Filtering probability of $P(r10|u1:10)$ is : 0.717086
Mean LW estimate of $P(r10|u1:10)$ after 100 repeated runs is: 0.693756
Variance after 100 repeated runs of likelihood weighting : 0.0218256

For number of samples = 1000 :

Evidence Sequence 1 : T,T,T,T,T,F,F,F,F,F
Filtering probability of $P(r10|u1:10)$ is : 0.056163
Mean LW estimate of $P(r10|u1:10)$ after 100 repeated runs is: 0.0576702

Variance after 100 repeated runs of likelihood weighting : 0.0003094

Evidence Sequence 2 : F,F,F,F,F,F,F,T,T,T

Filtering probability of P($r10|u1:10$) is : 0.890216
Mean LW estimate of P($r10|u1:10$) after 100 repeated runs is: 0.886153
Variance after 100 repeated runs of likelihood weighting : 0.001320

Evidence Sequence 3 : F,T,F,T,F,T,F,T,F,T

Filtering probability of P($r10|u1:10$) is : 0.717086
Mean LW estimate of P($r10|u1:10$) after 100 repeated runs is: 0.707135
Variance after 100 repeated runs of likelihood weighting : 0.003453

Particle Filtering :

For number of particles = 100 :

Evidence Sequence 1 : T,T,T,T,T,F,F,F,F,F

Filtering probability of P($r10|u1:10$) is : 0.056163
Mean PF estimate of P($r10|u1:10$) after 100 repeated runs: 0.0554
Variance after 100 repeated runs of particle filtering : 0.00060084

Evidence Sequence 2 : F,F,F,F,F,F,F,T,T,T

Filtering probability of P($r10|u1:10$) is : 0.890216
Mean PF estimate of P($r10|u1:10$) after 100 repeated runs: 0.8918
Variance after 100 repeated runs of likelihood weighting : 0.001636

Evidence Sequence 3 : F,T,F,T,F,T,F,T,F,T

Filtering probability of P($r10|u1:10$) is : 0.717086
Mean PF estimate of P($r10|u1:10$) after 100 repeated runs is: 0.7195
Variance after 100 repeated runs of likelihood weighting : 0.003794

For number of particles = 1000 :

Evidence Sequence 1 : T,T,T,T,T,F,F,F,F,F

Filtering probability of P($r10|u1:10$) is : 0.056163
Mean PF estimate of P($r10|u1:10$) after 100 repeated runs is: 0.055910
Variance after 100 repeated runs of likelihood weighting : 6.57419e-05

Evidence Sequence 2 : F,F,F,F,F,F,F,T,T,T

Filtering probability of P($r10|u1:10$) is : 0.890216
Mean PF estimate of P($r10|u1:10$) after 100 repeated runs is: 0.89042
Variance after 100 repeated runs of likelihood weighting : 0.0001463

Evidence Sequence 3 : F,T,F,T,F,T,F,T,F,T
Filtering probability of P($r10|u1:10$) is : 0.717086
Mean PF estimate of P($r10|u1:10$) after 100 repeated runs is: 0.71553
Variance after 100 repeated runs of likelihood weighting : 0.0004513

We can see from the above outputs that particle filtering performs much better than likelihood weighting in estimating the probability of P($r10|u1:10$) for a given number of samples/particles. The variances are also much smaller for particle filtering than likelihood weighting. The probability estimate of particle filtering becomes even better as we increase the number of particles from 100 to 1000. Similarly, the performance of likelihood weighting also improves as we increase the number of samples from 100 to 1000. But in general the performance of particle filtering algorithm is superior in estimating the probability of P($r10|u1:10$) than likelihood weighting.

Problem 3

(a) Given , Value of Perfect information: (VPI) VPI$_E(Ej)$

To show, VPI is non negative : VPI$_E(Ej) \geq 0$, $\forall$ j, E.

Lets denote the possible values of E$_j$ as e$_{jk}$. From the definition of VPI$_E(Ej)$, we have :

$$\text{VPI}_E(Ej) = (\textstyle\sum_k P(E_j = e_{jk}|E) EU(\alpha_{e_{jk}}|E, E_j = e_{jk})) - EU(\alpha|E)$$

$$= (\textstyle\sum_k P(E_j = e_{jk}|E) max_a \sum_i U(S_i) P(S_i|E, a, E_j = e_{jk})) - max_a \sum_i U(S_i) P(S_i|E, a))$$

Suppose the best action that maximizes EU($\alpha|E$) is a^*

$$\text{VPI}_E(Ej) = (\textstyle\sum_k P(E_j = e_{jk}|E) max_a \sum_i U(S_i) P(S_i|E, a, E_j = e_{jk})) - \sum_i U(S_i) P(S_i|E, a^*))$$

$= (\sum_k P(E_j = e_{jk}|E) max_a \sum_i U(S_i) P(S_i|E, a, E_j = e_{jk})) - \sum_i U(S_i) \sum_k P(S_i|E, a^*, E_j = e_{jk}) P(E_j = e_{jk}|E)$

(Using marginal distribution)

$= \sum_k P(E_j = e_{jk}|E)(max_a \sum_i U(S_i) P(S_i|E, a, E_j = e_{jk}) - \sum_i U(S_i) P(S_i|E, a^*, E_j =$

$e_{jk}))$

Term $\sum_k P(E_j = e_{jk}|E)$ is positive. Hence we need to show that :

$$max_a \sum_i U(S_i)P(S_i|E, a, E_j = e_{jk}) \geq \sum_i U(S_i)P(S_i|E, a^*, E_j = e_{jk})$$

Given the fact, $\sum_i y_i max_{x_i} f_i(x_i) \geq max_x \sum_i y_i f_i(x)$ for any y_i and f_i

Using this:

$$max_a \sum_i U(S_i)P(S_i|E, a, E_j = e_{jk}) \geq \sum_i U(S_i)P(S_i|E, a^*, E_j = e_{jk})$$

Hence we can say that Value of Perfect Information is non negative.

(b) Given , Value of Perfect information: (VPI) $\text{VPI}_E(Ej, Ek)$

To show, VPI is order independent : $\textbf{VPI}_E(Ej, Ek) = \textbf{VPI}_E(Ek, Ej)$, $\forall$ j,k,E.

We want to show that:

$$\text{VPI}_E(Ej, Ek) = \text{VPI}_E(Ej) + \text{VPI}_{E,e_j}(Ek) = \text{VPI}_E(Ek) + \text{VPI}_{E,e_k}(Ej)$$

Lets denote the possible values of E_j as e_{jp} and the possible values of E_k as e_{kq}, we have :

$\text{VPI}_E(Ej) + \text{VPI}_{E,e_j}(Ek) = (\sum_p P(E_j = e_{jp}|E)EU(\alpha_{e_{jp}}|E, E_j = e_{jp})) - EU(\alpha|E) + (\sum_q \sum_p P(E_k = e_{kq}|E, E_j = e_{jp})EU(\alpha_{e_{kq}}|E, E_k = e_{kq}, E_j = e_{jp})) - EU(\alpha|E, E_j = e_{jp})$

$= (\sum_p P(E_j = e_{jp}|E)max_a \sum_i U(S_i)P(S_i|E, a, E_j = e_{jp})) - max_a \sum_i U(S_i)P(S_i|E, a)) + (\sum_q \sum_p P(E_k = e_{kq}|E, E_j = e_{jp})max_a \sum_i U(S_i)P(S_i|E, a, E_k = e_{kq}, E_j = e_{jp}))$
$- max_a \sum_i U(S_i)P(S_i|E, a, E_j = e_{jp}))$

Similarly, we can write $\text{VPI}_E(Ek) + \text{VPI}_{E,e_k}(Ej)$ as :

$\text{VPI}_E(Ek) + \text{VPI}_{E,e_k}(Ej) = (\sum_q P(E_k = e_{kq}|E)EU(\alpha_{e_{kq}}|E, E_k = e_{kq})) - EU(\alpha|E) + (\sum_p \sum_q P(E_j = e_{jp}|E, E_k = e_{kq})EU(\alpha_{e_{jp}}|E, E_j = e_{jp}, E_k = e_{kq})) - EU(\alpha|E, E_k = e_{kq})$

$= (\sum_q P(E_k = e_{kq}|E)max_a \sum_i U(S_i)P(S_i|E, a, E_k = e_{kq})) - max_a \sum_i U(S_i)P(S_i|E, a)) + (\sum_p \sum_q P(E_j = e_{jp}|E, E_k = e_{kq})max_a \sum_i U(S_i)P(S_i|E, a, E_j = e_{jp}, E_k = e_{kq}))$
$- max_a \sum_i U(S_i)P(S_i|E, a, E_k = e_{kq}))$

When the order of summation in this equation is switched we get :

$= (\sum_p P(E_j = e_{jp}|E)max_a \sum_i U(S_i)P(S_i|E, a, E_j = e_{jp})) - max_a \sum_i U(S_i)P(S_i|E, a)) +$

$(\sum_q \sum_p P(E_k = e_{kq}|E, E_j = e_{jp}) max_a \sum_i U(S_i) P(S_i|E, a, E_k = e_{kq}, E_j = e_{jp}))$
$- \, max_a \sum_i U(S_i) P(S_i|E, a, E_j = e_{jp}))$

which is the same as $\text{VPI}_E(Ej)$ + $\text{VPI}_{E,e_j}(Ek)$

Thus, $\text{VPI}_E(Ej, Ek) = \text{VPI}_E(Ej) + \text{VPI}_{E,e_j}(Ek) = \text{VPI}_E(Ek) + \text{VPI}_{E,e_k}(Ej)$

This shows that VPI is order independent.