CSCI 5521 - INTRO TO MACHINE LEARNING

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ASSIGNMENT 4

NOVEMBER 22, 2019

PROBLEM 1:

INTRO TO MACHINE LEARNING

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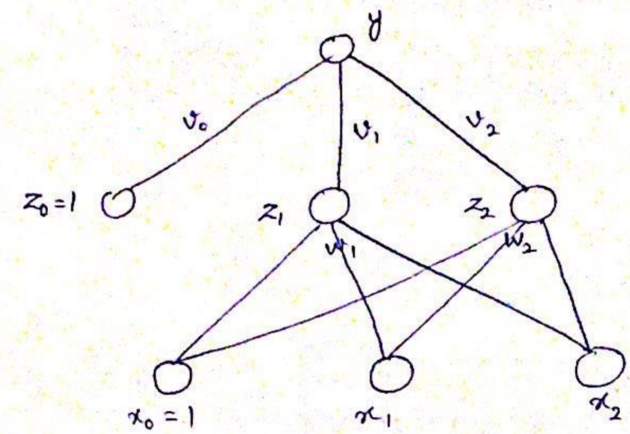
PROBLEM 1

SOLUTION:



Given, Error function

$$E(w_1, w_2, v \mid x) = -\sum_t r^t \log y^t + (1 - r^t)\log(1 - y^t)$$

$$y^t = \text{sigmoid}(v_2 * z_2 + v_1 * z_1 + v_0)$$

$$z_1^t = \tanh(w_{1,2}\, x_2^t + w_{1,1}\, x_1^t + w_{1,0})$$

$$z_2^t = \text{LRELU}(w_{2,2}\, x_2^t + w_{2,1}\, x_1^t + w_{2,0})$$

a) For updating v_i, we need $\frac{\partial E}{\partial v_i}$ and for that we need $\frac{\partial E}{\partial y^t}$

since,
$$\frac{\partial E}{\partial v_i} = \frac{\partial E}{\partial y^t} \cdot \frac{dy^t}{\partial v_i}$$

$$E(w_1, w_2, v \mid X) = -\sum_t \left(r^t \log y^t + (1 - r^t)\log(1 - y^t)\right)$$

$$\frac{\partial E}{\partial y^t} = -\sum_t \left(\frac{r^t}{y^t} - \frac{(1 - r^t)}{(1 - y^t)}\right)$$

$$\frac{\partial E}{\partial y^t} = -\Sigma_t \left(\frac{(1-y^t)r^t - y^t(1-r^t)}{y^t(1-y^t)} \right) = -\Sigma_t \left(\frac{r^t - y^t}{y^t(1-y^t)} \right)$$

$y^t = \text{sigmoid}\,(v_2 * z_2 + v_1 * z_1 + v_0)$

Assume $v_2 * z_2 + v_1 * z_1 + v_0 = h^t$

Then,

$$\frac{\partial E}{\partial v_i} = \frac{\partial E}{\partial y^t} \frac{\partial y^t}{\partial h^t} \frac{\partial h^t}{\partial v_i}$$

Since derivative of sigmoid $\frac{d\,sig(x)}{dx} = sig(x)\,(1 - sig\,x)$

$$\therefore \frac{\partial E}{\partial v_1} = -\Sigma_t \left(\frac{r^t - y^t}{y^t(1-y^t)} \right) y^t(1-y^t)\, z_1^t \qquad \left(\frac{\partial h^t}{\partial v_1} = z_1^t \right)$$

$$\frac{\partial E}{\partial v_1} = -\Sigma_t \, (r^t - y^t)\, z_1^t$$

Hence, the update to v_1:

$$\boxed{\Delta v_1 \quad -\eta \; \Sigma_t \, (r^t - y^t)\, z_1^t}$$

(η : learning rate)

Similarly, the update to v_2:

$$\boxed{\Delta v_2 = -\eta \; \Sigma_t \, (r^t - y^t)\, z_2^t} \qquad \left(\frac{\partial h^t}{\partial v_2} = z_2^t \right)$$

& the update to v_0:

$$\boxed{\Delta v_0 = -\eta \; \Sigma_t \, (r^t - y^t)} \qquad \left(\text{since } \frac{\partial h^t}{\partial v_0} = 1 \right)$$

For update to w_1 and w_2 :-

We have to calculate $\frac{\partial E}{\partial w_{10}}$, $\frac{\partial E}{\partial w_{11}}$ and $\frac{\partial E}{\partial w_{12}}$ for w_1 as well as $\frac{\partial E}{\partial w_{20}}$, $\frac{\partial E}{\partial w_{21}}$, $\frac{\partial E}{\partial w_{22}}$ for w_2.

<u>w_1:</u>

$$\frac{\partial E}{\partial w_{10}} = \frac{\partial E}{\partial y^t} \cdot \frac{\partial y^t}{\partial h^t} \cdot \frac{\partial h^t}{\partial z_1^t} \frac{\partial z_1^t}{\partial g^t} \cdot \frac{\partial g^t}{\partial w_{10}}$$

where,

$$y^t = \text{sigmoid}\,(v_2 * z_2 + v_1 * z_1 + v_0)$$
$$h^t = (v_2 * z_2 + v_1 * z_1 + v_0)$$
$$z_1^t = \tanh(w_{12} x_2^t + w_{11} x_1^t + w_{10})$$
$$g^t = (w_{12} x_2^t + w_{11} x_1^t + w_{10})$$

$$\frac{\partial E}{\partial w_{10}} = -\Sigma_t \left(\frac{r^t - y^t}{y^t(1-y^t)}\right) \cdot y^t(1-y^t) \cdot v_1 \frac{\partial z_1^t}{\partial g^t} \cdot \frac{\partial g^t}{\partial w_{10}}$$

$$\frac{\partial E}{\partial w_{10}} = -\Sigma_t (r^t - y^t) v_1 \cdot (1 - (z_1^t)^2) \cdot 1$$

$$\left(\text{since } \frac{\partial \tanh(x)}{\partial x} = 1 - \tanh^2(x) \text{ \& } \frac{\partial g^t}{\partial w_{10}} = 1\right)$$

Hence the update to w_{10}:

$$\boxed{\Delta w_{10} = -\eta\, \Sigma_t (r^t - y^t)\, v_1 \,(1 - z_1{t^2})}$$

Similarly the update to w_{11}:

$$\boxed{\Delta w_{11} = -\eta\, \Sigma_t (r^t - y^t)\, v_1 \,(1 - z_1{t^2})\, x_1^t} \quad \left(\text{since } \frac{\partial g^t}{\partial w_{11}} = x_1^t\right)$$

Similarly the update to w_{12}:

$$\boxed{\Delta w_{12} = -\eta\, \Sigma_t (r^t - y^t)\, v_1 \,(1 - z_1{t^2})\, x_2^t} \quad \left(\text{since } \frac{\partial g^t}{\partial w_{12}} = x_2^t\right)$$

$\underline{W_2}$:

$$\frac{\partial E}{\partial w_{20}} = \frac{\partial E}{\partial y^t} \cdot \frac{\partial y^t}{\partial h^t} \cdot \frac{\partial h^t}{\partial z_2^t} \cdot \frac{\partial z_2^t}{\partial g^t} \cdot \frac{\partial g^t}{\partial w_{20}}$$

where,

$$y^t = \text{sigmoid}\,(v_2 * z_2 + v_1 * z_1 + v_0)$$
$$h^t = (v_2 * z_2 + v_1 * z_1 + v_0)$$
$$z_2^t = \text{LRELU}\,(w_{22} x_2^t + w_{21} x_1^t + w_{20})$$
$$g^t = w_{22} x_2^t + w_{21} x_1^t + w_{20}$$

$$\text{LRELU}(x) = \begin{cases} 0.01x & \text{for } x < 0 \\ x & \text{otherwise} \end{cases} \qquad \text{LRELU}'(x) = \begin{cases} 0.01 & \text{for } x < 0 \\ 1 & \text{otherwise} \end{cases}$$

$$\frac{\partial E}{\partial w_{20}} = -\sum_t \left(\frac{r^t - y^t}{y^t(1-y^t)}\right) y^t(1-y^t) \cdot v_2 \, \frac{\partial z_2^t}{\partial g^t} \cdot \frac{\partial g^t}{\partial w_{20}} \qquad \left(\frac{\partial g^t}{\partial w_{20}} = 1\right)$$

$$\frac{\partial E}{\partial w_{20}} = \begin{cases} -\sum_t (r^t - y^t) \cdot v_2 \cdot (0.01) \cdot 1 & \text{when } g^t < 0 \\ -\sum_t (r^t - y^t) \cdot v_2 \cdot 1 \cdot 1 & \text{when } g^t > 0 \end{cases}$$

Hence the update to w_{20}:

$$\boxed{\Delta w_{20} = \begin{cases} -\eta \sum_t (r^t - y^t)\, v_2 \times (0.01) & \text{for } g^t < 0 \\ -\eta \sum_t (r^t - y^t)\, v_2 & \text{for } g^t > 0 \end{cases}}$$

Similarly the update to w_{21}:

$$\boxed{\Delta w_{21} = \begin{cases} -\eta \sum_t (r^t - y^t)\, v_2 \times (0.01)\, x_1^t & \text{for } g^t < 0 \\ -\eta \sum_t (r^t - y^t)\, v_2\, x_1^t & \text{for } g^t > 0 \end{cases}} \qquad \left(\frac{\partial g^t}{\partial w_{21}} = x_1^t\right)$$

Also update to w_{22}:

$$\boxed{\Delta w_{22} = \begin{cases} -\eta \sum_t (r^t - y^t)\, v_2 \times (0.01)\, x_2^t & \text{for } g^t < 0 \\ -\eta \sum_t (r^t - y^t)\, v_2\, x_2^t & \text{for } g^t > 0 \end{cases}} \qquad \left(\frac{\partial g^t}{\partial w_{22}} = x_2^t\right)$$

b) When $w = w_1 = w_2$.

The equations for updating remains unchanged

Thus,

$$\boxed{\begin{aligned} \Delta v_1 &= -\eta \textstyle\sum_t (r^t - y^t) z_1^t \\ \Delta v_2 &= -\eta \textstyle\sum_t (r^t - y^t) z_2^t \\ \Delta v_0 &= -\eta \textstyle\sum_t (r^t - y^t) \end{aligned}}$$

Now since $w = w_1 = w_2$,

$\Rightarrow \quad w_{10} = w_{20}, \quad w_{11} = w_{21}, \quad w_{12} = w_{22}$

Hence when we update values from the previous calculation, we are updating the same value twice. So we need to take an average of the two updates.

Therefore,

$$\boxed{\begin{aligned} \Delta w_0 &= \frac{\Delta w_{10} + \Delta w_{20}}{2} \\ \Delta w_1 &= \frac{\Delta w_{11} + \Delta w_{21}}{2} \\ \Delta w_2 &= \frac{\Delta w_{12} + \Delta w_{22}}{2} \end{aligned}}$$

PROBLEM 2 :

SOLUTION :

H	TRAINING ERROR RATE	VALIDATION ERROR RATE
3	0.615	0.631
6	0.462	0.496
9	0.444	0.502
12	0.207	0.214
15	0.061	0.0938
18	0.0344	0.0569

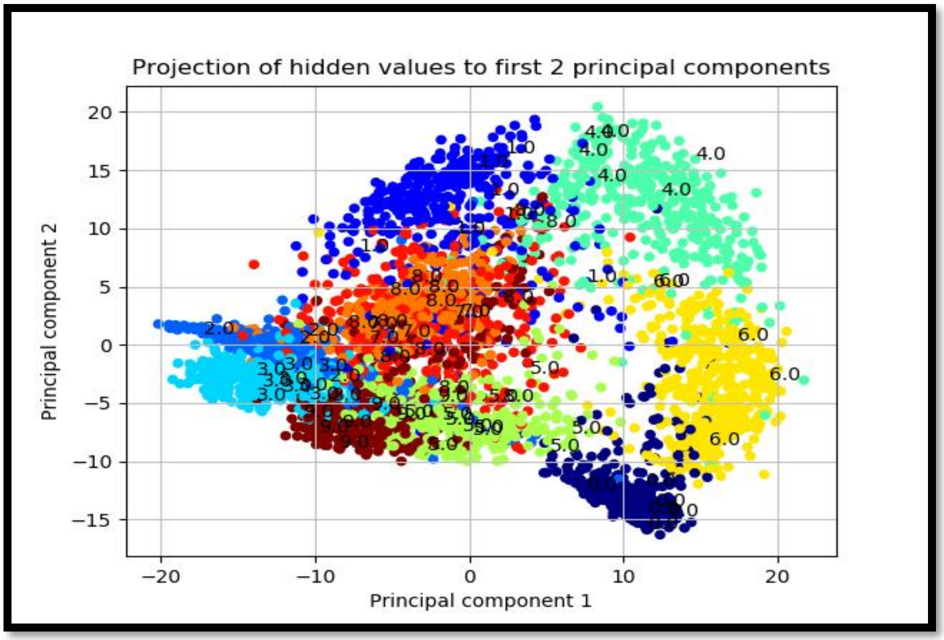
a) Number of hidden units(H) which give the lowest training and validation error rates: 18

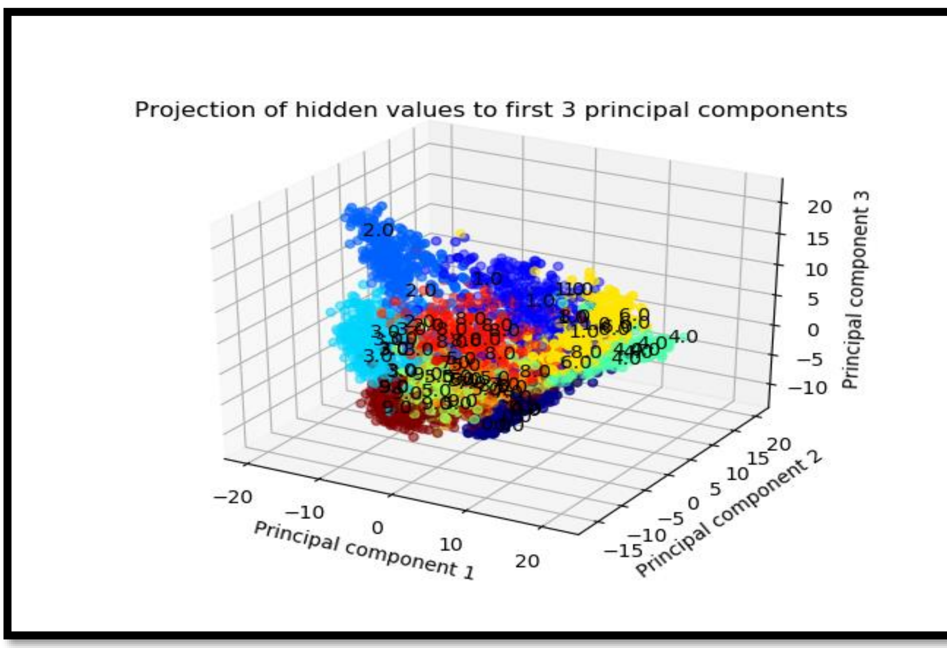
Test set error rate with 18 hidden units : 0.0746

Plot of the training and validation error rates by the number of hidden units(H) :

Because of random initialized weight matrices W and V, the error rates mentioned and the error rate observed after running the scripts may differ.

b) Visualization of the hidden units by projecting them to 2D and 3D using PCA :





The 3D plot offers better visualization than the 2D plot because it can be rotated along any of the 3 axes which makes the distinctions between the classes more recognizable. In the 2D plot we can see a lot of overlap between the classes and the distinctions can't be seen by rotating the axes as in the case of the 3D plot.

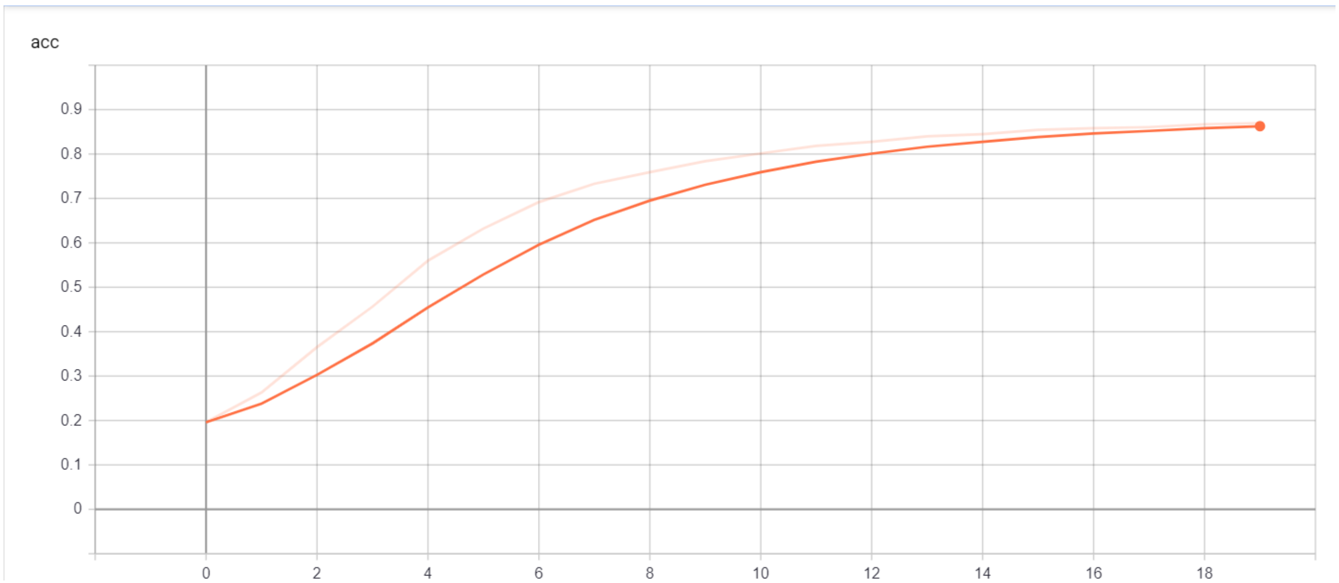
PROBLEM 3:

SOLUTION:

First model accuracy : 86.01%

Second moddel accuracy : 97.33 %

Accuracy plot for model 1 :



Accuracy plot for model 2 :