

Rao

Comilla University
Department of Statistics

3rd Year 2nd Semester B.Sc. (Hon's) Final Examination-2023

Course Title: Statistical Inference-II, Course Code: Stat-321

Session: 2019-2020

Full Marks: 60

Time: 3 hours

Answer any five (05) of the following questions

- ✓ 1. a) What do you mean by point estimation? When would you say that estimate of a parameter is good? In particular, discuss the requirements of consistency and unbiasedness of an estimate. 06
- b) If X_1, \dots, X_n be random sample from the normal density with mean μ and variance unity: Find the sufficient statistic for μ and σ^2 . 06
2. a) What do you mean by minimum variance unbiased estimator? Explain the important properties that a minimum variance unbiased estimator must satisfy? 04
- b) Let, X_1, \dots, X_n be iid $G(1, \frac{1}{\alpha})$ random variables. Show that the estimator $T = \frac{(X_1 + \dots + X_n) - \frac{(n-1)}{n}\bar{X}}{\frac{\alpha^2}{n}}$ is the UMVUE for α with variance $\frac{\alpha^2}{(n-2)}$. 05
- c) State the Bhattacharya inequality. 3
- ✓ 3. a) Define Shrinkage estimator and James-Stein estimator with calculation procedure. 05
- b) Show that $1 - \beta \geq \alpha$. 04
- c) Write down the different types of loss function used in point estimation. 03
- ✓ 4. a) Define minimal sufficient statistic and ancillary statistic. State and proof Rao-Cramer inequality with applications. 06
- b) Distinguish between i) Unbiased and biased estimators; ii) Sufficient and minimal sufficient statistic; iii) Estimator and minimax estimator. 06
5. a) Define confidence interval and confidence co-efficient. What is the necessity of interval estimation? 04

- b) What do you mean by pivotal quantity? Write its importance in determination of 04
 confidence interval Determine a confidence interval for the mean of a normal distribution when its variance is unknown. 04
- c) State and prove Lehman-Scheffe theorem. 04
6. a) Define the likelihood ratio test (LRT). Write down the properties of LRT test 05
 statistics. Derive the LRT testing procedure for testing equality of means of two normal populations.
- b) Let, X_1, \dots, X_n be a random sample from $f(x; \theta) = \theta e^{-\theta x} I(0, \infty)(x)$ where, $\bar{\theta} = \frac{1}{n} \sum_{i=1}^n x_i$. Test $H_0: \theta \leq \theta_0$ against $H_1: \theta > \theta_0$ using LRT. 07
- ✓ a) Define the following terms: 06
 i) Test statistic, ii) Type-I error iii) Level of significance iv) Simple hypothesis v)
 Composite hypothesis vi) P-value.
- b) The managing director of a firm claims that his firm produces 110 items on average 06
 daily. A random sample of 15 days gives the following data set:
 $110, 118, 130, 140, 142, 146, 112, 100, 95, 98, 96, 122, 123, 124, 130$.
 It is known that the number of items produced by the firm follows normal distribution with variance 300.
 Can we conclude at 5% level of significance that the average daily production of items of that firm is 110 items?
8. a) What is Sequential Probability Ratio Test (SPRT)? Write down the steps of SPRT. 04
 b) State and prove Wald's equation. Determine the expected sample size i) if the null 05
 hypothesis is true ii) if the alternative hypothesis is true. Give an example.
- c) What do you mean by Operating Characteristics function? 03

Comilla University

Department of Statistics

3rd Year 2nd Semester B. Sc (Hon's) Final Examination- 2022

Course Title: Order Statistics and Nonparametric Methods

Session-2019-2020 Course Code : Stat-322

Total Marks 60

Time 3Hours

Answer Any Five of the Following Questions.

- 1✓ a) Define order statistics. Mention some of its importance and applications in practical field. 3

- b) State the properties of order statistics and show that, the marginal distribution of the r^{th} order statistics is 4

$$f_{r:n}(x) = \frac{n!}{(r-1)!(n-r)!} [F(x)]^{r-1} [1 - F(x)]^{n-r} f(x)$$

- c) Let $x_1, x_2, x_3, \dots, x_n$ be a random sample from $f(x) = 1, 0 < x < 1$, find 5
(1) $f(x_{(1)})$ (2) $f((x_{(n)}))$. Also the mean and variance of (1) $x_{(1)}$ and (2) $x_{(n)}$

2. a) Let $x_{(1)} = y_1, x_{(2)} = y_2, x_{(3)} = y_3, \dots, x_{(n)} = y_n$, represent the order statistics of the 4
random sample (x_1, x_2, \dots, x_n) from a cdf $F(x)$. Show that, the marginal cdf of r^{th} order
statistics $x_{(r)} = y_r$ is given by,

$$F_r(y) = \sum_{j=r}^n \binom{n}{j} [F(y)]^j [1 - F(y)]^{n-j}$$

- b) Let $Y_1 < Y_2 < Y_3 < Y_4$ be an order statistics of a random sample of size 4 from a 5
distribution having pdf,

$$f(x) = \begin{cases} 2x, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

Find the probability $p(Y_3)$

- c) If $Y_1 \leq Y_2 \leq Y_3$ are the order statistics, find the correlation 3
between Y_2 and Y_3

3. a) Find the joint distribution of three order statistics. 4

- b) Let Let $x_1, x_2, x_3, \dots, x_n$ be a random sample from $f(x) = \frac{1}{\theta}, 0 \leq x \leq \theta$. Find the 4
distribution of a) maximum order statistics b) Range

- c) Also find the joint distribution of 5th order statistics. 4

- 4✓ a) What do you mean by goodness of fit? Mention the condition for the validity of 4
chi-square goodness of fit test.

- b) Define and derive the Kolmogorov-Smirnov (k-s) goodness of fit test 5

- c) Distinguish between chi-square and Kolmogorov- Smirnov goodness of fit test 3
with their uses.

5. a) Define Non-parametric test with their assumptions. Also write down the difference between parametric and non-parametric test. 3
- b) Define and derive Empirical distribution function with usual notations 4
- c) Show that, $S_n(x)$ is a consistent estimator of $F_x(x)$ 5
6. a) Let R_1 and R_2 denote the respective number of runs of n_1 objects of type - 1 and n_2 objects of type - 2 in a random sample of size $n = n_1 + n_2$. Show that the joint probability distribution of R_1 and R_2 is, 4
- $$f_{(R_1 R_2)}(r_1 r_2) = \frac{c \binom{n_1-1}{r_1-1} \binom{n_2-1}{r_2-1}}{\binom{n_1+n_2}{n_1}} ; r_1 = 1, 2, \dots, n_1 \text{ and } r_2 = 1, 2, \dots, n_2$$
- Where $c=2$ when $r_1 = r_2$ and $c=1$ when $r_1 = r_2 \pm 1$
- b) Derive the single sample sign test. How sign test is carried out in case of large sample? 5
- c) Distinguish between sign test and Wilcoxon signed-rank test. 3
- ✓ a) Derive the distribution of median for both odd and even sample size. 6
- b) Suppose that a sample of size n (n is odd) drawn from a standard population. Find the m th moment of sample median, also find the mean and variance of it. 6
8. a) Why we prefer to use Mann-whitney U test instead of Kruskal-walli's test? 3
- b) Distinguish between Mann-whitney U test and t-test. 3
- c) Derive Mann-whitney U-test. derive the confidence interval for this test. 6

Comilla University

Department of Statistics

3rd Year 2nd Semester B.Sc. (Hon's) Final Examination-2022

Course Title: Linear Programming and Operation Research

Course Code: Stat-323, Session: 2019-2020

Full Marks: 60

Time: 3 hours

Answer any five (05) of the following questions

1. a) What is operation research? Discuss the application and scope of operation research in modern management. Also, discuss the characteristics of significant features of operation research. 4
- b) What are the different types of variables used in operations research? Briefly discuss the scientific methodology of operation research. 4
- c) What are the different types of models used in operation research? Discuss the application of mathematical, languages, and concrete models in operation research. 4
- ✓ a) Define general linear programming problem. Discuss some objectives of linear programming problem. 4
- b) A company sells two different products A and B. The company makes a profit of Rs. 40 and Rs. 30 per unit on products A and B respectively. The two products are produced in a common production process and are sold in two different markets. The production process has a capacity of 30000 man hours. It takes 3 hours to produce one unit of A and one hour to produce one unit of B. The market has been surveyed, and company officials feel that the maximum number of units of A that can be sold is 8000 and the maximum of B is 12000 units. Subject to these limitations, the products can be sold in any convex combination. Formulate the problem as an Linear programming problem. 5
- c) What are advantages and applications of linear programming problem? 3
- ✓ a) (Discuss types of graphical methods for solving linear programming problems) Show that the set of feasible solution of linear programming problem in a convex set. 4
- b) A manufacture produces two types of models M_1 and M_2 . Each M_1 model requires 4 hours of grinding and 2 hours of polishing; whereas each M_2 model requires 2 hours of grinding and 5 hours of polishing. The manufacturer has 2 grinders and 3 polishers. Each grinder works for 40 hours a week and each polisher works 60 hours a week. Profit on a M_1 model is 3 tk and each M_2 model is 4 Tk. Whatever is produced in a week is sold in market. How should the manufacturer allocate his production capacity to the two types of models so that he may make the maximum profit in a week? Solve the linear programming problem by graphical method. 4

- c) Rewrite the following linear programming problem in the standard form:

4

$$\text{Minimize } z = 2x_1 + x_2 + 4x_3$$

Subject to constraints:

$$-2x_1 + 4x_2 \leq 4$$

$$x_1 + 2x_2 + x_3 \geq 5$$

$$2x_1 + 3x_3 \leq 2$$

x_1, x_2 and x_3 unrestricted in sign.

4. a) Discuss the basic steps in the simplex procedure for Big-M method of LPP.

4

- b) What is simplex method? Define slack and surplus variable with example.

3

- c) Solve the following LPP by Dual- simplex method-

5

$$\text{Minimize, } Z = 5x_1 + 6x_2$$

Subject to the constraint:

$$x_1 + x_2 \geq 2$$

$$4x_1 + x_2 \geq 4$$

$$x_1, x_2, \geq 0$$

5. a) Define transportation problem with real life example.

4

- b) Explain Column minima method for obtaining an initial basic feasible solution of a transportation problem.

4

- c) Determine an initial basic feasible solution to the transportation problem using north-west corner rule method:

Origin \ Destination	D_1	D_2	D_3	Supply
O_1	2	7	4	5
O_2	3	3	1	8
O_3	5	4	7	7
O_4	1	6	2	14
Demand	7	9	18	

Also calculate the minimum cost of transportation.

6. a) What is artificial variable? Write down the algorithm of maximization method to solve linear programming problem.

- b) Solve the following two-person zero sum game with 3×4 payoff matrix for player A and player B and find the value of game of the matrix

4

Player B

	-5	2	0	7
Player A	5	6	4	8
	4	0	2	-3

- c) Discuss the graphical method of solving linear programming problem.

3

7. a) Define the following terms:

3

i) Basis matrix ii) Feasible solution iii) Non-Degenerate solution

- b) Along with essential characteristics write down the standard form of a linear programming problem. Also express the following linear programming problem in standard form:

$$\text{Maximize, } z = 3x_1 + 2x_2$$

Subject to constraints:

$$3x_1 + 2x_2 \leq 6$$

$$x_1 - x_2 \leq -1$$

$$-x_1 - 2x_2 \geq 1$$

and $x_j \geq 0$.

- c) Show that the set of feasible solution of linear programming problem is a convex set.

3

8. a) Define degeneracy I transportation problem. Show that the transportation problem is a special case of linear programming problem.

4

- b) Explain row minima method for obtaining an initial basic feasible solution of a transportation problem.

4

- c) Show that a necessary and sufficient condition for the existence of a feasible solution to a transportation problem is that $\sum_{i=1}^m a_i = \sum_{j=1}^n b_j = \lambda$ (say).

Comilla University
Faculty of Science
Department of Statistics
3rd Year 2nd Semester B. Sc. (Hon's) Final examination-2022
Course Title: Environmental Statistics Course Code: Stat-324
Session: 2019-2020

Marks: 60

Time: 03 hour

(Answer any five questions from the following questions)

1. a) What is environmental statistics? What are the objectives of environmental statistics. 4
b) Why stochastic process is essential for making environmental prediction? Give examples. 4
c) Define deterministic process. How it is differ from random process? 4
2. a) Define Bernoulli process. Mention the conditions for Bernoulli process. Describe its application in environment in case of missing observation. 6
b) What is stationary probability? Discuss the application of Binomial distribution to environmental problem with example for 3 year time period and list out the probability distribution for the number of exceedances. 6
3. a) Define deterministic dilution with its assumptions. Show that successive deterministic dilution described as a proportionate process. 6
b) Suppose that five beakers are available. The first beaker contains 250 ml of water with a dissolved chemical pollutant at a concentration of $c_0 = 1000$ parts-per-million (ppm) by volume. A person carefully measures out 50 ml of this solution and pours it into the next beaker add clean water to this beaker to bring its volume up to 250 ml. Find out the final concentration. 6
4. a) What is indoor air quality? What are the five common measures of air quality? Mention some question's regarding air quality. 6
b) How successive random dilutions can be applied to pollutants released into other small micro-environments like houses and buildings? Discuss about the water quality. 6
5. a) What is environmental sampling? What are the steps involved in environmental sampling? 3
b) Mention the different types of environmental sampling. What must be considered in determining the number of samples to be taken in a particular study? 4
c) Suppose an investigator interested in estimating the number N of fish in a pond by capturing by trawl net a random sample of them, tagging them, releasing them and recapturing a second random sample. The initial sample was of size 300. The second sample of size 200 contained 50 previously

tagged specimens. Find out the Petersen and Chapman estimator along with their variance.

6. a) Define Wedge Machine. Let $n_0 = 1000$ particles to be released at the source and assume that they are released one at a time so that they do not interact with each other in any manner as they fall down the array. Since the probability that a given particle will arrive in channel A is $p(A) = \frac{1}{32} = 0.03125$. Find the expected number of arrivals for each channels. 6
- b) For each of the bottom channels, the probability that a particle arrives is fixed and arrivals are assumed independent. Thus N_A is the sum of n_0 possible events, each occurring with fixed probability $P(A) = \frac{1}{32}$ and the result is a Bernoulli process i.e. $B\left(n_0, \frac{1}{32}\right)$. Find the probability of bottom channels with respect to time. 6
7. a) Write about spatial point process models and methods. 3
b) Describe the general spatial process: prediction, interpolation, and kriging. 5
c) Briefly explain spatial sampling and spatial design. 4
8. a) What is stochastic dilution? Discuss the Successive Random Dilutions (SRD) theory. 7
b) Define Mass Balancer Model. Derive this model for infinite sample. 5

Comilla University
Department of Statistics
3rd Year 2nd Semester B. Sc. (Hon's) Final Examination – 2022
Course Title: Research Methodology
Session: 2019-20 Course Code: Stat – 325

Marks-60

Time 3Hours

Answer any Five questions from the following

1. a) Define qualitative and quantitative research with examples. Which research should we do, qualitative or quantitative? Explain your opinion. 6
- b) How does applied research differ from pure research? Give real life examples. 6
2. a) 'Research work is guided by reflecting thinking, not by traditional or conventional thinking.' Comment on this statement. 5
- b) Give the sources of research problem. How a problem is identified? Enumerate the criteria for the selection of a problem. 7
3. a) Explain the need and functions of review of literature. Enumerate the sources of review of literature. 6
- b) Indicate the main characteristics of a good hypothesis and uses of a hypothesis in various types of research studies. 6
4. a) Explain the meaning and significance of research design? What are the criteria for selecting a research design? Draw a flowchart for developing a good research design. 6
- b) What are the experimental and Non-experimental research design? Describe the main stages in an experimental research design. Give an example of the application of an experimental design. 6
5. a) Describe the steps involved in estimating reliability. Distinguish it from validity. 5
- b) What is Likert scale? How is scale score computed for each individual in such scaling? 4
- c) Compare and contrast with unidimensional and multidimensional scaling. 3
6. a) Distinguish between a bid and TOR and explain their significance and importance. Describe the main points as incorporated in a bid and a TOR. 6
- b) Present a diagram for the different stages of a proposal that shows an overview of the whole process. 6
7. a) Suggest a general format for a scientific report. Which component you consider most important in a report? Why? 5
- b) Explain the theoretical framework of SWOT analysis. Show with the help of this framework how a SWOT analysis fits into a situational taking into consideration the internal and external factors. 7
8. a) What is monitoring and evaluation (M & E)? What is the difference between an M&E plan, M&E framework and M&E system? 6
- b) Discuss how Monitoring and evaluation of development projects help to empower stakeholder communities. 6

Comilla University
 Department of Statistics
3rd Year 2nd Semester B.Sc. (Hon's) Final Examination-2023
Course Title: Statistical Data Analysis-VI, Course Code: Stat-326
Session: 2019-2020

Full Marks: 100

Time: 5 hours

Group A (Linear programming and operation research, Marks-35)

1. a) Solve the following linear programming problem by simplex method: 7.5
 Maximize, $z = 10x_1 + x_2 + 2x_3$
 Subject to constraints:
 $x_1 + x_2 - 2x_3 < 10$
 $4x_1 + x_2 + x_3 < 20$
 $x_1, x_2, x_3 \geq 0.$
2. a) Solve the following linear programming problem by simplex method: 7.5
 Minimize, $z = x_1 - 3x_2 + 2x_3$
 Subject to constraints:
 $3x_1 - x_2 + 2x_3 < 7$
 $-2x_1 + 4x_2 < 12$
 $-4x_1 + 3x_2 + 8x_3 < 10$
 $x_1, x_2, x_3 \geq 0.$
3. a) Solve the following transportation problem by using solver 10

From	To				
	D_1	D_2	D_3	D_4	
O_1	6	7	3	4	
O_2	7	9	1	2	
O_3	6	5	16	7	
O_4	18	9	10	2	

4. a) Solve the following transportation problem by using solver. 10

From	To				Available
	D_1	D_2	D_3	D_4	
O_1	10	0	20	11	15
O_2	1	7	9	20	25
O_3	12	14	16	18	5
Required	12	8	15	10	45

15

29/2

Group -B Order Statistics and Non-parametric Methods, Marks-35

1. The maximum level of a lake each year for a period of 20 years is given below. Use Run test to find out, Whether the sequence is generated by a random process or the process contains a trend?

Year	Level (above 190 meter)	Year	Level (above 190 meter)
1	6.6	11	6.0
2	6.5	12	5.8
3	6.4	13	5.9
4	6.5	14	5.6
5	6.4	15	5.5
6	6.4	16	5.3
7	6.3	17	5.1
8	6.2	18	5.3
9	6.1	19	5.4
10	5.9	20	5.2

2. The mark obtains in both the mock examination and A-level examination by a random sample of 13 students is shown below:

Candidate	1	2	3	4	5	6	7	8	9	10	11	12	13
Mock	40	65	53	79	87	42	80	63	51	82	27	71	29
A-level	45	68	47	75	88	60	77	69	60	88	30	73	35

Use sign test to see, did the students perform better in A-level than mock test at 5% level of significance?

3. An experiment was performed to determine if self-fertilized and crossed fertilized plants have different growth rate. Pair of plants, one self and other crossed fertilized, were planted in 15 pots their height were measured after specified period of time. Perform Wilcoxon signed rank test to determine crossed plants have higher growth rate 9

Sl no.	Crossed fertilized	Self fertilized
1	45.5	40.0
2	40.0	42.3
3	42.8	41.2
4	41.6	41.3
5	37.9	36.7
6	42.5	38.0
7	44.1	39.8
8	40.7	38.9
9	41.2	41.2
10	42.7	42.0
11	43.3	42.0
12	41.0	40.7
13	46.0	43.5
14	39.2	40.6
15	44.3	42.5

4. There is a movie rating of 20 male and female is given below. Use Mann Whitney U test to see whether any difference between the ratings. 9

Sl no.	Male	Female
1	6.35	5.90
2	6.54	6.28
3	6.11	5.76
4	6.97	6.66
5	6.78	6.71
6	5.83	5.58
7	6.48	6.17
8	6.84	6.79
9	6.35	6.04
10	6.59	6.16
11	7.00	6.60
12	7.43	7.01
13	6.05	8.68
14	6.46	6.23
15	6.65	6.40
16	6.95	6.62
17	6.76	6.41
18	7.30	6.92
19	6.86	6.49
20	7.15	6.84

accept

Group C (Statistical inference-II, Marks-30)

1. a) Suppose the daily number of items produced by a firm for randomly selected 15 days are as follows: 10
- 110, 118, 130, 140, 142, 146, 112, 100, 95, 98, 96, 122, 123, 124, 130
 Can we conclude at 5% level of significance that the average daily production of items of that firm is 110 items?
2. a) In study of relationship between expenditure(x) and sales volume (y), a sample of 10 firms yields the co-efficient of correlation $r = 0.93$, Can it be concluded on the basis of information that x and y are linearly related? (use $\alpha = 0.05$)
3. a) A regression of retail sales on disposable income provides with the following results. N=22, 10
 $b=0.3815$, $se(b)=0.0253$. Test the significance of regression co-efficient at 1% level of significance. Also, compute 99% confidence interval for regression parameter.