Introduction to Mathematical Thinking

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Question 8

Prove (from the definition of a limit of a sequence) that if the sequence $\{a_n\}_{n=1}^{\infty}$ tends to limit L as $n \to \infty$, then for any fixed number $M > 0$, the sequence $\{Ma_n\}_{n=1}^{\infty}$ tends to the limit ML.

Answer

1. Given $\mathcal{E} > 0$, $|a_n - L| \leq \mathcal{E}$ for $n \in \mathbb{N}$

   $M \in \mathbb{R} > 0, so$

   $\frac{\mathcal{E}}{M} > 0$
2. $\exists N \in \mathbb{N} \mid N > n, |a_N - L| \leq \frac{\mathcal{E}}{M}$
3. Multiplying by M on both sides,

$$M \times |a_N - L| \leq \frac{\mathcal{E}}{M} \times M$$

$$|Ma_N - ML| \leq \mathcal{E}$$

which is the *limit definition* of $\{Ma_N\}_{N=1}^{\infty}$

$\therefore$ the sequence $\{Ma_n\}_{n=1}^{\infty}$ tends to the limit ML by the definition of the limit of a sequence.