Introduction to Mathematical Thinking

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Question 10

Give an example of a family of intervals $A_n, n = 1, 2, ...,$ such that $A_{n+1} \subset A_n$ for all n and $\bigcap_{n=1}^{\infty} A_n$ consists of a single real number. Prove that your example has the stated property.

Answer

Let A_i be an infinite collection of sets of real line intervals $= [\frac{i}{i+1}, 1], i \in \mathbb{N}$

When...

$$i = 1, A_1 = [\frac{1}{2}, 1]$$

$$i = 2, A_2 = [\frac{2}{3}, 1]$$

$$i = 3, A_3 = [\frac{3}{4}, 1]$$

$$\vdots$$

$$i = \infty, A_\infty = [1, 1] = \{1\}$$

$A_1 \cap A_2 \cap A_3 \cap ... \cap \{1\} = \{1\}$

$\therefore \bigcap_{i=1}^{\infty} A_i = \{1\}$, which is a set of a single real number.