Introduction to Mathematical Thinking

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Question 3

Say whether the following is true or false and support your answer by a proof: For any integer n, the number $n^2 + n + 1$ is odd.

Answer

1. Let n=1...

   $(1^2) + (1) + 1 = 3$ is odd

2. Assuming the given statement is true for $n = k$,

   $k^2 + k + 1$ is odd.

3. let $n = k + 1$...
$$(k+1)^2 + (k+1) + 1$$
$$= k^2 + 2k + 1 + k + 1 + 1$$
$$= (k^2 + k + 1) + (2k + 2)$$
$$= (k^2 + k + 1) + 2(k + 1)$$

For all integers k, $2(k+1)$ is *even*, and from our assumption, $k^2 + k + 1$ is *odd*.

$$ODD + EVEN = ODD...$$

$\therefore$ the given statement, "$n^2 + n + 1$ is odd for any integer n", is *TRUE*.