Introduction to Mathematical Thinking

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Question 9

Given an infinite collection $A_n, n = 1, 2, ...$ of intervals of the real line, their *intersection* is defined to be

$$\bigcap_{n=1}^{\infty} A_n = \{x | (\forall n)(x \in A_n)\}$$

Give an example of a fmaily of intervals $A_n, n = 1, 2, ...$, such that $A_{n+1} \subset A_n$ for all n and $\bigcap_{n=1}^{\infty} A_n = \emptyset$. Prove that your example has the stated property.

Answer

Let A_i be an infinite collection of sets of real line intervals $= (0, \frac{1}{i}), i \in \mathbb{N}$

When...

$$i = 1, A_1 = (0, 1)$$
$$i = 2, A_2 = (0, \frac{1}{2})$$
$$i = 3, A_3 = (0, \frac{1}{3})$$
$$\vdots$$
$$i = \infty, A_\infty = \emptyset$$

$A_1 \cap A_2 \cap A_3 \cap ... \cap \emptyset = \emptyset$

$\therefore \bigcap_{i=1}^{\infty} A_i = \emptyset$