Introduction to Mathematical Thinking

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Question 4

Prove that every odd natural number is of one of the forms $4n + 1$ or $4n + 3$, where n is an integer.

Answer

1. Let $m \in \mathbb{N}$
2. By the Division Theorem, $\exists (n, r) \in \mathbb{Z} | m = 4n + r, 0 \leq r < 4$

So every natural number m can be represented as one of the following:

Case 1: $m = 4n = 2(2n)$

Case 2: $m = 4n + 1$

Case 3: $m = 4n + 2 = 2(2n + 1)$

Case 4: $m = 4n + 3$

Case 1 and Case 3 represent even natural numbers, so every odd natural number must be of the form $4n + 1$ or $4n + 3$.

$\therefore$ the given statement is *TRUE*.