Introduction to Mathematical Thinking

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Question 7

Prove that for any natural number n,

$$2 + 2^2 + 2^3 + ... + 2^n = 2^{n+1} - 2$$

Answer

1. Proving by induction, first let $n = 1$,

$$2^1 = 2^{1+1} - 2$$
$$2 = 2$$

which is *true*

2. Assuming the statment is true for $n = k$,

$$2 + 2^2 + 2^3 + ... + 2^k = 2^{k+1} - 2$$

3. Proving for $n = k + 1$,

$$(2 + 2^2 + 2^3 + ... + 2^k) + 2^{k+1} = 2^{k+2} - 2$$

$$(2^{k+1} - 2) + 2^{k+1} = 2^{k+2} - 2$$

$$2(2^{k+1}) - 2 = 2^{k+2} - 2$$

$$2^{k+2} - 2 = 2^{k+2} - 2$$

which is *true*

$\therefore$ $2 + 2^2 + 2^3 + ... + 2^n = 2^{n+1} - 2$ for any natural number n, and the given statement is *TRUE*