

Introduction to Mathematical Thinking

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Question 6

A classic unsolved problem in number theory asks if there are infinitely many pairs of ‘twin primes’, pairs of primes separated by 2, such as 3 and 5, 11 and 13, or 71 and 73. Prove that the only prime triple (i.e. three primes, each 2 from the next) is 3, 5, 7.

Answer

1. Since prime numbers besides 2 cannot be even, let our three primes be consecutive odd integers $2n + 1$, $2n + 3$, and $2n + 5$
2. Prime numbers cannot be multiples of three either. Since multiples of three occur every three integers, one of the following must be divisible by three...

$$\text{Case1 : } 2n + 1$$

$$\text{Case2 : } 2n + 2$$

$$\text{Case3 : } 2n + 3$$

3. If either Case 1 or Case 3 is divisible by three, then they are not prime, and the given statement is *TRUE*
 4. If Case 2 is divisible by 3, then $(2n + 2) + 3 = 2n + 5$ must also be divisible by 3, meaning it is not a prime number, and the given statement is *TRUE*
- \therefore there is no prime triple besides 3, 5, 7, and the given statement is *TRUE*