Weight of this project is 30%.

Project Description:

For single-phase problems, pressure and velocity fields are important to be solved (Saturation is 1, obviously). Based on the fluid and rock of our reservoir, we need to determine either

(1) incompressible $-\nabla \cdot \left(\lambda \cdot \nabla p\right) = q$,

(2) slightly compressible $\left(\phi c_e\right) \partial p / \partial t - \nabla \cdot \left(\lambda \cdot \nabla p\right) = q$, or

(3) general compressible $\partial\left(\rho\phi\right) / \partial t - \nabla \cdot \left(\rho\lambda \cdot \nabla p\right) = \rho q$

formulation is applicable. Only Eq. (3) is nonlinear, which should be first written in linear form (typically with Newton linearization method), and then be iteratively solved.

Equation (3) automatically reduces to Eq. (1) and Eq. (2) once the inputs (ρ, ϕ) are set as slightly compressible (constant compressibility coefficient) and incompressible (zero compressibility coefficient, constant density), respectively.

Task: Develop a 1D (7 points) and 2D (3 points) reservoir simulator for single-phase flow problems with general BC in homogeneous and heterogeneous fields. Show your simulator works correctly, i.e., converges to the exact solution with a proper order of consistency and it is stable.

Deliver: A neat and clear report + Code all in 1 single zipped file.

Have “Abstract”, “Introduction”, “Methodology”, “Results”, “Validation”, “Discussions”, “References”. Maximum number of pages is 8. We aim for a high-quality report with minimum text but more figures and plots.

Submission policy:

- Zipped File Name: FIRSTNAME_FAMILYNAME_ID.zip, when unzipped generates a folder called “FIRSTNAME_FAMILYNAME_ID”.
- Upload on Blackboard by 16th of May 2016.

Some typical parameters are provided below. It is your choice to use them (check AES1300):

Absolute permeability K = 10^{-13} - 10^{-14} m^2

Water viscosity $\mu_f = 10^{-3}$ [Pa . s]

Water density at atmospheric pressure: $\rho_f = 10^3$ [kg / m^3]

Porosity: 0.1-0.3

Effective compressibility (slightly compressible water/oil with porous media): $c_{eff} = 10^{-6}$ [Pa^{-1}]

Some hints on calculation of exact solution is provided in the next page.

Notes on the calculation of error:

For elliptic problems, if mobility is assumed to be constant, exact solution can be easily computed by applying twice an integration operator. The same strategy holds if the function of mobility with respect to the spatial coordinate (here, x) is known.

For parabolic problems, as seen also in AES1310, several analytical methods are proposed: (1) similarity solution and (2) Laplace Transform.

For your parabolic problem error study, please consider the following setup:

Consider a long reservoir [-L L], with L tending to infinity. For homogeneous domain, one can reformulate the parabolic problem for slightly compressible systems as:

$$\frac{\partial p}{\partial t} - D\frac{\partial^2 p}{\partial x^2} = 0 \qquad D = \frac{K}{\mu \phi c_{eff}} \quad \text{with } x \in (-\infty, +\infty)$$

Consider the boundary conditions of: $p(-\infty, t) = 0$ and $p(+\infty, t) = 0$, and a Dirac Pulse as initial condition: $p(x, t=0) = p_{init}\delta(x - x_0)$.

Take parameters as: L = 100m, K=10^{-14} m^2, $\mu = 10^{-3}$, $\phi = 0.3$, $c_{eff} = 10^{-6}$.

Verify that $p(x,t) = \frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{1}{2}\frac{x^2}{\sigma^2}\right)$ where $\sigma = \sqrt{2Dt}$ is an analytical solution of the equation subject to boundary conditions.

Solve this problem with your code, and prove it is a correct simulator.

Note that $\int_{x_{i-1/2}}^{x_{i+1/2}} f(x)\delta(x - x_i) = f(x_0)$ and not $f(x_0)\Delta x$!