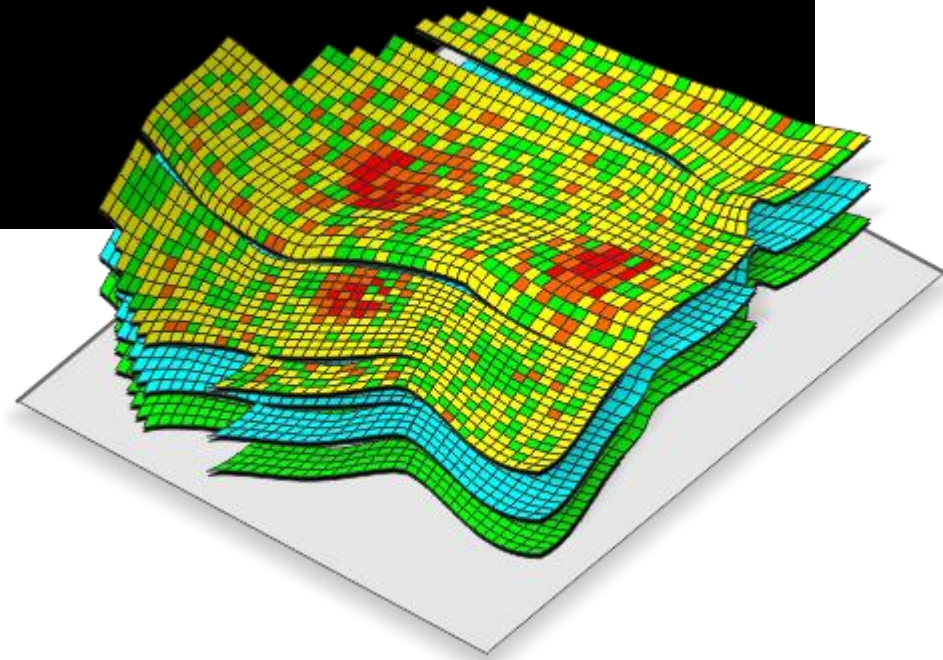


Lab 4



Incompressible pressure

Singe phase

$$-\nabla \cdot (\lambda \cdot \nabla p) = q$$

$$-\begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{bmatrix} \cdot \left(\begin{bmatrix} \lambda_x & \lambda_{xy} & \lambda_{xz} \\ \lambda_{yx} & \lambda_y & \lambda_{yz} \\ \lambda_{zx} & \lambda_{zy} & \lambda_z \end{bmatrix} \cdot \begin{bmatrix} \frac{\partial p}{\partial x} \\ \frac{\partial p}{\partial y} \\ \frac{\partial p}{\partial z} \end{bmatrix} \right) = q$$

Incompressible pressure

Singe phase, **1D**

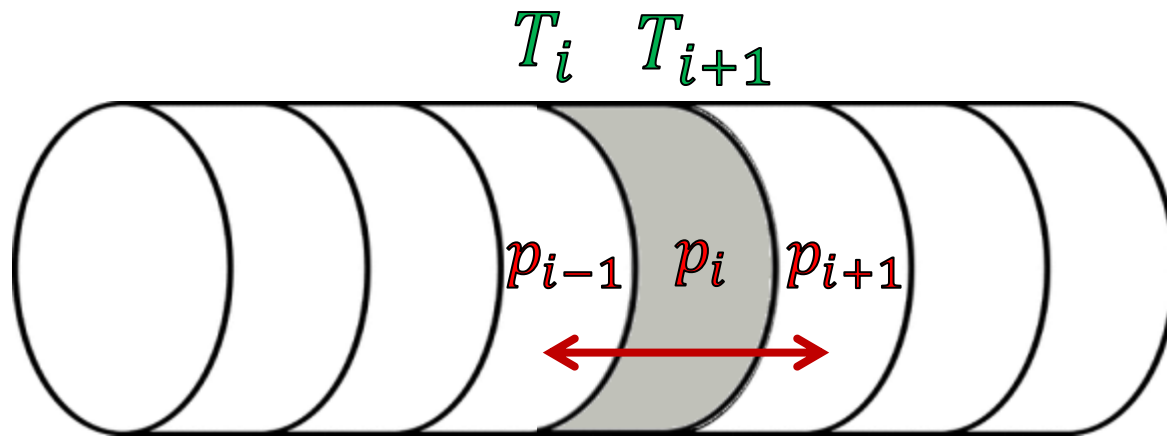
$$-\nabla \cdot (\lambda \cdot \nabla p) = q$$

$$-\begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{bmatrix} \cdot \left(\begin{bmatrix} \lambda_x & \lambda_{xy} & \lambda_{xz} \\ \lambda_{yx} & \lambda_y & \lambda_{yz} \\ \lambda_{zx} & \lambda_{zy} & \lambda_z \end{bmatrix} \cdot \begin{bmatrix} \frac{\partial p}{\partial x} \\ \frac{\partial p}{\partial y} \\ \frac{\partial p}{\partial z} \end{bmatrix} \right) = q$$

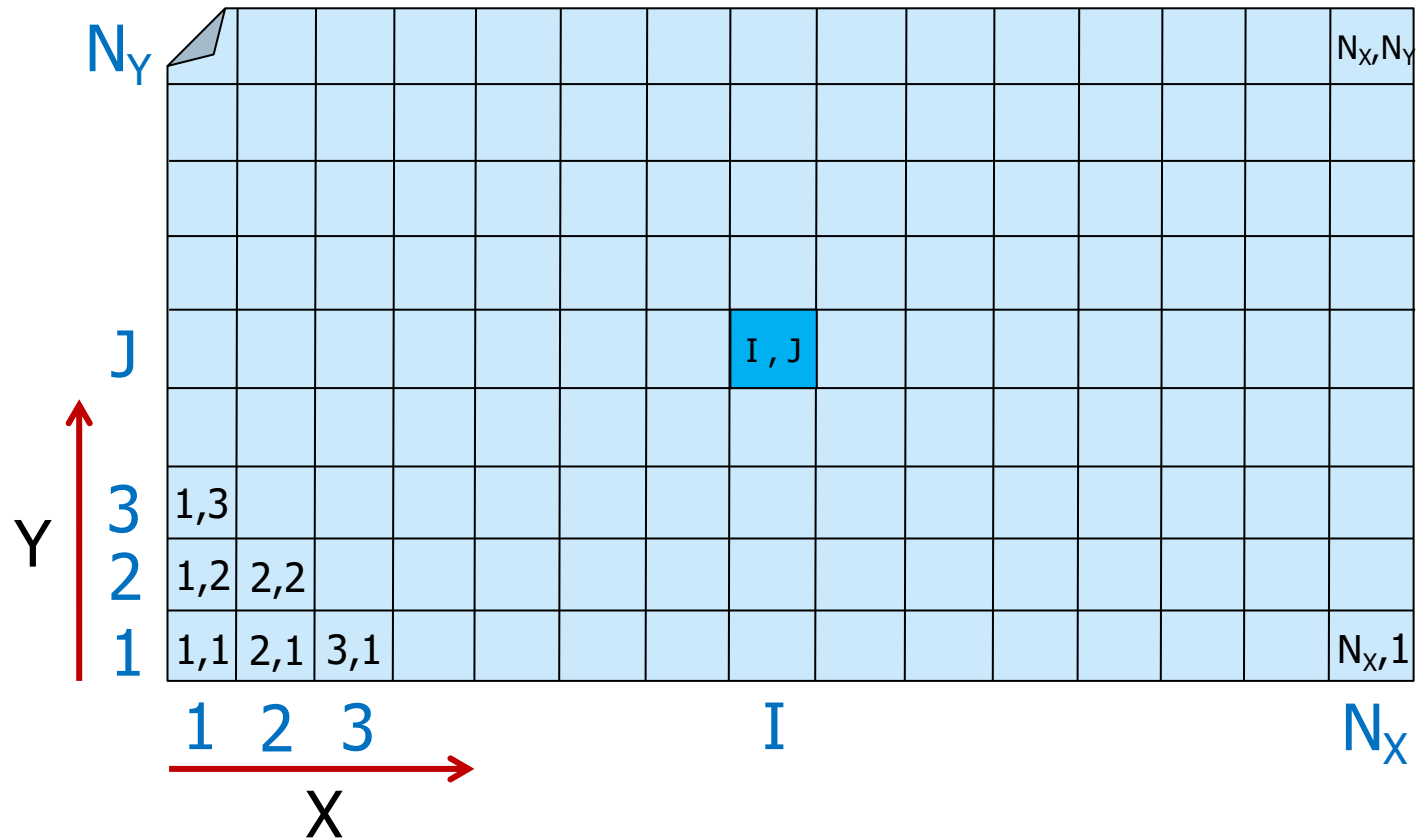
Incompressible pressure

Single phase, 1D

$$-\frac{\partial}{\partial x} \left(\lambda_x \frac{\partial p}{\partial x} \right) = q$$

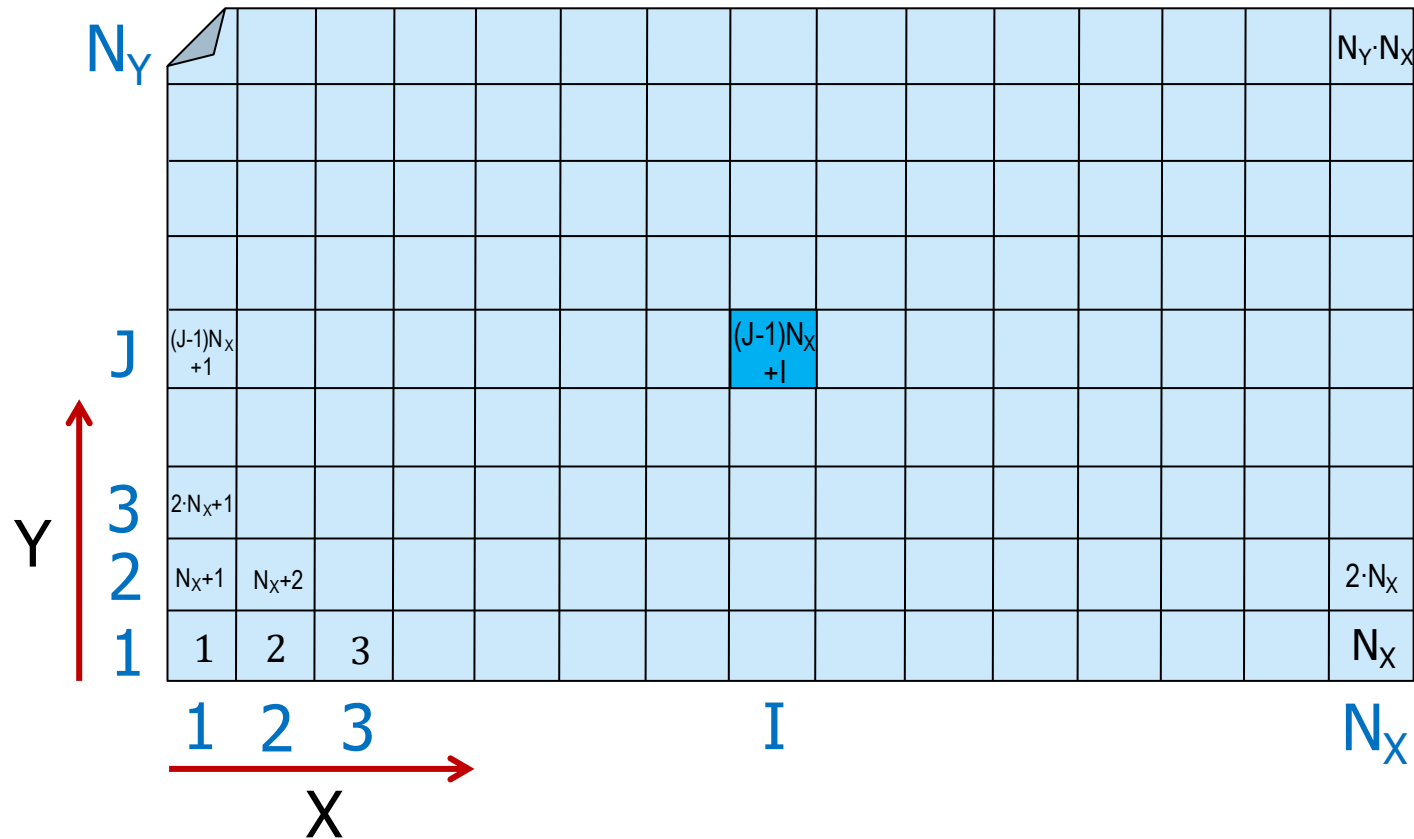


2D domain



2D domain

$$\mathbf{A_p} = \mathbf{q}$$



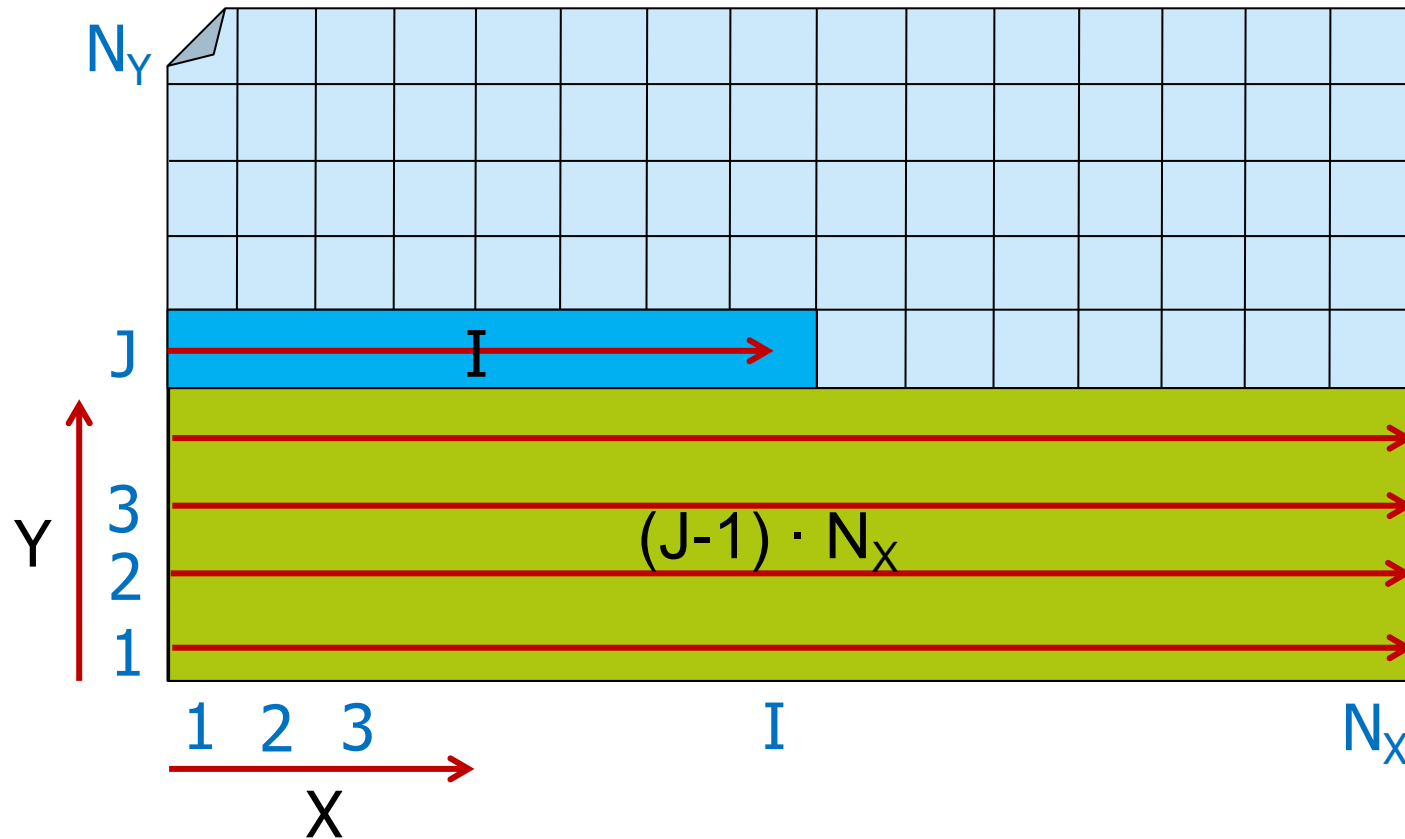
Cell index

$I, J \rightarrow \text{Index}: \quad \text{Index} = (J-1) \cdot N_x + I$

$\text{Index} \rightarrow I, J: \quad ?$

Cell index

$$I, J \rightarrow \text{Index:} \quad \text{Index} = (J-1) \cdot N_x + I$$



Cell index

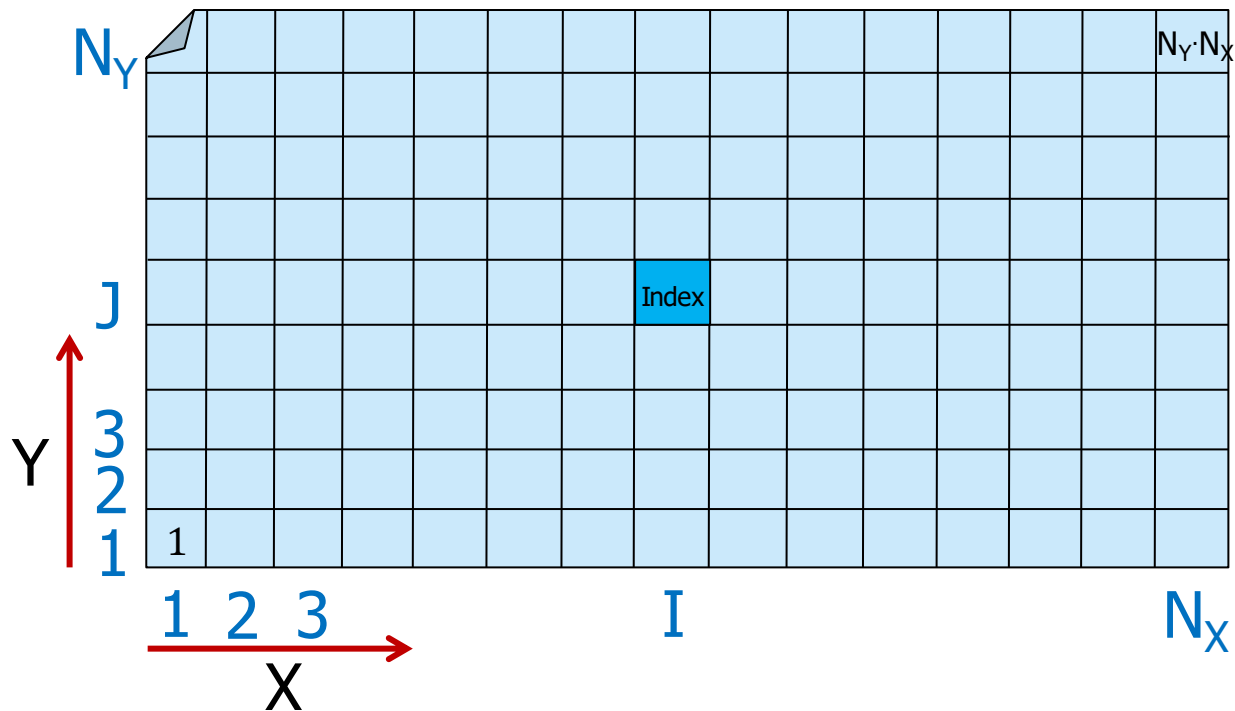
$$I, J \rightarrow \text{Index:} \quad \text{Index} = (J-1) \cdot N_x + I$$

$$\text{Index} \rightarrow I, J: \quad \begin{cases} I = \text{rem}(\text{Index}, N_x) \\ J-1 = (\text{Index} - I) / N_x \end{cases}$$

Let's implement these as functions in MATLAB!

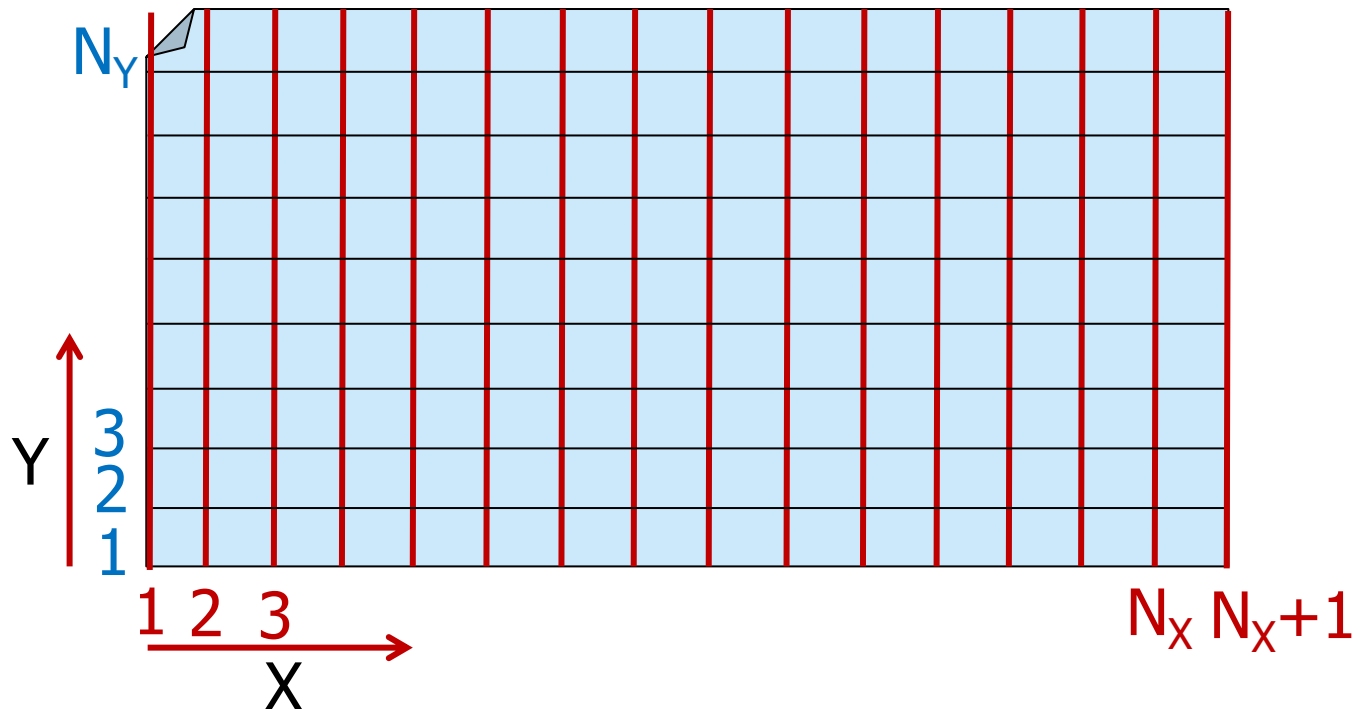
2D grid

$N_X * N_Y$ cells

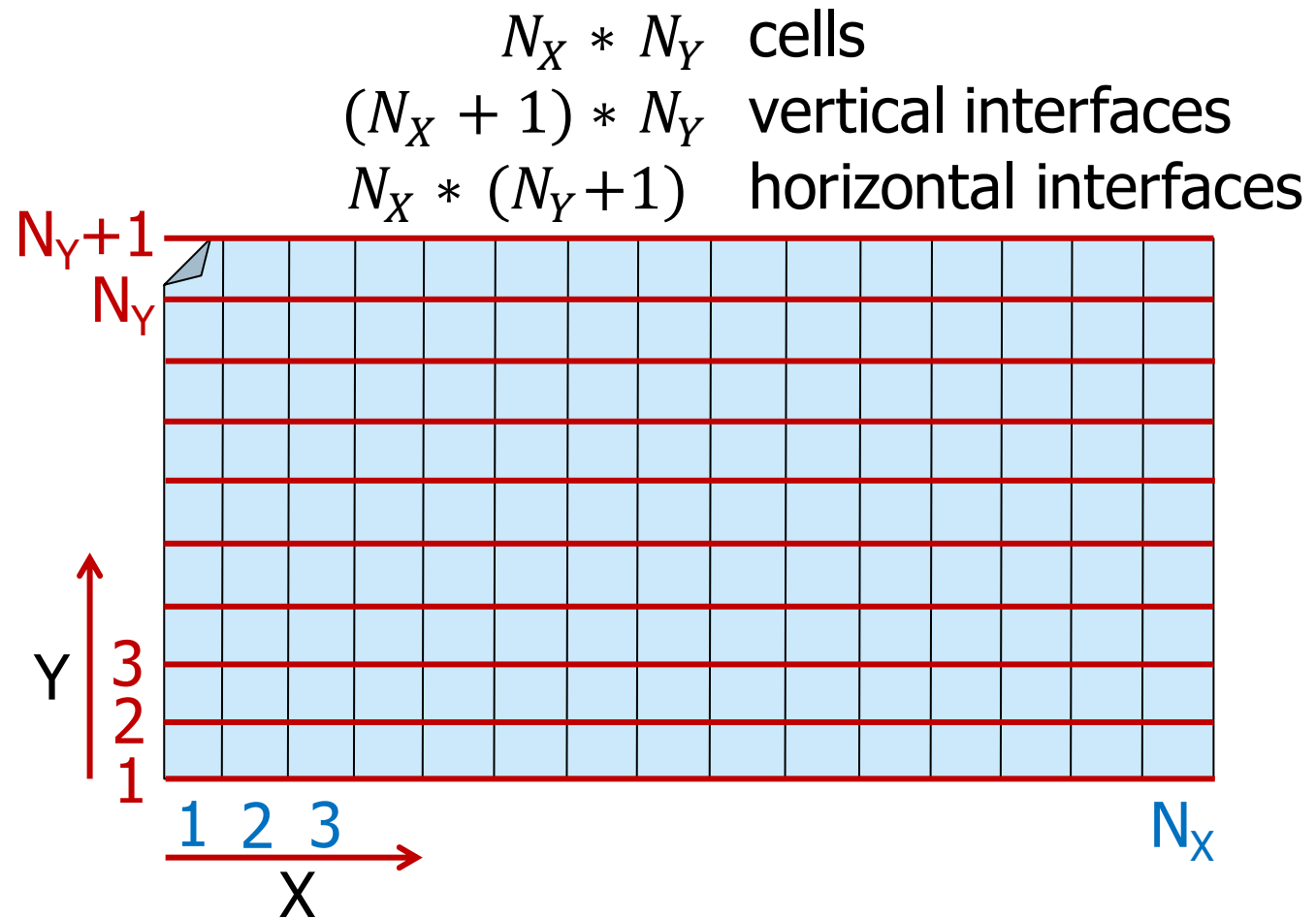


2D grid

$N_X * N_Y$ cells
 $(N_X + 1) * N_Y$ vertical interfaces



2D grid



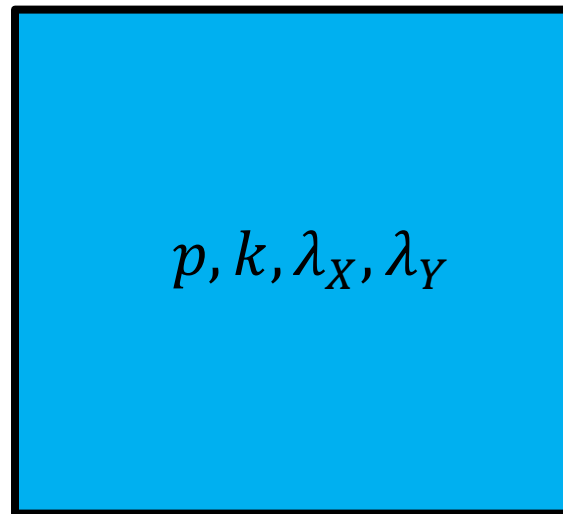
2D grid

$N_X * N_Y$ cells

$(N_X + 1) * N_Y$ vertical interfaces

$N_X * (N_Y + 1)$ horizontal interfaces

$\lambda_{vert}^H, T_{vert}, u_{vert}$



$\lambda_{horiz}^H, T_{horiz}, u_{horiz}$



Incompressible pressure

Singe phase, **2D**

$$-\nabla \cdot (\lambda \cdot \nabla p) = q$$

$$-\begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{bmatrix} \cdot \left(\begin{bmatrix} \lambda_x & \lambda_{xy} & \lambda_{xz} \\ \lambda_{yx} & \lambda_y & \lambda_{yz} \\ \lambda_{zx} & \lambda_{zy} & \lambda_z \end{bmatrix} \cdot \begin{bmatrix} \frac{\partial p}{\partial x} \\ \frac{\partial p}{\partial y} \\ \frac{\partial p}{\partial z} \end{bmatrix} \right) = q$$

Incompressible pressure

Singe phase, **2D**

$$-\nabla \cdot (\lambda \cdot \nabla p) = q$$

$$-\begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \cancel{\frac{\partial}{\partial z}} \end{bmatrix} \cdot \left(\begin{bmatrix} \lambda_x & \mathbf{0} & \cancel{\lambda_{xz}} \\ \mathbf{0} & \lambda_y & \cancel{\lambda_{yz}} \\ \cancel{\lambda_{zx}} & \cancel{\lambda_{zy}} & \cancel{\lambda_z} \end{bmatrix} \cdot \begin{bmatrix} \frac{\partial p}{\partial x} \\ \frac{\partial p}{\partial y} \\ \cancel{\frac{\partial p}{\partial z}} \end{bmatrix} \right) = q$$

Incompressible pressure

Singe phase, **2D**

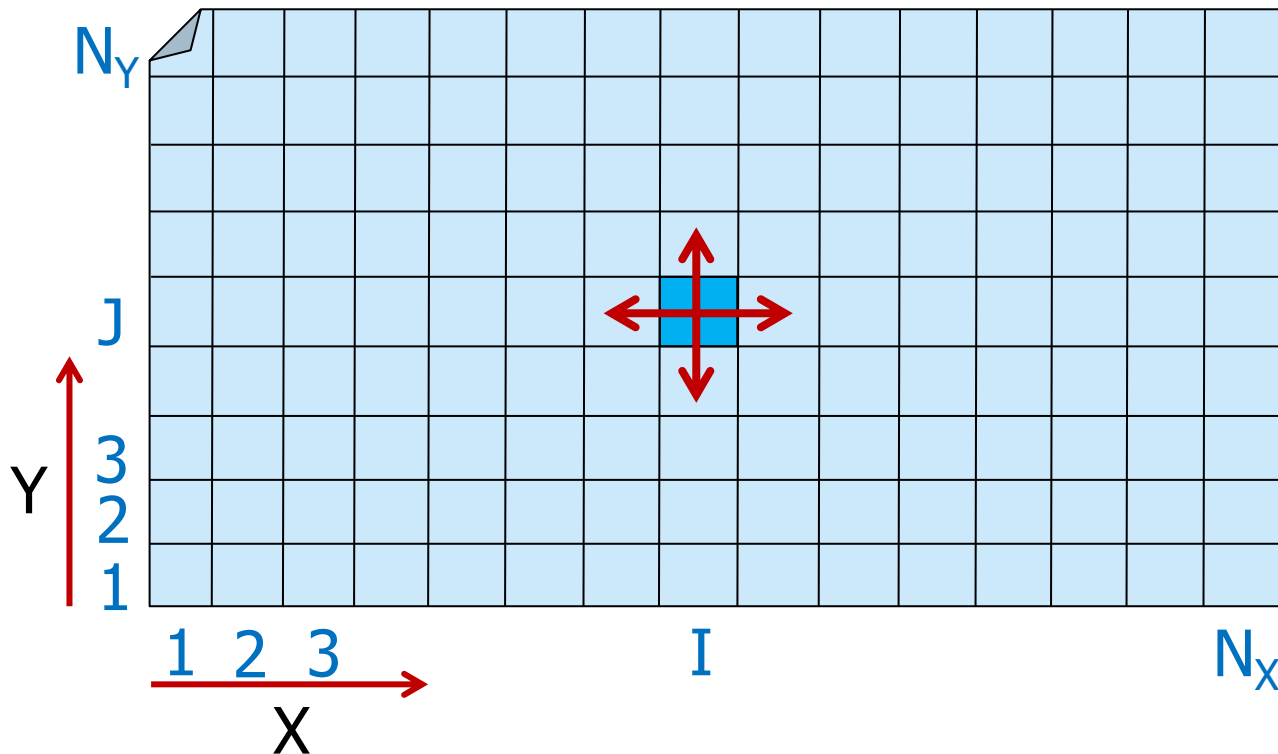
$$-\nabla \cdot (\lambda \cdot \nabla p) = q$$

$$-\begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{bmatrix} \cdot \begin{bmatrix} \lambda_x \frac{\partial p}{\partial x} \\ \lambda_y \frac{\partial p}{\partial y} \end{bmatrix} = q$$

Incompressible pressure

Singe phase, **2D**

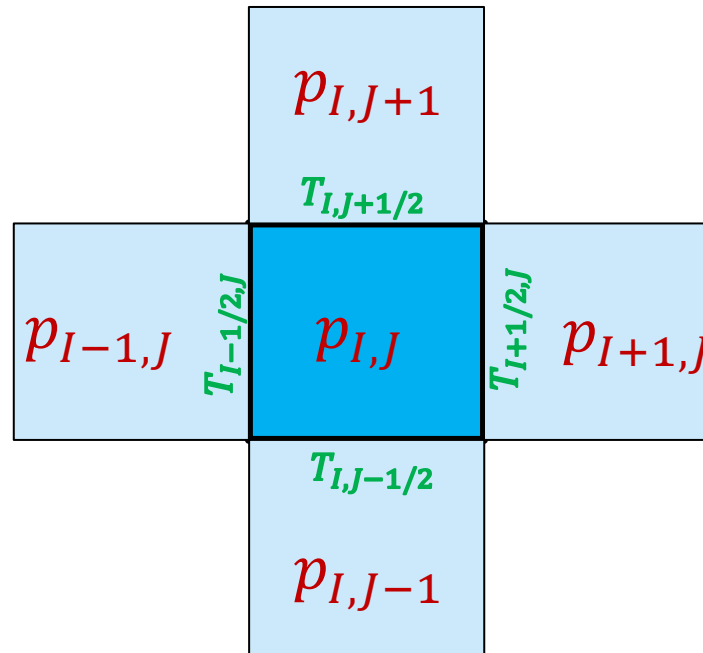
$$-\frac{\partial}{\partial x} \left(\lambda_x \frac{\partial p}{\partial x} \right) - \frac{\partial}{\partial y} \left(\lambda_y \frac{\partial p}{\partial y} \right) = q$$



Incompressible pressure

Singe phase, **2D**

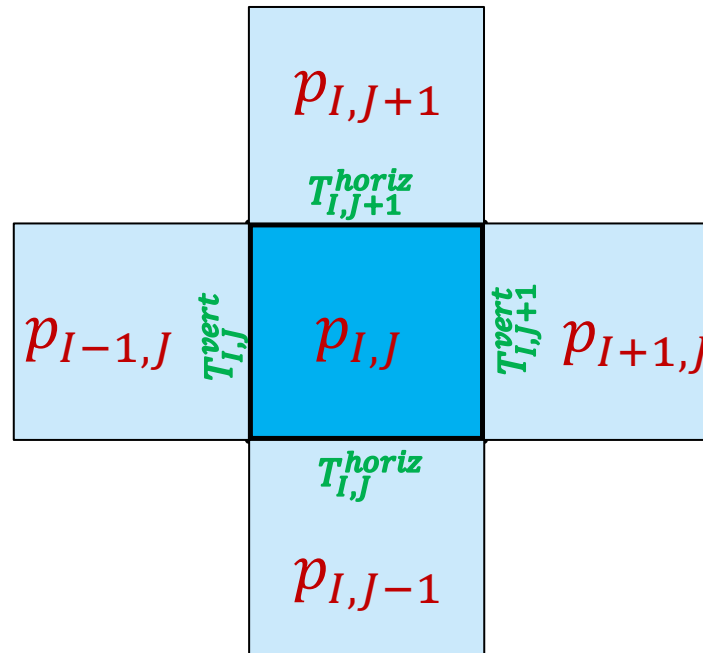
$$-\frac{\partial}{\partial x} \left(\lambda_x \frac{\partial p}{\partial x} \right) - \frac{\partial}{\partial y} \left(\lambda_y \frac{\partial p}{\partial y} \right) = q$$



Incompressible pressure

Singe phase, **2D**

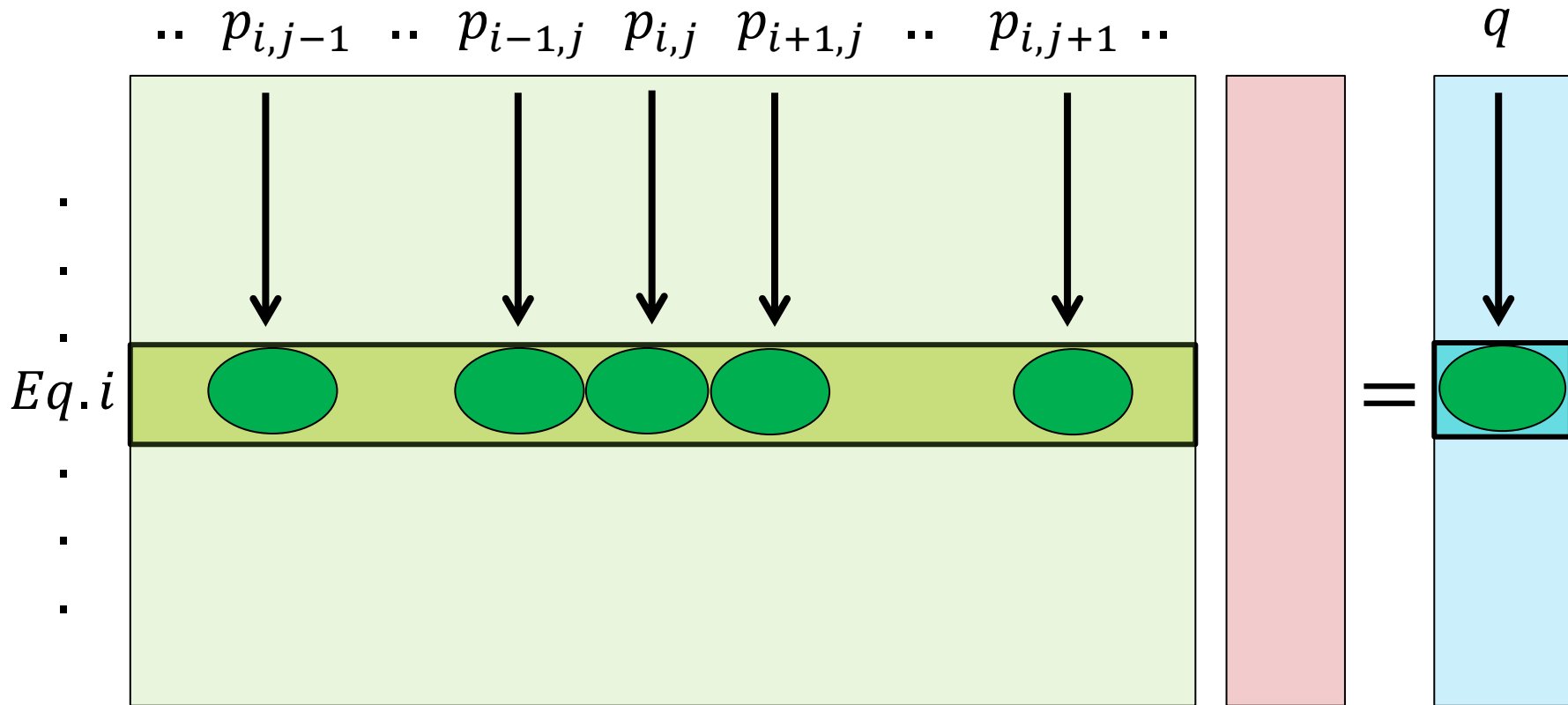
$$-\frac{\partial}{\partial x} \left(\lambda_x \frac{\partial p}{\partial x} \right) - \frac{\partial}{\partial y} \left(\lambda_y \frac{\partial p}{\partial y} \right) = q$$



Incompressible system

$$\begin{array}{cccccccc} & p_1 & p_2 & \cdots & p_{i-1} & p_i & p_{i+1} & \cdots & p_{N-1} & p_N & & q \\ \begin{array}{l} Eq. 1 \\ \vdots \\ Eq. i \\ \vdots \\ Eq. N \end{array} & \begin{array}{c} \boxed{A} \end{array} & & & & & & & & & \begin{array}{c} \boxed{p} \end{array} = \begin{array}{c} \boxed{q} \end{array} \end{array}$$

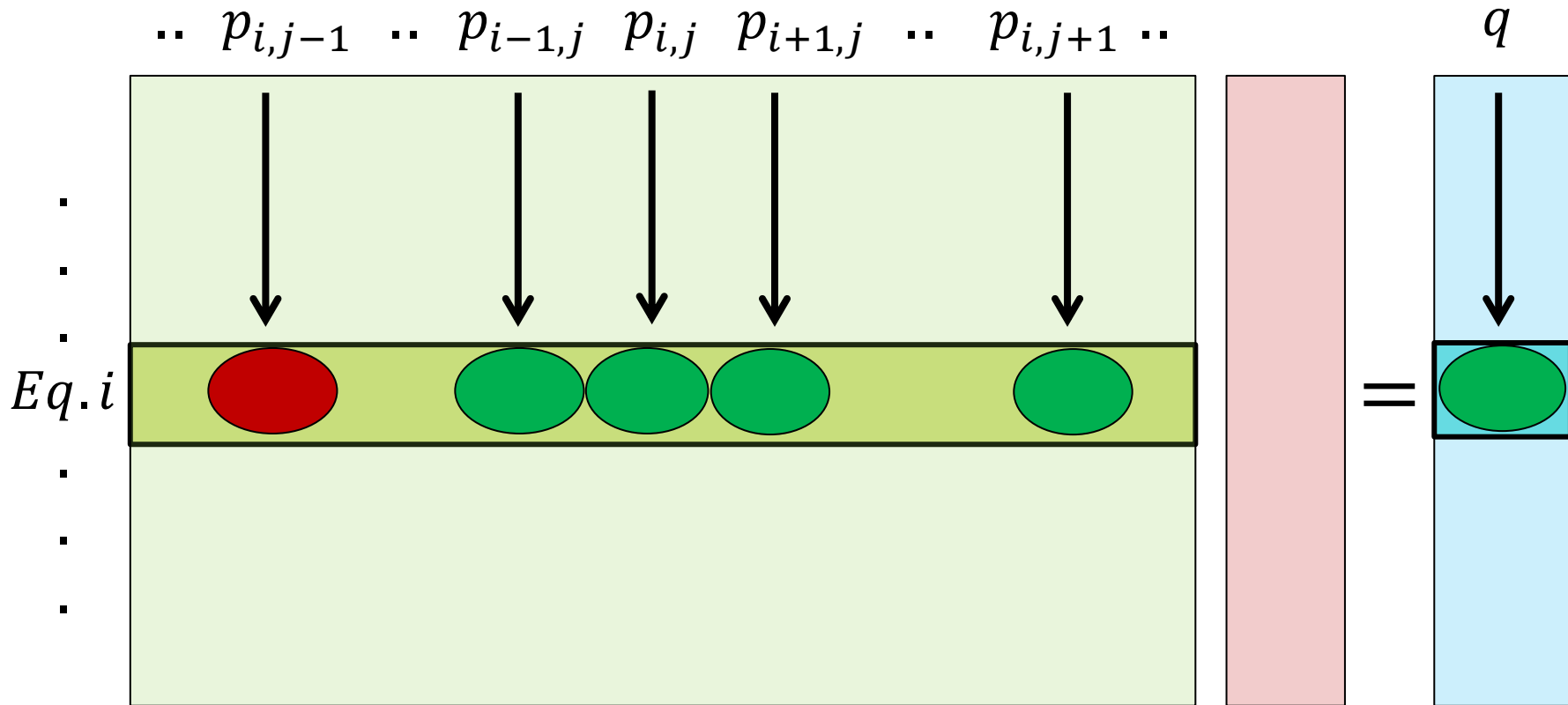
Incompressible system



$$T_{i,j}^{vert}(p_{i,j} - p_{i-1,j}) + T_{i+1,j}^{vert}(p_{i,j} - p_{i+1,j}) + T_{i,j}^{horiz}(p_{i,j} - p_{i,j-1}) + T_{i,j+1}^{horiz}(p_{i,j} - p_{i,j+1}) = 0$$

$$-T_{i,j}^{horiz} \mathbf{p}_{i,j-1} - T_{i,j}^{vert} \mathbf{p}_{i-1,j} + \left(\sum T \right) \mathbf{p}_{i,j} - T_{i+1,j}^{vert} \mathbf{p}_{i+1,j} - T_{i,j+1}^{horiz} \mathbf{p}_{i,j+1} = 0$$

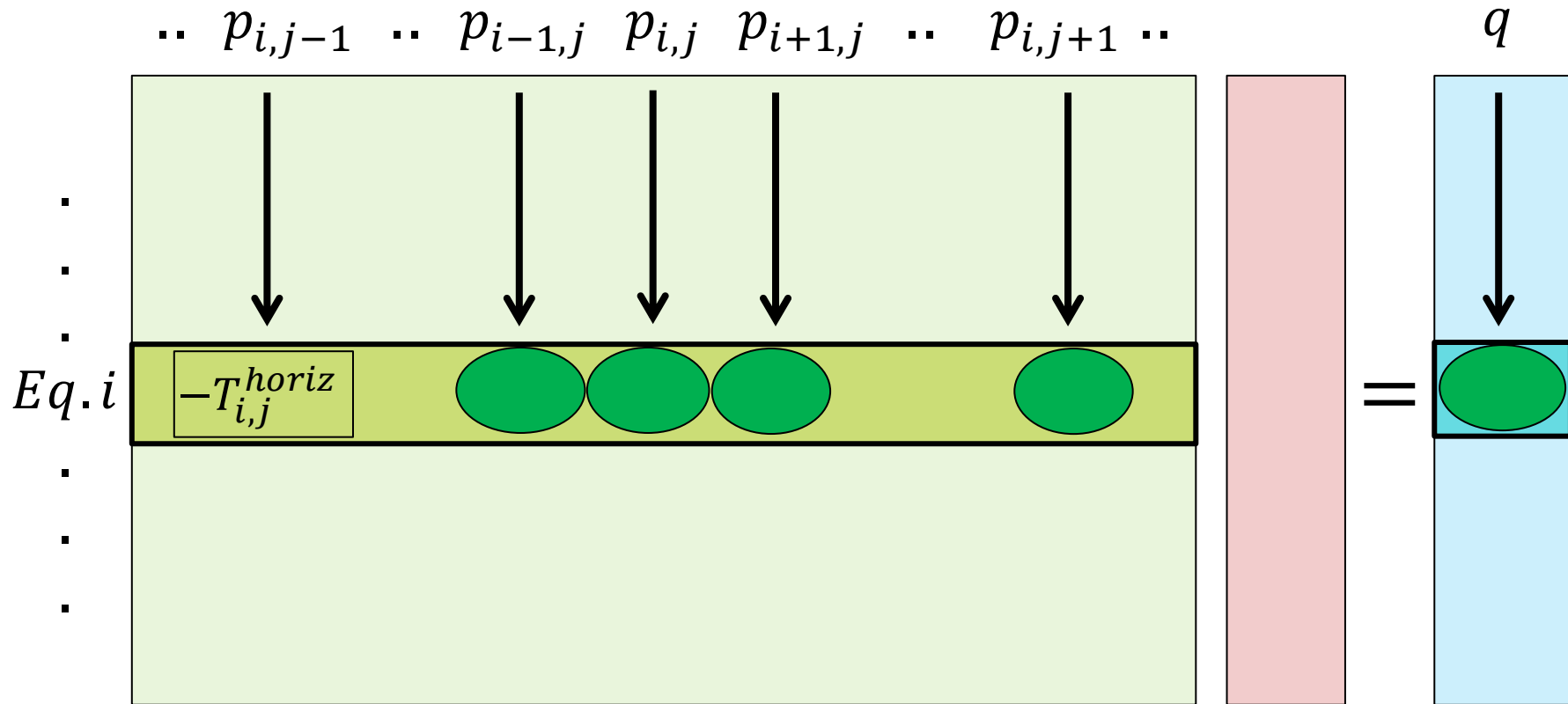
Incompressible system



$$T_{i,j}^{vert}(p_{i,j} - p_{i-1,j}) + T_{i+1,j}^{vert}(p_{i,j} - p_{i+1,j}) + T_{i,j}^{horiz}(p_{i,j} - p_{i,j-1}) + T_{i,j+1}^{horiz}(p_{i,j} - p_{i,j+1}) = 0$$

$$-T_{i,j}^{horiz} \mathbf{p}_{i,j-1} - T_{i,j}^{vert} \mathbf{p}_{i-1,j} + \left(\sum T \right) \mathbf{p}_{i,j} - T_{i+1,j}^{vert} \mathbf{p}_{i+1,j} - T_{i,j+1}^{horiz} \mathbf{p}_{i,j+1} = 0$$

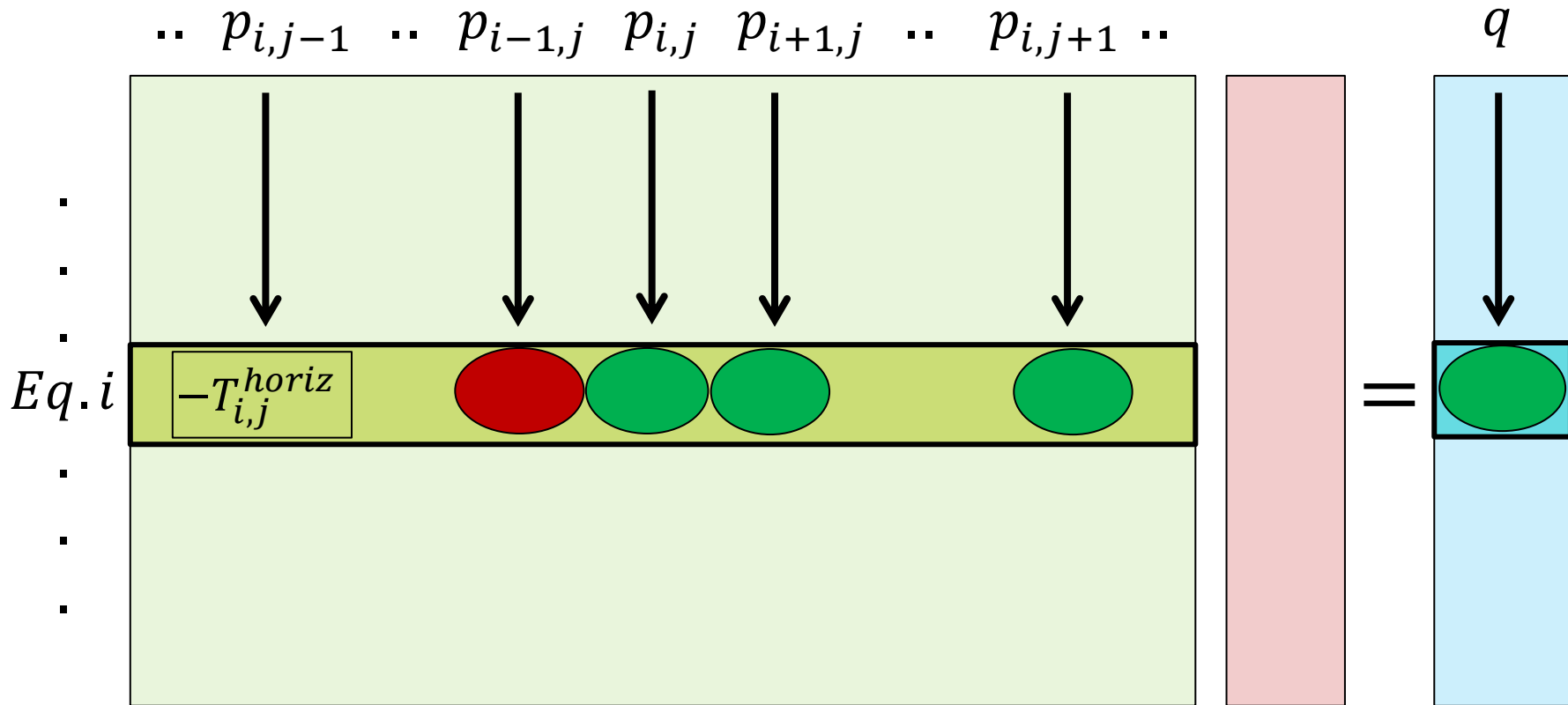
Incompressible system



$$T_{i,j}^{vert}(p_{i,j} - p_{i-1,j}) + T_{i+1,j}^{vert}(p_{i,j} - p_{i+1,j}) + T_{i,j}^{horiz}(p_{i,j} - p_{i,j-1}) + T_{i,j+1}^{horiz}(p_{i,j} - p_{i,j+1}) = 0$$

$$-T_{i,j}^{horiz} \mathbf{p}_{i,j-1} - T_{i,j}^{vert} \mathbf{p}_{i-1,j} + \left(\sum T \right) \mathbf{p}_{i,j} - T_{i+1,j}^{vert} \mathbf{p}_{i+1,j} - T_{i,j+1}^{horiz} \mathbf{p}_{i,j+1} = 0$$

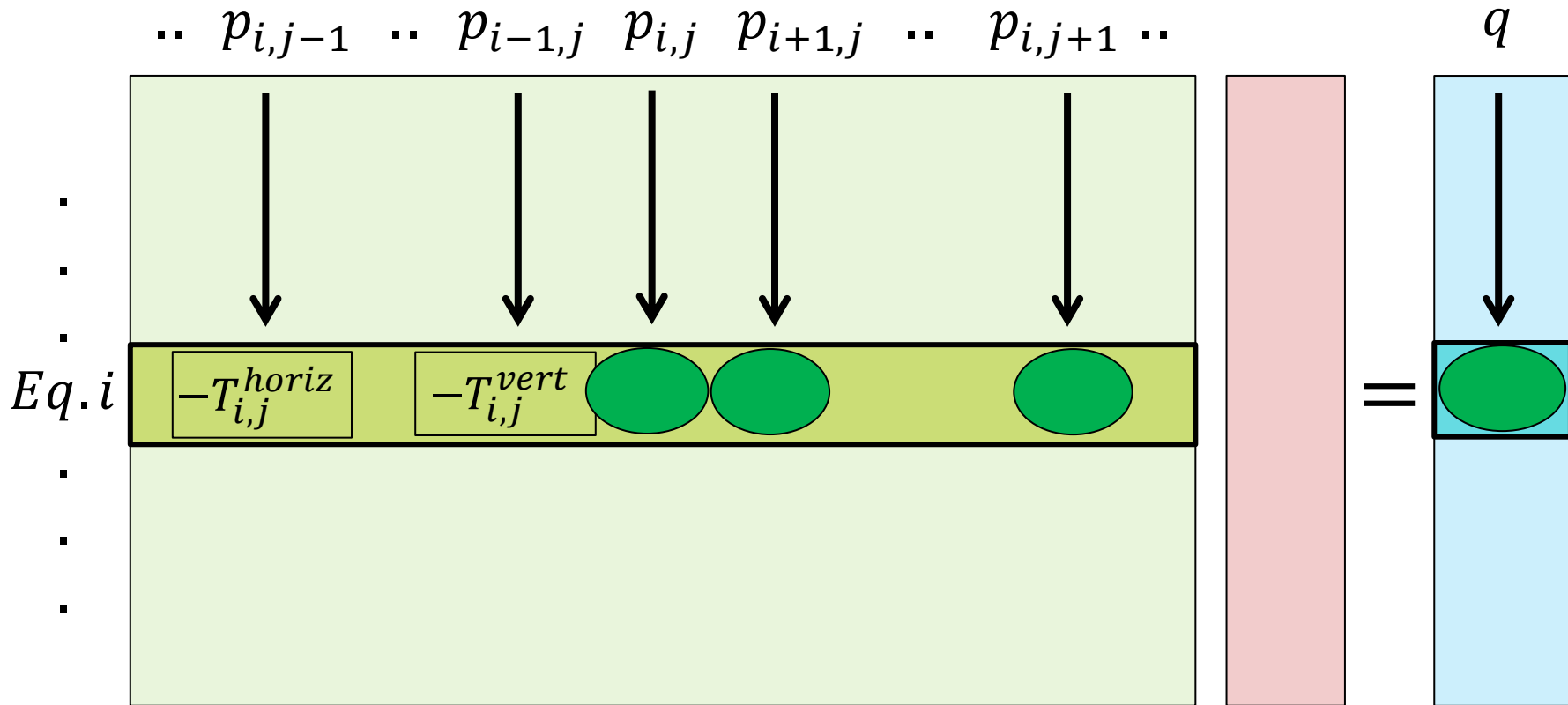
Incompressible system



$$T_{i,j}^{vert}(p_{i,j} - p_{i-1,j}) + T_{i+1,j}^{vert}(p_{i,j} - p_{i+1,j}) + T_{i,j}^{horiz}(p_{i,j} - p_{i,j-1}) + T_{i,j+1}^{horiz}(p_{i,j} - p_{i,j+1}) = 0$$

$$-T_{i,j}^{horiz} \mathbf{p}_{i,j-1} - T_{i,j}^{vert} \mathbf{p}_{i-1,j} + \left(\sum T \right) \mathbf{p}_{i,j} - T_{i+1,j}^{vert} \mathbf{p}_{i+1,j} - T_{i,j+1}^{horiz} \mathbf{p}_{i,j+1} = 0$$

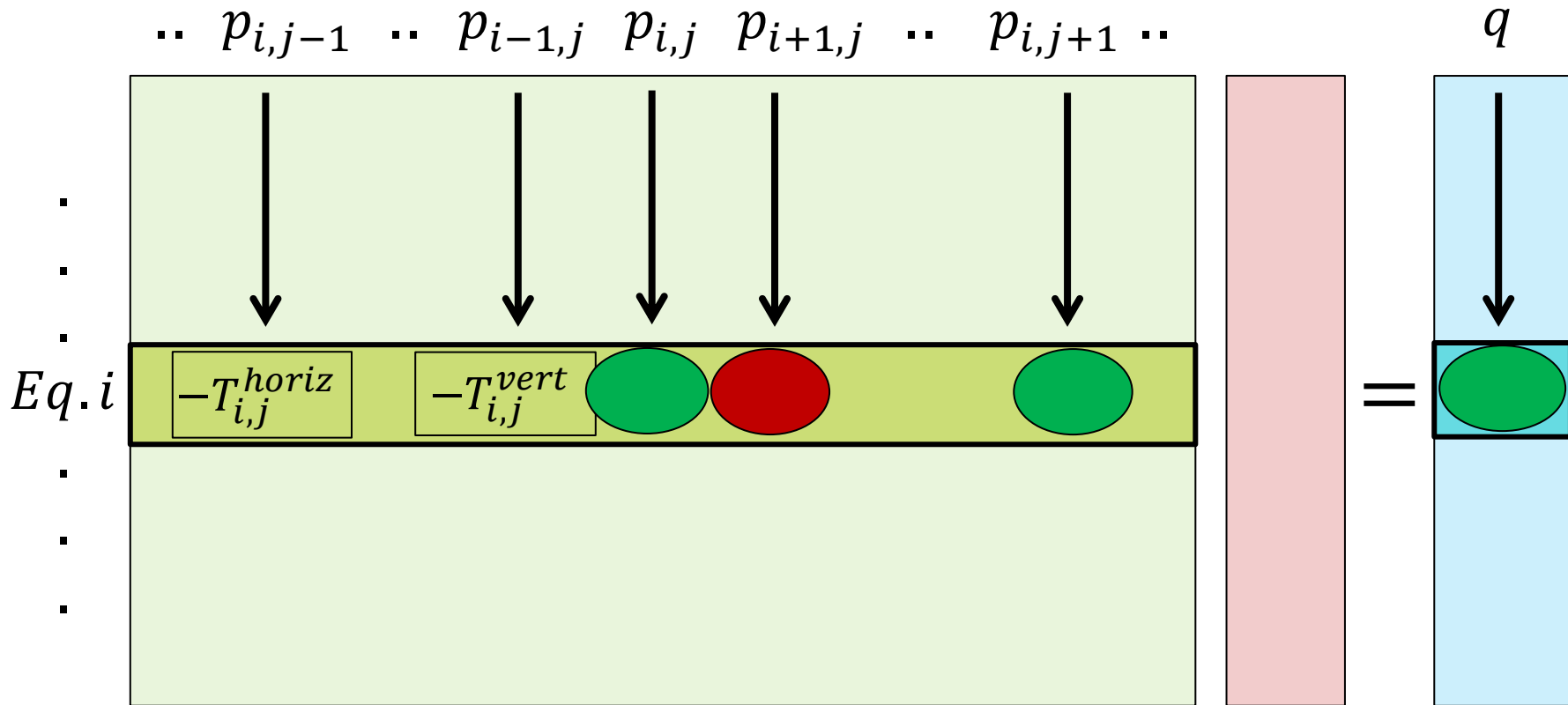
Incompressible system



$$T_{i,j}^{vert}(p_{i,j} - p_{i-1,j}) + T_{i+1,j}^{vert}(p_{i,j} - p_{i+1,j}) + T_{i,j}^{horiz}(p_{i,j} - p_{i,j-1}) + T_{i,j+1}^{horiz}(p_{i,j} - p_{i,j+1}) = 0$$

$$-T_{i,j}^{horiz} \mathbf{p}_{i,j-1} - T_{i,j}^{vert} \mathbf{p}_{i-1,j} + \left(\sum T \right) \mathbf{p}_{i,j} - T_{i+1,j}^{vert} \mathbf{p}_{i+1,j} - T_{i,j+1}^{horiz} \mathbf{p}_{i,j+1} = 0$$

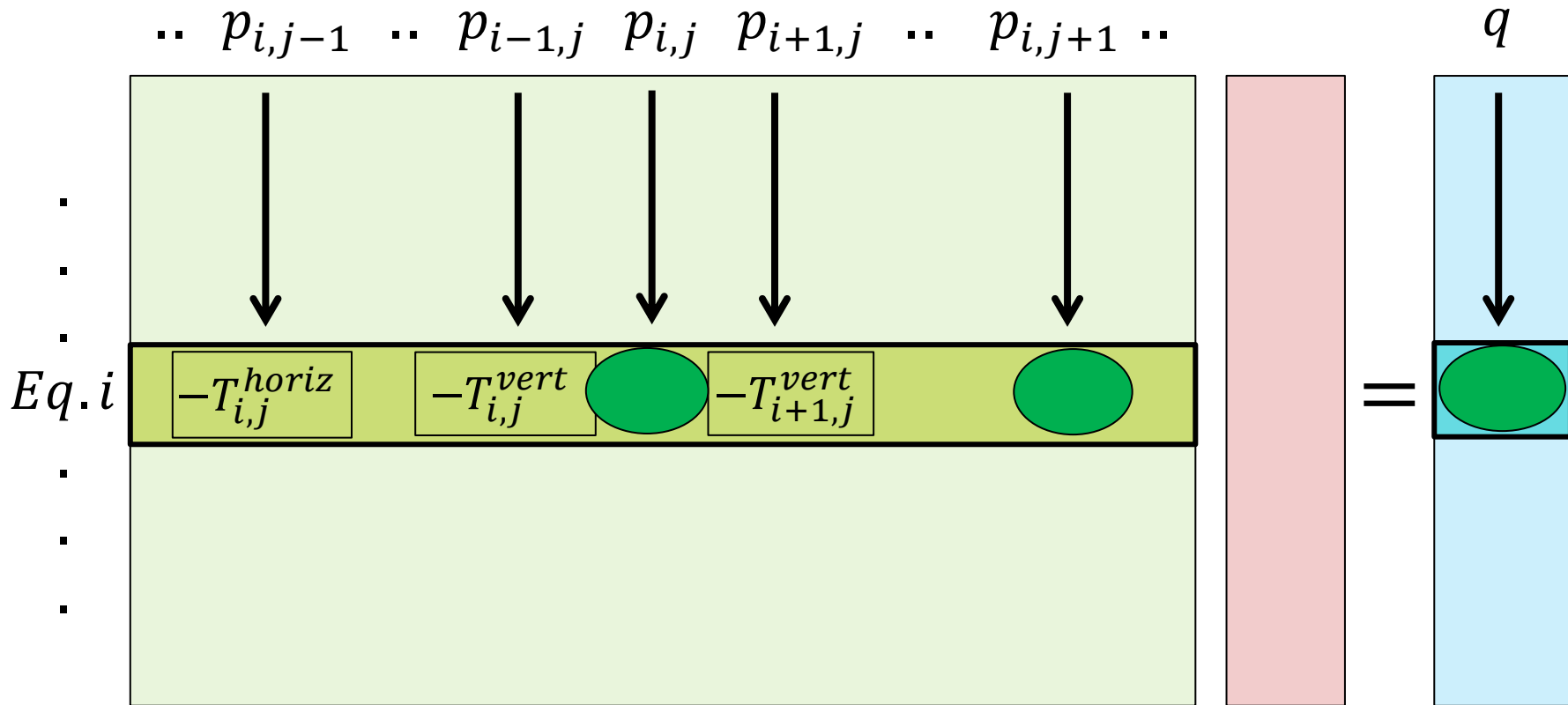
Incompressible system



$$T_{i,j}^{vert}(p_{i,j} - p_{i-1,j}) + T_{i+1,j}^{vert}(p_{i,j} - p_{i+1,j}) + T_{i,j}^{horiz}(p_{i,j} - p_{i,j-1}) + T_{i,j+1}^{horiz}(p_{i,j} - p_{i,j+1}) = 0$$

$$-T_{i,j}^{horiz} \mathbf{p}_{i,j-1} - T_{i,j}^{vert} \mathbf{p}_{i-1,j} + \left(\sum T \right) \mathbf{p}_{i,j} - T_{i+1,j}^{vert} \mathbf{p}_{i+1,j} - T_{i,j+1}^{horiz} \mathbf{p}_{i,j+1} = 0$$

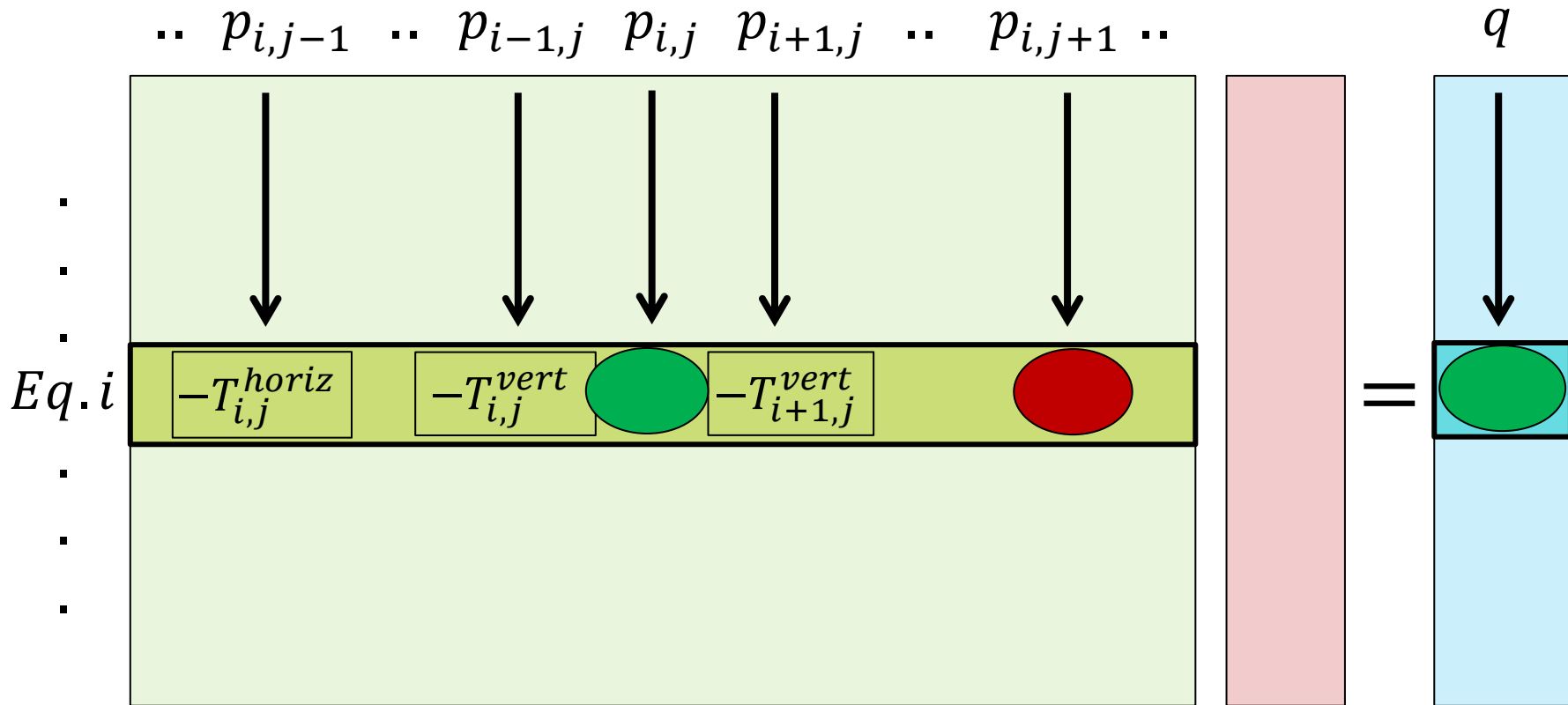
Incompressible system



$$T_{i,j}^{vert}(p_{i,j} - p_{i-1,j}) + T_{i+1,j}^{vert}(p_{i,j} - p_{i+1,j}) + T_{i,j}^{horiz}(p_{i,j} - p_{i,j-1}) + T_{i,j+1}^{horiz}(p_{i,j} - p_{i,j+1}) = 0$$

$$-T_{i,j}^{horiz} \mathbf{p}_{i,j-1} - T_{i,j}^{vert} \mathbf{p}_{i-1,j} + \left(\sum T \right) \mathbf{p}_{i,j} - T_{i+1,j}^{vert} \mathbf{p}_{i+1,j} - T_{i,j+1}^{horiz} \mathbf{p}_{i,j+1} = 0$$

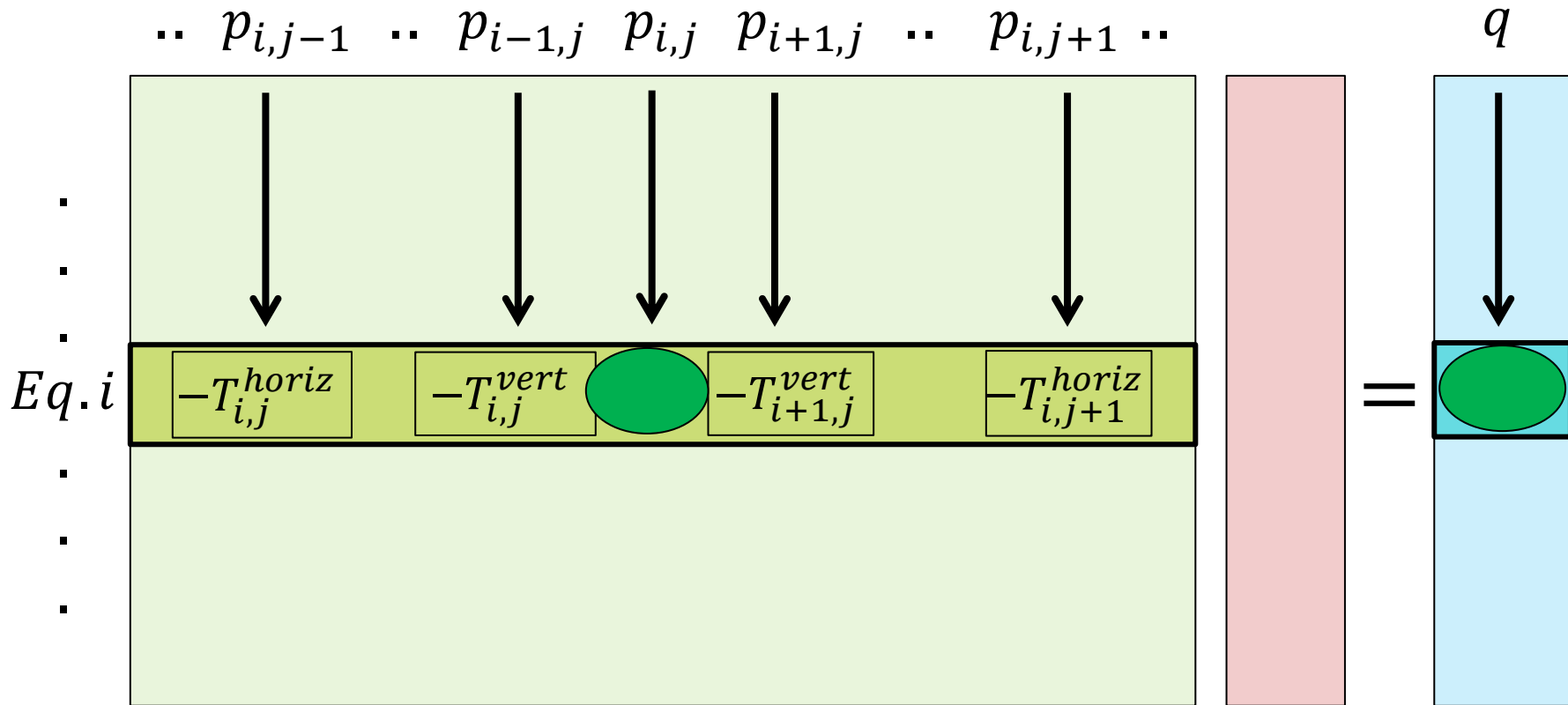
Incompressible system



$$T_{i,j}^{vert}(p_{i,j} - p_{i-1,j}) + T_{i+1,j}^{vert}(p_{i,j} - p_{i+1,j}) + T_{i,j}^{horiz}(p_{i,j} - p_{i,j-1}) + T_{i,j+1}^{horiz}(p_{i,j} - p_{i,j+1}) = 0$$

$$-T_{i,j}^{horiz} \mathbf{p}_{i,j-1} - T_{i,j}^{vert} \mathbf{p}_{i-1,j} + \left(\sum T \right) \mathbf{p}_{i,j} - T_{i+1,j}^{vert} \mathbf{p}_{i+1,j} - T_{i,j+1}^{horiz} \mathbf{p}_{i,j+1} = 0$$

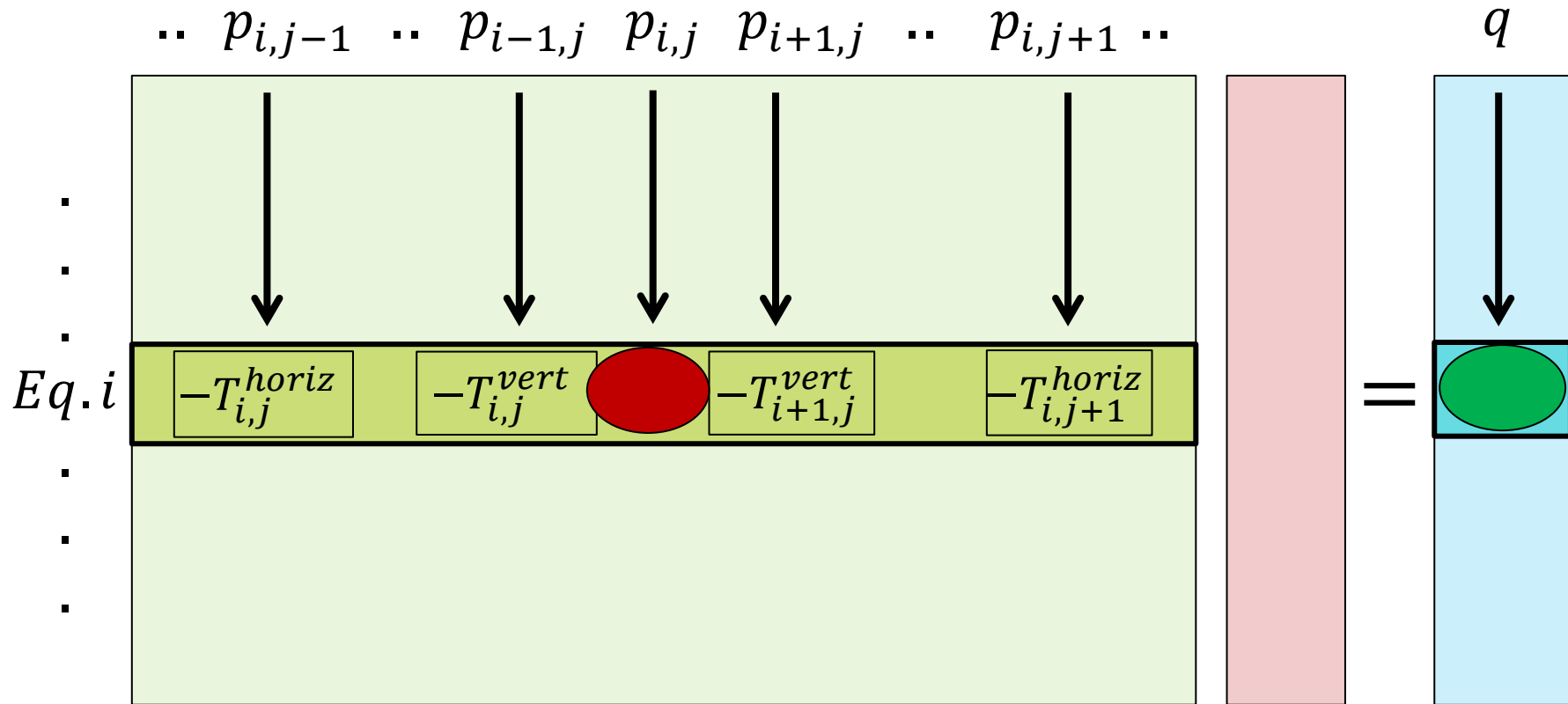
Incompressible system



$$T_{i,j}^{vert}(p_{i,j} - p_{i-1,j}) + T_{i+1,j}^{vert}(p_{i,j} - p_{i+1,j}) + T_{i,j}^{horiz}(p_{i,j} - p_{i,j-1}) + T_{i,j+1}^{horiz}(p_{i,j} - p_{i,j+1}) = 0$$

$$-T_{i,j}^{horiz} \mathbf{p}_{i,j-1} - T_{i,j}^{vert} \mathbf{p}_{i-1,j} + \left(\sum T \right) \mathbf{p}_{i,j} - T_{i+1,j}^{vert} \mathbf{p}_{i+1,j} - T_{i,j+1}^{horiz} \mathbf{p}_{i,j+1} = 0$$

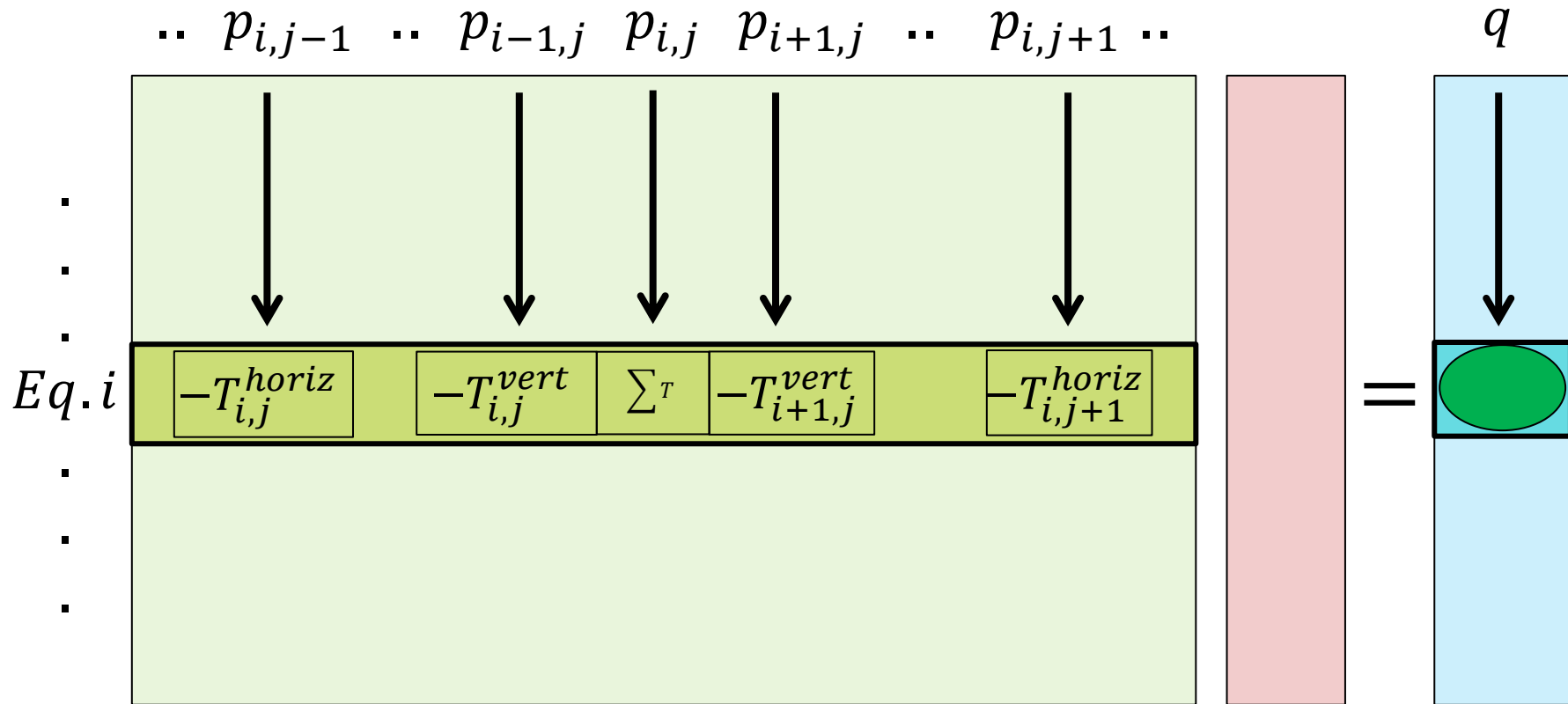
Incompressible system



$$T_{i,j}^{vert}(p_{i,j} - p_{i-1,j}) + T_{i+1,j}^{vert}(p_{i,j} - p_{i+1,j}) + T_{i,j}^{horiz}(p_{i,j} - p_{i,j-1}) + T_{i,j+1}^{horiz}(p_{i,j} - p_{i,j+1}) = 0$$

$$-T_{i,j}^{horiz} \mathbf{p}_{i,j-1} - T_{i,j}^{vert} \mathbf{p}_{i-1,j} + \left(\sum T \right) \mathbf{p}_{i,j} - T_{i+1,j}^{vert} \mathbf{p}_{i+1,j} - T_{i,j+1}^{horiz} \mathbf{p}_{i,j+1} = 0$$

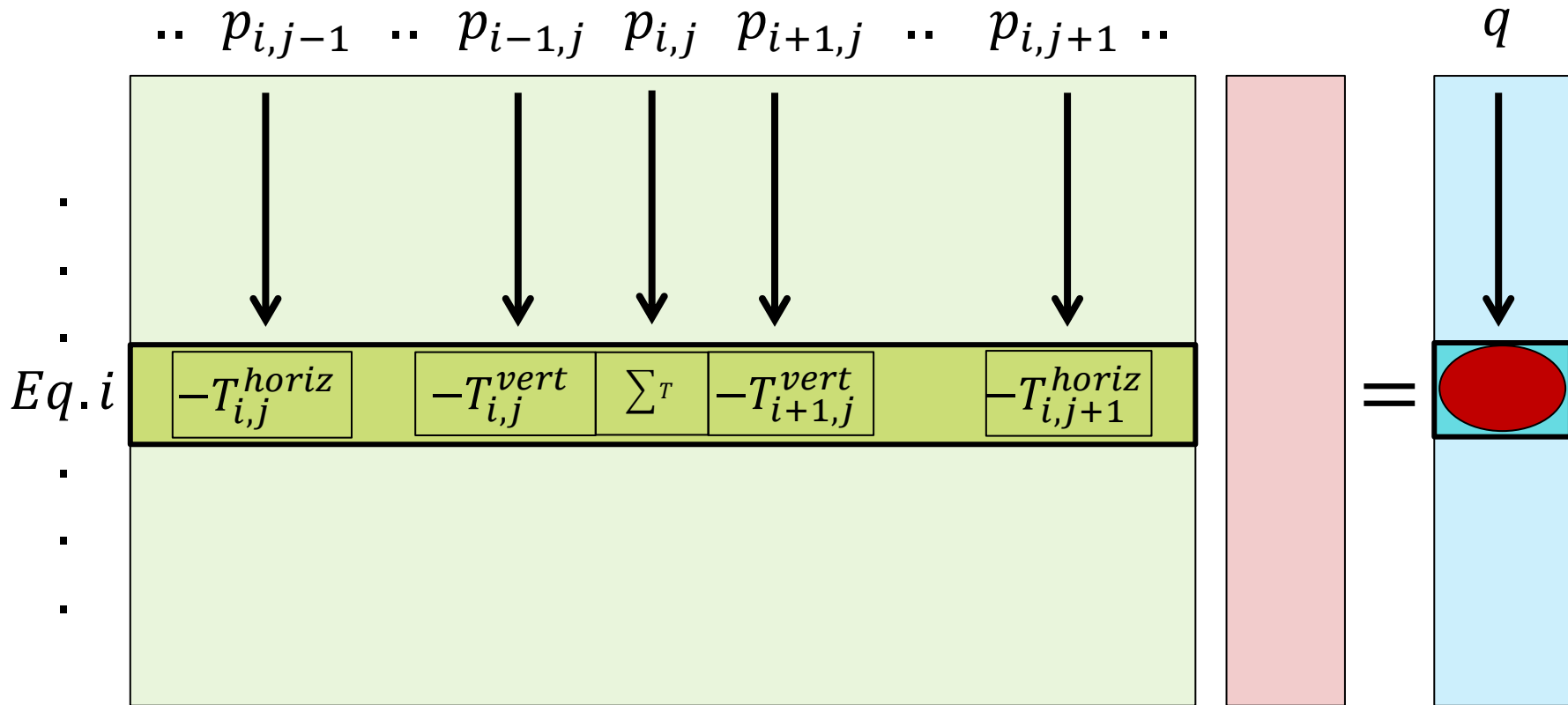
Incompressible system



$$T_{i,j}^{vert}(p_{i,j} - p_{i-1,j}) + T_{i+1,j}^{vert}(p_{i,j} - p_{i+1,j}) + T_{i,j}^{horiz}(p_{i,j} - p_{i,j-1}) + T_{i,j+1}^{horiz}(p_{i,j} - p_{i,j+1}) = 0$$

$$-T_{i,j}^{horiz} \mathbf{p}_{i,j-1} - T_{i,j}^{vert} \mathbf{p}_{i-1,j} + \left(\sum T \right) \mathbf{p}_{i,j} - T_{i+1,j}^{vert} \mathbf{p}_{i+1,j} - T_{i,j+1}^{horiz} \mathbf{p}_{i,j+1} = 0$$

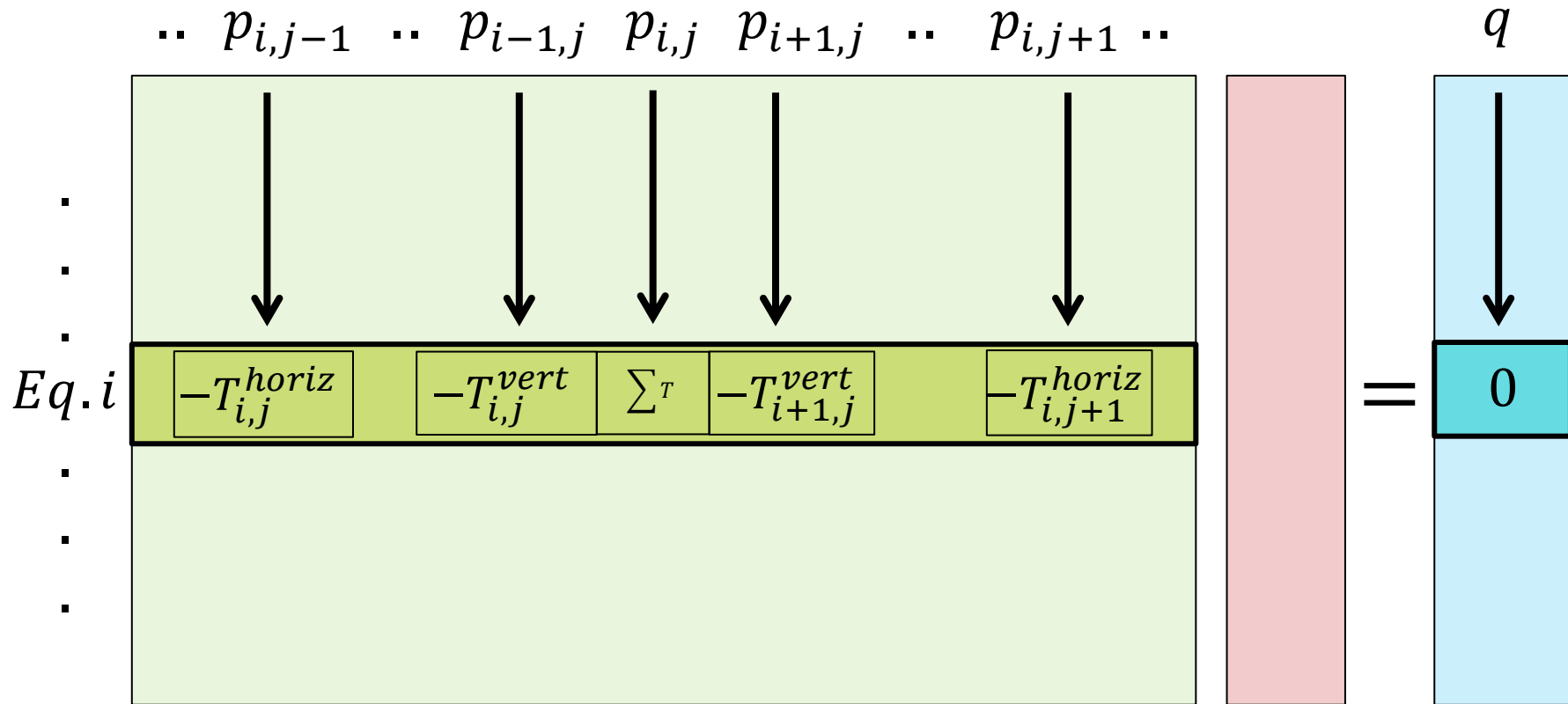
Incompressible system



$$T_{i,j}^{vert}(p_{i,j} - p_{i-1,j}) + T_{i+1,j}^{vert}(p_{i,j} - p_{i+1,j}) + T_{i,j}^{horiz}(p_{i,j} - p_{i,j-1}) + T_{i,j+1}^{horiz}(p_{i,j} - p_{i,j+1}) = 0$$

$$-T_{i,j}^{horiz} \mathbf{p}_{i,j-1} - T_{i,j}^{vert} \mathbf{p}_{i-1,j} + \left(\sum T \right) \mathbf{p}_{i,j} - T_{i+1,j}^{vert} \mathbf{p}_{i+1,j} - T_{i,j+1}^{horiz} \mathbf{p}_{i,j+1} = 0$$

Incompressible system

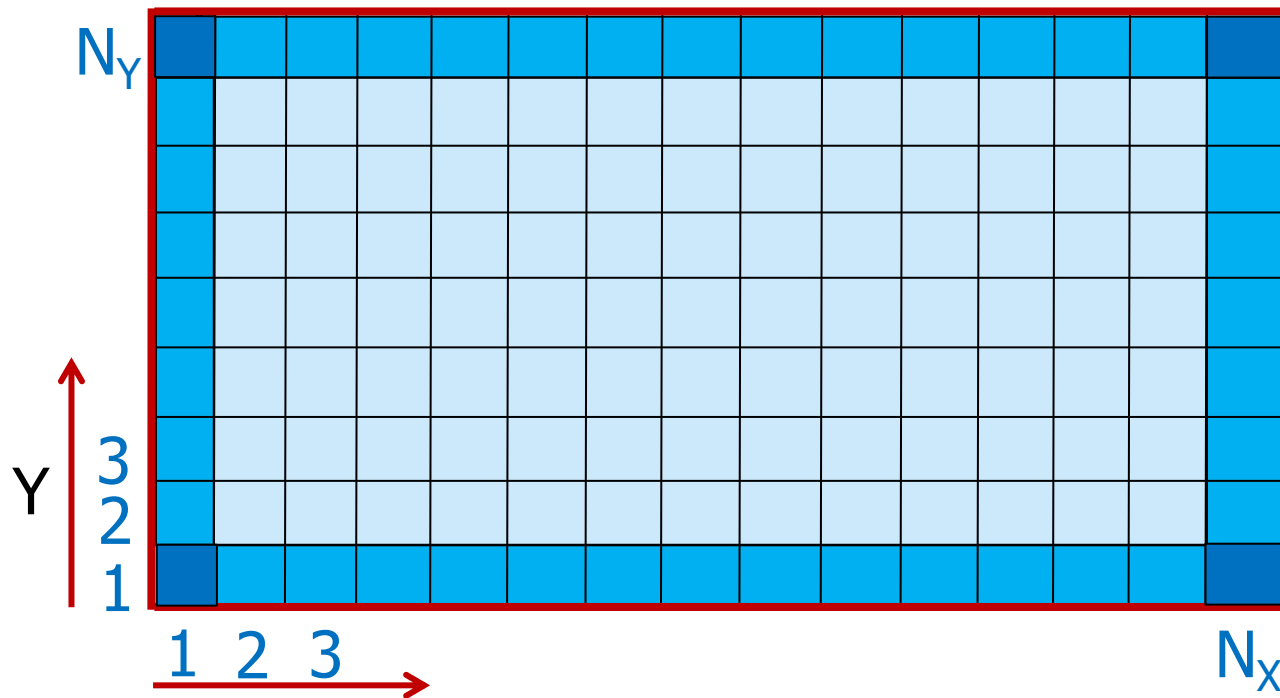


$$T_{i,j}^{vert}(p_{i,j} - p_{i-1,j}) + T_{i+1,j}^{vert}(p_{i,j} - p_{i+1,j}) + T_{i,j}^{horiz}(p_{i,j} - p_{i,j-1}) + T_{i,j+1}^{horiz}(p_{i,j} - p_{i,j+1}) = 0$$

$$-T_{i,j}^{horiz} \mathbf{p}_{i,j-1} - T_{i,j}^{vert} \mathbf{p}_{i-1,j} + \left(\sum T \right) \mathbf{p}_{i,j} - T_{i+1,j}^{vert} \mathbf{p}_{i+1,j} - T_{i,j+1}^{horiz} \mathbf{p}_{i,j+1} = 0$$

Boundary cells

“if” statements



Boundary conditions

