

Module 4 Lab Assessment: Q3

Group No.: 14

$$\text{tol} = 10^{-10}$$

a)

Expected eigenvalues are:

$$\begin{bmatrix} 8.5844e+00 \\ 1.2207e+00 \\ 2.1949e+00 \end{bmatrix}$$

for m = 1 No. of iteration: 4, B:

$$\begin{bmatrix} 8.5887e+00 & 1.3560e+00 & 3.3425e+00 \\ -2.3253e-02 & 1.2164e+00 & -9.6205e-01 \\ 0 & 9.6083e-13 & 2.1949e+00 \end{bmatrix}$$

We can see that the entry $|B_{3,2}| < \text{tol}$, so the matrix can be deflated into the upper 2x2 diagonal block:

$$\begin{bmatrix} 8.5887e+00 & 1.3560e+00 \\ -2.3253e-02 & 1.2164e+00 \end{bmatrix}$$

and the lower 1x1 diagonal block:

$$[2.1949e+00]$$

The 1x1 diagonal block has eigenvalue 2.1949e+00.

Eigenvalues of the 2x2 diagonal block:

$$\begin{bmatrix} 8.5844e+00 \\ 1.2207e+00 \end{bmatrix}$$

The eigenvalues of a block upper triangular matrix (B) are the union of the eigenvalues of the diagonal blocks. Hence, we have retrieved all eigenvalues of A through our code.

for m = 2

No. of iteration: 4, B:

$$\begin{bmatrix} 1.2207e+00 & 1.3768e+00 & 1.5533e+00 \\ 1.1050e-14 & 8.5796e+00 & 3.1230e+00 \\ 0 & 9.8533e-03 & 2.1997e+00 \end{bmatrix}$$

We can see that the entry $|B_{2,1}| < tol$, so the matrix can be deflated into the lower 2x2 diagonal block:

$$\begin{bmatrix} 8.5796e+00 & 3.1230e+00 \\ 9.8533e-03 & 2.1997e+00 \end{bmatrix}$$

and the upper 1x1 diagonal block:

$$[1.2207e+00]$$

The 1x1 diagonal block has eigenvalue 1.2207e+00.

Eigenvalues of the 2x2 diagonal block:

$$\begin{bmatrix} 8.5844e+00 \\ 2.1949e+00 \end{bmatrix}$$

The eigenvalues of a block upper triangular matrix (B) are the union of the eigenvalues of the diagonal blocks. Hence, we have retrieved all eigenvalues of A through our code.

b) Expected eigenvalues are::

$$\begin{bmatrix} 2.3801e+00 + 2.5364e+00i \\ 2.3801e+00 - 2.5364e+00i \\ 1.2399e+00 + 0.0000e+00i \end{bmatrix}$$

for m = 1

No. of iteration: 6, B:

$$\begin{bmatrix} 2.1269e+00 & -2.4424e+00 & 3.4363e+00 \\ 2.6603e+00 & 2.6333e+00 & -3.9348e-01 \\ 0 & 4.1936e-21 & 1.2399e+00 \end{bmatrix}$$

We can see that the entry $|B_{3,2}| < tol$, so the matrix can be deflated into the upper 2x2 diagonal block:

$$\begin{bmatrix} 2.1269e+00 & -2.4424e+00 \\ 2.6603e+00 & 2.6333e+00 \end{bmatrix}$$

and the lower 1x1 diagonal block:

$$[1.2399e+00]$$

The 1x1 diagonal block has eigenvalue 1.2399e+00.

Eigenvalues of the 2x2 diagonal block:

$$\begin{bmatrix} 2.3801e+00 + 2.5364e+00i \\ 2.3801e+00 - 2.5364e+00i \end{bmatrix}$$

The eigenvalues of a block upper triangular matrix (B) are the union of the eigenvalues of the diagonal blocks. Hence, we have retrieved all eigenvalues of

A through our code.

for $m = 2$

No. of iteration: 6, B:

$$\begin{bmatrix} 2.1045e+00 & -2.5553e+00 & -3.4427e+00 \\ 2.5474e+00 & 2.6557e+00 & -3.3261e-01 \\ 0 & -1.5874e-13 & 1.2399e+00 \end{bmatrix}$$

We can see that the entry $|B_{3,2}| < tol$, so the matrix can be deflated into the upper 2x2 diagonal block:

$$\begin{bmatrix} 2.1045e+00 & -2.5553e+00 \\ 2.5474e+00 & 2.6557e+00 \end{bmatrix}$$

and the lower 1x1 diagonal block:

$$[1.2399e+00]$$

The 1x1 diagonal block has eigenvalue 1.2399e+00.

Eigenvalues of the 2x2 diagonal block:

$$\begin{bmatrix} 2.3801e+00 + 2.5364e+00i \\ 2.3801e+00 - 2.5364e+00i \end{bmatrix}$$

The eigenvalues of a block upper triangular matrix (B) are the union of the eigenvalues of the diagonal blocks. Hence, we have retrieved all eigenvalues of A through our code.