Lab Sheet 8

Q2

Observations are taken for k=20.

a) Observations:

Part (a)

The value of lambda_1: 8.584428e+00

The value of lambda_2: 2.194882e+00

The vector v is:

1.098874252077420e-01

9.826756686582475e-01

-1.492423666667051e-01

The value of lambda returned by Powermethod is 8.584428e+00

The value of v returned by Powermethod is:

1.098874252086898e-01

9.826756686583382e-01

-1.492423666654101e-01

The value of |lambda_2|/|lambda_1| is 2.556818e-01

The value of |iter(:,j+1)-v|/|iter(:,j)-v| are:

2.496008e-01

2.518779e-01

2.542407e-01

2.551022e-01

2.554195e-01

2.555515e-01

2.556134e-01

2.556448e-01

2.556614e-01

2.556705e-01

2.556755e-01

2.556783e-01

2.556798e-01

2.556807e-01

2.556811e-01

2.556810e-01

2.556796e-01

2.556742e-01

2.556544e-01

Analysis:

We see that practical convergence rate is approximately equal to the theoretical convergence rate for large enough j and the values of λ and v agree quite well for eig and Powermethod.

b) Observations:

Part (b)

The value of lambda_1: 8.455873e+00

The value of lambda_2: 2.455873e+00

The vector v is:

-1.016822854981159e-01

-9.716357737105396e-01

2.135055878936208e-01

The value of lambda returned by Powermethod is 8.455873e+00

The value of v returned by Powermethod is:

1.016822854856139e-01

9.716357737055653e-01

-2.135055879222119e-01

The value of |lambda_2|/|lambda_1| is 2.904340e-01

The value of |iter(:,j+1)-v|/|iter(:,j)-v| are:

1.002945e+00

1.001445e+00

1.000246e+00

1.000013e+00

1.000000e+00

1.000000e+00

1.000000e+00

1.000000e+00

1.000000e+00

1.000000e+00

1.000000e+00

1.000000e+00

1.000000e+00

1.000000e+00

1.000000e+00

1.000000e+00

1.000000e+00

1.000000e+00

1.000000e+00

Analysis:

We see that practical convergence rate does not agree with the theoretical convergence rate even though the values of λ and v agree quite well for eig and Powermethod. The reason for disagreement in the convergence rates is that the matrix in (b) does not have 3 independent eigen vectors, it has only 2. Since, $|\lambda_1| > |\lambda_2| = |\lambda_3|$, the convergence is not linear.

c) Observations:

Part (c)

The value of lambda_1: 3.478227e+00

The value of lambda_2: 3.478227e+00

The vector v is:

-6.422511290267512e-02 - 3.154888868032283e-01i

-2.184797049459368e-02 - 5.982964576162619e-01i

7.334206924851507e-01 + 0.000000000000000e+00i

The value of lambda returned by Powermethod is 3.593669e+00

The value of v returned by Powermethod is:

-4.478419169077836e-01

-7.930081695547689e-01

4.130080634560986e-01

The value of |lambda_2|/|lambda_1| is 1.000000e+00

The value of |iter(:,j+1)-v|/|iter(:,j)-v| are:

1.101521e+00

9.579816e-01

8.049284e-01

6.793814e-01

7.456379e-01

1.334040e+00

1.509337e+00

1.216206e+00

1.046778e+00

9.161912e-01

7.556671e-01

6.663134e-01

8.685444e-01

1.503425e+00

1.416420e+00

1.150329e+00

1.008313e+00

8.673616e-01

7.138103e-01

Analysis:

We see that practical convergence rate does not agree with the theoretical convergence rate. Infact, the values of λ and v also disagree for eig and Powermethod. The reason for disagreement in the convergence rates is that the matrix in (c) does not have 3 independent eigen vectors, it has only 2. Since, $|\lambda_1| = |\lambda_2| > |\lambda_3|$, the assumption of the Powermethod that we have dominant eigenvalue is not satisfied by λ_1, thus, we don't see convergence in Powermethod.