1.1.4

a) $\int 2x(x^2+1)^{21}\,dx$

$u = x^2+1$

$\therefore du = 2x\,dx$

$= \int u^{21}\,du$

$= \frac{u^{22}}{22} + c$

$= \frac{1}{22}(x^2+1)^{22} + c$ (Ans)

b) $\int \frac{\sec^2(\ln x)}{x}\,dx$

$u = \ln x$

$\therefore du = \frac{1}{x}\,dx$

$= \int \sec^2 u\,du$

$= \tan u + c$

$= \tan(\ln x) + c$ (Ans)

c) $\int \frac{e^{3x}}{e^{3x}+5}$

$u = e^{3x}+5$

$\therefore du = 3e^{3x}\,dx$

$\therefore e^{3x}\,dx = \frac{du}{3}$

$= \int \frac{1}{u} \cdot \frac{du}{3}$

$= \frac{1}{3}\int \frac{du}{u}$

$= \frac{1}{3}\ln u + c$

$= \frac{1}{3}\ln|e^{3x}+5| + c$ (Ans)

d) $\int \cos^3 x \sin x \, dx$

$= \int u^3(-du)$

$= -\int u^3 du$

$= -\frac{u^4}{4} + C$

$= -\frac{1}{4}\cos^4 x + C$ (Ans)

$u = \cos x$

$\therefore du = -\sin x \, dx$

$\therefore \sin x \, dx = -du$

e) $\int \frac{x^3}{(x^4+1)^5} dx$

$= \int \frac{1}{u^5} \cdot \frac{du}{4}$

$= \frac{u^{-5+1}}{-5+1} + C$

$= (x^4+1)^{-4} + C$ (Ans)

$u = x^4 + 1$

$\therefore du = 4x^3 dx$

$\therefore x^3 dx = \frac{du}{4}$

f) $\int \frac{(1+\ln x)^3}{x} dx$

$= \int u^3 \cdot du$

$= \frac{u^4}{4} + C$

$= \frac{(1+\ln x)^4}{4} + C$

(Ans)

$u = 1 + \ln x$

$du = \frac{1}{x} \cdot dx$

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g) $\int \frac{\cos x}{(1+\sin x)^5}\,dx$

$u = 1+\sin x$

$\therefore du = \cos x\, dx$

$= \int \frac{du}{u^5}$

$= \frac{u^{-5+1}}{-5+1} + c$

$= -\frac{1}{4}(1+\sin x)^{-4} + c$ (Ans)

h) $\int \sin 3x \sqrt{2+\cos 3x}\,dx$

$u = 2+\cos 3x$

$\therefore du = (-\sin 3x)\cdot 3\,dx$

$\therefore \sin 3x\, dx = -\frac{du}{3}$

$= \int \sqrt{u}\cdot\left(-\frac{du}{3}\right)$

$= -\frac{1}{3}\int \sqrt{u}\, du$

$= -\frac{1}{3}\,\frac{u^{3/2}}{3/2} + c$

$= -\frac{2}{9}u^{3/2} + c$

$= -\frac{2}{9}(2+\cos 3x)^{3/2} + c$ (Ans)

i) $\int \frac{\cos(2/x)}{x^2}\,dx$

$u = \frac{2}{x}$

$\therefore du = 2\cdot(-x^{-2})\,dx$

$\therefore x^{-2}\,dx = -\frac{du}{2}$

$= \int \cos(2/x)\, x^{-2}\,dx$

$= \int \cos u\cdot\left(-\frac{du}{2}\right)$

$= -\frac{1}{2}\int \cos u\, du.$

$$= -\frac{1}{2}\sin u + c$$

$$= -\frac{1}{2}\sin\left(\frac{2}{x}\right) + c \quad \text{(Ans)}$$

j) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$

$u = \sqrt{x} = x^{1/2}$

$\therefore du = \frac{1}{2}x^{-1/2}\,dx$

$\therefore x^{-1/2}\,dx = 2du$

$$= \int e^u \cdot x^{-1/2}\,dx$$

$$= \int e^u (2du)$$

$$= 2\int e^u\,du$$

$$= 2\cdot e^u + c$$

$$= 2e^{\sqrt{x}} + c \quad \text{(Ans)}$$

k) $\int \frac{1}{x(1+\ln x)^3}\,dx$

$u = 1 + \ln x$

$\therefore du = \frac{1}{x}\,dx$

$$= \int \frac{1}{(1+\ln x)^3}\cdot\frac{1}{x}\,dx$$

$$= \int \frac{1}{u^3}\cdot du$$

$$= \frac{u^{-3+1}}{-3+1} + c$$

$$= -\frac{1}{2}u^{-2} + c$$

$$= -\frac{1}{2}(1+\ln x)^{-2} + c$$

l) $\int \frac{e^{-3x}}{\sqrt{3+e^{-3x}}}dx$

$u = 3+e^{-3x}$

$\therefore du = (-3)e^{-3x}\,dx$

$\therefore e^{-3x}dx = -\frac{1}{3}du$

$= \int \frac{1}{\sqrt{u}} \cdot \left(-\frac{1}{3}du\right)$

$= -\frac{1}{3}\int u^{-1/2}du$

$= -\frac{1}{3} \cdot \frac{u^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + c$

$= -\frac{1}{3} \cdot \frac{2}{1} \cdot u^{1/2} + c$

$= -\frac{2}{3}\sqrt{u} + c$

$= -\frac{2}{3}\sqrt{3+e^{-3x}} + c$ (An)

m) $\int \frac{e^{m(\arctan x)}}{1+x^2}dx$

$u = \arctan x$

$\Rightarrow u = \tan^{-1}x$

$\therefore du = \frac{1}{1+x^2}dx$

$= \int e^{mu} \cdot du$

$= \frac{e^{mu}}{m} + c$

$= \frac{1}{m}e^{m(\arctan x)} + c$

(An)

n) $\int \frac{e^x}{e^x+1}\,dx$

$u = e^x + 1$

$\therefore du = e^x\,dx$

$= \int \frac{du}{u}$

$= \ln|u| + c$

$= \ln|e^x + 1| + c$ (Ans)

o) $\int 4\tan^3 x \sec^2 x\,dx$

$u = \tan x$

$\therefore du = \sec^2 x\,dx$

$= \int 4 u^3 \cdot du$

$= 4 \cdot \frac{u^4}{4} + c$

$= \tan^4 x + c$

(Ans)