2.2.1

2. a) $y = x^2$

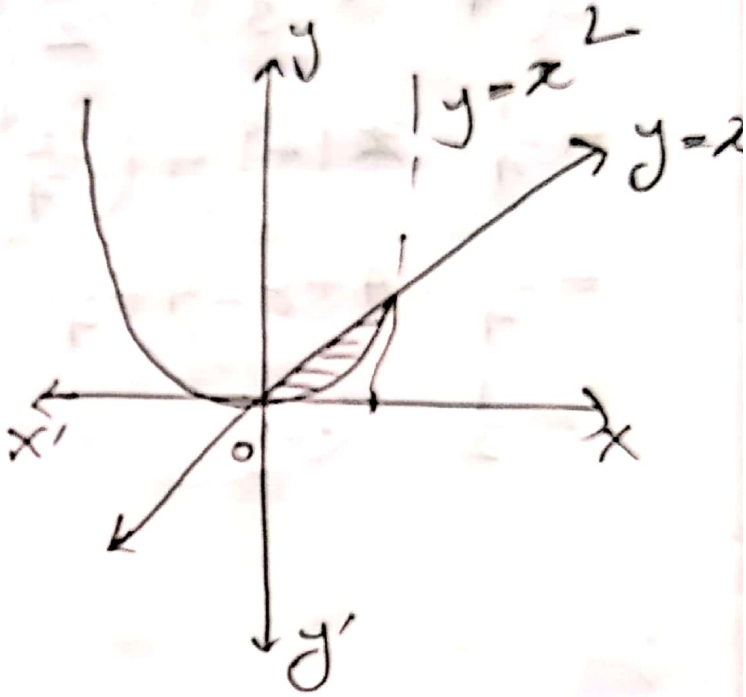
$y = x$

$\therefore x^2 = x$

$\Rightarrow x^2 - x = 0$

$\Rightarrow x(x-1) = 0$

$\therefore x = 0, 1$



$$A = \int_0^1 (x - x^2)\, dx$$

$$= \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1$$

$$= \left(\frac{1}{2} - \frac{1}{3}\right) - 0$$

$$= \frac{1}{6} \quad \text{(Ans)}$$

b) $y = x(x-3)$

$= x^2 - 3x$

$$A = -\int_0^3 (x^2 - 3x)\,dx + \int_3^5 (x^2 - 3x)\,dx$$

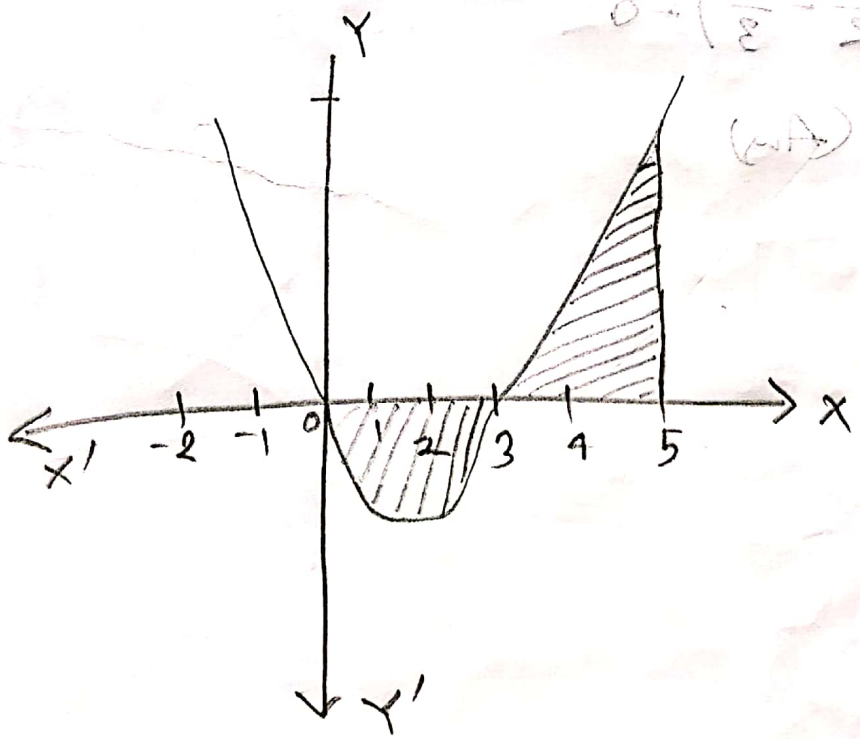
$$= -\left[\frac{x^3}{3} - \frac{3x^2}{2}\right]_0^3 + \left[\frac{x^3}{3} - \frac{3x^2}{2}\right]_3^5$$

$$= \left\{\left(\frac{125}{3} - \frac{75}{2}\right) - \left(\frac{27}{3} - \frac{27}{2}\right)\right\} - \left\{\left(\frac{27}{3} - \frac{27}{2}\right) - 0\right\}$$

$$= \frac{79}{6}$$

for graph,

x	0	-1	1	-2	2	-3	3	4	-4	5	-5
y	0	4	-2	10	-2	18	0	4	32	10	40



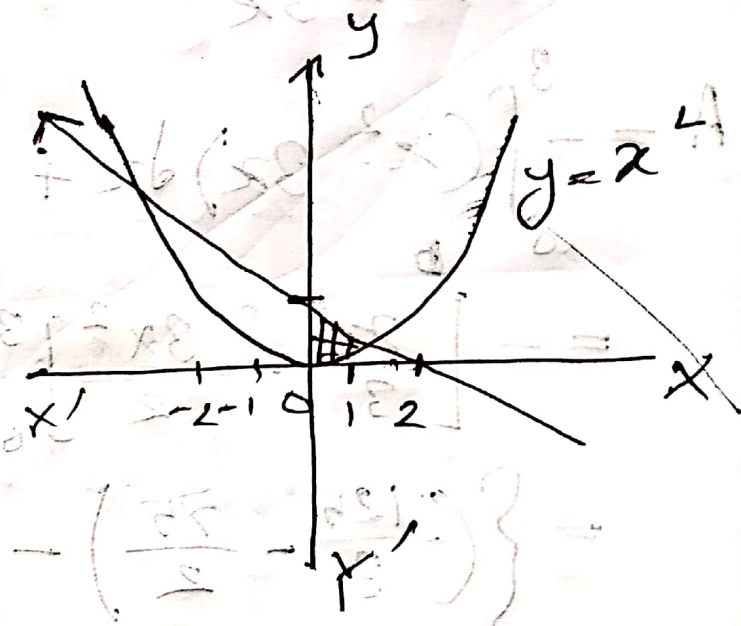
e) $y = x^2$ $[x=0, x \geq 0]$

$y = 2 - x$

$x^2 = 2 - x$

$\Rightarrow x^2 + x - 2 = 0$

$\Rightarrow (x+2)(x-1) = 0$



$$A = \int_0^1 \{(2-x) - x^2\}\, dx$$

$$= \int_0^1 (2 - x - x^2)\, dx$$

$$= \left[2x - \frac{x^2}{2} - \frac{x^3}{3}\right]_0^1$$

$$= \left(2 - \frac{1}{2} - \frac{1}{3}\right) - 0$$

$$= \frac{7}{6} \quad \text{(Ans)}$$

d) $y = 3x - x^2$

$y = x$

$3x - x^2 = x$

$\Rightarrow 3x - x - x^2 = 0$

$\Rightarrow x(x-2) = 0$

$\Rightarrow x = 0, 2$

for graph

x	0	1	2	3	4	5
y	0	2	2	0	-4	-10

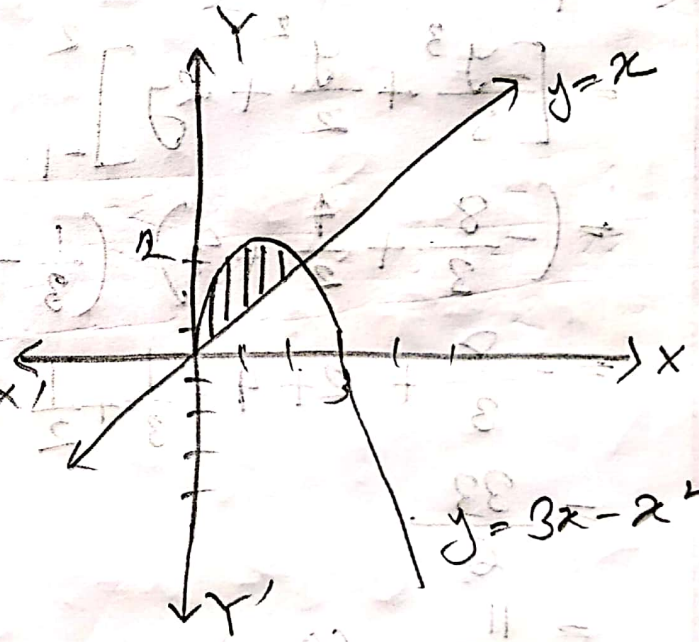
$$A = \int_0^2 (3x - x^2 - x)\,dx$$

$$= \int_0^2 (2x - x^2)\,dx$$

$$= \left[\frac{2x^2}{2} - \frac{x^2}{3}\right]_0^2$$

$$= \left(4 - \frac{8}{3}\right) - 0$$

$$= \frac{4}{3} \quad (\text{Ans})$$

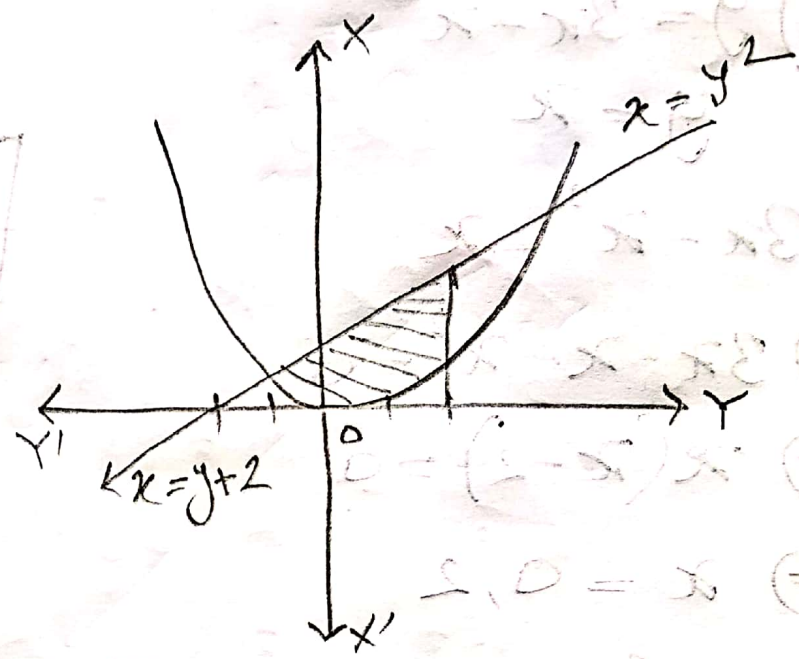


e) $x = y^2$

and $y = x - 2$

$\Rightarrow x = y + 2 = 0$

$\Rightarrow y = 2, -1$



$$A = \int_{-1}^{2} \{(y+2) - y^2\} dx$$

$$= \left[\frac{-y^3}{3} + \frac{y^2}{2} + 2y\right]_{-1}^{2}$$

$$= \left(-\frac{8}{3} + \frac{4}{2} + 4\right) - \left(\frac{1}{3} - \frac{1}{2} - 2\right)$$

$$= -\frac{8}{3} + 2 + 4 - \frac{1}{3} + \frac{1}{2} + 2$$

$$= \frac{33}{6}$$

$$= \frac{11}{2} \quad \text{(Ans)}$$

2.2.2

1. a) $y = 3 - 2x$

$\Rightarrow x = \frac{3}{2} - \frac{y}{2}$

$a = 0$

$b = 2$

$$V_y = \int_a^b \pi x^2 \, dy$$

$$= \frac{\pi}{4} \int_0^2 (3-y)^2 \, dy$$

$$= \frac{\pi}{4} \int_0^2 (9 - 6y + y^2) \, dy$$

$$= \frac{\pi}{4} \left[9y - \frac{6y^2}{2} + \frac{y^3}{3} \right]_0^2$$

$$= \frac{\pi}{4} \left\{ \left(18 - 12 + \frac{8}{3} \right) - 0 \right\}$$

$$= \frac{13\pi}{6} \quad \text{(Ans)}$$

b) $y = \sqrt{\cos x}$

$a = \frac{\pi}{4}$

$b = \frac{\pi}{2}$

$$V_x = \int_a^b \pi y^2 \, dx$$

$$V_x = \pi \int_{\pi/4}^{\pi/2} (\sqrt{\cos x})^2 \, dx$$

$$= \pi \left[\sin x \right]_{\pi/4}^{\pi/2}$$

$$= \pi \left(1 - \frac{1}{\sqrt{2}} \right)$$

$$= \pi - \frac{\pi}{\sqrt{2}} \quad \text{(Ans)}$$

c) $x = \sqrt{1+y}$

$a = -1$

$b = 3$

$$V_y = \int_a^b \pi x^2 \, dy$$

$$V_y = \int_{-1}^{3} \pi (\sqrt{1+y})^2 \, dy$$

$$= \pi \int_{-1}^{3} (1+y) \, dy$$

$$= \pi \left[y + \frac{y^2}{2} \right]_{-1}^{3}$$

$$= \pi \left\{ \left(3 + \frac{9}{2}\right) - \left(-1 + \frac{1}{2}\right) \right\}$$

$$= \pi \left(3 + \frac{9}{2} + 1 - \frac{1}{2}\right)$$

$$= 8\pi \quad \text{(Ans)}$$

21 a) $y = \sqrt{x}$, $x = 9$

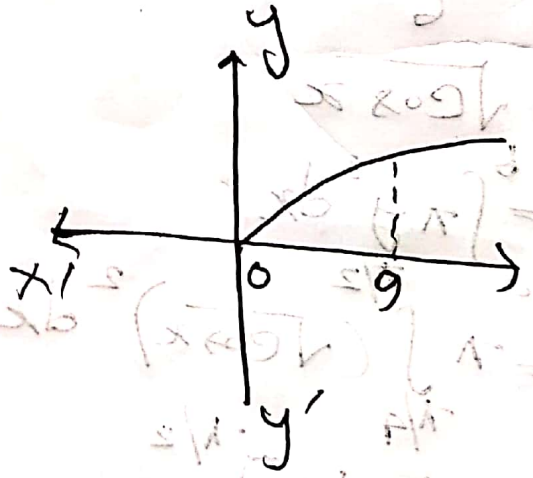
$$V_x = \int_0^9 \pi (\sqrt{x})^2 \, dx$$

$$= \pi \int_0^9 x \, dx$$

$$= \pi \left[\frac{x^2}{2} \right]_0^9$$

$$= \pi \left(\frac{81}{2} - 0 \right)$$

$$= \frac{81\pi}{2} \quad \text{(Ans)}$$



b) $y = x^2$; $x = 0$, $x = 2$

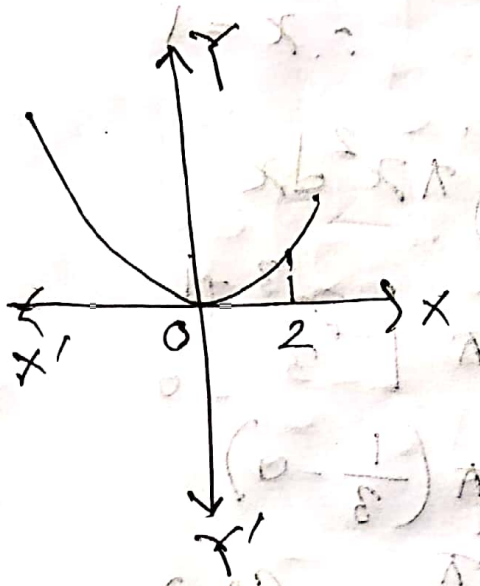
$$V = \int_0^2 \pi y^2 \, dx$$

$$= \pi \int_0^2 x^4 \, dx$$

$$= \pi \left[\frac{x^5}{5} \right]_0^2$$

$$= \pi \left(\frac{32}{5} - 0 \right)$$

$$= \frac{32\pi}{5} \quad \text{(Ans)}$$



c) $y = x^2 - 4x + 5$, $x = 1$, $x = 4$

$$V = \int_1^4 \pi (x^2 - 4x + 5)^2 \, dx$$

$$= \pi \int_1^4 \{ x^4 - 2x^2(4x - 5) + (4x - 5)^2 \} dx$$

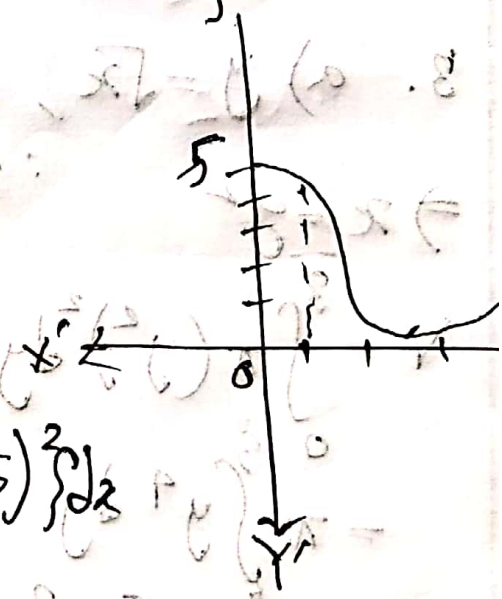
$$= \pi \int_1^4 (x^4 - 8x^3 + 10x^2 + 16x^2 - 40x + 25) dx$$

$$= \pi \int_1^4 (x^4 - 8x^3 + 26x^2 - 40x + 25) dx$$

$$= \pi \left[\frac{x^5}{5} - \frac{8x^4}{4} + \frac{26x^3}{3} - \frac{40x^2}{2} + 25x \right]_1^4$$

$$= \pi \{ \left(\frac{1024}{5} - 512 + \frac{2048}{4} + -320 + 100 \right) - \left(\frac{1}{5} - 20 + 25 \right) \}$$

$$= \frac{78\pi}{5} \quad \text{(Ans)}$$



d) $y = x$, $y = 1$, $x = 0$

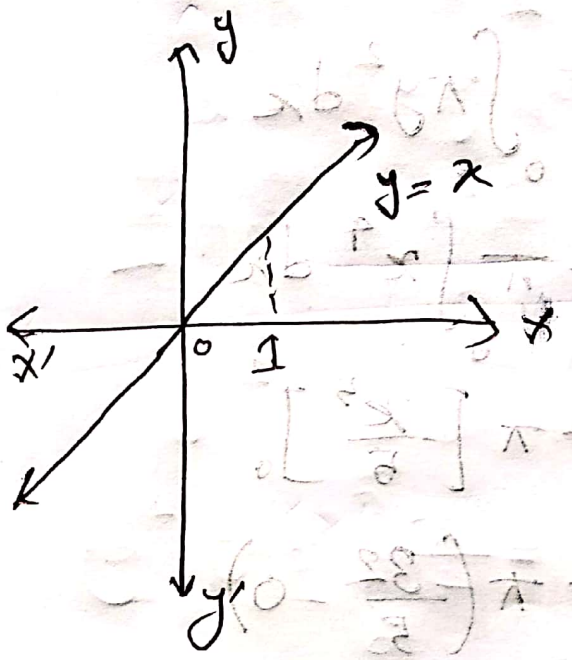
$\therefore x = 1$

$$V = \int_0^1 \pi x^2 \, dx$$

$$= \pi \left[\frac{x^3}{3} \right]_0^1$$

$$= \pi \left(\frac{1}{3} - 0 \right)$$

$$= \frac{\pi}{3} \quad \text{(Ans)}$$



3: a) $y = \sqrt{x}$, $x = 0$, $y = 3$

$\therefore y = 0$

$\Rightarrow x = y^2$

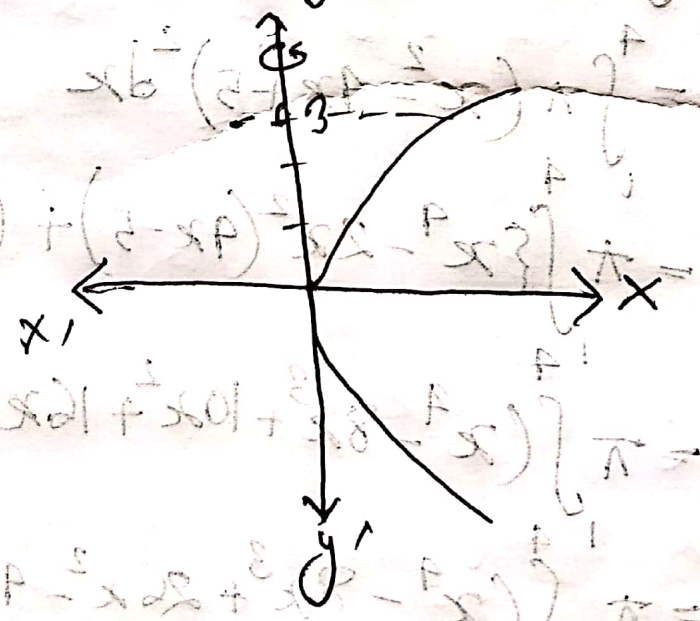
$$V = \int_0^3 \pi (y^2)^2 \, dy$$

$$= \pi \int_0^3 y^4 \, dy$$

$$= \pi \left[\frac{y^5}{5} \right]_0^3$$

$$= \pi \left(\frac{243}{5} - 0 \right)$$

$$= \frac{243\pi}{5} \quad \text{(Ans)}$$



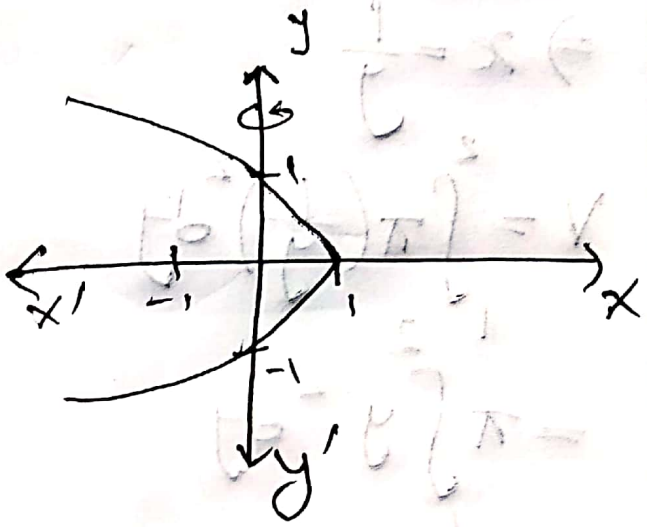
b) $x = 1 - y^2$,

$x = 0$

$1 - y^2 = 0$

$\Rightarrow y^2 = 1$

$\Rightarrow y = \pm 1$



$$V = \int_{-1}^{1} \pi (1-y^2)^2 \, dy$$

$$= \pi \int_{-1}^{1} (1 - 2y^2 + y^4) \, dy$$

$$= \pi \left[y - \frac{2y^3}{3} + \frac{y^5}{5} \right]_{-1}^{1}$$

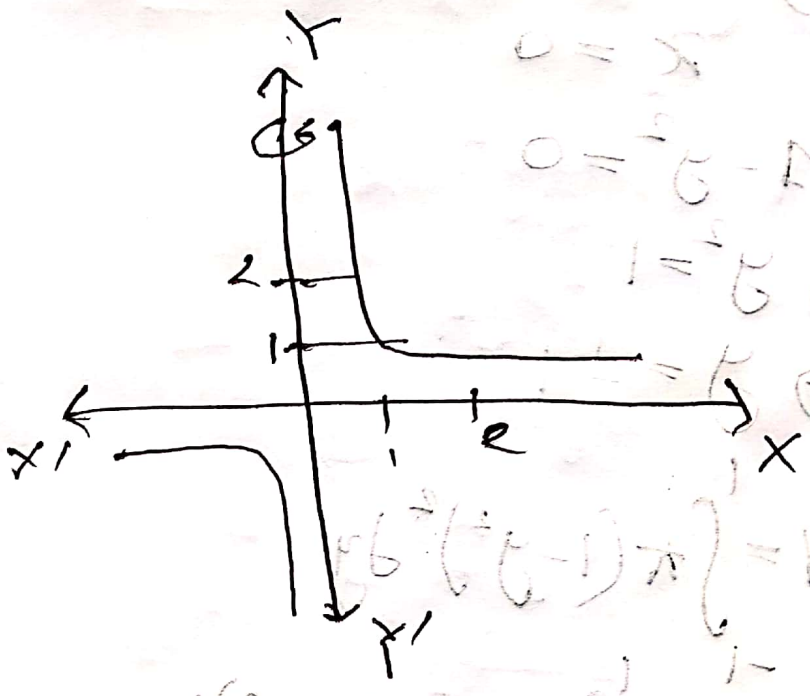
$$= \pi \left[\left(1 - \frac{2}{3} + \frac{1}{5}\right) - \left(-1 + \frac{2}{3} - \frac{1}{5}\right) \right]$$

$$= \pi \left(2 - \frac{4}{3} + \frac{2}{5}\right)$$

$$= \frac{16\pi}{15} \quad \text{(Ans)}$$

c) $y = \frac{1}{x}$, $y = 1$, $y = 2$

$\Rightarrow x = \frac{1}{y}$



$$V = \int_1^2 \pi \left(\frac{1}{y}\right)^2 dy$$

$$= \pi \int_1^2 y^{-2}\, dy$$

$$= \pi \left[-\frac{1}{y}\right]_1^2$$

$$= \pi \left(-\frac{1}{2} + 1\right)$$

$$= \frac{\pi}{2}$$