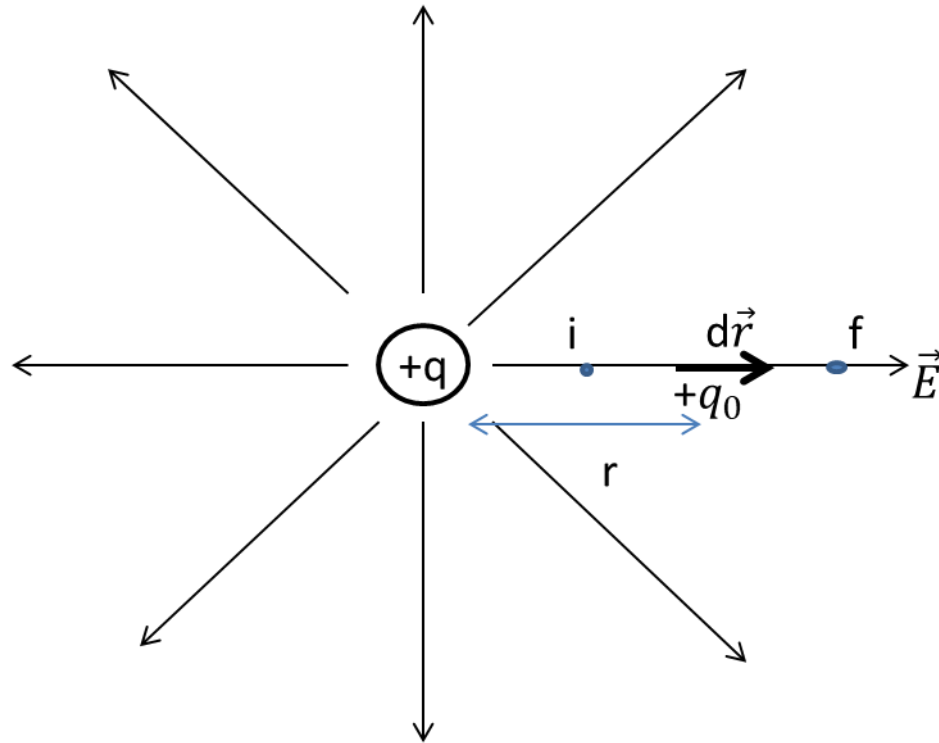


LESSON 4

BOOK CHAPTER 24

ELECTRIC POTENTIAL

Potential Due to a Point Charge:



We have $V_f - V_i = - \int_i^f \vec{E} \cdot d\vec{s} \dots\dots\dots (iv)$

$$V_f - V_i = - \int_i^f \vec{E} \cdot d\vec{r} \quad [\text{here } d\vec{s} = d\vec{r}]$$

$$V_f - V_i = - \int_i^f E \cos 0^\circ dr$$

$$V_f - V_i = - \int_i^f E dr$$

$$V_f - V_i = - \frac{q}{4\pi\epsilon_0} \int_i^f \frac{1}{r^2} dr$$

$$V_f - V_i = - \frac{q}{4\pi\epsilon_0} \left(- \left| \frac{1}{r} \right|_{r=r_i}^{r=r_f} \right)$$

$$V_f - V_i = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{r_f} - \frac{1}{r_i} \right]$$

If r_f goes to infinity (∞), then $V_f = 0$ and $\frac{1}{r_f} = 0$

Hence, $0 - V_i = \frac{q}{4\pi\epsilon_0} \left[0 - \frac{1}{r_i} \right]$

$$V_i = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{r_i} \right]$$

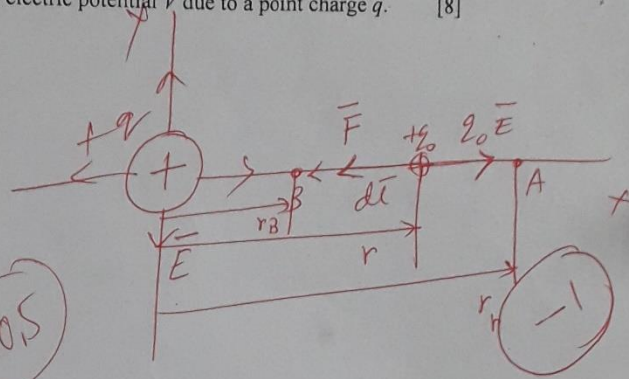
Finally we get,

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

1. Derive an expression for the electric potential V due to a point charge q . [8]

$$V_B - V_A = \frac{W_{AB}}{q_0} = \frac{1}{q_0} \int_A^B \vec{F} \cdot d\vec{l}$$

$$= \frac{1}{q_0} \int_A^B (-q_0 \vec{E}) \cdot d\vec{l}$$



$$V_B - V_A = - \int_A^B \vec{E} \cdot d\vec{l}$$

OS

$$= - \int_A^B E dl \cos 180^\circ = + \int_A^B E dl = - \int_{r_A}^{r_B} E dr$$

$$= - \int_{r_A}^{r_B} \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} dr = - \frac{q}{4\pi\epsilon_0} \int_{r_A}^{r_B} r^{-2} dr$$

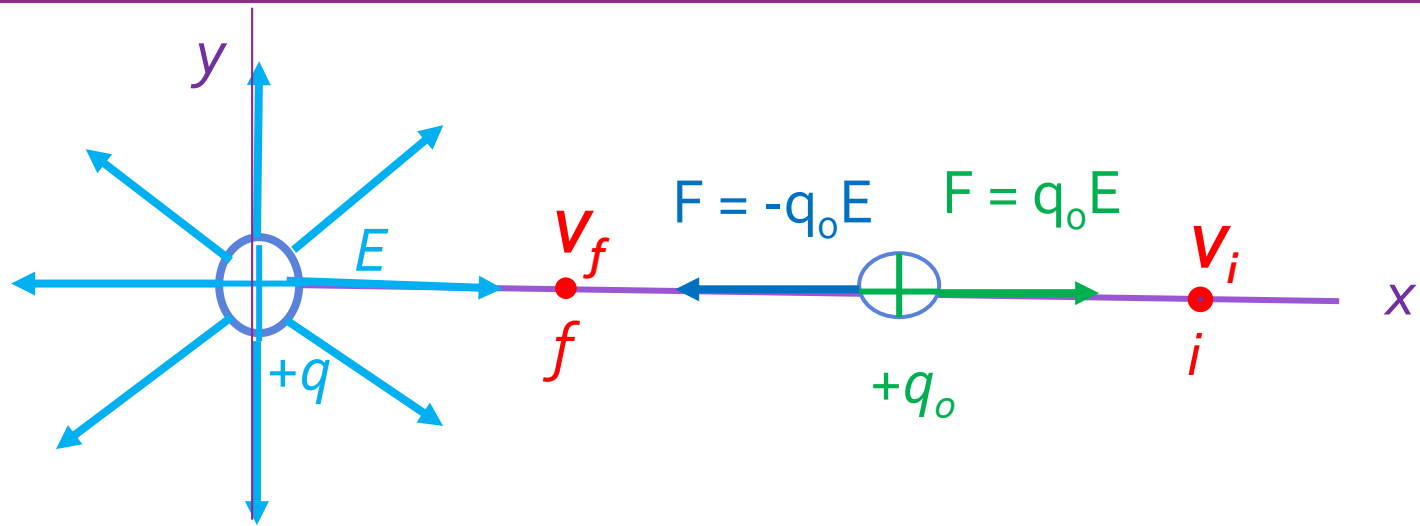
$$= - \frac{q}{4\pi\epsilon_0} \left[\frac{-2+1}{-2+1} \right]_{r_A}^{r_B} = - \frac{q}{4\pi\epsilon_0} \left[\frac{r^{-1}}{-1} \right]_{r_A}^{r_B}$$

$$= \frac{q}{4\pi\epsilon_0} \left[\frac{1}{r} \right]_{r_A}^{r_B} = \frac{q}{4\pi\epsilon_0} \left(\frac{1}{r_B} - \frac{1}{r_A} \right)$$

$$r_A \rightarrow \infty, V_A = 0, \left(\frac{1}{r_B} - \frac{1}{\infty} \right) = V_B = \frac{q}{4\pi\epsilon_0} \frac{1}{r_B}$$

$$V_B - 0 = \frac{q}{4\pi\epsilon_0} \left(\frac{1}{r_B} - \frac{1}{\infty} \right) = V_B = \frac{q}{4\pi\epsilon_0} \frac{1}{r_B}$$

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{r} \quad \text{where } V_B = V, r_B = r$$

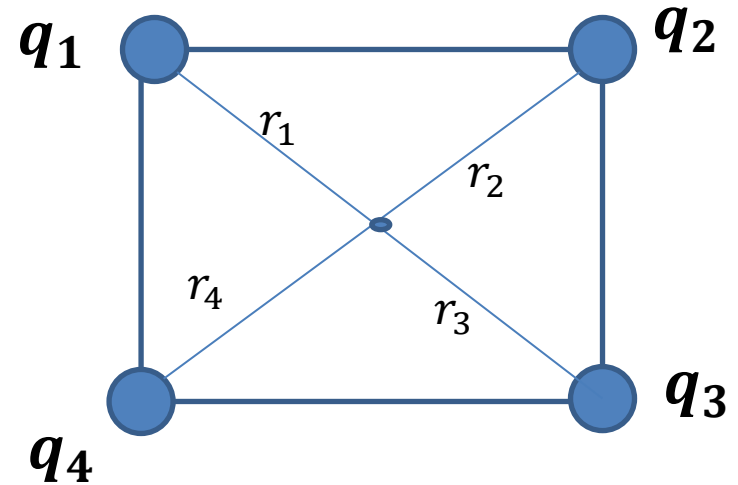


Positive point charge: $+q$

Potential Due to a Group of Point Charges:

General Formula:

$$V = \sum_{i=1}^n V_i = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^n \frac{q_i}{r_i}$$



For an example:
$$V = \sum_{i=1}^4 V_i = \frac{1}{4\pi\epsilon_0} \left[\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \frac{q_4}{r_4} \right]$$

Problem: 4 (Book chapter 24)

Two large, parallel, conducting plates are 12 cm apart and have charges of equal magnitude and opposite sign on their facing surfaces. An electric force of $3.9 \times 10^{-15} \text{ N}$ acts on an electron placed anywhere between the two plates. (Neglect fringing.) (a) Find the electric field at the position of the electron. (b) What is the potential difference between the plates?

Given

$$d = |\Delta x| = 12 \text{ cm} = 0.12 \text{ m}$$

$$|F| = 3.9 \times 10^{-15} \text{ N}$$

$$|e^-| = |q| = 1.6 \times 10^{-19} \text{ C}$$

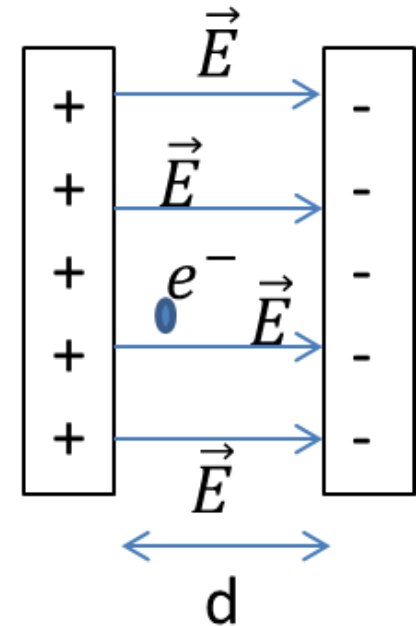
(a) $|F| = qE$

$$E = \frac{3.9 \times 10^{-15}}{1.6 \times 10^{-19}} = 2.437 \times 10^4 \frac{\text{N}}{\text{C}}$$

(b) The potential difference between the plates

$$E = \frac{|\Delta V|}{|\Delta x|}$$

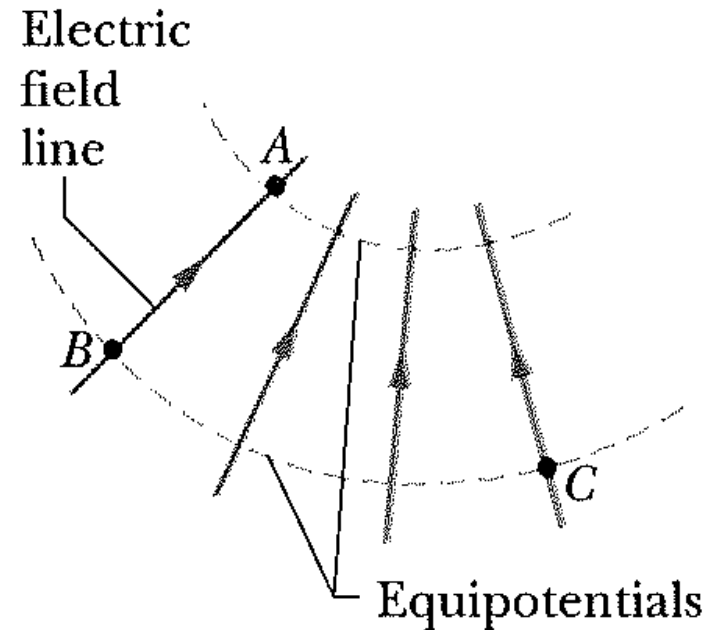
$$|\Delta V| = E|\Delta x| = (2.437 \times 10^4)(0.12) = 0.293 \times 10^4 \text{ Volt}$$



Problem: 6 (Book chapter 24)

When an electron moves from A to B along an electric field line in the adjacent figure, the electric field does $3.94 \times 10^{-19} \text{ J}$ of work on it. What are the electric potential differences

(a) $V_B - V_A$, (b) $V_C - V_A$, and (c) $V_C - V_B$?



Answer:

$$(a) \quad V_B - V_A = \frac{-W}{q} = \frac{-3.94 \times 10^{-19}}{-1.6 \times 10^{-19}} = 2.45 \text{ Volt}$$

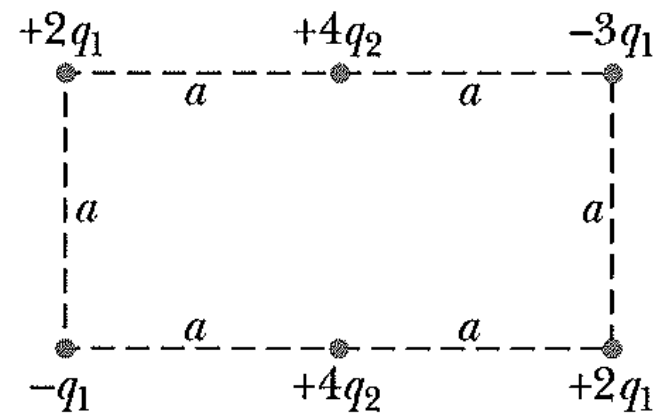
$$(b) \quad V_C - V_A = V_B - V_A = 2.45 \text{ Volt}$$

$$(c) \quad V_C - V_B = 0 \text{ Volt}$$

[Because points B and C are on the same equipotential surface, $V_C = V_B$]

Problem: 16 (Book chapter 24)

Figure shows a rectangular array of charged particles fixed in place, with distance $a = 39 \text{ cm}$ and the charges shown as integer multiples of $q_1 = 3.4 \text{ pC}$ and $q_2 = 6.0 \text{ pC}$. With $V = 0$ at infinity, what is the net electric potential at the rectangle's center?



The net potential at the rectangle center is

Given $a = 39 \text{ cm} = 0.39 \text{ m}$

$$V = V_1 + V_2 + V_3 + V_4 + V_5 + V_6 \quad [\text{algebraic sum}]$$

$$q_1 = 3.40 \text{ pC} = 3.4 \times 10^{-12} \text{ C}$$

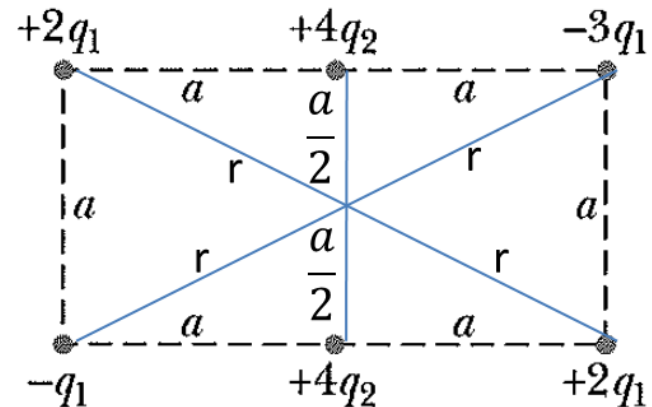
$$q_2 = 6.0 \text{ pC} = 6.0 \times 10^{-12} \text{ C}$$

$$V = \frac{1}{4\pi\epsilon_0} \left[\frac{+2q_1}{r} + \frac{-3q_1}{r} + \frac{+2q_1}{r} + \frac{-q_1}{r} \right] + \frac{1}{4\pi\epsilon_0} \left[\frac{+4q_2}{\frac{a}{2}} + \frac{+4q_2}{\frac{a}{2}} \right]$$

$$V = 0 + \frac{1}{4\pi\epsilon_0} \left[\frac{4q_2}{0.195} + \frac{4q_2}{0.195} \right]$$

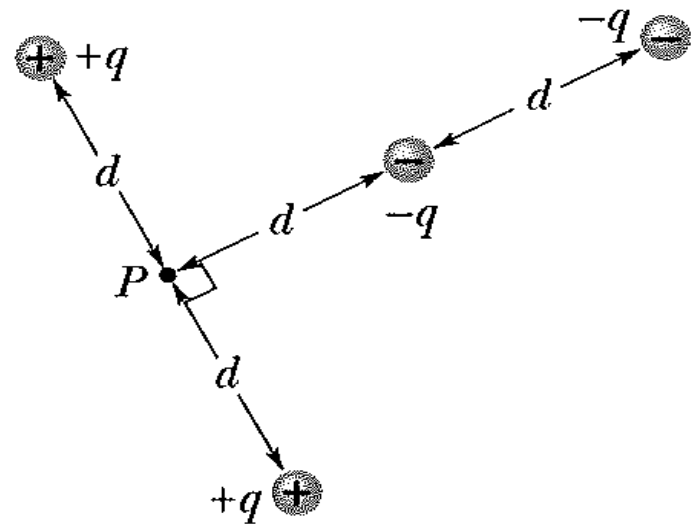
$$V = \frac{1}{4\pi\epsilon_0} \left[\frac{8q_2}{0.195} \right] = \frac{9 \times 10^9 \times 8 \times 6 \times 10^{-12}}{0.195}$$

$$V = 2.215 \text{ Volt}$$



Problem: 17 (Book chapter 24)

In the adjacent Figure, what is the net electric potential at point P due to the four particles if $V = 0$ at infinity, $q = 5 \text{ fC}$, and $d = 4 \text{ cm}$?



Answer:

Given

$$q = 5 \text{ fC} = 5 \times 10^{-15} \text{ C} \quad \text{and} \quad d = 4 \text{ cm} = 0.04 \text{ m}$$

The net electric potential at the point P is

$$V = V_1 + V_2 + V_3 + V_4 \quad [\text{algebraic sum}]$$

$$V = \frac{1}{4\pi\epsilon_0} \left[\frac{+q}{d} + \frac{+q}{d} + \frac{-q}{d} + \frac{-q}{2d} \right] = \frac{1}{4\pi\epsilon_0} \frac{q}{d} \left[1 + 1 - 1 - \frac{1}{2} \right]$$

$$V = \frac{9 \times 10^9 \times 5 \times 10^{-15}}{0.04} \left(\frac{1}{2} \right)$$

$$V = 562.5 \times 10^{-6} \text{ Volt}$$

Potential Due to an Electric Dipole:

The net potential at P is given by

$$V = V_{(+)} + V_{(-)} = \frac{1}{4\pi\epsilon_0} \left[\frac{+q}{r_{(+)}} + \frac{-q}{r_{(-)}} \right] = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{r_{(+)}} - \frac{1}{r_{(-)}} \right]$$

$$V = \frac{q}{4\pi\epsilon_0} \left(\frac{r_{(-)} - r_{(+)}}{r_{(-)}r_{(+)}} \right)$$

(1) If $r \gg d$: $r_{(-)} - r_{(+)} = d \cos \theta$ and $r_{(-)}r_{(+)} \approx r^2$

$$V = \frac{q}{4\pi\epsilon_0} \left(\frac{d \cos \theta}{r^2} \right)$$

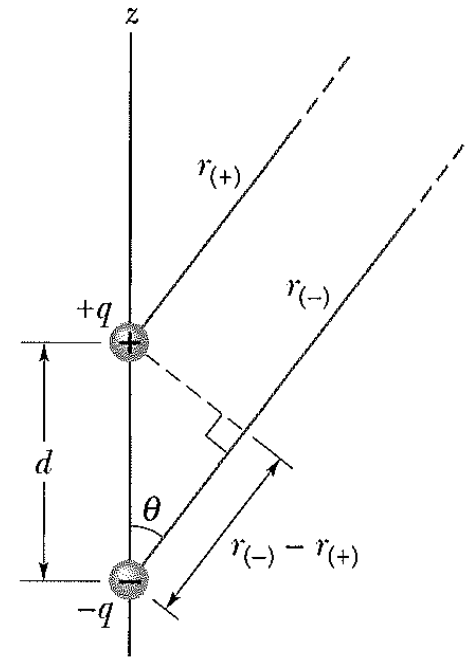
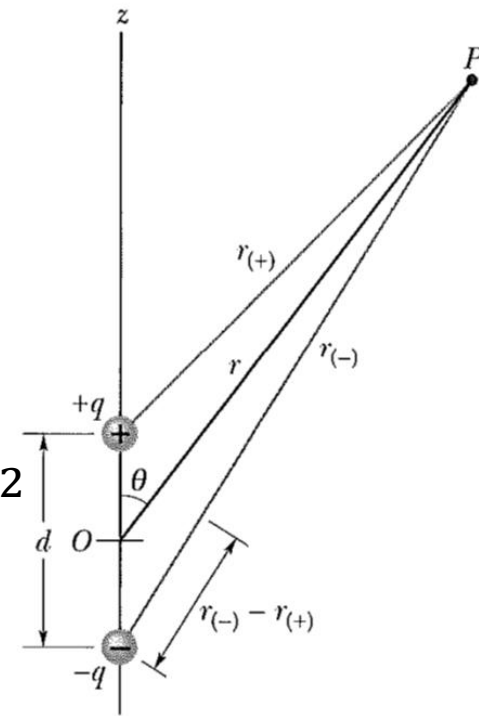
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where, θ is measured from the direction of the electric dipole moment.

(2) V in terms of electric dipole moment: $p = qd$

$$V = \frac{1}{4\pi\epsilon_0} \left(\frac{p \cos \theta}{r^2} \right)$$

The vector \vec{p} is directed along the dipole axis, from the negative to the positive charge.



THANK YOU