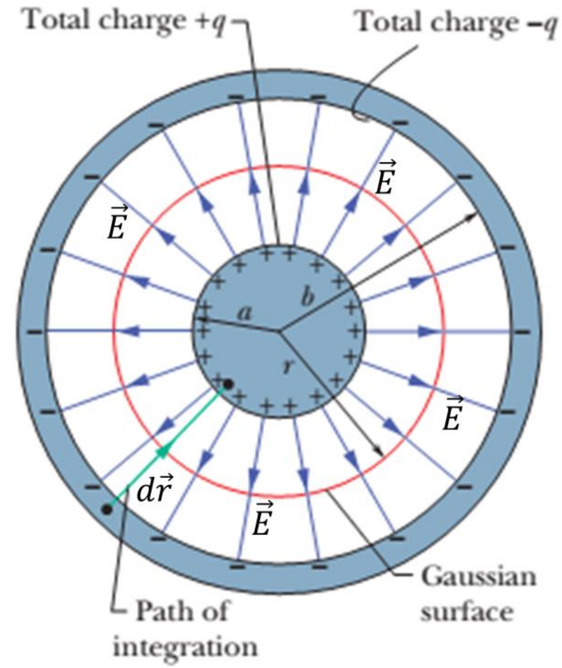
LESSON 7

BOOK CHAPTER 25

CAPACITANCE

A Spherical Capacitor:

Consider a spherical Gaussian surface of radius r concentric with two shells of radii a and b (b>a).



Gauss' law:

$$\varepsilon_0 \oint \vec{E}.\, d\overrightarrow{A} = q_{\,enc}$$

$$\varepsilon_0 \oint EdA\cos 0 = +q$$

$$\varepsilon_0 \oint EdA = q$$

$$\varepsilon_0 E \oint dA = q$$

$$\varepsilon_0 E(4\pi r^2) = q$$

$$E = \frac{q}{4\pi\varepsilon_0 r^2}$$

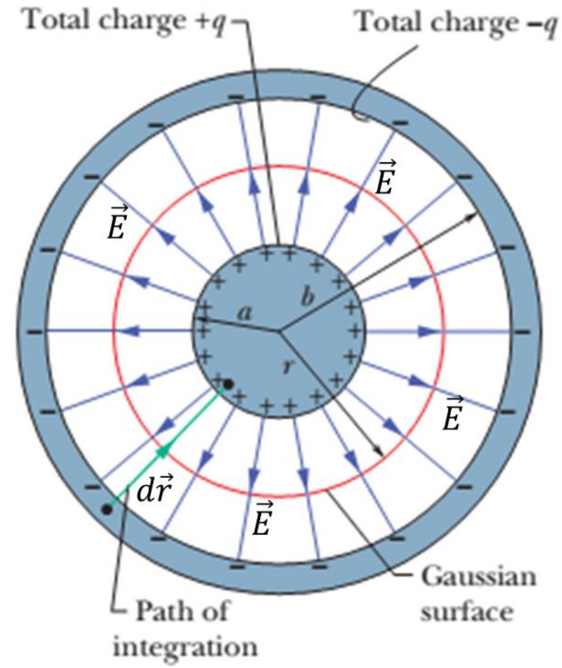
$$V = \int_{-}^{+} E\, ds = -\int_{-}^{+} E\, dr \qquad \text{[since } ds = -dr\text{]}$$

$$V = -\frac{q}{4\pi\varepsilon_0}\int_{r=b}^{r=a} \frac{dr}{r^2} = \frac{q}{4\pi\varepsilon_0}\left(\left|\frac{1}{r}\right|_{r=b}^{r=a}\right)$$

$$V = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{q(b-a)}{4\pi\varepsilon_0\, ab}$$

$$\boxed{C = \frac{q}{V} = \frac{4\pi\varepsilon_0\, ab}{b-a}}$$

$V = V_+ - V_- = -\int_-^+ \boldsymbol{E}.d\boldsymbol{s} = -\int_-^+ Edscos180 = \int_-^+ Eds$



$$\int_-^+ \boldsymbol{E}.d\boldsymbol{s} = -\int_-^+ Edscos180 = \int_-^+ Eds$$

$$V = \int_b^a E(-dr)$$

$$\varepsilon_0 E \oint dA = q$$

[since $ds = -dr$]

$$\varepsilon_0 E(4\pi r^2) = q$$

$$V = -\frac{q}{4\pi\varepsilon_0}\int_{r=b}^{r=a}\frac{dr}{r^2} = \frac{q}{4\pi\varepsilon_0}\left(\left|\frac{1}{r}\right|_{r=b}^{r=a}\right)$$

$$E = \frac{q}{4\pi\varepsilon_0 r^2}$$

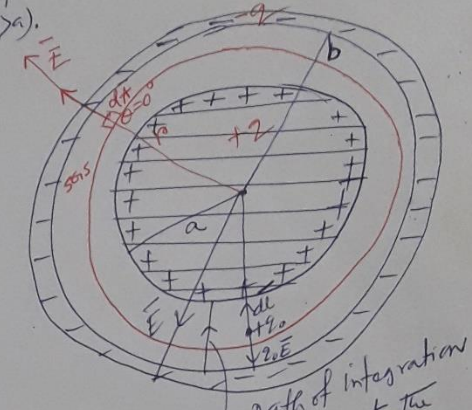
$$V = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{q(b-a)}{4\pi\varepsilon_0\, ab}$$

$$\boxed{C = \frac{q}{V} = \frac{4\pi\varepsilon_0\, ab}{b-a}}$$

20/6/16: New

(2) Capacitance for a Spherical capacitor: $C = 4\pi\epsilon_0\left(\frac{ab}{b-a}\right)$ | where $b>a$

Consider a spherical Gaussian surface of radius r concentric with two shells of radii a and b $(b>a)$.



path of integration from -ve to the +ve plates.

(1) Gauss' law:

$$\epsilon_0 \oint \bar{E}\cdot d\bar{A} = q_{enc}$$

$$\epsilon_0 \oint E\,dA\cos 0° = q$$

$$\epsilon_0 \oint E\,dA = q$$

$$\epsilon_0 E \oint dA = q$$

$$\epsilon_0 E\left(4\pi r^2\right) = q$$

$$\boxed{E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}}$$

(2) $V = V_+ - V_- = -\int_-^+ \bar{E}\cdot d\bar{l} = -\int_b^a E\,dl\cos 180°$ ok

$$V = +\int_b^a E\,dl = \int_b^a E(-dr) \quad \Big|\ \underset{\text{increasing}}{dl} = \underset{\text{decreasing}}{-dr}$$

$$= -\int_b^a E\,dr = -\int_b^a \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\,dr$$

$$= -\frac{q}{4\pi\epsilon_0}\int_b^a \frac{dr}{r^2} \quad \Big|\ \text{variable} = r$$

$$= -\frac{q}{4\pi\epsilon_0}\int_b^a r^{-2}\,dr = -\frac{q}{4\pi\epsilon_0}\left|\frac{r^{-2+1}}{-2+1}\right|_b^a$$

$$= -\frac{q}{4\pi\epsilon_0}\left|\frac{r^{-1}}{-1}\right|_b^a = +\frac{q}{4\pi\epsilon_0}\left|\frac{1}{r}\right|_b^a = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right)$$

$$V = \frac{q}{4\pi\epsilon_0}\left(\frac{b-a}{ab}\right)$$

(3) $q = CV \Rightarrow C = \frac{q}{V} = \frac{q}{\frac{q}{4\pi\epsilon_0}\left(\frac{b-a}{ab}\right)}$

$\boxed{\therefore C = 4\pi\epsilon_0\left(\frac{ab}{b-a}\right)} \Leftarrow C = \cancel{q}\times\frac{4\pi\epsilon_0}{\cancel{q}}\times\frac{ab}{(b-a)} = 4\pi\epsilon_0\left(\frac{ab}{b-a}\right)$

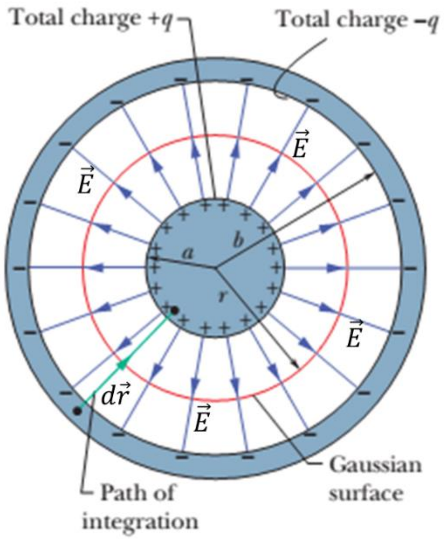
An isolated sphere:

We can assign a capacitance to a *single* isolated spherical conductor of radius $a = R$ by assuming that the "missing plate" is a conducting sphere of infinite radius $b = \infty$.

The capacitance of the spherical capacitor,

$$C = \frac{4\pi\varepsilon_0\, ab}{b - a}$$ [Dividing both numerator and denominator by b]

$$C = \frac{4\pi\varepsilon_0\, a}{1 - \frac{a}{b}}$$



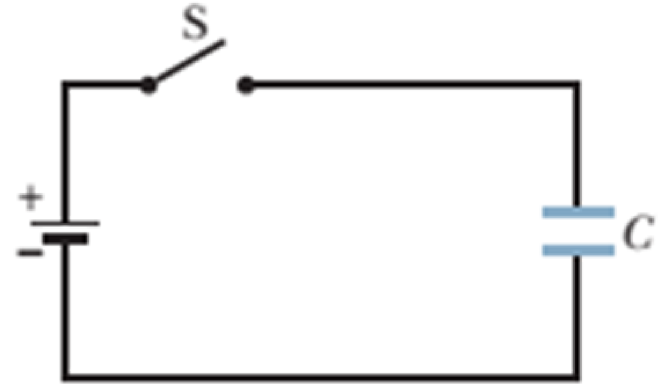
If $b \to \infty$ (infinity), $C = \frac{4\pi\varepsilon_0\, a}{1 - 0} = 4\pi\varepsilon_0 a$

By substituting $a = R$,

$\boxed{C = 4\pi\varepsilon_0 R}$ (capacitance for isolated sphere)

Problem 2 (Book chapter 25):

The capacitor in the adjacent Fig. has a capacitance of $25\,\mu F$ and is initially uncharged. The battery provides a potential difference of 120 V. After switch S is closed, how much charge will pass through it?



Given: $C = 25\,\mu F = 25 \times 10^{-6} F$

$$V = 120\,V$$

$$q = ?$$

$$q = CV$$

$$q = 25 \times 10^{-6} \times 120 = 3 \times 10^{-3}\ \text{C} = 0.003\ \text{C}$$

Problem 3 (Book chapter 25):

A parallel-plate capacitor has circular plates of 8.20 cm radius and 1.30 mm separation. (a) Calculate the capacitance. (b) Find the charge for a potential difference of 120 V.

Given: $r = 8.20\ cm = 0.082\ m$

$$d = 1.30\ mm = 1.3 \times 10^{-3}\ m$$

$$V = 120\ V$$

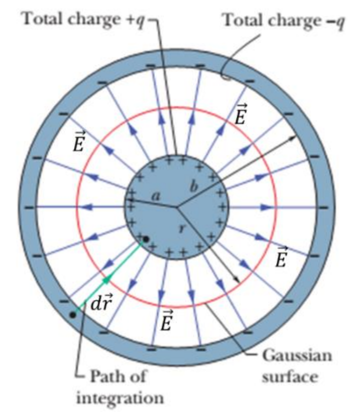
(a) $$C = \frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0 (\pi r^2)}{d}$$

$$C = \frac{8.854 \times 10^{-12} \times 3.1416 \times (0.082)^2}{1.3 \times 10^{-3}} = 143.87 \times 10^{-12}\ F$$

(b) $$q = CV = 143.87 \times 10^{-12} \times 120 = 17.26 \times 10^{-9}\ C$$

Problem 4 (Book chapter 25):

The plates of a spherical capacitor have radii 38.0 mm and 40.0 mm. (a) Calculate the capacitance. (b) What must be the plate area of a parallel-plate capacitor with the same plate separation and capacitance?



Given: $a = 38\, mm = 38 \times 10^{-3}\, m$

$$b = 40\, mm = 40 \times 10^{-3}\, m$$

$$b - a = (40 - 38) mm = 2\, mm = 2 \times 10^{-3}\, m$$

(a) Capacitance for a Spherical capacitor: $C = \frac{4\pi\varepsilon_0\, ab}{b - a} = \frac{1}{9 \times 10^9} \frac{38 \times 40 \times 10^{-6}}{2 \times 10^{-3}}$

$$C = 84.44 \times 10^{-12}\, F$$

(b) Here: $d = b - a = 2\, mm = 2 \times 10^{-3}\, m$

$$A = ?$$

Parallel plate capacitor, $C = \frac{\varepsilon_0 A}{d}$

$$A = \frac{Cd}{\varepsilon_0} = \frac{84.44 \times 10^{-12} \times 2 \times 10^{-3}}{8.854 \times 10^{-12}}$$

$$A = 19.074 \times 10^{-3}\, m^2$$

Problem 6 (Book chapter 25):

You have two flat metal plates, each of area 1.00 m^2, with which to construct a parallel-plate capacitor. (a) If the capacitance of the device is to be 1.00 F, what must be the separation between the plates? (b) Could this capacitor actually be constructed?

Answer:

Given

$$A = 1.00\ m^2$$
$$C = 1.00\ F$$

(a) $d = ?$

(b) Could this capacitor actually be constructed?

(a) We know

$$C = \frac{\varepsilon_0 A}{d}$$

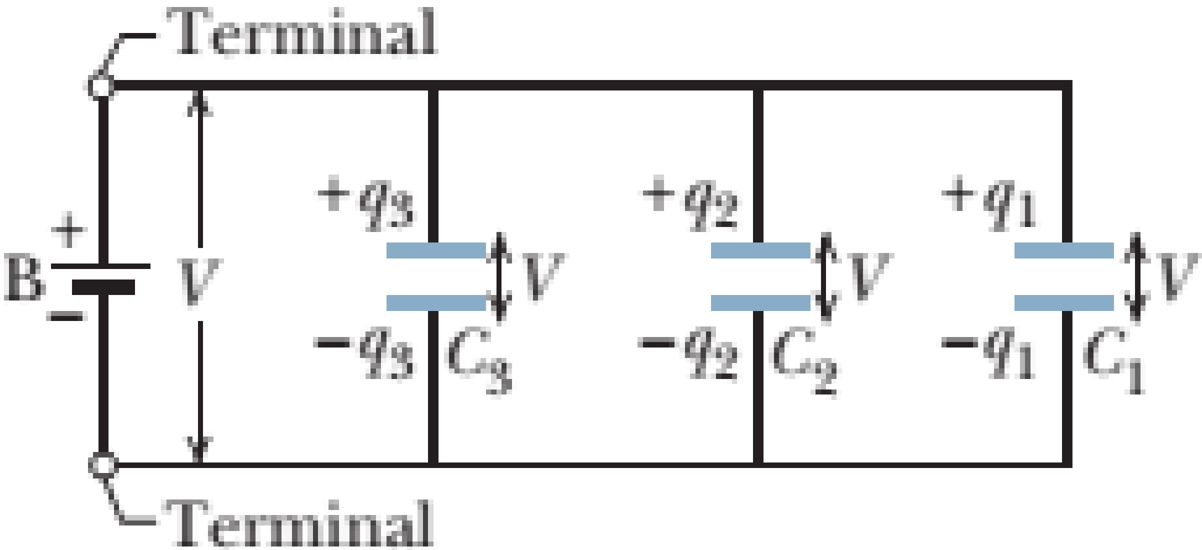
$$d = \frac{\varepsilon_0 A}{C} = \frac{8.854 \times 10^{-12} \times 1}{1} = 8.854 \times 10^{-12}\ m$$

d = 0.000000000008854 m

(b)

No: It is not possible to construct a capacitor by the separation distance, $d = 8.854 \times 10^{-12}\ m$, because d value is less than the minimum size of an atom ($10^{-10}\ m$)

Capacitors in parallel combination:



Charge on each capacitor:

$$q_1 = C_1 V$$

$$q_2 = C_2 V$$

$$q_3 = C_3 V$$

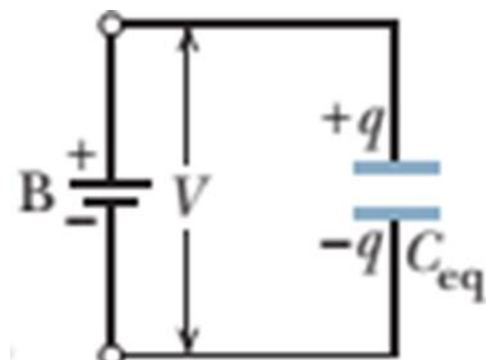
The total charge on the parallel combination is then

$$q = q_1 + q_2 + q_3 = (C_1 + C_2 + C_3)V$$

The equivalent capacitance, with the same total charge *q* and applied potential difference V as the combination, is then

$$C_{eq} = \frac{q}{V} = \frac{(C_1 + C_2 + C_3)V}{V} = C_1 + C_2 + C_3$$

$$\boxed{C_{eq} = C_1 + C_2 + C_3}$$



Thank You