

# Regular Expression

Course Code: CSC3220

Course Title: Compiler Design



**Dept. of Computer Science**  
**Faculty of Science and Technology**

<b>Lecturer No:</b>		<b>Week No:</b>		<b>Semester:</b>	
<b>Lecturer:</b>	<i>Md Masum Billah, <a href="mailto:billah.masumcu@aiub.edu">billah.masumcu@aiub.edu</a></i>				

# Lecture Outline



1. Definition of a Regular Expression
2. Rules of a Regular Expression
3. Examples
4. Exercises

# Objectives and Outcomes



## Objectives:

- Understand the basic concept of Regular expression
- Understand the regular expression algorithm

## Outcome:

- Students should be able to design the nondeterministic finite automate from regular expression.
- Students should be able to know the applications of a regular expression.

# Regular Expression



**Definition:** A sequence of symbols and characters expressing a string or pattern to be searched for within a longer piece of text.

Another words to say a regular expression is a method used in programming for pattern matching. Regular expressions provide a flexible and concise means to match strings of text.

The regular expressions are built recursively out of smaller regular expressions, using some rules.

Each regular expression  $r$  denotes a language  $L(r)$ , which is also defined recursively from the languages denoted by  $r$ 's subexpressions.

# Regular Expression



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# Rules of Regular Expression

Here are the rules that define the regular expressions over some alphabet  $\Sigma$  and the languages that those expressions denote.

- Basis
- Induction
- Precedence



# Rules of Regular Expression

**BASIS:** There are two rules that form the basis:

- $E$  is a regular expression, and  $L(E)$  is  $\{\epsilon\}$ , that is, the language whose sole member is the empty string.
- If  $a$  is a symbol in  $E$ , then  **$a$**  is a regular expression, and  $L(\mathbf{a}) = \{a\}$ , that is, the language with one string, of length one, with  $a$  in its one position. Here italics is used for symbols, and boldface for their corresponding regular expression.



# Rules of Regular Expression

**INDUCTION:** There are four parts to the induction. Suppose  $r$  and  $s$  are regular expressions denoting languages  $L(r)$  and  $L(s)$ , respectively.

- $(r)|(s)$  is a regular expression denoting the language  $L(r) \cup L(s)$ .
- $(r)(s)$  is a regular expression denoting the language  $L(r)L(s)$ .
- $(r)^*$  is a regular expression denoting  $(L(r))^*$ .
- $(r)$  is a regular expression denoting  $L(r)$ . The last rule says that we can add additional pairs of parentheses around expressions without changing the language they denote.





## Example of a Regular expression

Let  $E = \{a, b\}$ .

- 1. The regular expression  **$a|b$**  denotes the language  $\{a, b\}$ .
- 2.  **$(a|b)(a|b)$**  denotes  $\{aa, ab, ba, bb\}$ , the language of all strings of length two over the alphabet  $E$ .
- Another regular expression for the same language is  **$aa|ab|ba|bb$** .
- 3.  **$a^*$**  denotes the language consisting of all strings of zero or more  $a$ 's, that is,  $\{E, a, aa, aaa, \dots\}$ .



## Example of a Regular expression

Let  $E = \{a, b\}$ .

- 4.  $(a|b)^*$  denotes the set of all strings consisting of zero or more instances of  $a$  or  $b$ , that is, all strings of  $a$ 's and  $b$ 's:  $\{E, a, b, aa, ab, ba, bb, aaa, \dots\}$ .
- Another regular expression for the same language is  $(a^*b^*)^*$ .
- $a|a^*b$  denotes the language  $\{a, b, ab, aab, aaab, \dots\}$ , that is, the string  $a$  and all strings consisting of zero or more  $a$ 's and ending in  $b$ .



# Operations of a Regular expression

## Operations:

The various operations on languages are:

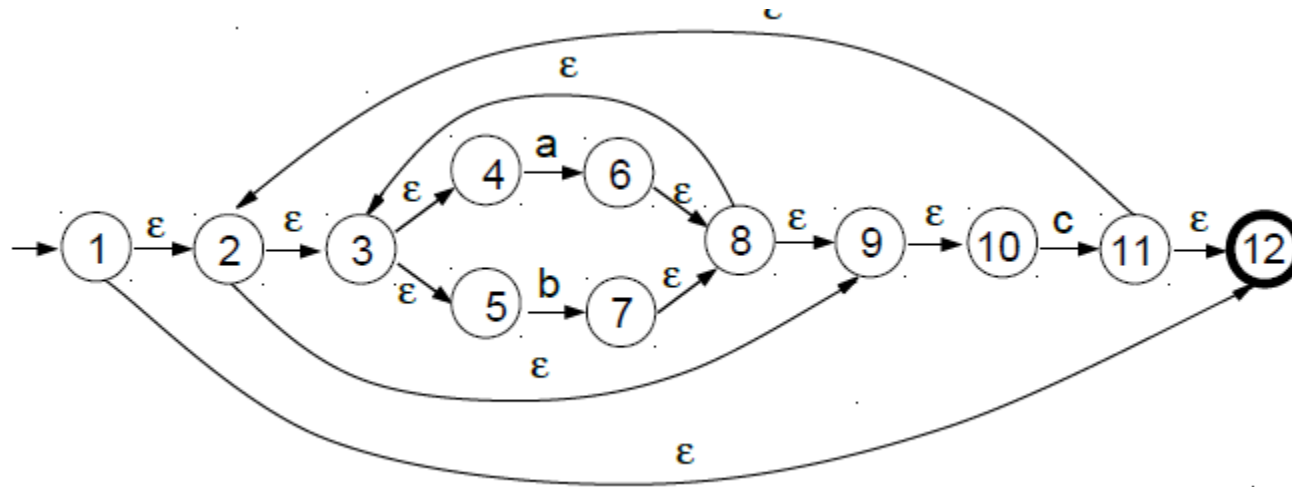
- Union of two languages L and M is written as  
 $L \cup M = \{s \mid s \text{ is in } L \text{ or } s \text{ is in } M\}$
- Concatenation of two languages L and M is written as  
 $LM = \{st \mid s \text{ is in } L \text{ and } t \text{ is in } M\}$
- The Kleene Closure of a language L is written as  
 $L^* = \text{Zero or more occurrence of language } L.$

# Regular Expression To NFA



Outline the NFA generated by the construction of Thompson relevant to the following regular expression:

**Example:  $((a \mid b)^*c)^*$**

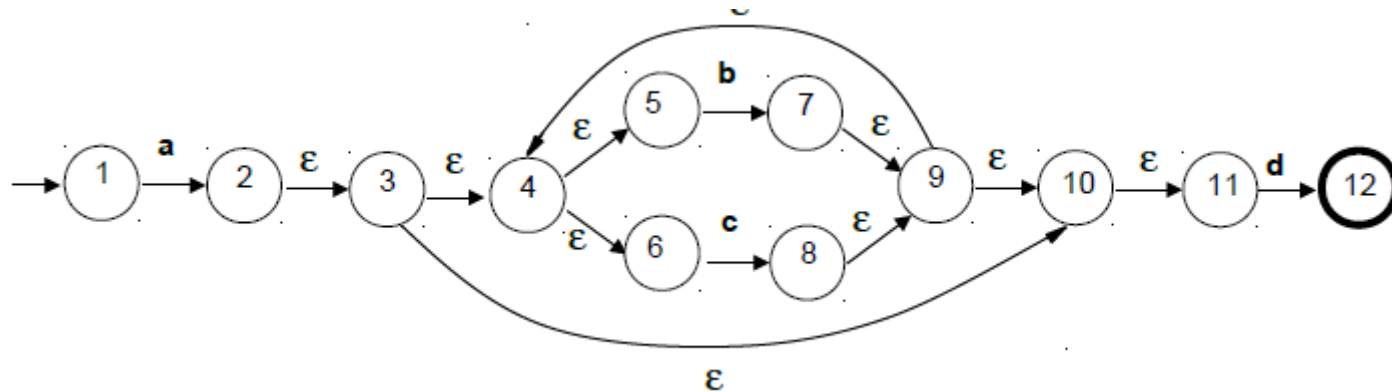


# Regular Expression To NFA



By means of the construction of Thompson, outline the NFA relevant to the following regular expression:

**Example:  $a(b \mid c)^*d$**



# Regular Expression To NFA



By means of the construction of Thompson, outline the NFA relevant to the following regular expression:

**Example:  $a(b \mid c)^*d$**



# Class Exercises

1.  $(aUb)^*abc$

1.  $(abUbc(abUc)^*)^*$



# Lecture References

A. Aho, R. Sethi and J. Ullman, ***Compilers: Principles, Techniques and Tools***  
(The Dragon Book), [ Second Edition]





# References

1. A. Aho, R. Sethi and J. Ullman, ***Compilers: Principles, Techniques and Tools***(The Dragon Book), [ Second Edition]
2. **Principles of Compiler Design** (2nd Revised Edition 2009) A. A. Puntambekar
3. Basics of Compiler Design Torben Mogensen

# NFA to DFA Conversion (Subset Construction Method)

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# Lecture Outline



1. NFA TO DFA (Subset Construction Method)
2. Subset Construction Algorithm
3. DFA Designing
4. Example
5. Exercise
6. References

# Objective and Outcome



## Objective:

- To explain the subset construction algorithm/method for converting a Non deterministic machine to deterministic machine.
- Provide necessary example and explanation of NFA to DFA conversion method using subset construction method.
- To explain and practice Deterministic Finite Automata (DFA) Machine Design for a given Grammar.

## Outcome:

- After this lecture the students will be capable of demonstrating the subset construction algorithm
- After this lecture the student will be able to convert an NFA to relevant DFA by following subset construction method.
- After this class student will be able to design and demonstrate DFA construction from a given Grammar.

# NFA to DFA Conversion

## Subset Construction Algorithm



**Input:** An NFA  $N$

**Output:** A DFA  $D$  accepting the same language

**Method:** Constructs a transition table  $D_{tran}$  for  $D$ . Each DFA state is a set of NFA states and construct  $D_{tran}$  so that  $D$  will simulate “in parallel” all possible moves  $N$  can make on a given input string

OPERATION	DESCRIPTION
$\epsilon\text{-closure}(s)$	Set of NFA states reachable from NFA state $s$ on $\epsilon$ -transitions alone.
$\epsilon\text{-closure}(T)$	Set of NFA states reachable from some NFA state $s$ in $T$ on $\epsilon$ -transitions alone.
$move(T, a)$	Set of NFA states to which there is a transition on input symbol $a$ from some NFA state $s$ in $T$ .

# NFA to DFA Conversion

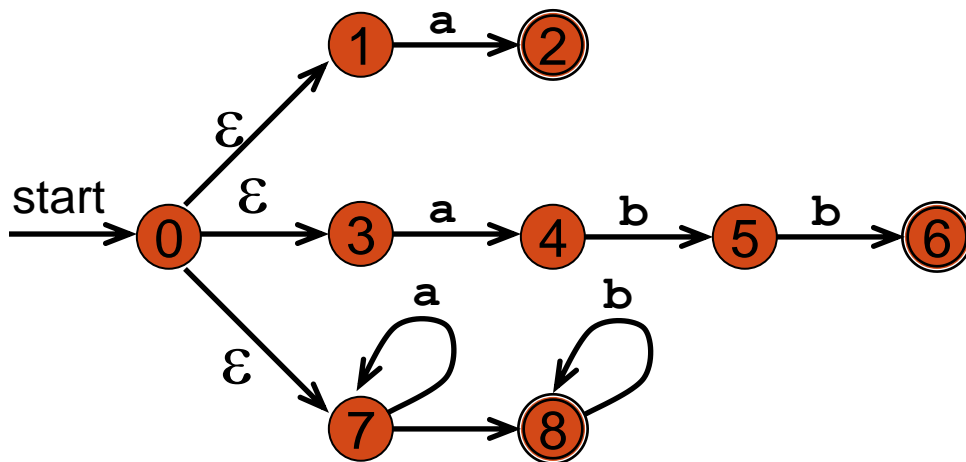
## Subset Construction Algorithm



```
initially,  $\epsilon$ -closure( $s_0$ ) is the only state in  $Dstates$  and it is unmarked;  
while there is an unmarked state  $T$  in  $Dstates$  do begin  
    mark  $T$ ;  
    for each input symbol  $a$  do begin  
         $U := \epsilon$ -closure(move( $T, a$ ));  
        if  $U$  is not in  $Dstates$  then  
            add  $U$  as an unmarked state to  $Dstates$ ;  
         $Dtran\{T, a\} := U$   
    end  
end
```

# NFA to DFA Conversion

$\epsilon$ -closure and move Examples



$\epsilon$ -closure( $\{0\}$ ) =  $\{0, 1, 3, 7\}$

$move(\{0, 1, 3, 7\}, a) = \{2, 4, 7\}$

$\epsilon$ -closure( $\{2, 4, 7\}$ ) =  $\{2, 4, 7\}$

$move(\{2, 4, 7\}, a) = \{7\}$

$\epsilon$ -closure( $\{7\}$ ) =  $\{7\}$

$move(\{7\}, b) = \{8\}$

$\epsilon$ -closure( $\{8\}$ ) =  $\{8\}$

$move(\{8\}, a) = \emptyset$

Alphabet / Symbol =  $\{a, b\}$

# Subset Construction Algorithm

## Subset Construction Algorithm



The *subset construction algorithm* converts an NFA into a DFA using:

$$\varepsilon\text{-closure}(s) = \{s\} \cup \{t \mid s \rightarrow_{\varepsilon} \dots \rightarrow_{\varepsilon} t\}$$

$$\varepsilon\text{-closure}(T) = \bigcup_{s \in T} \varepsilon\text{-closure}(s)$$

$$\text{move}(T, a) = \{t \mid s \rightarrow_a t \text{ and } s \in T\}$$

The algorithm produces:

- $D_{states}$  is the set of states of the new DFA consisting of sets of states of the NFA
- $D_{tran}$  is the transition table of the new DFA



# Subset Construction Algorithm

## Algorithm Explained



1. Create the start state of the DFA by taking the  $\varepsilon$ -closure of the start state of the NFA
2. Perform the following for the DFA state:
  - Apply move to the newly-created state and the input symbol; this will return a set of states.
  - Apply the  $\varepsilon$ -closure to this set of states, possibly resulting in a new set.  
This set of NFA states will be a single state in the DFA.
3. Each time we generate a new DFA state, we must apply step 2 to it. The process is complete when applying step 2 does not yield any new states.
4. The finish states of the DFA are those which contain any of the finish states of the NFA

# Subset Construction Algorithm

## Algorithm with while Loop



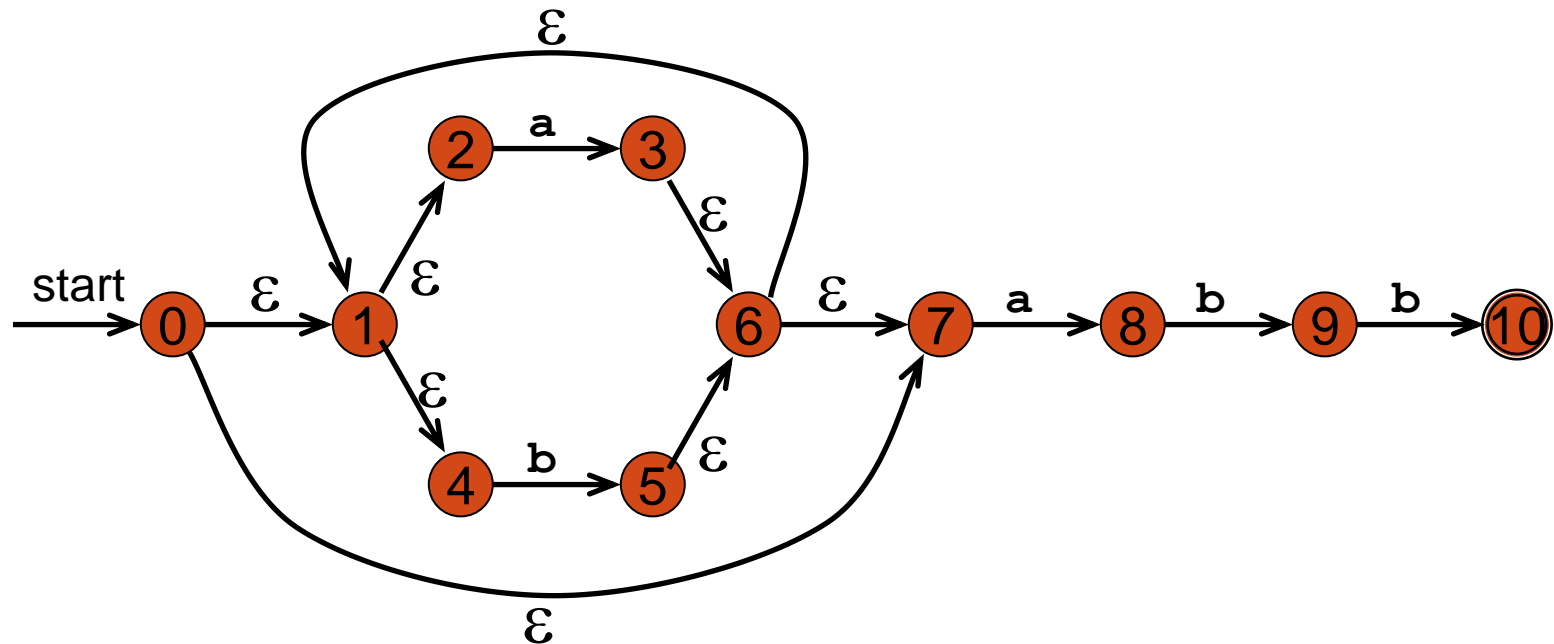
```
fun nfa2dfa start edges =  
  let val chars = nodup(sigma edges)  
      val s0 = eclosure edges [start]  
      val worklist = ref [s0]  
      val work = ref []  
      val old = ref []  
      val newEdges = ref []  
  in while (not (null (!worklist))) do  
    ( work := hd(!worklist)  
    ; old := (!work) :: (!old)  
    ; worklist := tl(!worklist)  
    ; let fun nextOn c = (Char.toString c  
                        , eclosure edges (nodesOnFromMany (Char c) (!work) edges))  
      val possible = map nextOn chars  
      fun add ((c,[]):xs) es = add xs es  
        | add ((c,ss)::xs) es = add xs ((!work,c,ss)::es)  
        | add [] es = es  
      fun ok [] = false  
        | ok xs = not(exists (fn ys => xs=ys) (!old)) andalso  
                  not(exists (fn ys => xs=ys) (!worklist))  
      val new = filter ok (map snd possible)  
      in worklist := new @ (!worklist);  
        newEdges := add possible (!newEdges)  
      end  
    );  
    (s0,!old,!newEdges)  
  end;  
end;
```

# NFA to DFA Conversion

Subset Construction Method (Example-1)

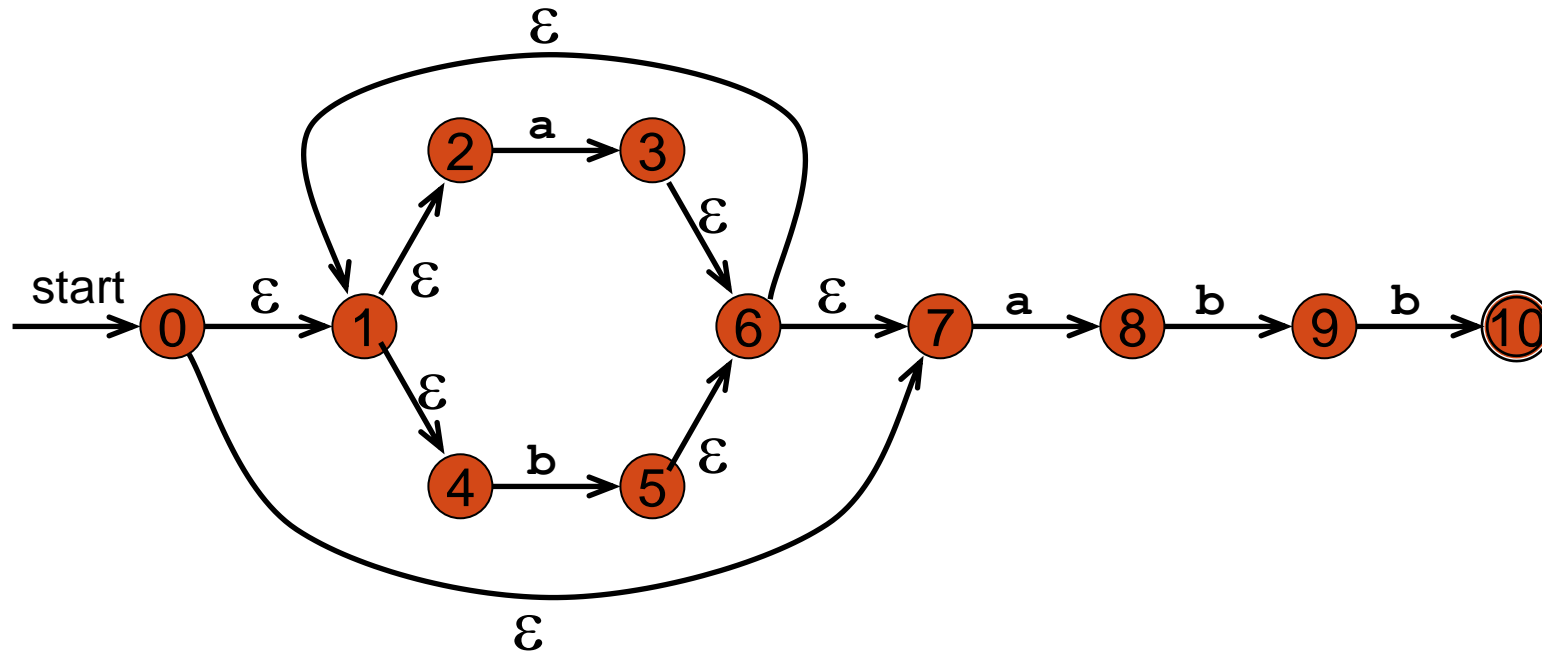


NFA:



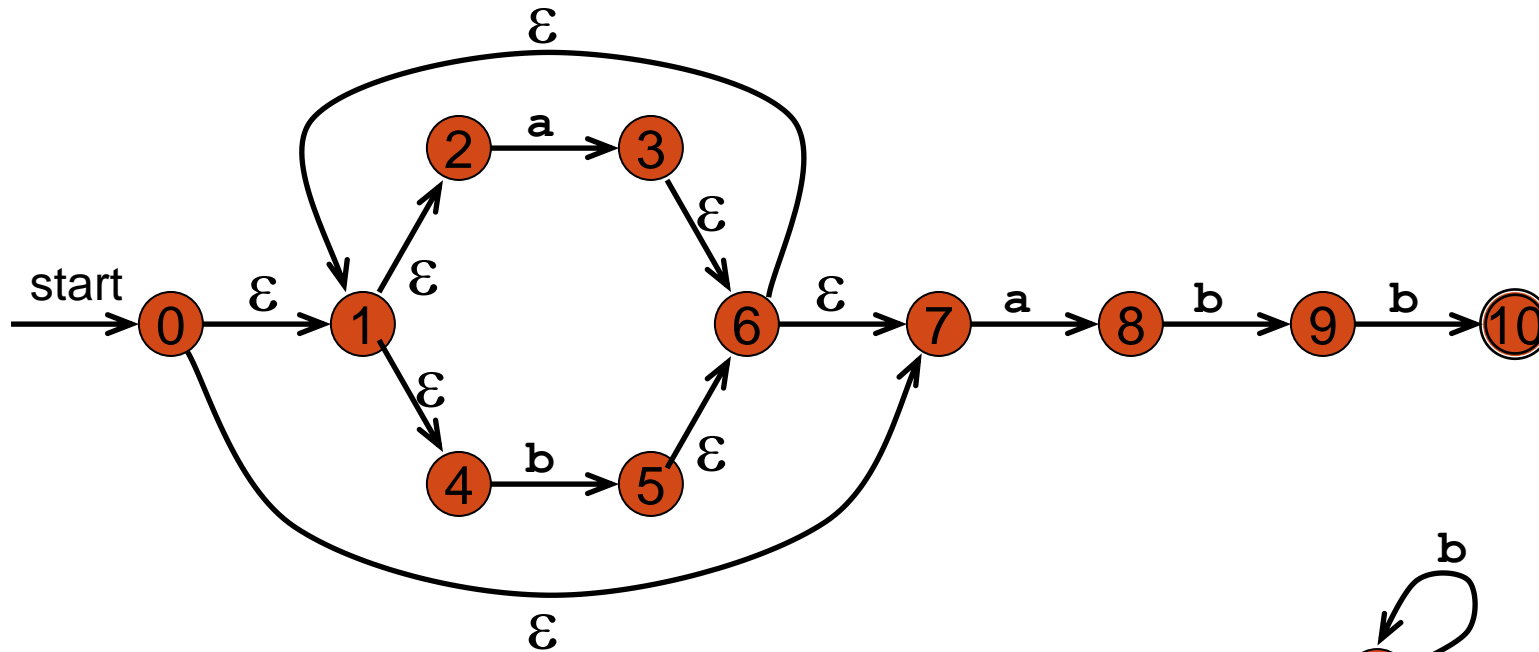
Regular Expression:  $(a \mid b)^* abb$

# Subset Construction Method (Example-1)

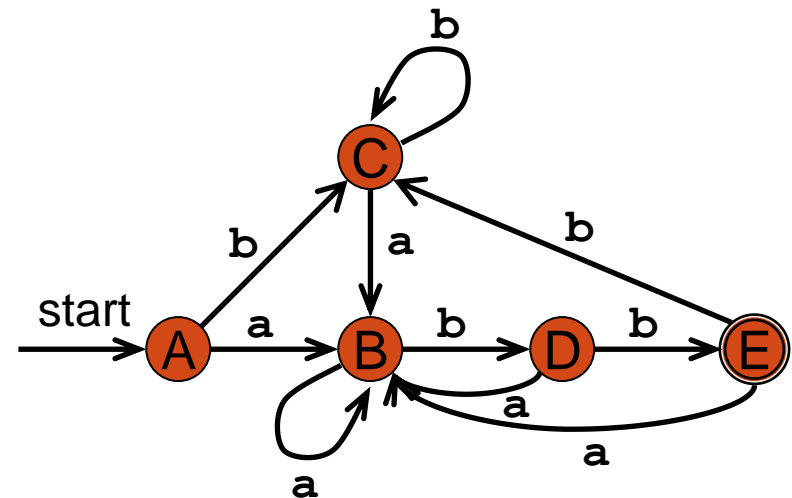


DFA State	E-closure of	E-closure outcome states	NFA States	DFA State	a	b
A	E-closure ({0})	0,1,2,4,7	0,1,2,4,7	A	B	C
B	E-closure ({3,8})	1,2,3,4,6,7,8	1,2,3,4,6,7,8	B	B	D
C	E-closure ({5})	1,2,4,5,6,7	1,2,4,5,6,7	C	B	C
D	E-closure({5,9})	1,2,4,5,6,7,9	1,2,4,5,6,7,9	D	B	E
E	E-closure({5,10})	1,2,4,5,6,7,10	1,2,4,5,6,7,10	E	B	C

# Subset Construction Method (Example-1 Cont.)



NFA State	DFA State	a	b
0,1,2,4,7	A	B	C
1,2,3,4,6,7,8	B	B	D
1,2,4,5,6,7	C	B	C
1,2,4,5,6,7,9	D	B	E
1,2,4,5,6,7,10	E	B	C

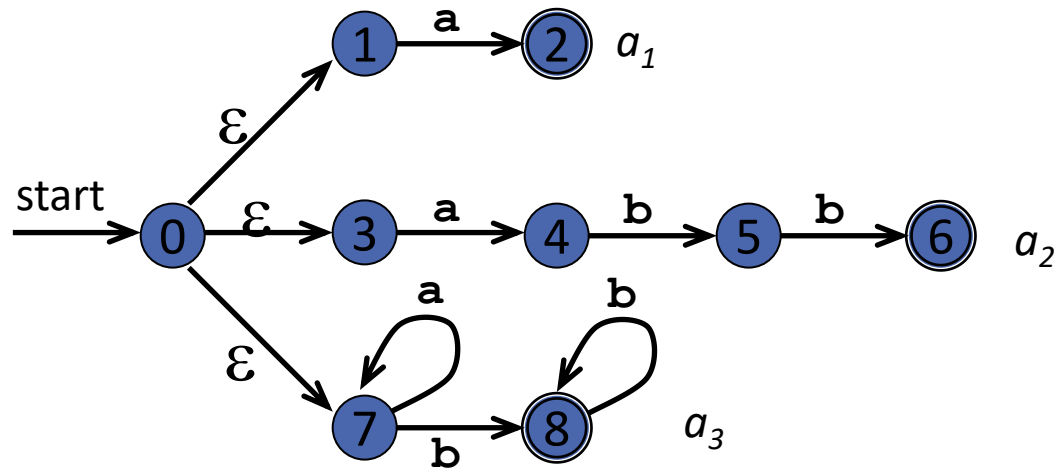


# NFA to DFA Conversion

Subset Construction Method (Exercise 1)



NFA



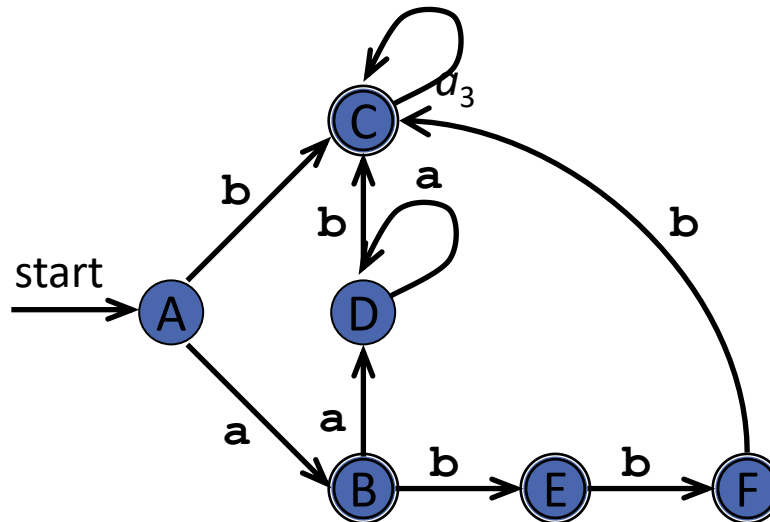
Converted DFA in the next Slide

# NFA to DFA Conversion

Subset Construction Method (Exercise 1)



DFA



*Dstates*

A = {0,1,3,7}

B = {2,4,7}

C = {8}

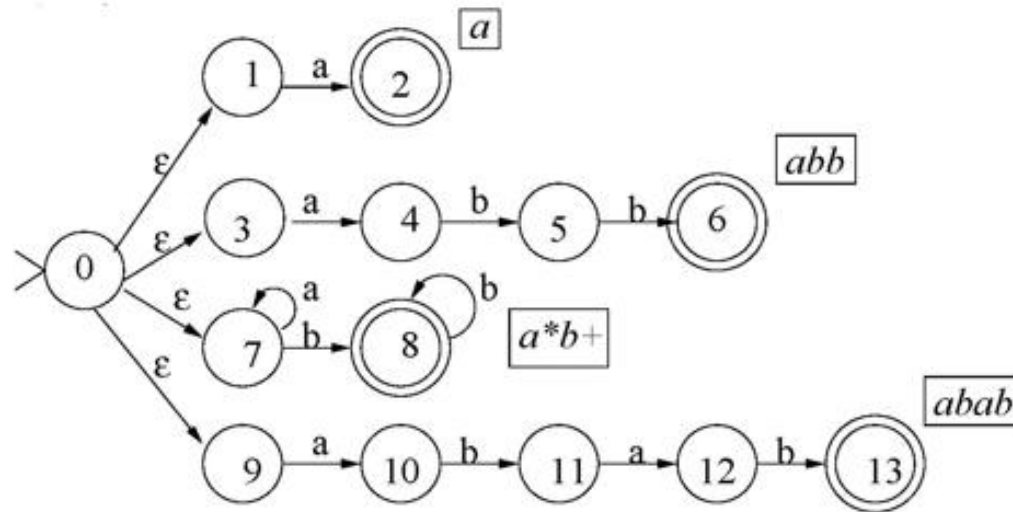
D = {7}

E = {5,8}

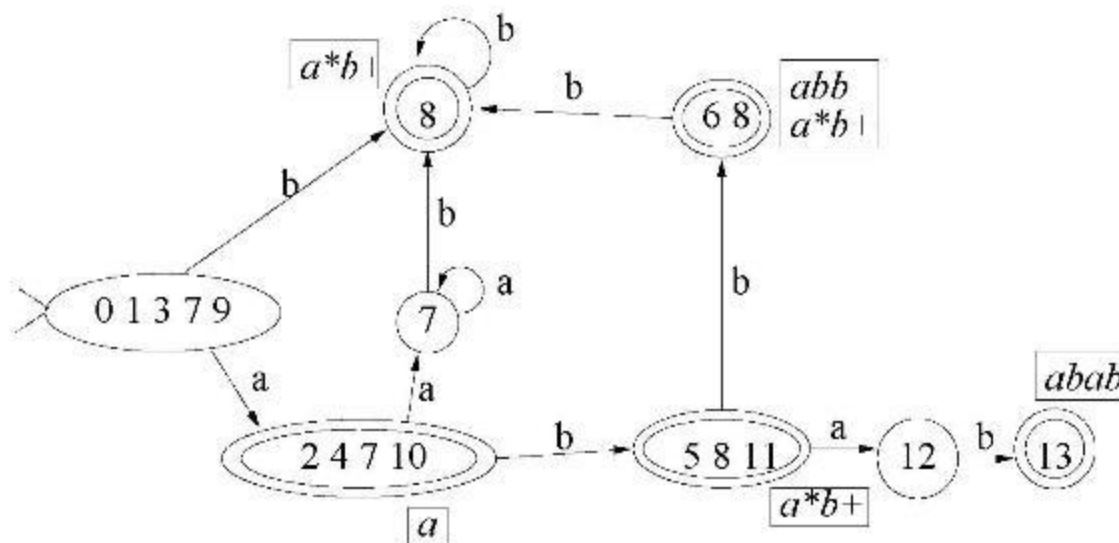
F = {6,8}

# NFA to DFA / Subset Construction Method (Exercise 2)

NFA



DFA  
Hints





# Deterministic Finite Machine

## DFA DESIGN



- A finite automaton is a 5-tuple  $(Q, \Sigma, \delta, q_0, F)$ , where
  - $Q$  is a finite set called the **states**,
  - $\Sigma$  is a finite set called the **alphabet**,
  - $\delta: Q \times \Sigma \rightarrow Q$  is the **transition function**,
  - $q_0 \in Q$  is the **start state**,
  - $F \subseteq Q$  is the set of **accept (final) states**.
- If  $A$  is the set of all strings that a machine  $M$  accepts, we say that  $A$  is the **language of machine  $M$**  and write  $L(M)=A$ ,  **$M$  recognizes  $A$**  or  **$M$  accepts  $A$** .

# Deterministic Finite Machine

## DFA Example 1

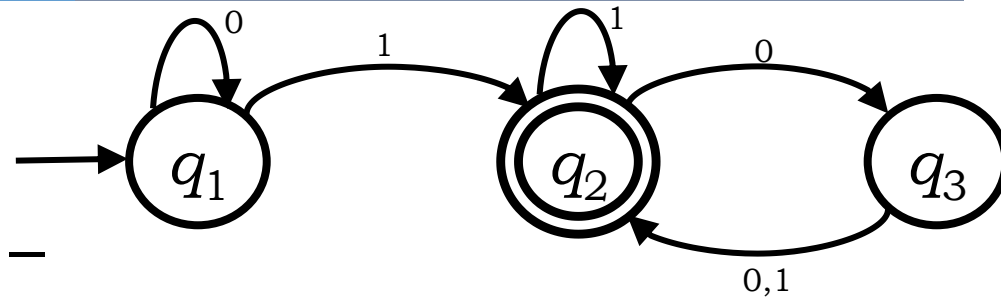


Figure: Finite Automaton  $M_1$

■  $M_1 = (Q, \Sigma, \delta, q_0, F)$ , where –

■  $Q = \{q_1, q_2, q_3\}$ ,

■  $\Sigma = \{0, 1\}$ ,

■  $\delta$  is describe as –

■  $q_0 = q_1$ ,

■  $F = \{q_2\}$ .

$\delta$	0	1
$q_1$	$q_1$	$q_2$
$q_2$	$q_3$	$q_2$
$q_3$	$q_2$	$q_2$

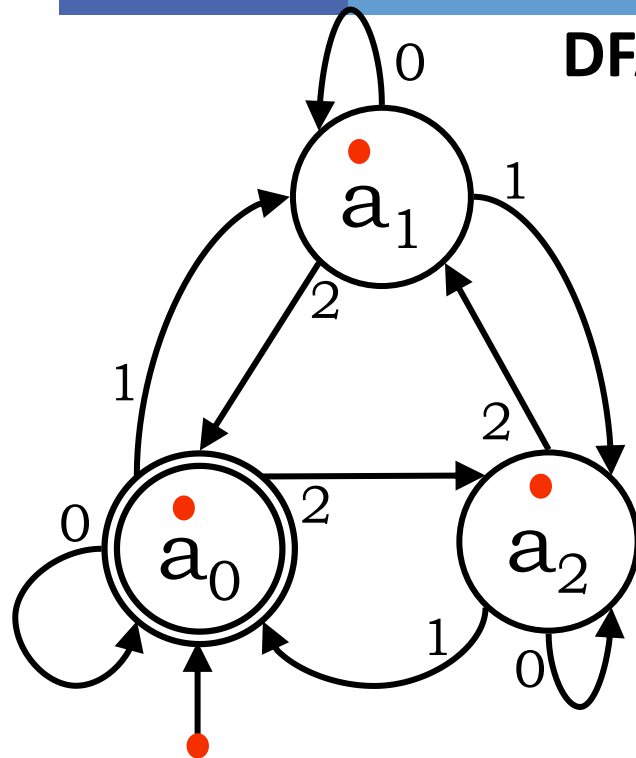
or

$\delta(q_1, 0) = q_1, \delta(q_1, 1) = q_2,$

$\delta(q_2, 0) = q_3, \delta(q_2, 1) = q_2,$

$\delta(q_3, 0) = q_2, \delta(q_3, 1) = q_2.$

# DFA Design Example



- Alphabet  $\Sigma = \{0, 1, 2\}$ .
- Language  $A_1 = \{w : \text{the sum of all the symbols in } w \text{ is multiple of } 3\}$ .

Can be represented as follows –

- $S =$  the sum of all the symbols in  $w$ .
- If  $S \text{ modulo } 3 = 0$  then the sum is multiple of 3.
- So the sum of all the symbols in  $w$  is 0 modulo 3.
- Here,  $a_i$  is modeled as  $S \text{ modulo } 3 = i$ .

The finite state machine  $M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ , where –

- $Q_1 = \{a_0, a_1, a_2\}$ ,
- $q_1 = a_0$ ,
- $F_1 = \{a_0\}$ ,
- $\delta_1$

	0	1	2
$a_0$	$a_0$	$a_1$	$a_2$
$a_1$	$a_1$	$a_2$	$a_0$
$a_2$	$a_2$	$a_0$	$a_1$

Input example: 01120101

Present State:

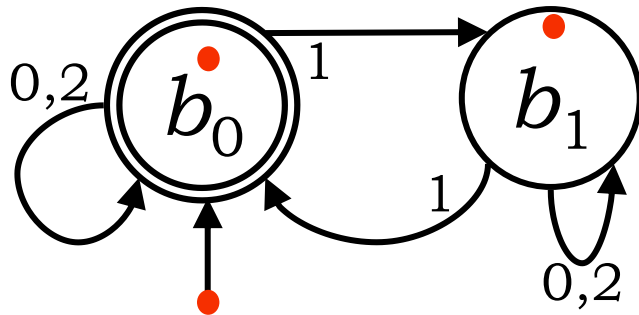
$a_2$

Input symbol:

$\epsilon$

**Accepted**

# DFA Design Example



Alphabet  $\Sigma = \{0, 1, 2\}$ .

Language  $A_1 = \{w : \text{the sum of all the symbols in } w \text{ is an even number}\}$ .

Can be represented as follows –

- $S =$  the sum of all the symbols in  $w$ .
- If  $S \text{ modulo } 2 = 0$  then the sum is even.
- Here,  $b_i$  is modeled as  $S \text{ modulo } 2 = i$ .

The finite state machine  $M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ , where –

$Q_2 = \{b_0, b_1\}$ ,

$q_2 = b_0$ ,

$F_2 = \{b_0\}$ ,

$\delta_2$

	0	1	2
$b_0$	$b_0$	$b_1$	$b_0$
$b_1$	$b_1$	$b_0$	$b_1$

Input example: 01120101

Present State:

$b_1$

Input symbol:

$\epsilon$

**Accepted**



## DFA Design Example (Type 1)

The construction of DFA for languages consisting of strings ending with a particular substring.

- Determine the minimum number of states required in the DFA.
  - Calculate the length of substring.
  - All strings ending with 'n' length substring will always require minimum  $(n+1)$  states in the DFA.
- Draw those states.
- Decide the strings for which DFA will be constructed.
- Construct a DFA for the decided strings
  - While constructing a DFA, Always prefer to use the existing path. Create a new path only when there exists no path to go with.
- Send all the left possible combinations to the starting state.
- Do not send the left possible combinations over the dead state.



# DFA Design Example and Exercise

- Draw a DFA for the language accepting strings ending with 'abb' over input alphabets  $\Sigma = \{a, b\}$
- Draw a DFA for the language accepting strings starting with 'ab' over input alphabets  $\Sigma = \{a, b\}$
- Draw a DFA for the language accepting strings 'ab' in the middle (sub string) over input alphabets  $\Sigma = \{a, b\}$



# Lecture References

- Portland State University Lectures ([Link](#))
- Power set Construction Wikipedia ([Link](#))
- Maynooth University Lectures ([Link](#))



# References/Books

- 1. Compilers-Principles, techniques and tools (2nd Edition) V. Aho, Sethi and D. Ullman
- 2. Principles of Compiler Design (2nd Revised Edition 2009) A. A. Puntambekar
- 3. Basics of Compiler Design Torben Mogensen



# FIRST and FOLLOW

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# Lecture Outline



1. Review of Subset Construction Rule (NFA to DFA conversion)
2. Overview of First and Follow
3. First and Follow set Rules
4. Examples
5. Exercises

# Objective and Outcome



## Objective:

- To Explain the necessity or requirement of FIRST and FOLLOW set calculation.
- To elaborate the method/algorithm of FIRST and FOLLOW calculation from a given CFG.
- To provide necessary example and exercise of FIRST and FOLLOW calculation from a given CFG

## Outcome:

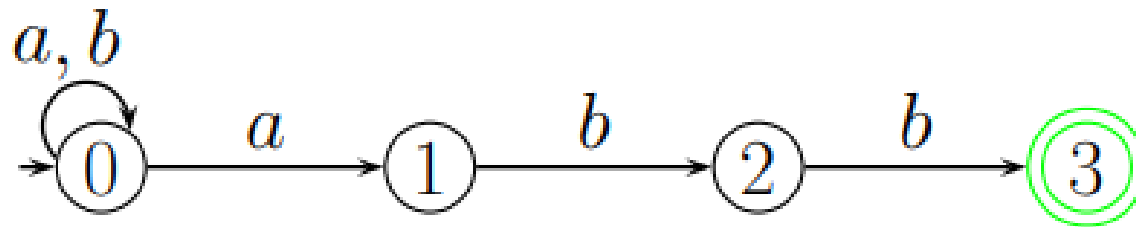
- After this class the students will know the necessity of FIRST and FOLLOW calculation
- After this class the students will be able to demonstrate the FIRST and FOLLOW calculation method.
- The students will also be capable of calculating FIRST and FOLLOW set from a given CFG

# Review on NFA to DFA

## Example



A NFA for the language,  $L3 = \{a, b\}^*\{abb\}$ .



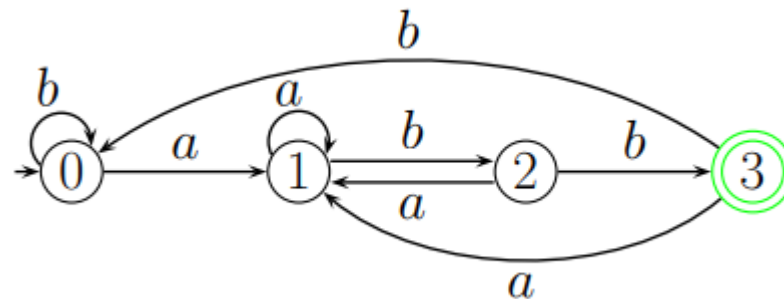
Given NFA

# Review on NFA to DFA

## Example



names	states	<i>a</i>	<i>b</i>
<i>A</i>	$\{0\}$	<i>B</i>	<i>A</i>
<i>B</i>	$\{0, 1\}$	<i>B</i>	<i>C</i>
<i>C</i>	$\{0, 2\}$	<i>B</i>	<i>D</i>
<i>D</i>	$\{0, 3\}$	<i>B</i>	<i>A</i>



Converted DFA

# FIRST and FOLLOW Overview



The basic problem in parsing is choosing which production rule to use at any stage during a derivation.

- Lookahead

Means attempting to analyze the possible production rules which can be applied, in order to pick the one most likely to derive the current symbol(s) on the input.

- FIRST and FOLLOW

We formalize the task of picking a production rule using two functions, FIRST and FOLLOW. we need to find FIRST and FOLLOW sets for a given grammar, so that the parser can properly apply the needed rule at the correct position.

# FIRST Set Calculation

## Rules



1. If  $X$  is terminal,  $\text{FIRST}(X) = \{X\}$ .
2. If  $X \rightarrow \epsilon$  is a production, then add  $\epsilon$  to  $\text{FIRST}(X)$ .
3. If  $X$  is a non-terminal, and  $X \rightarrow Y_1 Y_2 \dots Y_k$  is a production, and  $\epsilon$  is in all of  $\text{FIRST}(Y_1), \dots, \text{FIRST}(Y_k)$ , then add  $\epsilon$  to  $\text{FIRST}(X)$ .
4. If  $X$  is a non-terminal, and  $X \rightarrow Y_1 Y_2 \dots Y_k$  is a production, then add  $a$  to  $\text{FIRST}(X)$  if for some  $i$ ,  $a$  is in  $\text{FIRST}(Y_i)$ , and  $\epsilon$  is in all of  $\text{FIRST}(Y_1), \dots, \text{FIRST}(Y_{i-1})$ .

Applying rules 1 and 2 is obvious. Applying rules 3 and 4 for  $\text{FIRST}(Y_1 Y_2 \dots Y_k)$  can be done as follows:

Add all the non- $\epsilon$  symbols of  $\text{FIRST}(Y_1)$  to  $\text{FIRST}(Y_1 Y_2 \dots Y_k)$ . If  $\epsilon \in \text{FIRST}(Y_1)$ , add all the non- $\epsilon$  symbols of  $\text{FIRST}(Y_2)$ . If  $\epsilon \in \text{FIRST}(Y_1)$  and  $\epsilon \in \text{FIRST}(Y_2)$ , add all the non- $\epsilon$  symbols of  $\text{FIRST}(Y_3)$ , and so on. Finally, add  $\epsilon$  to  $\text{FIRST}(Y_1 Y_2 \dots Y_k)$  if  $\epsilon \in \text{FIRST}(Y_i)$ , for all  $1 \leq i \leq k$ .

# First Set

The algorithm to compute the firsts set of a symbol  $X$ :



```
if( $X$  is a terminal symbol):  
    first( $X$ ) =  $X$ ;  
    break;  
if ( $X \rightarrow \epsilon \in$  productions of the grammar):  
    first( $X$ ).add({  $\epsilon$  });  
foreach( $X \rightarrow Y_1 \dots Y_n \in$  productions of the grammar):  
     $j = 1$ ;  
    while ( $j \leq n$ ):  
        first( $X$ ).add({  $b$  }),  $\forall b \in \text{first}(Y_j)$  ;  
        if (  $\epsilon \in \text{first}(Y_j)$ ):  
             $j++$ ;  
        else:  
            break;  
    if( $j = n+1$ ):  
        first( $X$ ).add({  $\epsilon$  });
```

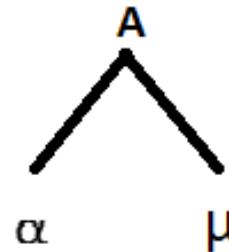


## First Set (Case 1)

- For a Production, if the first thing is terminals that terminal (left most) would be considered as a 'First'
- If the Left most thing is a terminals then that terminals will be 'First'
- Don't worry about the rest of the things residing on the right side of the first terminals

### Case 1

$A \rightarrow \alpha\mu$



$\text{FIRST}(A) = \{ \alpha \}$



## First Set (Case 2)

- For a Production, if the first things is epsilon ( $\epsilon$ ) then 'FIRST' is epsilon ( $\epsilon$ )



## First Set (Case 3)

- For a Production, if the first thing is Non-Terminals, then we should continue until we found a terminals.
- Look for the next production and next until we encounter a terminals

## First Set (Example 1)

### Problem

```
E  -> TE'  
E' -> +T E' |  $\epsilon$   
T  -> FT'  
T' -> *F T' |  $\epsilon$   
F  -> (E) | id
```

### Solution

```
FIRST(E) = FIRST(T) = { ( , id }  
FIRST(E') = { +,  $\epsilon$  }  
FIRST(T) = FIRST(F) = { ( , id }  
FIRST(T') = { *,  $\epsilon$  }  
FIRST(F) = { ( , id }
```

## First Set (Example 2)

### Problem

```
S -> ACB | Cbb | Ba
A -> da | BC
B -> g | ε
C -> h | ε
```

### Solution

FIRST sets

```
FIRST(S) = FIRST(A) U FIRST(B) U FIRST(C)
          = { d, g, h, ε, b, a }
```

```
FIRST(A) = { d } U FIRST(B) = { d, g, h, ε }
```

```
FIRST(B) = { g, ε }
```

```
FIRST(C) = { h, ε }
```

# Follow Set

## Rules



- Follow should be look for right side of anything
- Follow always starts with \$
- **Follow(X)** to be the set of terminals that can appear immediately to the right of Non-Terminal X in some sentential form.
- $FOLLOW(S) = \{ S \}$  // where S is the starting Non-Terminal
- If  $A \rightarrow pBq$  is a production, where p, B and q are any grammar symbols, then everything in  $FIRST(q)$  except  $\epsilon$  is in  $FOLLOW(B)$
- If  $A \rightarrow pB$  is a production, then everything in  $FOLLOW(A)$  is in  $FOLLOW(B)$
- If  $A \rightarrow pBq$  is a production and  $FIRST(q)$  contains  $\epsilon$ , then  $FOLLOW(B)$  contains  $\{ FIRST(q) - \epsilon \} \cup FOLLOW(A)$

# Follow Set

## Rules



Apply the following rules:

1. If  $\$$  is the input end-marker, and  $S$  is the start symbol,  $\$ \in \text{FOLLOW}(S)$ .
2. If there is a production,  $A \rightarrow \alpha B \beta$ , then  $(\text{FIRST}(\beta) - \epsilon) \subseteq \text{FOLLOW}(B)$ .
3. If there is a production,  $A \rightarrow \alpha B$ , or a production  $A \rightarrow \alpha B \beta$ , where  $\epsilon \in \text{FIRST}(\beta)$ , then  $\text{FOLLOW}(A) \subseteq \text{FOLLOW}(B)$ .

**Note** that unlike the computation of FIRST sets for non-terminals, where the focus is *on what a non-terminal generates*, the computation of FOLLOW sets depends upon *where the non-terminal appears on the RHS of a production*



## Follow Set (Case 1-a)

- Follow means something right behind of it.
- Follow means the next one
- If the next of a thing (whos Follow should be calculated) **terminal**/nonterminal then we must find the 'FIRST' of that terminal/nonterminal
- That particular 'FIRST' would be the designated 'FOLLOW' of the things (whos Follow should be calculated)





## Follow Set (Case 1-b)

- Follow means something right behind of it.
- Follow means the next one
- If the next of a thing (whos Follow should be calculated) terminal/**nonterminal** then we must find the 'FIRST' of that terminal/nonterminal
- That particular 'FIRST' would be the designated 'FOLLOW' of the things (whos Follow should be calculated)



## Follow Set (Case 2)

- We never write epsilon ( $\epsilon$ ) in 'FOLLOW'
- If we do not have anything on right side
- That is, if we do not have an 'FOLLOW' then we will take the 'FOLLOW' (all FOLLOW) of its parent (non-terminal) (from which the production came)



# Follow Set (Example 1)

## Problem

### Production Rules:

$E \rightarrow TE'$

$E' \rightarrow +T E' \mid \epsilon$

$T \rightarrow F T'$

$T' \rightarrow *F T' \mid \epsilon$

$F \rightarrow (E) \mid id$

## Solution

### FIRST set

$FIRST(E) = FIRST(T) = \{ (, id \}$

$FIRST(E') = \{ +, \epsilon \}$

$FIRST(T) = FIRST(F) = \{ (, id \}$

$FIRST(T') = \{ *, \epsilon \}$

$FIRST(F) = \{ (, id \}$

### FOLLOW Set

$FOLLOW(E) = \{ \$, ) \}$  // Note ')' is there because of 5th rule

$FOLLOW(E') = FOLLOW(E) = \{ \$, ) \}$  // See 1st production rule

$FOLLOW(T) = \{ FIRST(E') - \epsilon \} \cup FOLLOW(E') \cup FOLLOW(E) = \{ +, \$, ) \}$

$FOLLOW(T') = FOLLOW(T) = \{ +, \$, ) \}$

$FOLLOW(F) = \{ FIRST(T') - \epsilon \} \cup FOLLOW(T') \cup FOLLOW(T) = \{ *, +, \$, ) \}$

## Follow Set (Example 2)

### Problem

Production Rules:

$S \rightarrow ACB \mid Cbb \mid Ba$

$A \rightarrow da \mid BC$

$B \rightarrow g \mid \epsilon$

$C \rightarrow h \mid \epsilon$

### Solution

FIRST set

$FIRST(S) = FIRST(A) \cup FIRST(B) \cup FIRST(C) = \{ d, g, h, \epsilon, b, a \}$

$FIRST(A) = \{ d \} \cup FIRST(B) = \{ d, g, \epsilon \}$

$FIRST(B) = \{ g, \epsilon \}$

$FIRST(C) = \{ h, \epsilon \}$

FOLLOW Set

$FOLLOW(S) = \{ \$ \}$

$FOLLOW(A) = \{ h, g, \$ \}$

$FOLLOW(B) = \{ a, \$, h, g \}$

$FOLLOW(C) = \{ b, g, \$, h \}$

# First and Follow Set

## Example



Grammar	First	Follow
$S \rightarrow ABCDE$	$\{a, b, c\}$	$\{\$ \}$
$A \rightarrow a/\epsilon$	$\{a, \epsilon\}$	$\{b, c\}$
$B \rightarrow b/\epsilon$	$\{b, \epsilon\}$	$\{c\}$
$C \rightarrow c$	$\{c\}$	$\{d, e, \$\}$
$D \rightarrow d/\epsilon$	$\{d, \epsilon\}$	$\{e, \$ \}$
$E \rightarrow e/\epsilon$	$\{e, \epsilon\}$	$\{\$ \}$



# Lecture References

- Online Tool:  
<http://jsmachines.sourceforge.net/machines/ll1.html>
- Online Tutorial  
<https://www.geeksforgeeks.org/why-first-and-follow-in-compiler-design/>
- Maynooth University Material  
<http://www.cs.nuim.ie/~jpower/Courses/Previous/parsing/node48.html>
- StackOverflow Explanation  
<https://stackoverflow.com/questions/3720901/what-is-the-precise-definition-of-a-lookahead-set>



## References/ Books

- 1. Compilers-Principles, techniques and tools (2nd Edition) V. Aho, Sethi and D. Ullman
- 2. Principles of Compiler Design (2nd Revised Edition 2009) A. A. Puntambekar
- 3. Basics of Compiler Design Torben Mogensen