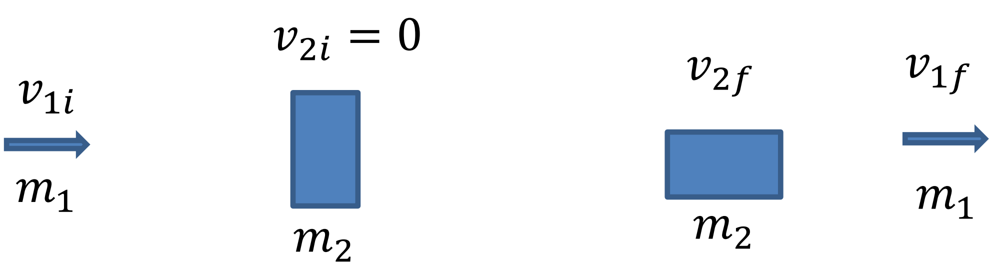
LECTURE 10

THE BOOK CHAPTER 9

Problem 50 (Book chapter 9):

A 5.20 g bullet moving at 672 m/s strikes a 700 g wooden block at rest on a frictionless surface. The bullet emerges, traveling in the same direction with its speed reduced to 428 m/s. (a) What is the resulting speed of the block? (b) What is the speed of the bullet–block center of mass?

Answer:



Before collision **After collision**

Given

$m_1 = 5.2\ g = 5.2 \times 10^{-3} kg$

$m_2 = 700\ g = 700 \times 10^{-3} kg$

$v_{1i} = 672\ m/s$

$v_{2i} = 0\ m/s$

$v_{1f} = 428\ m/s$

$\boldsymbol{(a)\ v_{2f} = ?}$

$\boldsymbol{(b) v_c = ?}$

Inelastic collision: conservation of liner momentum, Pi = Pf:

$$m_1 v_{1i} + 0 = m_1 v_{1f} + m_2 v_{2f}$$

$$m_2 v_{2f} = m_1 v_{1i} - m_1 v_{1f} = m_1 (v_{1i} - v_{1f})$$

$$v_{2f} = \frac{m_1 (v_{1i} - v_{1f})}{m_2} = \frac{5.2 \times 10^{-3} (672 - 428)}{700 \times 10^{-3}} = \mathbf{1.81\ m/s}$$

(b) The center of mass of a closed, isolated system of two colliding bodies is not affected by a collision. That is, the velocity of center of mass is not changed by the collision.

$$P = Mv_{com}$$

$P_i = P_f$

$$Pi = (m_1 + m_2)v_{com}$$

$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2)v_{com}$$

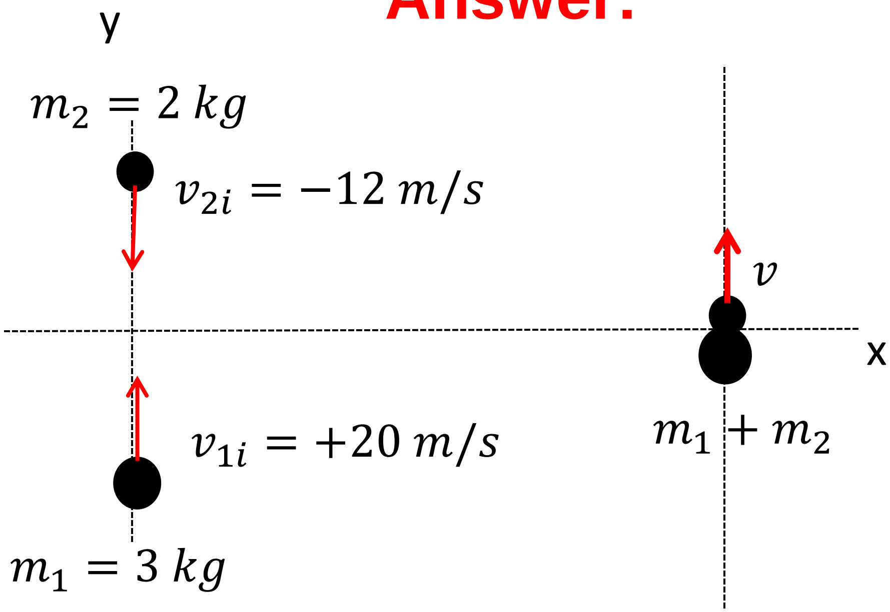
$$m_1 v_{1i} + m_2(0) = (m_1 + m_2)v_{com}$$

$$v_{com} = \frac{m_1 v_{1i}}{m_1+m_2} = \frac{5.2\times10^{-3}\times672}{10^{-3}(5.2+700)} = \frac{5.2\times672}{705.2} = 4.955 \text{ m/s}$$

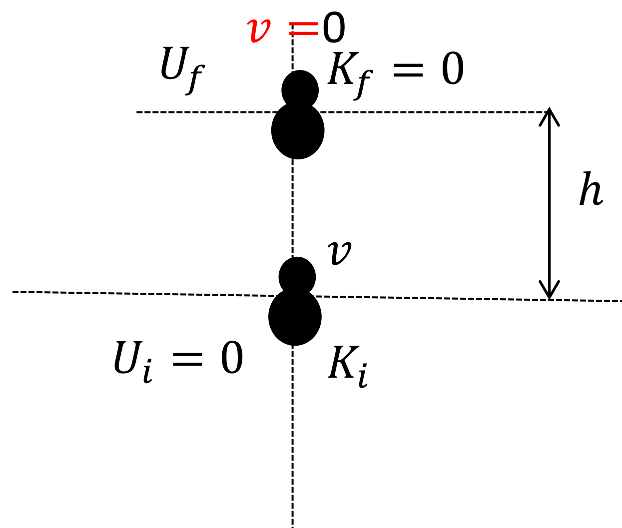
Problem 54 (Book chapter 9):

A completely inelastic collision occurs between two balls of wet putty that move directly toward each other along a vertical axis. Just before the collision, one ball, of mass 3.0 kg, is moving upward at 20 m/s and the other ball, of mass 2.0 kg, is moving downward at 12 m/s. How high do the combined two balls of putty rise above the collision point? (Neglect air drag.)

Answer:



Before collision After collision



After collision

Inelastic collision: Pi = Pf

$$m_1 v_{1i} + m_2 v_{2i} = v\,(m_1 + m_2)$$

$$v = \frac{m_1 v_{1i} + m_2 v_{2i}}{(m_1 + m_2)} = \frac{(3)(+20) + (2)(-12)}{3 + 2}$$

$$v = 7.2\ m/s$$

$E_i = E_f$

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}(m_1 + m_2)v^2 + 0 = 0 + (m_1 + m_2)gh$$

$$\frac{1}{2}(7.2)^2 = 9.8\ h$$

$$25.92 = 9.8\ h$$

$$h = 2.644\ m$$

Problem 65 (Book chapter 9):

A body of mass 2.0 kg makes an elastic collision with another body at rest and continues to move in the original direction but with one-fourth of its original speed. (a) What is the mass of the other body? (b) What is the speed of the two-body center of mass if the initial speed of the 2.0 kg body was 4.0 m/s?

Answer:

(a) We know

$$\boldsymbol{v_{1f} = \frac{(m_1 - m_2)v_{1i}}{m_1 + m_2}}$$

$$\frac{1}{4}v_{1i} = \frac{(2 - m_2)v_{1i}}{2 + m_2}$$

$$\frac{1}{4} = \frac{2 - m_2}{2 + m_2}$$

$$2 + m_2 = 8 - 4m_2$$

$$5m_2 = 6$$

$$\boxed{\boldsymbol{m_2 = 1.2\ kg}}$$

(b) We know

$$\boldsymbol{v_c = \frac{m_1 v_{1i}}{m_1 + m_2}}$$

$$v_c = \frac{(2)(4)}{2 + 1.2} = \frac{8}{3.2}$$

$$\boxed{\boldsymbol{v_c = 2.5\ m/s}}$$

Given

$m_1 = 2\ kg$

v_{1i}

$v_{2i} = 0$

$v_{1f} = \frac{1}{4}v_{1i}$

$(a)\ m_2 = ?$

(b) If $v_{1i} = 4\ m/s$

$v_c = ?$

Problem 61 (Book chapter 9):

A cart with mass 340 g moving on a frictionless linear air track at an initial speed of 1.2 m/s undergoes an elastic collision with an initially stationary cart of unknown mass. After the collision, the first cart continues in its original direction at 0.66 m/s. (a) What is the mass of the second cart? (b)What is its speed after impact? (c) What is the speed of the two-cart center of mass?

Homework

Similar to the problem 65 (book chapter 9)

BOOK CHAPTER 10

ROTATION

Book chapter 10

ROTATION

A **rigid body** is a body that can rotate with all its parts locked together and without any change in its shape. A **fixed axis** means that the rotation occurs about an axis that does not move.

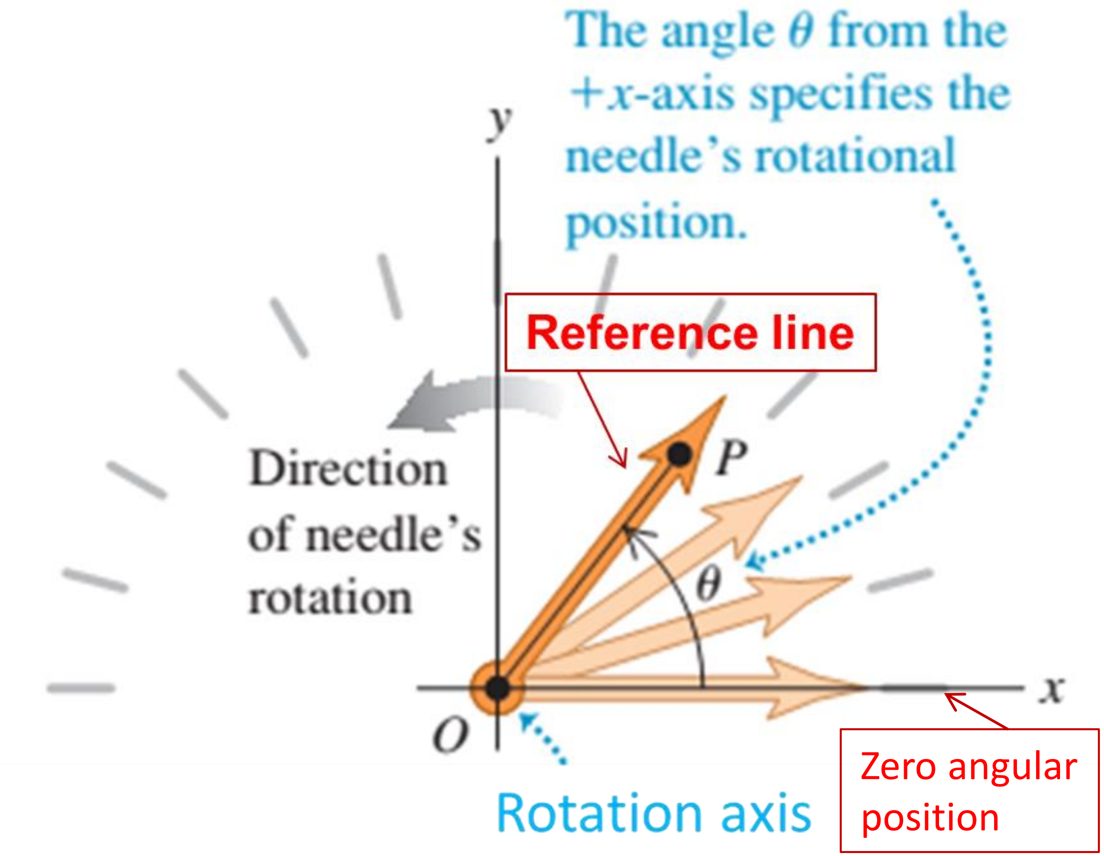


Figure 1

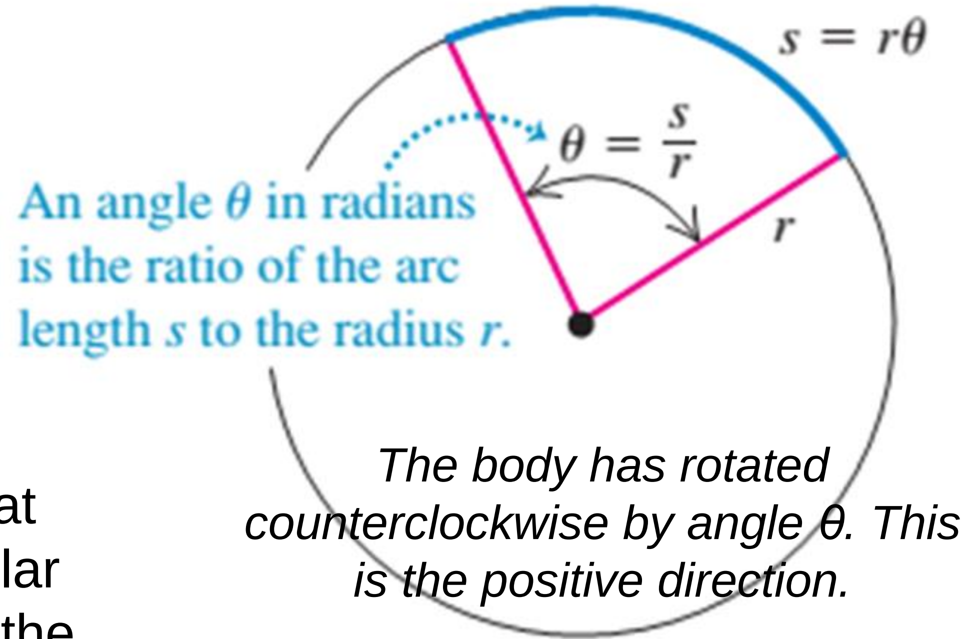
Angular position:

Figure1(a) shows a *reference line*, fixed in the body, perpendicular to the rotation axis and rotating with the body. The **angular position** of this reference line is the angle of the line relative to a fixed direction, which we take as the **zero angular position.**

Angular position:

The angular position θ is measured relative to the positive direction of the *x* axis. From geometry, we know that θ is given by

$$\boldsymbol{\theta = \frac{s}{r}} \qquad \text{(radian measure)}$$



The body has rotated counterclockwise by angle θ. This is the positive direction.

Here s is the length of a circular arc that extends from the *x* axis (the zero angular position) to the reference line, and *r* is the radius of the circle.

NOTE:

An angle defined in this way is measured in **radians** (rad) rather than in revolutions (rev) or degrees. The radian, being the ratio of two lengths, is a pure number and thus has no dimension. Because the circumference of a circle of radius *r* is $2\pi r$, there are 2π radians in a complete circle:

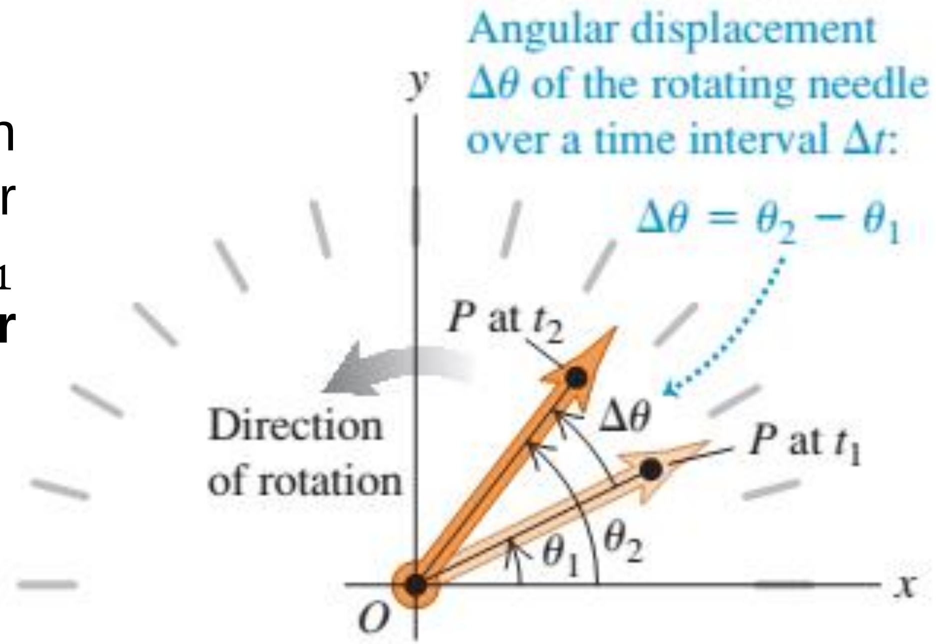
$$1\,rev = 360^0 = \frac{2\pi r}{r} = 2\pi\,rad$$

Thus $$1\,rad = 57.3^0 = 0.159\,rev$$

Angular displacement:

If the body rotates about the rotation axis as in Figure, changing the angular position of the reference line from θ_1 to θ_2 , the body undergoes an **angular displacement** $\Delta\boldsymbol{\theta}$ given by

$$\Delta\theta = \theta_2 - \theta_1$$



Angular velocity:

Suppose that the rotating body is at angular position θ_1 at time t_1 and at angular position θ_2 at time t_2 as in Figure. We define the **average angular velocity** of the body in the time interval Δt from t_1 to t_2 to be

$$\omega_{avg} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}$$

where $\Delta\theta$ is the angular displacement during Δt (ω is the lowercase omega).

The (instantaneous) **angular velocity** ($\boldsymbol{\omega}$) is defined as

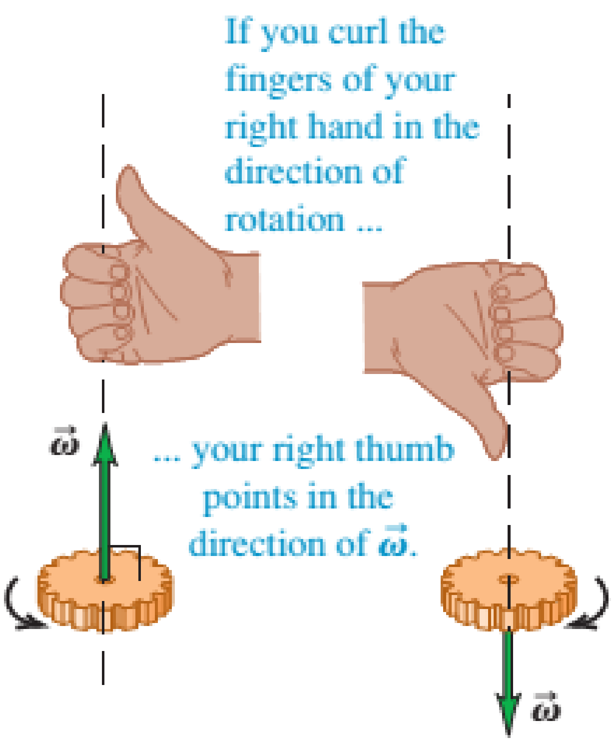
$$\omega = \frac{d\theta}{dt}$$

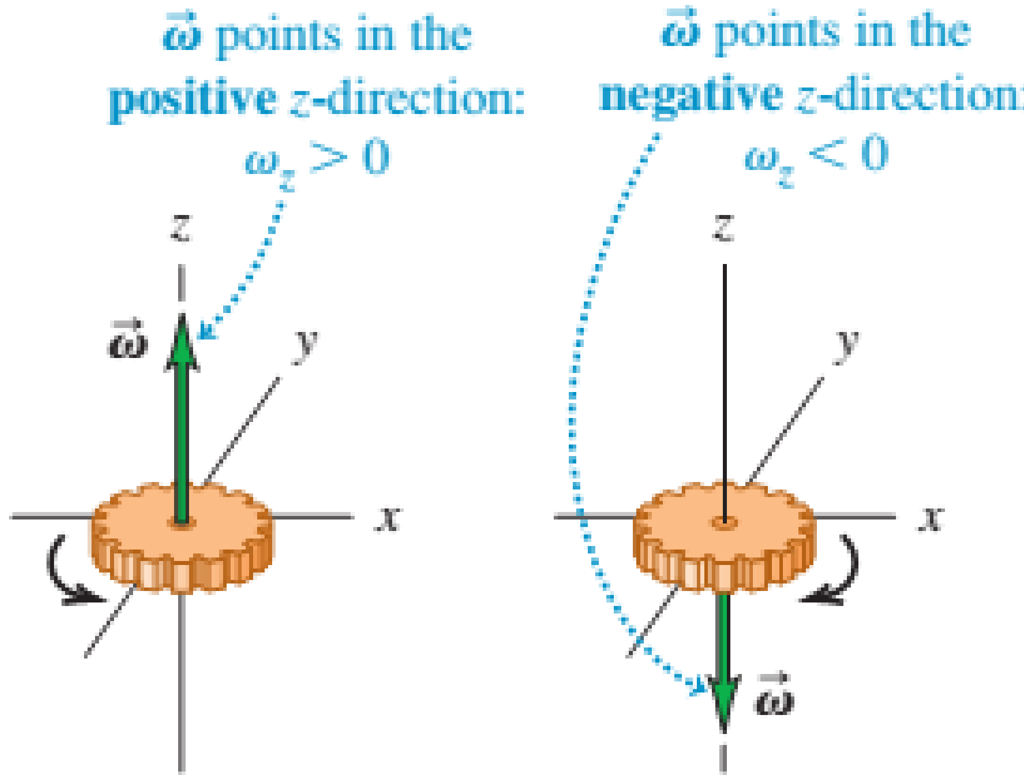
If we know $\theta(t)$, we can find the angular velocity ω by differentiation

The unit of angular velocity is commonly the radian per second (rad/s) or the revolution per second (rev/s).

Another unit is rev/min, which is known as rpm.

Angular velocity as a vector:





Angular acceleration

If the angular velocity of a rotating body is not constant, then the body has an angular acceleration. Let ω_2 and ω_1 be its angular velocities at times t_1 and t_1, respectively. The **average angular acceleration** of the rotating body in the interval from t_1 to t_2 is defined as

$$\alpha_{avg} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t}$$

The (instantaneous) **angular acceleration (α)** is defined as

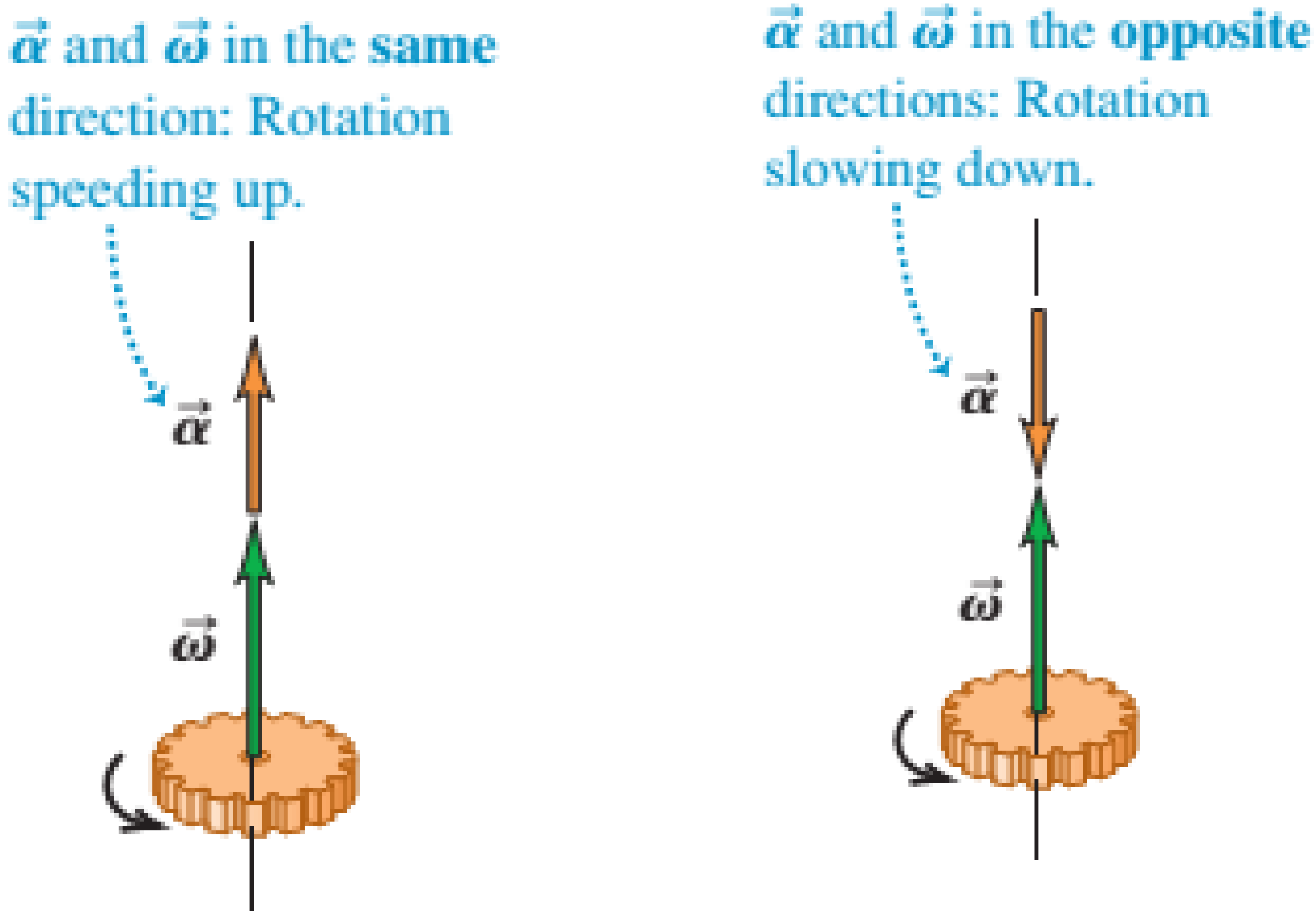
$$\alpha = \frac{d\omega}{dt} \qquad \text{OR} \qquad \alpha = \frac{d}{dt}\frac{d\theta}{dt}$$

in which $\Delta\omega$ is the change in the angular velocity that occurs during the time Interval Δt.

If we know $\omega(t)$, we can find the angular acceleration α by differentiation.

The unit of angular acceleration is commonly the radian per second-squared (rad/s^2) or the revolution per second-squared (rev/s^2).

Angular acceleration as a vector:



NOTE:

In rotational motion, if the angular acceleration α is positive, then the angular velocity ω is increasing; if α is negative, then ω is decreasing. The rotation is speeding up if α and ω have the same sign and slowing down if α and ω have opposite signs.

The Kinematic Equations for Constant Angular Acceleration:

$$\omega = \omega_0 + \alpha t,$$

$$\theta - \theta_0 = \omega_0 t + \tfrac{1}{2}\alpha t^2,$$

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0),$$

$$\theta - \theta_0 = \tfrac{1}{2}(\omega_0 + \omega)t,$$

$$\theta - \theta_0 = \omega t - \tfrac{1}{2}\alpha t^2.$$

Relating the Linear and Angular Variables:

If a reference line on a rigid body rotates through an angle θ, a point within the body at a position *r* from the rotation axis moves a distance s along a circular are, where s is given by

$$s = r\theta \qquad \text{-----------------------------} \qquad (1)$$

Differentiating equation (1) with respect to time with *r* held constant leads to

$$\frac{ds}{dt} = \frac{d\theta}{dt} r$$

$$v = \omega r \qquad \text{----------------} \qquad (2)$$

Where $\frac{ds}{dt} = v$ and $\frac{d\theta}{dt} = \omega$

If the angular speed ω of the rigid body is constant, then Eq.1 tells us that the linear speed v of any point within it is also constant. Thus, each point within the body undergoes uniform circular motion. The period of revolution T for the motion of each point and for the rigid body itself is given

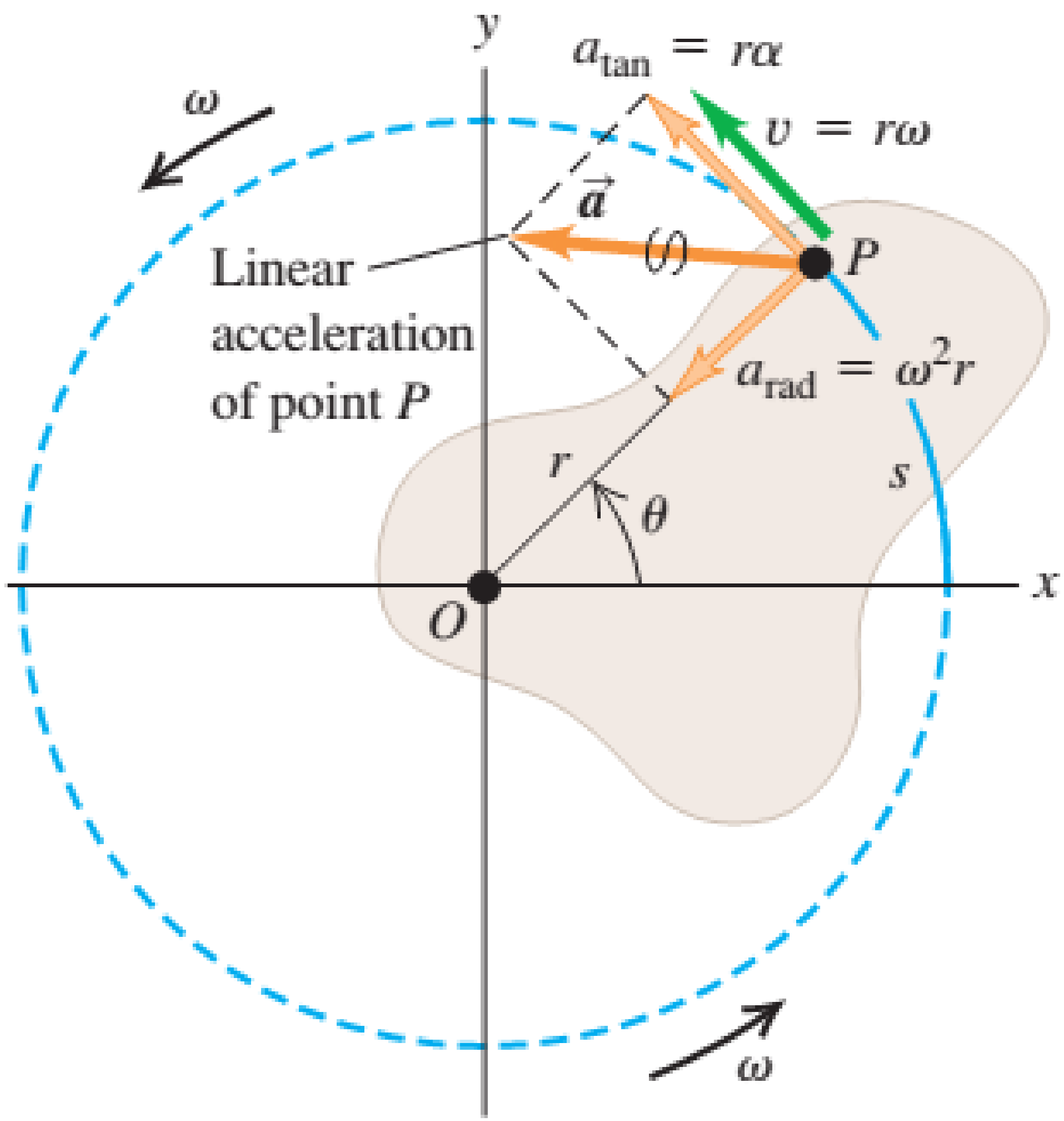
$$T = \frac{2\pi r}{v} \qquad \text{----------------} \qquad (3)$$

This equation tells us that the time for one revolution is the distance $2\pi r$ traveled in one revolution divided by the speed at which that distance is traveled.

From equations (2) and (3), we get

$$T = \frac{2\pi r}{\omega r} = \frac{2\pi}{\omega}$$

Thus $T = \frac{2\pi}{\omega}$



We can represent the acceleration of a particle moving in a circle in terms of its radial (centripetal) and tangential components, a_r and α_t, recpectively.

Differentiating equation (2) with respect to time with *r* held constant leads to

$$\frac{dv}{dt} = \frac{d\omega}{dt}\, r$$

$$\frac{dv}{dt} = \frac{d\omega}{dt}\, r$$

$\boldsymbol{a_t = \alpha r}$ Where, $\alpha = \frac{d\omega}{dt}$ is the angular acceleration of the body.

This component (a_t) of a particle's acceleration is always tangent to the circular path of the particle.

A particle moving in a circular path has a *radial component* of linear acceleration, $\boldsymbol{a_r}$ (directed radially inward), that is responsible for changes in the *direction* of the linear velocity $\vec{v}$.

We can express a_r as

$$a_r = \frac{v^2}{r} = \frac{(\omega r)^2}{r} = \omega^2 r$$

Kinetic Energy of Rotation:

When a rigid body rotates about a fixed axis, the speed of the *i*th particle is given by $v_i = r_i\omega$, where is the body's angular speed. Different particles have different values of *r*, but ω is the same for all (otherwise, the body wouldn't be rigid). The kinetic energy of the *i*th particle can be expressed as

$$K = \frac{1}{2}m_i v_i^2 = \frac{1}{2}m_i r_i^2 \omega^2$$

The *total* kinetic energy of the body is the sum of the kinetic energies of all its particles:

$$K = \frac{1}{2}m_1 r_1^2 \omega^2 + \frac{1}{2}m_2 r_2^2 \omega^2 + \cdots = \sum_I \frac{1}{2}m_i r_i^2 \omega^2$$

$$K = \frac{1}{2}(m_1 r_1^2 + \frac{1}{2}m_2 r_2^2 + \cdots)\omega^2 = \frac{1}{2}\sum_I (m_i r_i^2)\omega^2$$

The quantity in parentheses, obtained by multiplying the mass of each particle by the square of its distance from the axis of rotation and adding these products, is denoted by *I* and is called the **moment of inertia** of the body for this rotation axis:

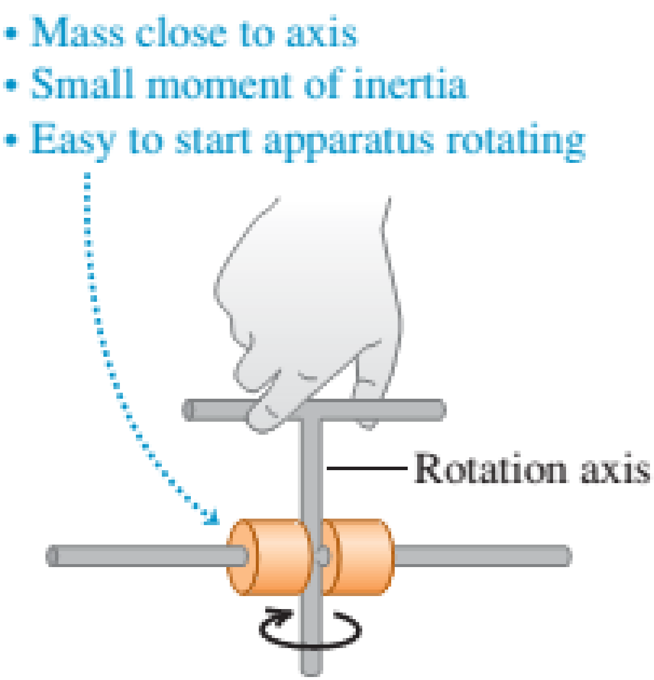
$$I = m_1 r_1^2 + m_2 r_2^2 + \cdots = \sum_i m_i r_i^2$$

For a body with a given rotation axis and a given total mass, the greater the distance from the axis to the particles that make up the body, the greater the moment of inertia. In a rigid body, the distances r_i are all constant and *I* is independent of how the body rotates around the given axis.

The SI unit of moment of inertia is the $kg.m^2$

In terms of moment of inertia *I*, the **rotational kinetic energy** *K* of a rigid body is

$$K = \frac{1}{2} I \omega^2$$



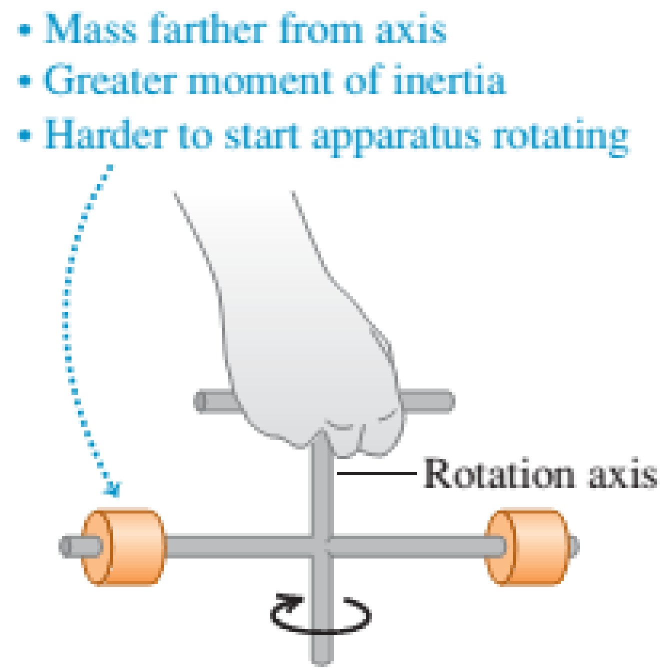


Figure: An apparatus free to rotate around a vertical axis. To vary the moment of inertia, the two equal-mass cylinders can be locked into different positions on the horizontal shaft.

Thank You