



ASSIGNMENT : 04

**Course : Computational Statistics
And
Probability**

Submitted By,

**Group A
Section : K**

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Q.2 - Let, $\mu_1 \sim N(\mu_1, \sigma_1^2)$, $\mu_2 \sim N(\mu_2, \sigma_2^2)$
Also, let $\sigma_1^2 = \sigma_2^2 = \sigma^2$

We need to test, $H_0: \mu_1 = \mu_2$ vs $H_1: \mu_1 \neq \mu_2$
Both $n_1 = 7$ and $n_2 = 11$ are small (< 30) and σ^2 is not known

Test statistic $t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}} \sim t(n_1 - 1) + (n_2 - 1)$

$$\begin{aligned}\bar{x}_1 &= \frac{1}{n_1} \sum x_1 = \frac{114}{7} = 16.29; s_1^2 = \frac{1}{n_1 - 1} \left[\sum x_1^2 - \frac{(\sum x_1)^2}{n_1} \right] \\ &= \frac{1}{6} (1898 - \frac{114^2}{7}) \\ &= 6.905\end{aligned}$$

$$\bar{x}_2 = \frac{1}{n_2} \sum x_2 = \frac{163}{11} = 14.82; s_2^2 = \frac{1}{10} (2569 - \frac{163^2}{11}) = 15.364$$

$$t = \frac{16.29 - 14.82}{\sqrt{12.192 \left(\frac{1}{7} + \frac{1}{11} \right)}} = 0.87$$

$$s_{\text{or}}^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{(n_1 - 1) + (n_2 - 1)} = \frac{6(6.905) + 10(15.364)}{16} = 12.192$$

Since $|t|/t_{16} = 2.12$, H_0 is accepted. Employment facility for both university is same

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Q.4 We need to test $H_0: P = P_0 \equiv 0.10$ vs $H_1: P \neq P_0$

$$\text{Now, } P = \frac{\alpha}{n} = \frac{2}{15} = 0.13$$

$$\text{test statistic: } Z = \frac{P - P_0}{\sqrt{\frac{P_0(1-P_0)}{n}}} = \frac{0.13 - 0.10}{\sqrt{\frac{0.10 \times 0.90}{15}}} \\ = 0.39$$

Since, $|Z| < 1.96$, H_0 is accepted. It can be considered that 10% students got A

Q.5 test stat

We need to test, $H_0: P_1 = P_2$ vs $H_1: P_1 \neq P_2$

$$\text{test statistic: } Z = \frac{P_1 - P_2}{\sqrt{P_0\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \sim N(0,1)$$

$$P_0 = \frac{a_1 + a_2}{n_1 + n_2}, Q = 1 - P$$

$$P_1 = \frac{39}{85} = 0.4, P_2 = \frac{14}{70} = 0.2 \quad P = \frac{48}{155} = 0.31 \quad Q = 0.69$$

$$Z = \frac{0.4 - 0.2}{\sqrt{0.31(0.69)\left(\frac{1}{85} + \frac{1}{70}\right)}} = 2.68$$

Since $|Z| > 1.96$, H_0 is rejected. We conclude that gender does make a difference for drug use

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Q.6 We need to test $H_0: P_1 = P_2 = P_3 = P_4$ vs H_1
 $H_1: At least one of them does not hold$

$$\text{Test statistic } \chi^2 = \sum \frac{O_i - E_i}{E_i} - n$$

$$= \frac{1}{10} [8^2 + 15^2 + 5^2 + 12^2] - 40 \\ = 45.8 - 40 = 5.8$$

$$E_i = \frac{n}{k} = \frac{40}{4} = 10$$

Since, $\chi^2 < \chi_{k-1}^2 = \chi_3^2 = 7.81$, H_0 is accepted. Hence the proportions of defective computers of different laboratories are similar.

Test of Independence:

Here, a, b, c are observed frequencies in different cell

$$n = \sum O_{ij}$$

O_{ij} = Observation of i^{th} row and j^{th} column recorded from the experiment.

$E_{ij} = \frac{R_i C_j}{n}$ = expected frequency corresponding to i^{th} row and j^{th} column under H_0

We need to test, H_0 : the characters P and Q are independent vs H_1 , they are not independent

$$\text{Test statistic: } \chi^2 = \sum \frac{O_{ij}^2}{E_{ij}} - n \sim \chi^2_{(r-1)(c-1)}$$

Here r = no. of rows and c = no. of columns

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9.1 H_0 : Job satisfaction does not depend on good result

H_1 : The good result and job satisfaction are associated

Test statistics : $n^{\sqrt{n}} \frac{(ad - bc)^{\sqrt{v}}}{(a+b)(a+c)(b+d)(c+d)} =$

$$= \frac{150(22 \times 50 - 22 \times 58)^{\sqrt{v}}}{80 \times 70 \times 70 \times 108} = 0.02$$

Since $n^{\sqrt{v}} < \chi_{0.05}^2 = 3.84$, H_0 is accepted. So, job satisfaction does not depend on good result.

Ex 9.2 We need to test $H_0: p_1 = p_2 = p_3 = p_4$ vs
 H_1 : At least one of them not hold

$$E_i = \frac{n}{k} = \frac{206}{4} = 51.5 ; \chi^2 = \sum \frac{O_i - E_i}{E_i}^2 = n$$

$$n^{\sqrt{v}} = \sqrt{\sum \frac{O_i^2}{E_i}} - n$$

$$= \frac{1}{51.5} [50^{\sqrt{v}} + 42^{\sqrt{v}} + 32^{\sqrt{v}} + 82^{\sqrt{v}}] - 206 = 27.24$$

Since, $n^{\sqrt{v}} > \chi_{0.05}^2 = \chi_3^2 = 7.81$.

so, H_1 accepted

the proportion of road accidents not similar

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107 We need to test, $H_0: P_0 = 0.40$ vs $H_1: P_0 \neq 0.40$

$$P = \frac{a}{n} = \frac{8}{25} = 0.32$$

$$P_0 = 0.40 \therefore Q = 0.60$$

~~$\therefore Z = \frac{P - P_0}{\sqrt{\frac{P_0 Q}{n}}}$~~

$$\therefore Z = \frac{P - P_0}{\sqrt{\frac{P_0 Q}{n}}} = \frac{0.32 - 0.40}{\sqrt{\frac{0.40 \times 0.60}{25}}} = 0.816$$

Since $|Z| < 1.96$, so H_0 accepted

The overall proportion of female students are 0.40

108 We need to test, $H_0: P_1 = P_2$ vs $H_1: P_1 \neq P_2$

$$P_1 = \frac{25}{100} = 0.25 \quad P = \frac{25+18}{100+125} = 0.191$$

$$P_2 = \frac{18}{125} = 0.144 \quad Q = 1 - 0.191 = 0.808$$

$$\therefore Z = \frac{P_1 - P_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{0.25 - 0.144}{\sqrt{0.191 \times 0.808 \left(\frac{1}{100} + \frac{1}{125}\right)}} = 2.011$$

Since, $|Z| > 1.96$ H_0 rejected

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10.11		B.P	Heart problem	Total
		Yes	No	
High	150.	120	270	
(not high)	122	158	280	
	272	278	550	

H_0 : High BP not associated with Heart problem

H_1 : High BP associated with Heart problem

$$\chi^2 = \frac{n \left(ad - bc \right)^2}{(a+b)(a+c)(b+d)(c+d)} = \frac{550 \left[(150 \times 158) - (120 \times 122) \right]^2}{270 \times 280 \times 272 \times 278} = 7.89 \approx 7.90$$

$$\chi^2 > \chi^2_{\alpha} = 3.84, H_1 \text{ is accepted}$$

10.14 We need to test,

$H_0: \mu_1 = \mu_2$ vs $H_1: \mu_1 \neq \mu_2$

both n_1 and n_2 are small and σ^2 is unknown

test statistics: $t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}} \sim t(n_1-1) + t(n_2-1)$

$$\bar{x}_1 = \frac{1}{n_1} \times 15 = \frac{16875}{15} = 1125 \text{ and } s_1^2 = 5625$$

$$\bar{x}_2 = \frac{26500}{20} = 1325 \text{ and } s_2^2 = 50625$$

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$$S^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{(n_1-1) + (n_2-1)} = \frac{(14 \times 5625) + (10 \times 50625)}{14+10} \\ = 31534.09$$

$$t = \frac{1725 - 1325}{\sqrt{31534.09} \left(\frac{1}{15} + \frac{1}{20} \right)} = -3.297$$

$$t(n_1-1) + (n_2-1) = 20.0.2.035$$

Since, $t > t_3 = 2.035$, H_0 is rejected
Salinity information not similar.

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Example:

19.2

$$\text{let. } x_1 \sim N(\mu_1, \sigma^2)$$

$$x_2 \sim N(\mu_2, \sigma^2)$$

$$\text{Also, } \sigma^2 = \sigma_1^2 = \sigma^2$$

We need to test, $H_0: \mu_1 = \mu_2$ vs $H_1: \mu_1 \neq \mu_2$
Both $n_1 = 7$ and $n_2 = 11$ are small (< 30) and
 σ^2 is unknown.

$$\text{Test statistic } t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}} \sim t(n_1 - 1) + (n_2 - 1)$$

$$\bar{x}_1 = \frac{1}{n_1} \sum x_1 = \frac{114}{7} = 16.29$$

$$s_1^2 = \frac{1}{n_1 - 1} \left[\sum x_1^2 - \frac{(\sum x_1)^2}{n_1} \right] = \frac{1}{6} \left(1898 - \frac{114^2}{7} \right) = 6.905$$

$$\bar{x}_2 = \frac{1}{n_2} \sum x_2 = \frac{163}{11} = 14.82$$

$$s_2^2 = \frac{1}{n_2 - 1} \left[\sum x_2^2 - \frac{(\sum x_2)^2}{n_2} \right] = \frac{1}{10} \left(2569 - \frac{163^2}{11} \right) = 15.364$$

$$\text{Now } s^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{(n_1 - 1) + (n_2 - 1)} = \frac{6(6.905) + 10(15.364)}{16} = 12.192$$

$$t = \frac{16.29 - 14.82}{\sqrt{22.192 \left(\frac{1}{7} + \frac{1}{22} \right)}} = 0.87$$

Since $|t| < t_{16} = 2.12$

H_0 is accepted. Employment facility for students of both universities are same.

Q.9 we need to test

$$H_0: P = P_0 = 0.10 \text{ vs } H_1: P \neq P_0$$

$$\text{Now, } p = \frac{a}{n} = \frac{2}{15} = 0.13$$

$$\text{Test statistic } z = \frac{p - P_0}{\sqrt{\frac{P_0 Q_0}{n}}} = \frac{0.13 - 0.10}{\sqrt{\frac{0.10 \times 0.1590}{15}}} = 0.39$$

Since $|z| < 1.96$, H_0 is accepted. It can be considered that 10% students got grade A.

Q.5 we need to test $H_0: P_1 = P_2$ vs $H_1: P_1 \neq P_2$

$$\text{Test statistic } z = \frac{P_1 - P_2}{\sqrt{PQ \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}} \sim N(0, 1)$$

$$P = \frac{a_1 + a_2}{n_1 + n_2}, \quad Q = 1 - P$$

$$P_1 = \frac{34}{85} = 0.4 \quad P_2 = \frac{14}{70} = 0.2 \quad P = \frac{48}{155} = 0.31$$

$$z = 0.69$$

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9.7 H_0 : Job satisfaction does not depend on good result
 H_1 : The good result and job satisfaction are associated.

$$\text{Test statistic: } \chi^2 = \frac{n(ad - bc)^2}{(a+b)(a+c)(b+d)(c+d)}$$

$$= \frac{150(22 \times 50 - 22 \times 58)^2}{80 \times 70 \times 42 \times 108} = 0.02$$

Since $\chi^2 < \chi^2_{\alpha/2} = 3.84$, H_0 is accepted. So job satisfaction does not depend on good result.

Exercise

9.2 we need to test,

 $H_0: P_1 = P_2 = P_3 = P_4 \text{ vs } H_1: \text{At least one of them doesn't hold}$

$\text{hence, } E_i = \frac{n}{k} = \frac{206}{4} = 51.5$

$$\begin{aligned} \chi^2 &= \sum \frac{(O_i - E_i)^2}{E_i} - n \\ &= \frac{1}{51.5} [50^2 + 42^2 + 32^2 + 82^2] - 206 \\ &= 27.24 \end{aligned}$$

Since $\chi^2 > \chi^2_{k-1} = \chi^2_3 = 7.81$. So, H_1 is accepted
 hence the proportions of road accidents doesn't similar.

$$Z = \frac{0.4 - 0.2}{\sqrt{(0.31)(0.67) \left(\frac{1}{35} + \frac{1}{20} \right)}} = 2.68$$

Since $|z| > 1.96$, H_0 is rejected we conclude
that gender does make a difference for drug use.

9.6 we need to test $H_0: p_1 = p_2 = p_3 = p_4$ vs H_1 : At least one of them doesn't hold.

Test statistic,

$$\begin{aligned} \chi^2 &= \sum \frac{(O_i - E_i)^2}{E_i} - n \\ &= \frac{1}{10} [8^2 + 15^2 + 5^2 + 12^2] - 40 \\ &= 458 - 40 = 58 \end{aligned}$$

$$E_i = \frac{n}{k} = \frac{40}{4} = 10$$

Since $\chi^2 < \chi^2_{K-1} = \chi^2_3 = 7.81$. H_0 is accepted

- ② Hence the proportion of defective computers of different laboratories are similar.

Q.7

we need to test.

$$H_0: P_0 = 0.40 \text{ vs } H_1: P_0 \neq 0.40$$

$$P = \frac{x}{n} = \frac{8}{25} = 0.32$$

$$P_0 = 0.40 \quad \therefore Q = 0.60$$

$$\therefore Z = \frac{P_0 - P_0}{\sqrt{\frac{P_0 Q_0}{n}}} = \frac{0.32 - 0.40}{\sqrt{\frac{0.40 \times 0.60}{25}}} = -0.816$$

Since $|Z| < 1.96$, so H_0 is accepted

Hence the overall proportion of female students are 0.32

Q.9
we need to test $H_0: P_1 = P_2 \text{ vs } H_1: P_1 \neq P_2$

$$P_1 = \frac{25}{100} = 0.25$$

$$P = \frac{25+18}{100+125} = 0.191$$

$$Q = 1 - 0.191 = 0.808$$

$$\therefore Z = \frac{P_1 - P_2}{\sqrt{PQ\left(\frac{1}{m_1} + \frac{1}{m_2}\right)}}$$

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Date: / /

$$= \frac{0.25 - 0.144}{\sqrt{0.191 \times 0.808 \left(\frac{1}{100} + \frac{1}{125} \right)}} = 2.022$$

Since $|z| > 1.96$, H_0 is rejected.

Q21

H_0 : High blood pressure isn't associated with heart problem

H_1 : High blood pressure is associated with heart problem

$$\begin{aligned} \chi^2 &= \frac{n(ad - bc)^2}{(a+b)(a+c)(b+d)(c+d)} \\ &= \frac{550 \left[(50 \times 158) - (120 \times 122) \right]^2}{270 \times 280 \times 272 \times 278} \\ &= 7.897 \sim 7.9 \end{aligned}$$

Since $\chi^2 > \chi^2_{0.05} = 3.84$ so, H_1 is accepted

High blood pressure is associated with heart problems.

9.14 we need to test

$$H_0: \mu_1 = \mu_2 \text{ vs } H_1: \mu_1 \neq \mu_2$$

both n_1 and n_2 are sample (< 30) and σ is unknown.

$$\text{Test statistic: } t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}} \sim t(n_1-1) + t(n_2-1)$$

$$\bar{x}_1 = \frac{1}{n_1} \sum x_1 = \frac{16875}{15} = 1125 \text{ and } s_1^2 = 5625$$

$$\bar{x}_2 = \frac{1}{n_2} \sum x_2 = \frac{26500}{20} = 1325 \text{ and } s_2^2 = 50625$$

$$s^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{(n_1-1) + (n_2-1)}$$

$$= \frac{(19 \times 5625) + (19 \times 50625)}{19 + 19}$$

$$= 31534.09$$

$$t = \frac{1125 - 1325}{\sqrt{31534.09 \left(\frac{1}{15} + \frac{1}{20} \right)}} \sim t(n_1-1) + t(n_2-1)$$

$$= -3.297 \quad t_{33} = 2.09$$



$$= -3.297$$

since $|t| > t_{33}$

So, H_0 are rejected

: salary information are not similar.

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10.14

We need to test

$$H_0: \mu_1 = \mu_2 \text{ vs } H_1: \mu_1 \neq \mu_2$$

Both n_1 and n_2 are small (< 30) and σ^2 is unknown

unknown

$$\text{Test Statistic } t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}} \sim t(n_1 - 1) + (n_2 - 1)$$

$$\bar{x}_1 = \frac{1}{n_1} \sum x_1 = \frac{16825}{15} = 1125 \text{ and } s_1^2 = 5625$$

$$\bar{x}_2 = \frac{1}{n_2} \sum x_2 = \frac{2650}{20} = 1325 \text{ and } s_2^2 = 50625$$

$$s^2 = \frac{n_1(n_1-1)s_1^2 + (n_2-1)s_2^2}{(n_1-1) + (n_2-1)}$$
$$= \frac{14 \times 5625 + (12 \times 50625)}{14 + 12}$$

- 50625

$$= 31534.09$$

$$\therefore t = \frac{1125 - 1325}{\sqrt{31534.09} \left(\frac{1}{15} + \frac{1}{20} \right)} \quad \left| \begin{array}{l} t(n_1-1) + (n_2-1) \\ t_3 = 3.035 \end{array} \right.$$



10-II

H_0 : High blood pressure is not associated with heart problem

H_1 : High blood pressure is associated with heart problem

$$\chi^2 = \frac{n(ad - bc)^2}{(a+b)(a+c)(b+d)(c+d)}$$

$$= \frac{550 [(150 \times 158) - 120 \times 122]^2}{270 \times 280 \times 222 \times 228}$$

$$= 2.897 \sim 2.90$$

B.P	Heart Problem		Total
	Yes	No	
High	150	120	270
Not High	122	158	280
Total	272	278	550

Since, $\chi^2 > \chi_{0.05}^2 = 3.84$

So H_0 is accepted

Since $|t| > t_{\alpha/2} = 2.035$

So, H_0 is rejected

∴ H_1 is accepted
i.e. Salary information are not similar

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{50 - 45}{\sqrt{\frac{1}{10} + \frac{1}{10}}} = 2.236$$

$$\text{Margin of error} = z_{\alpha/2} \cdot \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = 2.035 \cdot \sqrt{\frac{1}{10} + \frac{1}{10}} = 0.407$$

$$\text{Range} = \frac{36.3}{0.407} = 89.3 \cdot \frac{1}{0.407} = 220.3$$

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(50 - 45)}{\sqrt{\frac{1}{10} + \frac{1}{10}}} = 2.236$$

$$\text{Margin of error} = z_{\alpha/2} \cdot \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = 2.035 \cdot \sqrt{\frac{1}{10} + \frac{1}{10}} = 0.407$$



~~10.6~~

We need to test $H_0: P_1 = P_2 = P_3 = P_4$ vs. At least one of them doesn't hold

$$\text{Test statistics } \chi^2 = \sum_i \frac{(O_i - E_i)^2}{E_i} - n$$

$$= \frac{1}{10} [8^2 + (S^2 + S^2 + R^2)] - 40$$

$$\text{Hence no break } b = 15.8 - 10 = 5.8$$

$$E_i = \frac{n}{k} = \frac{40}{4} = 10$$

$$\text{Since } \chi^2 < \chi^2_{k-1} = \chi^2 = 7.81 \quad H_0 \text{ is accepted}$$

10.9

10.1

we need to test $H_0 : P_1 = P_2$ vs $H_1 : P_1 \neq P_2$

$$P_1 = \frac{25}{100} = 0.25$$

$$P_2 = \frac{18}{135} = 0.144$$

$$P = \frac{25 + 18}{100 + 135} = 0.191$$

$$Q = 1 - 0.191 = 0.808$$

$$\therefore Z = \frac{P_1 - P_2}{\sqrt{PQ \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}}$$

$$= \frac{0.25 - 0.144}{\sqrt{0.191 \times 0.808}} \left(\frac{1}{100} + \frac{1}{135} \right) = 2.011$$

Here $|Z| > 1.96$

So

H_0 is rejected



10.8

9-9: all but of less are 2.01

H_0 : Job satisfaction doesn't depend on good result

H_1 : The good result and job satisfaction are associated

$$\text{Test statistics } \chi^2 = \frac{n(ad - bc)^2}{(a+b)(a+c)(c+d)} \\ = \frac{150(22 \times 50 - 22 - 58)^2}{80 \times 70 \times 72 \times 108} \\ = \frac{150}{127} = 0.62$$

So, H_0 is accepted

so, job satisfaction doesn't depend on good result

$$0.1 = \frac{0.1}{N} = \frac{1}{8} = 0.125$$



Q.5

We need to test $H_0: P_1 = P_2 \text{ vs } H_1: P_1 \neq P_2$

$$\text{Test statistic } Z = \frac{P_1 - P_2}{\sqrt{PQ} \left(\frac{1}{n_1} + \frac{1}{n_2} \right)} \sim N(0.1);$$

$$P = \frac{a_1 + a_2}{n_1 + n_2} - \alpha = \frac{8}{15} - 0.1 = \frac{0.8 - 0.1}{15} = 0.05 \text{ well}$$

$$P_1 = \frac{34}{85} = 0.4 \quad P_2 = \frac{14}{20} = 0.2$$

$$P = \frac{48}{155} = 0.31 \quad q = 0.69$$

$$\therefore Z = \frac{0.4 - 0.2}{\sqrt{(0.31)(0.69)} \left(\frac{1}{85} + \frac{1}{20} \right)}$$

Since $(Z > 1.96)$ H_0 is rejected



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Example

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9.2

Let, $x_1 \sim N(\mu_1, \sigma^2)$

$x_2 \sim N(\mu_2, \sigma^2)$ and let $\sigma_1^2, \sigma_2^2 = \sigma^2$

We need to test, $H_0: \mu_1 = \mu_2$ vs $H_1: \mu_1 \neq \mu_2$

Both $n_1 = 7$ and $n_2 = 11$ are small (< 30) and σ^2 isn't known

Test statistic: $t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}} \sim t(n_1-1) + (n_2-1)$

$$\bar{x}_1 = \frac{1}{n_1-1} \sum x_1 = \frac{114}{7} = 16.29$$

$$\bar{x}_2 = \frac{1}{n_2-1} \sum x_2 = \frac{163}{11} = 14.82$$

$$s_1^2 = \frac{1}{n_1-1} \left[\sum x_1^2 - \frac{(\sum x_1)^2}{n_1} \right] = \frac{1}{6} \left(1898 - \frac{114^2}{7} \right) = 6.905$$

$$s_2^2 = \frac{1}{n_2-1} \left[\sum x_2^2 - \frac{(\sum x_2)^2}{n_2} \right] = \frac{1}{10} \left(2569 - \frac{163^2}{11} \right) = 15.364$$

$$t = \frac{16.29 - 14.82}{\sqrt{12.192 \left(\frac{1}{7} + \frac{1}{11} \right)}} = 0.87$$

$$s^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{(n_1-1) + (n_2-1)} = \frac{6(6.905) + 10(15.364)}{16} = 12.192$$

9.4

We need to test, $H_0: P = P_0 = 0.10$ vs $H_1: P \neq P_0$

$$\text{Now, } P = \frac{\alpha}{n} = \frac{2}{15} = 0.13$$

$$\text{Test statistic: } z = \frac{P - P_0}{\sqrt{\frac{P_0(1-P_0)}{n}}} = \frac{0.13 - 0.10}{\sqrt{\frac{0.1 \times 0.9}{15}}} = 0.39$$

Since $|z| < 1.96$, H_0 is accepted, it can be considered that 10% students got grade A.

9.5

We need to test $H_0: P_1 = P_2$ vs $H_1: P_1 \neq P_2$

$$\text{Test statistic, } z = \frac{P_1 - P_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \sim N(0, 1)$$

$$P = \frac{\alpha_1 + \alpha_2}{n_1 + n_2}, Q = 1 - P$$

$$P_1 = \frac{34}{85} = 0.4, P_2 = \frac{14}{10} = 0.2, P = \frac{98}{155} = 0.31, Q = 0.69$$

$$z = \frac{0.4 - 0.2}{\sqrt{(0.31)(0.69)\left(\frac{1}{85} + \frac{1}{10}\right)}} = 2.68$$

Since, $|z| > 1.96$, H_0 is rejected. We conclude that gender does make a difference for drug use.

9.6

We need to test $H_0: p_1 = p_2 = p_3 = p_4$ vs H_1 : at least one of them doesn't hold.

$$\text{Test statistic, } \chi^2 = \sum \frac{(O_i - E_i)^2}{E_i} - n$$

$$\Rightarrow \frac{1}{10} [8^2 + 15^2 + 5^2 + 12^2] - 40$$

$$\Rightarrow 45.8 - 40 = 5.8$$

$$E_1 = \frac{n}{k} = \frac{40}{4} = 10$$

Since $\chi^2 < \chi_{k-1}^2 = \chi_3^2 = 7.81$, H_0 is accepted

Hence, the proportions of defective computers of different laboratories are similar.

9.7

H_0 : Job satisfaction doesn't depend on good result

H_1 : The good result and job satisfaction are associated.

$$\text{Test statistic: } \chi^2 = \frac{n(ad-bc)^2}{(a+b)(a+c)(b+d)(c+d)} = \frac{150(22 \times 50 - 22 \times 58)^2}{80 \times 70 \times 92 \times 108} = 0.02$$

since, $\chi^2 < \chi_1^2 = 3.84$, H_0 is accepted. So, job satisfaction does not depend on good result.

Exercise

9.2

We need to test, $H_0: p_1 = p_2 = p_3 = p_4$ vs H_1 : at least one of them doesn't hold

$$\text{Here } E_i = \frac{n}{k} = \frac{206}{4} = 51.5$$

$$\chi^2 = \sum \frac{O_i - E_i}{E_i} = -n$$

$$= -\frac{1}{51.5} [50 - 42 + 32 + 82] = 206 = 27.24$$

Since, $\chi^2 > \chi_{k-1}^2 = \chi_3^2 = 7.81$, so H_1 is accepted.

Hence, the proportions of road accidents aren't similar.

9.7

We need to test, $H_0: p_0 = 0.4$ vs $H_1: p_0 \neq 0.4$

$$p = \frac{\alpha}{n} = \frac{8}{25} = 0.32$$

$$p_0 = 0.4 \text{, so, } Q = 1 - 0.4 = 0.6$$

$$\therefore z = \frac{-p_0 + p}{\sqrt{\frac{p_0 Q_0}{n}}} = \frac{0.32 - 0.4}{\sqrt{\frac{0.4 \times 0.6}{25}}} = -0.816$$

since $|z| < 1.96$, so H_0 is accepted. Hence the overall proportion of female students are 0.4.

9.9

We need to test, $H_0: p_1 = p_2$ vs $H_1: p_1 \neq p_2$

$$p_1 = \frac{25}{100} = 0.25$$

$$p_2 = \frac{18}{125} = 0.144$$

$$P = \frac{25+18}{100+125} = 0.191$$

$$\begin{aligned} & \text{Here, } a_1 = 25, a_2 = 18 \\ & n_1 = 100, n_2 = 125 \end{aligned}$$

$$Q = |1 - 0.191| = 0.808$$

$$\therefore z = \frac{p_1 - p_2}{\sqrt{pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{0.25 - 0.144}{\sqrt{0.191 \times 0.808 \left(\frac{1}{100} + \frac{1}{125}\right)}} = 2.011$$

since $|z| > 1.96$, H_0 is rejected.

9.11

H_0 : High blood pressure isn't associated with heart problem

H_1 : High blood pressure is associated with heart problem

$$\chi^2 = \frac{n(ad-bc)^2}{(a+b)(a+c)(b+d)(c+d)}$$
$$= \frac{550[(150 \times 158) - (120 \times 122)]^2}{270 \times 180 \times 272 \times 278}$$

$$\therefore \chi^2 \approx 7.90$$

Since, $\chi^2 > \chi_{0.05}^2 = 3.84$, so, H_1 is accepted

Hence, High blood pressure is associated with heart problem.

B.P	Heart Problem		Total
	Yes	No	
High	150	120	270
Normal	122	158	280
Total	272	278	550

9.14

We need to test

$$H_0: \mu_1 = \mu_2 \text{ vs } H_1: \mu_1 \neq \mu_2$$

both n_1 and n_2 are small (< 30) and σ^2 is unknown

$$\text{Test statistic: } t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s^2(\frac{1}{n_1} + \frac{1}{n_2})}} \sim t(n_1-1) + t(n_2-1)$$

$$\bar{x}_1 = \frac{1}{n_1} \sum x_1 = \frac{16875}{15} = 1125 \text{ and } s_1^2 = 5625$$

$$\bar{x}_2 = \frac{1}{n_2} \sum x_2 = \frac{26500}{20} = 1325 \text{ and } s_2^2 = 50625$$

$$\begin{aligned} s^2 &= \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{(n_1-1) + (n_2-1)} \\ &= \frac{(14 \times 5625) + (19 \times 50625)}{14 + 19} = 31534.09 \end{aligned}$$

$$\therefore t = \frac{1125 - 1325}{\sqrt{31534.09} \left(\frac{1}{15} + \frac{1}{20} \right)} \quad \begin{cases} t(n_1-1) + t(n_2-1) \\ t_{33} = 2.035 \end{cases}$$
$$\approx -3.297$$

since $|t| > t_{33} = 2.035$

so, H_0 is rejected

Hence, Salary information aren't similar

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P: 1

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Example:

9.2

$$H_0: \mu_1 = \mu_2 \text{ vs } H_1: \mu_1 \neq \mu_2$$

both $n_1 = 7$ and $n_2 = 11$ are small (< 30) and σ^2 is not known.

$$\bar{x}_1 = \frac{114}{7} = 16.29$$

$$S_1^2 = \frac{1}{6} (1898 - \frac{114^2}{7}) = 6.905$$

$$\bar{x}_2 = \frac{163}{11} = 14.82$$

$$S_2^2 = \frac{1}{10} (2569 - \frac{163^2}{11}) = 15.364$$

$$t = \frac{16.29 - 14.82}{\sqrt{12.192(\frac{1}{7} + \frac{1}{11})}} = 0.87$$

$$S^2 = \frac{6(6.905) + 10(15.364)}{16}$$

$$= 12.192$$

~~(A)~~

$\therefore |t| < t_{16}$. H_0 accepted

9.4 $H_0: P = P_0 = 0.1$ vs $H_1: P \neq P_0$

$$P = \frac{a}{n} = \frac{2}{15} = 0.13$$

$$Z = \frac{0.13 - 0.10}{\sqrt{\frac{0.10 \times 0.9}{15}}} = 0.39$$

$|Z| < 1.96$, H_0 is accepted.

9.5 $H_0: P_1 = P_2$ vs $H_1: P_2 \neq P_1$

$$P = \frac{a_1 + a_2}{n_1 + n_2}, Q = 1 - P$$

$$\text{Data given } P_1 = \frac{34}{85} = 0.4, P_2 = \frac{14}{70} = 0.2$$

$$P = \frac{48}{155} = 0.31, Q = 0.69$$

$$Z = \frac{0.4 - 0.2}{\sqrt{(0.31)(0.69)\left(\frac{1}{85} + \frac{1}{70}\right)}} = 2.68$$

$|Z| > 1.96$. H_0 is rejected.

SaiKart

P: 3

9.6

$H_0: P_1 = P_2 = P_3 = P_4$ vs $H_1:$ At least one of them doesn't same.

$$\chi^2 = \frac{1}{10} (8^2 + 5^2 + 5^2 + 12^2) - 40 \\ = 45.8 - 40 = 5.8$$

$$E_i = \frac{n}{K} = \frac{40}{4} = 10$$

Since, $\chi^2 < \chi_{k-1}^2 = \chi_3^2 = 7.81$.

H_0 is accepted.

9.7

$H_0:$ Job satisfaction does not depend on good result.

$H_1:$ The good result and job satisfaction are associated.

$$\chi^2 = \frac{150(22 \times 50 - 22 \times 58)^2}{80 \times 70 \times 42 \times 108} = 0.02$$

$\chi^2 < \chi_4^2 = 3.84$. H_0 is accepted.

Srikar

P: 4

Exercise:

9.2

$H_0: P_1 = P_2 = P_3 = P_4$ vs $H_1:$ At least one of them doesn't hold.

$$E_i = \frac{n}{k} = \frac{206}{4} = 51.5$$

$$\chi^2 = \frac{1}{51.5} (50^2 + 42^2 + 32^2 + 82^2) - 206 \\ = \cancel{229.24} \quad 27.24$$

Since $\chi^2 > \chi^2_{\alpha} = 7.81$.

H_0 is rejected.

SaiKarth

P: 5

9.7

$$H_0: P = P_0 = 0.4 \text{ vs } H_1: P \neq P_0$$

$$P = \frac{8}{25} = 0.32$$

$$Z = \frac{P - P_0}{\sqrt{\frac{P_0(1-P_0)}{n}}} = \frac{0.32 - 0.4}{\sqrt{\frac{0.4 \times 0.6}{25}}} \\ = -0.816$$

$$|Z| = 0.816$$

$|Z| < 1.96$. H_0 is accepted.

9.9

$$H_0: P_1 = P_2 \text{ vs } H_1: P_1 \neq P_2$$

$$P_1 = \frac{25}{100} = 0.25, P_2 = \frac{18}{125} = 0.144$$

$$P = \frac{43}{225} = 0.1911, q = 0.8089$$

$$Z = \frac{0.25 - 0.144}{\sqrt{0.1911 \times 0.8089 \left(\frac{1}{100} + \frac{1}{125}\right)}} = 2.009$$

$|Z| > 1.96$ - H_0 is rejected.

SaiKarth

P: 6

Q.11

H_0 : blood pressure is not associated with heart problem

H_1 : associated.

$$\cancel{272 \times 278} \quad \chi^2 = \frac{550[(150 \times 158 - 122 \times 120)]}{272 \times 270 \times 280 \times 278} = 7.89$$

$$\chi^2 > \chi^2_{\text{crit}} = 3.84.$$

H_1 is accepted.

Q.14

$H_0: \mu_1 = \mu_2$ vs $H_1: \mu_1 \neq \mu_2$

$$\bar{\mu}_1 = \frac{16875}{15} = 1125, \bar{\mu}_2 = \frac{26500}{20} = 1325$$

$$S^2 = \frac{5625 \times 14 + 50625 \times 19}{33} = 31534.1$$

$$t = \frac{1125 - 1325}{\sqrt{31534.1 \left(\frac{1}{15} + \frac{1}{20} \right)}} = -3.29$$

$$\therefore |t| > t_{33} = 2.035. \quad \begin{matrix} \leftarrow & \text{value from random} \\ & \text{table} \end{matrix}$$

H_0 is rejected.

Image

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Assignment - 4

Example:

Q.2

Let, $x_1 \sim N(\mu_1, \sigma^2)$, $x_2 \sim N(\mu_2, \sigma^2)$.

Also,

$$\text{let } \sigma^2_1 = \sigma^2_2 = \sigma^2$$

We need to test, $H_0: \mu_1 = \mu_2$ vs $H_1: \mu_1 \neq \mu_2$

Both $n_1 = 7$ and $n_2 = 11$ are small (< 30) and σ^2 is not known.

Test statistic: $t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}} \sim t_{(n_1-1) + (n_2-1)}$

$$\bar{x}_1 = \frac{1}{n_1} \sum x_1 = \frac{109}{7} = 16.29 \quad s_1^2 = \frac{1}{n_1-1} \left[\sum x_1^2 - \frac{(\sum x_1)^2}{n_1} \right] = \frac{1}{6} \left(1898 - \frac{114^2}{7} \right) = 6.905$$

$$\bar{x}_2 = \frac{1}{n_2} \sum x_2 = \frac{163}{11} = 14.82 \quad s_2^2 = \frac{1}{n_2-1} \left[\sum x_2^2 - \frac{(\sum x_2)^2}{n_2} \right] = \frac{1}{10} \left(2569 - \frac{163^2}{11} \right) = 15.364$$

$$t = \frac{16.29 - 14.82}{\sqrt{12.192 \left(\frac{1}{7} + \frac{1}{11} \right)}} = 0.87 \quad s^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{(n_1-1) + (n_2-1)} = \frac{6(6.905) + 10(15.364)}{16} = 12.102$$

Since $|t| < t_{0.05} = 2.12$, H_0 is accepted. Employment facility for students of both universities is same.

(2)

Q.9

We need to test, $H_0: P = P_0 = 0.10$ vs $H_1: P \neq P_0$.

$$\text{Now, } p = \frac{a}{n} = \frac{2}{15} = 0.13 \text{ with } n=15.$$

$$\text{Test statistic: } z = \frac{P - P_0}{\sqrt{\frac{P_0 Q_0}{n}}} = \frac{0.13 - 0.10}{\sqrt{\frac{0.10 \times 0.90}{15}}} = 0.39$$

Since $|z| < 1.96$, H_0 is accepted. It can be considered that 10% students got grade 'A'.

Q.5

We need to test $H_0: P_1 = P_2$ vs $H_1: P_1 \neq P_2$.

$$\text{Test statistic, } z = \frac{P_1 - P_2}{\sqrt{PQ(\frac{1}{n_1} + \frac{1}{n_2})}} \sim N(0,1); P = \frac{a_1 + a_2}{n_1 + n_2}, Q = 1 - P$$

$$P_1 = \frac{39}{85} = 0.4, P_2 = \frac{14}{70} = 0.2, P = \frac{48}{155} = 0.31, Q = 0.69$$

$$z = \frac{0.4 - 0.2}{\sqrt{(0.31)(0.69)(\frac{1}{85} + \frac{1}{70})}} = 2.68$$

Since $|z| > 1.96$, H_0 is rejected. We conclude that gender does make a difference for drug use.

(3)

Q.6

We need to test $H_0: p_1 = p_2 = p_3 = p_4$ vs H_1 : At least one of them doesn't hold test statistic.

$$\begin{aligned} \chi^2 &= \sum \frac{(O_i - E_i)^2}{E_i} \\ &= \frac{1}{10} [8^2 + 15^2 + 5^2 + 12^2] - 40 = 95.8 - 40 = 55.8 \end{aligned}$$

$$E_i = \frac{n}{k} = \frac{40}{4} = 10$$

since $\chi^2 < \chi^2_{k-1} = \chi^2_3 = 7.81$, H_0 is accepted. Hence, the proportions of defective computers of different laboratories are similar.

Q.7

H_0 : Job satisfaction does not depend on good result.

H_1 : The good result and job satisfaction are associated.

$$\text{Test statistic: } \chi^2 = \frac{n(ad - bc)^2}{(a+b)(a+c)(b+d)(c+d)} = \frac{150(22 \times 50 - 22 \times 58)^2}{80 \times 70 \times 42 \times 108} = 0.02$$

Since, $\chi^2 < \chi^2_1 = 3.84$, H_0 is accepted. So, Job satisfaction does not depend on good result.

(4)

Exercise:

Q.2 We need to test, $H_0: P_1 = P_2 = P_3 = P_4$ vs $H_1: \text{At least one of them doesn't hold.}$

Hence,

$$Ei = \frac{n}{K} = \frac{206}{4} = 51.5$$

$$\chi^2 \geq \frac{\alpha^2}{Ei} - n = \frac{1}{51.5} [50^2 + 42^2 + 32^2 + 82^2] - 206 = 27.24$$

Since $\chi^2 > \chi^2_{k-1} = \chi^2_3 = 7.81$, H_1 is accepted.

Hence the proportion of road accidents ~~are not~~ are similar.

Q.4

We need to test,

$$H_0: P_0 = 0.40 \text{ vs } H_1: P_0 \neq 0.40$$

$$P = \frac{a}{n} = \frac{8}{25} = 0.32. \text{ This company sold light bulb}$$

$P_0 = 0.40$ is the proportion of ordinary light bulb

$$\therefore Q_0 = 0.60$$

$$Z = \frac{P - P_0}{\sqrt{\frac{P_0 Q_0}{n}}} = \frac{0.32 - 0.40}{\sqrt{\frac{0.40 \times 0.60}{25}}} = -0.816$$

Since $|Z| < 1.96$, H_0 is accepted

Hence, the overall proportion of female students are 0.40.

(5)

Q.9

We need to test, $H_0: p_1 = p_2$ vs $H_1: p_1 \neq p_2$

$$P_1 = \frac{25}{100} = 0.25$$

$$P = \frac{25+125}{100+125} = 0.191$$

$$Q = 1 - 0.191 = 0.808$$

$$P_2 = \frac{18}{125} = 0.144$$

Hence,

$$n_1 = 100$$

$$n_2 = 125$$

$$a_1 = 25$$

$$n_1 = 100$$

$$a_2 = 18$$

$$\therefore Z = \frac{P_1 - P_2}{\sqrt{PQ(\frac{1}{n_1} + \frac{1}{n_2})}}$$

$$= \frac{0.25 - 0.144}{\sqrt{0.191 \times 0.808 (\frac{1}{100} + \frac{1}{125})}}$$

Since $|Z| > 1.96$, H_0 is rejected.

Q.11

H_0 : High blood pressure isn't associated with heart problem

H_1 : High blood pressure is associated with heart problem.

$$\chi^2 = \frac{n(ad-bc)^2}{(a+b)(a+c)(b+d)(c+d)}$$

$$= \frac{550 [(150 \times 122) - (120 \times 128)]^2}{270 \times 280 \times 272 \times 278}$$

$$= 7.807 \approx 7.9$$

since, $\chi^2 > \chi_{0.05}^2 = 3.84$, H_1 is accepted.

High blood pressure associated with heart.

Blood Pressure	Heart problem		Total
	Yes	No	
High	150	120	270
Not high	122	158	280
Total	272	278	550

(6)

Q.14

We need to test

$$H_0: \mu_1 = \mu_2 \quad vs \quad H_1: \mu_1 \neq \mu_2$$

$$H_0: n_1 = n_2 \quad vs \quad H_1: n_1 \neq n_2$$

both n_1 and n_2 are small (< 30) and σ^2 is unknown.

$$\text{Test statistic: } t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s^2(\frac{1}{n_1} + \frac{1}{n_2})}} \sim t(n_1-1) + t(n_2-1)$$

$$\bar{x}_1 = \frac{1}{n_1} \sum x_1 = \frac{16875}{15} = 1125 \quad \text{and} \quad s_1^2 = 5625$$

$$\bar{x}_2 = \frac{1}{n_2} \sum x_2 = \frac{26500}{20} = 1325 \quad \text{and} \quad s_2^2 = 50625$$

$$s^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{(n_1-1)(n_2-1)} = \frac{(14 \times 5625) + (19 \times 50625)}{19+19} = 31534.00$$

$$\therefore t = \frac{1125 - 1325}{\sqrt{31534.00} (\frac{1}{15} + \frac{1}{20})} \quad \left| \begin{array}{l} \therefore t(n_1-1) + t(n_2-1) \\ t_{33} = 2.035 \end{array} \right.$$

$$= -3.207$$

since, $|t| > t_{33} = 2.035$

so, H_0 is rejected

\therefore Salary information are not similar.

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Examples

10.2 Let $x_1 \sim N(\mu_1, \sigma^2)$
 $x_2 \sim N(\mu_2, \sigma^2)$

Also let $\sigma^2 = \sigma_1^2 = \sigma_2^2$

We need to test - $H_0: \mu_1 = \mu_2$ vs $H_1: \mu_1 \neq \mu_2$

Both $n_1 = 7$ and $n_2 = 11$ are small (< 30) and σ^2 is known.

Test statistic $t_2 = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}} \sim t_{(n_1-1) + (n_2-1)}$

$\bar{x}_1 = \frac{1}{n_1} \sum x_i = \frac{114}{7} = 16.22$

$s_1^2 = \frac{1}{n_1-1} \left[\sum n_i - \frac{(\sum n_i)^2}{n_1} \right] = \frac{1}{6} \left(1898 - \frac{114^2}{7} \right) = 6.905$

$\bar{x}_2 = \frac{1}{n_2} \sum x_i = \frac{163}{11} = 14.82$

corrected values and as H₁ testing $\mu_1 > \mu_2$

A strong MC test statistic

$$S_1 = \frac{1}{n_1 - 1} \left[\sum n_1 - \frac{\sum n_1 \bar{x}}{n_1} \right] = \frac{1}{10} \left(15.369 - \frac{14.3}{11} \right)$$

Now

$$S_2 = \frac{(n_1 - 1)S_1 + (n_2 - 1)S_2}{(n_1 - 1) + (n_2 - 1)} = \frac{6(4.909) + 10(15.369)}{16} = 15.369$$

$$t = \frac{16.29 - 14.82}{\sqrt{12.192 \left(\frac{1}{2} + \frac{1}{11} \right)}} = 0.87$$

$$\text{since } |t| < t_{10}$$

Hence accepted Employment faculty for students to both university are same.

10.4 (1) We need to test

$$H_0: P = P_0 = 0.10 \text{ vs } H_1: P \neq P_0$$

$$\text{Now } \hat{P} = \frac{A}{n} = \frac{2}{15} = 0.13$$

$$\text{Test statistic } Z = \frac{P - P_0}{\sqrt{\frac{P_0 Q_0}{n}}} = \frac{0.13 - 0.10}{\sqrt{\frac{0.10 \times 0.90}{15}}} = 0.39$$

$$\text{since } |Z| < 1.96$$

Hence is accepted It can be considered that $\frac{2}{15}$ student got grade A.

10.5

We need to test $H_0: p_1 = p_2$ vs $H_1: p_1 \neq p_2$

Test statistics, $Z = \frac{p_1 - p_2}{\sqrt{pq(\frac{1}{n_1} + \frac{1}{n_2})}} \sim N(0,1)$

$$p = \frac{a_1 + a_2}{n_1 + n_2} = Q = 1 - p$$

$$p_1 = \frac{39}{85} = 0.61 \quad p_2 = \frac{19}{70} = 0.27 \quad p = \frac{48}{155} = 0.31, q = 0.69$$
$$Z = \frac{0.61 - 0.27}{\sqrt{(0.31)(0.69)\left(\frac{1}{85} + \frac{1}{70}\right)}} = 2.68$$

Since $|Z| > 1.96$, H_0 is rejected

We conclude that gender does make a difference for drug use

10.6 We need to test $H_0: p_1 = p_2 = p_3 = p_4$ vs 201

$H_1:$ At least one of them doesn't hold

(1) Test statistics χ^2 - suitable test

$$\begin{aligned} \chi^2 &= \sum \frac{(O_i - E_i)^2}{E_i} \\ &= \frac{1}{10} [8^2 + 15^2 + 5^2 + 12^2] = 40.22 \end{aligned}$$

$$E_i = \frac{n}{k} = \frac{40}{4} = 10$$

Since,

$$\chi^2 < \chi_{\alpha/2}^2 = 7.81 \quad H_0 \text{ is accepted}$$

Hence the properties of different laboratories are similar

so can't say anything

10.7

H_0 : Job satisfaction doesn't depend on good result.

H_1 : The good result and job satisfaction are associated.

$$\text{Test statistics } \chi^2 = \frac{n(ad - bc)^2}{(a+b)(c+d)(b+c)(d+a)}$$
$$= \frac{150(22 \times 59 - 22 \times 58)^2}{80 \times 70 \times 92 \times 108} = 0.02$$

Since $\chi^2 < \chi_{0.05}^2 = 3.84$ H_0 is accepted.

So, job satisfaction doesn't depend on good results.

Exercise

F. 01

10.2 ~~containing back to back dot~~ . H

We need to test $H_0: P_1 = P_2 = P_3 = P_4$ vs H_1 : At least one of them doesn't hold

Here,

$$E_1 = \frac{n}{4} = \frac{206}{4} = 51.5$$

$$x^2 = \sum \frac{(O_i - E_i)^2}{E_i}$$

$$\begin{aligned} x^2 &= \frac{1}{51.5} [(50 - 51.5)^2 + (42 - 51.5)^2 + (32 - 51.5)^2 + (82 - 51.5)^2] - 206 = 27.29 \end{aligned}$$

Since $n^2 > n_{k-1} = n_3 = 7.81$

~~Hence H_0 is accepted.~~

~~Similar, we can take with the other categories.~~

10.7:

We need to test

$H_0: P_0 = 0.40$ vs $H_1: P_0 \neq 0.40$

$$P = \frac{9}{25} = \frac{8}{25} = 0.32$$

$$P = 0.40 \therefore Q = 0.60$$

$$\therefore Z = \frac{p_1 - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.32 - 0.40}{\sqrt{\frac{0.40 \times 0.60}{25}}} = 0.816$$

since $|Z| < 1.96$ so H_0 is accepted

Hence the overall proportion of female students

$$\text{approx} (0.40) \cdot (1 + 0.816^2)$$

Q.9 We need to test $H_0 : p_1 = p_2 : p_1 \neq p_2$

$$p_1 = \frac{25}{100} = 0.25$$

$$p_2 = \frac{18}{125} = 0.144$$

$$Q = 1 - 0.191 = 0.808$$

$$p_2 = \frac{18}{125} = 0.144$$

Hence

$$a_1 = 25$$

$$a_2 = 125$$

$$Z = \frac{p_1 - p_2}{\sqrt{pq \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}}$$

$$= \frac{0.25 - 0.144}{\sqrt{0.191 \times 0.808 \left(\frac{1}{100} + \frac{1}{125} \right)}} = 2.011$$

since $|Z| > 1.96$, H_0 is rejected

10.14 We need to test

$$H_0: \mu_1 = \mu_2, \mu_3, H_1: \mu_1 \neq \mu_2, \mu_3$$

both n_1 and n_2 are small (< 30) and/or σ unknown

Test statistic: $t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s^2(\frac{1}{n_1} + \frac{1}{n_2})}}$ ~ $t(n_1-1) + t(n_2-1)$

$$\bar{x}_1 = \frac{1}{n_1} \sum x_1 = \frac{168.75}{15} = 11.25 \text{ ad } s_1 = 5.25$$
$$s^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{(n_1-1) + (n_2-1)} = \frac{25.0 + 25.0}{20} = 2.5$$
$$= \frac{(19 \times 5.25) + (19 \times 5.0625)}{19+19} = 1.0$$

$$t_1 = \frac{112.5 - 132.5}{\sqrt{31.534.09(\frac{1}{15} + \frac{1}{20})}} \quad \begin{cases} \therefore t(n_1-1) + t(n_2-1) \\ t_{37} = 2.035 \end{cases}$$
$$= -3.297$$

Since $|t| > t_{37} = 2.035$

So H_0 is rejected

∴ Salary information are not similar

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P-1

Example 9.2

Let $x_1 \sim N(\mu_1, \sigma^2)$, $(x_2 \sim N(\mu_2, \sigma^2))$.

Also, let $\sigma^2 = \sigma_1^2 = \sigma_2^2$.
we need to test, $H_0: \mu_1 = \mu_2$ vs
 $H_1: \mu_1 \neq \mu_2$.

Both $n_1 = 7$ and $n_2 = 11$ are small (< 30) and σ^2 is not known.

Test statistic: $t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s^2(\frac{1}{n_1} + \frac{1}{n_2})}} \sim t(n_1 - 1) + (n_2 - 1)$

$$\bar{x}_1 = \frac{1}{n_1} \sum x_1 = \frac{114}{7} = 16.29$$

$$s_1^2 = \frac{1}{n_1 - 1} \left[\sum x_1^2 - \frac{(\sum x_1)^2}{n_1} \right] \\ = \frac{1}{6} (18.98 - \frac{114^2}{7}) = 6.905$$

$$\bar{x}_2 = \frac{1}{n_2} \sum x_2 = \frac{163}{11} = 14.82$$

$$s_2^2 = \frac{1}{n_2 - 1} \left[\sum x_2^2 - \frac{(\sum x_2)^2}{n_2} \right] \\ = \frac{1}{10} (2569 - \frac{163^2}{11}) = 15.364$$

$$t = \frac{16.29 - 14.82}{\sqrt{12.102 (\frac{1}{7} + \frac{1}{11})}} \\ = 0.87$$

$$s^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{(n_1 - 1) + (n_2 - 1)} \\ = \frac{6(6.905) + 10(15.364)}{16} \\ = 12.102$$

since $|t| < t_{0.05/2, 16} = 2.12$, we

is ^{not} student of both

accepted Employment facility is same. universities

P-2

Q.4

We need to test, $H_0: P_1 = P_2 = 0.10$ vs $H_1: P_1 \neq P_2$

$$\text{Now, } P = \frac{\alpha}{n} = \frac{2}{15} = 0.13$$

$$\text{Test statistic } H_2, Z = \frac{P - P_0}{\sqrt{\frac{P_0(1-P_0)}{n}}} = \frac{0.13 - 0.10}{\sqrt{\frac{0.1 \times 0.9}{15}}} = 0.387$$

since $|Z| < 1.96$, H_0 is accepted. It can be considered that 10% students get grade A

Q.5

We need to test $H_0: P_1 = P_2$ vs $H_1: P_1 \neq P_2$

$$\text{Test statistic } H_2, Z = \frac{P_1 - P_2}{\sqrt{PQ \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}} \sim N(0, 1)$$

$$; P = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2}, Q = 1 - P$$

$$P_1 = \frac{34}{85} = 0.4, P_2 = \frac{14}{70} = 0.2, P = \frac{48}{155} = 0.31, Q = 1 - P = 0.69$$

$$0.4 - 0.2$$

$$Z = \frac{0.4 - 0.2}{\sqrt{(0.31)(0.69) \left(\frac{1}{85} + \frac{1}{70} \right)}} = 2.679$$

P-3

Q. 6

We need to test $H_0: P_1 = P_2 = P_3 = P_4$
vs $H_1: \text{At least one of them doesn't hold}$

Test statistics,

$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i} = n$$
$$= \frac{1}{10} \cdot [8^2 + 15^2 + 5^2 + 12^2] - 40$$
$$= 5.8$$

$$E_i = \frac{n}{k} = \frac{40}{4} = 10$$

Since $\chi^2 < \chi^2_{k-1} = \chi^2_{3,0.05} = 7.81$, H_0 is accepted. Hence, the proportion of defective computers of different laboratories are similar.

P-4

Q.7

H_0 : Job satisfaction does not depend on good result.

H_1 : The good result and job satisfaction are associated.

Test statistic: $\chi^2 = \frac{n(ad - bc)}{(a+b)(a+c)(b+d)(c+d)}$

$$\frac{150(22 \times 50 - 22 \times 58)^2}{80 \times 70 \times 42 \times 108}$$
$$= 0.02$$

since $\chi^2 < \chi^2_{0.05} = 3.84$, H_0 is accepted
so job satisfaction does not depend on good result.

Job satisfaction does not depend on good result.

P5 Exercise

broadband 9.2 in positive dot : 04

we need to test $H_0: p_1 = p_2 = p_3 = p_4$

vs H_1 - At least one of them doesn't hold.

Test statistic

$$\chi^2 = \frac{\sum (O_i - E)^2}{E}$$

$$= \frac{\sum O_i^2}{n/k} - n = \frac{50^2 + 42^2 + 32^2 + 82^2}{206/4} - 206$$

$$= 27.24$$

As $\chi^2 > \chi^2_{k-1} = \chi^2_3 = 7.81$, H_0 is rejected

Hence the proportion of road accidents are not similar in various highways of Bangladesh

P-6

9.7

We need to test, $H_0: p = p_0 = 0.4$ vs.

$H_1: p \neq p_0$

$$P = \frac{a}{n} = \frac{8}{25} = 0.32$$

$$\text{Test statistic: } z = \frac{p - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{0.32 - 0.4}{\sqrt{\frac{0.4 \times 0.6}{25}}} = -0.82$$

As, $|z| < 1.96$, H_0 is accepted. It can be considered that 0.40 is the overall proportion of female student in ATUB.

9.8

We need to test $H_0: p_1 = p_2$ vs $H_1: p_1 \neq p_2$

$$\text{Test statistic, } z = \frac{p_1 - p_2}{\sqrt{\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}}}$$

$$P = \frac{a_1 + a_2}{n_1 + n_2} = \frac{25 + 18}{100 + 25} = 0.19; p_1 = \frac{25}{100} = 0.25$$

$$\approx 0.25$$

P - 7

$$\theta = 1 - P = 1 - 0.19 = 0.81; P_2 = \frac{18}{125} = 0.14$$

$$Z = \frac{0.25 - 0.4}{\sqrt{0.19 \times 0.81 \times (\frac{1}{100} + \frac{1}{125})}} = 2.09$$

As, $|Z| > 1.96$, H_0 is rejected. We conclude that probation is not same for boys and girls at ATUB.

(Q. 1)

H_0 : High blood pressure does not associated with heart problem.

H_1 : High pressure and Heart problem are associated

$$\text{Test Statistic: } \chi^2 = \frac{n(ad - bc)}{(a+b)(a+c)(b+d)(c+d)}$$
$$= \frac{550(150 \times 158) - (22 \times 120)}{272 \times 270 \times 280 \times 278}$$

$$\approx 7.897$$

As $\chi^2 < \chi^2_1 = 3.84$. H_0 is accepted so High blood pressure does not associate with heart problem.

P-8

Q.117

Institution 1: $n_1 = 15 \sum x_1 = 16875; s_1^2 = 5625$

Institution 2: $n_2 = 20 \sum x_2 = 26500; s_2^2 = 50625$

We need to test, $H_0: \mu_1 = \mu_2$ vs $H_1: \mu_1 \neq \mu_2$

$$\bar{x}_1 = \frac{\sum x_1}{n_1} = \frac{16875}{15} = 1125; s_1^2 = 5625$$

$$\bar{x}_2 = \frac{\sum x_2}{n_2} = \frac{26500}{20} = 1325; s_2^2 = 50625$$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s_1^2 / (n_1 - 1) + s_2^2 / (n_2 - 1)}}$$

$$= \frac{1125 - 1325}{\sqrt{5625 / (15 - 1) + 50625 / (20 - 1)}} = 3.29$$

$$\sqrt{5625 / (15 - 1) + 50625 / (20 - 1)} = 3.29$$

$$s^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{(n_1 - 1) + (n_2 - 1)} = \frac{(14 \times 5625) + (19 \times 50625)}{14 + 19}$$

$$= 31534.09$$

As $|t| > t_{0.05/2, 33} = 2.035$, H_1 is accepted.

The salary information of both institutions are not similar.

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19-41759 (P-1)

Ex : 10.2

$$X_1 \sim N(\mu_1, \sigma_1^2)$$

$$X_2 \sim N(\mu_2, \sigma_2^2)$$

$$\text{Also, let } \sigma_1^2 = \sigma_2^2 = \sigma^2$$

We need to test, $H_0: \mu_1 = \mu_2$ vs $H_1: \mu_1 \neq \mu_2$

Both $n_1=7$ and $n_2=11$ are small (< 30) and σ^2 is unknown

$$\text{Test statistic: } t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{s^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}} \sim t(n_1-1) + (n_2-1)$$

$$\bar{X}_1 = \frac{1}{n_1} \sum n_1 = \frac{114}{7} = 16.29$$

$$s_1^2 = \frac{1}{n_1-1} \left[\sum n_{1,i}^2 - \frac{(\sum n_{1,i})^2}{n_1} \right] = \frac{1}{6} \left(1808 - \frac{114^2}{7} \right) = 6.905$$

$$\bar{X}_2 = \frac{1}{n_2} \sum n_2 = \frac{163}{11} = 14.82$$

$$s_2^2 = \frac{1}{n_2-1} \left[\sum n_{2,i}^2 - \frac{(\sum n_{2,i})^2}{n_2} \right] = \frac{1}{10} \left(2560 - \frac{163^2}{11} \right) = 15.369$$

$$s^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{(n_1-1) + (n_2-1)} = \frac{6(6.905) + 10(15.369)}{16} = 12.192$$

$$t = \frac{16.29 - 14.82}{\sqrt{12.192 \left(\frac{1}{7} + \frac{1}{11} \right)}} = 0.87$$

Since, $|t| < t_{0.05} = 2.12$

H_0 is accepted. Employment facility for student
of both universities are same

10.4

We need to test,

$$H_0: p = p_0 = 0.10 \text{ vs } H_1: p \neq p_0$$

$$\text{Now, } p = \frac{\alpha}{n} = \frac{2}{15} = 0.13$$

$$\text{Test statistic: } z = \frac{p - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.13 - 0.10}{\sqrt{\frac{0.10 \times 0.90}{15}}} = 0.39$$

Since $|z| < 1.96$

H_0 is accepted. It can be considered that 10% students got grade it.

10.5

We test $H_0: p_1 = p_2 \text{ vs } H_1: p_1 \neq p_2$

$$\text{Test statistic, } z = \frac{p_1 - p_2}{\sqrt{p_0(\frac{1}{n_1} + \frac{1}{n_2})}} \sim N(0.1);$$

$$p = \frac{a_1 + a_2}{n_1 + n_2}, q = 1 - p$$

$$p_1 = \frac{34}{85} = 0.4, p_2 = \frac{14}{70} = 0.2, p = \frac{48}{155} = 0.31, q = 0.69$$

$$z = \frac{0.4 - 0.2}{\sqrt{(0.31)(0.69)(\frac{1}{85} + \frac{1}{70})}} = 2.68$$

Since $|z| > 1.96$, H_0 is rejected.

We conclude that gender does make a difference for drug use.

10.6

We need to test $H_0: P_1 = P_2 = P_3 = P_4$ vs H_1 at least one of them doesn't hold.

Test statistics,

$$\begin{aligned} \chi^2 &= \sum_i \frac{(O_i - E_i)^2}{E_i} \\ &= \frac{1}{10} [8^2 + 15^2 + 5^2 + 12^2] - 40 \\ &= 45.8 - 40 = 5.8 \end{aligned}$$

$$E_i = \frac{n}{k} = \frac{40}{4} = 10$$

Since,

$$\chi^2 < \chi_{k-1}^2 = \chi_3^2 = 7.81. H_0 \text{ is accepted.}$$

Hence, the proportions of defective computer of different laboratories are similar.

10.7

H_0 : Job satisfaction doesn't depend on good result.

H_1 : The good result and job satisfaction are associated

$$\begin{aligned} \text{Test statistics. } \chi^2 &= \frac{n(ad - bc)^2}{(a+b)(a+c)(b+d)(c+d)} \\ &= \frac{150(22 \times 50 - 22 \times 58)^2}{30 \times 70 \times 42 \times 108} = 0.02 \end{aligned}$$

Since, $\chi^2 < \chi_{1}^2 = 3.84$. H_0 is accepted.

So, Job satisfaction doesn't depend on good result.

10.2

We need to test, $H_0: P_1 = P_2 = P_3 = P_4$ Vs H_1 : at least one of them doesn't hold.

$$\text{Here, } E_i = \frac{n}{k} = \frac{206}{4} = 51.5$$

$$\begin{aligned} \chi^2 &= \sum \frac{(O_i - E_i)^2}{E_i} - n \\ &= \frac{1}{51.5} [50^2 + 42^2 + 32^2 + 82^2] - 206 = 27.29 \end{aligned}$$

Since, $\chi^2 > \chi^2_{k-1} = \chi^2_3 = 7.81$, so H_1 is accepted.
Hence the proportions of road accidents aren't similar.

10.7

We need to test,

$$H_0: P_0 = 0.40 \text{ Vs } H_1: P_0 \neq 0.40$$

$$P = \frac{\alpha}{n} = \frac{3}{25} = 0.12$$

$$P_0 = 0.40$$

$$\therefore Q = 0.60$$

$$\therefore Z = \frac{P_0 - P}{\sqrt{\frac{P_0 Q}{n}}} = \frac{0.32 - 0.40}{\sqrt{\frac{0.40 \times 0.60}{25}}} = -0.816$$

Since $|Z| < 1.96$. So H_0 is accepted.

Hence, the overall proportion of female students are 0.40 (Ans)

10.9

We need to test, $H_0: P_1 = P_2$ vs $H_1: P_1 \neq P_2$

$$P_1 = \frac{25}{100} = 0.25 \quad P_2 = \frac{18}{125} = 0.144$$

$$P = \frac{25+18}{100+125} = 0.191$$

$$\delta = 1 - 0.191 = 0.808$$

Hence,
 $a_1 = 25$
 $n_1 = 100$
 $n_2 = 125$
 $a_2 = 18$

$$\therefore z = \frac{P_1 - P_2}{\sqrt{P(1-P)} \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}$$

$$= \frac{0.25 - 0.144}{\sqrt{0.191 \times 0.808} \left(\frac{1}{100} + \frac{1}{125} \right)} = 2.011$$

Since $|z| > 1.96$, H_0 is rejected.

10.11

H_0 = High blood pressure isn't associated with heart problem

H_1 = High blood pressure is associated with heart problem

$$\chi^2 = \frac{n(ad - bc)^2}{(a+b)(a+c)(b+d)(c+d)}$$

$$= \frac{550 \left[(150 \times 138) - (120 \times 122) \right]^2}{270 \times 280 \times 272 \times 278}$$

$$= 7.897 \sim 7.90$$

Since, $\chi^2 > \chi_{0.05}^2 = 3.84$, so, H_1 is accepted

High blood pressure is associated with heart problem.

10.14

We need to test

$$H_0: \mu_1 = \mu_2 \text{ vs } H_1: \mu_1 \neq \mu_2$$

both n_1 and n_2 are small (< 30) and σ^2 is unknown

Test statistics: $t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}} \sim t(n_1-1) + t(n_2-1)$

$$\bar{x}_1 = \frac{1}{n_1} \sum x_1 = \frac{16875}{15} = 1125 \text{ and } s_1^2 = 5625$$

$$\bar{x}_2 = \frac{1}{n_2} \sum x_2 = \frac{26500}{20} = 1325 \text{ and } s_2^2 = 50625$$

$$\begin{aligned} s^2 &= \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{(n_1-1) + (n_2-1)} \\ &= \frac{(14 \times 5625) + (19 \times 50625)}{14+19} = 31534.09 \end{aligned}$$

$$\begin{aligned} \therefore t &= \frac{1125 - 1325}{\sqrt{31534.09 \left(\frac{1}{15} + \frac{1}{20} \right)}} \\ &= -3.297 \quad \left| \begin{array}{l} \therefore t(n_1-1) + (n_2-1) \\ t_{33} = 2.035 \end{array} \right. \end{aligned}$$

Since, $|t| > t_{33} = 2.035$

So, H_0 is rejected

\therefore Salaries information are not similar.

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Pg: 01

Assignment: 04

Example 10.2:

Let,

$X_1 \sim N(\mu_1, \sigma^2)$, $X_2 \sim N(\mu_2, \sigma^2)$. Also
let $\sigma^2 = \sigma_1^2 = \sigma_2^2$

We need to test,

$$H_0: \mu_1 = \mu_2 \text{ vs } H_1: \mu_1 \neq \mu_2$$

Both, $n_1 = 7$ and $n_2 = 11$ are < 30 and σ^2 is not known.

Test statistics $t_2 = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s^2(\frac{1}{n_1} + \frac{1}{n_2})}} \sim t(n_1 - 1) + (n_2 - 1)$

From

For x_1 ,

$$\bar{x}_1 = \frac{1}{n_1} \sum x_1 = \frac{114}{7} = 16.29$$

$$s_1^2 = \frac{1}{n_1 - 1} \left[\sum x_1^2 - \frac{(\sum x_1)^2}{n} \right] = \frac{1}{6} \left[1898 - \frac{114^2}{7} \right]$$

$$= 6.905.$$

For n_2 ,

$$\bar{u}_2 = \frac{1}{n_2} \sum u_2 = \frac{163}{11} = 14.82$$

$$S_2^2 = \frac{1}{n_2 - 1} \left[\sum u_2^2 - \frac{(\sum u)^2}{n_2} \right]$$

$$= \frac{1}{10} \left(2562569 - \frac{163^2}{11} \right)$$

$$= 15.364$$

Now,

$$S^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{(n_1 - 1) + (n_2 - 1)}$$

$$= \frac{6(6.905) + 10(15.364)}{16}$$

$$= 12.192$$

$$t = \frac{16.29 - 14.82}{\sqrt{12.192 \left(\frac{4}{7} + \frac{1}{11} \right)}}$$

$$= 0.87$$

Since,

$$|t| t_{18} = 2.12$$

H_0 is accepted Employment Isacity for students of both Universities is same.

Example 10.4:

We need to test $H_0: p = p_0 = 0.10$ vs $p \neq p_0$.

Now,

$$p = \frac{a}{n} = \frac{2}{15} = 0.13$$

$$\text{Test statistic: } z = \frac{p - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.13 - 0.10}{\sqrt{\frac{0.10 \times 0.90}{15}}} = 0.39$$

Since,

($|z| < 1.96$, H_0 is accepted. It can be considered that 10% students got grade A.)

Example 10.5:

We need to test $H_0: p_1 = p_2$ vs $H_1: p_1 \neq p_2$

Test statistic,

$$Z = \frac{p_1 - p_2}{\sqrt{pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \sim N(0, 1)$$

$$p = \frac{a_1 + a_2}{n_1 + n_2}, q = 1 - p$$

$$p_1 = \frac{a_1}{n_1} = \frac{39}{85}, p_2 = \frac{14}{70}$$

$$p_2 = \frac{39 + 14}{85 + 70} = \frac{98}{155} = 0.31$$

$$q = 1 - p \Rightarrow 0.69$$

$$Z = \frac{0.4 - 0.2}{\sqrt{(0.31)(0.69)\left(\frac{1}{85} + \frac{1}{70}\right)}} = 2.68$$

Since,

$|Z| > 1.96$, H_0 is rejected, we conclude that gender does make a difference for drug use.

Example 10.6:

We need to test

$H_0: P_1 = P_2 = P_3 = P_4$ vs H_1 : at least one of them does not hold.

Test statistic,

$$\begin{aligned} \chi^2 &= \sum \frac{(O_i - E_i)^2}{E_i} = n \\ &= \frac{1}{10} [8^2 + 15^2 + 5^2 + 12^2] - 40 \\ &= 45.8 - 40 \\ &= 5.8 \end{aligned}$$

$$E_i = \frac{n}{k} = \frac{40}{4} = 10$$

Since,

$\chi^2 < \chi_{k-1}^2 = \chi_3^2 = 7.81$ H_0 is accepted.

Hence,

The proportion of refedion computers of different laboratories are similar.

Example 10.7:

H_0 : job satisfaction does not depend on good result.

H_1 : The good result and good job satisfaction are associated.

Test statistic:

$$\chi^2 = \frac{n(ad - bc)^2}{(a+b)(b+c)(b+d)(c+d)}$$

$$= \frac{150(22 \times 56 - 20 \times 58)^2}{80 \times 70 \times 92 \times 108}$$

$$= 0.02$$

Since,

$\chi^2 < \chi_{0.05}^2 = 3.89$ H_0 is accepted.

So, job satisfaction does not depend on good result.

Exercise: 10.4

We need to test,

$H_0: p_1 = p_2 = p_3 = p_4$ Vs H₁: at least one of
the proportions does not hold.

$$E_i = \frac{n}{K} = \frac{1000}{4} = 250$$

$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}$$

$$= \frac{(250^2 + 450^2 + 150^2 + 150^2)}{250} - 1000$$

$$= 1240 - 1000$$

$$\Rightarrow 240$$

Since,

$$\chi^2 > \chi_{\alpha/2}^2 \quad \text{at } \alpha = 0.05 \quad \chi_{0.05}^2 = 7.81, H_0 \text{ is not accepted}$$

accepted. Hence, The proportion of Female students is not similar in various department in AIUB.

Exercise 10.7:

We need to test,

$$H_0: p = p_0 = 0.90 \text{ vs } H_1: p \neq p_0$$

Now, $p = \frac{8}{25} = 0.32$

Test statistic, $Z = \frac{p - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$

$$= \frac{0.32 - 0.90}{\sqrt{\frac{(0.90)(0.60)}{25}}} = \frac{-0.58}{\sqrt{0.0216}} = -0.81$$

Since,

$|Z| < 1.96$, H_0 is accepted. It can be

considered that the overall proportion of female students are 0.90 in AIUB.

19-91829-3

Exercise 10.9:

We need to test,

$$H_0: p_1 = p_2 \text{ vs } H_1: p_1 \neq p_2$$

$$p_1 = \frac{25}{100} = 0.25$$

$$p_2 = \frac{18}{125} = 0.144$$

$$p_2 = \frac{43}{225} = 0.191$$

$$\therefore q = 1 - p = 0.808$$

$$\therefore z = \frac{0.25 - 0.191}{\sqrt{(0.191)(0.808)} \left(\frac{1}{100} + \frac{1}{125} \right)}$$

$$z = 2.011$$

Since,

$|z| > 1.96$. H_0 is rejected. We conclude

that the probation problem is different
for boys and girls at AIUB.

Exercise: 10.12

H_0 : learning does not depend residential origin

H_1 : Residential origin and full attention is associated with learning.

$$\text{Test statistic: } \chi^2 = \frac{n(ad - bc)^2}{(a+b)(a+c)(b+d)(c+d)}$$

$$= \frac{350(138 \times 89) - (64 \times 64)}{(130+64)(138+64)(64+89)(64+89)} \\ = 22.003$$

Since, $\chi^2 > \chi^2_{0.05} = 3.84$, H_0 is rejected.

So residential origin and full attention is associated with learning.

Exercise 10.14:

We need to test, $H_0: \mu_1 = \mu_2$ vs $H_1: \mu_1 \neq \mu_2$

Here,

$n_1 = 15$ and $n_2 = 20$ are small (< 30) and

σ^2 is not known.

$$\bar{x}_1 = \frac{1}{n_1} \sum x_1 = \frac{1}{15} \times 16875 = 1125$$

$$\bar{x}_2 = \frac{1}{n_2} \sum x_2 = \frac{1}{20} \times 26500 = 1325$$

$$S^2 = \frac{19(5625) + 19(50625)}{14 + 19}$$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{S^2}} = \frac{1125 - 1325}{\sqrt{50625}} \approx -3.297$$

Since,

$|t| > t_{33}$, H_0 is rejected, salary information of both institute are not similar.

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Example

9.2 | Let, $x_1 \sim N(\mu_1, \sigma^2)$

$$x_2 \sim N(\mu_2, \sigma^2)$$

Also,

$$\sigma^2 = \sigma_1^2 = \sigma_2^2$$

We need to test, $H_0: \mu_1 = \mu_2$ vs $H_1: \mu_1 \neq \mu_2$

Both $n_1 = 7$ and $n_2 = 11$ are small (< 30) and σ^2 is unknown.

$$\text{Test : } t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s^2(\frac{1}{n_1} + \frac{1}{n_2})}} \sim t(n_1-1) + (n_2-1)$$

$$\bar{x}_1 = \frac{1}{n_1} \sum x_{1i} = \frac{119}{7} = 16.29$$

$$s_1^2 = \frac{1}{n_1-1} \left[\sum x_{1i}^2 - \frac{(\sum x_{1i})^2}{n_1} \right] = \frac{1}{6} (1889 - \frac{119^2}{7}) = 6.905$$

$$\bar{x}_2 = \frac{1}{n_2} \sum x_{2i} = \frac{163}{11} = 14.82$$

$$s_2^2 = \frac{1}{n_2-1} \left[\sum x_{2i}^2 - \frac{(\sum x_{2i})^2}{n_2} \right] = \frac{1}{10} (2869 - \frac{163^2}{11}) = 15.369$$

$$\text{So, } s^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{(n_1-1) + (n_2-1)} = \frac{6(6.905) + 10(15.369)}{16} = 12.192$$

$$t = \frac{16.29 - 14.82}{\sqrt{12.192 (\frac{1}{7} + \frac{1}{11})}} \\ = 0.87$$

Since $|t| < t_{16} = 2.12$ (Ans)

9.4] We need to test,

$$H_0: p = p_0 = 0.10 \quad \text{vs} \quad H_1: p \neq p_0$$

$$\text{Now, } p = \frac{\alpha}{n} = \frac{2}{15} = 0.13$$

$$\begin{aligned} \text{Test statistic: } z &= \frac{p - p_0}{\sqrt{\frac{p_0 \alpha_0}{n}}} = \frac{0.13 - 0.10}{\sqrt{\frac{0.10 \times 0.70}{15}}} \\ &= 0.39 \end{aligned}$$

Since $|z| < 1.96$

H_0 is accepted. It can be considered that 10% student got grade A.

9.5]

We know, $H_0: p_1 = p_2 \text{ vs } H_1: p_1 \neq p_2$

$$\text{Test statistic: } z = \frac{p_1 - p_2}{\sqrt{pq(\frac{1}{n_1} + \frac{1}{n_2})}} \sim N(0, 1).$$

$$p = \frac{\alpha_1 + \alpha_2}{n_1 + n_2}, q = 1 - p$$

$$p_1 = \frac{31}{85} = 0.37, \quad p_2 = \frac{19}{70} = 0.27, \quad p = \frac{48}{155} = 0.31, \quad q = 0.69$$

$$z = \frac{0.37 - 0.27}{\sqrt{(0.31)(0.69)(\frac{1}{85} + \frac{1}{70})}} = 2.68$$

since ($|z| > 1.96$, H_0 is rejected).

Q.6 We know, $H_0: P_1 = P_2 = P_3 = P_4$ vs $H_1: At least one of them doesn't hold.$

Test statistics,

$$\begin{aligned} \chi^2 &= \sum \frac{(O_i - E_i)^2}{E_i} - n \\ &= \frac{1}{10} [8^2 + 15^2 + 5^2 + 12^2] - 90 \\ &= 95.8 - 90 = 5.8 \end{aligned}$$

$$E_i = \frac{n}{k} = \frac{90}{4} = 10$$

Since,

$$\chi^2 < \chi^2_{\alpha=0.05} = 7.81 \quad H_0 \text{ is accepted.}$$

Hence the properties of defective computers of different machines are similar.

Q.7

$H_0:$ Job satisfaction doesn't depend on good result.

$H_1:$ The good result and job satisfaction are associated

We know,

$$\chi^2 = \frac{n(ad - bc)^2}{(a+b)(a+c)(b+d)(c+d)} = \frac{150(22 \times 10 - 22 \times 8)^2}{80 \times 70 \times 92 \times 10} = 0.02$$

Since, $\chi^2 < \chi^2_{\alpha=0.05} = 3.89$. H_0 is accepted.

So, job satisfaction doesn't depend on good result.

Exercise

9.2] We need to test, $H_0: p_1 = p_2 = p_3 = p_4$ vs H_1 : At least one of them doesn't hold.

Hence,

$$E_i = \frac{n}{k} = \frac{206}{4} = 51.5$$

$$\begin{aligned} \chi^2 &= \sum \frac{(O_i - E_i)^2}{E_i} - n \\ &= \frac{1}{51.5} [50^2 + 42^2 + 32^2 + 82^2] - 206 = 27.29 \end{aligned}$$

Since, $\chi^2 > \chi^2_{k-1} = \chi^2_3 = 7.81$, H_0 is accepted.
Hence, the proportions of road accidents are almost similar.

9.7]

We need to test, $H_0: p_0 = 0.90$ vs $H_1: p_0 \neq 0.90$

$$P = \frac{a}{n} = \frac{8}{25} = 0.32$$

$$P_0 = 0.90 \therefore Q = 0.60$$

$$\therefore Z = \frac{P_0 - P}{\sqrt{\frac{P_0 Q_0}{n}}} = \frac{0.32 - 0.90}{\sqrt{\frac{0.90 \times 0.60}{25}}}$$

$$= -0.816$$

Since, $|Z| < 1.96$. So H_0 is accepted.

Hence, the overall proportion of female students are 0.90
(Ans).

Q.21 We need to test, $H_0: P_1 = P_2$ vs $H_1: P \neq P_2$

$$P_1 = \frac{25}{100} = 0.25$$

$$P = \frac{25+18}{100+125} = 0.171$$

$$\phi = 1 - 0.171 = 0.808$$

$$\therefore Z = \frac{P_1 - P_2}{\sqrt{\phi(\frac{1}{n_1} + \frac{1}{n_2})}}$$

$$= \frac{0.25 - 0.199}{\sqrt{0.191 \times 0.808 (\frac{1}{100} + \frac{1}{125})}} = 2.011$$

Hence,

$$\begin{cases} \alpha_1 = 25 \\ n_1 = 100 \\ n_2 = 125 \\ \alpha_2 = 10 \end{cases}$$

Since, $|Z| > 1.96$, H_0 is rejected.

Q.21 H_0 : High blood pressure isn't associated with heart problem.

H_1 : High blood pressure is associated with heart problem.

$$\chi^2 = \frac{n(ad - bc)^2}{(a+b)(a+c)(b+d)(c+d)}$$

$$= \frac{550[(150 \times 158) - (120 \times 122)]^2}{270 \times 280 \times 272 \times 278}$$

Since,

$\chi^2 > \chi^2_{0.05} = 3.89$. So, H_1 is accepted.

B.P	Heart Problem		Total
	Yes	No	
High	150	120	270
Not High	122	158	280
Total	272	278	550

Q.19 We need to test,

$$H_0: \mu_1 = \mu_2 \text{ vs } H_1: \mu_1 \neq \mu_2$$

both n_1 and n_2 are small (< 30) and σ^2 is unknown.

Test statistic: $t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s^2(\frac{1}{n_1} + \frac{1}{n_2})}} \sim t(n_1 - 1) + t(n_2 - 1)$

$$\bar{x}_1 = \frac{1}{n_1} \sum x_1 = \frac{16875}{15} = 1125 \text{ and } s_1^2 = 5625$$

$$\bar{x}_2 = \frac{1}{n_2} \sum x_2 = \frac{26540}{20} = 1325 \text{ and } s_2^2 = 50523$$

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{(n_1 - 1) + (n_2 - 1)}$$
$$= \frac{(19 \times 5625) + (19 \times 50523)}{19 + 19}$$
$$= 31539.09$$

$$\therefore t = \frac{1125 - 1325}{\sqrt{31539.09(\frac{1}{15} + \frac{1}{20})}}$$
$$= -3.297$$

Hence,
 $t = (n_1 - 1) + t(n_2 - 1)$
 $t_{33} = 2.035$

Since, $|t| > t_{33} = 2.035$

so, H_0 is rejected.

Nayef (20-42048-1)

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Examples.

10.2

$$\text{Let, } x_1 \sim N(\mu_1, \sigma_1^2)$$

$$x_2 \sim N(\mu_2, \sigma_2^2)$$

$$\text{Also, let } \sigma_1^2 = \sigma_2^2 = \sigma^2$$

We need to test, $H_0: \mu_1 = \mu_2$ vs $H_1: \mu_1 \neq \mu_2$

Both $n_1 = 7$ and $n_2 = 11$ are small (< 30) and σ^2 is unknown.

$$\text{Test statistic: } t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}} \sim t(n_1 - 1) + (n_2 - 1)$$

$$\bar{x}_1 = \frac{1}{n_1} \sum x_1 = \frac{119}{7} = 16.29$$

$$s_1^2 = \frac{1}{n_1 - 1} \left[\sum x_1^2 - \frac{(\sum x_{1i})^2}{n_1} \right] = \frac{1}{6} \left(1898 - \frac{119^2}{7} \right) = 6.905$$

$$\bar{x}_2 = \frac{1}{n_2} \sum x_2 = \frac{163}{11} = 14.82$$

$$s_2^2 = \frac{1}{n_2 - 1} \left[\sum x_2^2 - \frac{(\sum x_{2i})^2}{n_2} \right] = \frac{1}{10} \left(2569 - \frac{163^2}{11} \right) = 15.369$$

Now,

$$s^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{(n_1 - 1) + (n_2 - 1)} = \frac{6(6.905) + 10(15.369)}{16} = 12.192$$

$$t = \frac{16.29 - 14.82}{\sqrt{12.192 \left(\frac{1}{7} + \frac{1}{11} \right)}} = 0.87$$

since, $|t| < t_{16} = 2.12$,

H_0 is accepted. Employment facility for students of both universities are same.

10.4

We need to test,

$$H_0: P = P_0 = 0.10 \text{ vs } H_1: P \neq P_0$$

$$\text{Now, } p = \frac{a}{n} = \frac{2}{15} = 0.13$$

$$\text{Test statistic: } z = \frac{p - P_0}{\sqrt{\frac{P_0 Q_0}{n}}} = \frac{0.13 - 0.10}{\sqrt{\frac{0.10 \times 0.90}{15}}} = 0.39$$

Since $|z| < 1.96$

H_0 is accepted. It can be considered that 10% students got grade A.

10.5

We need to test $H_0: P_1 = P_2$ vs $H_1: P_1 \neq P_2$

Test statistics, $Z = \frac{P_1 - P_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \sim N(0, 1);$

$$P = \frac{\alpha_1 + \alpha_2}{n_1 + n_2}, Q = 1 - P$$

$$P_1 = \frac{34}{85} = 0.4, P_2 = \frac{19}{70} = 0.2, P = \frac{48}{155} = 0.31, Q = 0.69$$

$$Z = \frac{0.4 - 0.2}{\sqrt{(0.31)(0.69)\left(\frac{1}{85} + \frac{1}{70}\right)}} = 2.68$$

since $|Z| > 1.96$, H_0 is rejected.

We conclude that gender does make a difference for drug use.

10.6

We need to test $H_0: P_1 = P_2 = P_3 = P_4$ vs H_1 : At least one of them doesn't hold.

Test statistics,

$$\begin{aligned} \chi^2 &= \sum_i \frac{(O_i - E_i)^2}{E_i} - n \\ &= \frac{1}{10} [8^2 + 15^2 + 5^2 + 12^2] - 10 \\ &= 45.8 - 10 = 35.8 \end{aligned}$$

$$E_i = \frac{n}{k} = \frac{40}{4} = 10$$

Since,

$$\chi^2 < \chi_{k-1}^2 = \chi_3^2 = 7.81. H_0 \text{ is accepted.}$$

Hence the properties of defective computers of different laboratories are similar.

10.7

H_0 : Job satisfaction doesn't depend on good result.

H_1 : The good result and job satisfaction are associated.

Test statistics, $\chi^2 = \frac{n(ad-bc)^2}{(a+b)(a+c)(b+d)(c+d)} = \frac{150(22+50)}{(22+50)(22+50)(22+50)(22+50)}$

$$\chi^2 = \frac{150 (22 \times 50 - 22 - 58)^2}{80 \times 70 \times 12 \times 108} = 0.02$$

since, $\chi^2 < \chi^2_{0.05} = 3.84$. H_0 is accepted.

So, job satisfaction does not depend on good result.

Exercise

10.2

We need to test, $H_0: P_1 = P_2 = P_3 = P_4$ vs H_1 : At least one of them doesn't hold.

$$\text{Here, } E_i = \frac{n}{k} = \frac{206}{4} = 51.5$$

$$\chi^2 = \sum \frac{O_i^2}{E_i} - n$$

$$= \frac{1}{51.5} [50^2 + 92^2 + 32^2 + 82^2] - 206 = 27.29$$

since, $\chi^2 > \chi^2_{0.05} = \chi^2_3 = 7.81$. So, H_1 is accepted.

Hence the proportions of road accidents are not similar.

10.7

We need to test,

$$H_0: P_0 = 0.40 \text{ vs } H_1: P_0 \neq 0.40$$

$$P = \frac{Q}{n} = \frac{8}{25} = 0.32$$

$$P_0 = 0.40 \quad \therefore Q = 0.60$$

Nayaf (20-92098-3)

$$\therefore z = \frac{P_0 - P_0}{\sqrt{\frac{P_0 Q_0}{n}}} = \frac{0.32 - 0.40}{\sqrt{\frac{0.40 \times 0.60}{25}}} = -0.816$$

Since $|z| < 1.96$. So H_0 is accepted.

Hence, the overall proportion of female students are

10.9

0.40

(Ans)

We need to test, $H_0 : P_1 = P_2$ vs $H_1 : P_1 \neq P_2$

$$P_1 = \frac{29}{100} = 0.25$$

$$P_2 = \frac{18}{125} = 0.144$$

$$P = \frac{25+18}{100+125} = 0.191$$

$$Q = 1 - 0.191 = 0.808$$

Here,
 $a_1 = 25$
 $n_1 = 100$
 $n_2 = 125$
 $a_2 = 18$

$$\therefore z = \frac{P_1 - P_2}{\sqrt{PQ \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}}$$

$$= \frac{0.25 - 0.144}{\sqrt{0.191 \times 0.808 \left(\frac{1}{100} + \frac{1}{125} \right)}} = 2.011$$

Since $|z| > 1.96$, H_0 is rejected.

10.11

H_0 : High blood pressure isn't associated with heart problem

H_1 : High blood pressure is associated with heart problem.

$$\chi^2 = \frac{n(ad - bc)^2}{(a+b)(a+c)(b+d)(c+d)}$$

$$= \frac{550 \left[(150 \times 158) - (120 \times 122) \right]^2}{270 \times 280 \times 272 \times 278}$$

$$= 7.897 \sim 7.90$$

B. P	Heart Problem		Total
	Yes	No	
High	150	120	270
Not high	122	158	280
Total	272	278	550

since,

$$\chi^2 > \chi_1^2 = 3.81 \text{. So, } H_1 \text{ is accepted.}$$

High blood pressure is associated with heart problems.

10.14

We need to test

$$H_0 : \mu_1 = \mu_2 \text{ vs } H_1 : \mu_1 \neq \mu_2$$

both n_1 and n_2 are small (< 30) and σ^2 is unknown.

Test statistic : $t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}} \sim t(n_1 - 1) + t(n_2 - 1)$

$$\bar{x}_1 = \frac{1}{n_1} \sum x_1 = \frac{16875}{15} = 1125 \quad \text{and} \quad s_1^2 = 5625$$

$$\bar{x}_2 = \frac{1}{n_2} \sum x_2 = \frac{26500}{20} = 1325 \quad \text{and} \quad s_2^2 = 50625$$

$$s^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{(n_1 - 1) + (n_2 - 1)}$$

$$= \frac{(19 \times 5625) + (19 \times 50625)}{14 + 19} = 31534.09$$

$$\therefore t = \frac{1125 - 1325}{\sqrt{31534.09 \left(\frac{1}{15} + \frac{1}{20} \right)}} \quad \left| \begin{array}{l} \therefore t(n_1 - 1) + t(n_2 - 1) \\ t_{33} = 2.035 \end{array} \right.$$

$$= -3.297$$

Since, $|t| > t_{33} = 2.035$

So, H_0 is rejected.

\therefore Salary information are not similar.

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Example 9.2:

Let, $x_1 \sim N(\mu_1, \sigma^2)$, $x_2 \sim N(\mu_2, \sigma^2)$. Also

Also let, $\sigma^2 = \sigma_1^2 = \sigma_2^2$.

We need to test, $H_0: \mu_1 = \mu_2$ vs $H_1: \mu_1 \neq \mu_2$

Both $n_1 = 8$ and $n_2 = 11$ are small (< 30) and

σ^2 is not known between the two universities.

Test statistic: $t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}} \sim t(n_1-1) + (n_2-1)$

$\bar{x}_1 = \frac{1}{n_1} \sum x_1 = \frac{114}{8} = 16.29$	$s_1^2 = \frac{1}{n_1-1} \left[\sum x_1^2 - \frac{(\sum x_1)^2}{n_1} \right] = \frac{1}{6} (1898 - \frac{114^2}{8}) = 6.905$
$\bar{x}_2 = \frac{1}{n_2} \sum x_2 = \frac{163}{11} = 14.82$	$s_2^2 = \frac{1}{n_2-1} \left[\sum x_2^2 - \frac{(\sum x_2)^2}{n_2} \right] = \frac{1}{10} (2569 - \frac{163^2}{11}) = 15.36$
$t = \frac{16.29 - 14.82}{\sqrt{12.192 \left(\frac{1}{8} + \frac{1}{11} \right)}} = 0.87$	$s^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{(n_1-1) + (n_2-1)} = \frac{6(6.905) + 10(15.36)}{16} = 12.19$

Since $|t| < t_{0.05} = 2.12$, H_0 is accepted. Employment facility for students of both universities is same.

Example 9.4

We need to test, $H_0: P = P_0 = 0.10$ vs $H_1: P \neq P_0$

Now, $p = \frac{a}{n} = \frac{2}{15} = 0.13$

Test statistic: $Z = \frac{p - P_0}{\sqrt{\frac{P_0(1-P_0)}{n}}} = \frac{0.13 - 0.10}{\sqrt{\frac{0.10 \times 0.90}{15}}} = 0.39$

Since $|Z| < 1.96$, H_0 is accepted. It can be considered that 10% students got grade A.

Example: 9.5

We need to test $H_0: P_1 = P_2$ vs $H_1: P_1 \neq P_2$

Test statistic, $Z = \frac{P_1 - P_2}{\sqrt{P_0(\frac{1}{m_1} + \frac{1}{m_2})}} \sim N(0,1)$;

$$P = \frac{a_1 + a_2}{m_1 + m_2}, Q = 1 - P$$

$$P_1 = \frac{34}{85} = 0.4, P_2 = \frac{14}{20} = 0.2, P = \frac{48}{155} = 0.31, Q = 0.69$$

$$Z = \frac{0.4 - 0.2}{\sqrt{(0.31)(0.69)(\frac{1}{85} + \frac{1}{20})}} = 2.68$$

Since $|Z| > 1.96$, H_0 is rejected. We conclude that gender does make a difference for drug use.

Example 9.6:

We need to test $H_0: P_1 = P_2 = P_3 = P_4$ vs H_1 : At least one of them doesn't hold.

Test statistics.

$$\begin{aligned} \chi^2 &= \sum \frac{O_i - E_i}{E_i} - n \\ &= \frac{1}{10} [8^2 + 15^2 + 5^2 + 12^2] - 40 = 5.8 \end{aligned}$$

$$E_i = \frac{n}{K} = \frac{40}{4} = 10$$

Since, $\chi^2 < \chi_{k-1}^2 = \chi_3^2 = 7.81$, H_0 is accepted. Hence, the proportions of defective computers of different laboratories are similar.

Example 9.7

H_0 : Job satisfaction does not depend on good result.

H_1 : The good result and job satisfaction are associated.

$$\text{Test statistic: } \chi^2 = \frac{n(ad - bc)^2}{(a+b)(a+c)(b+d)(c+d)} = \frac{150(22 \times 50 - 22 \times 58)^2}{80 \times 70 \times 42 \times 108} = 0.02$$

Since $\chi^2 < \chi_1^2 = 3.84$, H_0 is accepted. So, job satisfaction does not depend on good result.

Exercise 9.2:

$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i} - n = \frac{50^2 + 42^2 + 32^2 + 82^2}{\frac{206}{4}} - 206 = 27.24$$

Since $\chi^2 > \chi_{k-1}^2$. So, H_0 is not accepted.

Hence, the proportion of road accident is not similar in various highways of Bangladesh.

Exercise 9.7

We need to test, $H_0: P = P_0 = 0.40$ vs $H_1: P \neq P_0$.

$$P = \frac{a}{n} = \frac{8}{25} = 0.32 \quad Q_0 = 1 - 0.40 = 0.6$$

Test statistic $(Z = \frac{P - P_0}{\sqrt{\frac{P_0 Q_0}{n}}} = \frac{0.32 - 0.40}{\sqrt{\frac{0.40 \times 0.60}{25}}} = -0.82)$

Since, $|Z| < 1.96$. H_0 is accepted. It can be considered that 0.40 is the overall proportion of female students in AIUB.

Exercise 9.9 :

We need to test $H_0 : P_1 = P_2$ vs $H_1 : P_1 \neq P_2$

Here, $P_1 = \frac{25}{100} = 0.25$ $P_2 = \frac{18}{125} = 0.144$

$$P = \frac{25+18}{100+125} = 0.19 \quad Q = 1 - 0.19 = 0.81$$

Test statistic, $Z = \frac{P_1 - P_2}{\sqrt{PQ \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}}$

$$\text{So, } Z = \frac{0.25 - 0.14}{\sqrt{0.19 \times 0.81 \left(\frac{1}{100} + \frac{1}{125} \right)}} = 2.089$$

Since, $|Z| > 1.96$, H_0 is rejected.

Exercise 9.11

Test statistic, $\chi^2 = \frac{n(ad - bc)^2}{(a+b)(a+c)(b+d)(c+d)}$

$$= \frac{550 \left[(150 \times 158) - (120 \times 122) \right]^2}{280 \times 280 \times 282 \times 288}$$

$$= 2.89$$

Since, $\chi^2 > \chi^2_{0.05} = 3.84$, H_0 is rejected

Exercise 9.14

Here, $n_1 = 15$, $n_2 = 20$

$$\bar{x}_1 = \frac{1}{n_1} \sum x_1 = \frac{16875}{15} = 1125 \quad S_1^2 = 5625$$

$$\bar{x}_2 = \frac{1}{n_2} \sum x_2 = \frac{26500}{20} = 1325 \quad S_2^2 = 50625$$

$$S_p^2 = \frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{(n_1-1) + (n_2-1)} = \frac{(14 \times 5625) + (19 \times 50625)}{14 + 19} \\ = 31534.09$$

Test statistic, $t_1 = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{S_p^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}}$

$$= \frac{1125 - 1325}{\sqrt{31534.09 \left(\frac{1}{15} + \frac{1}{20} \right)}} \\ = -3.29$$

$|t| > t_{0.05} = 2.035$, $\therefore H_0$ is accepted.