

LESSON 2

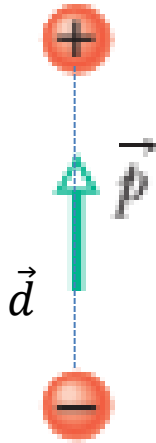
BOOK CHAPTER 22

ELECTRIC FIELDS

Electric Dipoles:

An electric dipole is a pair of point charges with equal magnitude and opposite sign (a positive charge $+q$ and a negative charge $-q$) separated by a small distance d .

Fig.1



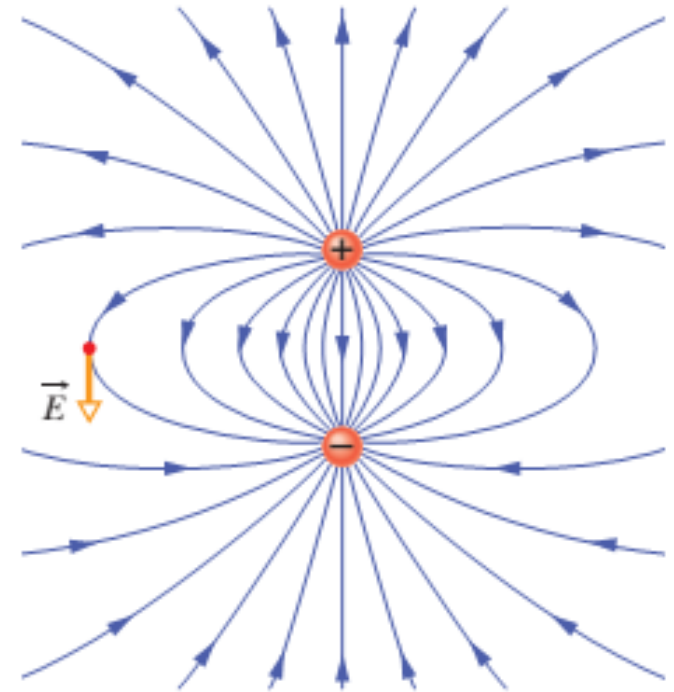
The product of the charge q and the separation d is the magnitude of a quantity called the **electric dipole moment**, denoted by p .

$$\text{That is, } p = qd$$

$$\text{In vector form, } \mathbf{p} = qd \hat{k}$$

The direction of \vec{p} is from negative charge to positive charge as shown in figure 1.

Fig.2



The pattern of electric field lines around an electric dipole, with an electric field vector \vec{E} shown (Figure 2) at one point (tangent to the field line through that point).

The Electric Field Due to an Electric Dipole:

The net magnitude of the electric field at point P is

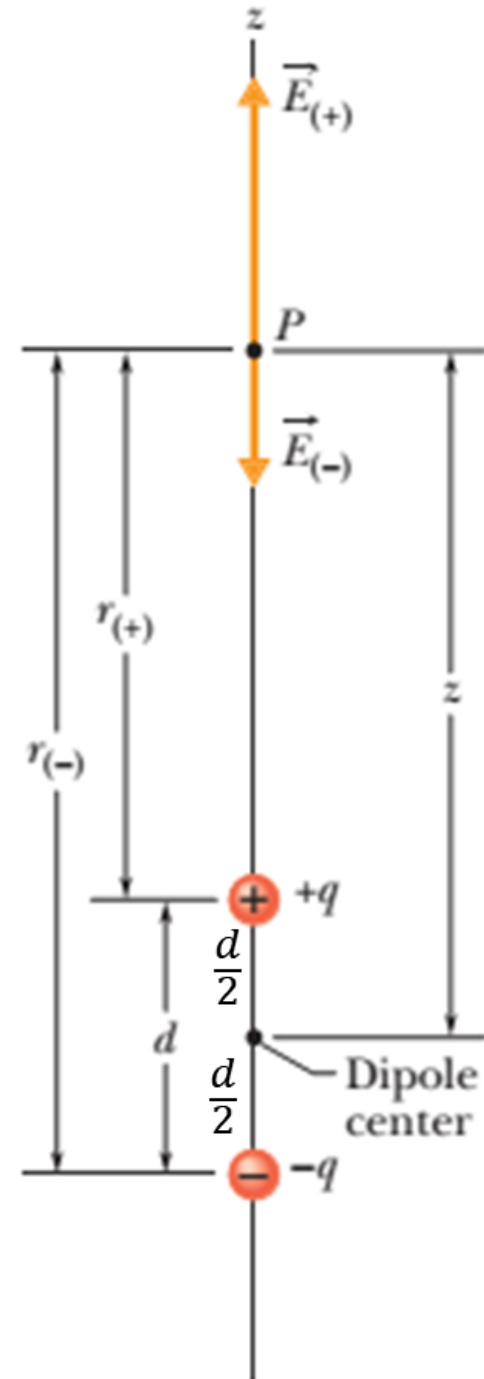
$$\mathbf{E} = \mathbf{E}_{(+)} + \mathbf{E}_{(-)}$$

$$E = +\frac{1}{4\pi\epsilon_0} \frac{q}{r_{(+)}^2} + \left(-\frac{1}{4\pi\epsilon_0} \frac{q}{r_{(-)}^2}\right)$$

$$E = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{\left(z - \frac{d}{2}\right)^2} - \frac{1}{\left(z + \frac{d}{2}\right)^2} \right]$$

$$E = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{\left\{z\left(1 - \frac{d}{2z}\right)\right\}^2} - \frac{1}{\left\{z\left(1 + \frac{d}{2z}\right)\right\}^2} \right]$$

$$E = \frac{q}{4\pi\epsilon_0 z^2} \left[\frac{1}{\left(1 - \frac{d}{2z}\right)^2} - \frac{1}{\left(1 + \frac{d}{2z}\right)^2} \right]$$



$$E = \frac{q}{4\pi\epsilon_0 z^2} \left[\left(\left\{ 1 + \left(-\frac{d}{2z} \right) \right\}^{-2} - \left(1 + \frac{d}{2z} \right)^{-2} \right] \text{ For } z \gg d, \text{ we have } \frac{d}{2z} \ll 1$$

We use the form of binomial theorem,

$$(1+x)^n = 1 + \frac{nx}{1!} + \frac{n(n-1)x^2}{2!} + \dots \quad (x^2 < 1)$$

$$(1+x)^n \approx 1 + nx \quad \text{for } x \ll 1$$

$$E = \frac{q}{4\pi\epsilon_0 z^2} \left[\left\{ \left(1 + (-2) \left(-\frac{d}{2z} \right) \right) \right\} - \left\{ \left(1 + (-2) \left(\frac{d}{2z} \right) \right) \right\} \right]$$

$$E = \frac{q}{4\pi\epsilon_0 z^2} \left[\left(1 + \frac{d}{z} \right) - \left(1 - \frac{d}{z} \right) \right]$$

$$E = \frac{q}{4\pi\epsilon_0 z^2} \left[1 + \frac{d}{z} - 1 + \frac{d}{z} \right]$$

$$E = \frac{q}{4\pi\epsilon_0 z^2} \left[2 \frac{d}{z} \right] \quad [p = qd]$$

$$E = \frac{qd}{2\pi\epsilon_0 z^3} = \frac{p}{2\pi\epsilon_0 z^3}$$

$$E = \frac{p}{2\pi\epsilon_0 z^3} \hat{k}$$

Linear charge density:

When charge is distributed along a line (such as a long, thin, charged plastic rod), we use (the Greek letter lambda λ) to represent the charge per unit length known as **linear charge density**.

That is

$$\lambda = \frac{\text{Amount of charge distributed on the rod}}{\text{Length of the rod}} \quad [\text{For uniform linear charge density}]$$

The SI unit of λ is Coulomb/meter; simply, we use C/m.

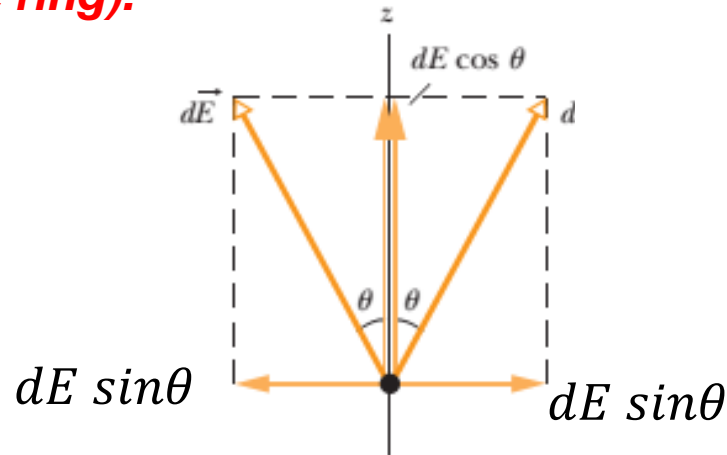
Note:

Some Measures of Electric Charge

Name	Symbol	SI Unit
Charge	q	C
Linear charge density	λ	C/m
Surface charge density	σ	C/m ²
Volume charge density	ρ	C/m ³

Electric field due to a line of charge (ring): $E = \frac{qz}{4\pi\epsilon_0(z^2+R^2)^{3/2}}$

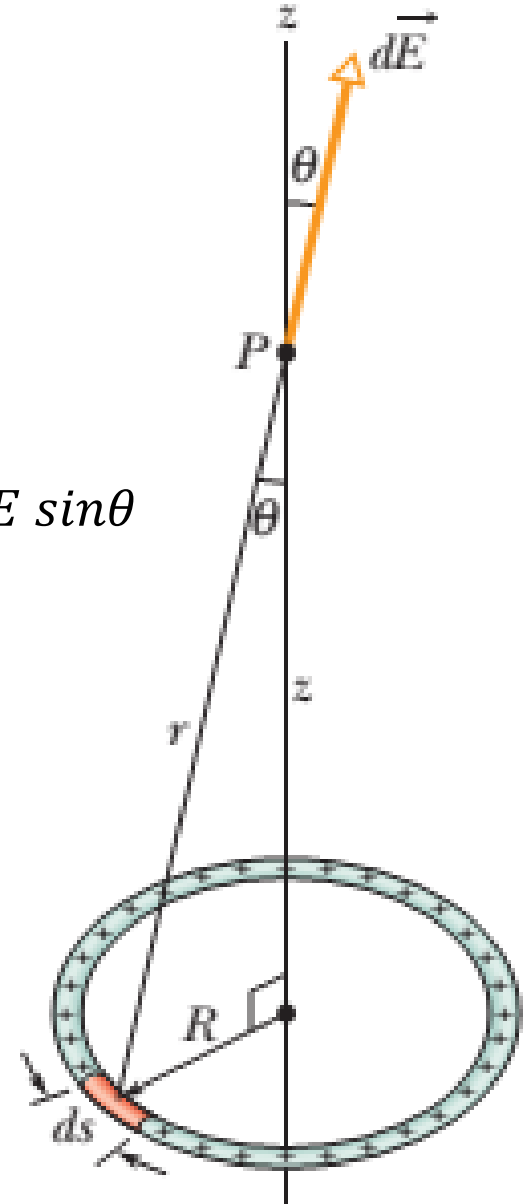
For charge that is distributed uniformly over a ring, determine the net electric field at a given point on the axis of the ring (distance z from the center of the ring).



Let ds be the (arc) length of any differential element of the ring. Since λ is the charge per unit (arc) length, the element has a charge of magnitude

Linear charge density, $\lambda = dq/ds$

$$dq = \lambda ds$$



This differential charge (dq) sets up a differential electric field $d\vec{E}$ at point P , which is a distance r from the element. Treating the element as a point charge.

$$dE = \frac{1}{4\pi\epsilon_0} \frac{dq}{r^2} = \frac{1}{4\pi\epsilon_0} \frac{\lambda ds}{r^2} = \frac{1}{4\pi\epsilon_0} \frac{\lambda ds}{(\sqrt{z^2 + R^2})^2} = \frac{1}{4\pi\epsilon_0} \frac{\lambda ds}{(z^2 + R^2)}$$

Since the components perpendicular to the z axis ($dE \sin\theta$) cancel and the parallel components ($dE \cos\theta$) add, the net electric field along z -axis is

$$E_z = E = \int dE \cos\theta = \int_{s=0}^{s=2\pi R} ds \int \frac{1}{4\pi\epsilon_0} \frac{\lambda ds}{(z^2 + R^2)} \left[\frac{z}{\sqrt{z^2 + R^2}} \right] \left[\cos\theta = \frac{z}{r} = \frac{z}{\sqrt{z^2 + R^2}} \right]$$

$$E = \frac{\lambda z}{4\pi\epsilon_0 (z^2 + R^2)^{3/2}} \int_{s=0}^{s=2\pi R} ds = \frac{\lambda z}{4\pi\epsilon_0 (z^2 + R^2)^{3/2}} [s]_0^{2\pi R}$$

$$E = \frac{\lambda z}{4\pi\epsilon_0(z^2 + R^2)^{3/2}}(2\pi R - 0)$$

$$[\lambda = q/2\pi R]$$

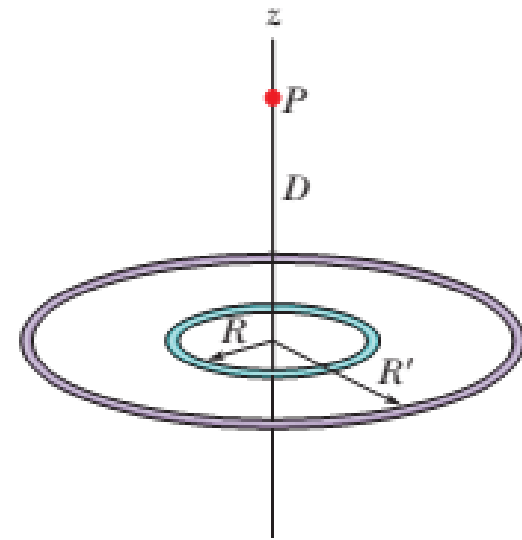
Total charge of the ring,
 $q = \lambda(2\pi R)$

$$E = \frac{\{\lambda(2\pi R)\} z}{4\pi\epsilon_0(z^2 + R^2)^{3/2}}$$

$$E = \frac{qz}{4\pi\epsilon_0(z^2 + R^2)^{3/2}}$$

Problem 30 (Book chapter 22)

Figure shows two concentric rings, of radii R and $R' = 3R$, that lie on the same plane. Point P lies on the central z axis, at distance $D = 2R$ from the center of the rings. The smaller ring has uniformly distributed charge $+Q$. In terms of Q , what is the uniformly distributed charge on the larger ring if the net electric field at P is zero?



$$\mathbf{E}_{+Q} + \mathbf{E}_{-q} = 0$$

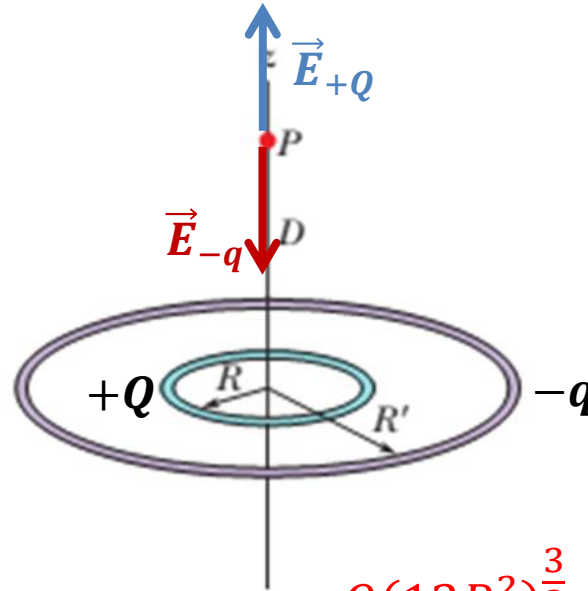
$$+E_{+Q} + (-E_{-q}) = 0$$

$$E_{+Q} = E_{-q}$$

$$\frac{QD}{4\pi\epsilon_0(D^2 + R^2)^{3/2}} = \frac{qD}{4\pi\epsilon_0(D^2 + (3R)^2)^{3/2}}$$

$$\frac{Q(2R)}{(4R^2 + R^2)^{3/2}} = \frac{q(2R)}{(4R^2 + 9R^2)^{3/2}}$$

$$\frac{Q}{(5R^2)^{3/2}} = \frac{q}{(13R^2)^{3/2}}$$



$$q = \frac{Q(13R^2)^{3/2}}{(5R^2)^{3/2}} = \left(\frac{13R^2}{5R^2}\right)^{3/2} Q$$

$$q = \left(\frac{13}{5}\right)^{3/2} Q = 4.19Q$$

$$q = -4.19Q$$

THANK YOU