

# Numerical Methods for Science and Engineering

Chapter one  
Solutions to Linear System

## Solution of Linear System by Elimination

**Objective:** Solve the system of linear equations using Elimination Method

**Gaussian Elimination:** The process which eliminates an unknown from succeeding equations using elementary operations

### Gaussian Elimination with Partial Pivoting

- Select pivotal equation for a variable for elimination.
- Eliminate the chosen variable from the remaining equations with respect to the pivotal equation.
- Repeat the process for the subsystem.
- Using back-substitution find solutions by using pivotal equations.

**Output :** By applying Gaussian Elimination with partial pivoting, system of linear equations can be solved to find roots (approximately) of the system.

**Example :**

The currents running through an electrical system are given by the following system of equations. The three currents,  $x$ ,  $y$ , and  $z$ , are measured in amps.

$$8x + 3y - 5z = 3$$

$$10x + 7y + 2z = 4 .$$

$$6x + 4y + 7z = 8$$

- a. Solve the above system to find the currents in this circuit using Gaussian elimination with partial pivoting.
- b. Justify your result by direct substitution in the original equation.
- c. Write MATLAB codes to solve by left division (backslash) operator.

Operation	x	y	z	Const	Eq	Remark
	8	3	-5	3	Eq1	
	10	7	2	4	Eq2	
	6	4	7	8	Eq3	
Eq2/10	1.0000	0.7000	0.2000	0.4000	E4*	pivot in x
Eq1/8	1.0000	0.3750	-0.6250	0.3750	E5	
Eq3/6	1.0000	0.6667	1.1667	1.3333	E6	
E5-E4	0.0000	-0.3250	-0.8250	-0.0250	E7	
E6-E4	0.0000	-0.0333	0.9667	0.9333	E8	
E7/(-0.325		1.0000	2.5385	0.0769	E9*	Pivot in y
E8(-/0.033		1.0000	-29.0003	-28.0003	E10	
E10-E9		0.0000	-31.5388	-28.0772	E11	
E11/-31.538				0.8902	E12*	Pivot in z
z=	0.8902					
y=	-2.1829					
x=	1.7500					

Pivot --- the element which  
is highest in magnitude

**Do yourself**

Operation	x	y	z	Constant	Eqn #	
	8	3	-5	3	E1	
	10	7	2	4	E2	
	6	4	7	8	E3	
E2/10	1.000				E4*	1 <sup>st</sup> Piv eq in x
E1/8	1.000				E5	
E3/6	1.000				E6	
E5-E4	0				E7	
E6-E4	0				E8	
E7/(-0.325)	0	1			E9*	2 <sup>nd</sup> Piv eq in y
E8/(-0.03)	0	1			E10	
E10-E9	0	0			E11	
E11/(-31.5388)	0	0	1		E12*	3 <sup>rd</sup> Piv eq in z

The **upper triangular form** of the system can be summarized as

$$x + 0.7y + 0.2z = 0.4$$

$$y + 2.5385z = 0.0769$$

$$z = 0.8902$$

Solving by **back substitution**, we have,  $z = 0.8902$

$$y = 0.0769 - 2.538(0.8902) = -2.1829$$

$$x = 0.4 - 0.7(-2.1829) - 0.2(0.8902) = 1.7500$$

b.

Justification:

$$\begin{aligned} E1: \quad & 8(1.750) + 3(-2.1829) - 5(0.8902) \cong 3 \\ E2: \quad & 10(1.750) + 7(-2.1829) + 2(0.8902) \approx 4 \\ E3: \quad & 6(1.750) + 4(-2.1829) + 7(0.8902) \approx 8 \end{aligned}$$

c. MATLAB

```
>> A=[8 3 -5; 10 7 2; 6 4 7];
```

```
>> b=[3; 4; 8];
```

```
>> Solution = A\b
```

Solution = 1.7500, -2.1829, 0.8902

$$\begin{aligned} 8x + 3y - 5z &= 3 \\ 10x + 7y + 2z &= 4 \\ 6x + 4y + 7z &= 8 \end{aligned}$$

$$Ax=b$$

$$x = A^{-1}b$$

## Sample MCQ

1. To solve the system of linear equation which method we use?
  - a) Gaussian elimination
  - b) Iterative method
  - c) Both
  - d) None
2. The process which eliminates an unknown from succeeding equations using elementary operations is known as
  - a) Gaussian Elimination
  - b) Gauss Jacobi Method
  - c) Gauss-Seidal Iterative Method
  - d) None
3. One such system is the upper triangular form which method is then used to solve the system?
  - a) Substitution
  - b) Back-substation
  - c) Both
  - d) None
4. The equation with numerical coefficient, highest in magnitude, of that variable in the system is called
  - a) Main equation
  - b) Pivotal equation
  - c) Iterative equation
  - d) None

# Exercise

1. a.  $5x - 4y + 3z = 21$ ,  $2x + 3y + 2z = 20$ ,  $8x + 2y + z = 13$ .

b.  $3x - 4y + 6z = 12$ ,  $7x + y - 5z = 18$ ,  $2x + 9y + 4z = 14$ .

c.  $2x + y - 3z = 17$ ,  $5x - 2y + 3z = 6$ ,  $x - 8y + z = 11$ .

d.  $6x + 5y - 8z = 24$ ,  $10x + 3y + 4z = 11$ ,  $8y + 3z = 10$

e.  $5x + 2y + z = 7$ ,  $2x - 4y + 3z = 6$ ,  $3x + 5y + 7z = 6$

f.  $6x + 5y - 8z = 14$ ,  $8x + 3y + 4z = 16$ ,  $2x + 7y + 3z = 12$ .

i. **Solve the above linear system using Gaussian elimination with pivoting (partial/total).**

ii. **Justify your result by direct substitution in the original equations.**

iii. **Write MATLAB codes to solve by left division (backslash) operator.**



2. Cory, Josh and Dan went shopping for Halloween treats. Cory bought 3 chocolate pumpkins, 4 masks and 8 candy witches. He spent \$36.65. Josh bought 5 chocolate pumpkins, 3 masks and 10 candy witches. He spent \$37.50. Dan bought 4 chocolate pumpkins, 5 masks and 6 candy witches. He spent \$43.45.

- i. Write a system of equations to represent this problem
- ii. Solve the system of linear equation using Gaussian elimination with partial pivoting.
- iii. Write MATLAB codes to solve by left division (backslash) operator.

3. A local computer company sells three types of laptop computers to three nearby stores. The number of laptops ordered by each store and the amount owing to the company for the order is shown in the following table:

Store	Laptop A	Laptop B	Laptop C	Amount Owing(\$)
Wal-Mart	10	8	6	21 200
Sears	7	9	5	18 700
Target	8	4	3	13 000

- i. Write a system of equations to represent the above information.
- ii. Solve the system of linear equation using Gaussian elimination with partial pivoting.
- iii. Write MATLAB codes to solve by left division (backslash) operator.