Ex: 1.1.1

a) $\int dx = x + C$ (Ans)

b) $\int x^5 dx = \frac{x^6}{6} + C$ (Ans)

c) $\int x^{3/2} dx = \frac{x^{3/2+1}}{3/2+1} + C$

$= \frac{2x^2\sqrt{x}}{5} + C$ (Ans)

d) $\int \sin(-3x) dx = \int -\frac{\sin(t)}{3} dt$

$= -\frac{1}{3}\int \sin(t) dt$

$= -\frac{1}{3}(-\cos(t))$

$= -\frac{1}{3}(-\cos(-3x))$ $\because t = -3x$

$= \frac{\cos(-3x)}{3} + C$ (Ans)

e) $\int \cos(2x)\,dx = \int \frac{\cos(t)}{2}\,dt$

$= \frac{1}{2}\int \cos(t)\,dt$

$= \frac{1}{2}\sin t$

$= \frac{1}{2}\sin 2x \quad \because t = 2t$

$= \frac{\sin(2x)}{2} + c$ (Ans)

f) $\int e^{5x}\,dx = \frac{1}{5}e^{5x}$

$= \frac{e^{5x}}{5} + c$ (Ans)

g) $\int e^{2x/3}\,dx = \int \frac{3e^{t}}{2}\,dt \quad \because t = \frac{2x}{3}$

$= \frac{3}{2}\int e^{t}\,dt$

$= \frac{3}{2}e^{t}$

$= \frac{3}{2}e^{\frac{2x}{3}}$

$= \frac{3e^{\frac{2x}{3}}}{2} + c$ (Ans)

h) $\int \exp(-3x)\,dx$ $\quad \therefore \int e^{ax}\,dx = \frac{1}{a}e^{ax}$

$= \frac{1}{-3}e^{-3x}$

$= -\frac{1}{3e^{3x}} + c$ (Ans)

i) $\int x^{-1}\,dx = \int \frac{1}{x}\,dx$

$= \ln(|x|) + c$ (Ans)

k) $\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}x + c$ (Ans)

l) $\int \sqrt[3]{y^2}\,dy = \int y^{2/3}\,dy$ $\quad \sqrt[n]{a^m} = a^{\frac{m}{n}}$

$= \frac{3y\sqrt[3]{y^2}}{5} + c$ (Ans)

m) $\int \frac{1}{r}\,dr = \ln(|r|) + c$ (An)

n) $\int \sinh(2x)\,dx = \int \frac{\sinh(t)}{2}\,dt$ $\quad \therefore t = 2x$

$= \frac{1}{2}\int \sinh(t)\,dt$

$= \frac{1}{2}\cosh(t)$

$= \frac{1}{2}\cosh(2x) + c$ (Ans)

o) $\int \cosh(-3x)\,dx = \int -\frac{\cosh(t)}{3}\,dt$

$= -\frac{1}{3}\int \cosh(t)\,dt$

$= -\frac{1}{3}\sinh(t) \quad \because t = -3x$

$= -\frac{1}{3}\sinh(-3x)$

$= \frac{\sinh(3x)}{3} + c$ (An)

p) $\int (2x+3)^{3/2}\,dx = \int \frac{t^{3/2}}{2}\,dt \quad \therefore t = 2x+3$

$= \frac{1}{2}\int t^{3/2}\,dt$

$= \frac{1}{2}\times\frac{2t^2\sqrt{t}}{5}$

$= \frac{1}{2}\times\frac{2(2x+3)^2\times\sqrt{2x+3}}{5}$

$= \frac{(4x^2+12x+9)\sqrt{2x+3}}{5} + c$

(An)

$= \frac{\cos(1-2x)}{2} + c$ (Ans)

w) $\int \exp(-3x+1)\,dx$

$= \int -\frac{e^t}{3}\,dt$

$= -\frac{1}{3}\int e^t\,dt$

$= -\frac{1}{3}e^t$

$= -\frac{1}{3}e^{-3x+1}$

$= -\frac{e^{-3x+1}}{3} + c$ (Ans)

x) $\int \frac{3}{9+(x-2)^2}\,dx$

$= \int \frac{3}{9+t^2}\,dt$

$= 3\int \frac{1}{9+t^2}\,dt$

~~$= 3\tan^{-1}\left(\frac{t}{3}\right)$~~

$= 3 \times \frac{1}{3}\tan^{-1}\left(\frac{t}{3}\right)$

$= \tan\frac{x-2}{3} + c$ (Ans)

Y) $\int \frac{1}{\sqrt{4-(x+1)^2}}dx$

$= \int \frac{1}{\sqrt{4-t^2}}dt \qquad t = x+1$

$= \sin^{-1}\left(\frac{t}{2}\right)$

$= \sin^{-1}\left(\frac{x+1}{2}\right) + c \qquad$ (Ans)

Z) $\int \frac{1}{\sqrt{4-9x^2}}dx$

$= \int \frac{1}{\sqrt{9\left(\frac{4}{9}-x^2\right)}}dx$

$= \frac{1}{\sqrt{9}} \int \frac{1}{\sqrt{\frac{4}{9}-x^2}}dx$

$= \frac{1}{3}\sin^{-1}\left(\frac{x}{2/3}\right)$

$= \frac{\sin^{-1}\left(\frac{3x}{2}\right)}{3} + c \qquad$ (Ans)

Exercise set - 1.1.2

i) $\int \left(\frac{1}{x} - 3\right) dx$

$\Rightarrow \int \frac{1}{x} dx - \int 3 \, dx$

$\Rightarrow \ln(|x|) - 3x + c$ (Ans)

ii) $\int (x^3 + x^{-3}) dx$

$= \int x^3 + \frac{1}{x^3} dx$

$= \frac{x^4}{4} - \frac{1}{2x^2} + c$

$\boxed{\int \frac{1}{x^n} dx = -\frac{1}{(n-1) \times x^{n-1}} ; n \neq 1}$

iii) $\int \left(x^{-1} + \frac{1}{e^x}\right) dx$

$= \int \left(\frac{1}{x} + \frac{1}{e^x}\right) dx$

$= \int \frac{1}{x} dx + \int \frac{1}{e^x} dx$

$= \ln(|x|) - \frac{1}{e^x} + c$

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iv) $\int (\sin(-3x) + \cos(2x))\,dx$.

$= -\int \sin(3x)\,dx + \int \cos(2x)\,dx$

$= \frac{\cos(3x)}{3} + \frac{\sin(2x)}{2} + c$ (Ans)

v) $\int (e^{-x} + \cos(3x-2))\,dx$

$= \int \frac{1}{e^x}\,dx + \int \cos(3x-2)\,dx$

$= -\frac{1}{e^x} + \frac{\sin(3x-2)}{3} + c$ (Ans)

vi) $\int (3e^{3x+1} + \frac{1}{2x+3}\,dx)$

$= \int 3e^{3x+1}\,dx + \int \frac{1}{2x+3}\,dx$

$= e^{3x+1} + \frac{1}{2}\ln(|2x+3|) + c$ (Ans)

vii) $\int \left(\frac{x^2+1}{x}\right)dx$

$= \int \frac{x^2}{x} + \frac{1}{x}\,dx$

$= \int x + \frac{1}{x}\,dx$

$= \int (x)\,dx + \int \frac{1}{x}\,dx$

$= \frac{x^2}{2} + \int \frac{1}{x} dx$

$= \frac{x^2}{2} + \ln(|x|) + c$ (An)

viii) $\int \left(\frac{\sqrt{x}+1}{\sqrt{x}}\right) dx$

$= \int \left(\frac{\sqrt{x}}{\sqrt{x}} + \frac{1}{\sqrt{x}}\right) dx$

$= \int 1\, dx + \int \frac{1}{x^{1/2}} dx$

$= x + 2\sqrt{x} + c$

(A)

$$\boxed{\begin{aligned} &\int \frac{1}{x^n} dx \\ &= -\frac{1}{(n-1)x^{n-1}}, \quad n \neq 1 \end{aligned}}$$

ix) $\int \left(4x^3 + \frac{2}{x^2} - 1\right) dx$

$= \int 4x^3 dx + \int \frac{2}{x^2} dx - \int 1\, dx$

$= x^4 - \frac{2}{x} - x + c$ (A)

x) $\int \left(1 + \frac{3}{x} - 7\sin 2x\right) dx$

$\Rightarrow \int 1\, dx + \int \frac{3}{x} dx - \int 7\sin(2x)\, dx$

$\Rightarrow x + 3\ln(|x|) + \frac{7\cos(2x)}{2} + c$ (An)

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xi) $\int(\cancel{3}\ 3e^{2x} + 3e^{-4x} + \sqrt[3]{x})\,\partial x$

$= \int\left(3e^{2x} + 3\cdot\frac{1}{e^{4x}} + \sqrt[3]{x}\right)\partial x$

$= \int 3e^{2x} + 3\cdot\frac{1}{e^{4x}} + x^{1/3}\,dx$

$= \int\left(3e^{2x} + \frac{3}{e^{4x}} + x^{1/3}\right)dx$

$= \int 3e^{2x}\,dx + \int\frac{3}{e^{4x}}\,dx + \int x^{1/3}\,dx$

$= \frac{3e^{2x}}{2} \cancel{+} - \frac{3}{4e^{4x}} + \frac{3x\sqrt[3]{x}}{4} + c$ (Ans)

xii) $\int(2\cos 2x - \sin 3x)\,\partial x$

$= \int 2\cos(2x)\,\partial x - \int \sin(3x)\,dx$

$= \sin(2x) + \frac{\cos(3x)}{3} + c$ (Ans)

xiii) $\int 5^{y+1}\,dy$

$= \int 5^{t}\,\partial t$ $\qquad t = y+1$

$= \frac{5^{t}}{\ln(5)}$

$= \frac{5^{y+1}}{\ln(5)} + c$ (Ans)

xiv) $\int\left(\frac{4x^3-2x^2+15x^5}{x^2}\right)dx$

$= \int \frac{x^2(4x-2+15x^3)}{x^2}dx$

$= \int 4x-2+15x^3 dx$

$= \int 4x dx - \int 2 dx + \int 15x^3 dx$

$= 2x^2 - 2x + \frac{15x^4}{4} + c$

1.1.3

a) $\int \frac{1}{x+2}dx$

$= \ln(|x+2|)+c$ (Ans)

b) $\int \frac{3x^2+2}{x^3+2x}dx$

$= \ln(|x^3+2x|)+c$ (Ans)

c) $\int \frac{2x-1}{x^2-x+3}dx$

$= \ln(|x^2-x+3|)+c$ (Ans)

d) $\int \frac{\cancel{x^2+2x}}{x}$

d) $\int \frac{2x+\sin x}{x^2-\cos x}\,dx$

$= \ln(|x^2-\cos(x)|)+c$ (Ans)

e) $\int \frac{1+e^{-t}}{t-e^{-t}}\,dt$

$= \ln(|t-e^{-t}|)+c$ (Ans)

f) $\int \frac{1}{2x+3}\,dx$

$= \frac{1}{2}\times \ln(|2x+3|)+c$ (Ans)

g) $\int \frac{x^2+2x}{x^3+3x^2+1}\,dx$

$= \frac{1}{3}\int \frac{1}{t}\,dx$ $\qquad t = x^3+3x^2+1$

$= \frac{1}{3}\times \ln(|x^3+3x^2+1|)+c$

(Ans)

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h) $\int \frac{\cos 3x}{3+\sin 3x} dx$

$= \frac{1}{3} \times \ln(|3+\sin(3x)|) + c$ (Ans)

i) $\int \frac{\sec^2 3x}{2+\tan 3x} dx$.

$= \frac{1}{3} \ln(|2+\tan(3x)|) + c$ (Ans).

j) $\int \frac{e^{3x}}{3-2e^{3x}} dx$

$= -\frac{1}{6} \times \int \frac{1}{t} dt$ $\quad t = 3-2e^{3x}$

$= -\frac{1}{6} \times \ln(|t|)$

$= -\frac{1}{6} \times \ln(|3-2e^{3x}|) + c$

(Ans)

k) $\int \cot 3z \, dz$

$= \int \frac{\cot(t)}{3} dt$

$= \frac{1}{3} \int \cot(t) \, dt$

$= \frac{1}{3} \int \frac{\cos(t)}{\sin(t)} dt$

$$= \frac{1}{3} \times \int \frac{1}{u} du \qquad \therefore u = \sin(t)$$

$$= \frac{1}{3} \times \ln(|u|)$$

$$= \frac{1}{3} \ln(|\sin(3z)|) + c \qquad \text{(Ans)}$$

l) $\int \frac{1}{y(1+\ln y)} dy$

$$= \ln|1+\ln y| + c \qquad \text{(Ans)}$$

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5. $\int (x^{1.3} + 7x^{2.5})dx$

$= \frac{1}{2.5}x^{2.3} + \frac{7}{3.5}x^{3.5} + c$

$= 0.43x^{2.3} + 2x^{3.5} + c$ (Ans)

6. $\int \sqrt[4]{x^5}\, dx$

$= \int x^{5/4} dx$

$= \frac{4}{9}x^{9/4} + c$ (Ans)

7. $\int (5 + \frac{2}{3}x^2 + \frac{3}{4}x^3)dx$

$= 5x + \frac{2}{3} \cdot \frac{1}{3} \cdot x^3 + \frac{3}{4} \cdot \frac{1}{4}x^4 + c$

$= 5x + \frac{2}{9}x^3 + \frac{3}{16}x^4 + c$ (Ans)

8. $\int u^6 - 2u^5 - u^3 + \frac{2}{7})du$

$= \frac{1}{7}u^7 - \frac{2}{6}u^6 - \frac{1}{4}u^4 + \frac{2}{7}x + c$

$= \frac{1}{7}u^7 - \frac{1}{3}u^6 - \frac{1}{4}u^4 + \frac{2}{7}x + c$ (Ans

9. $\int (u+4)\cdot(2u+1)\,du$

$= \int (2u^2 + u + 8u + 4)\,du$

$= \int (2u^2 + 9u + 4)\,du$

$= 2\cdot\frac{1}{3}u^3 + \frac{9}{2}u^2 + 4u + c$

$= \frac{2}{3}u^3 + \frac{9}{2}u^2 + 4u + c$ (Ans)

10. $\int \sqrt{t}\,(t^2 + 3t + 2)\,dt$

$= \int t^{1/2}(t^2 + 3t + 2)\,dt$

$= \int (t^{5/2} + 3t^{3/2} + 2t^{1/2})\,dt$

$= \frac{2}{7}t^{7/2} + 3\cdot\frac{2}{5}t^{5/2} + 2\cdot\frac{2}{3}t^{3/2} + c$

$= \frac{2}{7}t^{7/2} + \frac{6}{5}t^{5/2} + 4/3\,t^{3/2} + c$ (Ans)

11. $\int \frac{1+\sqrt{x}+x}{x}\,dx$

$= \int \frac{x(1/x + \frac{1}{\sqrt{x}} + 1)}{x}\,dx$

$= \int \frac{1}{x}\,dx + 2\int \frac{1}{2\sqrt{x}}\,dx + \int 1\,dx$

$= \ln x + 2\sqrt{x} + x + c$ (Ans)

16. $\int \sec t\,(\sec t + \tan t)\,dt$

$= \int (\sec^2 t + \sec t \tan t)\,dt$

$= \tan t + \sec t + C$

17. $\int 2^t (1+5^t)\,dt$

$= \int (2^t + 10^t)\,dt$

$= 2^t \frac{1}{\ln 2} + 10^t \frac{1}{\ln 10} + C$

18. $\int \frac{\sin 2x}{\sin x}\,dx$

$= \int \frac{2\sin x \cos x}{\sin x}\,dx$

$= 2\int \cos x\,dx$

$= 2\sin x + C$ (A)

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