LESSON 9

BOOK CHAPTERS 25 and 26

CAPACITANCE

And

CURRENT and RESISTANCE

Energy Density:

In a parallel-plate capacitor, the electric field has the same value at all points between the plates. Thus, **the energy density u that is, the potential energy per unit volume** between the plates should also be uniform. We can find *u* by dividing the total potential energy by the volume *Ad* of the space between the plates. Thus

$$u = \frac{U}{Ad} = \left(\tfrac{1}{2}CV^2\right)\frac{1}{Ad}$$

For parallel plate capacitor, the capacitance can be expressed as

$$C = \frac{\varepsilon_0 A}{d}$$

Therefore,

$$u = \left(\tfrac{1}{2}CV^2\right)\frac{1}{Ad} = \left(\frac{V^2}{2Ad}\right)\left(\frac{\varepsilon_0 A}{d}\right) = \frac{\varepsilon_0 V^2}{2d^2}$$

Finally, $\boxed{u = \frac{1}{2}\varepsilon_0\left(\frac{V}{d}\right)^2}$ OR $\boxed{u = \frac{1}{2}\varepsilon_0 E^2}$ [since $V = Ed$]

Problem 29 (Book chapter 25):

What capacitance is required to store an energy of 10 kW.h at a potential difference of 1000 V?

Problem 31 (Book chapter 25):

A 2.0 μF capacitor and a 4.0 μF capacitor are connected in parallel across a 300 V potential difference. Calculate the total energy stored in the capacitors.

Problem 32 (Book chapter 25):

A parallel-plate air-filled capacitor having area 40 cm^2 and plate spacing 1.0 mm is charged to a potential difference of 600 V. Find (a) the capacitance, (b) the magnitude of the charge on each plate, (c) the stored energy, (d) the electric field between the plates, and (e) the energy density between the plates.

Problem 33 (Book chapter 25):

A charged isolated metal sphere of diameter 10 cm has a potential of 8000 V relative to $V = 0$ at infinity. Calculate the energy density in the electric field near the surface of the sphere.

Sample problem 25.04 (page-730):

An isolated conducting sphere whose radius R is 6.85 cm has a charge $q = 1.25$ nC. (a) How much potential energy is stored in the electric field of this charged conductor? (b) What is the energy density at the surface of the sphere?

Problem 29 (Book chapter 25):

Given

$$U = 10\ kW.\ h = 10000 \times 60 \times 60\ W\ .s$$

$$U = 36 \times 10^6\ W\ .s$$

Potential difference, $V = 1000\ volt$

$$\boxed{C = ?}$$

Answer:

We know

$$U = \frac{1}{2} C V^2$$

$$C = \frac{2U}{V^2} = \frac{2 \times 36 \times 10^6}{(1000)^2} = 72\ F$$

Problem 31 (Book chapter 25):

Given

$$C_1 = 2\ \mu F \quad and\ C_2 = 4\ \mu F$$

$$V = 300\ volt$$

$$\boxed{U = ?}$$

Answer:

C_1 and C_2 $are\ in\ parallel$

$$C_{12} = C_1 + C_2 = 2 + 4 = 6\ \mu F = 6 \times 10^{-6}\ F$$

We know

$$U = \frac{1}{2} C_{12} V^2 = \frac{6 \times 10^{-6} \times (300)^2}{2}$$

$$U = 0.27\ J$$

Problem 32 (Book chapter 25):

Answer:

Given

$$A = 40\ cm^2 = 40 \times (10^{-2})^2\ m^2 = 4 \times 10^{-3}\ m^2$$

$$d = 1\ mm = 1 \times 10^{-3} m$$

$$V = 600\ volt$$

$$C = \frac{\varepsilon_0 A}{d} = \frac{8.854 \times 10^{-12} \times 4 \times 10^{-3}}{1 \times 10^{-3}}$$

(a) $\boxed{C = ?}$

$$\boxed{C = 35.416 \times 10^{-12}\ F}$$

(b) $\boxed{q = ?}$

$$\boxed{q = CV = 35.416 \times 10^{-12} \times 600 = 21.25 \times 10^{-9}\ C}$$

(c) $\boxed{U = ?}$

$$\boxed{U = \frac{1}{2} CV^2 = \frac{35.416 \times 10^{-12} \times (600)^2}{2} = 12.75 \times 10^{-6}\ J}$$

(d) $\boxed{E = ?}$

$$\boxed{E = \frac{V}{d} = \frac{600}{1 \times 10^{-3}} = 600 \times 10^3\ \frac{V}{m}}$$

(e) $\boxed{u = ?}$

$$\boxed{u = \frac{1}{2} \varepsilon_0 \left(\frac{V}{d}\right)^2 = \frac{8.854 \times 10^{-12} \times (600)^2}{2 \times (10^{-3})^2} = 1.59\ \frac{J}{m^3}}$$

Problem 33 (Book chapter 25):

Given

For isolated sphere:

$$Diameter, D = 10\ cm$$

$$Radius, R = 5\ cm = 0.05\ m$$

$$V = 8000\ volt \qquad \boxed{u = ?}$$

Answer:

$$u = \frac{1}{2}\varepsilon_0\left(\frac{V}{R}\right)^2 = \frac{8.854\times10^{-12}\times(8000)^2}{2\times(0.05)^2}$$

$$u = \frac{566.66\times10^{-6}}{5\times10^{-3}} = 113.33\times10^{-3}\ \frac{J}{m^3}$$

Sample problem 25.04 (page-730):

Given

For isolated sphere:

$$Radius, R = 6.85\ cm = 0.0685\ m$$

$$q = 1.25\ nC = 1.25\times10^{-9}\ C$$

(a) $\boxed{U = ?}$

(b) $\boxed{u = ?}$

Answer: We know $U = \frac{q^2}{2C}$

For isolated sphere, $C = 4\pi\epsilon_0 R$

$$U = \frac{q^2}{2\times4\pi\varepsilon_0 R} = \frac{9\times10^9\times(1.25\times10^{-9})^2}{2\times0.0685}$$

$$\boxed{U = 102.64\times10^{-9}\ J}$$

$$u = \frac{1}{2}\varepsilon_0 E^2 = \frac{8.854\times10^{-12}}{2}\left(\frac{q}{4\pi\varepsilon_0 R^2}\right)^2 = \frac{8.854\times10^{-12}}{2}\left(\frac{9\times10^9\times1.25\times10^{-9}}{(0.0685)^2}\right)^2$$

$$\boxed{u = \frac{1120.58\times10^{-12}}{4.403\times10^{-5}} = 25.450\times10^{-6}\ \frac{J}{m^3}}$$

BOOK CHAPTER 26

CURRENT and RESISTANCE

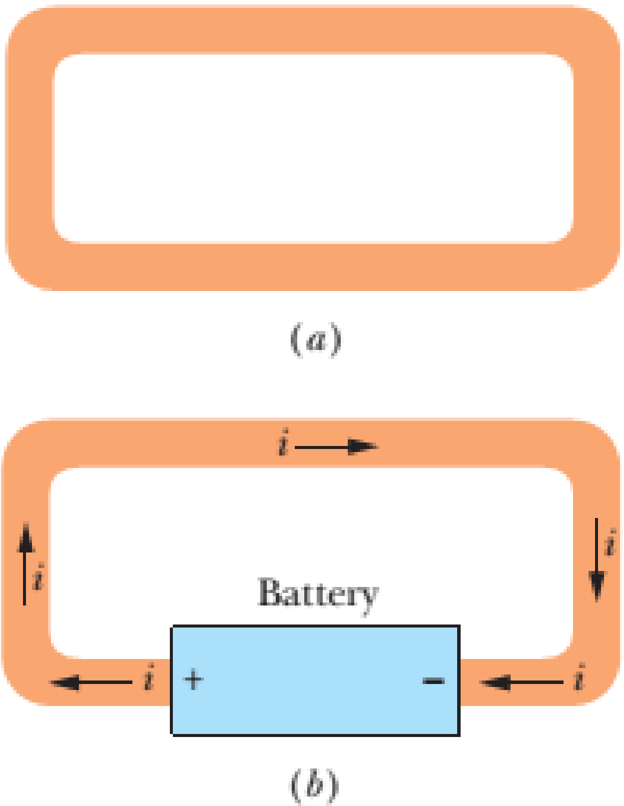
Electric Current:

An electric current *i* in a conductor is defined by

$$i = \frac{dq}{dt}$$

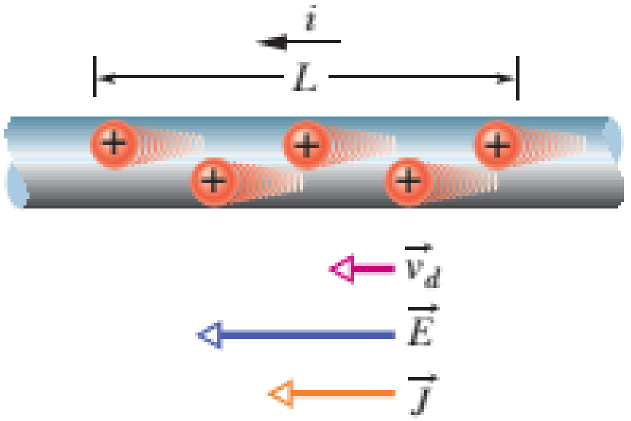
Here *dq* is the amount of (positive) charge that passes in time *dt* through a hypothetical surface that cuts across the conductor. By convention, the direction of electric current is taken as the direction in which positive charge carriers would move. The SI unit of electric current is the **ampere** (A): 1 A = 1 coulomb per second=1 *C/s.*

NOTE: A current arrow is drawn in the direction in which positive charge carriers would move, even if the actual charge carriers are negative and move in the opposite direction.



a) No electric field can exist within it or along its surface. Although conduction electrons are available, no net electric force acts on them and thus there is no current.(*b*) We insert a battery in the loop, the conducting loop is no longer at a single potential. Electric fields act inside the material making up the loop, exerting forces on the conduction electrons, causing them to move and thus establishing a current.

Current Density:



Current i (a scalar quantity) is related to current density $\vec{J}$ (a vector quantity) by

$$i = \int \vec{J} . d\vec{A}$$

Where $d\vec{A}$ is a vector perpendicular to a surface element of area dA and the integral is taken over any surface cutting across the conductor.

The current density $\vec{J}$ has the same direction as the velocity of the moving charges if they are positive and the opposite direction if they are negative.

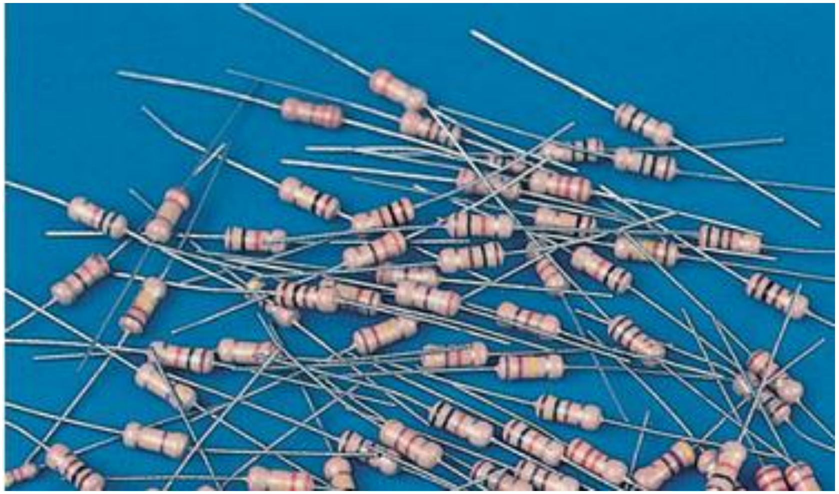
If the current is uniform across the surface and parallel to $d\vec{A}$ then $\vec{J}$ is also uniform and parallel to $d\vec{A}$ Then the above equation becomes

$$i = \int J \, dA = J \int dA = JA$$

Therefore, $$J = \frac{i}{A}$$

That is, for each element of the cross section, the magnitude J is equal to the current per unit area through that element

Resistance and Ohm's Law:



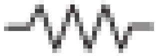
The r**esistance** *R* of a conductor is defined as

$$R = \frac{V}{i}$$

where *V* is the potential difference across the conductor and *i* is the current.

This relationship, called **Ohm's law**, was discovered in 1826 by the German physicist Georg Simon Ohm (1787–1854). The SI unit for resistance is the **ohm** (symbol Ω).

$$1\Omega = 1\, volt\ per\ ampere = 1\, V/A$$

A conductor whose function in a circuit is to provide a specified resistance is called a **resistor** (as shown in the adjacent figure). In a circuit diagram, we represent a resistor and a resistance with the symbol  .

Resistivity:

The current density $\vec{J}$ in a conductor depends on the electric field $\vec{E}$ and on the properties of the material. In general, this dependence can be quite complex. But for some materials, especially metals, at a given temperature, $\vec{J}$ is nearly directly proportional to $\vec{E}$ and the ratio of the magnitudes of E and J is constant.

We define the **resistivity *ρ*** of a material as the ratio of the magnitudes of electric field (E) and current density (J):

$$\rho = \frac{E}{J}$$

The greater the resistivity, the greater the field needed to cause a given current density.

Note:

The resistivity of a *metallic* conductor nearly always increases with increasing temperature as

$$\rho(T) = \rho_0[1 + \alpha(T - T_0)]$$

Where ρ_0 is the resistivity at a reference temperature T_0 (often taken as 0^0C or 20^0C) and $\rho(T)$ is the resistivity at temperature T, which may be higher or lower than T_0. The factor α is called the **temperature coefficient of resistivity.**

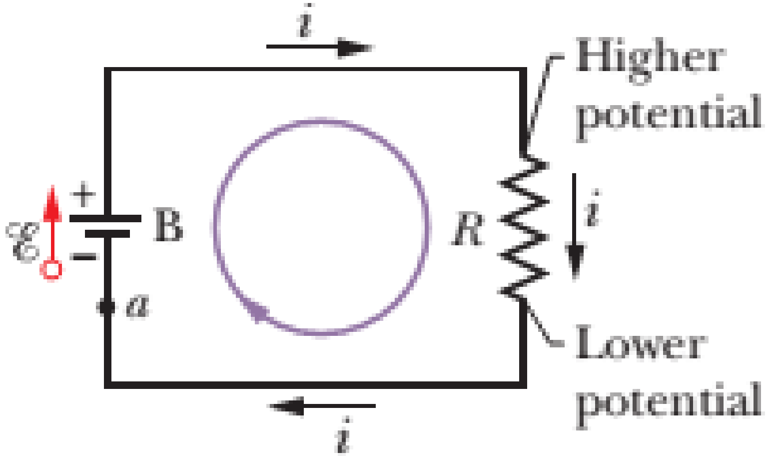
Loop Rule (Kirchhoff's voltage law): (Book Chapter 27):

The algebraic sum of the changes in potential encountered in a complete traversal of any loop of a circuit must be zero.

This is often referred to as *Kirchhoff's loop rule* (or *Kirchhoff's voltage law*), after German physicist Gustav Robert Kirchhoff.

That is

$$\varepsilon + (-iR) = 0$$



Note:

- The change in potential in traversing a resistance *R* in the direction of the current is $-iR$; in the opposite direction it is $+iR$ (resistance rule).

- The change in potential in traversing an ideal emf device in the direction of the emf arrow is $+\varepsilon$; in the opposite direction it is $-\varepsilon$ (emf rule).

Thank You