LECTURE 8

BOOK CHAPTER 9

(Center of Mass and Linear Momentum)

Center of Mass:

The center of mass of a system of particles is the point that moves as though (1) all of the system's mass were concentrated there and (2) all external forces were applied there.

How to find the center of mass?

The center of mass of a system of n particles is defined to be the point whose coordinates are given by

$$x_c = \frac{1}{M}\sum_{i=1}^{n} m_i x_i$$

$$y_c = \frac{1}{M}\sum_{i=1}^{n} m_i y_i \qquad OR \qquad \vec{r}_c = \frac{1}{M}\sum_{i=1}^{n} m_i \vec{r}_i$$

$$z_c = \frac{1}{M}\sum_{i=1}^{n} m_i z_i$$

Where M is the total mass of the system.

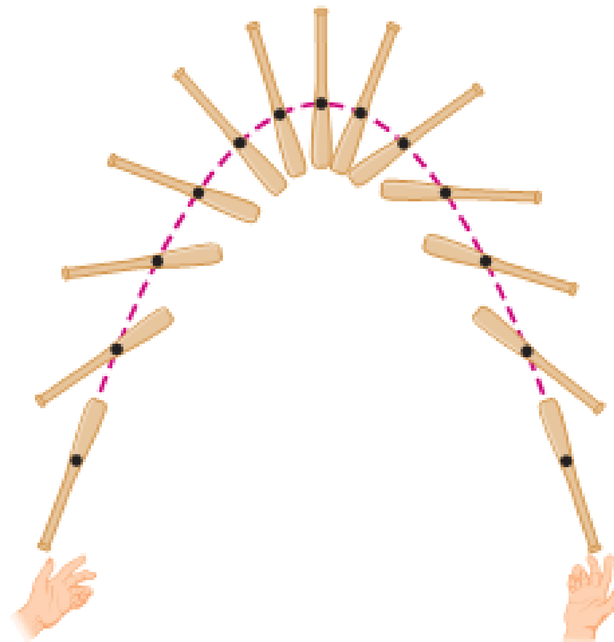


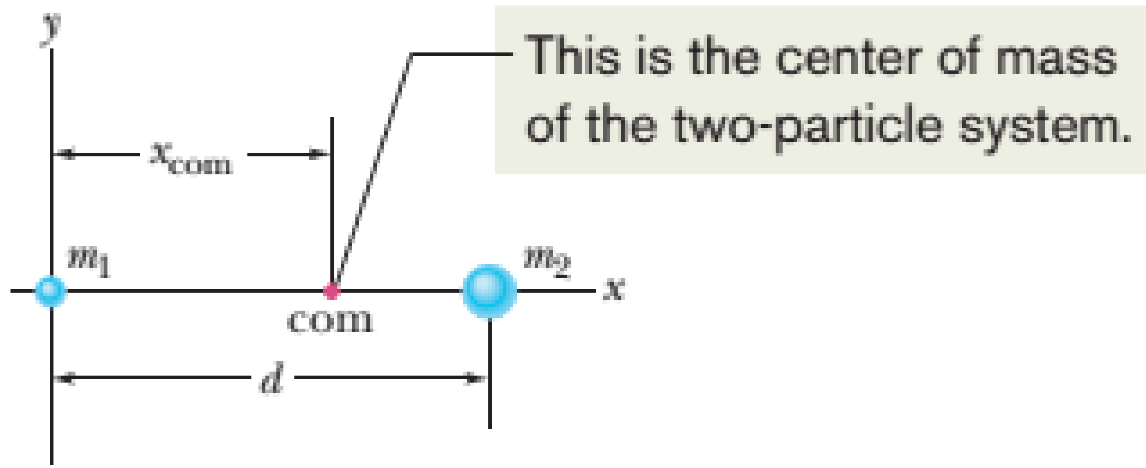
Figure. The center of mass (black dot) of a baseball bat flipped into the air follows a parabolic path, but all other points of the bat follow more complicated curved paths.

The center of mass of the two-particle system:

Case-1

The position of the center of mass of this two-particle system to be

$$x_c = \frac{m_2 d}{m_1 + m_2}$$

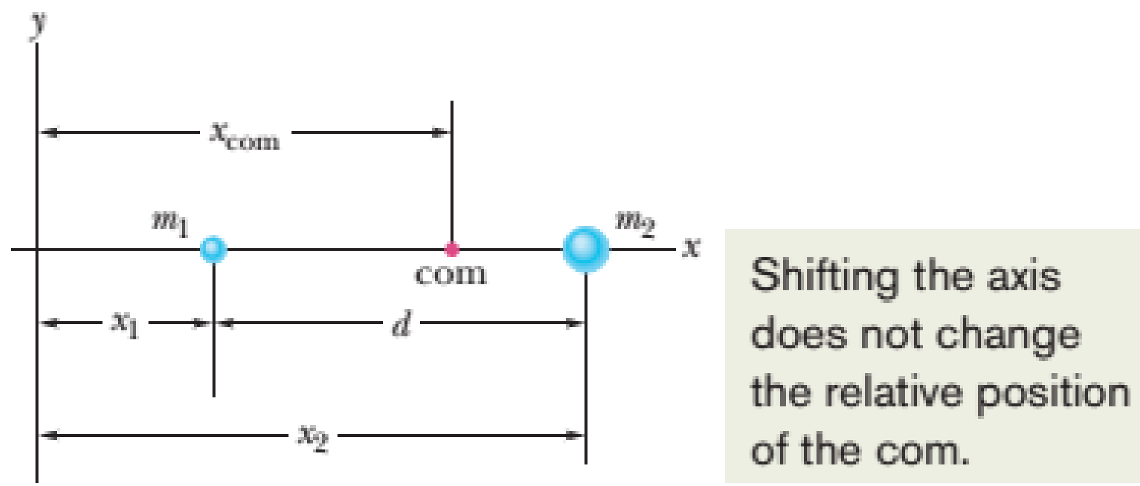


Case-1

Case-2

The position of the center of mass of this two-particle system to be

$$x_c = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$



Case-2

The location of the center of mass with respect to the particles is the same in both cases.

The velocity of the system's (two body system) center of mass:

$$x_c = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{m_1 x_1 + m_2 x_2}{M}$$

Where, $M = m_1 + m_2$

$$M x_c = m_1 x_1 + m_2 x_2$$

Differentiating with respect to time gives

$$M \frac{dx_c}{dt} = m_1 \frac{dx_1}{dt} + m_2 \frac{dx_2}{dt}$$

$$M v_c = m_1 v_1 + m_2 v_2$$

$$\boxed{v_c = \frac{m_1 v_1 + m_2 v_2}{M}}$$

Linear Momentum:

The **linear momentum** of a particle is a vector quantity $\vec{\boldsymbol{p}}$ that is defined as

$$\vec{\boldsymbol{p}} = \boldsymbol{m}\vec{\boldsymbol{v}}$$

in which *m* is the mass of the particle and $\vec{v}$ is its velocity.

A particle's momentum $\vec{p}$ has the same direction as its velocity $\vec{v}$.

The SI unit for momentum is the kilogram-meter per second ($kg.m/s$).

Force and Momentum:

Differentiating with respect to time gives

$$\frac{d\vec{p}}{dt} = m\frac{d\vec{v}}{dt} = m\vec{a}$$

Thus $$\vec{\boldsymbol{F}} = \frac{\boldsymbol{d}\vec{\boldsymbol{p}}}{\boldsymbol{dt}}$$

Where $\vec{F} = m\vec{a}$

Which is Newton's second law in terms of momentum.

In words, the time rate of change of the momentum of a particle is equal to the net force acting on the particle and is in the direction of that force.

9-4: Linear momentum: a single particle

$\bar{p} = m\,\bar{v}$, vector

m → +ve scalar

Same direction

SI unit: kg-m/s

Newton's 2nd law of motion: in terms of momentum

$$\bar{F}_{net} = \frac{d\bar{p}}{dt}$$

$\bar{F}_{net}$ → net external force

if $F_{net} = 0$ $\qquad \frac{d\bar{p}}{dt} = 0$

p = constant (cannot change)

$$\bar{F}_{net} = \frac{d\bar{p}}{dt} = \frac{d}{dt}(m\bar{v}) = m\,\frac{d\bar{v}}{dt}$$

$\bar{F}_{net} = m\,\bar{a}$ [Newton's 2nd law of motion for a particle]

9-5: Linear momentum: system of particles

Consider a system of n particles, each with its own mass, velocity and linear momentum.

Total linear momentum, $\bar{P} = \bar{p}_1 + \bar{p}_2 + \bar{p}_3 + \cdots + \bar{p}_n$

$$= m_1\bar{v}_1 + m_2\bar{v}_2 + m_3\bar{v}_3 + \cdots + m_n\bar{v}_n$$

$$\bar{P} = M\,\bar{v}_{com}$$

$\bar{v}_{com}$ → velocity of the center of mass

M → total mass of the system

$$\frac{d\bar{P}}{dt} = \frac{d}{dt}\left(M\bar{v}_{com}\right) = M\left(\frac{d\bar{v}_{com}}{dt}\right)$$

$$\cancel{\bar{F}_{net} = \frac{d\bar{P}}{dt}}$$

$$\bar{F}_{net} = M\,\bar{a}_{com}$$ Newton's 2nd law for a system of particles

$\bar{F}_{net}$ → net external force acting on the system

$$\bar{F}_{net} = \frac{d\bar{P}}{dt} \quad \text{(system of Particles)}$$

if $\bar{F}_{net} = 0$ (no net external force) $\quad \frac{d\bar{P}}{dt} = 0$

$\bar{P} =$ constant (cannot change)

The Linear Momentum of a System of Particles:

The linear momentum ($\vec{P}$) of a system of particles is equal to the product of the total mass *M* of the system and the velocity of the center of mass ($\vec{v}_c$).

That is $$\vec{P} = M\vec{v}_c$$

Collision and Impulse:

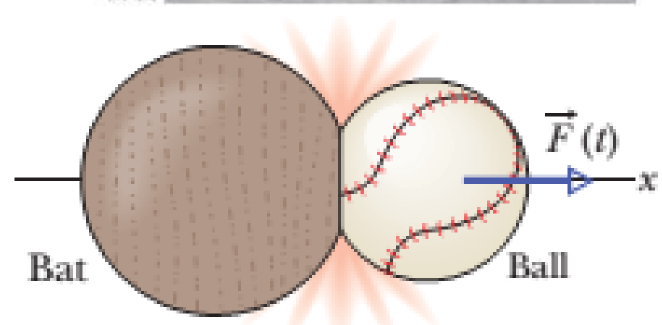
Newton's second law in terms of momentum,

$$\vec{F} = \frac{d\vec{p}}{dt}$$

In time interval dt, the change in the ball's momentum is

$$d\vec{p} = \vec{F}(t)dt$$

[Note: The ball experiences a force $\vec{F}(t)$ that varies during the collision and changes the linear momentum $\vec{p}$ of the ball.]



We can find the net change in the ball's momentum due to the collision if we integrate both sides of the equation ($d\vec{p} = \vec{F}(t)dt$) from a time t_i just before the collision to a time t_f just after the collision:

$$\int_{t_i}^{t_f} d\vec{p} = \int_{t_i}^{t_f} \vec{F}(t)dt$$

$$\vec{p}_f - \vec{p}_i = \int_{t_i}^{t_f} \vec{F}(t)dt$$

The left side of this equation gives us the change in momentum: $\Delta\vec{p} = \vec{p}_f - \vec{p}_i$. The right side, which is a measure of both the magnitude and the duration of the collision force, is called the **impulse ($\vec{J}$)** of the collision:

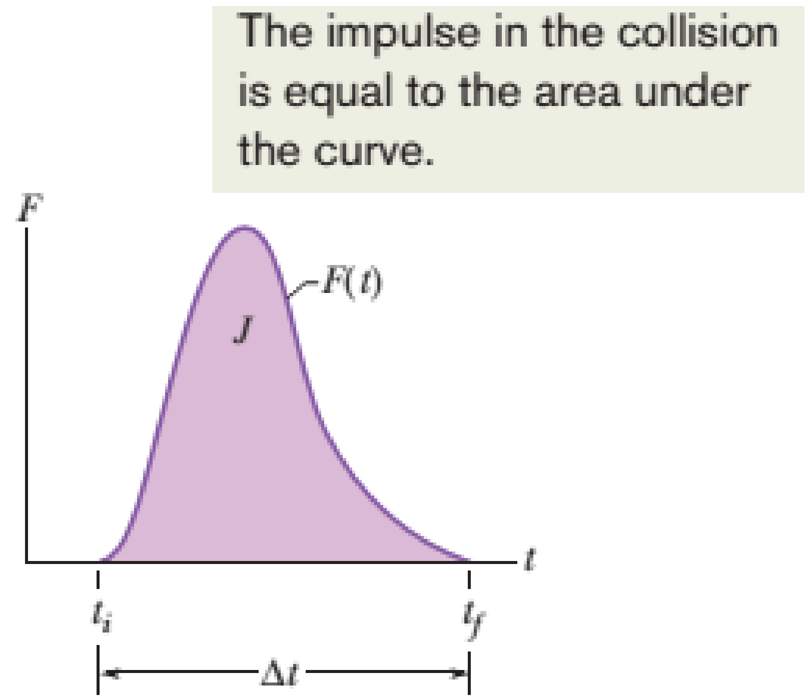
$$\vec{J} = \int_{t_i}^{t_f} \vec{F}(t)dt$$

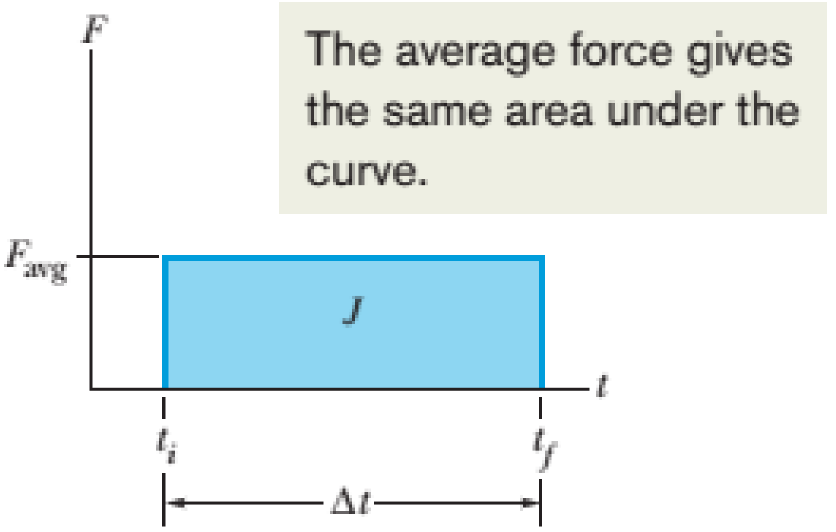
Thus, the change in an object's momentum is equal to the impulse on the object:

$$\boldsymbol{\vec{J} = \vec{p}_f - \vec{p}_i = \Delta\vec{p}}$$

If F_{avg} is the average magnitude of $\vec{F}(t)$ during the collision and Δt is the duration of the collision, then for one-dimensional motion

$$J = F_{avg}\,\Delta t$$





The law of conservation of linear momentum:

If a system is closed and isolated so that no net external force acts on it, then the linear momentum must be constant even if there are internal changes:

$$\vec{P} = constant$$

That means, $$\vec{P}_i = \vec{P}_f$$

In words, this equation says that, for a closed, isolated system,

$$\begin{pmatrix} Total\ linear\ momeentum\ at\ some \\ initial\ time\ t_i \end{pmatrix} = \begin{pmatrix} Total\ linear\ momeentum\ at\ some \\ later\ time\ t_f \end{pmatrix}$$

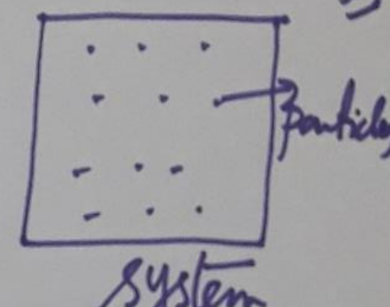
9.7: Conservation of linear momentum:

$$\bar{F}_{net} = \frac{d\bar{p}}{dt}$$

$\bar{F}_{net}=0$ [net external force acting on a system of particles is zero (the system is isolated)]

and

No particles leave or enter the system (system is closed)]



$$\frac{d\bar{p}}{dt} = 0$$

$\bar{p}$ = constant (cannot change), law of conservation of linear momentum

$\bar{p}_i = \bar{p}_f \rightarrow$ total linear momentum at some later time, t_f

$\rightarrow$ total linear momentum at some initial time, t_i

Thank You