chapter - 4

(1) $\int x^2 e^{2x}\,dx$

Here, $u = x^2$, $v = e^{2x}$

+	x^2	e^{2x}
−	$2x$	$\frac{1}{2}e^{2x}$
+	2	$\frac{1}{4}e^{2x}$
−	0	$\frac{1}{8}e^{2x}$

$$\int x^2 e^{2x}\,dx = +x^2 \cdot \frac{1}{2}e^{2x} - 2x \cdot \frac{1}{4}e^{2x} + 2 \cdot \frac{1}{8}e^{2x}$$

$$= \frac{x^2 \cdot e^{2x}}{2} - \frac{x \cdot e^{2x}}{2} + \frac{e^{2x}}{4} + C$$

$$= \frac{x^2 e^{2x} - xe^{2x}}{2} + \frac{e^{2x}}{4} + C$$

(Ans)

Scanned with CamScanner

$\boxed{2.}$ $\int x^2 \sin 2x\,dx$

Here $u = x^2$, $v = \sin 2x$

$+$	x^2	$\sin 2x$
$-$	$2x$	$-\frac{\cos 2x}{2}$
$+$	2	$-\frac{\sin 2x}{4}$
$-$	0	$\frac{\cos 2x}{8}$

$$\int x^2 \sin 2x\,dx = -\frac{x^2 \cos 2x}{2} + \frac{2x \cdot \sin 2x}{4} - \frac{2 \cdot \cos 2x}{8}$$

$$= -\frac{x^2 \cos 2x}{2} + \frac{x \sin 2x}{2} - \frac{\cos 2x}{4} + C$$

$$= -\frac{x^2 \cos(2x) + x \sin 2x}{2} + \frac{\cos 2x}{4} + C$$

(Ans)

$\boxed{3.}$ $\int x \sin(2x+1)\,dx$

Here, $u = x$, $v = \sin(2x+1)$

+	x	$\sin(2x+1)$
−	1	$-\frac{\cos(2x+1)}{2}$
+	0	$-\frac{\sin(2x+1)}{4}$

$$\int x \sin(2x+1) = -\frac{x\cos(2x+1)}{2} + \frac{\sin(2x}{4}$$

$+ c$

$\boxed{4}$ $\int_0^{\pi} (2x^2+1)\cos 2x \, dx$

$u = (2x^2+1)$, $v = \cos 2x$

+	$2x^2+1$	$\cos 2x$
−	$4x$	$\frac{1}{2} \cdot \sin 2x$
+	4	$-\cos 2x \cdot \frac{1}{4}$
−	0	$-\cos 2x \cdot \frac{1}{8}$

$$\int_0^{\pi} (2x^2+1)\cos 2x \, dx = \frac{(2x^2+1)\sin 2x}{2} + \frac{4x \cdot \cos 2x}{4} - \frac{4\cos 2x}{8} + c$$

(Ans)

Scanned with CamScanner

P - 472

1\. $\int x \sin x \, dx = x(-\cos x) - \int (-\cos x) dx$

$= -x\cos x + \int \cos x \, dx$

$= -x\cos x + \sin x + C$

(Ans)

2\. $\int \ln x \, dx$, Here, $u = \ln x$, $dv = dx$

$du = \frac{1}{x} dx$ $\therefore v = x$

$\int \ln x \, dx = x \ln x - \int x \frac{dx}{x}$

$= x \ln x - \int dx$

$= x \ln x - x + C$ (Ans)

$\boxed{3}$ $\int t^2 e^t dt$

Here, $u = t^2$, $dv = e^t dt$

$du = 2t dt$ $\quad v = e^t$

$$\int t^2 e^t dt = t^2 e^t - 2\int te^t dt$$

$$= t^2 e^t - 2(te^t - e^t + c)$$

$$= t^2 e^t - 2te^t + 2e^t - 2c$$

(Ans)

$\boxed{4}$ $\int e^x \sin x dx$

$$= -e^x \cos x + e^x \sin x - \int e^x \sin x dx$$

$$= \frac{1}{2} e^x (\sin x - \cos x) + c$$

(Ans)

P. 976

$\boxed{5.}$ $\int_0^1 \tan^{-1}x\,dx$

$u = \tan^{-1}x$, $dv = dx$

$du = \dfrac{dx}{1+x^2}$ $v = x$

$$\int_0^1 \tan^{-1}x\,dx = x\tan^{-1}x\Big|_0^1 - \int_0^1 \frac{x}{1+x^2}\,dx$$

$$= 1\cdot\tan^{-1}1 - 0\cdot\tan^{-1}0 - \int_0^1 \frac{x}{1+x^2}\,dx$$

$$= \frac{\pi}{4} - \int_0^1 \frac{x}{1+x^2}\,dx$$

$$= \frac{\pi}{4} - \frac{1}{2}\int_1^2 \frac{dt}{t}$$

$$= \frac{\pi}{4} - \frac{1}{2}\ln\left[t\right]_1^2$$

$$= \frac{\pi}{4} - \cancel{\int_0^1} \frac{1}{2}(\ln 2 - \ln 1)$$

$$= \frac{\pi}{4} - \frac{\ln 2}{2}$$

(Ans)

$t = 1+x^2$

$dt = 2x\,dx$

$x\,dx = \frac{1}{2}dt$

$\therefore x=0,\ t=1$

$\therefore x=1,\ t=2$