2.1

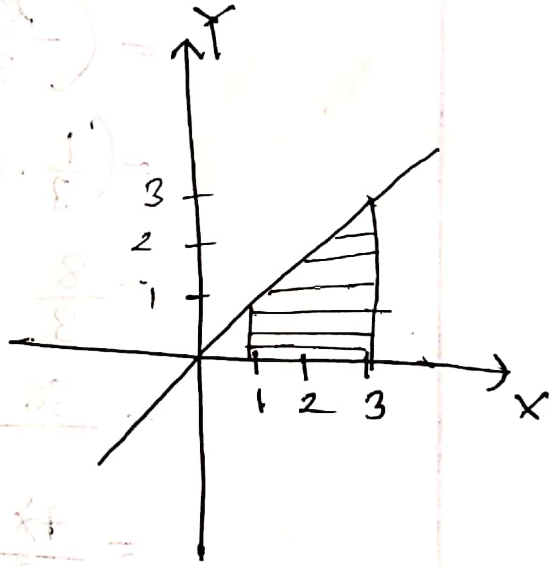
1. $\boxed{a)}$ Here, $y = x$ and $y = 0$ (x-axis) $(1 \le x \le 3)$

So, Area, $A = \int_1^3 (x-0)\,dx$

$= \left[\frac{x^2}{2}\right]_1^3$

$= \frac{3^2}{2} - \frac{1^2}{2}$

$= 4$ (Ans)



$\boxed{b)}$ Here, $y = x^3$ and $y = 0$ (x-axis) $(1 \le x \le 2)$

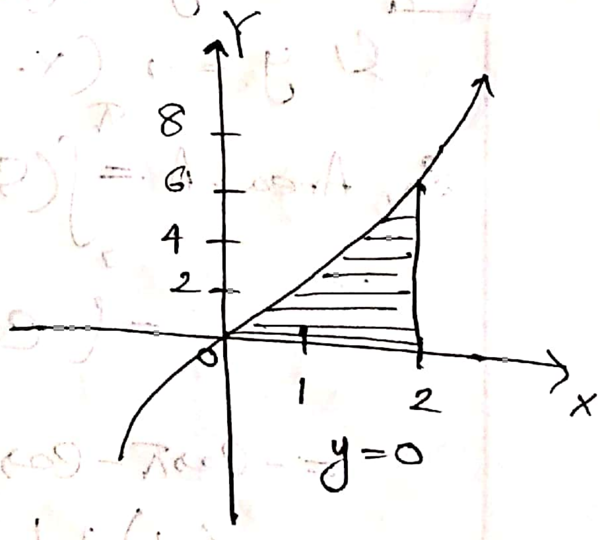
So, Area, $A = \int_1^2 (x^3 - 0)\,dx$

$= \left(\frac{x^4}{4}\right)\Big|_1^2$

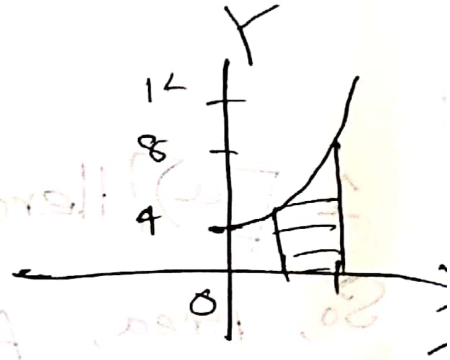
$= \frac{2^4}{4} - \frac{1^4}{4}$

$= \frac{1}{4}(16-1)$

$= \frac{15}{4}$ (Ans)



c) Here, $y = f(x) = x^2 + x + 4, \quad 1 \le x \le 2$



Area, $A = \int_1^2 (x^2 + x + 4)\, dx$

$= \left(\frac{x^3}{3} + \frac{x^2}{2} + 4x\right)\Big]_1^2$

$= \left(\frac{1}{3} \cdot 2^3 + \frac{2^2}{2} + 4.2\right) - \left(\frac{1}{3} \cdot 1^3 + \frac{1^2}{2} + 4.1\right)$

$= \frac{8}{3} + 10 - \frac{1}{3} - \frac{1}{2} - 4$

$= \frac{36 + 16 - 2 - 3}{6}$

$= \frac{47}{6}$ (Ans)

d) Here, $y = f(x) = \sin x, \quad 0 \le x \le \frac{3\pi}{2}$
& $y = 0$ (x-axis)

So, Area. $A = \int_0^{\pi} (\sin x - 0) + \int_{\pi}^{3\pi/2} (0 - \sin x)\, dx$

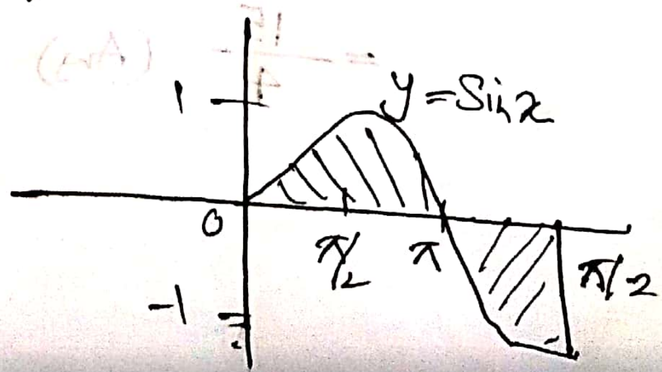
$= (-\cos x)\Big|_0^{\pi} - (-\cos x)\Big|_{\pi}^{3\pi/2}$

$= -\cos\pi - \cos 0 + \cos 3\pi/2 - \cos\pi$

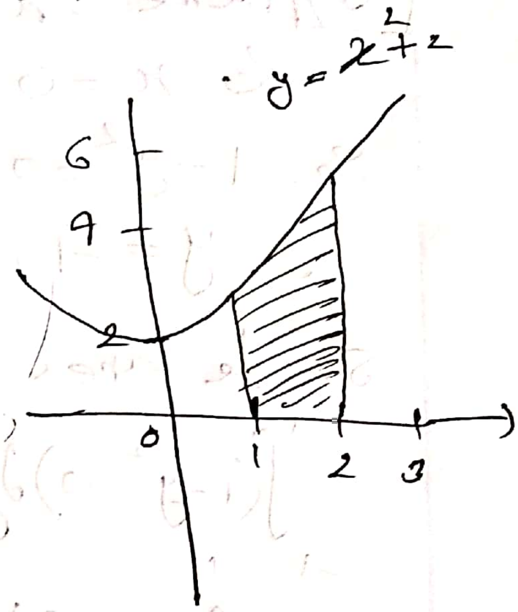
$= -(-1) + 1 + 0 - (-1)$

$= 3$

(Ans)



e) Here, $y = x^2 + 2$ and $x = 1$ and $x = 2$:



So, Area, $A = \int_1^2 (x^2 + 2)\,dx$

$= \left(\frac{x^3}{3} + 2x\right)\Big|_1^2$

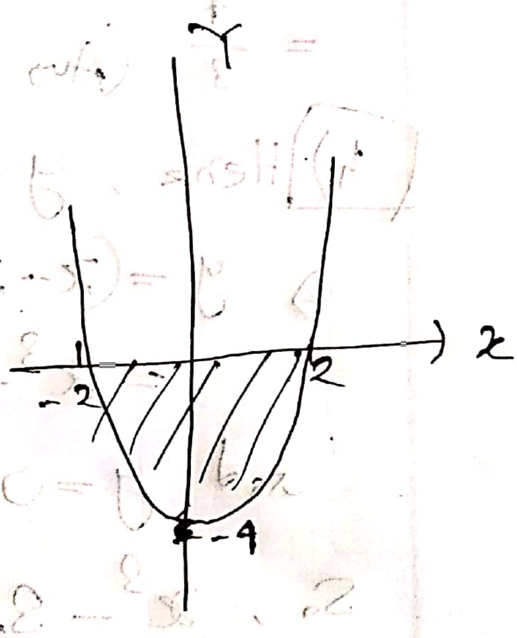
$= \left(\frac{2^3}{3} + 2.2 - \frac{1^3}{3} - 2.1\right)$

$= 2 + \frac{8}{3} - \frac{1}{3}$

$= \frac{6 + 8 - 1}{3}$

$= \frac{13}{3}$ (Ans)

f) Here, $y = x^2 - 4$ and x axis ($y = 0$) i.e. $y = 0$



So, $x^2 - 4 = 0$

$\therefore x = +2, -2$

So, Area, $A = \int_{-2}^{2} [0 - (x^2 - 4)]\,dx$

$= \int_{-2}^{2} (-x^2 + 4)\,dx$

$= 2\int_0^2 (-x^2 + 4)\,dx$

$= 2\left(-\frac{x^3}{3} + 4x\right)\Big|_0^2$

$= 2\left(-\frac{2^3}{3} + 4.2\right)$

$= 2.\ \frac{-8 + 24}{3}$

$= \frac{32}{3}$ (Ans)

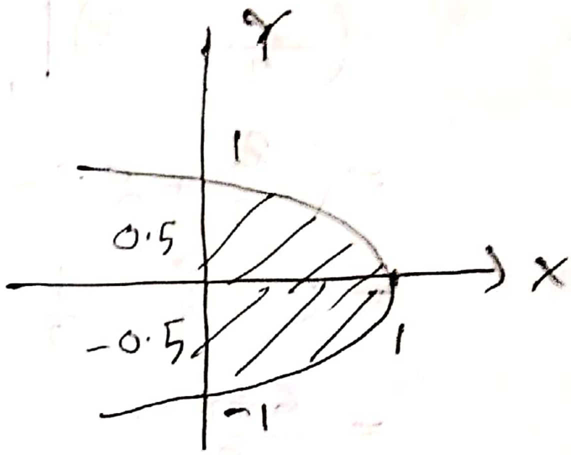
g) Here, $x = 1-y^2$ and y axis $(x=0)$

& $x = 0$

So, $1-y^2 = 0$

$\therefore y = -1, +1$

So, the area is, A



$$\int_{-1}^{1}(1-y^2-0)\,dy$$

$$= 2\int_0^1 (1-y^2)\,dy$$

$$= 2\left(y-\frac{y^3}{3}\right)\Big|_0^1$$

$$= \frac{4}{3} \text{ (Ans)}$$

h) Here, $y = f(x) = x(1-x)(2-x)$ and x axis

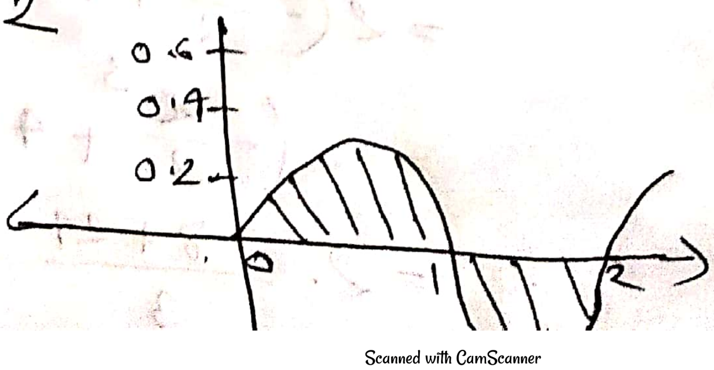
$\Rightarrow y = (x-x^2)(2-x)$

$= x^3 - 3x + 2x$

and $y = 0$

So, $x^3 - 3x^2 + 2x = 0$

$\therefore x = 0, 1, 2$



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$$\text{So, Area } A = \int_0^1 [x^3 - 3x^2 + 2x) - 0]\,dx + \int_1^2 [0 - (x^3 - 3x^2 + 2x)]\,dx$$

$$= \left(\frac{x^4}{4} - 3\cdot\frac{x^3}{3} + 2\cdot\frac{x^2}{2}\right)\Big|_0^1 + \left(-\frac{x^4}{4} + 3\cdot\frac{x^3}{3} - 2\cdot\frac{x^2}{2}\right)\Big|_0^2$$

$$= \frac{1}{4} - 1 + 1 + \left(-\frac{2^4}{4} + 2^3 - 2^2\right) - \left(-\frac{1}{4} + 1 - 1\right)$$

$$= \frac{1}{4} - 4 + 8 - 4 + \frac{1}{4}$$

$$= \frac{1}{2} \quad \text{(Ans)}$$