LESSON 5

BOOK CHAPTER 24

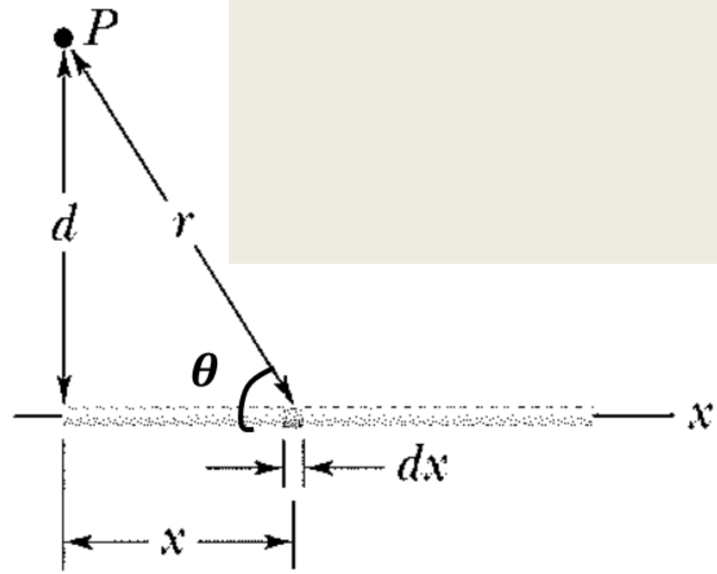
ELECTRIC POTENTIAL

Potential due to a Line of Charge:

A thin non-conducting rod of length *L* has a positive charge of uniform linear density *λ*. Let us determine the electric potential *V* due to the rod at point *P*, a perpendicular distance *d* from the left end of the rod.

Potential due to a Line of Charge

We consider a differential element *dx* of the rod as shown in Fig. This element of the rod has a differential charge *dq*.



Linear charge density, $\lambda = \frac{dq}{dx}$

$$dq = \lambda\, dx$$

$$r = \sqrt{d^2 + x^2}$$

Treating the element as a point charge, the potential dV at point P is

$$dV = \frac{1}{4\pi\epsilon_0}\frac{dq}{r} = \frac{1}{4\pi\epsilon_0}\frac{\lambda\, dx}{(d^2+x^2)^{1/2}}$$

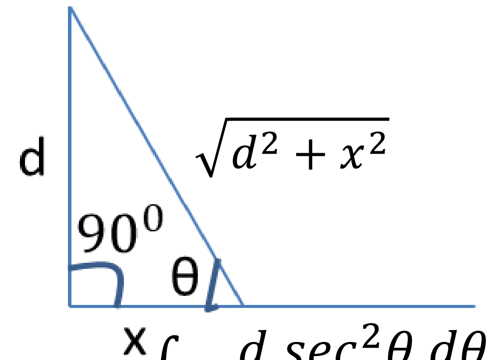
The total potential *V* produced by the rod at point *P* by integrating the above equation along the length of the rod, from *x* = 0 to *x* = *L*

$$V = \int dV = \int_{x=0}^{x=L} \frac{1}{4\pi\epsilon_0}\frac{\lambda\, dx}{(d^2+x^2)^{1/2}} = \frac{\lambda}{4\pi\epsilon_0}\int_0^L \frac{dx}{(d^2+x^2)^{1/2}}$$

Let $\tan\theta = \frac{x}{d}$

$$x = d\tan\theta$$

$$dx = d\ sec^2\theta\ d\theta$$



$$\int \frac{dx}{(d^2+x^2)^{1/2}} = \int \frac{d\ sec^2\theta\ d\theta}{(d^2+d^2tan^2\theta)^{1/2}} = \int \frac{d\ sec^2\theta\ d\theta}{\{d^2(1+tan^2\theta)\}^{1/2}} = \int \frac{d\ sec^2\theta\ d\theta}{\{d^2(sec^2\theta)\}^{1/2}}$$

$$= \int \frac{d\ sec^2\theta\ d\theta}{d\sec\theta} = \int sec\theta\ d\theta = \ln|sec\ \theta + \tan\theta| = \ln\left|\frac{1}{cos\ \theta} + \tan\theta\right|$$

$$= \ln\left|\frac{1}{\frac{d}{\sqrt{d^2+x^2}}} + \frac{x}{d}\right| = \ln\left|\frac{x+\sqrt{d^2+x^2}}{d}\right|$$

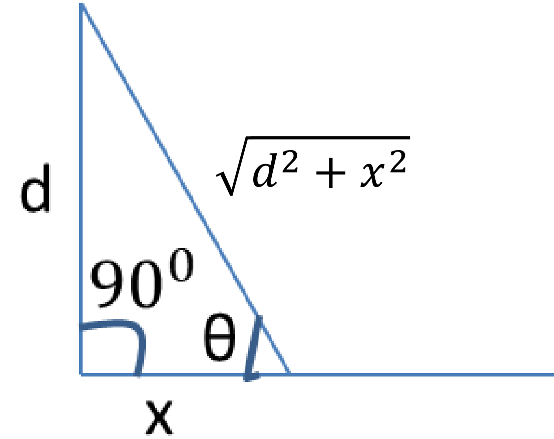
$$V = \frac{\lambda}{4\pi\epsilon_0}\int_{x=0}^{x=L} \frac{dx}{(d^2+x^2)^{1/2}} = \frac{\lambda}{4\pi\varepsilon_0}[\ln\left|\frac{x+\sqrt{d^2+x^2}}{d}\right|]_0^L$$

$$= \frac{\lambda}{4\pi\varepsilon_0}[\ln\left|\frac{L+\sqrt{d^2+L^2}}{d}\right| - ln\left|\frac{0+\sqrt{d^2+0^2}}{d}\right|]$$

$$= \frac{\lambda}{4\pi\varepsilon_0}[\ln\left|\frac{L+\sqrt{d^2+L^2}}{d}\right| - ln1] = \frac{\lambda}{4\pi\varepsilon_0}[\ln\left|\frac{L+\sqrt{d^2+L^2}}{d}\right| - 0]$$

$$\boxed{V = \frac{\lambda}{4\pi\varepsilon_0}\ln\left|\frac{L+\sqrt{d^2+L^2}}{d}\right|}$$

Let $\tan\theta = \frac{d}{x}$ $\quad and \;\; x = d\cot\theta$



$$dx = -d\ cosec^2\theta\ d\theta$$

$$\int \frac{dx}{(d^2 + x^2)^{1/2}} = \int \frac{-d\ cosec^2\theta\ \ d\theta}{(d^2 + d^2 cot^2\theta)^{1/2}} = -\int cosec\theta\ d\theta = \ln|cosec\ \theta + \cot\theta|$$

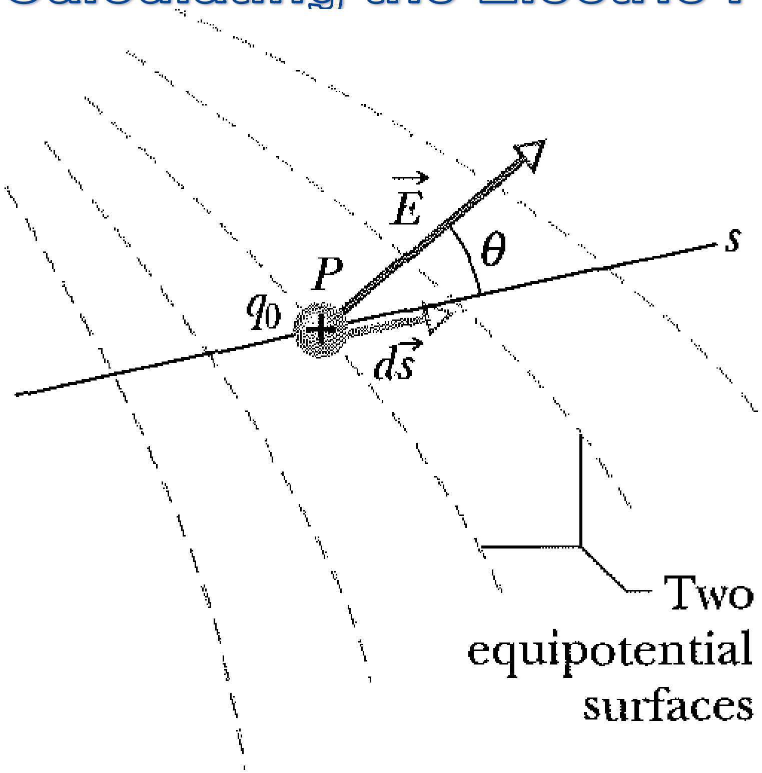
$$= \ln\left|\frac{\sqrt{d^2 + x^2}}{d} + \frac{x}{d}\right|$$

Now

$$V = \frac{\lambda}{4\pi\epsilon_0}\int_{x=0}^{x=L} \frac{dx}{(d^2 + x^2)^{1/2}} = \frac{\lambda}{4\pi\varepsilon_0}[\ln\left|\frac{\sqrt{d^2 + L^2}}{d} + \frac{L}{d}\right| - \ln\frac{d}{d}]$$

$$\boxed{V = \frac{\lambda}{4\pi\varepsilon_0}\ln\left|\frac{\sqrt{d^2 + L^2} + L}{d}\right|}$$

Calculating the Electric Field from the Electric Potential:



Suppose that a positive test charge q_0 moves through a displacement $\mathrm{d}\vec{s}$ from one equipotential surface to the adjacent surface.

The differential work done in terms of electric potential difference dV is

$dW = -q_0 \mathrm{dV}$ [external agent does not help]

The differential work done by the electric Field $\vec{E}$ is

$$dW = \vec{F}.d\vec{s} = q_0\vec{E}.d\vec{s} = q_0E(cos\theta)\,ds$$

Hence, we get $$-q_0dV = q_0E(cos\theta)\,ds$$

$$E(cos\theta) = -\frac{dV}{ds}$$

Since $E\cos\theta$ is the component of $\vec{E}$ in the direction of $\mathrm{d}\vec{s}$, then $E_s = -\dfrac{\partial V}{\partial s}$

If we take the s axis to be, in turn, the X, *y*, and z axes, we find that the X, *y*, and z components of $\vec{E}$ at any point are

$$E_x = -\frac{\partial V}{\partial x}\;; \qquad E_y = -\frac{\partial V}{\partial y}\;; \qquad E_z = -\frac{\partial V}{\partial z}$$

Thank You