LECTURE 9

BOOK CHAPTER 9

INELASTIC AND ELASTIC COLLISIONS

9-8: Momentum and Kinetic energy in Collisions:
The system is closed and isolated.

$\bar{F}_{net} = \frac{d\bar{p}}{dt}$ $\frac{d\bar{p}}{dt} = 0$

$\bar{p} =$ constant

Elastic collision: $P_i = P_f$ and $K_i = K_f$

Inelastic collision: $P_i = P_f$ but $K_i \neq K_f$

KE loss due to sound and heat etc

Completely inelastic collision: bodies stick together (greatest loss occurs)

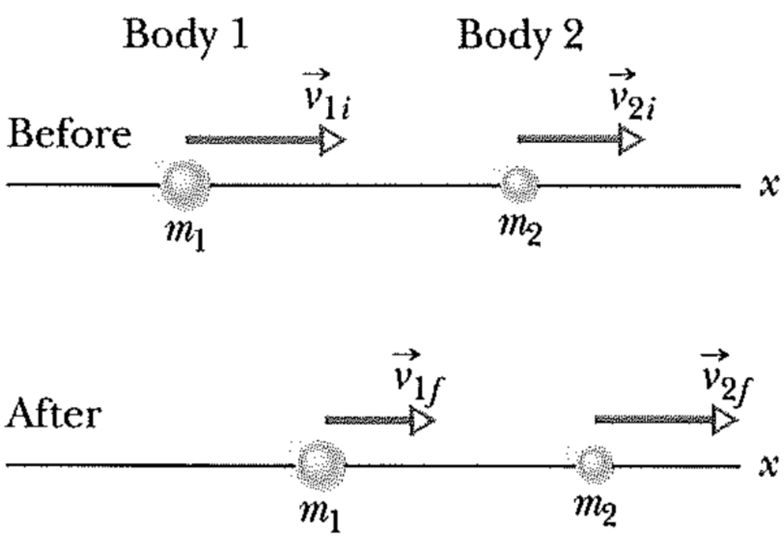
$\Downarrow$ ie

Wet putty ball + bat

Inelastic Collisions in One Dimension:

In an *inelastic collision* of two bodies, the kinetic energy of the two-body system is not conserved. If the system is closed and isolated (net external force is zero), the total linear momentum of the system *must* be conserved.

If the motion of the bodies is along a single axis, the collision is one-dimensional (as shown in the figure).



One-Dimensional Inelastic Collision

According to the principle of conservation of linear momentum, we can write from the figure

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

If the target is stationary, that is $v_{2i} = 0$, then

$$m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f}$$

9-9: Inelastic collision: one dimension

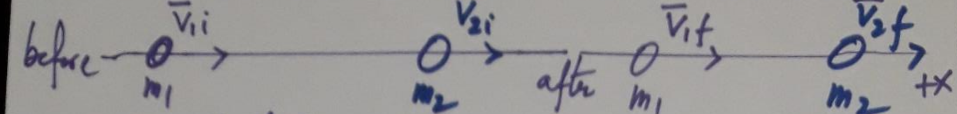


Fig: inelastic collision

Two bodes form the system (closed and isolated)

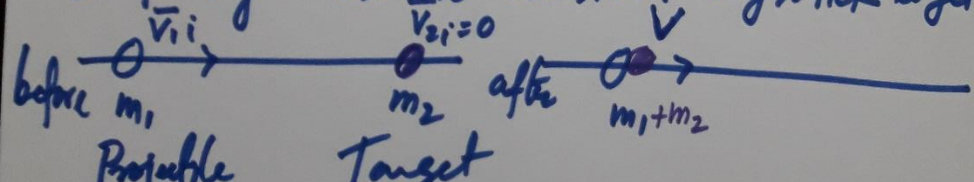
law of conservation of linear momentum for the two-body system:

$$\bar{P}_i = \bar{P}_f$$

$$\bar{P}_{1i} + \bar{P}_{2i} = \bar{P}_{1f} + \bar{P}_{2f}$$

$$m_1 V_{1i} + m_2 V_{2i} = m_1 V_{1f} + m_2 V_{2f}$$ [indicating direction with signs]

Completely inelastic collision: They stick together



$$\bar{P}_i = \bar{P}_f$$

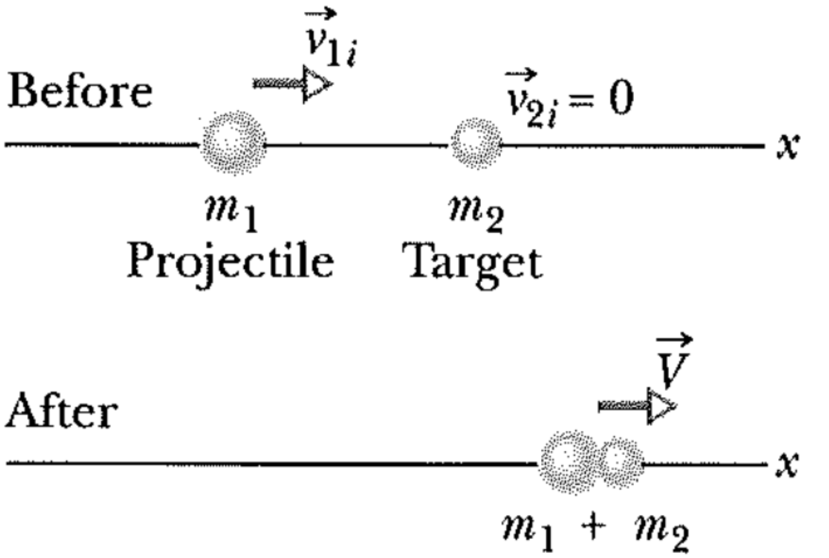
$$m_1 V_{1i} + m_2 V_{2i} = (m_1 + m_2) V$$

$$m_1 V_{1i} + 0 = (m_1 + m_2) V$$

$$V = \frac{m_1 V_{1i}}{m_1 + m_2} \qquad \Big| \; V < V_{1i}$$

Completely Inelastic Collisions in One Dimension:

If the bodies stick together, the collision is a completely inelastic collision, and the bodies have the same final velocity V (because they are stuck together).



One-Dimensional completely Inelastic Collision

According to the principle of conservation of linear momentum, we can write from the figure

$$m_1 v_{1i} = (m_1 + m_2)V$$

We can write

$$V = \frac{m_1 v_{1i}}{m_1 + m_2}$$

Velocity of the Center of Mass:

For a system of particles on which the net external force is zero, so that the total momentum is constant, the velocity of the center of mass (v_c) is also constant; which can not be changed by the collision.

We know from LESSON 8

$$v_c = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2} = \frac{P}{M}$$

Where, total momentum $P = m_1\, v_{1i} + m_2 v_{2i}$ and $M = m_1 + m_2$

Since $\frac{P}{M}$ is constant, $v_c\ is\ constant$.

9-9: Inelastic collision:

Velocity of the center of mass:

$$\bar{P} = \bar{P}_1 + \bar{P}_2 + \bar{P}_3 + \cdots + \bar{P}_n$$

$$= m_1\bar{v}_1 + m_2\bar{v}_2 + m_3\bar{v}_3 + \cdots + m_n\bar{v}_n$$

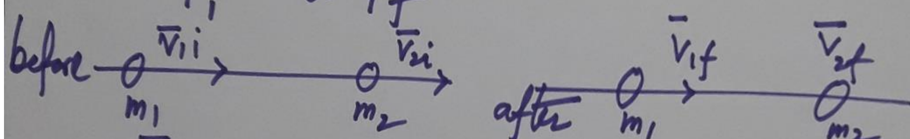
$$\bar{P} = M\,\bar{v}_{com}$$

Total linear momentum $\bar{p}$ of the two body system,

$$\bar{P} = (m_1 + m_2)\,\bar{v}_{com}$$

$\bar{p}$ is conserved during the collision,

P_i or P_f



$\bar{P}_i = m_1\bar{v}_{1i} + m_2\bar{v}_{2i}$ or, $\bar{P}_f = m_1\bar{v}_{1f} + m_2\bar{v}_{2f}$

$$\bar{P}_i = (m_1 + m_2)\,\bar{v}_{com}$$

$$\underset{\text{Constant}}{\bar{v}_{com}} = \frac{\bar{P}_i}{(m_1 + m_2)} = \underbrace{\frac{m_1\bar{v}_{1i} + m_2\bar{v}_{2i}}{(m_1 + m_2)}}_{\text{constant}}$$

or, $\bar{P}_f = (m_1 + m_2)\,\bar{v}_{com}$

$$\bar{v}_{com} = \frac{\bar{P}_f}{m_1 + m_2} = \frac{m_1\bar{v}_{1f} + m_2\bar{v}_{2f}}{m_1 + m_2}$$

also constant

Constant

Continue: velocity of the center of mass

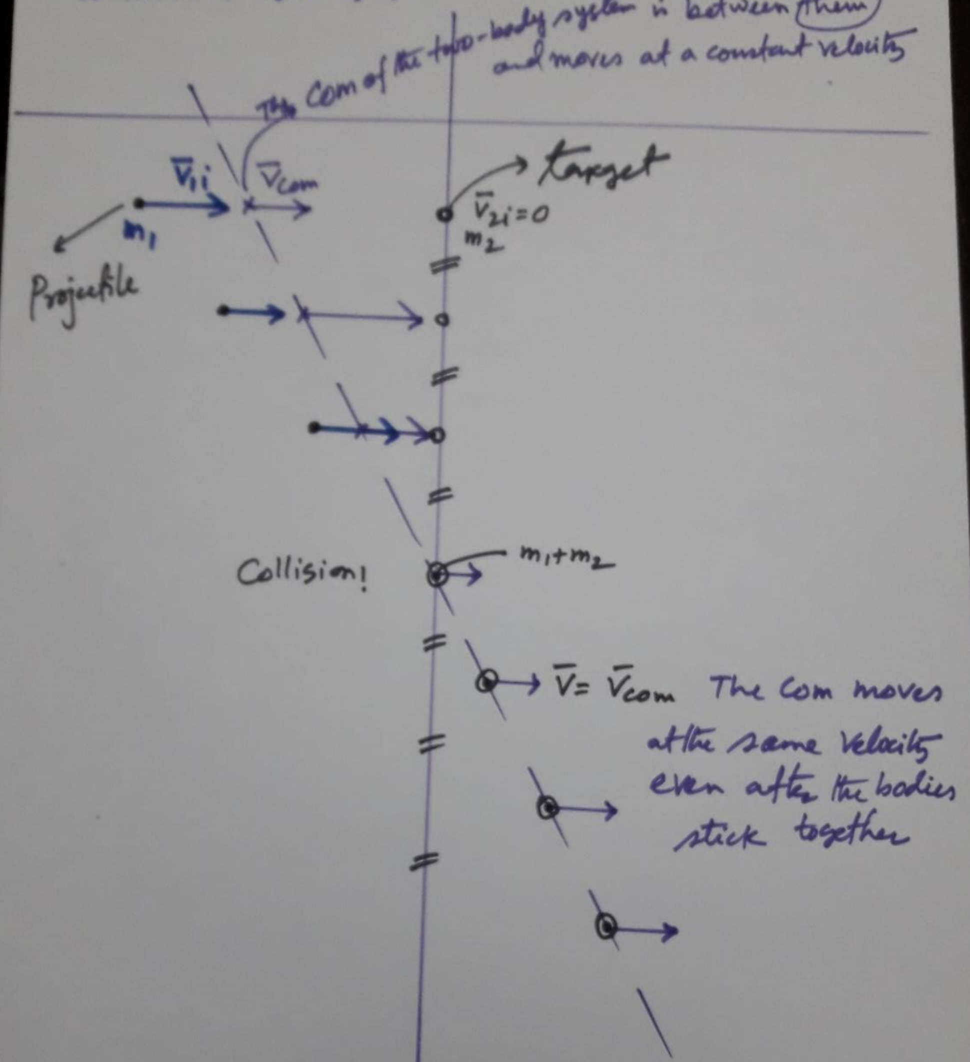


Fig 9-16: for Completely inelastic collision the system's com is shown (v_{com} is unaffected by the collision)

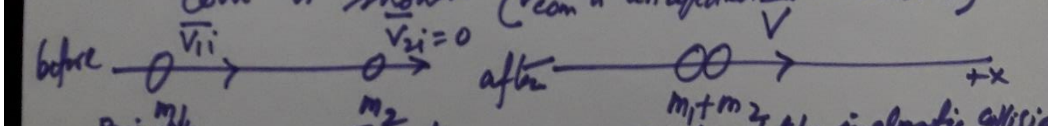


Fig: Completely inelastic collision

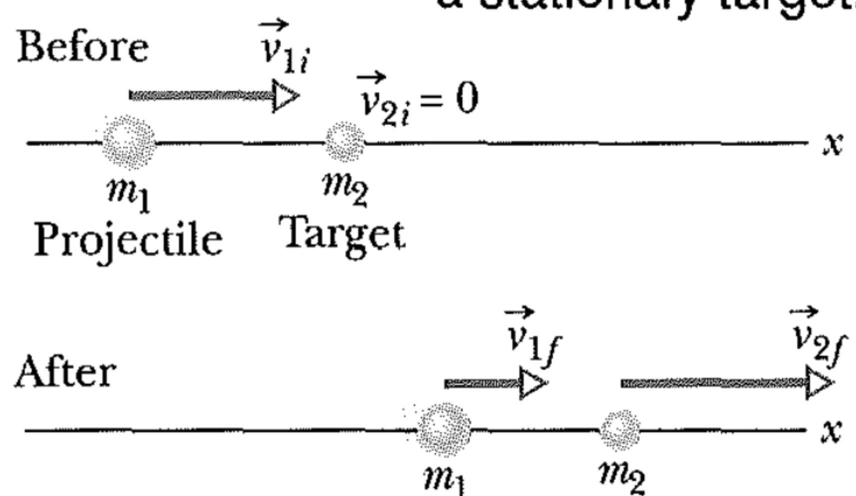
Elastic Collisions in One Dimension:

An *elastic collision* is a special type of collision in which the **kinetic energy of a system of colliding bodies is conserved**. If the system is closed and isolated, its **linear momentum is also conserved**. In an elastic collision, the kinetic energy of each colliding body may change, but the total kinetic energy of the system does not change.

For a one-dimensional collision (as shown in the figure) in which body 2 is a target and body 1 is an incoming projectile, conservation of kinetic energy and linear momentum yield the following expressions:

Total kinetic energy is conserved:

a stationary target.



$$\frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2$$

Since the target is stationary , $v_{2i} = 0$

$$\frac{1}{2}m_1v_{1i}^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2$$

$$m_1v_{1i}^2 = m_1v_{1f}^2 + m_2v_{2f}^2 \qquad \text{.................} \ (1)$$

Total linear momentum is conserved:

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

Since the target is stationary , $v_{2i} = 0$

$$m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f} \quad \text{............ (2)}$$

$$m_1 v_{1i} - m_1 v_{1f} = m_2 v_{2f}$$

$$m_1 (v_{1i} - v_{1f}) = m_2 v_{2f} \quad \text{............ (3)}$$

From equation (1), we can write

$$m_1 v_{1i}^2 - m_1 v_{1f}^2 = m_2 v_{2f}^2$$

$$m_1 (v_{1i}^2 - v_{1f}^2) = m_2 v_{2f}^2$$

$$m_1 (v_{1i} + v_{1f})(v_{1i} - v_{1f}) = m_2 v_{2f}^2$$

.... (4)

Dividing eq^n. (4) by eq^n. (3)

$$\frac{m_1 (v_{1i} + v_{1f})(v_{1i} - v_{1f})}{m_1 (v_{1i} - v_{1f})} = \frac{m_2 v_{2f}^2}{m_2 v_{2f}}$$

$$v_{1i} + v_{1f} = v_{2f} \quad \text{............ (5)}$$

From equations (3) and (5) we get

$$m_1 (v_{1i} - v_{1f}) = m_2 (v_{1i} + v_{1f})$$

$$m_1 v_{1i} - m_1 v_{1f} = m_2 v_{1i} + m_2 v_{1f}$$

$$m_1 v_{1i} - m_2 v_{1i} = m_1 v_{1f} + m_2 v_{1f}$$

$$v_{1f} (m_1 + m_2) = v_{1i} (m_1 - m_2)$$

$$\boldsymbol{v_{1f} = \frac{(m_1 - m_2) v_{1i}}{m_1 + m_2}} \quad \text{........ (6)}$$

From equations (4) and (5) we get

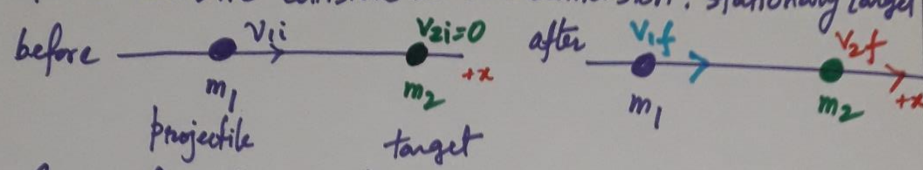
$$v_{2f} = v_{1i} + \frac{(m_1 - m_2)v_{1i}}{m_1 + m_2}$$

$$v_{2f} = \frac{v_{1i}(m_1 + m_2) + (m_1 - m_2)v_{1i}}{m_1 + m_2}$$

$$v_{2f} = \frac{v_{1i}m_1 + v_{1i}m_2 + v_{1i}m_1 - v_{1i}m_2}{m_1 + m_2}$$

$$\boldsymbol{v_{2f} = \frac{2m_1 v_{1i}}{m_1 + m_2}}$$

9:10 Elastic collisions in one dimension: stationary target



Conservation of momentum,

$$p_i = p_f$$

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \quad \cdots$$

$$m_1 v_{1i} + m_2 (0) = m_1 v_{1f} + m_2 v_{2f}$$

$$m_1 v_{1i} - m_1 v_{1f} = m_2 v_{2f} \quad \text{——} \; (1)$$

$$m_1 (v_{1i} - v_{1f}) = m_2 v_{2f} \quad \cdots \; (2)$$

Conservation of kinetic energy,

$$K_i = K_f$$

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 (0)^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

$$\frac{1}{2} m_1 v_{1i}^2 - \frac{1}{2} m_1 v_{1f}^2 = \frac{1}{2} m_2 v_{2f}^2$$

$$\frac{1}{2} m_1 (v_{1i}^2 - v_{1f}^2) = \frac{1}{2} m_2 v_{2f}^2$$

$$m_1 (v_{1i} + v_{1f})(v_{1i} - v_{1f}) = m_2 v_{2f}^2 \quad \text{——} \; (3)$$

(3) ÷ (2) $\quad v_{1i} + v_{1f} = v_{2f} \quad \cdots \; (3)$

From (1) $\quad m_1 v_{1i} - m_1 v_{1f} = m_2 (v_{1i} + v_{1f}) \quad$ [using (3)]

$$m_1 v_{1i} - m_1 v_{1f} = m_2 v_{1i} + m_2 v_{1f}$$

q-10: continue

$$m_1 v_{1i} - m_2 v_{1i} = m_1 v_{1f} + m_2 v_{1f}$$

$$(m_1 - m_2) v_{1i} = (m_1 + m_2) v_{1f}$$

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i}$$

From 3

$$v_{1i} + \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} = v_{2f}$$

$$\left(1 + \frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} = v_{2f}$$

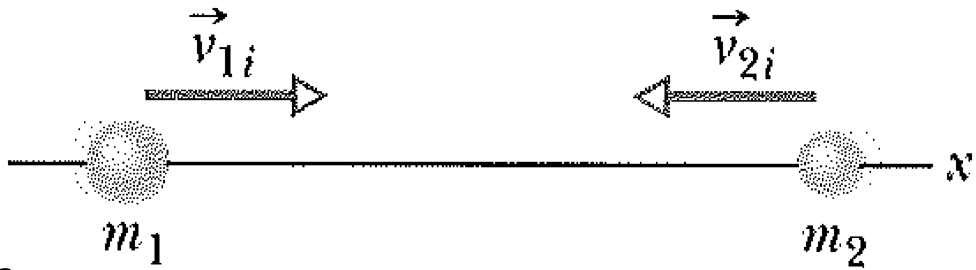
$$\left(\frac{m_1 + \cancel{m_2} + m_1 - \cancel{m_2}}{m_1 + m_2}\right) v_{1i} = v_{2f}$$

$$\left(\frac{2m_1}{m_1 + m_2}\right) v_{1i} = v_{2f}$$

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right) v_{1i}$$

Elastic Collisions in One Dimension (for a moving target):

For a one-dimensional collision (as shown in the figure) in which both bodies are moving before they undergo an elastic collision. Conservation of kinetic energy and linear momentum yield the following expressions:



Total kinetic energy is conserved:

$$\frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2$$

Total linear momentum is conserved:

$$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$$

$$v_{1f} = \frac{(m_1 - m_2)v_{1i}}{m_1 + m_2} + \frac{2m_2v_{2i}}{m_1 + m_2}$$

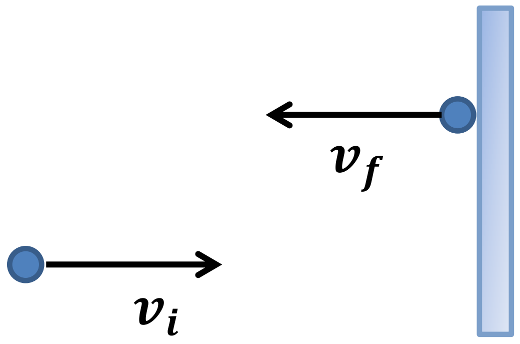
$$v_{2f} = \frac{2m_1v_{1i}}{m_1 + m_2} + \frac{(m_2 - m_1)v_{2i}}{m_1 + m_2}$$

Home Work

Problem 18 (Book chapter 9):

A 0.70 kg ball moving horizontally at 5.0 m/s strikes a vertical wall and rebounds with speed 2.0 m/s. What is the magnitude of the change in its linear momentum?

Answer:



Change in linear momentum:

$$\Delta\vec{p} = \vec{p}_f - \vec{p}_i$$

$$\Delta\vec{p} = m\vec{v}_f - m\vec{v}_i = m(\vec{v}_f - \vec{v}_i)$$

$$\Delta\vec{p} = 0.70[2(-\hat{\imath}) - 5(+\hat{\imath})] = 0.70(-7\hat{\imath}) = -4.9\hat{\imath}$$

Here

$v_i = 5\ m/s$

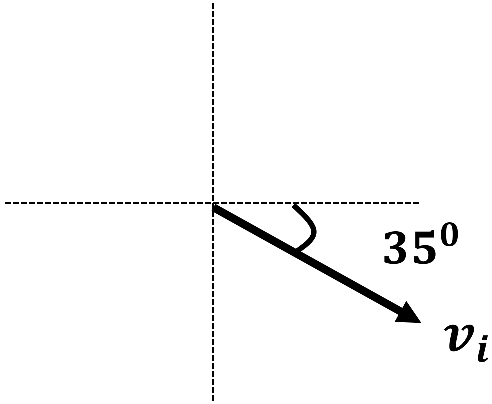
$v_f = 2\ m/s$

Therefore, magnitude of $\Delta\vec{p}$ is

$$\Delta p = 4.9\ kg - \mathrm{m/s}$$

Problem 21 (Book chapter 9):home work

A 0.30 kg softball has a velocity of 15 m/s at an angle of 35^0 below the horizontal just before making contact with the bat. What is the magnitude of the change in momentum of the ball while in contact with the bat if the ball leaves with a velocity of (a) 20 m/s, vertically downward, and (b) 20 m/s, horizontally back toward the pitcher?



Answer:

(a) Change in momentum along x-axis:

$$\Delta p_x = p_f - p_i = 0 - mv_i \cos 35^0 = -(0.30)(15)(0.8191)$$

$$\Delta p_x = -3.686\ kg.m/s$$

Change in momentum along y-axis:

$$\Delta p_y = p_f - p_i = -mv_f - (-mv_i \sin 35^0) = -(0.30)(20) + (0.30)(15)(0.5736)$$

$$\Delta p_y = -6.0 + 2.5812 = -3.4188\ kg.m/s$$

Therefore, the net change in momentum:

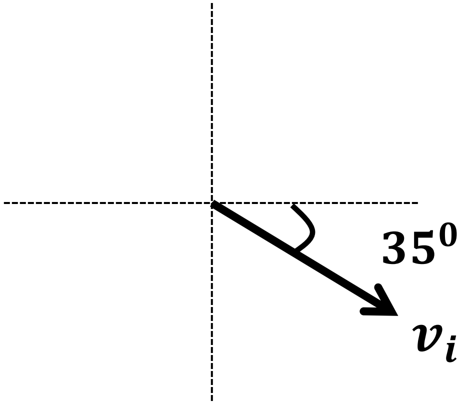
$$\Delta\vec{p} = \Delta p_x \hat{\imath} + \Delta p_y \hat{\jmath} = -3.686\ \hat{\imath} - 3.4188\ \hat{\jmath}$$

Magnitude of $\Delta\vec{p}\ is$

$$\Delta p = \sqrt{(-3.686)^2 + (-3.4188)^2}$$

$$\Delta p = 5.027\ kg.m/s$$

(a) Change in momentum along x-axis:



$$\Delta p_x = p_f - p_i = -mv_f - mv_i \cos 35^0$$

$$\Delta p_x = -(0.30)(20) - (0.30)(15)(0.8191) = -6 - 3.686$$

$$\Delta p_x = -9.686\ kg.m/s$$

Change in momentum along y-axis:

$$\Delta p_y = p_f - p_i = 0 - (-mv_i \sin 35^0) = (0.30)(15)(0.5736) = 2.5812\ kg.m/s$$

Therefore, the net change in momentum:

$$\Delta \vec{p} = \Delta p_x \hat{\imath} + \Delta p_y \hat{\jmath} = -9.686\ \hat{\imath} + 2.5812\ \hat{\jmath}$$

Magnitude of $\Delta \vec{p}\ is$

$$\Delta p = \sqrt{(-9.686)^2 + (2.5812)^2}$$

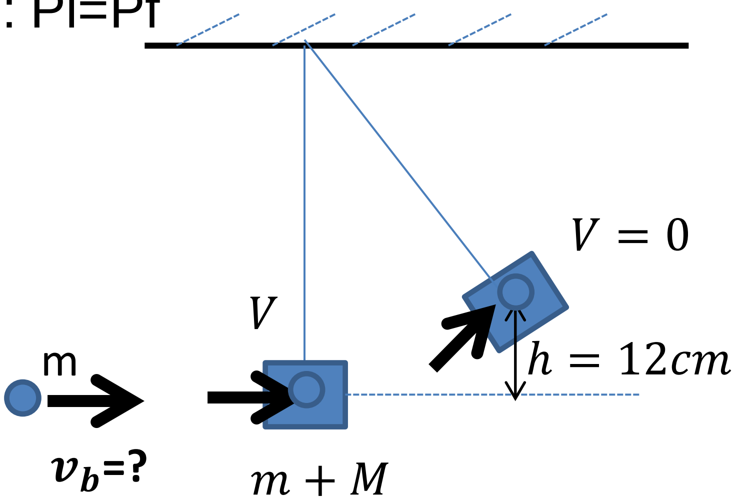
$$\Delta p = 10.024\ kg.m/s$$

Problem 49 (Book chapter 9):

A bullet of mass 10 g strikes a ballistic pendulum of mass 2.0 kg. The center of mass of the pendulum rises a vertical distance of 12 cm. Assuming that the bullet remains embedded in the pendulum, calculate the bullet's initial speed.

Answer:

Momentum is conserved throughout the process: Pi=Pf



$$mv_b + M(0) = (m + M)V$$

$$v_b = \frac{(m + M)V}{m} = \frac{(0.010 + 2)V}{0.010} = 201V$$

After collision,

conservation of mechanical energy: Ei=Ef

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}(m + M)V^2 + (m + M)g(0) = \frac{1}{2}(m + M)(0^2) + (m + M)gh$$

$$\frac{1}{2}(m + M)V^2 = (m + M)gh$$

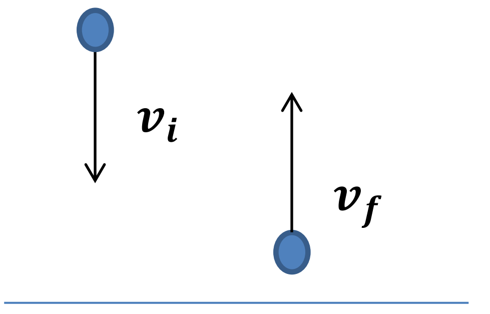
$$\frac{1}{2}V^2 = gh$$

$$V = \sqrt{2gh} = \sqrt{2(9.8)(0.12)} = 1.5336\ m/s$$

$$v_b = (201)(1.5336) = 308.25\ m/s$$

Problem 25 (Book chapter 9):

A 1.2 kg ball drops vertically onto a floor, hitting with a speed of 25 m/s. It rebounds with an initial speed of 10 m/s. (a) What impulse acts on the ball during the contact? (b) If the ball is in contact with the floor for 0.020 s, what is the magnitude of the average force on the floor from the ball?



Answer: (a) According to definition of Impulse $\vec{J}$:

$$\vec{J} = \Delta\vec{p} = \vec{p}_f - \vec{p}_i = m\boldsymbol{v_f} - m\boldsymbol{v_i}$$

$$\vec{J} = (1.2)(10)\,(+\widehat{j}) - (1.2)(25)\widehat{(-j)}$$

$$\vec{J} = \Delta\vec{p} = 12\,\hat{j} + 30\,\hat{j} = 42\,\hat{j}\;kg - m/s$$

$v_i = 25\text{ m/s}$

$v_f = 10\text{ m/s}$

Δt= 0.020 s

(b) we can write the magnitude of the impulse as

$$J = F_{avg}\,\Delta t$$

$$F_{avg} = \frac{J}{\Delta t} = \frac{42}{0.020} = 2100\,N$$

Thank You