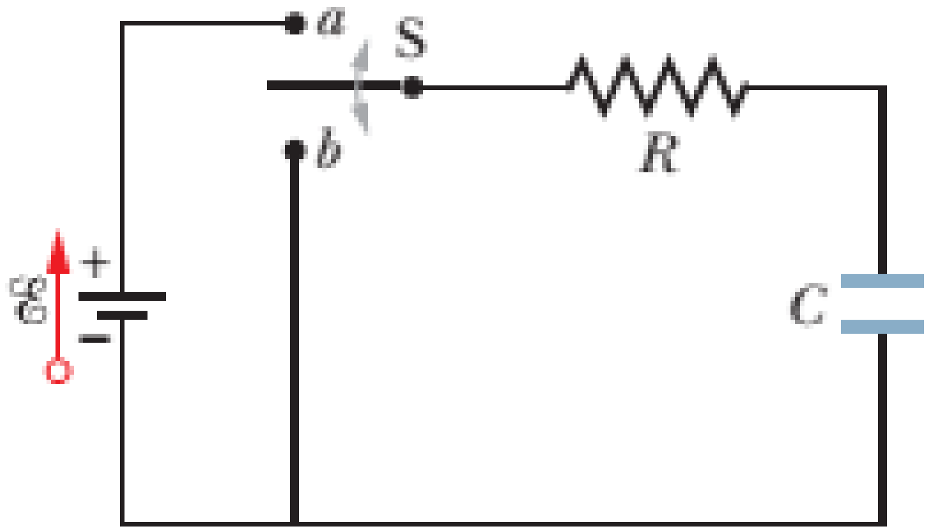
LESSON 10

BOOK CHAPTER 27

CIRCUITS

RC Circuit:

RC circuit is an electric circuit composed of resistors and capacitors driven by an emf (Electromotive Force) or power supply, as in figure with the switch at *a*, this type of circuit is called RC Circuit.



The Time Constant:

The Product of R and C is called capacitive time constant of the RC circuit. It has the dimensions of time and is represented with the symbol $\boldsymbol{\tau}$ (Tau):

$$\boldsymbol{\tau = RC}$$

If R is in ohms and C in farads, τ is in seconds.

Note:

$$\tau = \left(\frac{volt}{ampere}\right)\left(\frac{coulomb}{volt}\right) = \frac{coulomb}{ampere} = Time\ in\ seconds$$

Charging a Capacitor in RC circuit:

Let *q* represent the charge on the capacitor and *i* the current in the circuit at some time *t* after the switch has been closed.

According to Kirchhoff's loop theorem [*the algebraic sum of the changes in potential encountered in a complete traversal of any loop of a circuit must be zero*], We can write

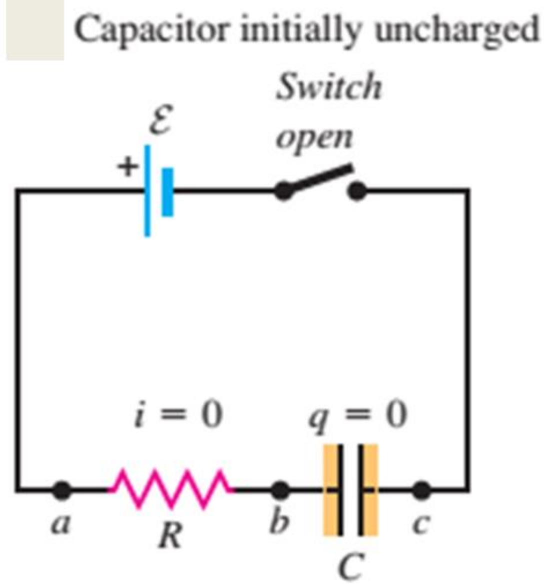
$$\varepsilon + (-iR) + \left(-\frac{q}{C}\right) = 0$$

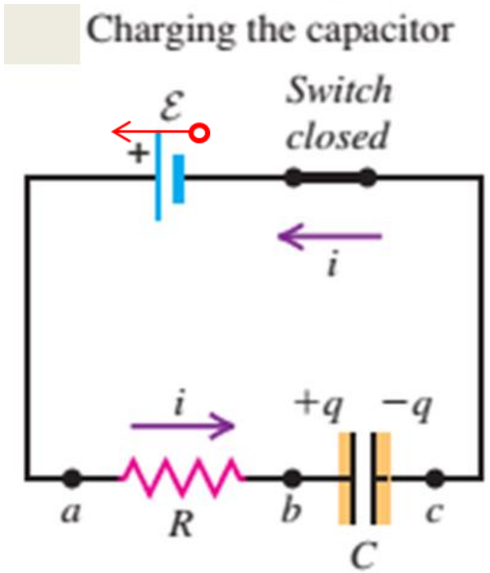
$$\varepsilon = iR + \frac{q}{C}$$

$$\varepsilon = R\frac{dq}{dt} + \frac{q}{C} \qquad \left[since\ i = \frac{dq}{dt}\right]$$

$$R\frac{dq}{dt} = \varepsilon - \frac{q}{C}$$

$$\frac{dq}{dt} = \frac{\varepsilon}{R} - \frac{q}{RC} \qquad [Dividing\ both\ sides\ by\ R]$$





When the switch is closed, the charge on the capacitor increases over time while the current decreases.

$$\frac{dq}{dt} = \frac{C\varepsilon - q}{RC}$$

$$\frac{dq}{C\varepsilon - q} = \frac{dt}{RC}$$

By integrating both sides , we get

$$\int_{q=0}^{q=q} \frac{dq}{C\varepsilon - q} = \int_{t=0}^{t=t} \frac{dt}{RC}$$

$$-|\ln(C\varepsilon - q)|_{q=0}^{q=q} = \frac{1}{RC}|t|_{t=0}^{t=t}$$

$$\ln(C\varepsilon - q) - \ln(C\varepsilon - 0) = -\frac{t}{RC}$$

$$\ln\frac{C\varepsilon - q}{C\varepsilon} = -\frac{t}{RC}$$

$$\frac{C\varepsilon - q}{C\varepsilon} = e^{-\frac{t}{RC}}$$

Note

Let

$$C\varepsilon - q = Q$$

$$0 - dq = dQ$$

$$dq = -dQ$$

$$\therefore \int \frac{dq}{C\varepsilon - q} = \int -\frac{dQ}{Q}$$

$$= -\ln Q = -\ln(C\varepsilon - q)$$

$$C\varepsilon - q = C\varepsilon e^{-\frac{t}{RC}}$$

$$C\varepsilon - C\varepsilon e^{-\frac{t}{RC}} = q$$

Therefore,

$$q(t) = C\varepsilon\left(1 - e^{-\frac{t}{RC}}\right)$$

Case 1 At $t = RC,$

$$q(t = RC) = C\varepsilon\left(1 - e^{-\frac{RC}{RC}}\right)$$

$$q(t = RC) = C\varepsilon(1 - e^{-1})$$

$$q(t = RC) = C\varepsilon\left(1 - \frac{1}{e}\right)$$

$$q(t = RC) = C\varepsilon\left(1 - \frac{1}{2.718}\right)$$

$$q(t = RC) = C\varepsilon(1 - 0.368) = 0.632\ \mathrm{C}\varepsilon$$

Case 2 At $t = 2RC,$

Case 3 At $t = 3RC,$

Case 4 At $t = 4RC,$

Case 5 At $t = 5RC,$

$q(t) = ?$

Home Task

Note: Draw a graph $q(t)$ vs t

Discharging a Capacitor in RC circuit:

We assume that the capacitor is fully charged to Q_0 (as shown in the top figure).
We remove the battery/power supply from the R-C circuit and the switch is closed as shown in the bottom figure. When the switch is closed, at $t = 0$, $q = q_0$. After some instant $t = t, q = q$.

According to Kirchhoff's loop theorem (clockwise, starting from point b):

$$-iR + \frac{q}{C} = 0$$

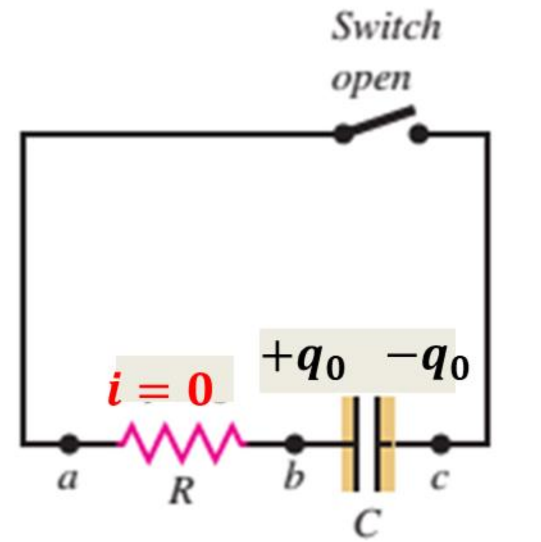
Since the positive charge q is leaving (decreasing) the positive plate of the capacitor,

$$i = -\frac{dq}{dt}$$

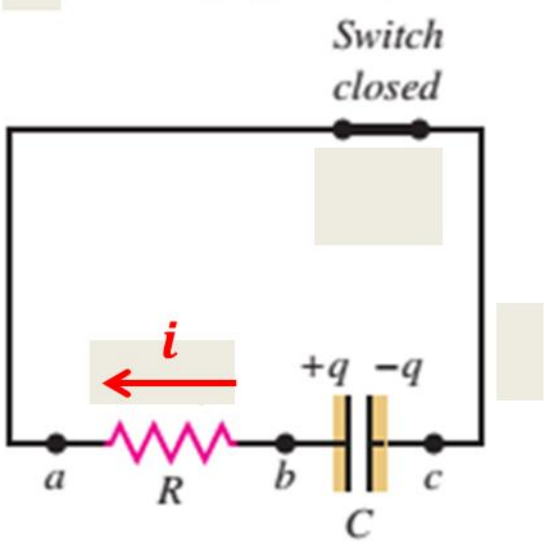
Thus,

$$R\frac{dq}{dt} + \frac{q}{C} = 0$$

Capacitor initially charged



Discharging the capacitor



When the switch is closed, the charge on the capacitor and the current both decrease over time.

$$\frac{dq}{dt} = -\frac{q}{RC}$$

$$\frac{dq}{q} = -\frac{dt}{RC}$$

By integrating both sides, we get

$$\int_{q=q_0}^{q=q} \frac{dq}{q} = -\frac{1}{RC}\int_{t=0}^{t=t} dt$$

$$|\ln q|_{q=q_0}^{q=q} = -\frac{1}{RC}|t|_{t=0}^{t=t}$$

$$\ln q - \ln q_0 = -\frac{t}{RC}$$

$$\ln\frac{q}{q_0} = -\frac{t}{RC}$$

$$\frac{q}{q_0} = e^{-\frac{t}{RC}}$$

$$q = q_0 e^{-\frac{t}{RC}}$$

Therefore, we can write

$$\boldsymbol{q(t) = q_0 e^{-\frac{t}{RC}}}$$

Case 1 At $t = RC$,

$$q(t = RC) = q_0 e^{-\frac{RC}{RC}}$$

$$q(t = RC) = q_0 e^{-1}$$

$$q(t = RC) = 0.368\, q_0$$

Home task

Case 2 At $t = 2RC$,

Case 3 At $t = 3RC$,

Case 4 At $t = 4RC$,

Case 5 At $t = 5RC$,

$q(t) =?$

Note: Draw a graph $q(t)$ vs t

Problem 58 (Book chapter 27):

In an *RC* series circuit, emf $\varepsilon = 12.0\ V$, resistance $R = 1.40\ M\Omega$, and capacitance $C = 1.8\ \mu F$. (a) Calculate the time constant. (b) Find the maximum charge that will appear on the capacitor during charging. (c) How long does it take for the charge to build up to $16\ \mu C$?

Answer:

Given

$\varepsilon = 12.0\ V$

$R = 1.40\ M\Omega = 1.4 \times 10^{6}\Omega$

$C = 1.8\ \mu F = 1.8 \times 10^{-6} F$

(a) $\tau = ?$

(b) Maximum charge $q_0 = ?$

(c) $t = ?$

(a) We know

$$\tau = RC = 1.4 \times 10^{6} \times 1.8 \times 10^{-6}$$

$$\boldsymbol{\tau = 2.52\ s}$$

(b) We know

$$q_0 = C\varepsilon = 1.8 \times 10^{-6} \times 12 = \mathbf{21.6 \times 10^{-6}\ C}$$

(c) We know $q(t) = C\varepsilon\left(1 - e^{-\frac{t}{RC}}\right)$

$$q = C\varepsilon - C\varepsilon e^{-\frac{t}{RC}} = q_0 - q_0 e^{-\frac{t}{RC}}$$

$$q = q_0 - q_0 e^{-\frac{t}{RC}}$$

$$q - q_0 = -q_0 e^{-\frac{t}{RC}}$$

$$-(q_0 - q) = -q_0 e^{-\frac{t}{RC}}$$

$$\frac{q_0 - q}{q_0} = e^{-\frac{t}{RC}}$$

$$\frac{q_0 - q}{q_0} = \frac{1}{e^{\frac{t}{RC}}}$$

$$\frac{q_0}{q_0 - q} = e^{\frac{t}{RC}}$$

$$\ln \frac{q_0}{q_0 - q} = \ln e^{\frac{t}{RC}} = \frac{t}{RC} \ln e$$

$$\ln \frac{q_0}{q_0 - q} = \ln e^{\frac{t}{RC}} = \frac{t}{RC} \ln e = \frac{t}{RC}$$

$$t = RC \ln \frac{q_0}{q_0 - q} = 2.52 \ln \frac{21.6 \times 10^{-6}}{21.6 \times 10^{-6} - 16 \times 10^{-6}}$$

$$t = 2.52 \ln \frac{21.6 \times 10^{-6}}{5.6 \times 10^{-6}} = 2.52 \ln 3.857$$

$$\boldsymbol{t = 2.52 \times 1.3499 = 3.402\ \mathrm{s}}$$

Problem 61 (Book chapter 27):

A 15.0 kΩ resistor and a capacitor are connected in series, and then a 12.0 V potential difference is suddenly applied across them. The potential difference across the capacitor rises to 5.00 V in 1.30 μs. (a) Calculate the time constant of the circuit. (b) Find the capacitance of the capacitor.

Answer:

(a) The potential difference $V_C(t)$ across the capacitor during the charging process is

$$\boldsymbol{V_C(t) = \frac{q(t)}{C} = \frac{C\varepsilon\left(1 - e^{-\frac{t}{RC}}\right)}{C} = \varepsilon\left(1 - e^{-\frac{t}{RC}}\right)}$$

$$5 = 12\left(1 - e^{-\frac{1.3\times10^{-6}}{\tau}}\right)$$

$$1 - e^{-\frac{1.3\times10^{-6}}{\tau}} = \frac{5}{12}$$

$$1 - \frac{5}{12} = e^{-\frac{1.3\times10^{-6}}{\tau}}$$

Given

$$R = 15.0\ k\Omega = 15 \times 10^3 \Omega$$

$$\varepsilon = 12\ V$$

$$V_C(t) = 5\ V$$

$$t = 1.3\ \mu s = 1.3 \times 10^{-6} s$$

(a) $\tau = RC = ?$

(b) $C = ?$

$$\frac{7}{12} = e^{-\frac{1.3\times10^{-6}}{\tau}} = \frac{1}{e^{\frac{1.3\times10^{-6}}{\tau}}}$$

$$\frac{12}{7} = e^{\frac{1.3\times10^{-6}}{\tau}}$$

$$\ln 1.7143 = \ln e^{\frac{1.3\times10^{-6}}{\tau}} = \frac{1.3\times10^{-6}}{\tau}\ln e = \frac{1.3\times10^{-6}}{\tau}$$

$$0.5390 = \frac{1.3\times10^{-6}}{\tau}$$

$$\tau = \frac{1.3\times10^{-6}}{0.5390}$$

$$\tau = 2.412\times10^{-6}\ s$$

(b) We know

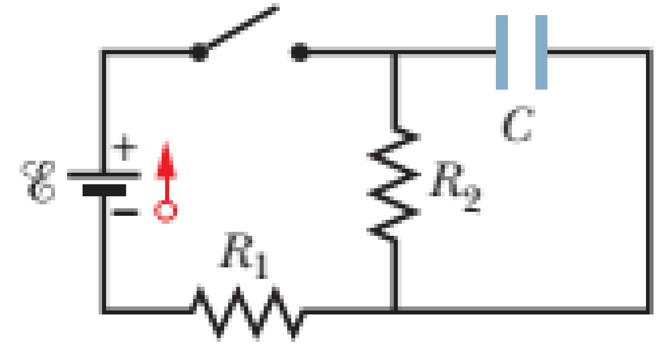
$$\tau = RC$$

$$C = \frac{\tau}{R} = \frac{2.412\times10^{-6}}{15\times10^{3}} = 0.1608\times10^{-9}F$$

$$C = 160.8\times10^{-12}F = 160.8\ pF$$

Problem 65 (Book chapter 27):

In the figure, $R_1 = 10.0\ k\Omega$, $R_2 = 15.0\ k\Omega, C = 0.400\ \mu F$, and the ideal battery has emf $\varepsilon = 20.0\ V$. First, the switch is closed a long time so that the steady state is reached. Then the switch is opened at time $t = 0$. What is the current in resistor 2 at $t = 4.00\ ms$?



Original Figure

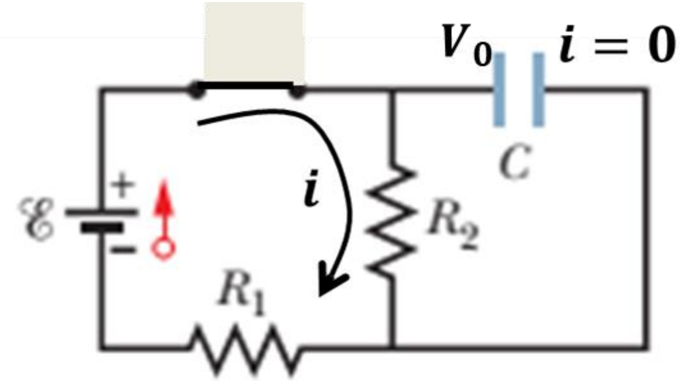
Answer:

Capacitor voltage during discharging:

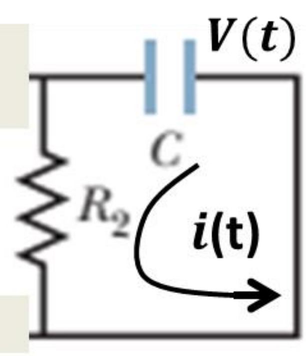
$$V(t) = \frac{q(t)}{C} = \frac{q_0 e^{-\frac{t}{R_2 C}}}{C} = V_0 e^{-\frac{t}{R_2 C}}$$

Step1: we need to find V_0 across the capacitor.

When the capacitor is fully charged, voltage drop across R_2 (iR_2) is equal to the voltage V_0 because $R_2\ and\ C$ are connected in parallel.



When the capacitor is fully charged



When the capacitor is discharging

Using Kirchhoff's loop theorem, we can write

$$\varepsilon - iR_2 - iR_1 = 0$$

$$\varepsilon = iR_2 + iR_1 = i(R_2 + R_1)$$

$$i = \frac{\varepsilon}{R_2 + R_1}$$

Now, we use

$$V_0 = iR_2 = \frac{\varepsilon R_2}{R_2 + R_1} = \frac{20 \times 15 \times 10^3}{(15 + 10) \times 10^3}$$

$$\boldsymbol{V_0 = 12\ V}$$

Given

$$R_1 = 10.0\ k\Omega = 10 \times 10^3 \Omega$$

$$R_2 = 15.0\ k\Omega = 15 \times 10^3 \Omega$$

$$\varepsilon = 20.0\ V$$

$$C = 0.400\ \mu F = 0.4 \times 10^{-6} F$$

$$V(t = 4 \times 10^{-3} s) = ?$$

$$\boldsymbol{I\left(t = 4 \times 10^{-3} s\right) = ?}$$

Step2: We need to find $V(t)$ during discharging the capacitor

$$V(t = 4 \times 10^{-3} s) = 12 \times e^{-\frac{4 \times 10^{-3}}{15 \times 10^3 \times 0.4 \times 10^{-6}}}$$

$$V(t = 4 \times 10^{-3} s) = 12 \times e^{-0.6666}$$

$$V(t = 4 \times 10^{-3} s) = 12 \times 0.5134 = 6.161\ V$$

Therefore, $\boldsymbol{I\left(t = 4 \times 10^{-3} s\right) = \frac{V\left(t = 4 \times 10^{-3} s\right)}{R_2} = \frac{6.161}{15 \times 10^3} = 0.411 \times 10^{-3} A}$

Thank You