1.2.1

a) Here, $f(x) = e^{-3x}$

$(a, b) = (0, 2)$

$n = 4$

$\therefore \Delta x = \frac{b-a}{n} = \frac{2-0}{4} = \frac{1}{2} = 0.5$

Riemann Sum $[x_{r-1}, x_r]$	Left R.S		Right R.S		Middle R.S	
	c_r	$f(c_r)$	c_r	$f(c_r)$	c_r	$f(c_r)$
0 – 0.5	0	1	0.5	0.2231	0.25	0.9724
0.5 – 1.0	0.5	0.2231	1.0	0.0498	0.75	0.1059
1.0 – 1.5	1.0	0.0498	1.5	0.0111	1.25	0.0235
1.5 – 2.0	1.5	0.0111	2.0	0.0025	1.75	0.0052
$\Sigma f(c_r$	1.284		0.2865		0.6065	
$\Delta x \Sigma f(c_r)$	0.642		0.1433		0.3033	

$\therefore T_n = \frac{0.5}{2}[1 + (2 \times 0.2231) + (2 \times 0.498) + (2 \times 0.0111) + 0.0025]$

$= 0.393$ (Ans)

b) Here, $f(x) = \frac{1}{\sqrt{x^2+1}}$

$(a, b) = (1, 7)$

$n = 6$

$\therefore \Delta x = \frac{b-a}{n} = \frac{7-1}{6} = \frac{6}{6} = 1$

Remann Sum $[x_{n-1}, x_n]$	Left R.S		Right R.S		middle R.S	
	e_n	$f(e_n)$	e_n	$f(e_n)$	e_n	$f(e_n)$
1, 2	1	0.707	2	0.3333	1.5	0.4781
2, 3	2	0.3333	3	0.189	2.5	0.2453
3, 4	3	0.189	4	0.1240	3.5	0.151
4, 5	4	0.1240	5	0.0891	4.5	0.1042
5, 6	5	0.0891	6	0.0697	5.5	0.0773
6, 7	6	0.0697	7	0.0539	6.5	0.0602
$\Sigma f(e_n)$	1.5104		0.8572		1.1161	
$\Delta \Sigma f(e_n)$	1.5104		0.8572		1.1161	

$\therefore T_n = \frac{1}{2}[0.7071 + (2\times0.3333) + (2\times0.189) + (2\times0.1240) + (2\times0.0891) + (2\times0.0697) + 0.0539]$

$= 1.1838$

$= 1.184$

(Ans)

c) Here, $f(x) = \frac{1}{1-\ln x}$

$(a,b) = (3,5)$

$n = 4$

$\therefore \Delta x = \frac{b-a}{n} = \frac{5-3}{4} = \frac{2}{4} = 0.5$

Riemann Sum $[x_{r-1}, x_r]$	Left R.S		Right R.S		middle R.S	
	c_r	$f(c_r)$	c_r	$f(c_r)$	c_r	$f(c_r)$
3, 3.5	3	-10.1907	3.5	-3.9563	3.25	-5.5974
3.5, 4	3.5	-3.9563	4	-2.5887	3.75	-3.1079
4, 4.5	4	-2.5887	4.5	-1.9837	4.25	-2.2375
4.5, 5	4.5	-1.9837	5	-1.6409	4.75	-1.7917
$\Sigma f(c_r)$	-18.6698		-10.1697		-12.7345	
$\Delta x \Sigma f(c_r)$	-9.3348		-5.0849		-6.3673	

$\therefore T_n = \frac{0.5}{2}[-10.1907 + \{2 \times (-3.9563)\} + 2(-2.5887) + 2(-1.9837) + (-1.6409)]$

$= -7.210$ (Ans)

d) Here, $f(x) = \sin(x)\cos(x^4)$

$(a, b) = (0, 1)$

$n = 4$

$\therefore \Delta x = \frac{b-a}{n} = \frac{1-0}{4} = \frac{1}{4} = 0.25$

Riemann Sum $[x_{r-1}, x_r]$	Left R.S		Right R.S		Middle R.S	
	c_r	$f(c_r)$	c_r	$f(c_r)$	c_r	$f(c_r)$
0, 0.25	0	0	0.25	0.2469	0.125	0.1247
0.25, 0.50	0.25	0.2469	0.50	0.4645	0.375	0.3627
0.50, 0.75	0.50	0.4645	0.75	0.5766	0.625	0.5410
0.75, 1	0.75	0.5766	1	0.4546	0.875	0.5334
$\Sigma f(x)$	1.288		1.7926		1.5818	
$\Delta x \Sigma f(x)$	0.322		0.4357		0.3955	

$$\therefore T_n = \frac{0.25}{2} \times [0 + (2 \times 0.2469) + (2 \times 0.4645) + (2 \times 0.5766) + 0.4546]$$

$$= 0.379$$

(Ans)

e) Here, $f(x) = \sin(x^2)$

$(a, b) = (0, 1)$

$n = 5$

$\therefore \Delta x = \frac{b-a}{n} = \frac{1-0}{5} = \frac{1}{5} = 0.2$

Riemann Sum $[x_{n-1}, x_n]$	Left R.S		Right R.S		Middle R.S	
	e_n	$f(e_n)$	e_n	$f(e_n)$	e_n	$f(e_n)$
0, 0.2	0	0	0.2	0.04	0.1	0.01
0.2, 0.4	0.2	0.04	0.4	0.15	0.3	0.0899
0.4, 0.6	0.4	0.15	0.6	0.3523	0.5	0.2474
0.6, 0.8	0.6	0.3523	0.8	0.5972	0.7	0.4706
0.8, 1.0	0.8	0.5972	1	0.8415	0.9	0.7243
$\Sigma f(e_n)$	1.1395		1.981		1.5422	
$\Delta x \Sigma f(e_n)$	0.2279		0.3962		0.3084	

$\therefore T_n = \frac{0.2}{2} \times [0 + (2 \times 0.04) + (2 \times 0.15) + (2 \times 0.3$
$+ (2 \times 0.5972) + 0.8415]$

$= 0.312$ (Ans)