1.2.3

1. $\boxed{a)}$ $f(x) = x + x^3$

$\therefore f(-x) = -x + (-x)^3$

$= -x - x^3$

$= -(x + x^3)$

$\therefore f(-x) = -f(x)$

So, $f(x)$ is an odd function.

$\boxed{b)}$ $f(x) = (x^2 + 25)^2$

$\therefore f(-x) = [(-x)^2 + 25]^2$

$= (x^2 + 25)^2$

$\therefore f(-x) = f(x)$

So, $f(x)$ is an even function.

$\boxed{c)}$ $f(x) = x^6 + e^{4x}$

$\therefore f(-x) = (-x)^6 + e^{4(-x)}$

$= (-1)^6 \cdot x^6 + e^{-4x}$

$= x^6 + e^{-4x}$

So, $f(x)$ is neither odd nor even function.

$\boxed{d)}$ $f(x) = \sin^3 x \cos^6 x$

$\therefore f(-x) = \sin^3(-x) \cos^6(-x)$

$= -\sin^3 x \cdot \cos^6 x$

$\therefore f(x) = -f(x)$

So, $f(x)$ is an odd function

$\boxed{e)}$ $f(x) = \sin^4 x \cos^5 x$

$\therefore f(-x) = \sin^4(-x) \cos^5(-x)$

$= \sin^4 x \cos^5 x$

$\therefore f(-x) = f(x)$

So, $f(x)$ is an even function.

$\boxed{f)}$ $f(x) = \tan x + \cot x$

$\therefore f(-x) = \tan(-x) + \cot(-x)$

$= -\tan x - \cot x$

$= -(\tan x + \cot x)$

$\therefore f(-x) = -f(x)$

So, $f(x)$ is an odd function.

2. $\boxed{a}$ $\int_{-1}^{1}(x^3+5x^4)dx$

Here, $f(x)=x^3+5x^4$

$\therefore f(-x)=(-x)^3+5.(-x)^4$

$=-x^3+5x^4$

So, $f(x)$ is neither even nor odd functio

Thus, $\int_{-1}^{1}(x^3+5x^4)dx$

$=\left[\frac{x^4}{4}+5.\frac{x^5}{5}\right]_{-1}^{1}$

$=\left(\frac{(1)^4}{4}+1^5\right)-\left[\frac{(-1)^4}{4}+(-1)^5\right]$

$=\left(\frac{1}{4}+1\right)-\left(\frac{1}{4}-1\right)$

$=\frac{1}{4}+1-\frac{1}{4}+1$

$=2$ (Ans)

$\boxed{b)}$ $\int_{-2}^{2} x(1+x+x^2)dx$

Here, $f(x) = x(1+x+x^2)$

$= x+x^2+x^3$

$\therefore f(-x) = (-x)+(-x)^2+(-x)^3$

$= -x+x^2-x^3$

So, $f(x)$ is neither even nor odd function.

Thus, $\int_{-2}^{2} x(1+x^2+x^3)dx$

$= \int_{-2}^{2}(x+x^2+x^3)dx$

$= \left[\frac{x^2}{2}+\frac{x^3}{3}+\frac{x^4}{4}\right]_{-2}^{2}$

$= \frac{2^2}{2}+\frac{2^3}{3}+\frac{2^4}{4} - \left\{\frac{(-2)^2}{2}+\frac{(-2)^3}{3}+\frac{(-2)^4}{4}\right\}$

$= 2+\frac{8}{3}+\frac{16}{4} - \left(\frac{4}{2}-\frac{8}{3}+\frac{16}{4}\right)$

$= 2 \cdot \frac{8}{3}$

$= \frac{16}{3}$ (Ans)

$\boxed{c)}$ $\int_{-4}^{4} (2+3x^2)\,dx$

$f(x) = 2+3x^2$

$\therefore f(-x) = 2+3(-x)^2$

$= 2+3x^2$

$\therefore f(-x) = f(x)$

So, $(2+3x^2)$ is an even function.

Thus, $\int_{-4}^{4} (2+3x^2)\,dx$

$= 2\int_{0}^{4} (2+3x^2)\,dx$

$= 2\left[x + 3\cdot\frac{x^3}{3}\right]_0^4$

$= 2\left[4 + 4^3 - 0 - 0\right]$

$= 136$ (Ans)

$\boxed{d)}$ $\int_{-5}^{5} x^5 e^{x^4}\,dx$

$f(x) = x^5 e^{x^4}$

$\therefore f(-x) = (-x)^5 \cdot e^{(-x)^4}$

$= -x^5 e^{x^4}$

$\therefore f(x) = -f(x)$

So, $x^5 \cdot e^{x^4}$ is odd function

Thus, $\int_{-5}^{5} x^5 e^{x^4}\,dx$

$= 0$ (Ans)

e) $\int_{-\pi}^{\pi} x^8 \sin x \, dx$

Here, $f(x) = x^8 \sin x$

$\therefore f(-x) = (-8x)^8 \sin(-x)$

$= x^8(-\sin x)$

$= -x^8 \sin x$

$\therefore f(-x) = -f(x)$

So, $x^8 \sin x$ is an odd function.

Thus, $\int_{-\pi}^{\pi} x^8 \sin x \, dx$

$= 0$ (Ans)

f) $\int_{-\pi}^{\pi} x \cos x \, dx$

Here, $f(x) = x \cos x$

$\therefore f(-x) = (-x)\cos(-x)$

$= -x \cdot \cos x$

$\therefore f(x) = -f(x)$

So, $x \cos x$ is an odd function.

Thus, $\int_{-\pi}^{\pi} x \cos x \, dx$

$= 0$ (Ans)

$\boxed{g)}$ $\int_{-\pi}^{\pi} \sin^3 x \cos^5 x \, dx$

Here, $f(x) = \sin^3 x \cos^5 x \, dx$

$\therefore f(-x) = \sin^3(-x) \cos^5(-x)$

$= -\sin^3 x \cos^5 x$

$\therefore f(-x) = -f(x)$

Thus, $\int_{-\pi}^{\pi} \sin^3 x \cos^5 x \, dx$

$= 0$ (Ans)

$\boxed{h)}$ $\int_{-\pi/2}^{\pi/2} x^4 \sin^3 x \cos^3 x \, dx$

$f(x) = x^4 \sin^3 x \cos^3 x$

$\therefore f(-x) = (-x)^4 \sin^3(-x) \cos^3(-x)$

$= x^4 (-\sin^3 x) \cos^3 x$

$= -x^4 \sin^3 x \cos^3 x$

$\therefore f(-x) = -f(x)$

So, $x^4 \sin^3 x \cos^3 x$ is an odd function.

Thus, $\int_{-\pi/2}^{\pi/2} x^4 \sin^3 x \cos^3 x$

$= 0$ (Ans)

$\boxed{\text{i)}}$ $\int_{-\pi}^{\pi} \frac{x^3}{\sqrt{1+x^2}}\,dx$

Here, $f(x) = \frac{x^3}{\sqrt{1+x^2}}$

$\therefore f(-x) = \frac{(-x)^3}{\sqrt{1+x^2}}$

$= \frac{-x^3}{\sqrt{1+x^2}}$

$\therefore f(-x) = -f(x)$.

So, $\frac{x^3}{\sqrt{1+x^2}}$ is an odd function.

Thus,

$\int_{-\pi}^{\pi} \frac{x^3}{\sqrt{1+x^2}}\,dx$

$= 0$.

(Ans)