Computer Science and Engineering Discipline
Khulna University, Khulna - 9208
Roll#:190208
Name:Tanvir Hassan
email:tanvir1908@cseku.ac.bd
Mobile:01989567104

PROBLEM :) $T(n) = 2\,T(n/2) + n\log_2 n$

Here $a = 2$, $b = 2$ and **$f(n) = n\log_2 n$, $p = 1$.**
$x = \log_b a$ = **$\log_2 2$** = 1. Here **$x=p$**, but **$n^x \neq f(n)$,**

Find out the solution of the recurrence in term of asymptotic notation.

Solution :

$n^{\log_b a} = n^{\log_2 2} = n^1 = n$
Comparing $n^{\log_b a} = n$ and $f(n) = n \log n$
Does not satisfy either Case 1 or 2 or 3 of the Master's theorem .
Case 3 states that f(n) should be polynomially larger but here it is asymptotically larger than n logb a only by a factor of log n
Let us take $n=2^m$. Then we have the recurrence

$$m) = 2T(2^{m-1}) + 2^m \log_2(2^m) = 2T(2^{m-1}) + m2^m$$

Calling $T(2^m)$ as f(m), we get that

$$\begin{aligned}
f(m) &= 2f(m-1)+m^{2m} \\
&= 2(2f(m-2)+(m-1)2^{m-1})+m2^m \\
&= 4f(m-2) + (m-1)2^m+m2^m \\
&= 4(2f(m-3) + (m-2)2^{m-2}) + (m-1)2^m + m2^m \\
&= 8f\,(m-3) + (m-2)2^m + (m-1)2^m + m2^m
\end{aligned}$$

Proceeding on these lines, we get that

$f(m) = 2^m f(0) + 2^m(1+2+3+....+m)$

$= 2^m f(0)+2^m(1+2+3+...+m) = 2^m\ f(0) + \frac{m(m+1)}{2}2^m +$

$= 2^m\ f(0) + m(m+1)2^{m-1}$

Hence , $T(n) = n(T1) + n\ (\frac{log2(n)(1+log2(n))}{2}) = \Theta(n\log 2n)$.

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