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Q.1)

given that,

$$f(z) = \log_e(1+z)$$

$$Z = x^T x, \quad x \in R^d$$

$$\therefore f'(z) = \frac{d}{dz} \log(1+z)$$

$$= \frac{1}{1+z} \frac{d}{dx}(z)$$

$$= \frac{1}{1+z} \frac{d}{dx}(x^T x)$$

$$= \frac{1}{1+x^T x} 2x$$

$$= \frac{2x}{1+x^T x}$$

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Q.2)

$$f(z) = e^{-z/2}$$

Where,

$$z = g(y) = y^T S^{-1} y$$

$$y = h(x) = x - \mu$$

$$x, \mu \in R^d, \; S \in R^{d \times d}$$

$$\therefore f'(z) = \frac{d}{dz}\left(e^{-z/2}\right)$$

$$= e^{-z/2} \cdot \frac{d}{dz}\left(-z/2\right)$$

$$= e^{-z/2}\left(-\frac{1}{2}\right) \frac{d}{dy} y^T S^{-1} y \cdot \frac{d}{dx}(x - \mu)$$

Now,

$$\frac{d}{dy}\left(y^T S^{-1} y\right) = \lim_{h \to 0} \frac{g(y+h) - g(y)}{h}$$

$$= \lim_{h \to 0} \frac{(y^T + h) S^{-1} (y + h) - y^T S^{-1} y}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{y^T S^{-1} y} + y^T S^{-1} h + h S^{-1} y + S^{-1} h^2 - \cancel{y^T S^{-1} y}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(y^T s^{-1} + s^{-1} y + s^{-1} h)}{\cancel{h}}$$

$$= y^T s^{-1} + s^{-1} y + \cancel{s^{-1} h} \lim_{h \to 0} s^{-1} h$$

$$= y^T s^{-1} + s^{-1} y$$

and $\frac{d}{dx}(x - \mu) = 1$

$$\therefore f'(z) = -\frac{1}{2} e^{-z/2} (y^T s^{-1} + s^{-1} y)$$

(Ans.)