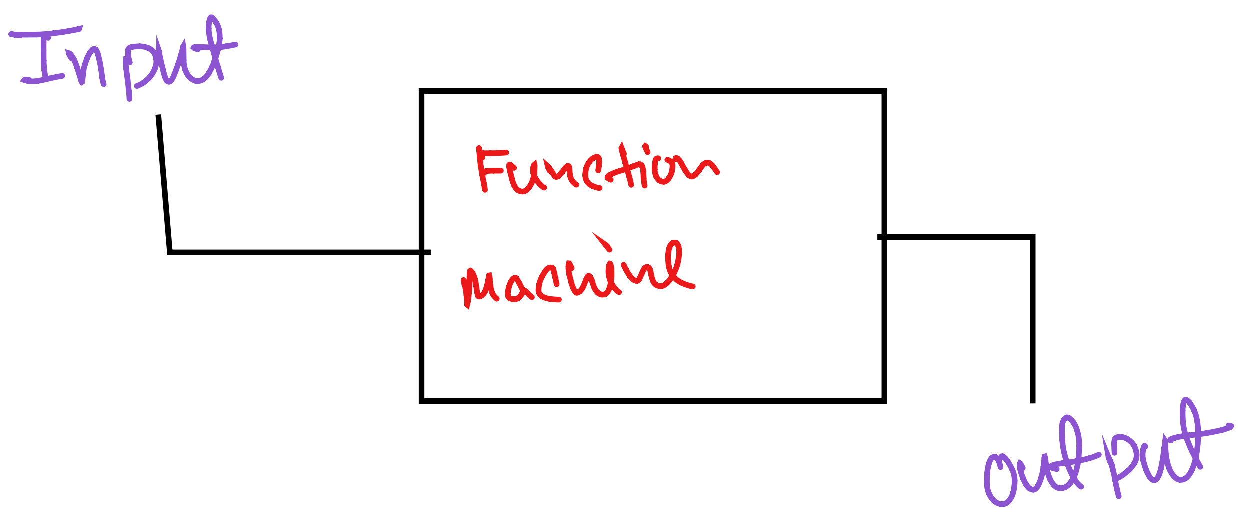
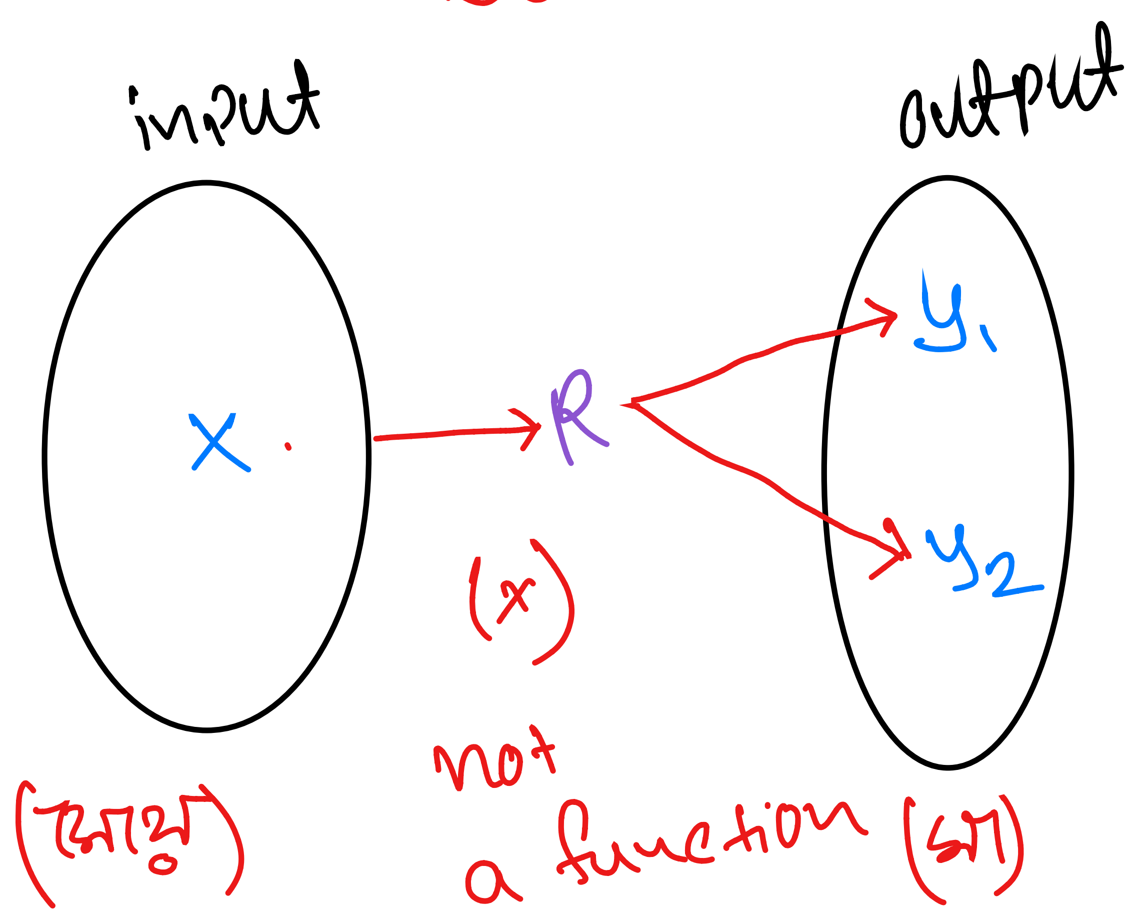
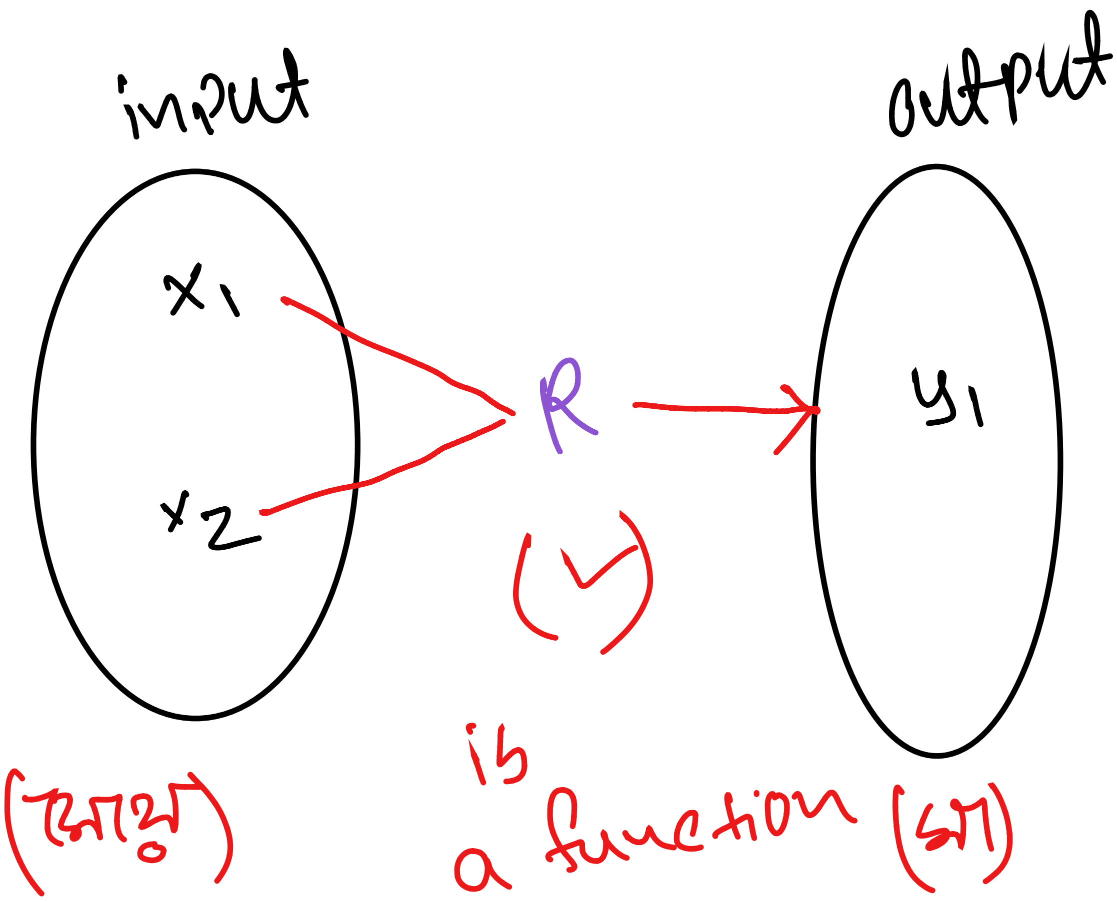
Function



Not a function





Ex. 1: Let $A = \{-1, 1, 2, 3\}$ and let $B = \{1, 2, 4, 5, 9, 10\}$ and let $f : A \to B$ be a relation such that $f(x) = x^2 + 1$ for each $x \in A$.
Is f represents a function? If so, find the domain and the range of f.

solve:

$f(x) = x^2+1$

$f(-1) = (-1)^2+1 = 2$

$f(1) = 1^2+1 = 2$

$f(2) = 2^2+1 = 5$

$f(3) = 3^2+1 = 10$

f is a function.

Domain: $\{-1, 1, 2, 3\}$

Range: $\{2, 5, 10\}$

Ex. 2: Let $A = \{-1, 1, 2, 3, 4\}$ and let $B = \{1, 2, 4, 5, 9, 10\}$ and let $f : A \to B$ be a relation such that $f(x) = x^2 + 1$ for each $x \in A$.
Is f represents a function? If so, find the domain and the range of f.

solve:

$f(x) = x^2 + 1$

$f(-1) = (-1)^2 + 1 = 2$

$f(1) = 1^2 + 1 = 2$

$f(2) = 2^2 + 1 = 5$

$f(3) = 3^2 + 1 = 10$

$f(4) = 4^2 + 1 = 17$

f is not a function. because for $x = 4$ we found $f(4) = 17$ which is apcent in B.

Representing a function

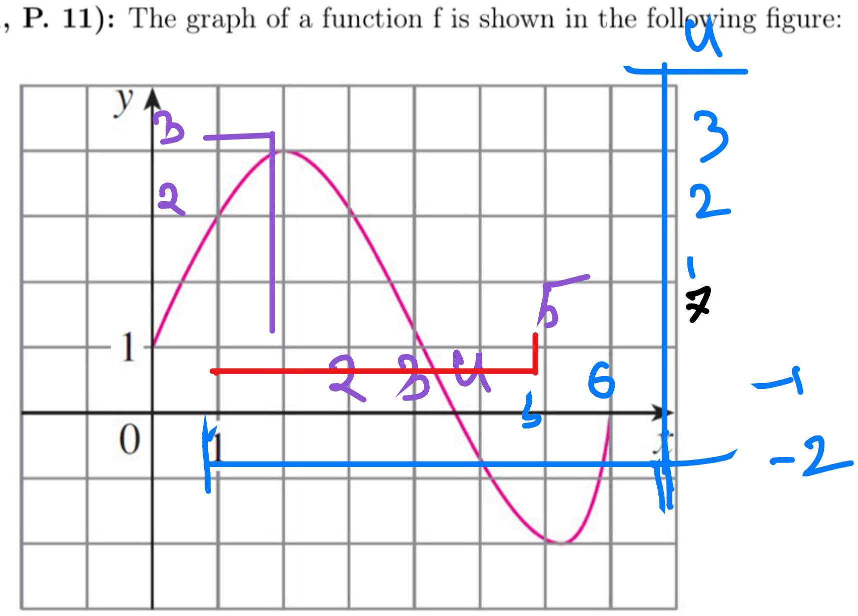
An Algebraic expression

$y = \sqrt{x+9}$

A graph

A table of selected pairs

Ex.3 (Example 1, P. 11): The graph of a function f is shown in the following figure:



(a) Find the values of $f(1)$ and $f(5)$.

(b) What are the domain and range of f ?

(a) $f(1) = 3$ [when the value of $x = 1$ the value of $y = 3$]

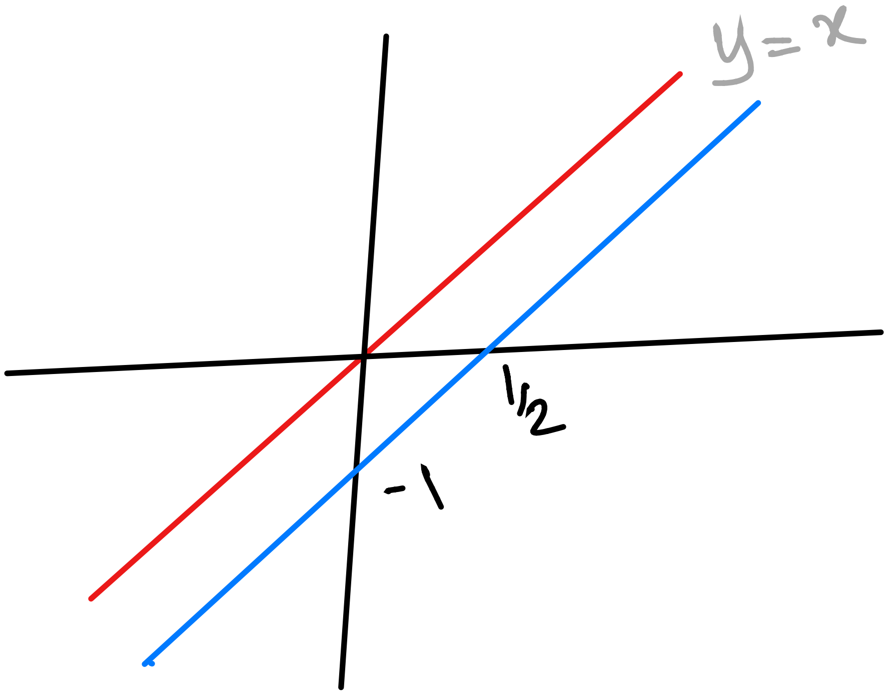
$f(5) = -0.8$

(b) Domain of $f = [0, 7]$

Range of $f = [-2, 4]$

Ex.4 (Example 2, P. 12): Sketch the graph and find the domain and range of each function:

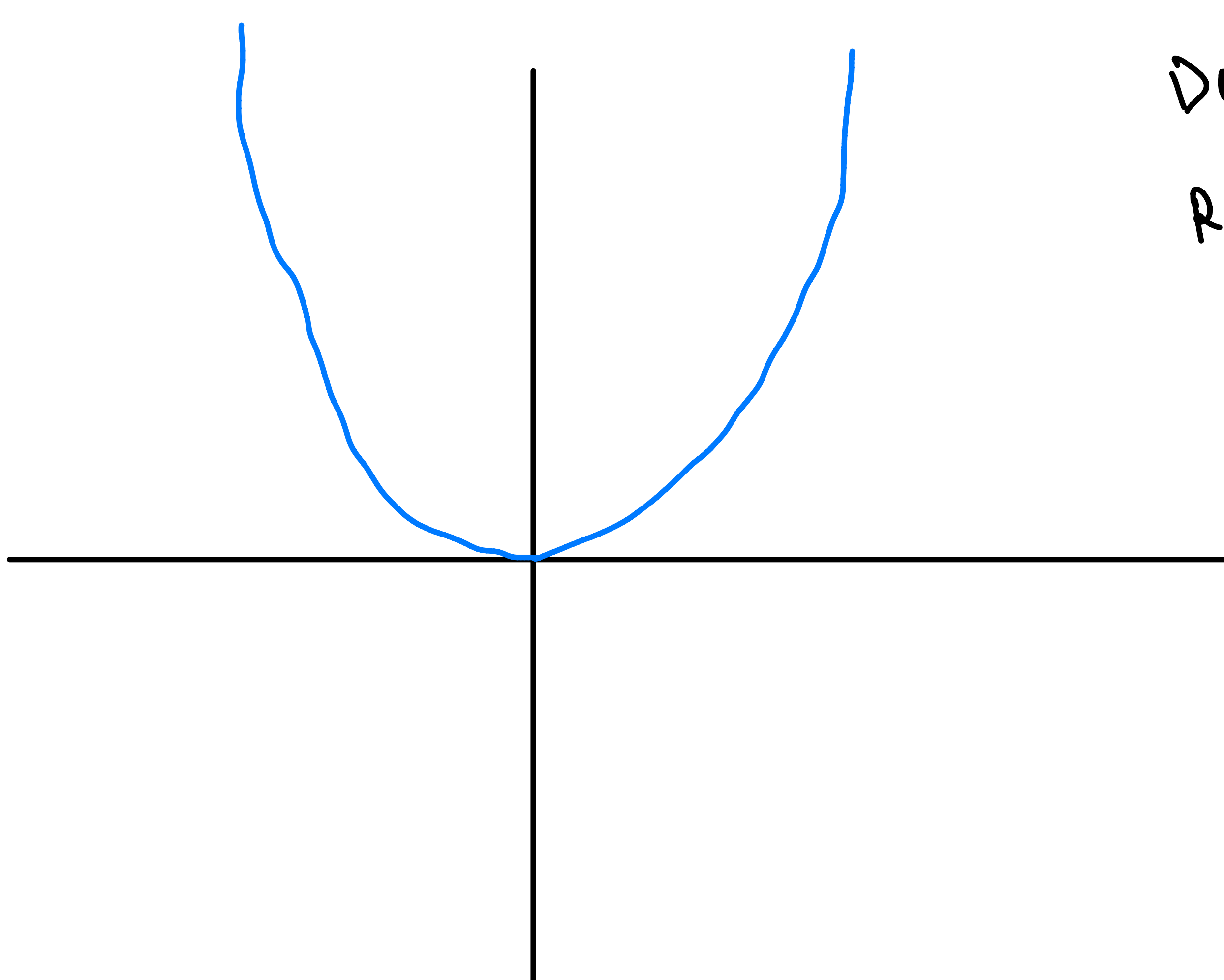
(a) $f(x) = 2x - 1$.



Domain: $\mathbb{R}$ / $(-\infty, \infty)$

Range: $\mathbb{R}$ / $(-\infty, \infty)$

(b) $g(x) = x^2$.



Domain: $\mathbb{R}$

Range: $[0, \infty)$

Ex.5 (Example 3, P. 12): If $f(x) = 2x^2 - 5x + 1$ and $h \neq 0$, evaluate $\dfrac{f(a+h) - f(a)}{h}$.

$$f(a+h) = 2(a+h)^2 - 5(a+h) + 1$$

$$= 2(a^2 + 2ah + h^2) - 5a - 5h + 1$$

$$= 2a^2 + 4ah + 2h^2 - 5a - 5h + 1$$

$$f(a) = 2a^2 - 5a + 1$$

now

$$\frac{f(a+h) - f(a)}{h} = \frac{\cancel{2a^2} + 4ah + 2h^2 - \cancel{5a} - 5h + \cancel{1} - \cancel{2a^2} + \cancel{5a} - \cancel{1}}{h}$$

$$= \frac{2h^2 + 4ah - 5h}{h}$$

$$= \frac{\cancel{h}(2h + 4a - 5)}{\cancel{h}}$$

$$= 2h + 4a - 5$$

Ex.6 (Example 6, P. 14): Find the domain of each function:

(a) $f(x) = \sqrt{x+2}$.

Solve:

Domain: $[-2, \infty)$

$x + 2 \geqslant 0$

$\Rightarrow x \geqslant -2$

(b) $g(x) = \dfrac{1}{x^2 - x}$.

Solve:

Domain: $\mathbb{R} - \{0, 1\}$

$x^2 - x \neq 0$

$\Rightarrow x(x-1) \neq 0$

$\Rightarrow x \neq 0$ and $x \neq 1$

Ex.7: Find the domain of each function:

(a) $f(x) = 4x - 8$.

(a) Domain: $\mathbb{R}$

(b) $h(x) = \sqrt{x-5}$.

(b) Domain: $[5, \infty)$

$x - 5 \geqslant 0$

$\Rightarrow x \geqslant 5$

Ex.8: Find the domain of each function:

(a) $f(x) = \dfrac{1}{\sqrt{1-x}}$.

Solve:

Domain: $(-\infty, 1)$

$1-x>0$

$\Rightarrow -x>-1$

$\Rightarrow x<1$

(b) $f(x) = \sqrt{3-\sqrt{x}}$.

Solve!

Domain: $[0,9]$

here,

$x\geq 0$

$[0,\infty)$

again,

$3-\sqrt{x}\geq 0$

$\Rightarrow -\sqrt{x}\geq -3$

$\Rightarrow \sqrt{x}\leq 3$

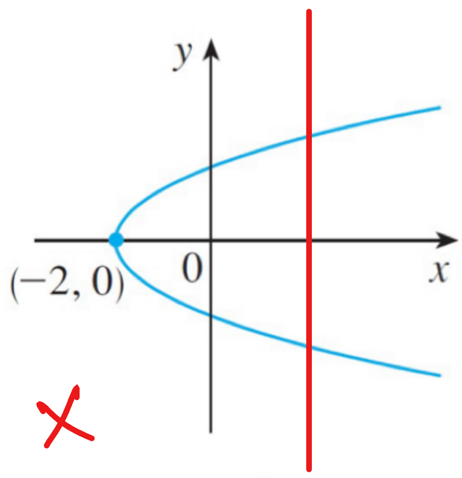
$\Rightarrow x\leq 9$

$(-\infty,9]$

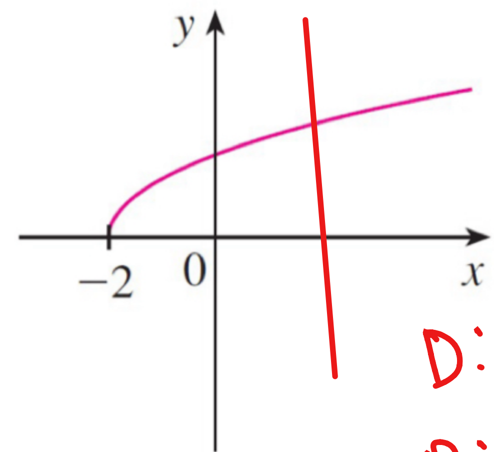
$[0,\infty)\cap(-\infty,9]$

$=[0,9]$

Ex.9 Determine whether the curve is the graph of a function of x. If it is, state the domain and range of the function.



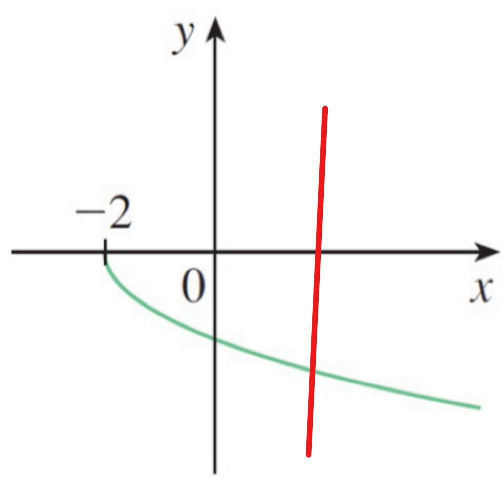
(a) $x = y^2 - 2$



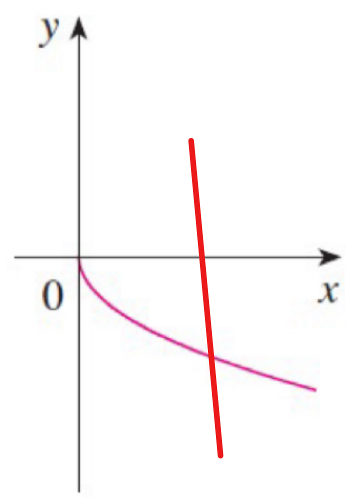
(b) $y = \sqrt{x+2}$

D: $[-2, \infty)$

R: $[0, \infty)$



(c) $y = -\sqrt{x+2}$



(d) $y = -\sqrt{x}$

D: $[-2, \infty]$ R: $(-\infty, 0]$

D: $[0, \infty)$

R: $(-\infty, 0]$

যদি $y = -x$ হয়ে যায় তাহলে graph

x অক্ষ বরাবর আড়াআড়ি ঘুরে যাবে।

Even and odd

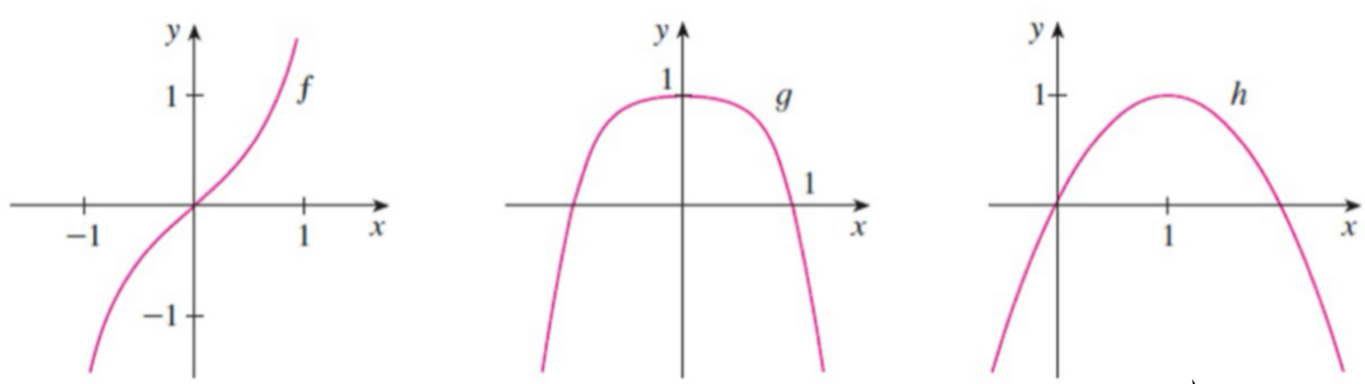
If,

$f(x)$ is converted to $f(-x)$ and we get

$f(-x) = f(x) \rightarrow$ Even function

$f(-x) = -f(x) \rightarrow$ odd function

Ex.10: Determine whether each of the following functions is even, odd, or neither even nor odd:



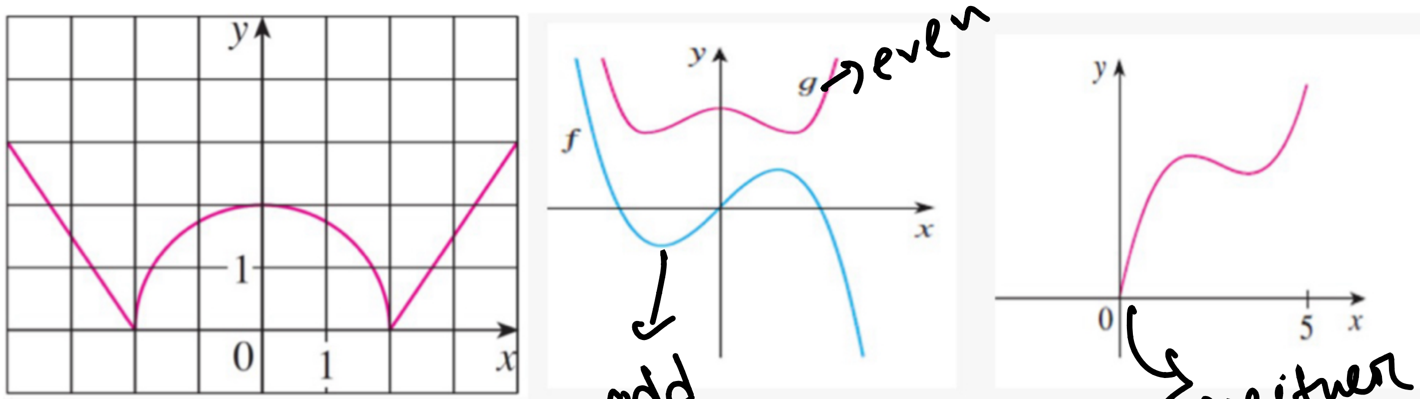
odd even neither

If the curve is symmetric about y axis (odd)

If the curve is symmetric about origin → (even)

Trick

Try to find out the mirror image



even

odd

neither

even

Ex.11 (Example 11, P. 18): Determine whether each of the following functions is even, odd, or neither even nor odd.
(a) $f(x) = x^5 + x$.

(a) $f(x) = x^5 + x$

$\Rightarrow f(-x) = (-x)^5 + (-x)$

$= -x^5 - x$

$= -(x^5 + x)$

$= -f(x)$

so it's an odd function

(b) $g(x) = 1 - x^4$.

(b) $g(x) = 1 - x^4$

$\Rightarrow g(-x) = 1 - (-x)^4$

$= 1 - x^4$

$= g(x)$

So, $g(x)$ is an even function.

(c) $h(x) = 2x - x^2$.

(c) $h(x) = 2x - x^2$

$h(-x) = 2 \times (-x) - (-x)^2$

$= -2x - x^2$

So, $h(x)$ is neither even nor odd

Ex.12: Determine whether each of the following functions is even, odd, or neither even nor odd.

(a) $f(x) = x^4 - 4x^2$.

(a) $f(x) = x^4 - 4x^2$

$\Rightarrow f(-x) = (-x)^4 - 4(-x)^2$

$= x^4 - 4x^2$

so, $f(x)$ is an even function.

(b) $g(x) = \dfrac{1}{x^3 - x}$.

(b) $g(x) = \dfrac{1}{x^3 - x}$

$\Rightarrow g(-x) = \dfrac{1}{(-x)^3 - (-x)}$

$= \dfrac{1}{-x^3 + x}$

$= -\dfrac{1}{x^3 - x}$

$= -g(x)$ so, $g(x)$ is an odd function.

(e) $h(x) = x^3+1$

$\Rightarrow h(-x) = -x^3+1$

so, $h(x)$ is neither even nor odd.

Trick

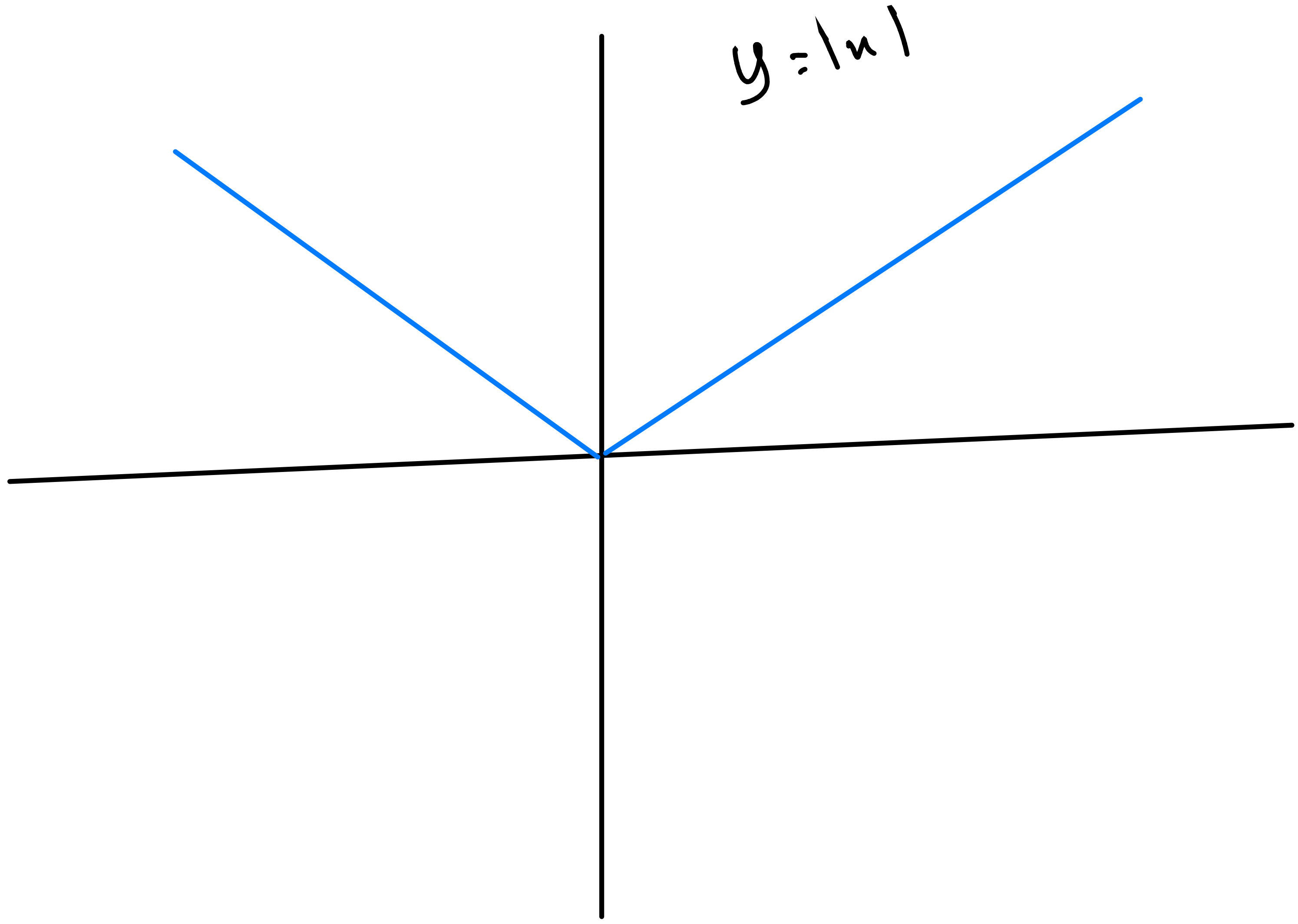
If, we have a function where all the power is even and has a constant it is an even function.

If we have only odd power and no constant then it is odd function.

Absolute value function

$$f(x) = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

$$\boxed{\sqrt{x^2} = |x|}$$



Rules

Suppose $a>0$ then,

$|x| = a$ if and only if $x = \pm a$

$|x| < a$ if and only if $-a < x < a$.

$|x| > a$ if and only if $x > a$ or $x < -a$.

$|x| = |b|$ if and only if $x = b$ or $x = -b$

Ex.14 (Example 7, P. 16): A function f is defined by

$$f(x) = \begin{cases} 1 - x & \text{if } \leq -1, \\ x^2 & \text{if } x > -1. \end{cases}$$

Evaluate $f(-2)$, $f(-1)$, and $f(0)$, then sketch the graph of f.

$$f(-2) = 1 - (-2)$$
$$= 1 + 2 = 3.$$

$$f(-1) = 1 - (-1)$$
$$= 1 + 1 = 2$$

$$f(0) = 0^2 = 0$$

Ex.13: Rewrite the expression without using the absolute value symbol:

(a) $f(x) = |x - 3|$.

$$f(x) = |x-3| = \begin{cases} x-3 & \text{if } x-3 \geqslant 0 \\ -(x-3) & \text{if } x-3 < 0 \end{cases}$$

(b) $g(x) = |2x - 1|$.

$$g(x) = |2x-1| = \begin{cases} 2x-1 & \text{if } 2x-1 \geqslant 0 \\ -(2x-1) & \text{if } 2x-1 < 0 \end{cases}$$