$$\frac{d}{dx}\left[\frac{1}{1+e^{-x}}\right] = \frac{d}{dx}\left[\left(1+e^{-x}\right)^{-1}\right] = -\frac{1}{(1+e^{-x})^2}\frac{d}{dx}\left(1+e^{-x}\right)$$

$$e^{-x}=e^{-x} \qquad -x=-1$$

$$= -\frac{1}{(1+e^{-x})^2}\frac{d}{dx}\left(e^{-x}\right) = -\frac{e^{-x}}{(1+e^{-x})^2}\frac{d}{dx}\left(-x\right) = \frac{e^{-x}}{(1+e^{-x})^2}$$

$$\frac{1\cdot e^{-x}}{(1+e^{-x})(1+e^{-x})} = \boxed{\frac{1}{1+e^{-x}}} \cdot \frac{e^{-x}}{1+e^{-x}} = \sigma(x)\left[\frac{e^{-x}}{1+e^{-x}}\right]$$

$$= \sigma(x)\left[\frac{1+e^{-x}-1}{1+e^{-x}}\right] = \sigma(x)\left[\frac{1+e^{-x}}{1+e^{-x}} - \frac{1}{1+e^{-x}}\right]$$

$$\sigma(x)\left[1-\sigma(x)\right] \Rightarrow \sigma'(x) = \boxed{\sigma(x)\left[1-\sigma(x)\right]}$$