

$x = \{x_1, x_2, x_3, \dots, x_n\} \rightarrow \text{column}$

$C_k = \{c_1, c_2, c_3, \dots, c_k\} \rightarrow \text{class}$

From previous example,

toss	value	outlook	result
x_1	x_2	x_3	$c_1 = \text{won}$
			$c_2 = \text{loss}$

Our final task would be find,

$$P(C_k | x)$$

We know,

$$P(C_k | x) = P(x | C_k) P(C_k) \dots \text{--- (1)}$$

according to conditional probability:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$\Rightarrow P(A|B) P(B) = P(A \cap B) \dots \textcircled{ii}$$

From \textcircled{i} and \textcircled{ii}

$$P(C_k | x) = P(x \cap C_k) \quad \text{we can replace } \cap \rightarrow ,$$

$$P(C_k | n) = P(n, C_k)$$

$$= P(\underbrace{n_1}_a, \underbrace{n_2, n_3, \dots, n_n}_b | C_k) = P(a | b) P(b)$$

$$= P(n_1 | n_2, n_3, \dots, n_n, C_k) P(b)$$

$$\Rightarrow P(C_k | x) = \alpha \times P(n_2, n_3, \dots, n_n, C_k)$$

same as, $P(n_2, n_3, \dots, n_n, C_k) \times \alpha$

$$= \alpha \times P(n_2 | n_3, \dots, n_n, C_k) P(n_3, \dots, n_n, C_k)$$

$$= \alpha \times \beta \times P(n_3, \dots, n_n, C_k)$$

$$= P(n_1 | n_2, \dots, n_n, C_k) P(n_2 | n_3, \dots, n_n, C_k)$$

$$P(n_3 | n_4 \dots n_n, C_k) \dots$$

$$P(n_n, C_k) P(C_k)$$

Now, we will assume n_1 is not depend on $n_2, n_3 \dots n_n$, it depends on only C_k . We make this assumption cause the event is rare, maximum times the event never exist. That's why we assume they are conditionally independent. and only depends on C_k .

Now we will simplify the entire equation and we have,

$$= P(n_1 | C_k) P(n_2 | C_k) P(n_3 | C_k) \dots P(C_k)$$

$$\text{So, } P(c_k | x) \propto P(c_k) \prod_{i=1}^n P(n_i | c_k)$$

\propto , cause we neglected the $P(n)$

$$\text{So, } \hat{y} = \underset{k \in \{1, 2, \dots, K\}}{\text{argmax}} \quad P(c_k) \prod_{i=1}^n P(n_i | c_k)$$

Maximum a posterior Rule.