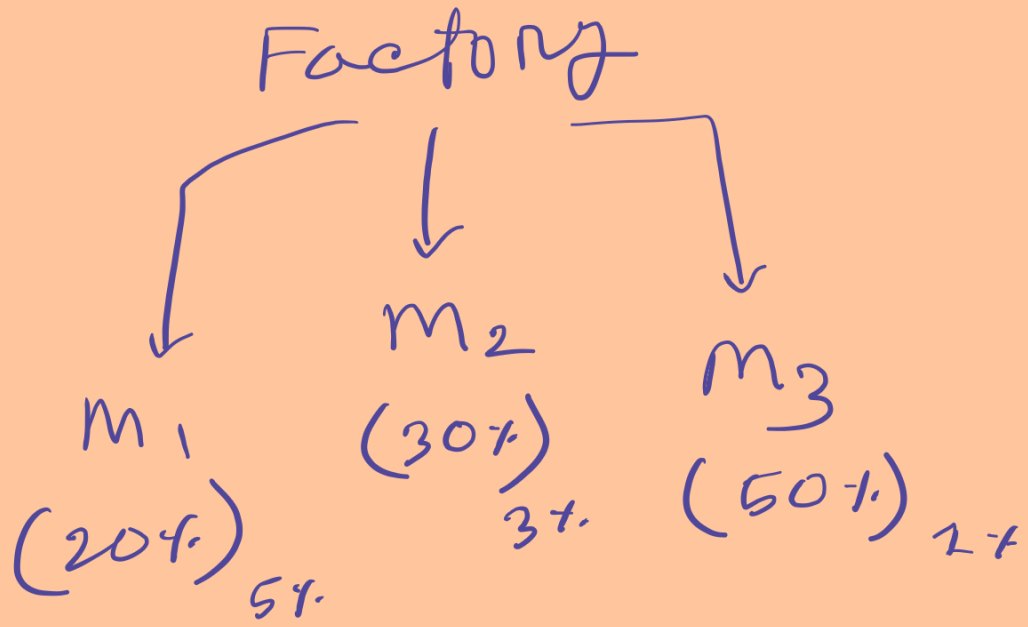
* A factory with 3 machines



→ M_1 producing 20% of the product and so on.

→ M_1 has 5% of manufacturing defects. and so on.

Question:

1 product is choosen and the product is defective.

*Find the probability that the product came from M_3.

Ans:

$$P(M_1) = \frac{1}{5}, \quad P(M_2) = \frac{3}{10}$$

$$P(M_3) = \frac{1}{2}$$

$$P(D|M_1) = \frac{1}{20}, \quad P(D|M_2) = \frac{3}{100}$$

$$P(D|M_3) = \frac{1}{100}$$

we need to find,

$$P(M_3|D) = ?$$

$$= \frac{P(D|M_3)P(M_3)}{P(D)} \quad \ldots (1)$$

we know

Now, $P(D) = P(D \cap M_1) + P(D \cap M_1) + P(D \cap M_3)$

We know, $P(A|B) = \frac{P(A \cap B)}{P(B)}$

So, $P(D) = P(D|M_1)P(M_1) + P(D|M_2)P(M_2) + P(D|M_3)P(M_3)$

$= \frac{1}{20} \times \frac{1}{5} + \frac{3}{100} \times \frac{3}{10} + \frac{1}{100} \times \frac{1}{2}$

$= \frac{1}{100} + \frac{9}{1000} + \frac{1}{200}$

$= \frac{10+9+5}{1000} = 0.024$

$\therefore$ (D=5) $\frac{\frac{1}{100} \times \frac{1}{2}}{0.024}$

$= 0.2083$