Assignment - 3

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MAT - 116.15

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32) $f(x) = x^3 - 3x^2 - 6x + 8$

$f(-2) = -8 - 12 + 12 + 8 = 0$

Synthetic division,

$$\begin{array}{r|rrrr} -2 & 1 & -3 & -6 & 8 \\ & & -2 & 10 & -8 \\ \hline & 1 & -5 & 4 & 0 \end{array}$$

$\therefore f(x) = (x+2)(x^2 - 5x + 4)$

$= (x+2)(x^2 - 4x - x + 4)$

$= (x+2)(x-1)(x-4)$

$\therefore (x+2)(x-1)(x-4) = 0$

$\therefore$ Real zeroes of $f(x) = -2, 1, 4$

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33) $f(x) = 4x^3 + 4x^2 - 7x + 2$

If $(x = -2)$,

$f(-2) = -32 + 16 + 14 + 2 = 0$

Synthetic Division :

$$\begin{array}{r|rrrr} -2 & 4 & 4 & -7 & 2 \\ & & -8 & -8 & -2 \\ \hline & 4 & -4 & 1 & 0 \end{array}$$

$\therefore f(x) = 0$

$\therefore (x+2)(4x^2 - 4x + 1) = 0$

$\Rightarrow (x+2)(4x^2 - 2x - 2x + 1) = 0$

$\Rightarrow (x+2)(2x-1)(2x-1) = 0$

$\Rightarrow (x+2)(2x-1)^2 = 0$

$\therefore$ Real zeroes $= -2, \frac{1}{2}$

(Ans.)

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34| $f(x) = x^4 - 4x^3 + 9x^2 - 20x + 20$

if $x = 2$,

$$f(2) = 16 - 32 + 36 - 40 + 20 = 0$$

Synthetic Division,

$$\begin{array}{r|rrrrr} 2 & 1 & -4 & 9 & -20 & 20 \\ & & 2 & -4 & 10 & -20 \\ \hline & 1 & -2 & 5 & -10 & 0 \end{array}$$

$\therefore f(x) = 0$

$\Rightarrow (x-2)(x^3 - 2x^2 + 5x - 10) = 0$

$\Rightarrow$ Suppose, $g(x) = x^3 - 2x^2 + 5x - 10$

$$g(2) = 8 - 8 + 10 - 10 = 0$$

Synthetic Division,

$$\begin{array}{r|rrrr} 2 & 1 & -2 & 5 & -10 \\ & & 2 & 0 & 10 \\ \hline & 1 & 0 & 5 & 0 \end{array}$$

$\therefore f(x) = (x-2)(x-2)(x^2+5) = 0$

$\Rightarrow (x-2)^2(x^2+5) = 0$

$x = 2$, multiciplity 2 (real zeroes)

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35) $f(x) = 2x^4 + 2x^3 - 11x^2 + x - 6 = 0$

Here,

$f(2) = 32 + 16 - 44 + 2 - 6 = 0$

Synthetic Division,

$$\begin{array}{r|rrrrr} 2 & 2 & 2 & -11 & 1 & -6 \\ & & 4 & 12 & 2 & 6 \\ \hline & 2 & 6 & 1 & 3 & 0 \end{array}$$

$\therefore f(x) = (x-2)(2x^3 + 6x^2 + x + 3) = 0$

Suppose, $g(x) = 2x^3 + 6x^2 + x + 3$

$g(-3) = -54 + 54 - 3 + 3 = 0$

Synthetic Division,

$$\begin{array}{r|rrrr} -3 & 2 & 6 & 1 & 3 \\ & & -6 & 0 & -3 \\ \hline & 2 & 0 & 1 & 0 \end{array}$$

$\therefore f(x) = (x-2)(x+3)(2x^2+1) = 0$

$\therefore$ real zeros: $x = 2, -3$

(Ans.)

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36) $f(x) = 2x^4 + 7x^3 + x^2 - 7x - 3 = 0$

$f(1) = 2 + 7 + 1 - 7 - 3 = 0$

$$\begin{array}{r|rrrrr} 1 & 2 & 7 & 1 & -7 & -3 \\ & & 2 & 9 & 10 & 3 \\ \hline & 2 & 9 & 10 & 3 & 0 \end{array}$$

$\therefore f(x) = (x-1)(2x^3 + 9x^2 + 10x + 3) = 0$

Suppose, $g(x) = 2x^3 + 9x^2 + 10x + 3$

$g(-1) = -2 + 9 - 10 + 3 = 0$

Synthetic Division,

$$\begin{array}{r|rrrr} -1 & 2 & 9 & 10 & 3 \\ & & -2 & -7 & -3 \\ \hline & 2 & 7 & 3 & 0 \end{array}$$

$\therefore f(x) = (x-1)(x+1)(2x^2 + 7x + 3) = 0$

$= (x-1)(x+1)(2x+1)(x+3) = 0$

$\therefore$ Real zeroes, $x = 1, -1, -\frac{1}{2}, -3$

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37| $f(x) = x^3 - x^2 - 4x + 2$

According to Bounds of Zeros Theorem:

bound: 1 : Absolute value of the largest co efficient + 1 = 4+1 = 5

bound .2: Sum of all absolute values of co efficients except the leading one = 1+4+2 = 7

The smallest of these two is 5

$\therefore \quad -5 \leq \text{zeroes} \leq 5$

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38| $f(x) = 2x^3 - 7x^2 - 10x + 35$

According to Bounds of Zero's theorem,

bound 1 : Absolute value of the largest coefficient $+ 1 = 10 + 1 = 11$

bound 2 : Sum of all absolute values of coefficients except the leading one $= 7 + 10 + 35$
$= 52$

The smallest of these two is 11

$\therefore \quad -11 \leq \text{zeroes} \leq 11$

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39] $f(x) = x^3 - 2x^2 + 3x - 1$ [0:2]

$f(0) = 0 - 0 + 0 - 1 = -1$

$f(2) = 8 - 8 + 6 - 1 = 5$

According to intermediate value theorem,

$f(0) < 0$ and $f(2) > 0$, so

the polynomial function has

at least one zero between 0 and 2

40] $f(x) = 8x^4 - 4x^3 - 2x - 1$; [0:1]

$f(0) = -1$

$f(1) = 8 - 4 - 2 - 1$

$= 1$

According to intermediate value theorem

$f(0) < 0$ and $f(1) > 0$, so the polynomial

function has at least one zero between

0 and 1

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41) $f(x) = x^3 - x - 2$

$f(1) = 1 - 1 - 2 = -2$

$f(2) = 8 - 2 - 2 = 4$

Here, $f(1) < 0$ and $f(2) > 0$,

there ~~zero~~ must be a positive zero between 1 and 2.

Solving for Sub intervals:

$f(1.1) = -1.769$

$f(1.3) = -1.103$

$f(1.5) = -0.125$

$f(1.6) < 0.496$

The ~~zo~~ positive zero must be between 1.5 and 1.6

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Solving for ~~into~~ sub intervals :

$f(1.51) = -0.067049$

$f(1.52) = -8.192 \times 10^{-3}$

$f(1.53) = 0.051577$

~~[1.0]~~

$\therefore$ The answer is 1.52

(Ans.)

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42) $f(x) = 8x^4 - 4x^3 - 2x - 1$

Here,

$f(0) = -1$

$f(1) = 8 - 4 - 2 - 1 = 1$

Here, $f(0) < 0$ and $f(1) > 1$, therefore there must be a positive zero between 0 and 1

Solving for sub intervals:

$f(.5) = -2$

$f(.7) = -1.3712$

$f(.9) = -.4672$

$f(1) = 1$

$f(.9) < 0$ and $f(1) > 0$, there must be one positive zero between them

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Solving for Sub intervals :

$f(.91) = -0.34826712$

$f(.93) = -0.09301192$

$f(.94) = 0.0436$

The zero lies between ~~1.23~~ 0.93 and ~~1.24~~ 0.94 and so, correct two decimal places, the zero is 0.93.

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42) Degree 3; Zeroes: $2+3i$, 4

According to the conjugate pair theorem,

the remaining zero is $2-3i$.

Now, $f(x) = a(x-4)(x-2-3i)(x-2+3i)$

$= a(-3ix + x^2 - 6x + 12i + 8)(x-2+3i)$

$= a(x^3 - 8x^2 + 29x - 52)$

$\therefore$ Remaining zeroes, $2-3i$

$f(x) = a(x^3 - 8x^2 + 29x - 52)$

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44) Degree 5, zeroes $6-i$, $2+5i$, 3

~~$f(x) = a(x-3$~~

According to Conjugate Pairs theorem, the remaining zeroes are: $6+i$, $2-5i$

$$f(x) = a(x-3)(x-6+i)(x-6-i)(x-2-5i)(x-2+5i)$$

$$= a(ix + x^2 - 9x - 3i + 18)(x-6-i)(x-2-5i)(x-2+5i)$$

$$= a(x^3 - 15x^2 + 73x - 111)(x-2-5i)(x-2+5i)$$

$$= a(x^4 - 5ix^3 - 17x^3 + 103x^2 + 75ix^2 - 365ix - 257x + 555i + 222)(x-2+5i)$$

$$= a(x^5 - 19x^4 + 162x^3 - 838x^2 + 2561x - 3219)$$

(Ans.)

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95] $f(x) = x^3 - 3x^2 - 6x + 8$

If $(x = -2)$

$f(-2) = -8 - 12 + 12 + 8 = 0$

Synthetic Division,

$$\begin{array}{r|rrrr} -2 & 1 & -3 & -6 & 8 \\ & & -2 & 10 & -8 \\ \hline & 1 & -5 & 4 & 0 \end{array}$$

$\therefore f(x) = (x+2)(x^2 - 5x + 4) = 0$

$\Rightarrow (x+2)(x^2 - 4x - x + 4) = 0$

$\Rightarrow (x+2)(x-1)(x-4) = 0$

$\therefore$ Complex zeroes: $-2, 1, 4$

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4b) $f(x) = 4x^3 + 4x^2 - 7x + 2$

If $x = -2$,

$f(-2) = -32 + 16 + 14 + 2 = 0$

Synthetic Division,

$$\begin{array}{r|rrrr} -2 & 4 & 4 & -7 & 2 \\ & & -8 & 8 & -2 \\ \hline & 4 & 4 & 1 & 0 \end{array}$$

$\therefore f(x) = (x+2)(4x^2 - 4x + 1) = 0$

$\Rightarrow (x+2)(4x^2 - 2x - 2x + 1) = 0$

$\Rightarrow (x+2)(2x-1)^2$

$\therefore$ Complex zeroes: $-2, \frac{1}{2}$; multiplicity 2

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47|

$f(x) = x^4 - 4x^3 + 9x^2 - 20x + 20$

$f(2) = 16 - 32 + 36 - 40 + 20 = 0$

Synthetic Division,

$$\begin{array}{r|rrrrr} 2 & 1 & -4 & 9 & -20 & 20 \\ & & 2 & -4 & 10 & -20 \\ \hline & 1 & -2 & 5 & -10 & 0 \end{array}$$

$\therefore f(x) = (x-2)(x^3 - 2x^2 + 5x - 10)$

Suppose, $g(x) = x^3 - 2x^2 + 5x - 10$

$g(2) = 8 - 8 + 10 - 10 = 0$

Synthetic Division,

$$\begin{array}{r|rrrr} 2 & 1 & -2 & 5 & -10 \\ & & 2 & 0 & 10 \\ \hline & 1 & 0 & 5 & 0 \end{array}$$

$\therefore f(x) = (x-2)(x-2)(x^2+5)$ | $x^2 + 5 = 0$, $x = \pm\sqrt{5}\,i$

$\therefore$, complex zeroes: 2; multiplicity 2, $\sqrt{5}\,i$, $-\sqrt{5}\,i$

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48) $f(x) = 2x^4 + 2x^3 - 11x^2 + x - 6$

$f(2) = 32 + 16 - 44 + 2 - 6 = 0$

Synthetic Division,

$$\begin{array}{r|rrrrr} 2 & 2 & 2 & -11 & 1 & -6 \\ & & 4 & 12 & 2 & 6 \\ \hline & 2 & 6 & 1 & 3 & 0 \end{array}$$

$\therefore f(x) = (x-2)(2x^3 + 6x^2 + x + 3) = 0$

Suppose, $g(x) = 2x^3 + 6x^2 + x + 3$

$g(-3) = -54 + 54 - 3 + 3 = 0$

Synthetic Division,

$$\begin{array}{r|rrrr} -3 & 2 & 6 & 1 & 3 \\ & & -6 & 0 & -3 \\ \hline & 2 & 0 & 1 & 0 \end{array}$$

$\therefore f(x) = (x-2)(x+3)(2x^2+1)$ | $2x^2+1=0$, $x = \pm\sqrt{\frac{1}{2}}\, i$, $= \pm \frac{\sqrt{2}}{2} i$

$\therefore$ Complex zeroes: $2, -3, \frac{\sqrt{2}}{2} i, -\frac{\sqrt{2}}{2} i$

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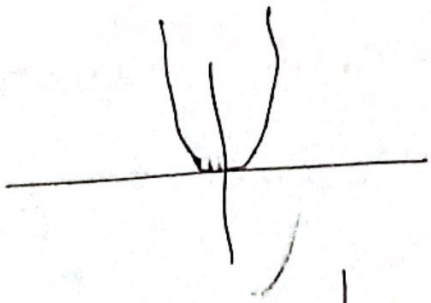
Chapter Test

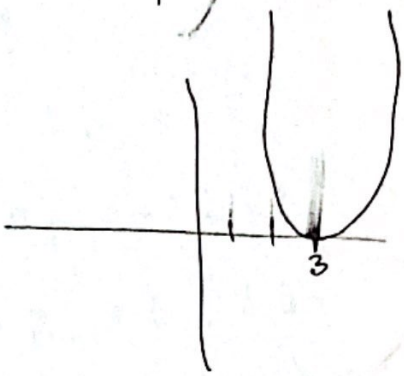
1. $\cancel{y =}\ f(x) = (x-3)^4 - 2$

$f(x) = x^4$



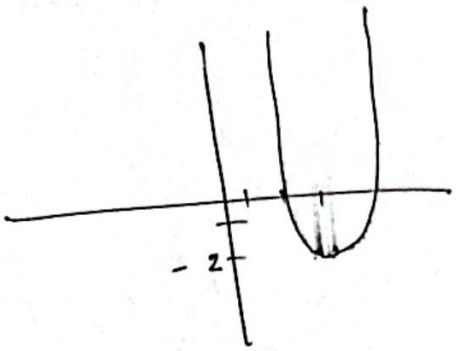
$f(x) = (x-3)^4$

shifted 3 units to the right



$f(x) = (x-3)^4 - 2$

shifted 2 units down



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2. $g(x) = 2x^3 + 5x^2 - 28x - 15$

(a) The maximum numbers of real zeroes is 3 because $g(x)$ is polynomial of degree 3.

(b) $2x^3 + 5x^2 - 28x - 15$

Here constant term $a_0 = -15 = P$

Factors of $P = \pm 1, \pm 3, \pm 5, \pm 15$

Factors of leading coefficients, 2

$\pm 1, \pm 2$

Form of all possible ratios, $\frac{P}{q}$

$\frac{\pm 1}{1}, \frac{\pm 3}{1}, \frac{\pm 5}{1}, \frac{\pm 15}{1}, \frac{\pm 1}{2}, \frac{\pm 3}{2}, \frac{\pm 5}{2}, \frac{\pm 15}{2}$

$\therefore$ So all zeroes are between ± 15.

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(C) $2x^3 + 5x^2 - 28x - 15 = g(x)$

$g(3) = 54 + 45 - 84 - 15 = 0$

Synthetic Division,

$$\begin{array}{r|rrrr} 3 & 2 & 5 & -28 & -15 \\ & & 6 & 33 & 15 \\ \hline & 2 & 11 & 5 & 0 \end{array}$$

$\therefore g(x) = (x-3)(2x^2 + 11x + 5) = 0$

$= (x-3)(2x^2 + 10x + x + 5) = 0$

$\Rightarrow (x-3)(2x+1)(x+5) = 0$

$\therefore x = 3, -5, -\frac{1}{2}$

$g(x) = (x-3)(2x+1)(x+5)$ (Ans:)

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(d)

From c,

$$g(x) = (x-3)(x+5)(2x+1) = 0$$

$\therefore$ x intercepts : $-5, -\frac{1}{2}, 3$

$$g(0) = -15$$

$\therefore$ y intercept $= -15$

(e) $g(x) = 2x^3 + 5x^2 - 28x - 15$

$= (x-3)(x+5)(2x+1)$

Here,

Each factor of $g(x)$ has odd power, so the graph crosses the x axis at every intercept.

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(f) $2x^3 + 5x^2 - 28x - 15$

The leading term of $g(x)$ is $2x^3$

So, the graph of $g(x)$ behaves like $y = 2x^3$ for large values of $|x|$

(g)

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(g) $g(x) = 2x^3 + 5x^2 - 28x - 15$

$\cancel{= (x+\frac{1}{2})(2x+5}$

$= (x+\frac{1}{2})(x+5)(x-3)$

x intercepts, $-\frac{1}{2}, -5, 3$

1. Factor of odd multiplicity, so crosses the graph of $g(x)$

2. Resembles the graph of $2x^3$

3. near $x = -5$,

$g(x) = (2\times(-5)+1)(x+5)(-5-3)$

$= 72(x+5)$ | $y = mx$

A line with slope 72

4. near $x = -\frac{1}{2}$,

$g(x) = (x+\frac{1}{2})(-\frac{1}{2}+5)(-\frac{1}{2}-3)$

$= -\frac{63}{2}(x+\frac{1}{2})$ | $y = mx$

A line with slope $-\frac{63}{2}$

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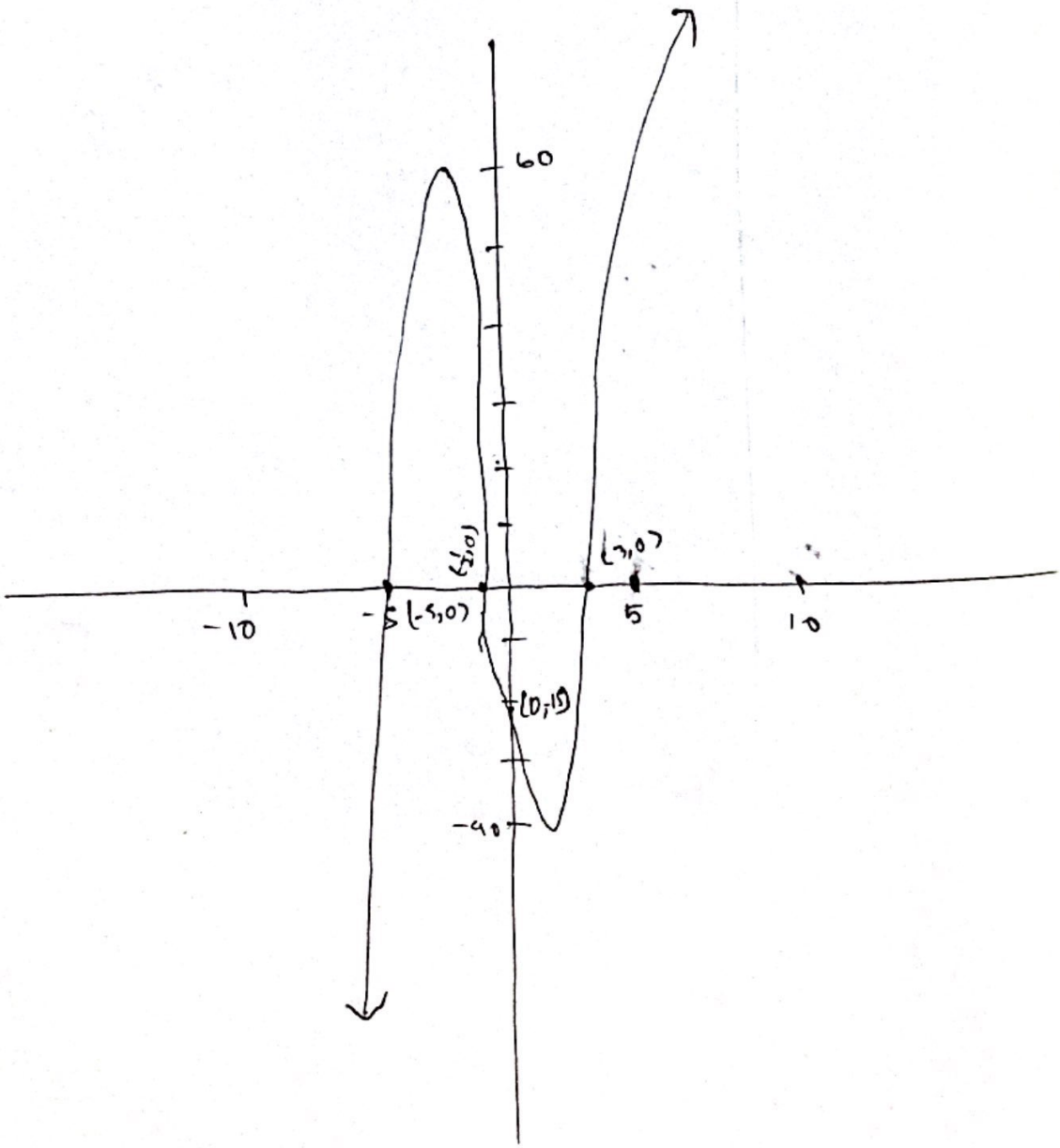
5. near $x = 3$,

$$g(x) = \left(3+\frac{1}{2}\right)(3+5)(x-3)$$

$$= 56(x-3) \quad \Big| \; y = mx$$

A line with slope 56



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3. $f(x) = x^3 - 4x^2 + 25x - 100$

$f(4) = 64 - 64 + 100 - 100$

$= 0$

Synthetic Division,

$$\begin{array}{r|rrrr} 4 & 1 & -4 & 25 & -100 \\ & & 4 & 0 & 100 \\ \hline & 1 & 0 & 25 & 0 \end{array}$$

$\therefore f(x) = (x-4)(x^2+25) = 0$

$x^2 + 25 = 0$ | $x - 4 = 0$

$\therefore x = \pm 5i$ | $x = 4$

$\therefore$ Complex zeroes $= 4, 5i, -5i$

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4. $3x^3 + 2x - 1 = 8x^2 - 4$

$\Rightarrow 3x^3 - 8x^2 + 2x + 3 = 0$

If, $g(x) = 3x^2 - 8x^2 + 2x + 3$

$g(1) = 3 - 8 + 2 + 3 = 0$

Synthetic division,

$$\begin{array}{r|rrrr} 1 & 3 & -8 & 2 & 3 \\ & & 3 & -5 & -3 \\ \hline & 3 & -5 & -3 & 0 \end{array}$$

$\therefore g(x) = (x-1)(3x^2 - 5x - 3) = 0$

Now,

$(x-1)(3x^2 - 5x - 3) = 0$

$\therefore x = 1$ or $x = \dfrac{5 \pm \sqrt{5^2 - 4 \times 3 \times (-3)}}{2 \times 3}$

$= \dfrac{5 \pm \sqrt{61}}{6}$

$\therefore x = 1, \dfrac{5 + \sqrt{61}}{6}, \dfrac{5 - \sqrt{61}}{6}$

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(5) $g(x) = \dfrac{2x^2 - 14x + 24}{x^2 + 6x - 40}$

Finding domain:

$$x^2 + 6x - 40 \neq 0$$

$$\Rightarrow x^2 + 10x - 4x - 40 \neq 0$$

$$\therefore (x-4)(x+10) \neq 0$$

$$\therefore x \neq 4, -10$$

$\therefore$ Domain all real numbers except $4, -10$.

Horizontal asymptotes:

The degree of the numerator and denominator are same.

Dividing the coefficients of the highest term:

$$\frac{2}{1} = 2$$

$\therefore$ Horizontal Asymptotes $y = 2$.

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Vertical Asymptote :

$$\frac{2x^2 - 14x + 24}{x^2 + 6x - 40}$$

$$\Rightarrow \frac{2(x-4)(x-3)}{(x-4)(x+10)} \Rightarrow \frac{2(x-3)}{x+10}$$

Here, $x + 10 = 0$

$\therefore x = -10$

$\therefore$ The vertical asymptote is $x = -10$.

Oblique asymptotes :

Degree of the numerator is not exactly 1 degree higher than the degree of denominator.

$\therefore$ No oblique asymptote.

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6. $r(x) = \dfrac{x^2+2x-3}{x+1}$

Finding Domain:

$$x+1 \neq 0$$

$$\therefore x \neq -1$$

$\therefore$ Domain $\mathbb{R} - \{-1\}$

Horizontal asymptote:

Here, numerator degree (2) is higher than the degree of the denominator by one.

$\therefore$ ~~Ho~~ No horizontal asymptote.

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Vertical Asymptote:

$$r(x) = \frac{x^2 + 2x - 3}{x+1}$$

$$= \frac{(x-3)(x-1)}{x+1}$$

Here, $x+1 = 0$

$\therefore x = -1$

$\therefore$ Vertical asymptote, $x = -1$

Oblique Asymptote:

$$\begin{array}{r|l} & x+1 \\ \hline x+1 & x^2+2x-3 \\ & x^2+x \\ \hline & x-3 \\ & x+1 \\ \hline & -4 \end{array}$$

The oblique asymptote is $y = x+1$

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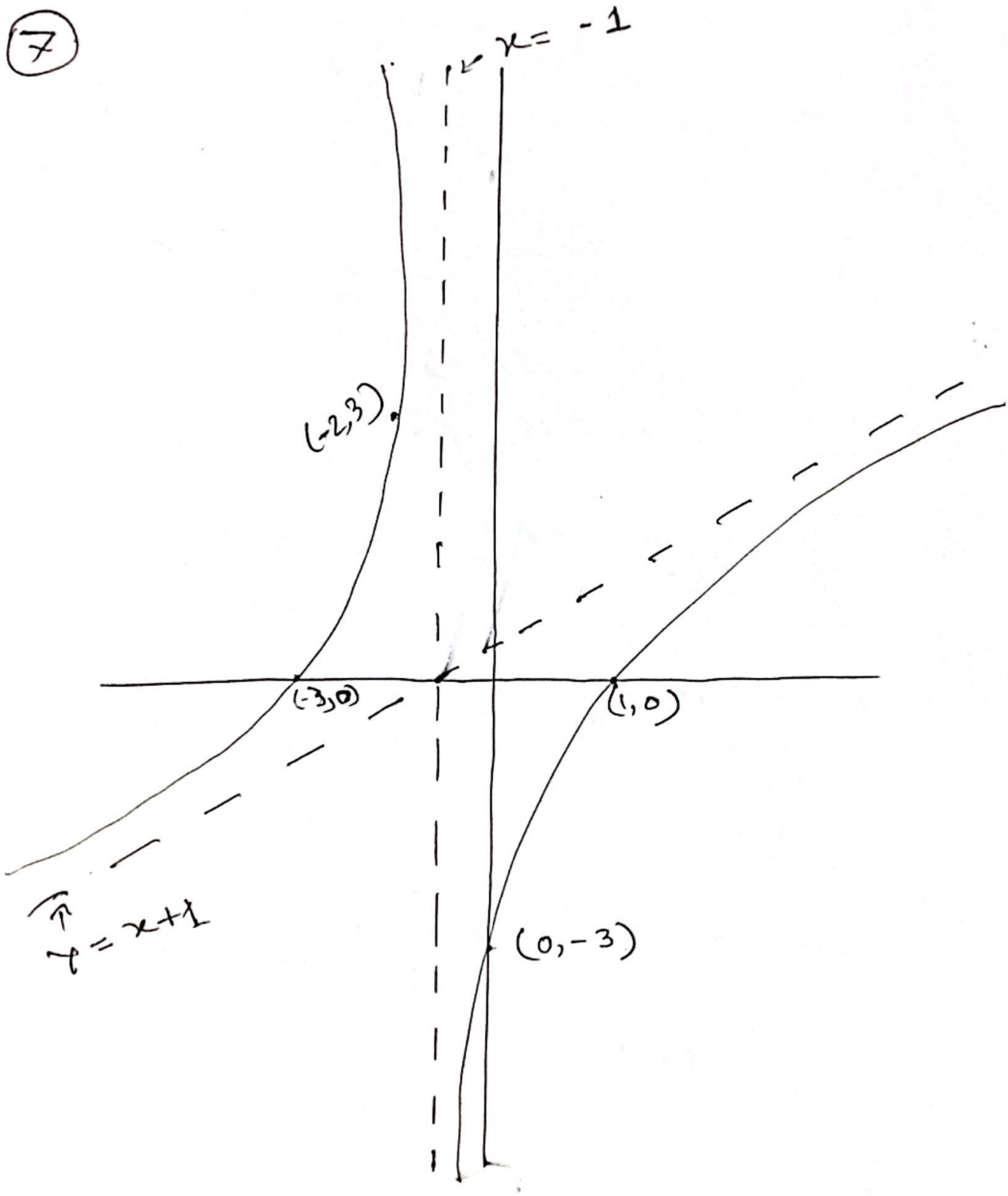

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8) Degree 4 ; Zeroes : $-2, 0, 3+i$

Remaining Zeroes : $3-i$

$$\therefore f(x) = x(x+2)(x-3-i)(x-3+i)$$

$$= (x^2+2x)(x^2-3x+xi-3x+9-3i-xi+3i-i^2)$$

$$= (x^2+2x)(x^2-6x+10)$$

$$= x^4-6x^3+10x^2+2x^3-12x^2+20x$$

$$= x^4-4x^3-2x^2+20x$$

$$\therefore f(x) = x^4-4x^3-2x^2+20x$$

(9) Rational function :

$$f(x) = \frac{2x(x-9)}{(x-4)(x-9)} = \frac{2x^2-18x}{x^2-13x+36}$$

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10) $f(x) = -2x^2 - 3x + 8$; $[0,4]$

$f(0) = -2(0)^2 - 3.0 + 8 = 8$

$f(4) = -2(4)^2 - 3.4 + 8 = -36$

Here,

$f(0) < 0$ and $f(4) > 0$; So,

According to the intermediate value theorem, the given function has at least one ~~ze~~ ~~o~~ zero ~~is~~ on the interval $[0,4]$

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$$\frac{x+2}{x-3} < 2 \quad ; \quad x \neq 3$$

$$\Rightarrow \frac{x+2}{x-3} - 2 < 0$$

$$\Rightarrow \frac{x+2-2(x-3)}{(x-3)} < 0$$

$$\Rightarrow \frac{x+2-2x+6}{x-3} < 0$$

$$= \frac{-x+8}{x-3} < 0$$

Here,

$-x+8 > 0$ and $x-3 < 0$

$\therefore x \in (-\infty, 3) \cup (8, \infty)$

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