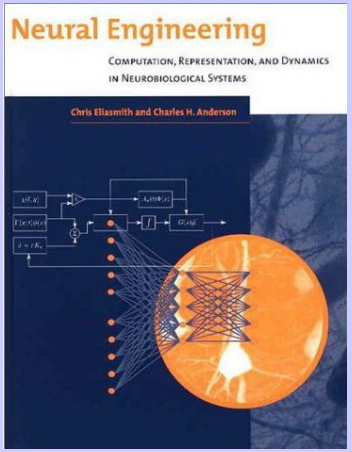
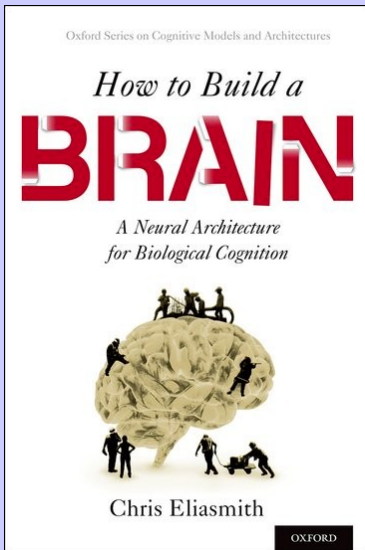
So far

- Neurons are good at algorithms of this form:
  - Vectors (with some random noise) x
  - Functions on vectors $y = f(x)$
  - Differential equations on vectors $\frac{dx}{dt} = f(x, u)$
  - Smooth functions are preferred





Symbols as Vectors

- Basic idea: each symbol is a vector
  - Pattern of numbers
  - Each vector is a different pattern of neural activity
  - BLUE, RED, CIRCLE, SQUARE, etc.
- Binding problem
  - How do we represent "red circle and blue square"?
  - Does just adding them together work?
  - What else could we do?

Vector Symbolic Architectures (Semantic Pointers)

- (Overview: Gayler, 2003)
- Use more than just addition to combine

$$\text{RED} \circledast \text{CIRCLE} + \text{BLUE} \circledast \text{TRIANGLE}$$

- The result is just a new vector pattern that can be stored
- Many possible choices for operation
  - We use Plate's suggestion of circular convolution
  - Neurons are good at computing it

Pattern Completion

1	11	111
4	44	444
5	55	?

S1 = ONE ⊛ P1

S2 = ONE ⊛ P1 + ONE ⊛ P2

S3 = ONE ⊛ P1 + ONE ⊛ P2 + ONE ⊛ P3

S4 = FOUR ⊛ P1

S5 = FOUR ⊛ P1 + FOUR ⊛ P2

S6 = FOUR ⊛ P1 + FOUR ⊛ P2 + FOUR ⊛ P3

S7 = FIVE ⊛ P1

S8 = FIVE ⊛ P1 + FIVE ⊛ P2

what is S9?

T1 = S2 ⊛ S1'

T2 = S3 ⊛ S2'

T3 = S5 ⊛ S4'

T4 = S6 ⊛ S5'

T5 = S8 ⊛ S7'

T = (T1 + T2 + T3 + T4 + T5)/5

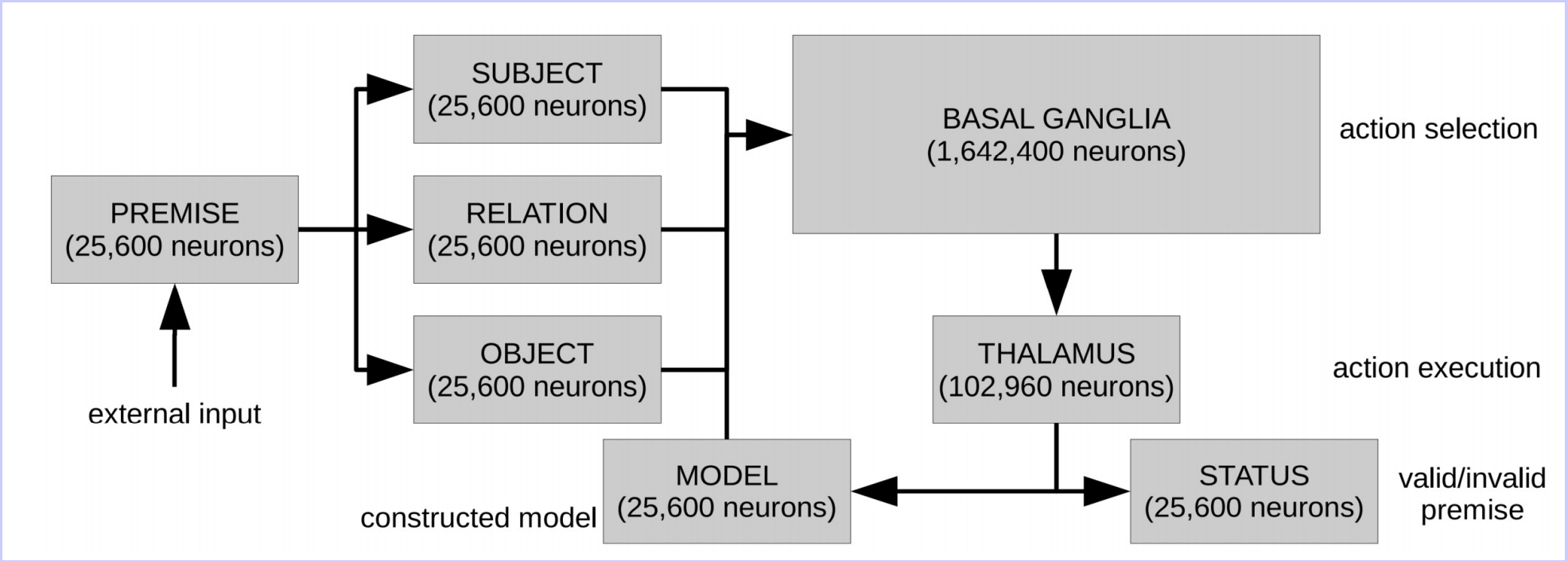
S9 = S8 ⊛ T

S9 = FIVE ⊛ P1 + FIVE ⊛ P2 + FIVE ⊛ P3

Relational Reasoning

- Julia Werthiem (Frieburg)
  - Spatial reasoning:
    - A is left of B
    - B is left of C
    - Is A left of C?
  - Preferred mental model theory
    - Build an internal representation of the two premises
    - Check if the last item is true in the mental model

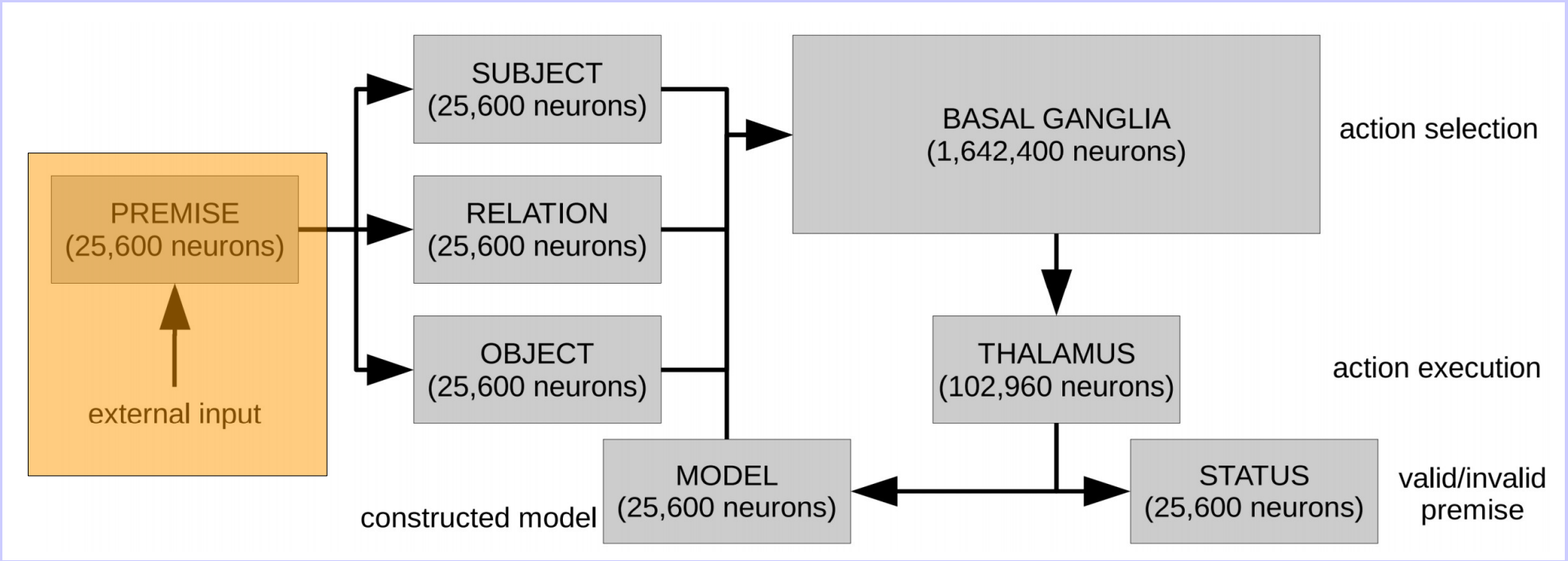
Relational Reasoning



- Vectors:

A B C D leftof rightof below above taller shorter subject relation object

Relational Reasoning

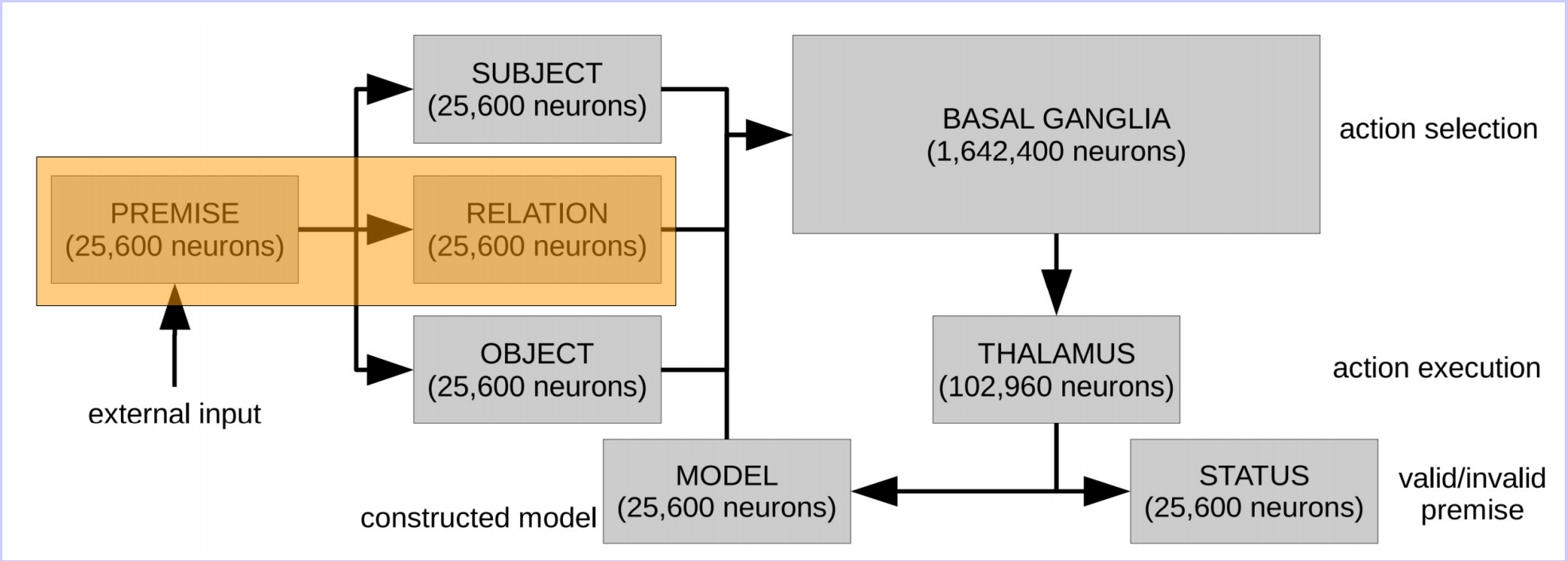


- Premise:

$$A \otimes subject + leftof \otimes relation + B \otimes object$$

(also just a vector!)

Relational Reasoning

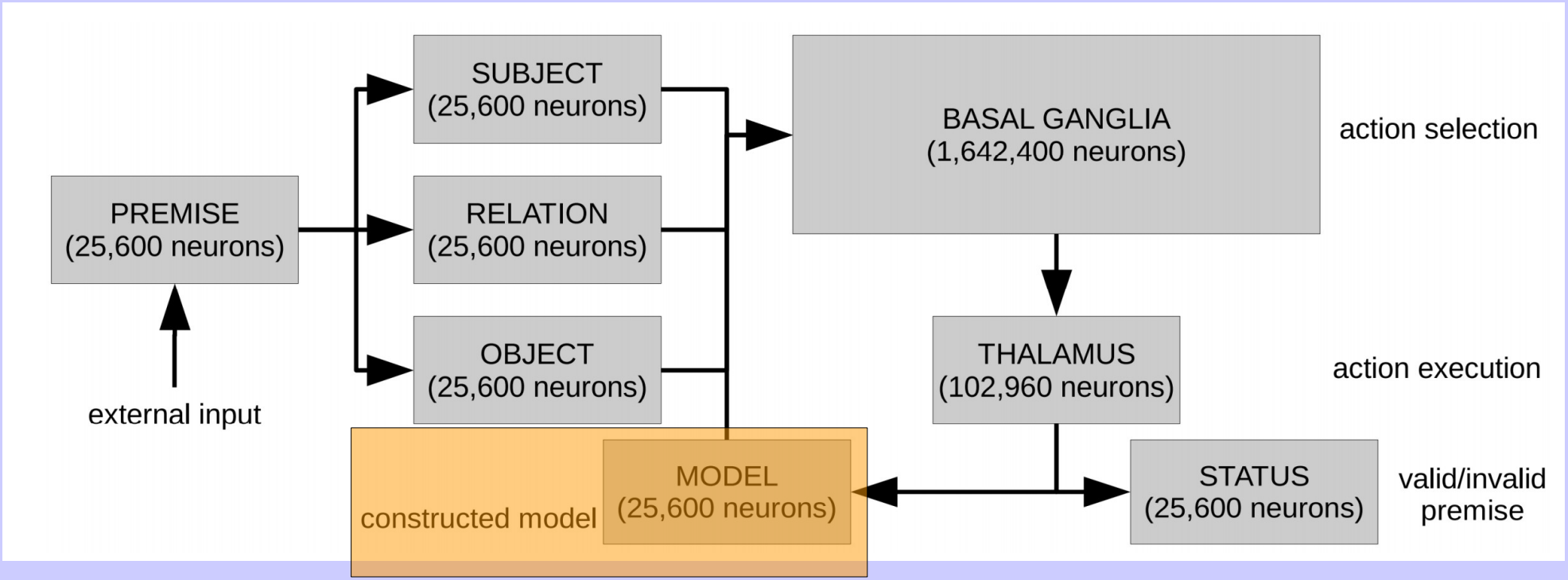


- Extract part of premise

$$relation = premise \otimes \mathbf{relation}^{-1}$$

  - optional: “round” the result to the nearest known vector
  - But that requires more neurons (scales linearly)

Relational Reasoning

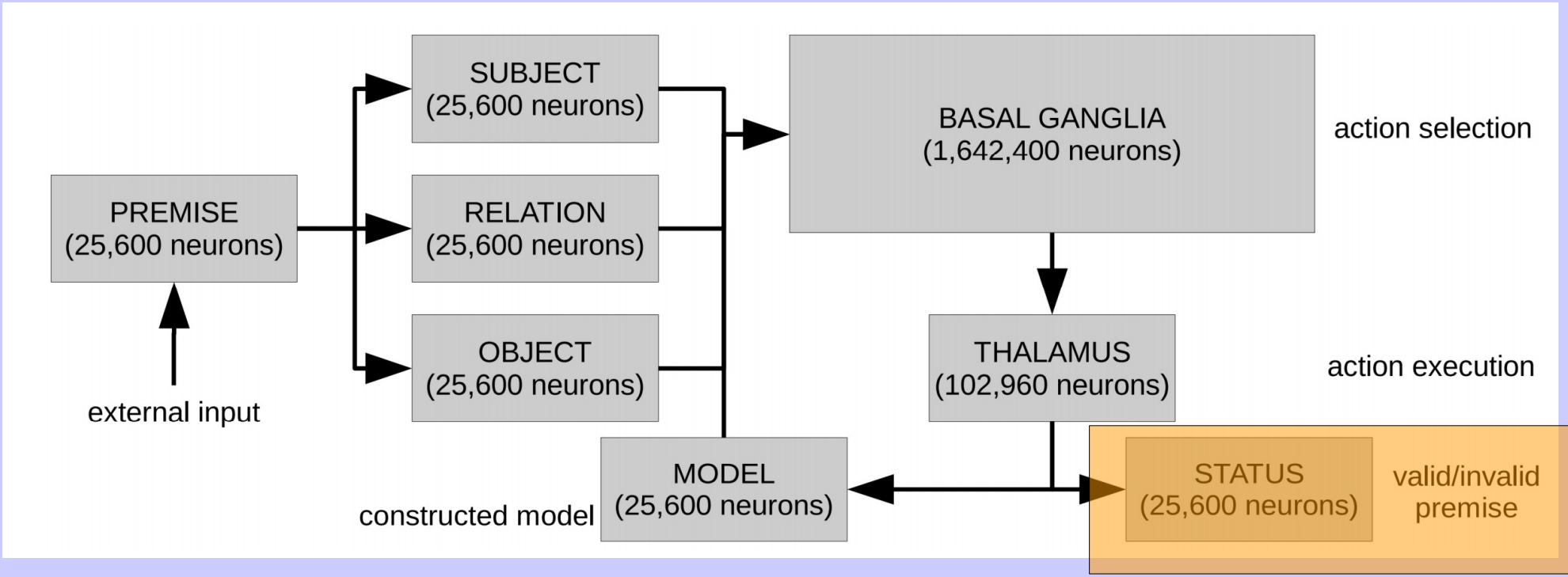


- The mental model
  - Multidimensional integrator

$$model = 0.3\mathbf{A} \otimes \mathbf{X} + 0.7\mathbf{B} \otimes \mathbf{X}$$

  - Handles as many objects and relations as desired!
  - (accuracy will gradually decrease….)

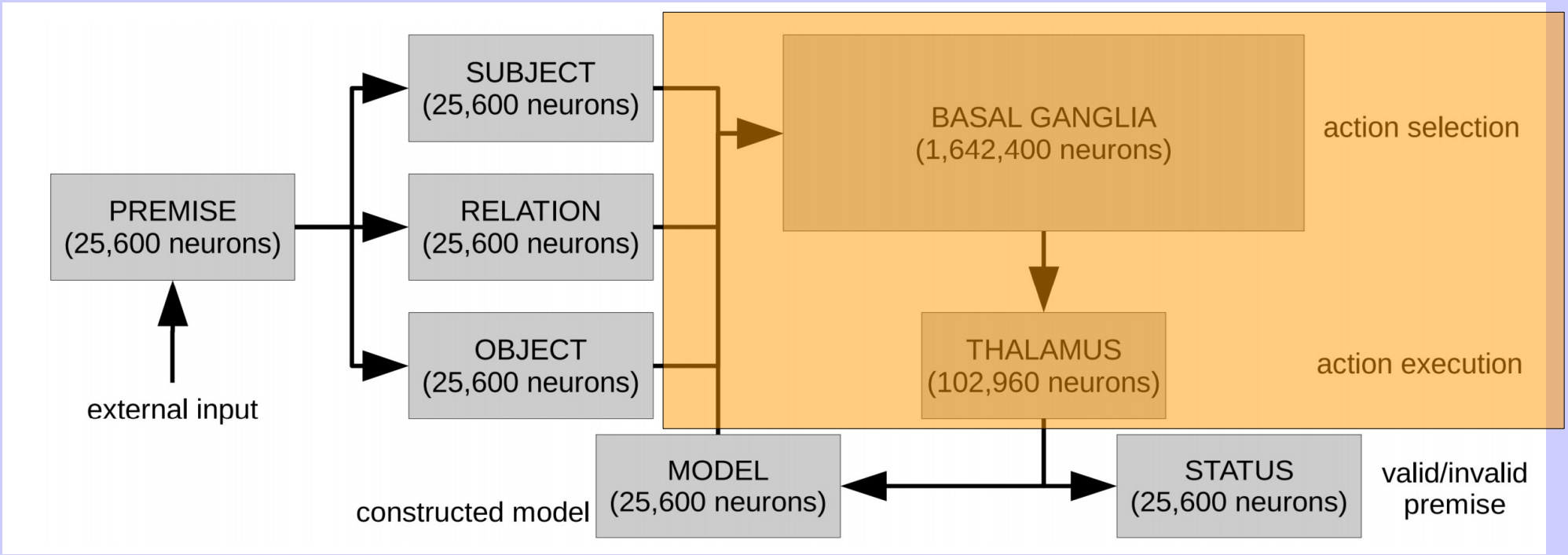
Relational Reasoning



- Status (reported output)

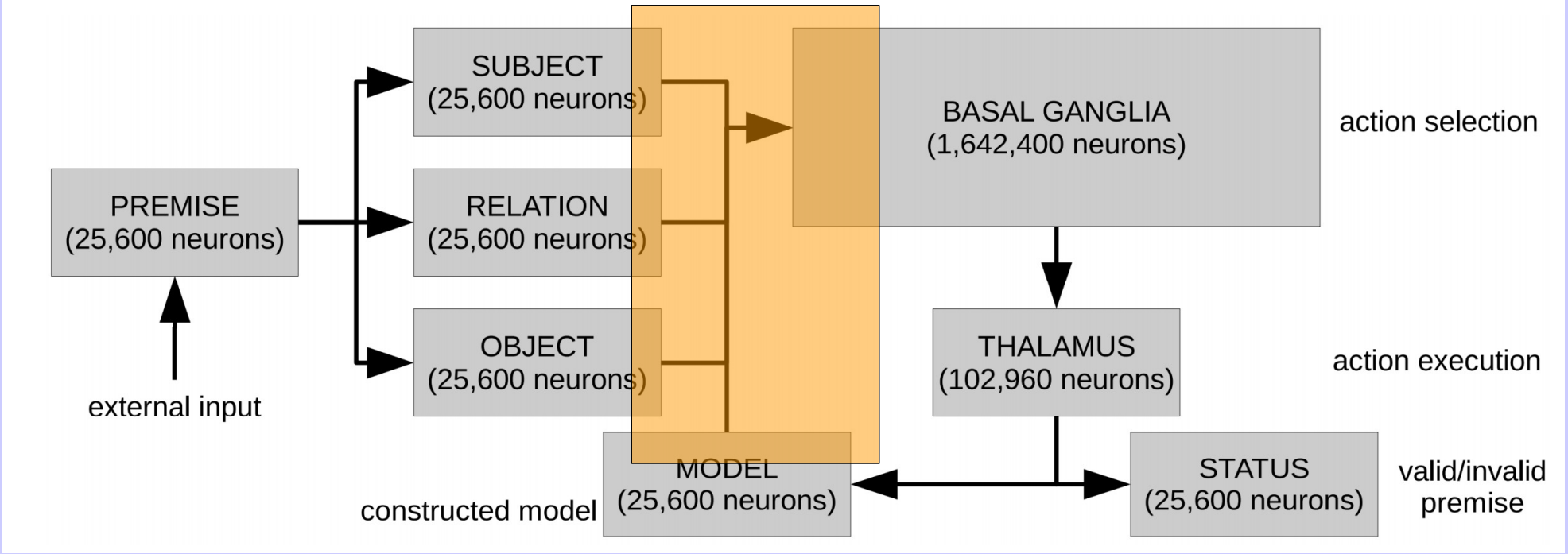
VALID **INVALID**

Relational Reasoning



- Cognitive control
  - If premise matches model
    - Set status to VALID
  - Otherwise
    - Adjust the model to make it more valid

Relational Reasoning

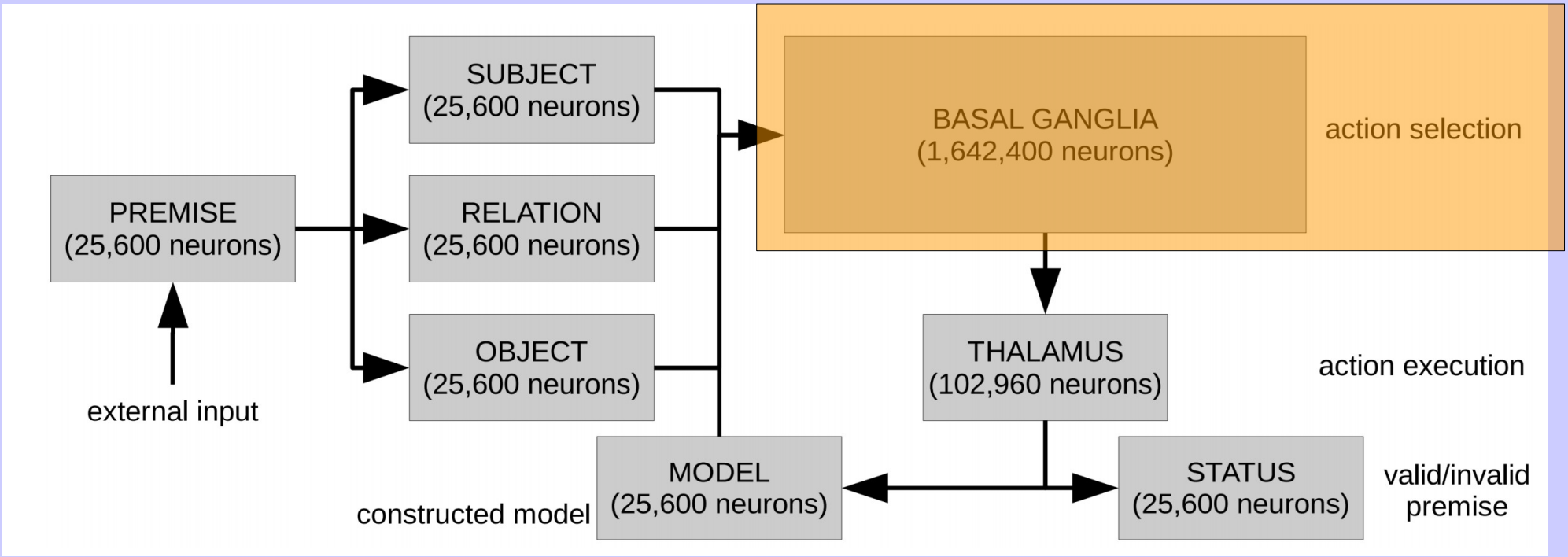


- Compute utility of actions

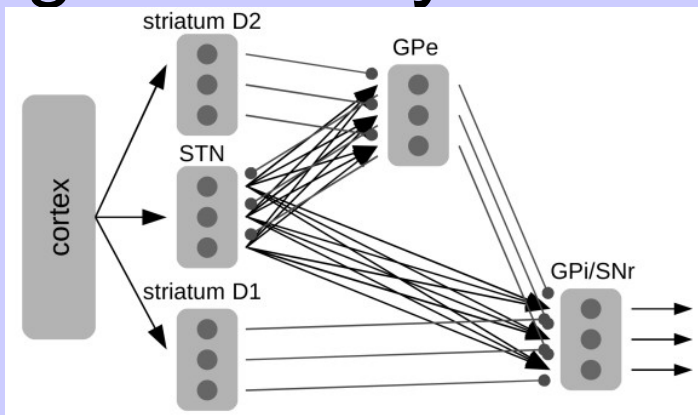
$$Q_{left,invalid} = relation \cdot \mathbf{leftof} + model \cdot (subject \otimes X) - model \cdot (object \otimes X)$$

$$Q_{left,valid} = relation \cdot \mathbf{leftof} + model \cdot (object \otimes X) - model \cdot (subject \otimes X)$$

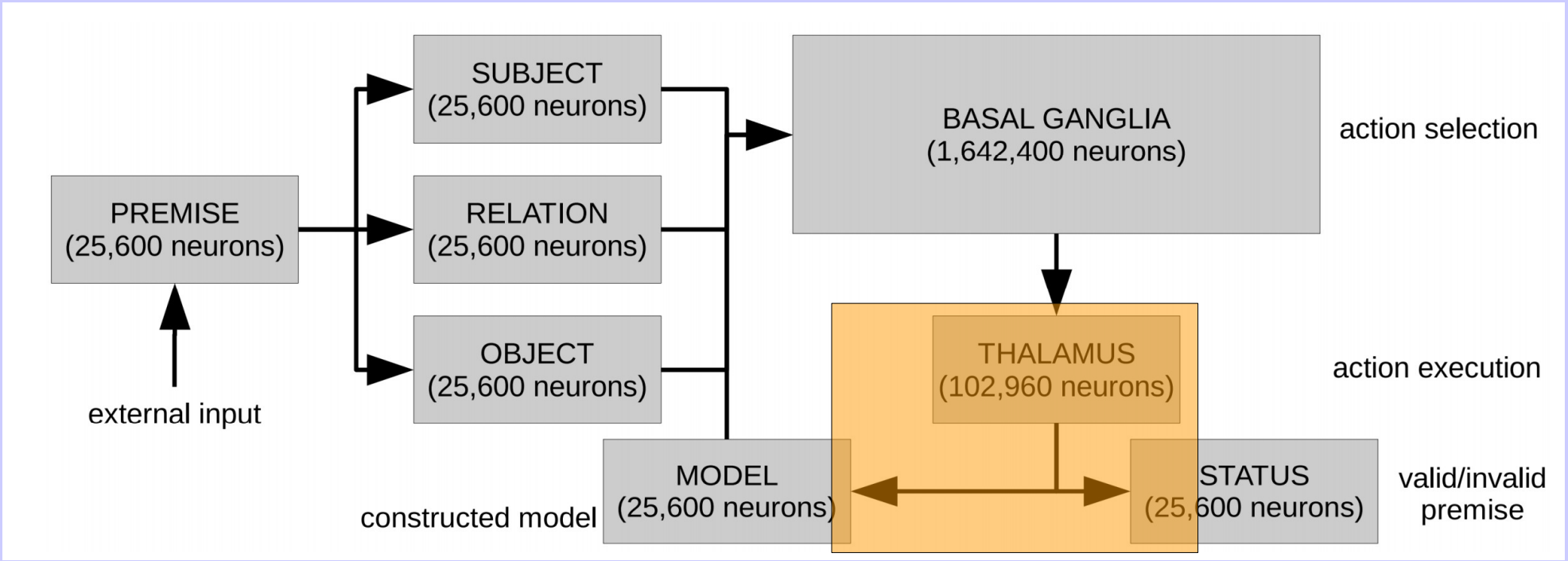
Relational Reasoning



- Determine which action has highest utility
  - (Gurney, Prescott, Redgrave, 2011) converted into spiking neurons



Relational Reasoning

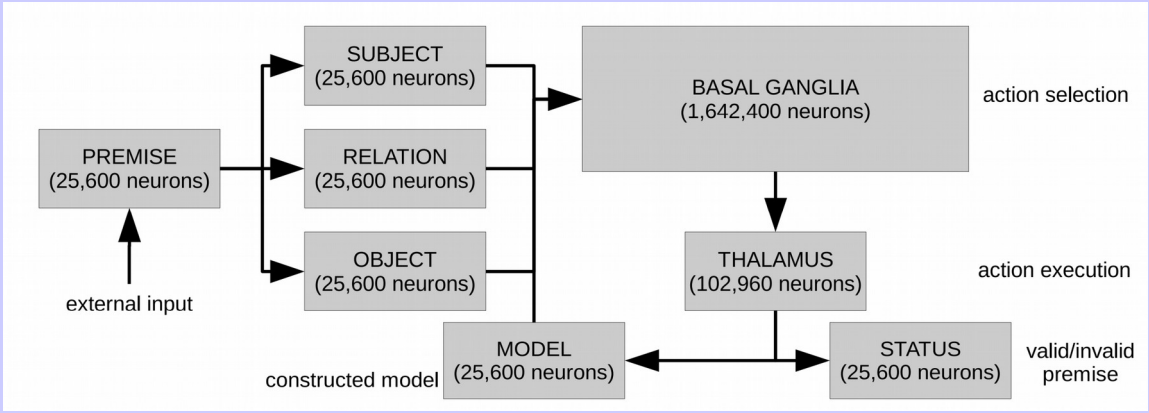


- Implement effect of actions

$$\frac{dmodel}{dt} = (subject \otimes X - object \otimes X) * A_{left,invalid} + \ldots$$

$$status = \mathbf{INVALID} * A_{left,invalid} + \mathbf{VALID} * A_{left,valid} + \ldots$$

Relational Reasoning



Task	Tested for	Correctness	
		Expe-riments	Similation [CI 95%]
Bucher et al., Exp. 1	Correct order	98.64%	93.34% [0.84, 0.98]
	Relocation LO	87.78%	94.00% [0.73, 0.99]
	Relocation last object	12.22%	5.56% [0.00, 0.27]
Nejasmic et al., Exp. 1, 4	Continuous	92%	78.33% [0.66, 0.88]
	Semicontin-uous	79%	6.67% [0.02, 0.16]
	Quasidis-continuous	81%	80.00% [0.67, 0.89]
	Discontin-uous	59% (exp. 1)/67% (exp. 4)	76.67% [0.64, 0.87]
Cognitive load	2 premises		100% [0.95, 1]
	3 premises		80.00% [0.68, 0.89]
	4 premises		40.00% [0.28, 0.53]
	5 premises		15.00% [0.07, 0.27]
	6 premises		1.67% [0.00, 0.09]