

Resolution Principle

AI

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Rules of Inference

Rule of inference	Tautology	Name
$\begin{array}{l} p \rightarrow q \\ \underline{p} \\ \hline \therefore q \end{array}$	$[p \wedge (p \rightarrow q)] \rightarrow q$	Modus ponens
$\begin{array}{l} \neg q \\ \underline{p \rightarrow q} \\ \hline \therefore \neg p \end{array}$	$[\neg q \wedge (p \rightarrow q)] \rightarrow \neg p$	Modus tollens
$\begin{array}{l} p \rightarrow q \\ \underline{q \rightarrow r} \\ \hline \therefore p \rightarrow r \end{array}$	$[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$	Hypothetical syllogism
$\begin{array}{l} p \vee q \\ \underline{\neg p} \\ \hline \therefore q \end{array}$	$((p \vee q) \wedge \neg p) \rightarrow q$	Disjunctive syllogism
$\begin{array}{l} \underline{p} \\ \hline \therefore p \vee q \end{array}$	$p \rightarrow (p \vee q)$	Addition
$\begin{array}{l} \underline{p \wedge q} \\ \hline \therefore p \end{array}$	$(p \wedge q) \rightarrow p$	Simplification
$\begin{array}{l} p \\ \underline{q} \\ \hline \therefore p \wedge q \end{array}$	$((p) \wedge (q)) \rightarrow (p \wedge q)$	Conjunction
$\begin{array}{l} p \vee q \\ \underline{\neg p \vee r} \\ \hline \therefore q \vee r \end{array}$	$[(p \vee q) \wedge (\neg p \vee r)] \rightarrow (q \vee r)$	Resolution

Inference Rules

Inference Rules

- The resolution principle is an **Inference Rule**
- **Inference Rule**: a rule that generates new clauses which are a logical consequence of some of the existing clauses
- New clauses can be used to turn some of the nodes in T to failure nodes.
- Thus number of nodes in T are reduced and \square will eventually appear.

The Resolution Principle

Resolution Principle

For any two clauses C_1 and C_2 if there is a literal L_1 in C_1 that is complementary to a literal L_2 in C_2 then delete L_1 and L_2 from C_1 and C_2 and generate a new clause C_3 as the disjunction of the remaining clauses.

C_3 is a **resolvent** for C_1 and C_2 .

Resolution Principle: Inference rule

$$\frac{L_1 \vee C_1 \quad \neg L_1 \vee C_2}{C_1 \vee C_2}$$

Example

The Resolution Principle

Example (Resolution Principle)

Consider the following clauses $C_1 = P \vee R$ and $C_2 = \neg P \vee Q$

$$\frac{P \vee R \qquad \neg P \vee Q}{}$$

Example

Example (Resolution Principle)

Consider the following clauses $C_1 = P \vee R$ and $C_2 = \neg P \vee Q$

$$\frac{P \vee R \qquad \neg P \vee Q}{R \vee Q}$$

Example

The Resolution Principle

Example (Resolution Principle)

Consider the following clauses $C_1 = P \vee R$ and $C_2 = \neg P \vee Q$

$$\frac{P \vee R \qquad \neg P \vee Q}{R \vee Q}$$

$C_3 = R \vee Q$ is the resolvent for C_1 and C_2 .

Example II

The Resolution Principle

Example (Resolution Principle)

Consider the following clauses $C_1 = \neg P \vee Q \vee R$ and $C_2 = \neg Q \vee S$

$$\frac{\neg P \vee Q \vee R \qquad \neg Q \vee S}{}$$

Example II

Example (Resolution Principle)

Consider the following clauses $C_1 = \neg P \vee Q \vee R$ and $C_2 = \neg Q \vee S$

$$\frac{\neg P \vee Q \vee R \qquad \neg Q \vee S}{\neg P \vee R \vee S}$$

Example II

Example (Resolution Principle)

Consider the following clauses $C_1 = \neg P \vee Q \vee R$ and $C_2 = \neg Q \vee S$

$$\frac{\neg P \vee Q \vee R \qquad \neg Q \vee S}{\neg P \vee R \vee S}$$

$C_3 = \neg P \vee R \vee S$ is the resolvent for C_1 and C_2 .

Example III

The Resolution Principle

Example (Resolution Principle)

Consider the following clauses $C_1 = \neg P \vee Q$ and $C_2 = \neg P \vee S$
There is no resolvent in this case as no complementary pair can be found in the clauses.

The Resolution Principle

The Resolution Principle

Robinson 1965

- **Aim:** directly test unsatisfiability of a set of clauses S **without** generating all possible associated ground clauses.
- **Basic idea:** test whether S contains the empty clause \square
 - If $\square \in S$ then S is unsatisfiable
 - Otherwise need to check whether $S \models \square$

Derivation of the empty clause

The Resolution Principle

Resolution and satisfiability

- If C_1 and C_2 are unit clauses then, if there is resolvent, that resolvent will necessarily be \square .
- If we can derive the empty clause from S , then S is unsatisfiable (correctness)
- If S is unsatisfiable using resolution we can always derive the empty clause (completeness)

Resolution Algorithm

- Resolution basically works by using the principle of proof by contradiction.
- To find the conclusion we should negate the conclusion.
- Then the resolution rule is applied to the resulting clauses.
- Each clause that contains complementary literals is resolved to produce a two new clause, which can be added to the set of facts (if it is not already present).
- This process continues until one of the two things happen:
 - ✓ There are no new clauses that can be added
 - ✓ An application of the resolution rule derives the empty clause
- An empty clause shows that the negation of the conclusion is a complete contradiction, hence the negation of the conclusion is invalid or false or the assertion is completely valid or true.

Resolution-Example

Propositional Resolution Example

Prove R

1	$P \vee Q$
2	$P \rightarrow R$
3	$Q \rightarrow R$

Step	Formula	Derivation

Lecture 7 • 9

So let's just do a proof. Let's say I'm given "P or Q", "P implies R" and "Q implies R". I would like to conclude R from these three axioms. I'll use the word "axiom" just to mean things that are given to me right at the moment.

Resolution-Example...

Propositional Resolution Example

Prove R

1	$P \vee Q$
2	$P \rightarrow R$
3	$Q \rightarrow R$

Step	Formula	Derivation
1	$P \vee Q$	Given
2	$\neg P \vee R$	Given
3	$\neg Q \vee R$	Given
4	$\neg R$	Negated conclusion

Resolution-Example...

Propositional Resolution Example

Prove R

1	$P \vee Q$
2	$P \rightarrow R$
3	$Q \rightarrow R$

Step	Formula	Derivation
1	$P \vee Q$	Given
2	$\neg P \vee R$	Given
3	$\neg Q \vee R$	Given
4	$\neg R$	Negated conclusion
5	$Q \vee R$	1,2
6	$\neg P$	2,4
7	$\neg Q$	3,4
8	R	5,7
9	\cdot	4,8

Types of Resolution

- Binary Resolution
- Unit Resulting (UR) Resolution
- Linear Resolution
- Linear Input Resolution

Reference: AI & Expert Systems
-- DAN. W. PATTERSON

THANKS