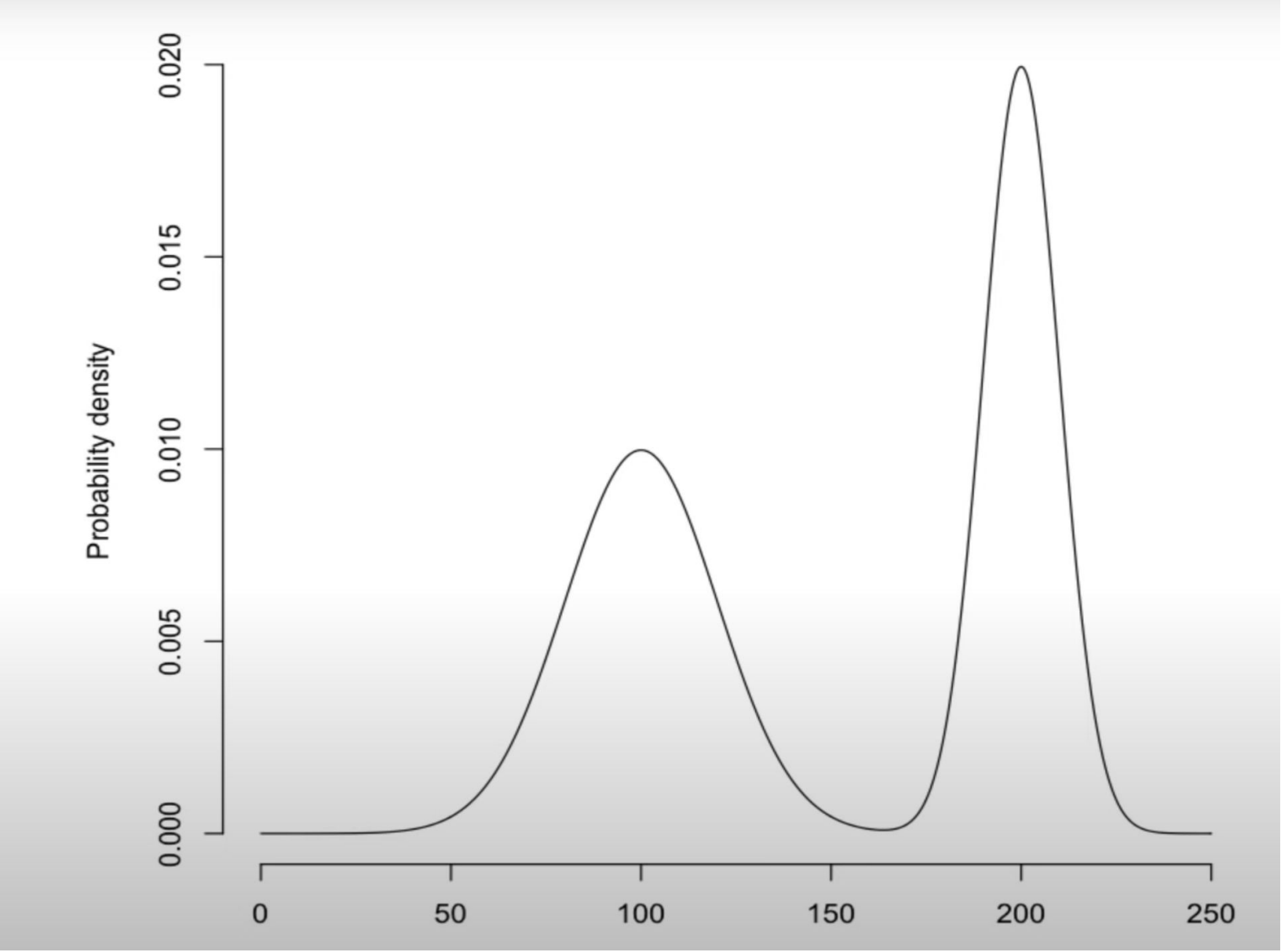
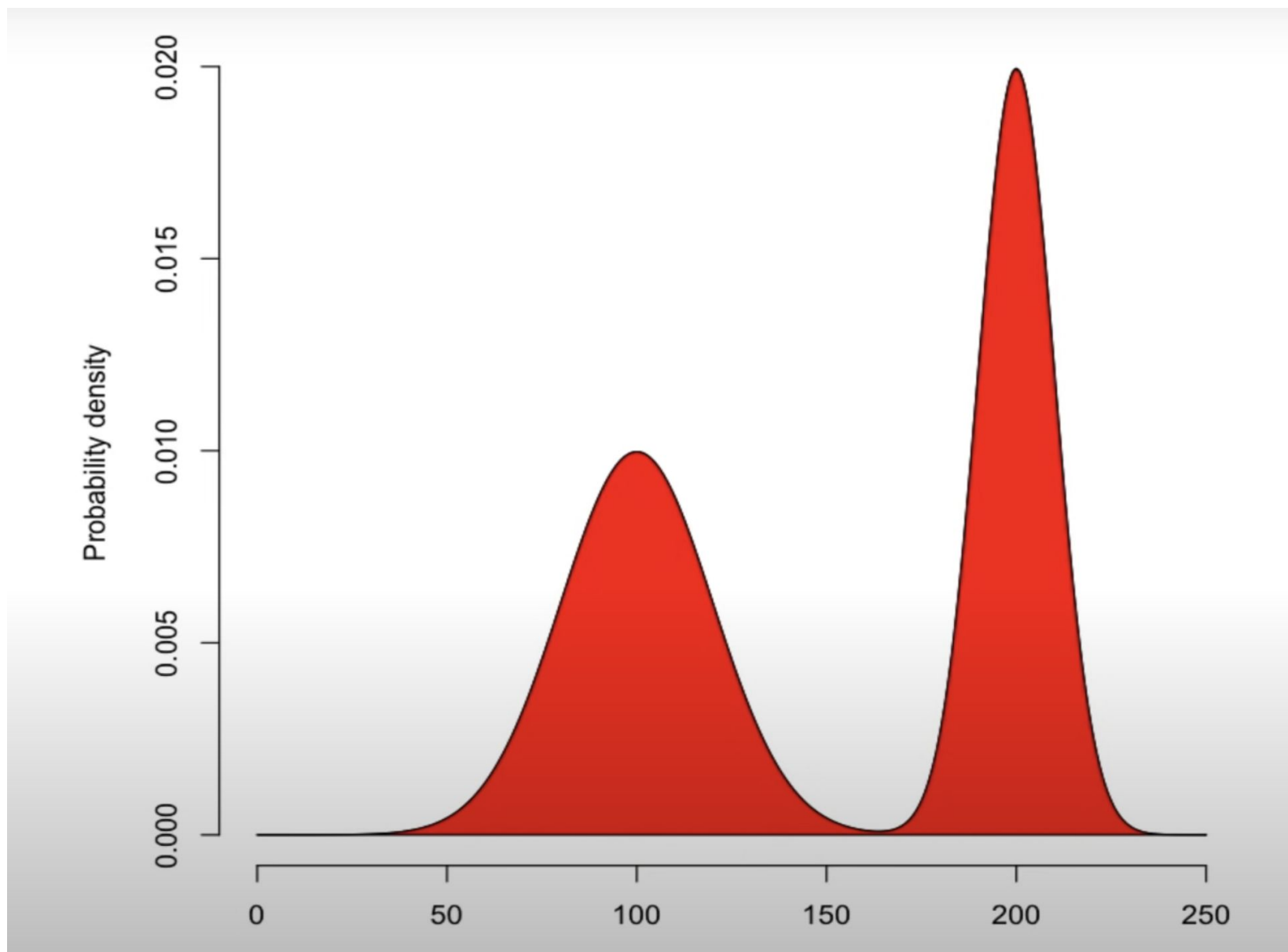


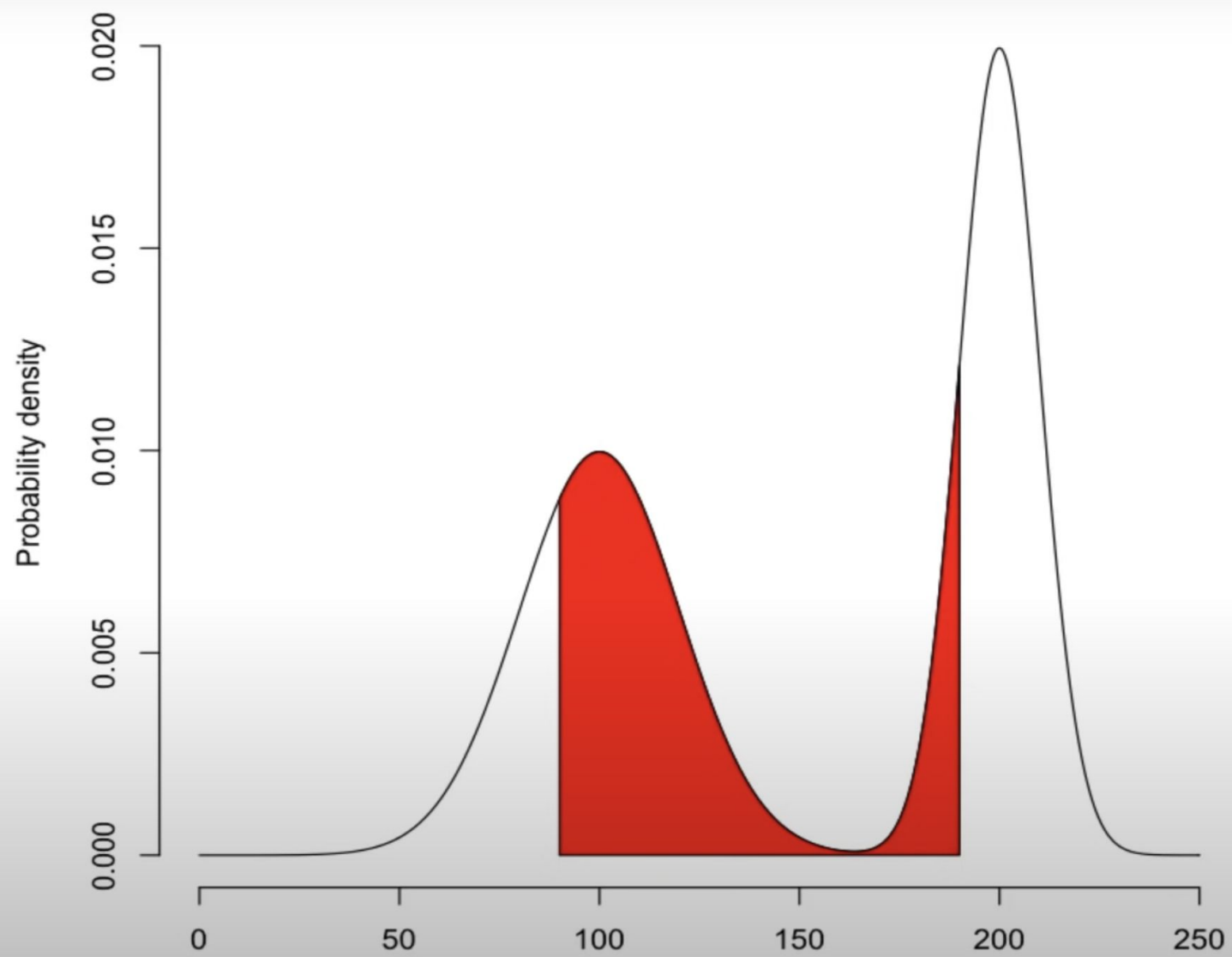
# PDF and CDF





The probability that  $X$  is between  $a$  and  $b$  is the area under the curve between  $a$  and  $b$ .

$$P(a \leq X \leq b) = \int_a^b f(x) dx$$



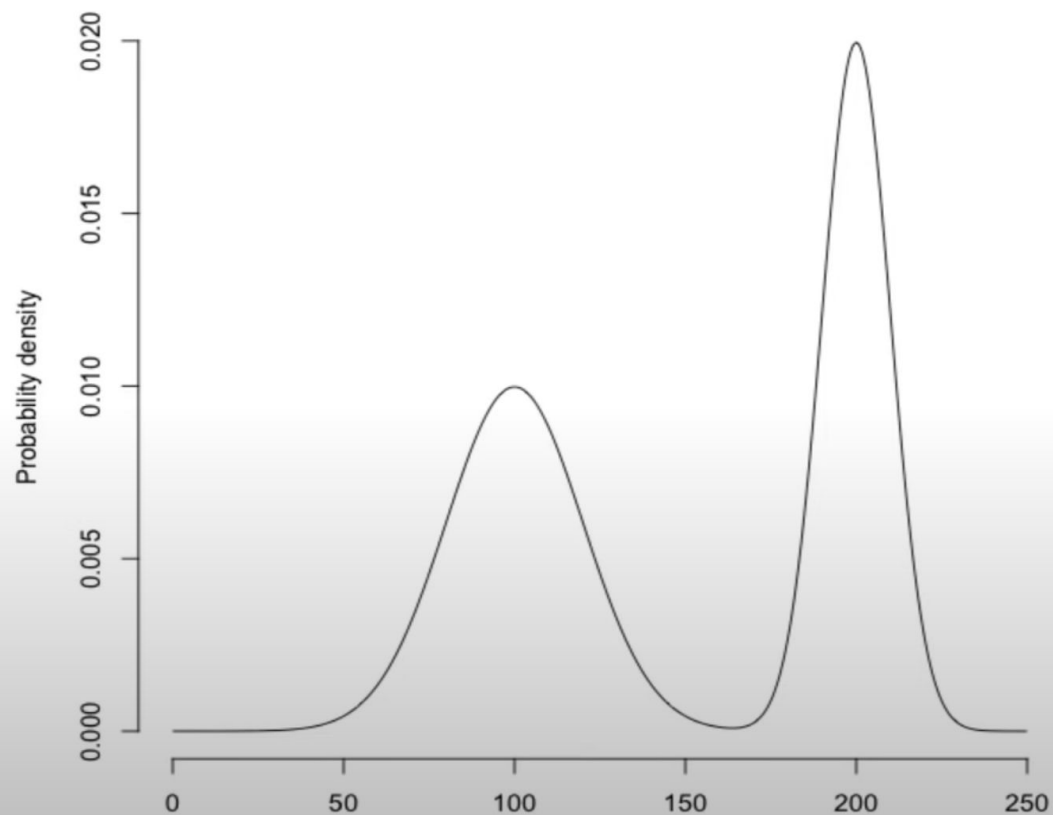
The probability density cannot be negative.

$$f(x) \geq 0 \text{ for all } x$$

The total area under the curve must be 1.

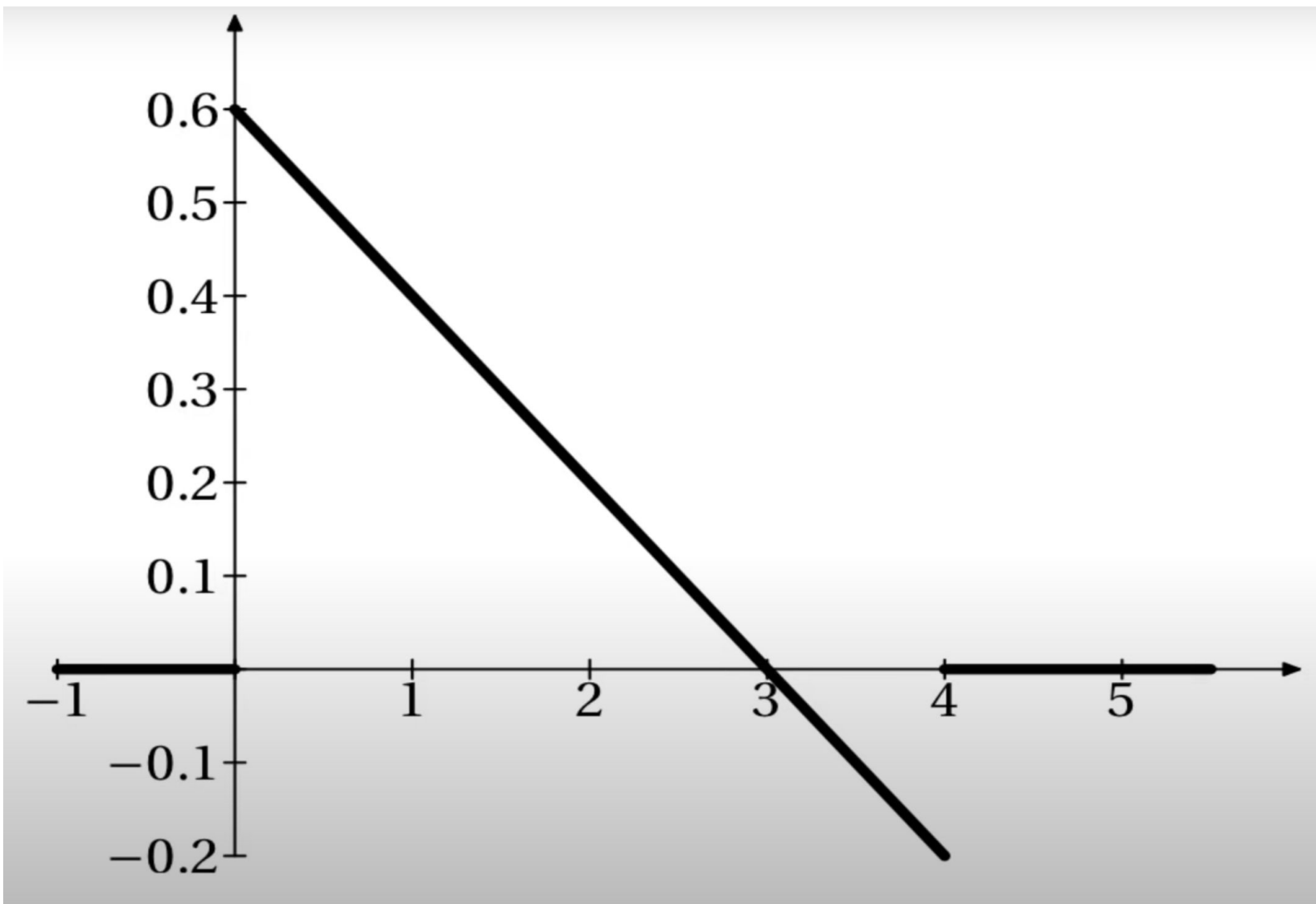
$$\int_{-\infty}^{\infty} f(x) dx = 1$$

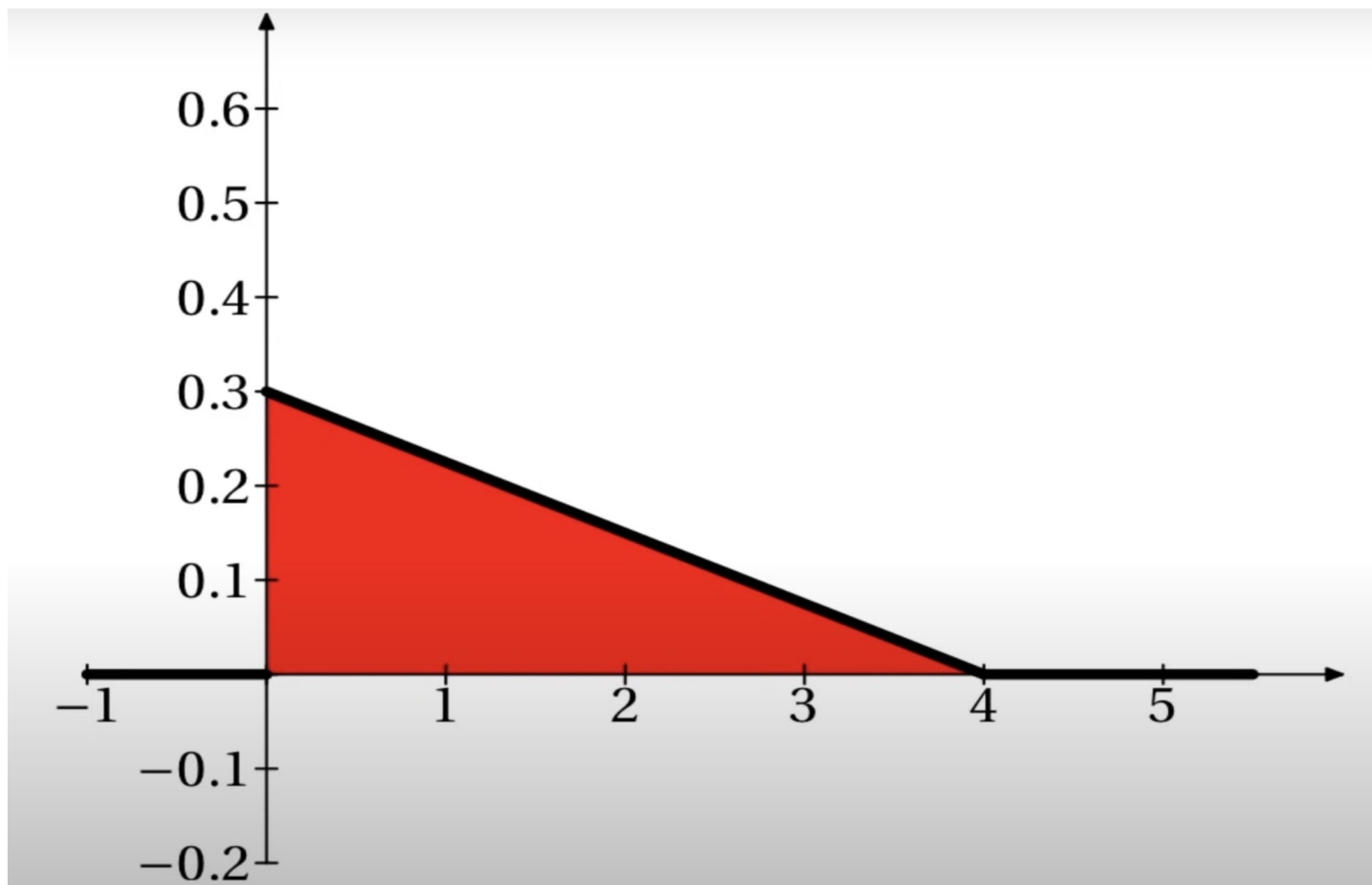
$$f(x) = \frac{1}{2} \left( \frac{1}{20\sqrt{2\pi}} e^{-\frac{(x-100)^2}{800}} + \frac{1}{10\sqrt{2\pi}} e^{-\frac{(x-200)^2}{200}} \right)$$

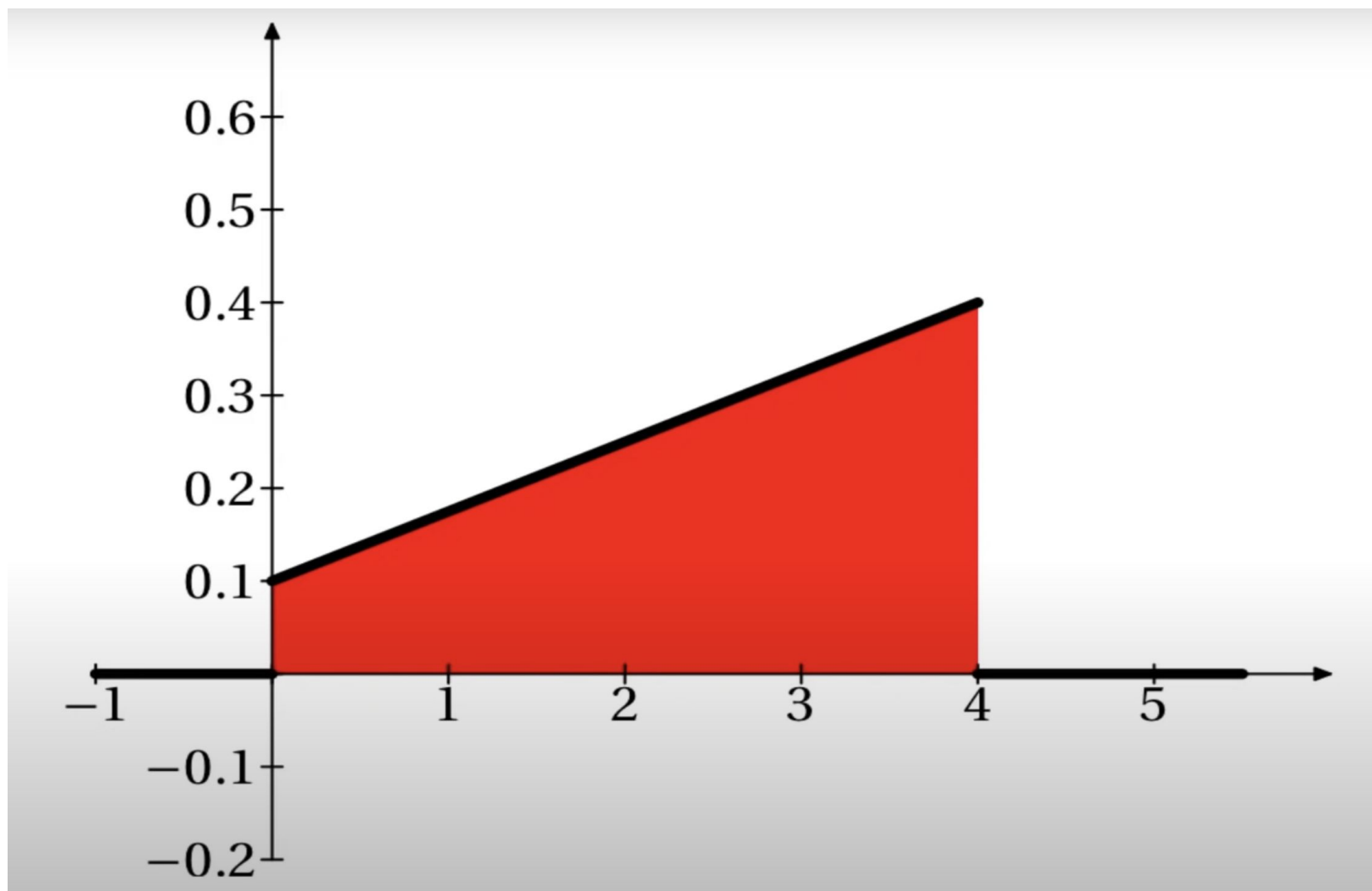


RECOGINZING PDF









Find the value of  $k$  for which the following function would be a valid probability density function.

$$f(x) = \begin{cases} k(x^2 + 1) & 1 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

$$f(x) = \begin{cases} k(x^2 + 1) & 1 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

$$\begin{aligned} \int_1^4 k(x^2 + 1) dx &= \left[ k \left( \frac{1}{3}x^3 + x \right) \right]_1^4 \\ &= k \left( \frac{1}{3} \times 4^3 + 4 \right) - k \left( \frac{1}{3} \times 1^3 + 1 \right) \\ &= k \left( \frac{64}{3} + 4 \right) - k \left( \frac{1}{3} + 1 \right) \\ &= k \left( \frac{63}{3} + 3 \right) \\ &= 24k \end{aligned}$$

$$24k = 1$$

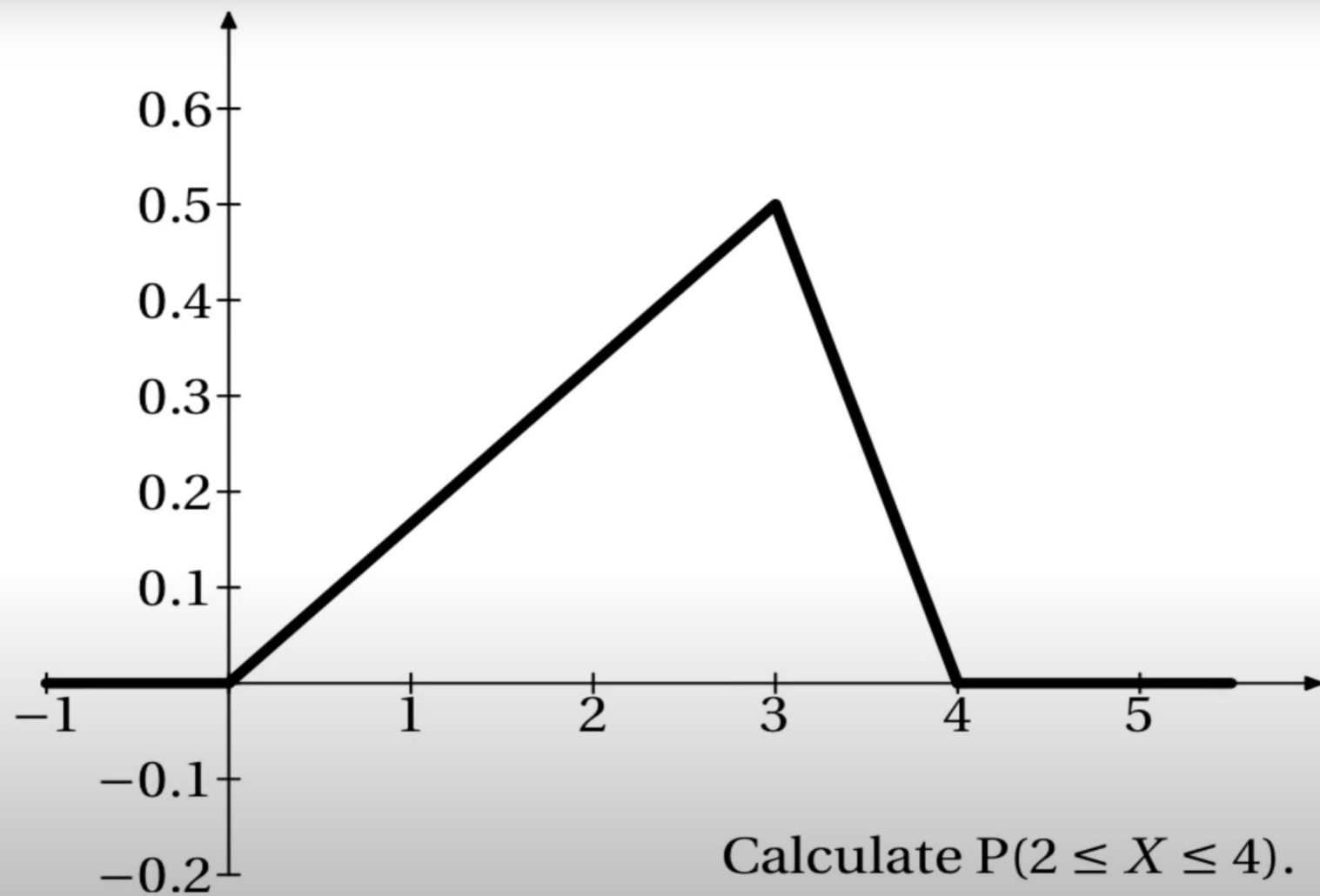
$$k = \frac{1}{24}$$

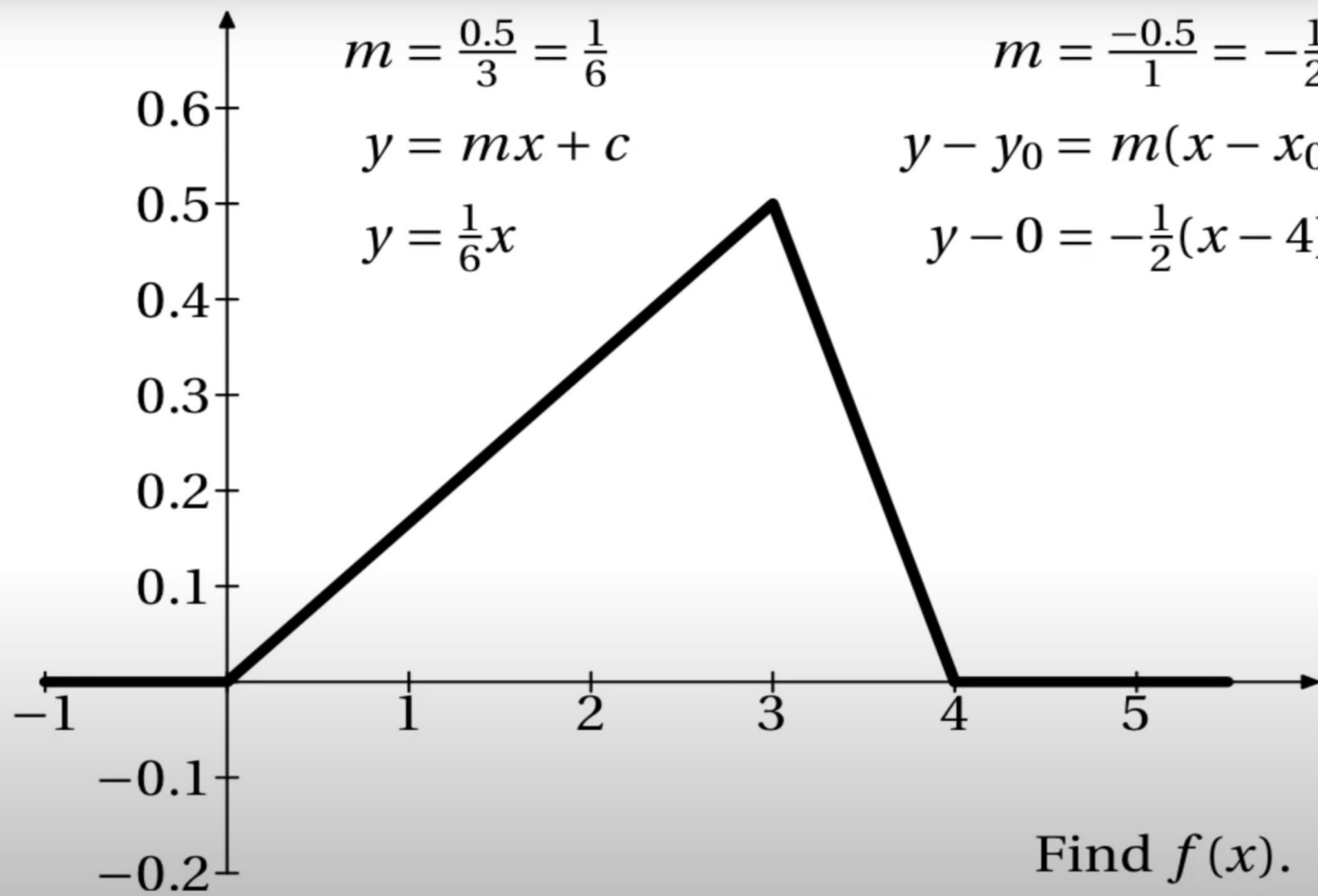
$$f(x) = \begin{cases} \frac{1}{24}(x^2 + 1) & 1 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

Calculate  $P(X = 3)$ . 0

Calculate  $P(2 < X \leq 3)$ .

$$\begin{aligned} \int_2^3 \frac{1}{24}(x^2 + 1)dx &= \left[ \frac{1}{24} \left( \frac{1}{3}x^3 + x \right) \right]_2^3 \\ &= \frac{1}{24} \left( \frac{1}{3} \times 3^3 + 3 \right) - \frac{1}{24} \left( \frac{1}{3} \times 2^3 + 2 \right) \\ &= \frac{1}{24} (9 + 3) - \frac{1}{24} \left( \frac{8}{3} + 2 \right) \\ &= \frac{11}{36} \end{aligned}$$

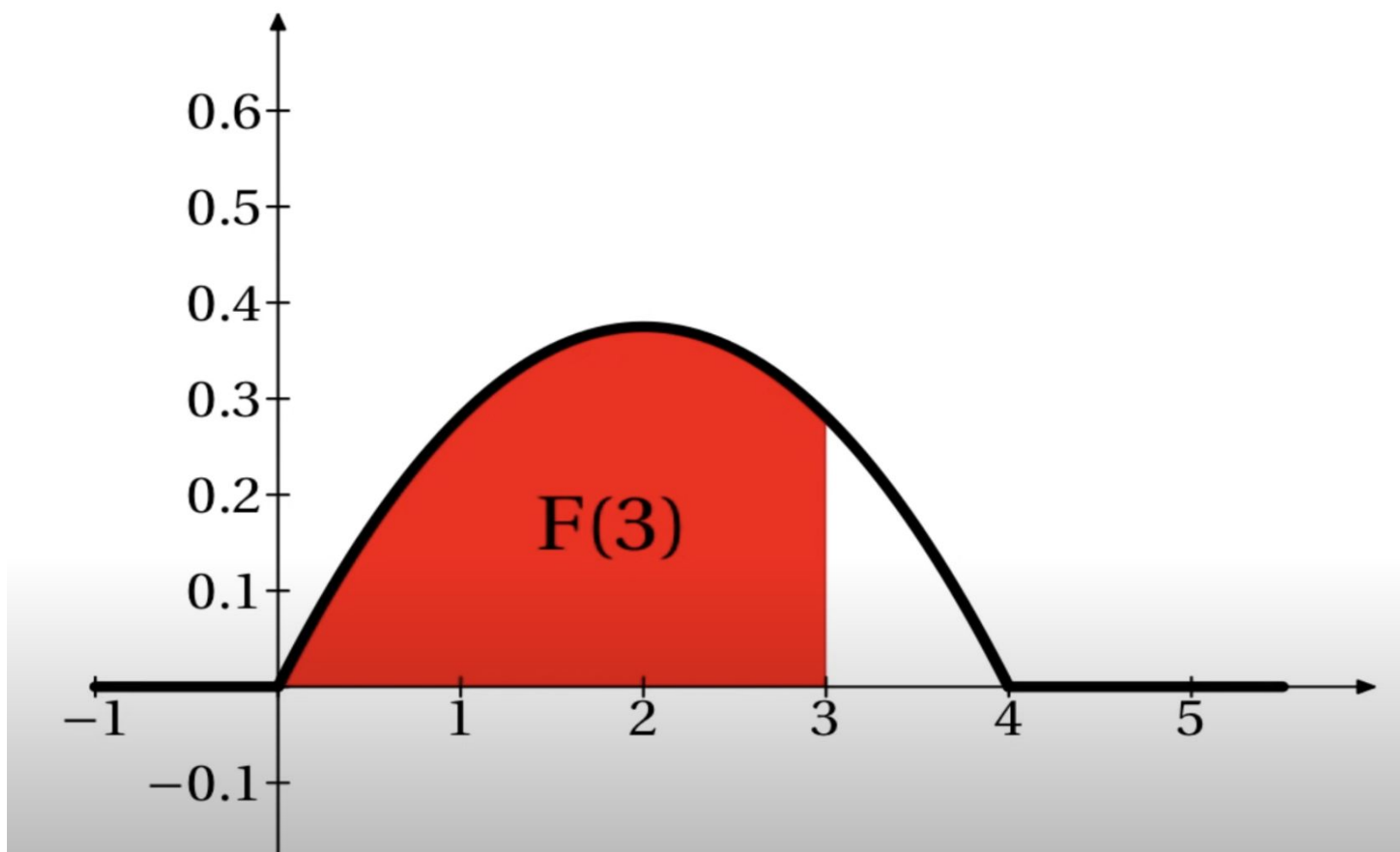




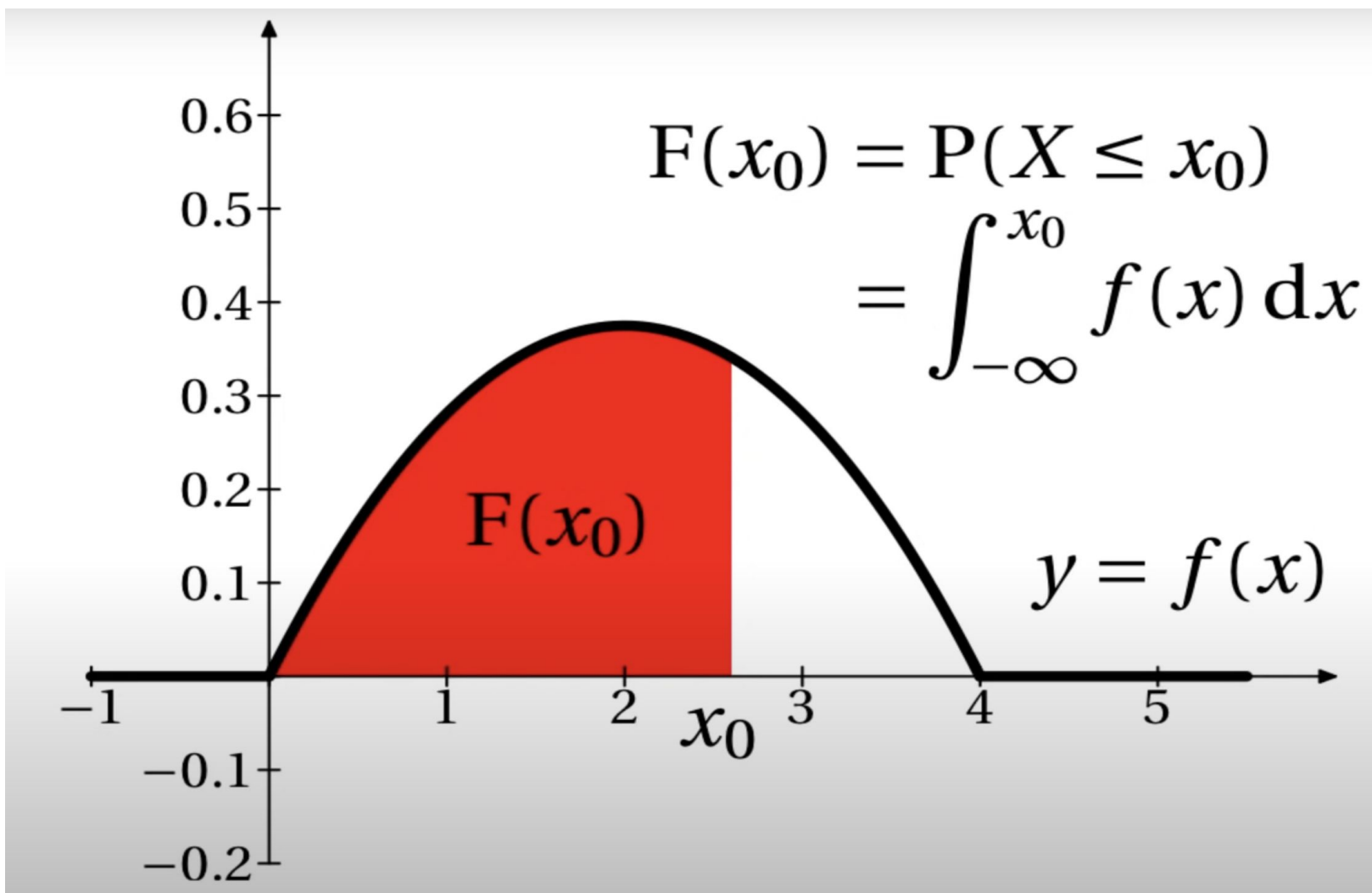


# Cumulative Density Function

$$F(x_0) = P(X \leq x_0)$$



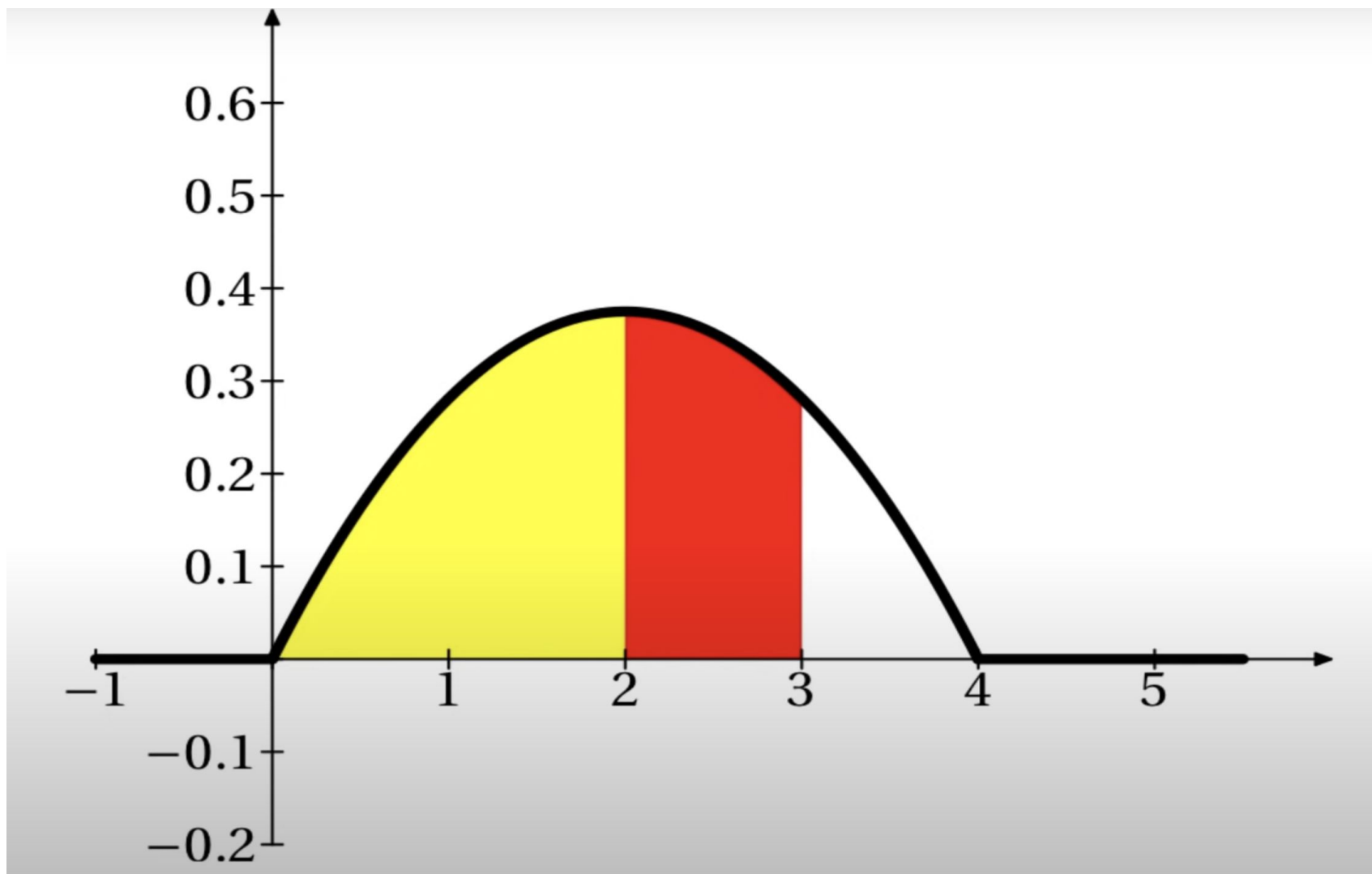
$$F(3) = P(X \leq 3)$$

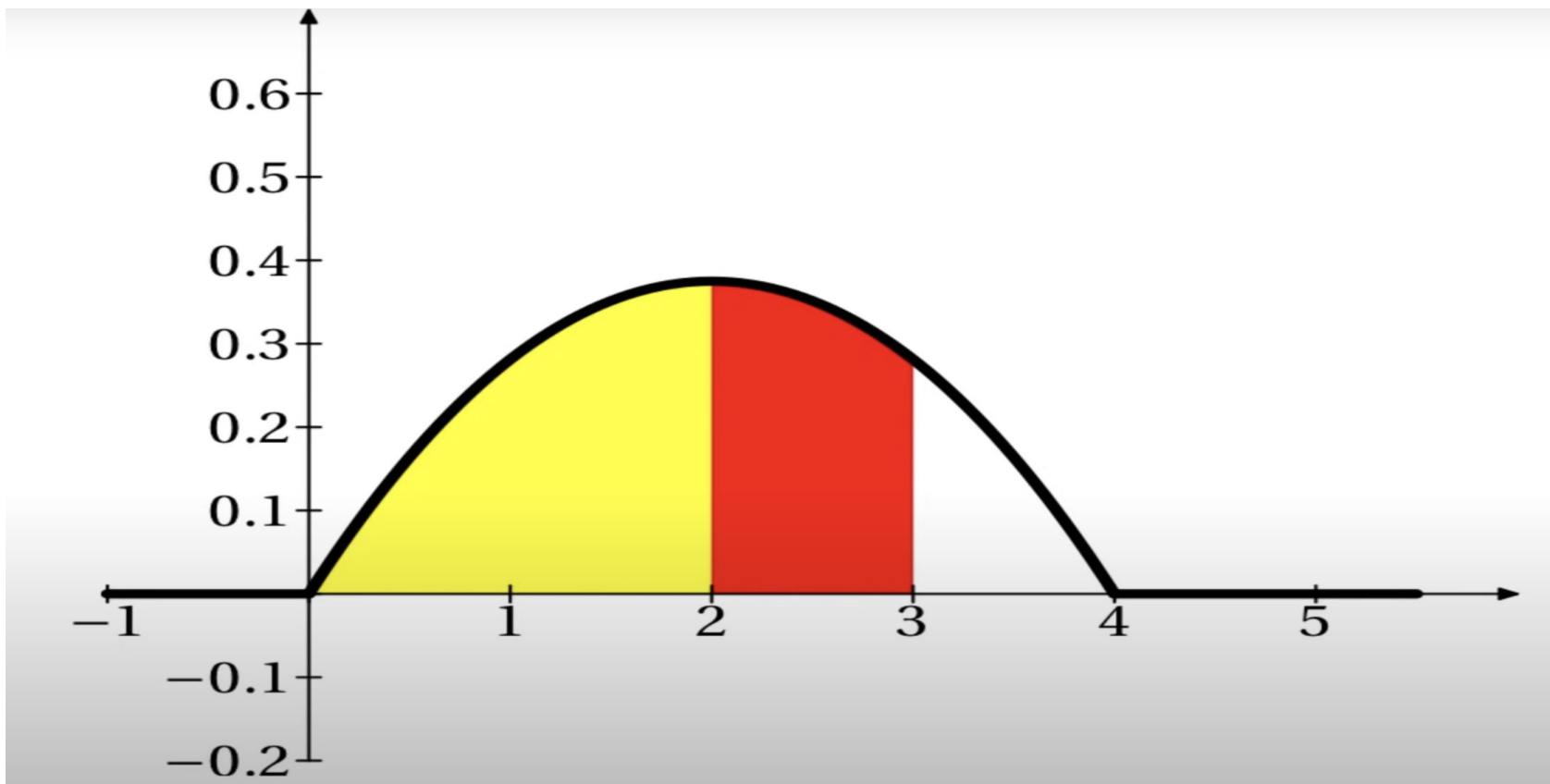


The cumulative distribution function for the random variable  $X$  is given by

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{32} (6x^2 - x^3) & 0 \leq x \leq 4 \\ 1 & x > 4 \end{cases}$$

Calculate  $P(2 < X < 3)$ .





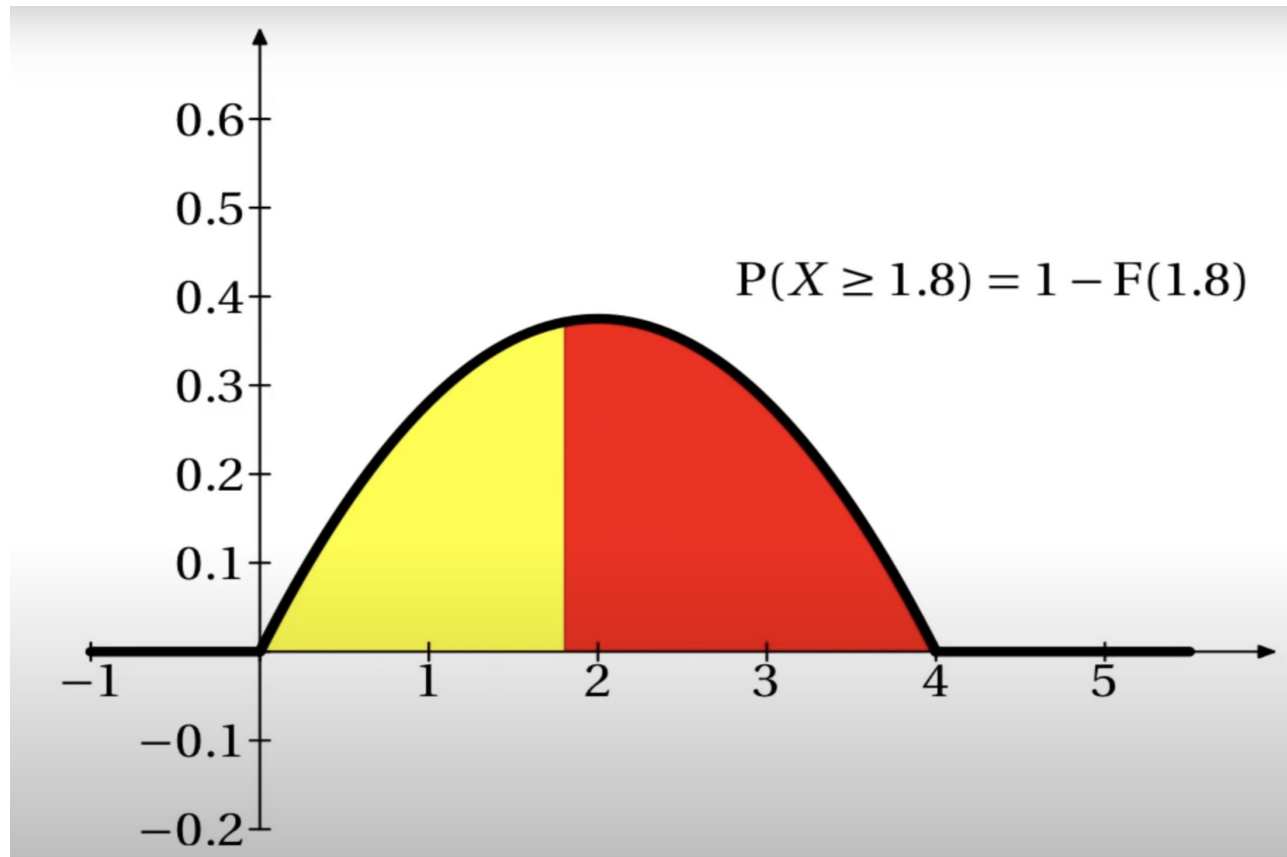
$$\begin{aligned} P(2 < X < 3) &= F(3) - F(2) \\ &= \frac{1}{32}(6 \times 3^2 - 3^3) - \frac{1}{32}(6 \times 2^2 - 2^3) \\ &= \frac{11}{32} \end{aligned}$$

The cumulative distribution function for the random variable  $X$  is given by

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{32}(6x^2 - x^3) & 0 \leq x \leq 4 \\ 1 & x > 4 \end{cases}$$

Calculate  $P(X \geq 1.8)$ .





$$P(X \geq 1.8) = 1 - F(1.8)$$

$$= 1 - \frac{1}{32}(6 \times 1.8^2 - 1.8^3)$$

$$= 0.57475$$

**PDF TO CDF**

The probability density function for the random variable  $X$  is given by

$$f(x) = \begin{cases} \frac{1}{8}x^2 & 0 \leq x \leq 2 \\ \frac{1}{8}x(4-x) & 2 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

Find  $F(x)$ .

$$f(x) = \begin{cases} \frac{1}{8}x^2 & 0 \leq x \leq 2 \\ \frac{1}{8}x(4-x) & 2 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

When  $0 \leq x \leq 2$ ,  $F(x) = \int \frac{1}{8}x^2 dx = \frac{1}{24}x^3 + C$

But  $F(0) = 0$ . Therefore,  $C = 0$ . So  $F(x) = \frac{1}{24}x^3$ .

When  $2 \leq x \leq 4$ ,  $F(x) = \int \frac{1}{8}x(4-x) dx = \frac{1}{4}x^2 - \frac{1}{24}x^3 + D$

But  $F(2) = \frac{1}{24} \times 2^3 = \frac{1}{3}$ . Therefore,  $D = -\frac{1}{3}$ .

So  $F(x) = \frac{1}{4}x^2 - \frac{1}{24}x^3 - \frac{1}{3} = \frac{1}{24}(6x^2 - x^3 - 8)$ .

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{24}x^3 & 0 \leq x \leq 2 \\ \frac{1}{24}(6x^2 - x^3 - 8) & 2 \leq x \leq 4 \\ 1 & x > 4 \end{cases}$$

**CDF TO PDF**

The cumulative distribution function for the random variable  $X$  is given by

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{24}x^3 & 0 \leq x \leq 2 \\ \frac{1}{24}(6x^2 - x^3 - 8) & 2 \leq x \leq 4 \\ 1 & x > 4 \end{cases}$$

Find  $f(x)$ .

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{24}x^3 & 0 \leq x \leq 2 \\ \frac{1}{24}(6x^2 - x^3 - 8) & 2 \leq x \leq 4 \\ 1 & x > 4 \end{cases}$$

$$\frac{d}{dx} \frac{1}{24}x^3 = \frac{1}{8}x^2$$

$$\frac{d}{dx} \frac{1}{24}(6x^2 - x^3 - 8) = \frac{1}{24}(12x - 3x^2) = \frac{1}{8}x(4 - x)$$

$$f(x) = \begin{cases} \frac{1}{8}x^2 & 0 \leq x \leq 2 \\ \frac{1}{8}x(4 - x) & 2 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

# **Mean and Variance of Continuous Random Variable**



$$\mu = \int_{-\infty}^{\infty} x f(x) \, dx$$

$$\sigma^2 = \int_{-\infty}^{\infty} x^2 f(x) \, dx - \mu^2$$

$$\mu = \int_{-\infty}^{\infty} x f(x) \, dx$$

$$\sigma^2 = \int_{-\infty}^{\infty} x^2 f(x) \, dx - \mu^2$$

The random variable  $X$  has probability density function given by

$$f(x) = \begin{cases} \frac{3}{10}(3x - x^2) & 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

Find the mean and variance of  $X$ .

$$f(x) = \begin{cases} \frac{3}{10}(3x - x^2) & 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

$$\begin{aligned} \mu &= \int_{-\infty}^{\infty} x f(x) \, dx \\ &= \int_0^2 x \times \frac{3}{10}(3x - x^2) \, dx \\ &= \int_0^2 \frac{9}{10}x^2 - \frac{3}{10}x^3 \, dx \\ &= \left[ \frac{3}{10}x^3 - \frac{3}{40}x^4 \right]_0^2 \\ &= \left( \frac{3}{10} \times 2^3 - \frac{3}{40} \times 2^4 \right) - 0 \\ &= 1\frac{1}{5} \end{aligned}$$

$$\begin{aligned}
 E(X^2) &= \int_{-\infty}^{\infty} x^2 f(x) \, dx \\
 &= \int_0^2 x^2 \times \frac{3}{10}(3x - x^2) \, dx \\
 &= \int_0^2 \frac{9}{10}x^3 - \frac{3}{10}x^4 \, dx \\
 &= \left[ \frac{9}{40}x^4 - \frac{3}{50}x^5 \right]_0^2 \\
 &= \left( \frac{9}{40} \times 2^4 - \frac{3}{50} \times 2^5 \right) - 0 \\
 &= \frac{42}{25}
 \end{aligned}$$

$$\begin{aligned}
 \sigma^2 &= E(X^2) - \mu^2 \\
 &= \frac{42}{25} - \left(\frac{6}{5}\right)^2 \\
 &= \frac{6}{25}
 \end{aligned}$$

The probability density function for the random variable  $X$  is given by

$$f(x) = \begin{cases} \frac{1}{4}x & 0 \leq x \leq 2 \\ \frac{1}{4}(4 - x) & 2 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

Find the mean and standard deviation of  $X$ .

The probability density function for the random variable  $X$  is given by

$$f(x) = \begin{cases} \frac{1}{4}x & 0 \leq x \leq 2 \\ \frac{1}{4}(4 - x) & 2 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

Find the mean and standard deviation of  $X$ .

The probability density function for the random variable  $X$  is given by

$$f(x) = \begin{cases} \frac{1}{4}x & 0 \leq x \leq 2 \\ \frac{1}{4}(4 - x) & 2 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

Calculate  $P(X < \mu - \sigma)$ .