06-03-2023

CSE 3107

Applied Statistics and Queing Theory

↓

used in Machine Learning

Statistics is concearned with

- collecting → data collect করা
- Organizing → unnecessary data, Invalid data বাদ দেওয়া
- Summarizing → Single value যেটা পুরো distribution কে represent করবে।
- Presenting → Data present করা।
- analyzing → কি সিদ্ধান্তে পৌঁছাতে পারবো। কি mean করবে।

$\boxed{\text{data}}$

Inferential Statistics → purpose: Make prediction → বর্তমান data র উপর ভিত্তি করে future value predict করা। (Machine Learning এ use করা হয়)

Descriptive → data কে describe করা।

↓

purpose: describing data → mean, median, std. deviation → parameters

Population

*population এর subset sample.

*population অনেক বেশি কাজ করা সবসময় feasible হয় না।

Sample → mean, median, std. deviation → statistics

→ Randomly sampling popular method.

*Data distribution যদি biased হয় তখন analyzing ফলপ্রসূ নাও হতে পারে।

Parameter estimation : Sample বা statistics state থেকে parameter state এ calculate করা, ideal এর সাথে তুলনা করে।

Books : Statistics by Murray R. Spiegel
Probability by Seymour Lipschutz.

11. 03.2023

Variable : A variable is an attribute that describes a person, place, thing or idea.

Variable changeable.

Variable

Categorical (Qualitative) Variable
- place of birth, Gender, Name

Numerical (Quantitative) Variable
- Height, weight, Temperature.

* Numerical থেকে categorical এ নেওয়া মানে কিছু কিছু data loss হয়।

Numerical

Discrete
takes finite number of values.
→ no. of child in a family.

continuous
Infinite domain
→ Height, weight, temp
- খুব সূক্ষ্ম value যেটা পরিমাপ করে।
- Fraction থাকলে value continuous.

*simplification এর জন্য continuous থেকে discrete এ আনা হয়।

* Level of Measurement :

* একটা variable থেকে কি পরিমান information পাবো।

1. Nominal → Nationality, Gender, Religion.
   → Categorical, without order. কথা info sort করা যাবে না, compare হবে না।
2. Ordinal → Categorical, with order. Level of customer satisfaction (excellent, good, fair, poor)
   → order আছে।
   → Agreement (agree, Neutral, disagree) সাজানো যায়।
3. Interval Scales → Numerical, No absolute zero. pH scale, Temp. in °F and °C
   zero দিয়েও কোনো value measure হবে।
   interval scale এ ratio নেওয়া valid না। অর্থাৎ reference নেওয়া যায় না।
4. Ratio Scales → Numerical, absolute zero exist. Temp. in K.
   একটা Reference value থাকে। ফলে compare করা easy.

Information বাড়বে

* Univariate data : looks at only variable.
যখন 1টি data নিয়ে বা 1টি Feature নিয়ে কাজ করা হয়।

কোনো data type না।
কি পরিমান data নিয়ে কাজ হয়

*Multivariate data : looks at multiple variable.
একাধিক data or feature নিয়ে কাজ করা হয়। Classification করা যায়।

13-03-2023

Graphs

Pie chart:

* categorical data visualization এর জন্য use করা হয়।

Bar graphs:

* Decreasing আকারে সাজানো হলে Pareto chart.

Multiple Bar Graph:

* একাধিক attribute একসাথে মিলে chart করতে হয়।

Line Graph:

* progression show করার জন্য use করা হয়।

Frequency Distribution:

* কোনো particular value র frequency user কে দেয়,

(ক্লাস 1 – 10 এর বিন্যস্ত-অবিন্যস্ত)

Left-end convention:

lower class interval ← 170 - 180 → upper class interval

180 - 190

a-b range এ input হবে

$a \leq x < b$ ।

*last number হলে include করতে হবে।

*এই convention আসছে overlap এর জন্য, ambiguity র জন্য।

class size = upper class interval − lower class interval.

*5 -10 টা class interval নেওয়া উচিত.

Ogive রেখা

* Histogram and bar charts are not the same.
↓
side থেকে পড়তে হবে.

* Bin width related to class interval.

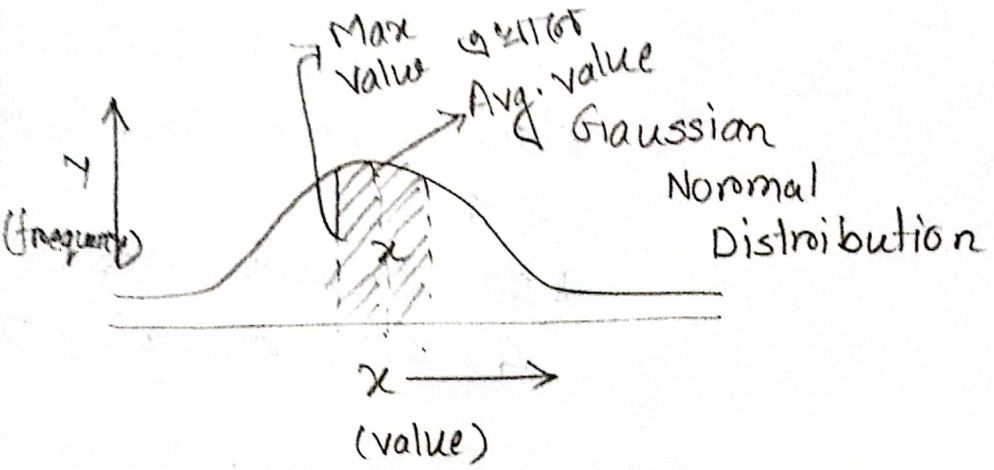
Scatterplot:

* Data distribution easily দেখা যায়। data র মধ্যে co-relation থাকলে বুঝা যায়।

15-03-2023

Measure of Central Tendency:

* central কে কেন্দ্র করে



* Avg এর আশে পাশে maximum value কেন্দ্রীভূত থাকে।

যেমন: 60 জন student এর marks distribution

* Arithmetic mean
* Geometric "
* Harmonic "
* Quartiles "
* Median "

① Arithmetic Mean :

$x_1, x_2, x_3, \dots x_n$

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

Tabular form এ গাণিতিক

$$\bar{x} = \frac{\sum f_i x_i}{n}$$

where, $n = \sum f_i$

x	f
10	2
20	3
30	1

from ungroup frequency distribution table

*Such of deviations from mean is equal to zero

$x_i \longrightarrow x_1, x_2, \dots x_n$

$d_i \longrightarrow d_1, d_2, \dots d_n$

$\sum d_i = 0$

$$\boxed{d_i = x_i - \bar{x}}$$

d_i → বিচ্যুতি

x_i → number

$\bar{x}$ → Avg.

Formal proof :

$$\sum_{i=1}^{n}(x_i - \bar{x})$$

$$= \sum_{i=1}^{n} x_i - \sum_{i=1}^{n} \bar{x}$$

$$\sum_{i=1}^{n} c = nc$$

a constant

$$= \sum_{i=1}^{n} x_i - n\bar{x}$$

particular সংখ্যক number এর জন্য avg ($\bar{x}$) constant

$$= \sum_{i=1}^{n} x_i - \cancel{n} \cdot \frac{\sum_{i=1}^{n} x_i}{\cancel{n}}$$

$$= 0$$

$x_1, x_2, x_3, \ldots x_n$

A → Assumed mean

$$d_i = x_i - A$$

Assumed mean actual mean এর সমান না হলে কখনই d_i 0 হবে না।

$$\bar{x} = \frac{\sum x_i}{n}$$

$$= \frac{\sum A + \sum d_i}{n}$$

$$= \frac{nA + \sum d_i}{n}$$

$$\boxed{\bar{x} = A + \frac{\sum d_i}{n}}$$

$$\frac{3+4+5}{3} = 4 \longrightarrow \text{True mean}$$

$A = 10 \longrightarrow$ Assume mean

$(3-10)+(4-10)+(5-10)$

$= -18$

$$\bar{x} = 10 + \frac{-18}{3}$$

$= 10-6 = 4$ true mean

For tabular / group data:

$$\boxed{\bar{x} = A + \frac{\Sigma f_i d_i}{\Sigma f_i}}$$

Example:

Class	Frequency, f_i	x_i	$d_i = x_i - A$	u_i
0-10	7	5	-20	-2
10-20	8	15	-10	-1
20-30	20	25	0	0
30-40	10	35	10	1
40-50	5	45	20	2

* A এর value assume করা হয় ঠিক মাঝের class এর x_i এর মান

25

$n = \Sigma f = 30$

$$\bar{x} = A + \frac{\Sigma f_i d_i}{n}, \quad d_i = x_i - A$$

class width, $c = 10$

* di এখানে 10 এর গুণিতক এর reason class width

shortest Formula

$$\boxed{\bar{x} = A + \frac{\sum f_i u_i}{n} \times c}$$

class width

class distribution even হলে,

$$u_i = \frac{d_i}{c}$$

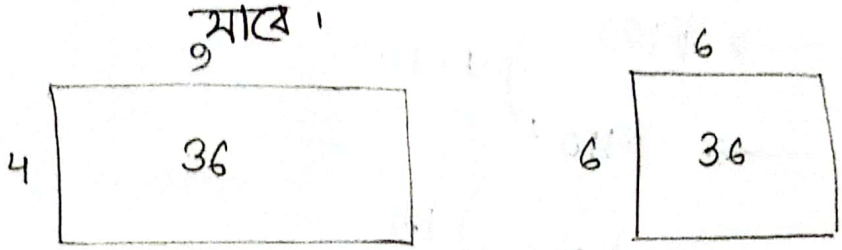
$$\boxed{u_i = \frac{x_i - A}{c}}$$

$$\bar{x} = 25 + \frac{(-14 - 8 + 0 + 10 + 10)}{50} \times 10$$

$$=$$

② Geometric Mean :

* Multiplicative in nature এরকম data র জন্য।
* এমন একটা value নিবো যেন একবার multiplication করলে তাকে একটা সংখ্যার square root করা যাবে।



$x_1, x_2, \ldots, x_n$

$$G = \sqrt[n]{x_1 \cdot x_2 \cdots x_n}$$

$$= \left(\prod_{i=1}^{n} x_i \right)^{1/n}$$

$$\log G = \frac{1}{n} \log (x_1 x_2 \cdots x_n)$$

$$= \frac{1}{n} (\log x_1 + \log x_2 + \cdots + \log x_n)$$

$$\boxed{\log G = \frac{\sum \log x_i}{n}}$$

$$\boxed{\log G = \frac{\sum f_i \log x_i}{\sum f_i}} \rightarrow \text{Group / Tabular data}$$

$$\therefore G = \text{anti}\log \left(\frac{\sum \log x_i}{n} \right)$$

18-03-2023

* Microsoft stock gained 10% in year 1, 20% in year 2 and 30% in year 3. What is the average yearly rate of return?

Year 0 → \$100

1.1

Year 1 → \$110

1.2

Year 2 → \$132

1.3

Year 3 → \$171.60

→ consider 3 sides of cube

$$G = \sqrt[3]{1.1 \times 1.2 \times 1.3} = 1.1972$$

→ Geometric mean এ সব side equal

$\$100 \times 1.1972 = \$119.72$

$\$119.72 \times 1.1972 = \$143.33$

$\$143.33 \times 1.1972 = \$171.59$

$\$100 \times (1.1972)^3 \rightarrow 19.72\%$

$1.1972 = (1 + 0.1972)$

→ \$100 থেকে

Arithmetic mean calculate করলে $\$100 \times (1.2)^3$
$= \$172.8$

which is quite accurate but geometric mean gives the actual rate.

Harmonic Mean :

$$\frac{1}{H} = \frac{1}{N} \sum \frac{1}{x}$$

$$\boxed{H = \frac{N}{\sum \frac{1}{x}}}$$

2, 3, 4 → 3টা সংখ্যা

$$\therefore H = \frac{3}{\frac{1}{2} + \frac{1}{3} + \frac{1}{4}} = 2.77$$

Harmonic series : $\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5} \ldots$

* কোনো particular frequency play করা হলে সেটা একা play করা হয় না কিছু over tone play হয় সেখানে একটা value তার previous ও next value র Harmonic mean. Harmonic series থেকে analysis করে Harmonic mean বের করা হয়।

For group / Tabular data :

$$H = \frac{\sum f}{\sum \frac{f}{x}}$$

* A car can travel 25 miles at 25 mph, 25 miles at 50 mph and 25 miles at 75 mph. What is the average velocity?

$$\text{Average Velocity} = \frac{\text{Total distance}}{\text{total time}}$$

$$t = \frac{s}{v} \longrightarrow = \frac{75}{\frac{25}{25} + \frac{25}{50} + \frac{25}{75}}$$

$$= \boxed{\frac{3}{\frac{1}{25} + \frac{1}{50} + \frac{1}{75}}}$$

→ Harmonic Mean এর formula.

$$= 40.9 \text{ mph}$$

Arithmetic mean : $\frac{25+50+75}{3} = 50 \text{ mph}$

↓ অনেক বেশি diff আসে.

* যখন উপরেরটা fixed (এখানে travel distance) এবং নিচেরটা variable (sub unit এ variable) হলে Harmonic mean use করা যাবে।

* swim 1 minute of freestyle at 3 km/h and
1 minute of buttenfly at 2 km/h speed

এখানে time fixed but distance variable ($\frac{\text{distance}}{\text{time}}$)
তাই arithmetic mean দিয়ে বের করা যাবে.

$$\frac{3+2}{2} = 2.5 \text{ km/h}$$

* swim one lap of free style at 3 km/h
and one lap of buttenfly at 2 km/h.

fixed distance

$$H = \frac{2}{\frac{1}{3} + \frac{1}{2}} = 2.4 \text{ km/h}$$

22 . 03 . 2023

Median :

Odd number : $\frac{N+1}{2} + h$

Even number : Avg. $\frac{N}{2} + h$ and $(\frac{N}{2}+1)+h$ value

* at most 50% data median এর চেয়ে বেশি হবে।

2, 4, 5, 10, 12

Median এর চেয়ে বড় Number আছে $= \frac{2}{5} \times 100$
$= 40\%$

" " " ছোট " " $= \frac{2}{5} \times 100$
$= 40\%$.

* Avg outline দ্বারা sensitive. but median না।

2, 4, 5, 10, 1000

Median = 5

avg. = অনেক huge.

For even number

2, 4, 5, 10, 12, 25

↓

Avg নিয়া হয়

Median = 7.5

Mode :

* frequency distribution এর mode বলা হয় সেই value যার frequency বেশি।

2 2 2 3 3 5

x	f
2	3
3	2
5	1

$\longrightarrow$ unimodal

mode = 2

2 2 2 3 3 3 $\longrightarrow$ uniform (সবগুলোর frequency same)

2, 2, 2, 3, 3, 3, 5, 7, 10 $\longrightarrow$ Bimodal.

x	f
2	3
3	3
5	1
7	1
10	1

$\longrightarrow$ Bimodal.

Median for group data:

weight	frequency	cummulative frequency
30 – 40	18	18
40 – 50	37	55 p.c.f
50 – 60	45 → f_m	100
60 – 70	27	127
70 – 80	15	142
80 – 90	8	150

median class → 50 – 60

$N = \Sigma f = 150$

Median হবে $\frac{150}{2}$ বা 75 no. value যে class এ আছে

$$\text{Median} = L + \frac{\frac{N}{2} - p.c.f}{f_m} \times c$$

L = lower class boundary of median class

f_m = frequency of median class

p.c.f = preceeding cummulative frequency / cummulative freq. of class before median

$(\Sigma f)_1$

c = class width

Lower class boundary

এটাই true lower boundary

49.5 ছোট value 40-49

" বড় " 50-59

Lower class interval

lower boundary নিলে	Weight
30-39	30-40
40-49	40-50
50-59	50-60

$\frac{50+49}{2} = 49.5$

এটা হলো (50-60) এর true lower boundary

$$\text{Median} = 49.5 + \frac{\frac{150}{2} - 55}{45} \times 10$$

$$= 53.494$$

* Median class খুঁজতে হতে হলে প্রথম class এর frequency অনেক বেশি হয়।

Mode for group data:

$$\text{Mode} = L + \frac{\Delta_1}{\Delta_1 + \Delta_2} \times C$$

L = lower class boundary of modal class.

Δ_1 = Difference between modal class and pre modal class

$= f_m - f_1$

Δ_2 = ~~no~~ difference between modal class and post modal class

$= f_m - f_2$

Weight	Frequency
30 - 40	18
40 - 50	37 → f_1
50 - 60	45 → f_m
60 - 70	27 → f_2
70 - 80	15
80 - 90	8

$\Delta_1 = f_m - f_1$
$= 45 - 37 = 8$

$\Delta_2 = f_m - f_2$
$= 45 - 27$
$= 18$

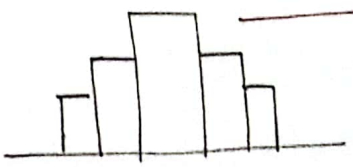
$$\text{Mode} = 50 + \frac{8}{8+18} \times 10$$

$$= 52.58$$

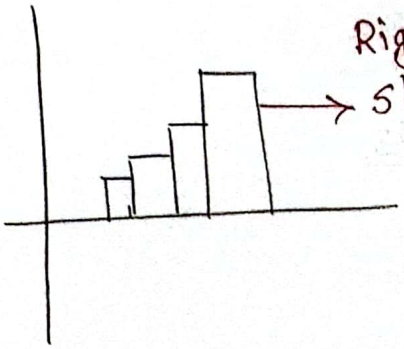
Empirical Relation of mode, Median and Mean:
For moderately skewed distribution

$$\text{Mode} = 3\,\text{Median} - 2\,\text{mean}$$

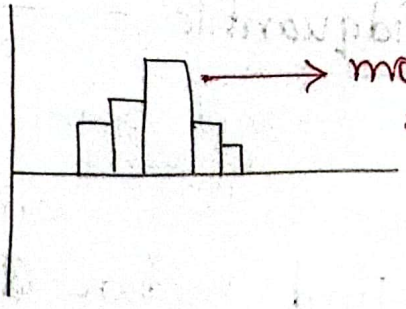
proof করা যায়নি, only observation করে পাওয়া



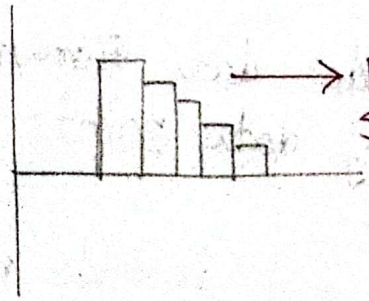
→ symmetrical distribution



Right (+ve) skewed distribution (এক side এ বেশি)



→ moderately skewed distribution



→ Left (-ve) skewed distribution

29.04.2023

Quantiles : Quantiles are values that divide total frequency into 4 parts

25%	25%	25%	25%

L Q_1 Q_2 Q_3 H

$Q_1 \rightarrow$ At most 25% data are smaller than Q_1 and at most 75% data are larger.

$Q_3 \rightarrow$ At most 75% data are smaller than Q_3 and ~~25~~ at most 25% data are larger

$Q_2 \rightarrow$ 50 - 50

$IQR = Q_3 - Q_1$

Midquantile $= \frac{Q_1 + Q_3}{2}$

↳ mid point যেখানে এটা মানে Q_2 না

Steps to find Quantiles:

* Find median Q_2
* Use median to divide data further.
  - → For odd no of data points includes median in both halves.
  - → For even, do not include median in either.

Q_1 Q_2 Q_3

2, 3, 5, 7, 8, 9, 10, 12, 15 → Sorted data

10, 12, 14, 15, 14, 16, 17, 18, 10, 19, 17, 17

$Q_1 = 13$ $Q_2 = 15.5$ $Q_3 = 17$

Sorted: 10, 10, 12, 14, 14, 15, 16, 17, 17, 17, 18, 19

Grouped Data :

$$Q_i = L + \frac{\frac{i \times N}{4} - p.c.f}{f} \times h$$

L = Lower class of boundary of quartiles class.

$$Q_2 = L + \frac{\frac{N}{2} - p.c.f}{f} \times h$$

* Quartile median এর generalization

f = frequency of quartile class

*quartile class is identified by $\frac{i \times N}{4}$ th observation.

Profit	No. of Companies	c.f
20-30	4	4
30-40	8	12
40-50	18	30
50-60	30	60
60-70	15	75
70-80	10	85
80-90	8	93
90-100	7	100

$N = 100$

$$Q_2 = 49.5 + \frac{\frac{2 \times 100}{4} - 30}{30} \times 10$$

$$= 56.17$$

$$Q_1 = 39.5 + \frac{\frac{1 \times 100}{4} - 12}{18} \times 10$$

$$= 46.72$$

$$Q_3 = 59.5 + \frac{\frac{3 \times 100}{4} - 60}{15} \times 10$$

$$= 69.5$$

Percentiles :

$$P_i = L + \frac{\frac{i \times N}{100} - p.c.f}{f} \times h$$

$\frac{i \times N}{100}$ th ~~th~~ observation

$P_{75} = Q_3$

$P_{50} = Q_2$

$P_{25} = Q_1$

Deciles :

$$D_i = L + \frac{\frac{i \times N}{10} - p.c.f}{f} \times h$$

$i = 1, 2, 3, \ldots 9$

$D_2 = P_{20}$

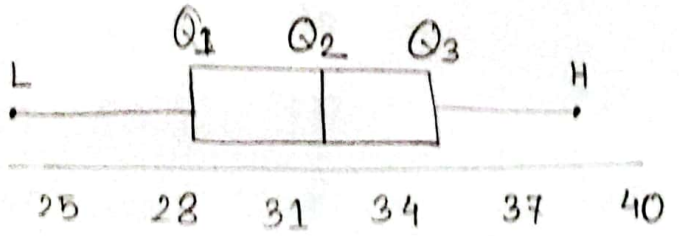
06 - 05 - 2023

Box Whisker plot / 5 number Summary :

$L, H, \underbrace{Q_1, Q_2, Q_3}_{\text{Quartiles}}$

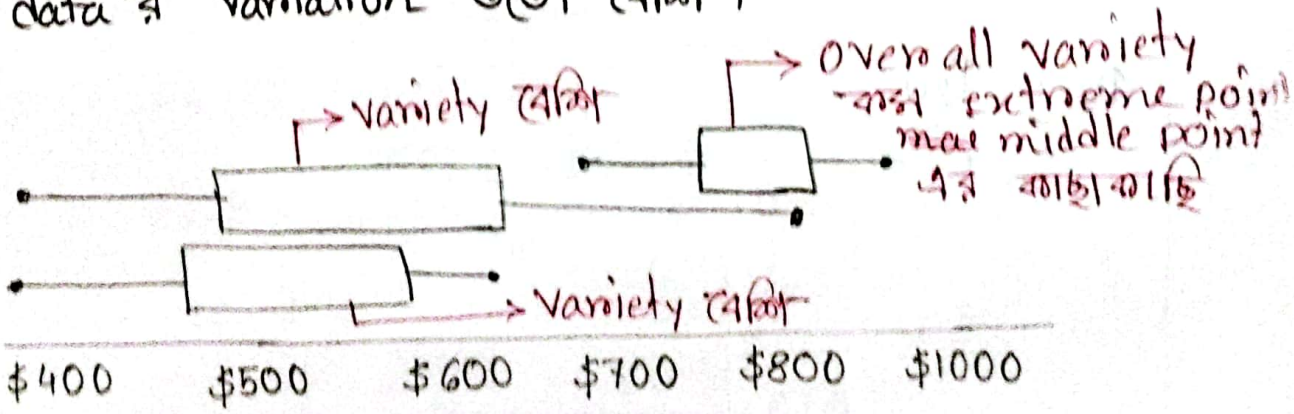
Data : 25, 28, 29, 29, 30, 34, 35, 35, 37, 38

$L = 25$, $H = 38$, $Q_2 = 32$, $Q_1 = 29$, $Q_3 = 35$



* Multiple box whisker plot কে একে অপরের against এ plot করা করে

* Data টা central point থেকে যত দূর আছে থাকবে data র variation ততো বেশি ।



* box যতো বড় variation ততো বেশি

* ~~Outlier~~ Outlier ~~Die~~ Detection using IQR

* Outlier → Particular value distribution follow না করলে তা outlier value.
Outlier highest value দেখে খালি বের করা যাবে না, comparison লাগবে।

* Outlier বের করতে Inter Quantile Range use করা হয়।

$$IQR = Q_3 - Q_1$$

$$x > Q_3 + 1.5 \times IQR$$
$$x < Q_1 - 1.5 \times IQR$$

→ এই condition মিললে data কে Outlier বলা হয়।

$Q_1 = 2.5$ $\quad$ Q_2 $\quad$ $Q_3 = 27.5$

$-30 \quad -5 \quad 10 \quad 20 \quad 25 \quad 30 \quad 200$

$Q_1 = 2.5$

$Q_2 = 20$

$Q_3 = 27.5$

$IQR = 27.5 - 2.5 = 25$

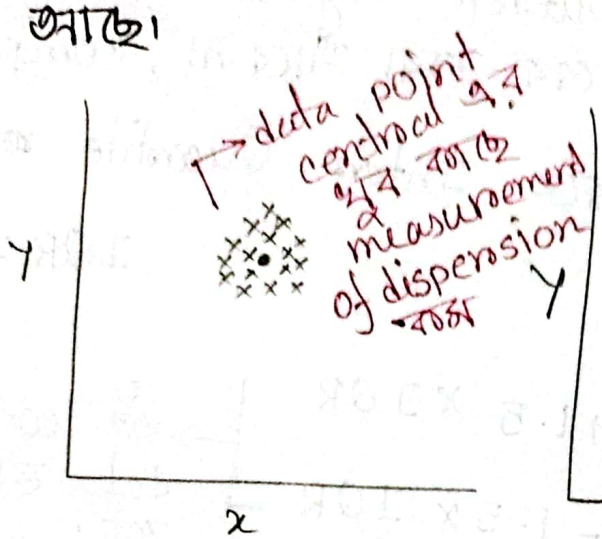
$$Q_3 + 1.5 \times IQR = 65$$
$$Q_1 - 1.5 \times IQR = -35$$

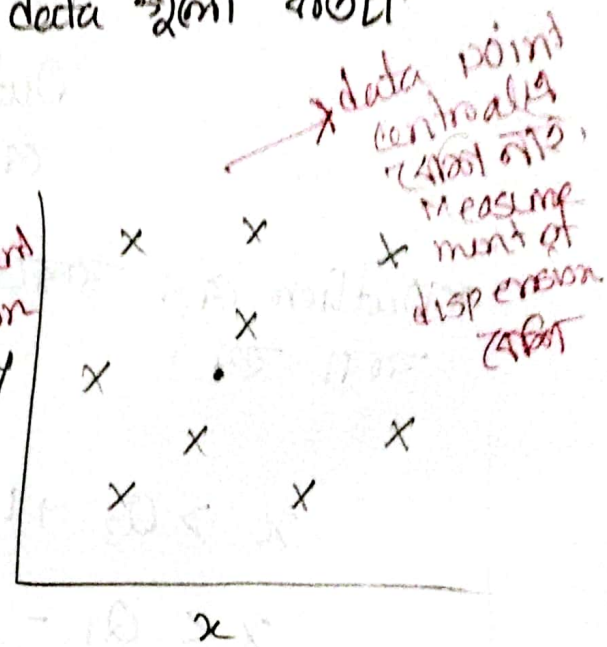
→ 65 এর চেয়ে বড় ও -35 এর চেয়ে ছোট value outlier হবে।

∴ 200 এখানে outlier.

Measure of Dispersion

* Central point থেকে অন্য data গুলো কতটা ছড়িয়ে আছে।





* Range = H-L ⟶ Measurement of dispersion এ use করা হয় না।

* Measure of dispersion data র variation বেশি হবে। Range এর ক্ষেত্রে always এমন হয় না।

* ~~Quartile Deviation, Q.D~~ = $\frac{Q_3 - Q_1}{2}$ = $\frac{IQR}{2}$

A: 6, 46, 46, 46, 46

B: 6, 6, 6, 6, 46

C: 6, 10, 15, 35, 46

R = 46 - 6 = 40

A, B, C এর range same হলেও A ও B তে data variation কম

* Quantile Deviation, $Q.D = \frac{Q_3 - Q_1}{2} = \frac{108}{2}$ lakh

Profit (In lakh)	# of company	cf
20 - 30	4	4
30 - 40	8	12
40 - 50	18	30
50 - 60	30	60
60 - 70	15	75
70 - 80	10	85
80 - 90	8	93
90-100	7	100

$$Q_i = L + \frac{\frac{i \times N}{4} - p.c.f}{f} \times h$$

$$Q_2 = 49.5 + \frac{\frac{2 \times 100}{4} - 30}{10} \times 10 = 56.17 \text{ lakh}$$

$Q_1 = 46.72$ lakh

$Q_3 = 69.50$ lakh

$Q.D = 11.39$ lakh

Mean Deviation :

$$M.D = \frac{\sum_{i=1}^{N} |(x_i - \bar{x})|}{N}$$

For grouped data, $M.D = \frac{\sum f_i |x_i - \bar{x}|}{N}$

	f	x	$\lvert x_i - \bar{x} \rvert$
20-30			
30-40			

10 68 90 40

$$\bar{x} = \frac{208}{4} = 52$$

$$MD = \frac{|10-52| + |68-52| + |90-52| + |40-52|}{4}$$

$$= 27$$

Variance,

$$\sigma^2 = \frac{\sum (x_i - \bar{x})^2}{N}$$ → MD এর কাছাকাছি just square করা হয়।

Standard Deviation,

$$\sigma = \sqrt{\frac{\sum (x_i - \bar{x})^2}{N}}$$

[Group data র টা দেখে নিতে হবে]

* standrand deviation এ unit করে যায় যা consistent হয় তাই use করা হয় বিভিন্ন operation. Analysis করার কাজে use হয়।
* বিভিন্ন problem এর distribution এ calculation এর জন্য use করা হয় variance.

Variance এর shortcut method for table,

$$\sigma^2 = \frac{\sum(x_i - \bar{x})^2}{n}$$

$$= \frac{\sum(x_i^2 - 2x_i\bar{x} + \bar{x}^2)}{n}$$

$$= \frac{\sum x_i^2}{n} - \frac{2\bar{x}\sum x_i}{n} + \frac{\sum \bar{x}^2}{n}$$

$$= \frac{\sum x_i^2}{n} - 2\bar{x}^2 + \frac{n\bar{x}^2}{n}$$

$$= \frac{\sum x_i^2}{n} - 2\bar{x}^2 + \bar{x}^2$$

$$= \frac{\sum x_i^2}{n} - \bar{x}^2$$

$$\sigma^2 = \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2$$

Assumed mean এর instance variance.

$$\sigma^2 = \frac{\sum f_d^2}{n} - \left(\frac{\sum f_d}{n}\right)^2 \qquad d_i = x_i - A, \quad u_i = \frac{x_i - A}{c}$$

$$= \left[\frac{\sum f_u^2}{n} - \left(\frac{\sum f_u}{n}\right)^2\right] \times c^2$$

$$s^2 = \frac{\sum (x_i - \bar{x})^2}{n-1}$$

Bessel's correction

* ~~Bas~~ Biased estimation : (n-1) দিয়ে ভাগ দিলে biased হয় না.

08-05-2023

Coefficient of variation:

↳ unitless.

Absolute measurement : একক থাকে (unit)

Relative " : comparison type করে & Different elements এর

Group 1 (lbs) : 120, 130, 140, 150, 160
$\mu_1 = 140$ $\sigma_1 = 14.14$

Group 2 (kg) : 60, 65, 70, 75, 80 $\mu_2 = 70$ $\sigma_2 = 7.07$

কোন group এ variation বেশি?

dispersion same হয়, unit আলাদা বলে দেখে মনে হচ্ছে double. এখানে dispersion দিয়ে measure করলে correct comparison আসবে না as unit আলাদা। এজন্য আলাদা unit এর data র জন্য coefficient of variation use করা হয়।

$$CV = \frac{\sigma}{\bar{x}} \times 100$$

$$CV_1 = \frac{14.14}{100} \times 100\% = 10.10\%$$

$$CV_2 = \frac{7.07}{70} \times 100\% = 10.10\%$$

→ দুইটা group এর data র dispersion same.

68-95 - 99.7 Rule, Empirical Rule for Normal distribution!

Z-score: standardization/standard score:

$$z = \frac{x - \mu}{\sigma}$$

* একটা particular value mean এর থেকে কত standard deviation এগিয়ে বা পিছিয়ে সেটা measure করবে, z-score এটা relative measurement

Q: Suppose mean of an exam is 80 and std. deviation 6.3. A student achieved 92.5. Student এর standard score কতো?

$$z = \frac{92.5 - 80}{6.3} = 1.97$$

Avg এর চেয়ে পরিমান std deviation এগিয়ে

$\mu = 80$

$\sigma = 5$

student A → 87

$$z_A = \frac{x - \mu}{\sigma} = \frac{87 - 80}{5} = 1.50$$

$\mu = 73$

$\sigma = 6$

student B → 82

$$z_B = \frac{x - \mu}{\sigma} = \frac{82 - 73}{6} = 1.50$$

* comperitively student B result ভালো করেছে।

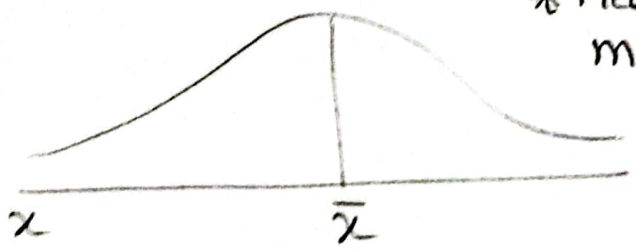
$x_i \longrightarrow 5, 7, 10, 12, 15, 16, 20, 25$ $\bar{x} = 13.75$

$\sigma = 6.24$

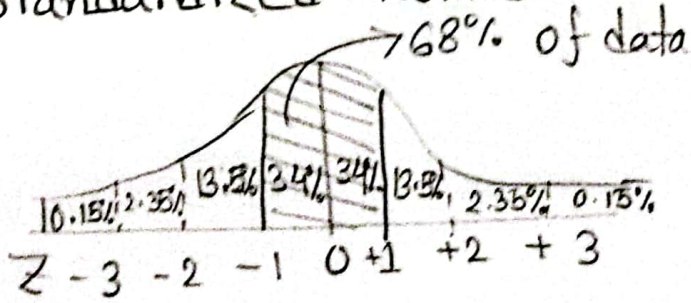
$z \longrightarrow -1.40, -1.08, -0.60, -0.28, 0.28, 0.36, 1.00, 1.80$

Normal distribution!

* Mean, median and mode সমান



যদি normal distribution কে z scale এ রাখা হয় সেটা standardized normal distribution



* কিছু data যদি এমন হয় যে normal distribution এর pattern follow করে তখন mean থেকে 1 standard deviation data (-1 to +1) র মধ্যে 68% data থাকবে।

* -2 থেকে +2 range এ 95% data থাকবে।

* -3 থেকে +3 range এ 99.7% data বা area পড়বে।

* এটাই 68 - 95 - 99.7 Rule

* $z > +3$, $z < -3$ হলে সে data extreme outlier.

10-05-2023

data standardized করার পর (Z-score এ convert করার পর)

mean $\longrightarrow 0$

Variance / SD $= 1$

$$N(x; \mu, \sigma) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

μ & σ $\longrightarrow$ parameters

$P(x)$ = Probability density function

standardized Normal distribution,

$$N(x; 0, 1) = \frac{1}{\sqrt{2\pi}} e^{-x/2}$$

$\int_{-\infty}^{0} N(x; 0, 1) = 0.5 \longrightarrow$ ∞ to 0 পর্যন্ত area under curve.

Q: Mean grade on final exam is 72 and standard deviation is 9. The top 10% student will get A+. What is the minimum mark required to get A+?

$$Z = \frac{x - \mu}{\sigma}$$

$\therefore x = \mu + Z\sigma$

$= 72 + Z \times 9$

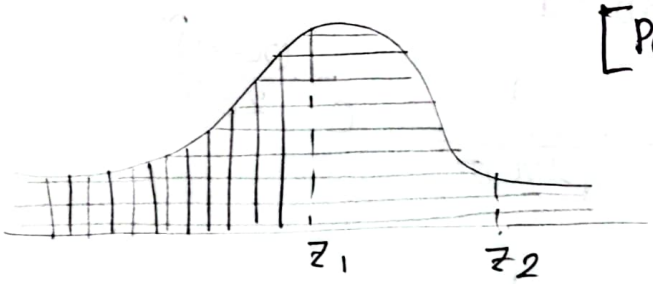
$= 72 + (1.28 \times 9)$ [$Z = 1.28$ from Z-score table]

$\downarrow$ Left sided Z-score table

row $\rightarrow$ 1.2
column $\rightarrow$ 0.08 } $\rightarrow$ $Z = 90\%$ almost

H.W:

range of marks in central 50%



[Percentile থেকে z-score বের করা calculator এ নাই] বাকিগুলো আছে

20.05.2023

Left sided z-value ⟶ percentile [Left, side এ যতগুলো area আছে]

↓

calculator এ করা যায়

1.0 ⟶ 84%

1.1

1.2

⋮

increment করলে area ও বাড়ে।

Chebyshev's Theorem

The proportion of any distribution that lies within K standard deviation of the mean is at least

$1 - \frac{1}{K^2}$, $K > 1$

↳ lower bound

K	1.7	2	2.5	3
$1 - 1/K^2$	0.65	0.75	0.84	0.89

Normal distribution এ 2 এর জন্য 95% হলেও, এখানে বলেছে at least 75% এর মধ্যেই থাকবে। তাছাড়া সব ধরনের distribution এর জন্য empirial rule নাই। সেক্ষেত্রে এই theorem কাজে লাগবে।

Moments :

$x_1, x_2, x_3, \ldots, x_n$

$$\overline{x_r} = \frac{\sum_{i=1}^{N} x_i^r}{N}$$

Central moment $\mu_r = \dfrac{\sum_{i=1}^{N} (x_i - \bar{x})^r}{N}$

$r = 0, \mu_0 = 1$

$r = 1 \ldots \mu \longrightarrow$ more important

$r = 1, \mu_1 = 0 \longrightarrow$ sum of dispersion

$r = 2, \mu_2 = \sigma^2 \longrightarrow$ variance

$r = 3, \longrightarrow$ data এর symmetry / Assymtry measure করে।

$r = 4 \longrightarrow$ data এর tale এবং peak এর ratio ~~meas~~ measure করে।

Raw moment, $\mu_r' = \dfrac{\sum_{i=1}^{N} (x_i - A)^r}{N}$

A = Assumed mean

A reduces calculation

for grouped data

$$\mu_r' = \frac{\sum f_i (x_i - A)^r}{N}$$

$$\mu_r = \frac{\sum f_i (x_i - \bar{x})^r}{N}$$

$\mu_1 = 0$

$$\mu_1' = \frac{\Sigma(x_i - A)}{N} = \frac{\Sigma x_i}{N} - \frac{\Sigma A}{N}$$

$$= \bar{x} - \frac{NA}{A}$$

$$= \bar{x} - A$$

$$\mu_2 = \frac{\Sigma(x_i - \bar{x})^2}{N}$$

$$= \frac{\Sigma\{(x_i - A) - (\bar{x}_i - A)\}^2}{N}$$

$$= \frac{\Sigma\{(x_i - A)^2 - 2(x_i - A)(\bar{x} - A) + (\bar{x} - A)^2\}}{N}$$

$$= \frac{\Sigma(x_i - A)^2}{N} - 2(\bar{x} - A)\frac{\Sigma x_i - A}{N} + \frac{\Sigma(\bar{x} - A)^2}{N}$$

$$= \mu_2' - 2\mu_1' \cdot \mu_1' + \frac{\Sigma \mu_1'^2}{N} \rightarrow \frac{N\mu_1'^2}{N}$$

$$= \mu_2' - 2\mu_1'^2 + \mu_1'^2$$

$$= \mu_2' - \mu_1'^2$$

$\therefore \boxed{\mu_2 = \mu_2' - \mu_1'^2}$ → Relation betn raw and ~~and~~ central moment

$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$$

$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$$

$$\mu_1' = \frac{\sum f_i(x_i - A)}{N}$$

$$= \frac{\sum(f_i u_i - h)}{N}$$

$$= \frac{\sum f_i u_i}{N} \times h$$

$$u_i = \frac{x_i - A}{h}$$

$$\Rightarrow x_i - A = u_i h$$

$$\therefore \mu_2' = \frac{\sum f_i u_i^2}{N} \times h^2$$

$$\mu_3' = \frac{\sum f_i u_i^3 \times h^3}{N}$$

$$\therefore \boxed{\mu_r' = \frac{\sum f_i u_i^r}{N} \times h^r}$$

Skewness, moments and kurtosis:

$\frac{\sum f_i u_i}{N} \times h$, $\frac{\sum f_i u_i^2}{N} \times h$; $u_i = \frac{x_i - A}{h}$

Assumed mean = raw moment

about the mean = central moment

আলাদা assumed mean ধরলে আলাদা raw moment but আলাদা central moment same থাকবে।

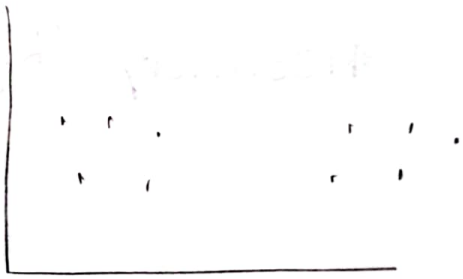
1st central moment = 0 ⟶ Always.

22-05-2023

Standardized moment:

* Transition Invariant:

  * value র সাথে constant term যোগ করলে value shift হবে but dispersion (ছড়িয়ে যাবে না) হবেনা, standard deviation change হবেনা।



Z score, $z = \frac{x-\bar{x}}{\sigma}$

std. deviation দিয়ে ভাগ

* standard deviation দিয়ে অনেক সময় data কে ভাগ করা যায়।

* distribution এর ক্ষেত্রে shape এর কিছু property Follow করে moment, skewness

* standard deviation দিয়ে কোনো data ভাগ দিলে unit less হবে। which will make the comparison easy and Dimentionless.

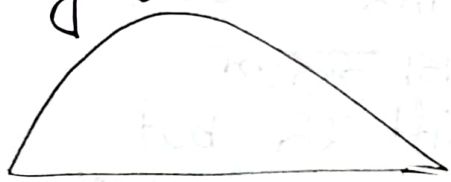
standardized moment:

$\frac{\mu}{\sigma}, \frac{\mu}{\sigma^2}, \frac{\mu}{\sigma^3}, \ldots \rightarrow$ std deviation দিয়ে ভাগ করা হলো

Skewness:

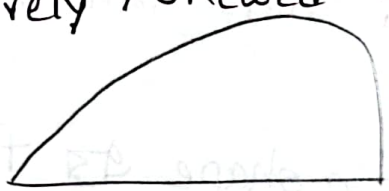
কোন দিকে data distribution asymmetric থাকবে।

*Positively / Right Distribution:



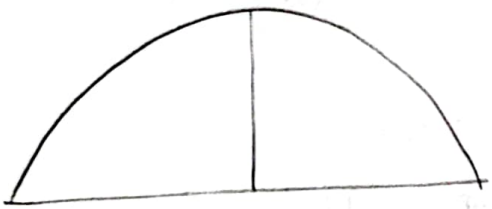
* Mean সবচেয়ে বড় হয় এরপর median and mode

*Negitively / Skewed Distribution:



* Mean সবচেয়ে ছোট mode বেশি।

*Symmetrical Distribution



* mean, mode, median same.

Coefficient of Skewness:

$$Skp = \frac{Mean - Mode}{\sigma}$$

→ Positive distribution (+)
→ Negative " (−)

↳ 1st skewness coefficient

* Skewness Factor same থাকতে হবে, size ছোট বড় করলেও, shape same থাকবে just scale up বা down হবে।

$$Skp = \frac{3(mean - median)}{\sigma}$$

↳ 2nd coefficient of skewness

~~* Quantile দিয়েও~~

* Quantile and Percentile দিয়ে calculate করা যায়

skewness range $-\infty$ to ∞, Ideally value (−3 to 3) থাকে because of std. deviation

* Purely symmetric value হলে $Skp = 0$

3rd central moment → (+) হলে (+)ve skewness
→ (−) " (−) " "

$$\frac{\mu_3}{\sigma^3}$$

Karl Pearson skewness :

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3}$$

$\mu_2 \rightarrow$ Variance
কখনও positive না

β_1 always (+)ve হবে, refers to magnitude, direction refer করতে পারে না।

$$\gamma_1 = \frac{\mu_3}{\sigma^3} = \frac{\mu_3}{(\sigma^2)^{3/2}} = \frac{\mu_3}{\mu^{3/2}} = \sqrt{\beta_1}$$

$\gamma_1 \rightarrow$ Karl Pearson moment coefficient of skewness

$\sigma^2 \rightarrow$ 2nd central moment
σ^2 = Variance

$\gamma_1 \rightarrow$ (+) হলে (+)ve skewed
$\rightarrow$ (−) " (−)ve "

Kurtosis $\rightarrow$ 4th standard central distribution.

23.05.2023

$$\gamma_1 = \frac{\sqrt{n}\sum_{i=1}^{n}(x_i-\bar{x})^3}{\left[\sum_{i=1}^{n}(x_i-\bar{x})^2\right]^{3/2}} = \frac{\mu_3}{\sigma^3}$$

* raw moment দিয়ে বের করা easy

Kurtosis:

* Skewness distribution এর দুইটা জিনিস নিয়ে concern

(i) Peak.

(ii) Tail

Peak and Tail এর উপর Kurtosis depend করে।

Kurtosis distribution

(1) leptokurtic:
Sharper than normal distribution, Fat tail than normal distribution

(2) Mesokurtic:
Normal distribution এর মতো peak ও tail

(3) Platykurtic:
Tail thinner than normal distribution,
Peak Flatter than normal distribution

* moment এর square power দিয়ে
* " " qubic " " Skewness বুঝায়
* " " forth " " kurtosis (Peak & tail) বুঝায়

$$\beta_2 = \frac{\mu_4}{\sigma^4} = \frac{\mu_4}{\mu_2^2}$$

$$= \frac{n \sum_{i=1}^{n} (x_i - \bar{x})^4}{\left[\sum (x_i - \bar{x})^2\right]^2}$$

mesokurtic $\longrightarrow \beta_2 = 3$
leptokurtic $\longrightarrow \beta_2 > 3$
playtokurtic $\longrightarrow \beta_2 < 3$

* moment generatic function $\longrightarrow$ polynomical এর power হলো

Excess Kurtosis, $\gamma_2 = \beta_2 - 3$

$\hookrightarrow$ playto (−) হবে
lepto (+) "

$\rightarrow$ kurtosis বের করতে হলে এটা হবে
(using 4th and 2nd central moment)

Corelation

* একটা বৃদ্ধি করলে আরেকটায় কি change করে।

Positive Negative:

Simple & multiple:

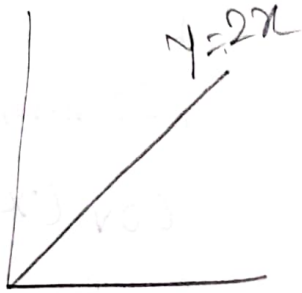
(i) simple → দুইটা variable এর মধ্যে

(ii) Multiple → কয়েকটার মধ্যে

Linear: Linear সম্পর্ক আছে কিনা। একটা change করলে আরেকটা ঐ scale এই change হচ্ছে কিনা

Method of estimation Corelation:

* perfect positive corelation:
  perfectly একটা line এ fit করবে
  ঢাল negative



* perfect negative corelation:
  perfectly একটা line এ fit করবে তবে ঢাল (-) হবে

* High degree of Positive

* High degree of Negative

* No corelation!

কোনো সম্পর্ক নাই একটা বাড়লে আরেকটা কমবে না বাড়বে ঠিক নাই।

* corelation হলেই যে cause and effect হবে এমন না।

* আবার causaion হলেই corelation হবে।

Covariance:

$$Cov(x, y) = \sigma_{x,y} = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{N}$$

$$\boxed{\sigma^2 = \frac{\Sigma(x_i - \bar{x})^2}{N}}$$

↓ Variance

* দুইটা variable এর জন্য calculate করা হয়।

* covariance দুইটা ~~var~~ ~~value~~ variable এর corelation নির্ণয় করে।

Covariance

- (+) positively corelated
- 0 No corelation
- (−) Negatively corelated

24.05.2023

$$Cov(x,y) = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{N}$$

*স্কেল down করলে calculation easy.

covariance → Linear corelation

range → $-\infty$ to $+\infty$

Karl Pearson's coefficient of corelation

$$r = \frac{cov(X,Y)}{\sigma_x \sigma_y}, \quad -1 \le r \le +1$$

এটা Linear corelation measure. করে।

non " " " " না।

Ex:

x	0	1	2	3
y	1	3	5	7

$\bar{x} = 1.5$

$\bar{y} = 4$

$$cov(x,y) = \frac{(0-1.5)\times(1-4) + (1-1.5)\times(3-4) + (2-1.5)\times(5-4) + (3-1.5)\times(7-4)}{4}$$

$= 1.81$

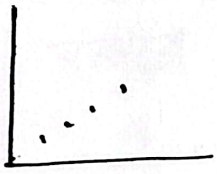
$$r = \frac{cov(x,y)}{\sigma_x \sigma_y} = \frac{2.5}{1.12 \times 2.24} = +1.00$$

∴ perfectly (+)vely corelated হলে $r = +1$

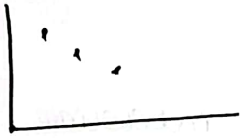
$y = 2x+1$; (~~pro~~ perfectly st line)

$$r = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\frac{\Sigma(x_i - \bar{x})^2}{N}} \sqrt{\frac{\Sigma(y_i - \bar{y})^2}{N}}}$$

$$r = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\Sigma(x_i - \bar{x})^2} \sqrt{\Sigma(y_i - \bar{y})^2}}$$



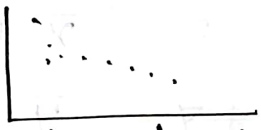
$r = +1$



$r = -1$

এই দুই ক্ষেত্রে st line এ fit করে

r close to 1



r close to -1

$r = 0$

corelation exist করে না

$r = 0$

$y = x^2$ (non linear parabolic, hyperbolic measure করতে পারে না)

Karl pearson coefficient of corelation linear relation এর জন্য, non linear এর জন্য $r = 0$ মানে $r = 0$ মানে এটা না যে তারা অন্যভাবে corelated না। তাই non-linear এর জন্য use করা যাবে না

~~30-05-2023~~
30-05-2023

Karl Pearson থেকে $r_s = 1 - \frac{6\Sigma D^2}{N^3 - N}$ → Spearman's Rank correlation coefficient

* mainly rank নিয়ে কাজ করে

D = দুইটা Rank এর বিয়োগফল

N = total number of elements

* correlation এখানে mean করে rank গুলোর correlation.

* Rank যতো বেশি ততো ভালো।

* Value গুলো repeat করলে

$$r_s = 1 - \frac{6\{\Sigma D^2 + \frac{1}{12}(m_1^3 - m_1) + \frac{1}{12}(m_2^3 - m_2) + \cdots\}}{(N^3 - N)}$$

* m = যতবার repeat হয়েছে

* সবচেয়ে বেশি N এর rank বেশি

* same value হলে rank avg করে rank দিবে।

80	60	40	28	20	20	15	12
Rank → 8	7	6	5	4	3	2	1

$\frac{4+3}{2} = 3.5$ প্রতিটি

~~Regression~~

~~*Data থেকে straight line বের করা।~~

from 24-05-2023

z-score	percentile
0	50%
1	84%
2	97%

left side এর z table এর জন্য

percentile থেকে z score ?

Q: 16% এর জন্য z এর value কত ?

Spearman's Rank corelation Coefficient

এই Karl pearson এর special case এটা Rank থেকে measure করা হয়।

$$r_s = 1 - \frac{6\sum D^2}{N(N^2-1)}$$

largaro data থেকে sample নেওয়া → sample

population > sample

→ question এ উল্লেখ না থাকলে এটা use

30-05-2023

Spearman's Rank Correlation Coefficient:

Karl Pearson এর formula থেকে এসেছে

$$r_s = 1 - \frac{6\sum D^2}{N^3 - N}$$

Example -(P-5)

সবচেয়ে ভালো যে তার rank সবচেয়ে বেশি।

Karl pearson এর formula তে ধরে নিবো x, y এর value same range এ থাকবে। তারা inclusive আকারে থাকবে। একের পর এক Karl Pearson formula দিলেও result আসবে। Spearman's formula comparatively easy.

-1 থেকে +1 ⟶ Karl Pearson & Spearman's Rank range

2টা manager 91% case এ agree করেছে।

$$D^2 = (R_1 - R_2)^2$$

manually rank ⟶ সবচেয়ে বড় value ⟶ 10 তারপরও...

Tie in Ranks:

$$r_s = 1 - \frac{6\left\{\sum D^2 + \frac{1}{12}(m_1^3 - m_1) + \frac{1}{12}(m_2^3 - m_2) + \dots\right\}}{N^3 - N}$$

$m_1, m_2 \dots$ চলতে থাকবে [ধারা]

80, 60, 40, 28, 20, 20, 15, 2

rank → 8, 7, 6, 5, 4, 3, 2, 1

3.5, 2টা

2টার average নিবো

rank এ কোন convention follow করছি তা matter করে না

→ desending / Ascending

D^r করার কারনে result same

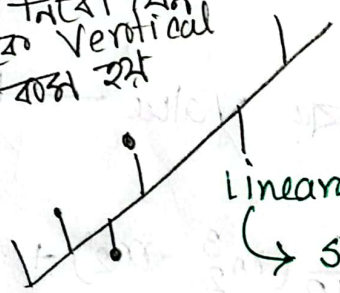
1টা num কতবার আছে তাই m

* co-relation = 0 মানে neutral. এই 2টার মাঝে কোনো সম্পর্ক নাই।

এগুলো linear co-relation

Regression Analysis:

এমন line নিবো যেন point থেকে Vertical distance কম হয়



Regression মানে কোনো কিছুকে যাওয়া point গুলো থেকে st line নিচ্ছি, source এখন line. data source regress.

Linear Regression

↳ simpliest form of regression

data point গুলোর মধ্যে linear corelation exist

↳ এখানে linear ধরে নিলে error আসবে

x → independent variable

Y → dependent "

router এর distance এর সাথে signal এর (-)ve co-relation.

$$\boxed{Y = \beta X + \alpha}$$

β = slope
α = intersect parameter

α না থাকলে সব ধরনের st line consider করা যেতে না। শুধু মূলবিন্দুগামী line consider করতে হতো, তাই intersect parameter important.

13-06-2023

Multiple Regression :

$$\hat{y} = b_1 x_1 + b_2 x_2 + a$$ > independent variable x_1, x_2

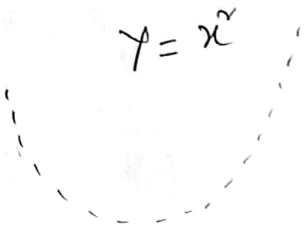
Shortcut Method:

$$\Sigma y = b_1 \Sigma x_1 + b_2 \Sigma x_2 + \Sigma a$$

$$\Sigma y = b_1 \Sigma x_1 + b_2 \Sigma x_2 + na \quad \text{— (I)}$$

$$\Sigma x_1 y = b_1 \Sigma x_1^2 + b_2 \Sigma x_1 x_2 + a \Sigma x_1 \quad \text{— (II)}$$

$$\Sigma x_2 y = b_1 \Sigma x_1 x_2 + b_2 \Sigma x_2^2 + a \Sigma x_2 \quad \text{— (III)}$$



$$\hat{y} = a + b_1 x + b_2 x^2$$ → polynomial regression model. Parabola র কাছাকাছি কোনো কিছু

$$\Sigma y = na + b_1 \Sigma x + b_2 \Sigma x^2$$

$$\Sigma xy = a \Sigma x + b_1 \Sigma x^2 + b_2 \Sigma x^3$$

$$\Sigma x^2 y = a \Sigma x^2 + b_1 \Sigma x^3 + b_2 \Sigma x^4$$

second degree polynomial → x^2 term থাকবে.

14.06.2023

Probability

SCHAUM'S Outline
Probability 2nd Edition
chap 1-7

Uncertinity tool হচ্ছে Probability:

Probability Theory:

Source of Probabilities:

Freequentists:
* experience থেকে

Objectivists:
* Real aspects of world. যেটা হবেই / সত্য ঘটনা (coin flip)

Subjectivists:
* আমার নিজের বিশ্বাস কোনো কিছু নিয়ে।
Probabilities characterize a person/Object belief

Sample Space / Outcome space:

S → outcome space → collection of all possible events

outcome → result of a random experiment
১টা dice মারলে 6টা possible outcomes.

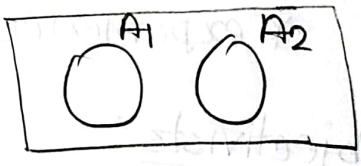
Events $\longrightarrow$ set of outcomes to which a probability is assigned.

$S = \{1, 2, 3, 4, 5, 6\}$

3 এর কম আসার probability set $P(A) = \{1, 2\}$

Mutually Exclusive Event:

যে event গুলো কখনো একসাথে হবে না।
(২টা result কখনো একসাথে আসবে না।
H/T এক থেকোনো একটা আসবে)



$A_i \cap A_j = \phi$

$A_1, A_2, \ldots, A_K$ are disjoint

Mutually Exhaustive Event:

$A_i \cup A_j \cup \ldots \cup A_K = S$

sample space

সবগুলো event এর result union করে সবগুলো possible event পাওয়া যায়।

The Axions of Probability :

1. $0 \leq P(A) \leq 1$
2. $P(\text{true}) = 1$ হলে $P(\text{false}) = 0$
3. disjoint event এর $P(A \cup B) = P(A) + P(B)$
4. infinite sequence of disjoint event

Empirical Probability :

Historical data এর ভিত্তি করে হয়।

$$\text{Probability} = \frac{\#\text{ of that event}}{\text{total numbers}}$$

Equiprobable Probability Space :

$$P(A) = \frac{|A|}{|S|}$$

কোনো random experiment চালালে, তার যেকোনো একটা result আসার probability equal হয়

$$P(6) = 1/6$$

Theoritical Probability :

$$P(A) = \frac{\text{\# of outcomes in } A}{\text{number of outcomes in } S}$$

$$= \frac{|A|}{|S|}$$

instant experiment করে তার probability historical data লাগে না।

Enough time experiment চালালে উহার probability.

Selecting a face card :

Jack, Queen, King from a standard deck of 52 cards

$$P = \frac{12}{52}$$

Simple Vs Joint Probability :

Simple (Marginal) Probability :

Simple event এর probability

King আসার $= \frac{4}{52}$

Joint probability:

2 or more events আসার probability.

$P(\text{King and spade}) = \frac{1}{52}$

OR হলে অন্যকিছু

Overlap থাকলে

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Additive Rule:

* Overlap হয়
* mutual exclusive হয় না

Complement Rule:

$P(A^c) = 1 - P(A)$

কোনো event না ঘটার probability

$P(A')$ ও লিখা হয়।

Q: 8টা coin flip করে at least ১টা head?

$P(\text{at least one head}) = 1 - P(\text{No head})$

$= 1 - \frac{1}{256}$ → 1 বার flip করলে $= 1/2$

8 টা independent probability $= (1/2)^8 = \frac{1}{256}$

08.07.2022

Technique of counting

↳ important for probability counting

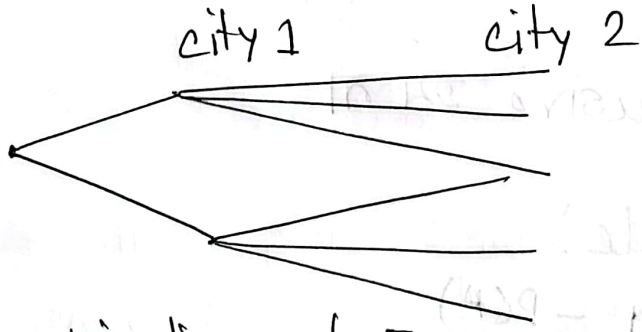
Sum Rule:

↳ event simultaneously occur না হলে $m+n$ way তে occur করতে পারে।

Two events E and F occurs in m and n way respectively

Product Rule:

↳ একটা event কয়েকটা event এর combination আকারে occur করতে পারে।



when both are independent events.

* combination of E, F can occur in mn way.
* E and F both independent. F doesn't depend on event E

Permutation:

* কত উপায়ে সাজাতে পারবো।

$P(n, r)$

↳ n সংখ্যক element থেকে r সংখ্যক element নেওয়া, Product Rule.

$$P(n,r) = \frac{n!}{(n-r)!}$$

$$n(n-1)(n-2)\ldots(n-r+1) = \frac{n!}{(n-r)!}$$

$$\boxed{P(n,n) = n!}$$

↳ সবগুলো object নিয়ে permutation করলে

Permutation with repeatation

$$P(5,3) = \frac{5!}{3!}$$

↳ যে কয়বার repeat হয়েছে

* Permutation এ order matter করে, combination এ করে না।

Orders of Samples:

abc
bac
cba

{a, b, c}

↳ combination এ order লাগে না

↳ permutation এ order লাগে।

Sampling with replacement:

$$\underbrace{n \cdot n \cdot n \cdot n \ldots n}_{r \text{ times}} = n^r$$

Sampling without replacement

10	9	8	7

$$\therefore n(n-1)(n-2)\ldots(n-r+1) = \frac{n!}{(n-r)!}$$

*Examples from book. = permutation

Combination :

$$P(n,r) = r!\, C(n,r)$$

Q1: In a lottery, players win a large prize when they pick four digits selected by a random mechanical process.

A smaller prize is won if only 3 digits are matched what is the probability that a player wins the large prize? What is the probability that a player wins small prize

Q2: Find the probability that hand of five cards in poker contains four cards of one kind.

Q3: What is the probability that a poker hand contains a full house, that is three of one kind and two of another kind?

Q4: What is the probability of there of a kind that is three of one kind two of another.

Q1 Ans:

$$\frac{1}{10^4} = 0.01\% \rightarrow \text{large price}$$

Let see for 1 case (abc): $\square_a \ \square_b \ \square_c \ \square_d$

a b c

$1 \times 1 \times 1 \times 9$

abc, bcd, acd, abd

equivalent to $C(4,3)$

For 4 case it will be $= 4 \times 1^3 \times 9$

$$\frac{4 \times 1^3 \times 9}{10^4}$$

from another explanation $= \dfrac{C(4,3) \times 1^3 \times 9}{10^4}$

$= 0.0036$

$= 0.36\%$

~~Q4 Ans:~~

$$\frac{C(4,2) \times 1^2 \times 9^2}{10^4}$$

$= 0.0486$

$= 4.86\%$

$1^2 \times 9^2$

1^2 → 2টা মিলবে

9^2 → 2টা মিলবেনা

∴ 1টিও match করেনি $\frac{04}{104}$

∴ at least 1টি match করেছে $= 1 - \frac{04}{104}$

$$\text{Odd} = \frac{P(A)}{1 - P(A)}$$

Odd → 1টি event occur হবার possibility তার
" না হবার " ratio

suit → ♣ ♢ ♡ ♠ → 4টি

Kind → A, J, Q, K, 1, 2, . . . 10 → 13টি

∴ $4 \times 13 = 52$ in a 1 set

Q3 Ans:

(13টি card থেকে particular 1টি kind select) × (4টি same card হওয়ার)
× (বাকি 48টি card থেকে যতো উপায় select)

52টি card থেকে বাকি 5টি card select

$$= \frac{C(13,1) \times C(4,4) \times C(48,1)}{C(52,5)}$$ Q-2

$= 0.02401\%$

Probability of full house

Q. 3 $$\frac{C(13,1) \times C(4,3) \times C(12,1) \times C(4,2)}{C(52,5)}$$

$= 0.1444\%$

[Rosent এর বই থেকে]

$$\frac{P(13,2) \quad C(4,3), \quad C(4,2)}{C(52,5)}$$ [example]

Q4 Ans:

Three of a kind

$$= \frac{C(13,1) \times C(4,3) \times C(12,2) \; C(4,1) \; C(4,1)}{C(52,5)}$$

09.07.2023

Q5. What is the probability that hand of five contains at least one ace.

$1 - P(\text{no Ace})$

$= 1 - \dfrac{C(48,5)}{C(52,5)}$

$= 0.34$

Q6. Suppose that we are given 20 tulip bulbs that are similar in apperance and told than 8 will bloom early 12 will bloom late 13 will be red 7 will be yellow, in accordance with various combination listed in -

	Early (E)	Late (L)	Total
Red (R)	5	8	13
yellow (Y)	3	4	7
Total	8	12	20

(1) If one bulb is selected at random find the probability that it'll be a red tulip

$P(R) = \dfrac{n(R)}{n(S)}$

$= \dfrac{13}{20}$

~~(ii) Given that the probability that hand~~

(ii) Given that a randomly selected bulb blooms early find the probability that it'll be Red

$$P(R|E) = \frac{n(R \cap E)}{n(E)} = \frac{5}{8} \text{ (Ans)}$$

R → Event of selecting Red tulip

E → " " " Early

Tree Diagram → Visualizing the branching but only useful for limited case

Chap-2

Example-2.13 → Mark & Eric

Chapter-4 (Important for ML & DL)

Conditional Probability & Independence

$P(Y|x)$ তে estimate conditional probability

$P(Y = +|x|$

$$P(A|E) = \frac{P(A \cap E)}{P(E)}$$

↳ event

এখানে possibility depends on condition

* Conditional Probability to sample set size কে reduce করছি।

↳ Difference between probability & conditional probability

$$P(A \cap E) = \frac{n(A \cap E)}{n(S)}$$

$$P(E) = \frac{n(E)}{n(S)}$$

$$\therefore P(A|E) = \frac{P(A \cap E)}{P(E)}$$

$$= \frac{n(A \cap E)}{n(E)}$$

Q8. A lot contains 12 items which 4 are defective 3 items are drawn at random from the lot one after the other. Find the probability p that all 3 are defective

Q9. Given a family with two children, find the probability that both are boys given at least one is by [Example 4.2]

Q10. A pap smear is screeing procedure used to detect cervical cancer. For women this cancer, there are about 16% False negative For women without cancer, There are about at 10% False positive. If pap semear test is positive what is the probability that someone actually has cervical cancer. Assuming 8 women in 100,000 have this cervical cancer.

Multiplication Theorem

* দুইটা event এর intersection calculation এ use করা হয়।

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \quad [A \cap B = B \cap A]$$

$$P(B|A) = \frac{P(B \cap A)}{P(A)}$$

$$P(A \cap B) = P(B)\, P(A|B)$$

$$\boxed{P(A \cap B \cap C) = P(A) \cdot P(B|A)\, P(C|A \cap B)}$$

একই সাথে তিনটা event হওয়ার possibility

Multiplication Theorem এটাই

Q8:

Total items $\longrightarrow 12$

Defective $\longrightarrow 4$

Non-Detective $\longrightarrow 8$

A $\longrightarrow$ 1st item non-detective হওয়ার event

B $\longrightarrow$ 2nd " " " " "

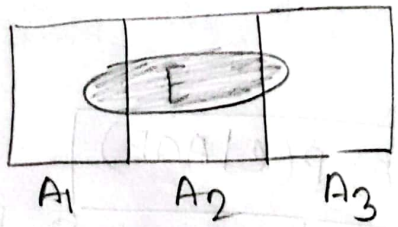
C $\longrightarrow$ 3rd " " " " "

$\longrightarrow$ intersection will be the probability of being non-detective

$$P(A \cap B \cap C) = \frac{8}{12} \times \frac{7}{11} \times \frac{6}{10}$$

Partitions, Total Probability and Biased Formula:

$A_1 \cup A_2 \cup A_3 = S$



$$E = E \cap S$$
$$= E \cap (A_1 \cup A_2 \cup \dots A_n)$$
$$= (E \cap A_1) \cup (E \cap A_2) \cup \dots \dots (E \cap A_n)$$

Law of Total Probability:

$$P(E) = P(E \cap A_1) + P(E \cap A_2) + \cdots + P(E \cap A_n)$$

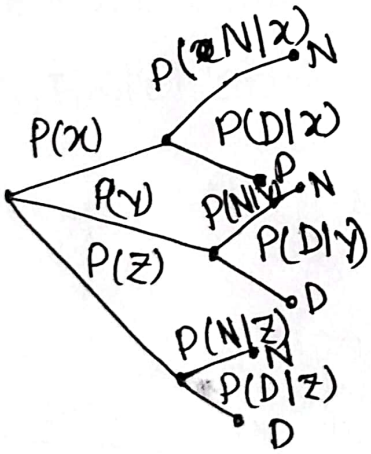
$$P(E \cap A_K) = P(A_K \cap E)$$

$$= P(A_K)\, P(E|A_K)$$

$$\therefore P(E) = P(A_1) \times P(E|A_1) + P(A_2) \times P(E|A_2) + \cdots + P(A_n)\, P(E|A_n)$$

$$= \sum_{K=1}^{n} P(A_K)\, P(E|A_K)$$

Example – 4.6: Machine – defectives



same as before

Baye's Theorem :

$$P(A_k | E) = \frac{P(A_k \cap E)}{P(E)} = \frac{P(A_k)\, P(E | A_k)}{P(E)}$$

$$= \frac{P(A_k)\, P(E | A_k)}{\sum_{i=1}^{n} P(A_i)\, P(E | A_i)}$$

$P(A_k) \longrightarrow$ Prior

$P(E | A_k) \longrightarrow$ Likelihood

$P(A_k | E) \longrightarrow$ Posterior

Examples all

$P(A^c) = 1 - P(A)$

$P(\bar{A} | B) = 1 - P(A | B)$

$P(\bar{A} | B) + P(A | B) = 1$

Ans

$S = \{bg, bb, gg, g^{(b)}\}$

$A \longrightarrow$ at least one boy

$B \longrightarrow$ both child are boy

$$P(A) = \frac{3}{4}$$

$$\therefore P(A \cap B) = \frac{1}{4}$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{1/4}{3/4} = \frac{1}{3}$$

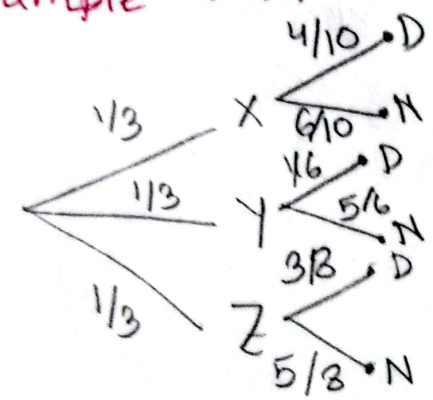
Finite stochastic Process and Tree diagram:

A (Finite) stochastic

Example 4.4: Box — ligth — defective

D → defective

N → Non-defective



$$P(N \cap x) = P(x)\, P(N|x)$$

$$= \frac{1}{3} \cdot 6/10$$

$$= \frac{2}{10} = 1/5$$

$\therefore P(N \cap Y) = P(Y) \cdot P(N|Y)$

$= \frac{1}{3} \cdot \frac{5}{6}$

$\therefore P(N \cap Z) = P(Z) \cdot P(N|Z)$

$= \frac{1}{3} \cdot \frac{5}{8}$

$\therefore P(N) = \left(\frac{1}{3} \times \frac{6}{10}\right) + \left(\frac{1}{3} \times \frac{5}{6}\right) + \left(\frac{1}{3} \times \frac{5}{8}\right)$

$= 0.686$ (Ans)

(ii) $P(Z|N) = \frac{P(Z \cap N)}{P(N)}$

$= \frac{\frac{1}{3} \cdot \frac{5}{8}}{0.686}$

$= 0.304$

10 Ans:

		Predicted +	Predicted −
Actual	+	TP	FN
	−	FP	TN

TP → True positive
FN → False Negative
FP → " Positive
TN → True Negative

Given that,

16% False negative
→ 84% True positive
10% False positive
→ 90% True negative

Recall / True positive rate (TRP) / sensitivity

$$= \frac{TP}{TP + FN}$$

(যাদের cancer আছে, তা detect করার power/rate)

Specifity / Negative Rates:

$$= \frac{TN}{FP + TN}$$

Prevalence (প্রাদুর্ভাব) $= \frac{8}{1000,000}$

$P(c^+) = 0.00008$

$P(c^-) = 1 - P(c^+) = 0.99992$

$P(T^+ | c^+) = 0.84$

$T^+ \longrightarrow$ Test positive

$c^+ \longrightarrow$ Cancer "

$\therefore P(T^- | c^+) = 1 - 0.84 = 0.16$

$P(T^- | c^-) = 0.90$

$P(T^+ | c^-) = 0.10$

Sample participate ~~করে~~ হয়েছে 2 ভাবে যাদের cancer আছে, যাদের cancer নাই।

$$\therefore P(c^+ | T^+) = \frac{P(T^+ | c^+) \times P(c^+)}{P(c^+)\, P(T^+ | c^+) + P(c^-)\, P(T^+ | c^-)}$$

$$= \frac{0.84 \times 0.00008}{0.00008 \times 0.84 + 0.99992 \times 0.10}$$

$$= 0.000672$$

$$= \frac{672}{10^6} \text{ (Ans)}$$

16-07-2023

$P(A \cap B | C) = P(A|C) \cdot P(B|C)$

↳ Conditional Independence

$P(A | B \cap C) = P(A|C)$

Naive Bayesian Classifier

$\vec{x}$ → Feature Vector → Multiple Value র collection

↳ conditionally independent otherwise independent না

মানে class দেওয়া না থাকলে independent হবেনা

* প্রত্যেকটা Feature সমান important

$$P(C_K | \vec{x}) = \frac{P(\vec{x}|C_K) \cdot P(C_K)}{P(\vec{x})}$$

↳ probability of a vector

$$P(\vec{x}|C_K) = P(x_1, x_2, \ldots, x_n | C_K) \cdot P(C_K)$$

$$= \frac{P(x_1, x_2, x_3, \ldots, x_n, C_K)}{P(C_K)} P(C_K)$$

$$= P(x_1, x_2, x_3, \ldots, x_n, C_K)$$

↳ Feature and class property র intersection

$$= P(x_1 | x_2, x_3, \ldots, x_n, C_K)$$ → chain Rule

$P(x_2, x_3, \ldots, x_n, C_K)$ → Given

$= P(x_1 | x_2, x_3, \ldots, x_n, C_k) \cdot P(x_2 | x_3, \ldots, x_n, C_k)$

$\ldots P(x_n | C_k) \cdot P(C_k) \rightarrow$ chain Rule

$= P(x_1 | C_k) \; P(x_2 | C_k) \ldots P(x_n | C_k) \cdot P(C_k)$

$$P(C_k | \vec{x}) = \frac{P(C_k) \prod_{i=1}^{n} P(x_i | C_k)}{P(\vec{x})}$$

$$P(C_k | \vec{x}) \propto P(C_k) \prod_{i=1}^{n} P(x_i | C_k)$$

Slide Math:

$P(C = \text{buys_computer} | x)$

$P(C = \text{don't buys} | x)$

$P(\text{buys_computer} = \text{yes} | x) = P(\text{Yes}) = \frac{9}{14}$

$P(\text{buys_computer} = \text{NO} | x) = P(\text{NO}) = \frac{5}{14}$

Yes এর case

$P(\text{age} <= 30 | \text{yes}) \cdot P(\text{income} = \text{medium} | \text{yes})$
$\cdot P(\text{student} = \text{yes} | \text{yes}) \; P(C.R = \text{Fair} | \text{yes})$
$P(\text{Yes})$

$\rightarrow P(C_k) \prod_{i=1}^{n} P(x_i | C_k)$ এই formula use করব।

$$= \frac{2}{9} \times \frac{4}{9} \times \frac{6}{9} \times \frac{6}{9} \times \frac{9}{14} = 0.028$$

$P(\text{buys_computer} = \text{yes} \mid x) \propto 0.028$

↓

buy করার probability

No এর case:

$P(\text{age} <= 30 \mid \text{No}) \cdot P(\text{income} = \text{medium} \mid \text{No})$ ~~P(Student = yes)~~

$P(\text{student} = \text{yes} \mid \text{No})\ P(\text{CR} = \text{Fair} \mid \text{No})$

$P(\text{No})$

$$= \frac{3}{5} \times \frac{2}{5} \times \frac{1}{5} \times \frac{2}{5} \times \frac{5}{14} = 0.007$$

$P(\text{buys_computer} = \text{No} \mid x) \propto 0.007$

* Feature Vector থেকে দেখছি computer কিনার সম্ভাবনা computer না কিনার চেয়ে বেশি

$$\boxed{P(C_k \mid x) = \frac{P(C_k) \prod_{i=1}^{n} P(x_i \mid C_k)}{P(\vec{x})}}$$

→ Law of total probability

$$P(\vec{x}) = \sum_k P(C_k)\, P(\vec{x} \mid C_k)$$

(law of total probability calculate করতে হবে Yes এর জন্য একবার No এর জন্য একবার)

$P(yes)\ P(\vec{x} \mid yes) + P(no)\ P(\vec{x} \mid no)$

$= \frac{9}{14}\ P(age \leq 30 \mid yes)\ P(income = medium \mid yes) \ldots$

$= 0.028 + 0.067$

$P(buys_c = Yes \mid x) = \frac{0.028}{0.028 + 0.007} = 0.8$

$P(buy_computer = No \mid x) = \frac{0.007}{0.028 + 0.007}$

$= 0.2$

* এই দুইটাতে probability বের করা হয়েছে। আর এর আগে পর্যন্ত 2টা category (হ্যাঁ / না)। যেকোনো 1টা ans হবে। হয় কিনবে / কিনবে না

Chap-4, Page = 92

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$\boxed{\begin{aligned} P(A \cap B) &= P(A|B) \cdot P(B) \\ &= P(A) \cdot P(B) \end{aligned}}$$

If A and B are independent then $P(A \cap B) = P(A) \cdot P(B)$

$P(A|B) = P(A)$

$P(B|A) = P(B)$

$$\boxed{P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C)}$$

[If A, B, C ও অনেকেই mutually independent হতে হবে।

AB
BC
CD
DA
- - -

→ এরা

Ex- 4.9:

$P(A \cap B) = P(A)\,P(B)$ → হলে independent

Ex- 4.10:

4.6: Independent Repeated ~~Trails~~. Trials

Q 11: Suppose only have a, b, c, d, race together yielding the sample space {a, b, c, d} and suppose the probability of winning are follows

$P(a) = 0.2$ $\quad$ $P(c) = 0.1$

$P(b) = 0.5$ $\quad$ $P(d) = 0.2$

(a) Describe and Find the numbers of ~~elements~~ elements in the product probability space S3.

(b) Find the probability that the same ~~house~~ horse wins all the ~~the~~ three race.

(c) Find the probability that ~~the~~ a, b, c, each win one race

(a) $S3 = S \times S \times S$

$= \{a, b, c, d\} \times \{a, b, c, d\} \times \{a, b, c, d\} \times \ldots$

এখন $4^3 = 64$ টা possible outcome সম্ভব।

$|S| = 64$

(b) aaa, bbb, ccc, ddd

[$\frac{4}{64} = X$ হবে না। কারন এরা finite equivalent space না।]

$\therefore P(aaa) = P(a) \cdot P(a) \cdot P(a)$

$= (0.2)^3$

(finite equi probable space হলে সবগুলো ঘটনা ঘটার সম্ভাবনা সমান)

$P(bbb) = (0.5)^3$

$P(ccc) = (0.1)^3$

$P(ddd) = (0.2)^3$

$\therefore P$(same ঘোড়া 3 টা race জিতবে) $= \emptyset$

$\Rightarrow P(aaa) + P(bbb) + P(ccc) + P(ddd) = 0.142$

(Ans)

(c)

abc
bac
cab
acb
cba
bca

a, b, c 1 বার করে জিতবে

$6 \times P(abc)$

$= 6\ P(a)\ P(b)\ P(c)$

$= 6\ (0.2)\ (0.5)(0.1)$

$= 0.06$ (Ans)

Chapter - 5

Random Variable

↳ basically a function

Random variable X is defined as a function from sample space in to real number.

$X : S \longrightarrow R$ (S কে R এর সাথে map করা)

Outcome কে real number এর সাথে map করা।

Range Space, $R_X \longrightarrow$ discrete random variable

$\longrightarrow$ continuous " "

$\{1, 2, 3, 4, 5\} \longrightarrow$ countable range space

$\{1, 2, 3, 4, 5, \ldots \ldots \infty\} \longrightarrow$ countably infinite

↓

count করা যাবে but infinite time লাগবে

continuous:

(0, 1)

↳ 0 থেকে 1 এর মধ্যে যেকোনো value.

Count করে শেষ করা যাবেনা কখনও

Example 5.1:

A pair of fair dice is tossed. The sample space consists of 36 ordered pairs (a, b) where a and b be any integers between 1 and 6

$6 \times 6 = 36$

$\longrightarrow$ a and b দুইটা dice

$S = \{(1,1), (1,2), \ldots (6,6)\}$

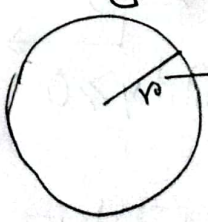
X এমন একটা random variable যা max value টা দিবে from sample variable

$X(a,b) = \max(a,b)$

$Y(a,b) = a+b \longrightarrow$ outcome এ real number যোগ করবে। Y এমন Random Variable

Example 5.2(c)

A point is chosen in a circle C of radius r. Let X denote the distance of the point from the center. Then X is a random variable whose value can be any number between 0 and r, inclusive. Thus Range Space



$\longrightarrow$ continuous value হবে random variable

1. X ও Y same sample space এর উপর দুইটা random variable হয়।

$$(X+Y)(s) = X(s) + Y(s)$$

(বাকিগুলো from book)

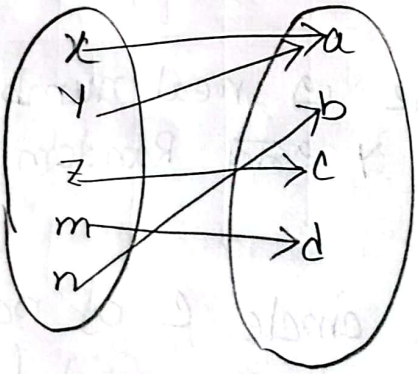
Important Notation:

$P(X=a)$ $P(a \le X \le b)$

1. $\boxed{P(\{s \in S : \underline{X(s) = a}\})}$ → probability set building notation

→ random variable এর outcome a হওয়ার probability কতো,

$P(\{x, y\})$



just like set.

2. $\boxed{P(\{s \in S : a \le x(s) \le b\})}$

A Finite Random Variable:

→ subset of discrete random variable

$f(x_k) = P(X = x_k)$

$= P(\{s \in S : x(s) = x_k\})$

→ range space এর সবগুলো element এর probability calculate করে এমন function.

(i) $f(x_k) \geq 0$ → probability ≥ 0

(ii) $\sum_k f(x_k) = 1$

→ সব probability র যোগফল $= 1$

Q: Toss a fair coin twice. X is a random variable denoting number of observed head, find R_X and probability distribution.

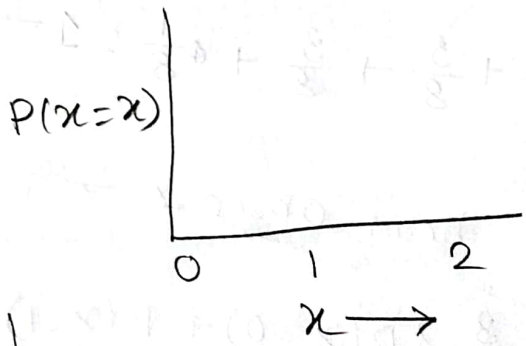
$S = \{HH, HT, TH, TT\}$

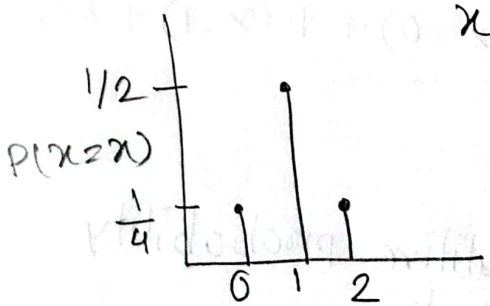
$R_X = \{0, 1, 2\}$ → head এর সংখ্যা

x	0	1	2
$f(x)$ / $P(x)$	$P(X=0) = \frac{1}{4}$	$P(X=1) = \frac{2}{4} = \frac{1}{2}$	$P(X=2) = \frac{1}{4}$

} condition (i) and (ii) satisfy করে।

Probability distribution function কে অনেক সময় এভাবে দেখানো হয়।





$P(X=x) = P_X(x)$

* probability distribution random variable গুলোর output এর probability বের করে function এর জন্য

19.07.2023

* probability distribution সবগুলো সম্ভাবনাকে তুলে আনে এবং যোগফল সবগুলোর সম্ভাবনা 1.

Random Variable X

Probability distribution $P_X(x)\ \{\ldots\}$

দুইভাবে লেখা যাবে $P(X=x)\ \{\ldots\}$

Q: A Fair coin is tossed 3 times. X denotes the number of head observed.

$R_X = \{0, 1, 2, 3\}$

x	0	1	2	3
$f(x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$1/8$

সবগুলো Tail আসে

$\frac{1}{8} + \frac{3}{8} + \frac{3}{8} + \frac{1}{8} = 1 \rightarrow$ সবগুলোর যোগফল 1.

$P(X=2) = \frac{3}{8}$ [2 বার head observe করা গেছে]

$P(x \le 2) = \frac{1}{8} + \frac{3}{8} + \frac{3}{8}\ \{P(x=0) + P(x=1) + P(x=2)\}$

$= \frac{7}{8}$

$\boxed{F(x) = P(x \le x)} \rightarrow$ commulitive probability distribution (Value x এর ছোট বা সমান)

$P(1 \le x \le 3) = \frac{3}{8} + \frac{3}{8} + \frac{1}{8} = \frac{7}{8}$

→ এটা cummilitive না

$P(x > 2) = 1 - P(x \le 2)$

$= 1 - \frac{7}{8}$

Expected Value:

*basically Average calculate করে।

x	x_1	x_2	x_3	x_n
$f(x)$	$f(x_1)$	$f(x_2)$	...	$f(x_n)$

$E[x] = x_1 f(x_1) + x_2 f(x_2) + \dots + x_n f(x_n)$

$$\boxed{\mu = \sum_{i=1}^{n} x_i f(x_i)}$$

→ Expected value of finite Random Variable. Weighted Average value/ Arithmetic mean

↳ সবগুলো property সমান হলে

$E[x] = 0 \times \frac{1}{8} + 1 \times \frac{3}{8} + 2 \times 3/8 + 3 \times 1/8$ μ

$= 1.5$ → এতো সংখ্যকবার head আসবে on average.

Variance of Random Variable X:

$$Var(x) = E[(x-\mu)^2]$$

$$= E[x^2 + \mu^2 - 2x\mu]$$

$$= E[x^2] + E[\mu^2] - E[2x\mu]$$

$$= E[x^2] + \mu^2 - 2\mu E[x]$$ → constant common নিয়ে

↳ constant এর expected value হয়ে

$$= E[x^2] + \mu^2 - 2\mu^2$$

$$\therefore \boxed{Var(x) = E[x^2] - (E[x])^2}$$

$$E[x^2] = \Sigma x^2 f(x)$$

$$E[x^2] = 0^2 \cdot \frac{1}{8} + 1^2 \cdot \frac{3}{8} + 2^2 \cdot \frac{3}{8} + 3^2 \frac{1}{8}$$

$$= 3$$

$$Var[x] = 3 - (1.5)^2$$

$$= 0.75$$

$$\sigma = \sqrt{Var[x]} = \sqrt{0.75} = 0.866$$

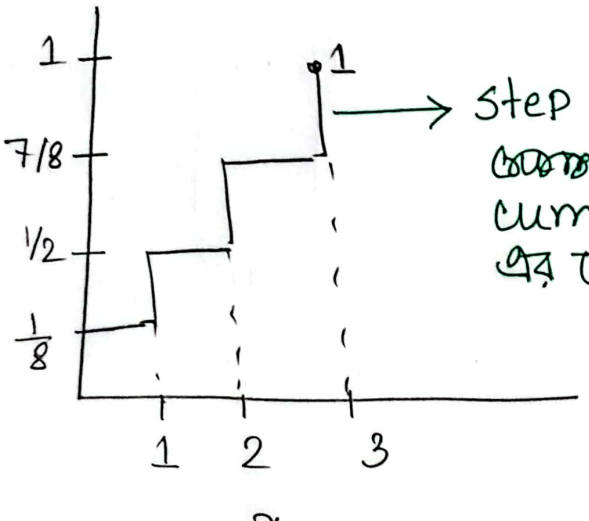
↳ Random variable এর standard deviation.

$F(x) = P(x \leq x)$ [commulitive distribution]

$P(x \leq 2)$

x	0	1	2	3
$F(x)$	$\frac{1}{8}$	$\frac{1}{2}$	$\frac{7}{8}$	1

→ Last case যেখানে সবগুলো যোগ করছি।



→ step এ step এ হবে ~~commulative~~ cummulitive distribution এর ক্ষেত্রে। শেষে 1 হয়ে যায়।

Standardized Random Variable:

* কিছু value কে z-score normalize করে যোগ করলে 0 আসবে মানে 0 centered হবে।

Sat Sun Mon Tue Wed Thu Fri

Date :
Time :

(i) $f(x) \geq 0$

→ probability output দিবেনা, density output দিবে।

(ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

এর ক্ষেত্রে,

$P(X=a) = 0$ → কোনো point এ specific point এর probability 0.

$P(a \leq x \leq b)$

$= \int_a^b f(x)\,dx$

$P(X=a) = P(a \leq x \leq b) = \int_a^a f(x)dx = [g(x)]_a^a$

$= g(x) - g(a)$

$= 0$

* particular point হওয়ার probability 0. Area র probability বের করা যাবে কিন্তু point এর না

$$\left.\begin{aligned} &P(a \leq x \leq b) \\ &= P(a < x < b) \\ &= P(a \leq x < b) \\ &= P(a < x \leq b) \end{aligned}\right\} \text{সব same}$$

Sat Sun Mon Tue Wed Thu Fri

Date :
Time :

$$E[X] = \int_{-\infty}^{\infty} x f(x)\, dx = \mu$$

$$Var(x) = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\, dx$$

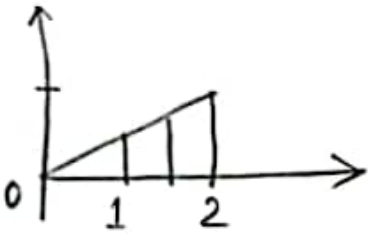
* continuous Random variable piecewise smooth. piece এ ভাগ করা থাকে প্রত্যেক piece এর জন্য একেক value দেওয়া থাকে।

Example 5.19:

$$f(x) = \begin{cases} \frac{1}{2}x & \text{if } 0 \le x \le 2 \\ 0 & \text{elsewhere} \end{cases}$$

$\left.\begin{array}{l} -\infty \text{ to } 0 \\ 2 \text{ to } +\infty \end{array}\right\}$ area under the curve 0.

$$\int_0^2 f(x)\, dx = 1$$



$P(1 \le x \le 1.5)$

$$= \int_1^{1.5} \frac{1}{2} x\, dx$$

$$= \frac{1}{2} \cdot \frac{1}{2} [x^2]_1^{1.5}$$

$$= \frac{1}{4}(1.5^2 - 1^2)$$

$$= \frac{5}{16}$$

Sat Sun Mon Tue Wed Thu Fri

Date :
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$$\sum_{k=1}^{\infty}(1-p)^{k-1}p$$

$$= p\sum_{k=1}^{\infty}(1-p)^{k-1}$$

$$= \frac{p}{1-(1-p)}$$

$$= \frac{p}{p}$$

$= 1$ [second property prove করে]

Formula:

$$\sum ar^k = \frac{a}{1-r} \text{ if } |r|<1$$

* probability always 0 - 1 এর মধ্যে হয়

If $p = 0.5$ then find $P(2 \le y \le 5)$

$$\therefore \sum_{k=2}^{5}(1-0.5)^{k-1}(0.5) =$$

Continuous Random Variable:

* height, weight, distance etc. range space continuous হবে। এগুলো perfectly measure করা যাবেনা

* এক probability ~~distribution~~ density function বলা হয়

continuous এর ক্ষেত্রে → PDF বলা হয়।

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* maximum likelyhood distribution → basically probability distribution
* density বেশি থাকলে probability ও বেশি হবে।
* Random variable ই mainly probability distribution mean করে।
* Real world এ continuous probability use বেশি করা হয়।
* Normal distribution use করা হয় calculation এ.

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Important properties for probability distribution

(i) $f(x_i) \geq 0$

(ii) $\sum_{i=1}^{\infty} f(x_i) = 1$

Discrete Random Variable:

Q12 # unfair coin এর head পাওয়ার probability

$$P(H) = P$$

coin is tossed repeatedly.

$$P(Y=1) = P(H) = P$$

$$P(Y=2) = P(TH) = (1-P) \cdot P$$

$$P(Y=3) = P(TTH) = (1-P)^2 P$$

$$P(Y=K) = (1-P)^{K-1} \cdot P$$

$$\boxed{P_Y(K) = \begin{cases} (1-P)^{K-1} P & \text{for } Y = 1, 2, 3, \ldots, \infty \\ 0, & \text{otherwise.} \end{cases}}$$

* Discrete case এ probability distribution কে probability mass function ও বলা হয়, PMF বলা হয়।

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Q 13.

(a) $\int_0^2 c(4x-2x^2)dx = 1$

$\Rightarrow c \left[\frac{4x^2}{2} - \frac{2}{3}x^3\right]_0^2 = 1$

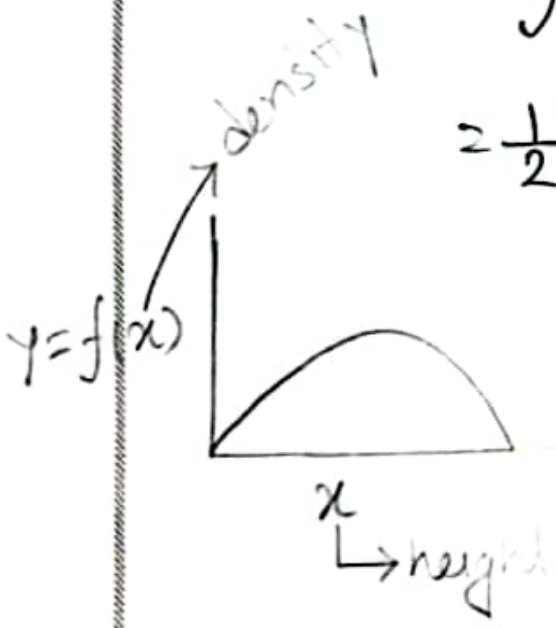
$\Rightarrow c = \frac{3}{8}$

(b) Find $x > 1$

$P(x>1) = \int_1^\infty f(x)\,dx$

$= \int_1^2 \frac{3}{8}(4x-2x^2)\,dx$ → 2 এর পর থেকে ∞ পর্যন্ত area 0

$= \frac{1}{2}$



*probability density র value কখনও negative হয়না.

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Q:14

$P(2 \le Y \le 4)$

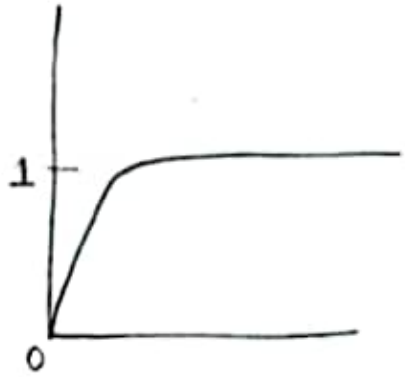
$= \int_2^4 f(x)\, dx$

$= \int_2^3 \frac{1}{6} x\, dx + \int_3^4 (2 - \frac{1}{2}x)\, dx = \frac{2}{3}$

Q15:

$\boxed{F(x) = \int_{-\infty}^{x} f(x)\, dx}$ → cummilitive probability Distribution Function $F(x)$

$\frac{d}{dx} F(x) = f(x)$ → probability distribution পাওয়া যাবে CPDF কে differentiate করলে।



Probability distribution এর property 0 থেকে শুরু করে 1 এ গিয়ে saturated হয়ে যাবে।

$F(x)$

$= \int \frac{1}{8} x^2 = \frac{1}{8} \cdot \frac{1}{3} x^3 + c$

$= \frac{1}{24} x^3 + c, \; [0 \le x \le 2]$

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$$F(0) = \int_{-\infty}^{0} f(x)\,dx = 0$$

$$\Rightarrow \frac{1}{24} \cdot 0^3 + c = 0$$

$$\therefore c = 0$$

$$F(x) = \int \left(\frac{x}{2} - \frac{x^2}{8}\right) dx$$

আগেরটা থেকে

$$= \frac{1}{4}x^2 - \frac{x^3}{24} + D$$

$$F(2) = \frac{1}{24} \cdot 2^3 = \frac{8}{24} = \frac{1}{3}$$

$$F(2) = \frac{1}{3} \Rightarrow \frac{2^2}{4} - \frac{2^3}{24} + D = 1/3$$

$$\therefore D = -\frac{1}{3}$$

$$F(x) = \begin{cases} 0, & x < 0 \\ \frac{1}{24}x^3, & 0 \le x \le 2 \\ \frac{x^2}{4} - \frac{x^3}{24} - \frac{1}{3}, & 2 \le x \le 4 \\ 1, & x > 4 \end{cases}$$

saturation হয়ে যাবে এরপর 1

Bernouli Distribution:

* Random Variable এর ২টা possible outcome আছে Success & Failure.
* Binary distribution as দুইটা case এর মধ্যে একটা হবে।

$$f(k; p) = \begin{cases} p & \text{if } k = 1 / \text{success} \\ q = 1 - p & \text{if } k = 0 / \text{failure} \end{cases}$$

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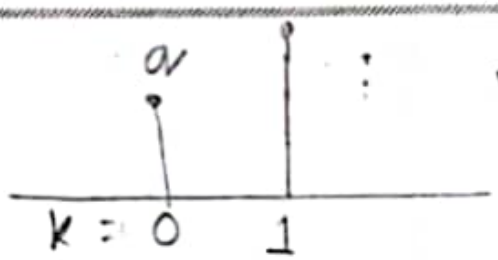

Mass Distribution of Bernouli Distribution

এখানে success এর probability বেশি আবার Failure এর probability ও বেশি আসতে পারতো।

Multinomial Distribution → Not in syllabus that way.

$$E[x] = \sum k f(x) = 1 \cdot p + 0 \cdot q = p$$

$$\sigma^2 = E[x^2] - (E[x])^2$$

variance

$$= p - p^2$$

$$= p(1-p) = p \cdot 0$$

$$= pq$$ → success ও Failure এর probability গুণ করলে তা variance

$$E[x^2] = \sum k^2 f(x)$$
$$= 1^2 p + 0^2 \cdot q$$
$$= p$$

* Bernouli কে n সংখ্যক বার trail (repeat) করলে তা Binomial Distribution

* Binomial Distribution → discrete Random variable
* Binomial distribution success পাওয়ার probability যেহেতু n সংখ্যক trial এ
* shortcut method

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Example:

The probability that a certain kind of component will survive a shock is 3/4. Find the probability that exactly 2 of the next 4 components tested survive.

$p = 3/4$

$n = 4$

$x = 2$

$$P(2; 4, 3/4) = \binom{4}{2}\left(\frac{3}{4}\right)^2\left(1 - 3/4\right)^{4-2}$$

$$= \frac{27}{128}$$

* $(p+q)^n$ কে expansion করলে binomial distribution এর n টা case পাওয়া যাবে → From book.

Example:

The probability that a patient recovers from a rare blood diease is 0.4. If

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(a)

$$P(X \geq 10) = 1 - P(X < 10)$$

$$= 1 - \sum_{x=0}^{9} B(x; 15, 0.4)$$

$$= 1 - \sum_{x=0}^{9} \binom{15}{x} (0.4)^x (0.6)^{15-x}$$

$$P(x \geq 10) = \sum_{x=10}^{15} B(x; 15, 0.4) - 0.0338$$

(b)

$$\sum_{x=3}^{8} B(x, 15, 0.4) = 0.8779$$

আবার, $\sum_{x=0}^{8} - \sum_{x=0}^{2}$ থেকেও করা যায় যেটা বইয়ে আছে

(c)

$$P(X=5) = B(5; 15, 0.4) \longrightarrow$$ বইয়ের অনুভাবে

$$= \binom{15}{5} (0.4)^5 (0.6)^{15-5}$$

$$= 0.1859$$

*multiple case আসতে পারে like mild success, mild Failure যেটা multinomial distribution এ পরে,

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(a)
at least one defective item

$P = 3\% = 0.03$

$n = 20$

$P(X \geq 1) = 1 - P(X = 0)$

$= 1 - B(0; 20, 0.03)$

$= 0.4562$

∴ 45% chance আছে একটা item defective হবে।

(b)

$\binom{10}{3} (0.4562)^3 (1 - 0.462)^{10-3}$

$n = 10$
$p = 0.4562$
$x = 3$

$= 0.160$

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Mean & Variance of Binomial Distribution:

* n সংখ্যক bernouli trial এ binomial distribution.
* Indepent trails হলে

$$X = I_1 + I_2 + \cdots + I_n$$

$$E[X] = E[I_1] + E[I_2] + \cdots + E[I_n]$$

$E[X+Y] = E[X] + E[Y]$ → sample space same হলে

$= np \longrightarrow$ n সংখ্যকবার p কে যোগ করছি

$$Var(aX + bY) = a^2 Var(X) + b^2 Var(Y)$$

$$Var[X] = Var[I_1] + Var[I_2] + \cdots + Var[I_n]$$

$$= npq$$

Poisson Distribution:

↓

probability mass function.

Trail অসংখ্য কিন্তু success কম।

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$$= \frac{n(n-1)(n-2)\cdots(n-x+1)(n-x)!}{(n-x)!\;x!}\;\frac{\lambda^x}{n^x}\left(1-\frac{\lambda}{n}\right)^n\left(1-\frac{\lambda}{n}\right)^{-x}$$

$$= \frac{\lambda^x}{x!}\cdot\underbrace{\left[\frac{n}{n}\cdot\frac{n-1}{n}\;\frac{n-2}{n}\cdots\frac{n-x+1}{n}\right]}_{\lim \text{ নিলে } 1 \text{ আসবে}}\underbrace{\left(1-\frac{\lambda}{n}\right)^n}_{\lim \text{ নিলে } e^{-\lambda}}\underbrace{\left(1-\frac{\lambda}{n}\right)^{-x}}_{\lim \text{ নিলে } 1}$$

$\frac{\lambda}{n} = 0$,

$(1-0)^{-x}$

$= 1^{-x} = 1$

$$= e^{-\lambda}\frac{\lambda^x}{x!}$$

* λ থেকে যতো দূরে যাবে probability কমতে থাকবে।

Ex: $\lambda = 4$, $\lambda t = 4$

$t = 1$, $x = 6$

$$P(6;4) = \frac{e^{-4}\,4^6}{6!} = 0.1042$$

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*N সংখ্যকবার trail করার পর x সংখ্যক বার success পেতে চাইলে

Binomial Distribution,

$$B(x; n, P) = \binom{n}{x} \cdot p^x q^{n-x}$$

$$= \binom{n}{x} p^x (1-P)^{n-x} \; ; \; x = 0, 1, 2, \ldots,$$

Q: 10 বার coin flip করলে 6 বার head আসার বা 6 বার success

$P(\text{head}) = P = 0.5$

$q(\text{tail}) = 1 - P = q = 0.5$

$$B(6; 10, 0.5) = \binom{10}{6} (0.5)^6 (0.5)^4$$

$$= 0.205$$

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Poission Distribution :

$$X \sim B(x; n, p)$$

$$n \to \infty \qquad \boxed{\mu = np}$$
$$p \to 0$$

* যখন trail এর সংখ্যা অসংখ্য হবে কিন্তু probability low হবে তা poission distribution দিয়ে করা যাবে,

Properties of poission distribution: → From book

একটা distribution poission distribution হলে

$X \sim P(\lambda)$

$P(x; \lambda) \qquad \lambda > 0$

$$= \frac{e^{-\lambda} \lambda^x}{x!} \qquad \left[e^x = \lim_{n \to \infty} \left(1 + \frac{x}{n}\right)^n \right]$$

↳ moment generating function দিয়ে prove করা যায়।

$\lambda \to$ mean, variance

$\lambda = np$

$\Rightarrow p = \frac{n}{\lambda}$

$$X \sim B(x; n, p)$$
$$= \binom{n}{x} p^x (1-p)^{n-x}$$
$$= \frac{n!}{(n-x)!\,x!} \left(\frac{\lambda}{n}\right)^x \left(1 - \frac{\lambda}{n}\right)^{n-x}$$
$$= \frac{n(n-1)(n-2)(n-x+1)\,(n-x)!}{(n-x)!\,x!} \cdot \frac{\lambda^x}{n^x} \left(1 - \frac{\lambda}{n}\right)^n \left(1 - \frac{\lambda}{n}\right)^{-x}$$

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Normal Distribution:

Parameter : σ, μ

*Parameter define করে peak কোথায় থাকবে আর shape কেমন হবে।

$$\boxed{N(x; \mu, \sigma) = \frac{1}{\sqrt{2\pi\sigma}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}}$$

$[-\infty < x < \infty]$

probability density return দেয়

* mean এর কারণে পার্থক্য হয় shape এ. mean same হলে σ^2 কম যার সে বেশি কেন্দ্রীভূত

Normal distribution থেকে

mean $= \mu$
Variance $= \sigma^2$ } সরাসরি calculate করা যায় না

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Ex 5.18 :

$P(x>15)$?

$\lambda = 10 \rightarrow$ per day tanker আসে

$t = 1$

$$P(x>15) = 1 - P(x \leq 15)$$

$$= 1 - \sum_{x=0}^{15} \frac{e^{-\lambda t} (\lambda t)^x}{x!}$$

$$= 1 - \sum_{x=0}^{15} \frac{e^{-10} \, 10^x}{x!}$$

$$= 0.0487$$

*Bionomial এর মতো poission distribution quality assurance, quality control এ use হয়।

Ex 5.19 :

$p = 0.005$

$n = 400 \rightarrow$ trail (sucess = accident)

(a)

$B(1; 400, 0.005)$

$$= \binom{400}{1} (0.005)^1 (1 - 0.005)^{399}$$

$$= 0.2707$$

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$E[X-\mu] = E[x] - \mu = 0$

$E[x] = \mu$

$z = \frac{x-\mu}{\sigma}$ odd function property use করে করা যায় direct করা যায়না

$dx = \sigma dz$

$$P(x_1 < x < x_2) = \int_{x_1}^{x_2} n(x; \mu, \sigma)$$

*z-score এ convert করে বের করা যাবে। সেটা easy

Example:

$P(45 < x < 62)$ $\quad X \sim N(\mu = 50, \sigma = 10)$

$= P(-0.5 < z < 1.2)$

↳ z value র আপেক্ষিক

$= P(z < 1.2) - P(z < -0.5)$

$= 0.88493 - 0.3085$ → from table

$= 0.5764$

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$np = 2 \rightarrow$ (2 দিনে on average accident হবে)

$\lambda t = np = 2$

by poission distribution

$$\frac{e^{-2}\, 2^1}{1!} = 0.271$$

(b)

$$P(x \leq 3) = \sum_{x=0}^{3} \frac{e^{-2}\, 2^x}{x!}$$

at most 3 days of accident

Ex 5.20:

$$p = \frac{1}{1000} = 0.001$$

$n = 8000$

$np = 8 \rightarrow$ on average 8টা glass এ

$P(x < 7) \approx P(x; 8)$

$\rightarrow$ binomial দিয়েও solve করা যাবে।

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Example:

$P(x > 362)$

$= P(Z > 1.24)$

$= 1 - P(Z < 1.24)$ → complement করে so that table থেকে পাওয়া যায় cummilative মান

$= 1 - 0.8925$

$= 0.1075$

Example 6.3:

$P(Z > k)$

$= 1 - P(Z < k)$ $= 0.3015$

$= \cancel{1 - 0.6985}$ $\Rightarrow P(Z < k) = 1 - 0.3015$

$= 0.6985$

$\cancel{0.3015}$

$k = 0.52$

↳ বের করতে হবে।

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10 দিনের std:

$$B \to P \to B \to H \to B \to H \to H \to H \to B \to P$$

$$\left.\begin{aligned} P(B) &= \frac{4}{10} \\ P(\text{Pizza}) &= \frac{2}{10} \\ P(H) &= \frac{4}{10} \end{aligned}\right\} \text{Based on 10 days}$$

stationary state:

$$\left.\begin{aligned} &0.2591 \\ &0.21245 \\ &0.43564 \end{aligned}\right\}$$ certain property থেকে long term basis এ নেওয়া

Transition matrix:

$$A = \begin{array}{c} \\ B \\ P \\ H \end{array} \begin{array}{c} \begin{array}{ccc} B & P & H \end{array} \\ \begin{bmatrix} 0.2 & 0.6 & 0.2 \\ 0.3 & 0 & 0.7 \\ 0.5 & 0 & 0.5 \end{bmatrix} \end{array}$$

$$\pi_0 = \begin{array}{c} \begin{array}{ccc} B & P & H \end{array} \\ \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \end{array}$$

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Uniform Distribution:

* সবগুলো probability equal.

* $f(x) = \frac{1}{n}$, discrete uniform distribution
  $\hookrightarrow$ n বার ঘটার probability

$X \sim \text{Uniform}(a, b)$

$$f(x) = \begin{cases} \frac{1}{b-a}, & a \le x \le b \\ 0, & \text{otherwise} \end{cases}$$

$\rightarrow$ certain range এর মধ্যে সবগুলোর probability সমান
probability density ও সমান

$$\int_a^b \frac{1}{b-a}\, dx$$

$$= \frac{1}{b-a} [x]_a^b$$

$$= \frac{b-a}{b-a} = 1$$ $\rightarrow$ Area under the curve

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$X_t = i$

$X_{t+1} = j$

$P(\underline{X_{t+1} = j} \mid X_0 = k_0, X_1 = k_1, X_2 = k_2 \cdots X_t = i)$

↓

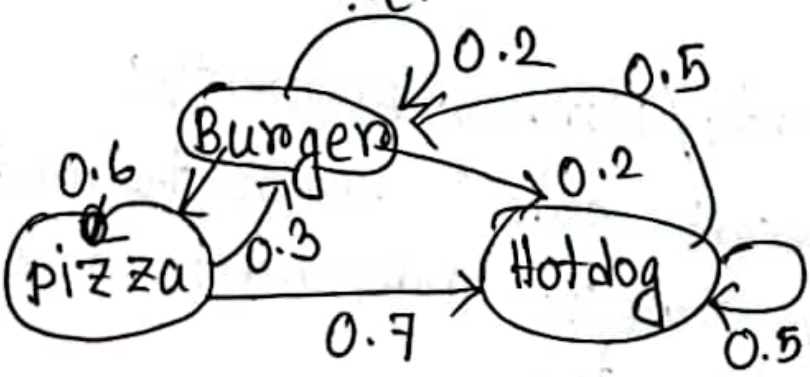
পরবর্তী যে time পাবে তার probability
markov বলে যে অতীতের ঘটনা মনে রাখবেনা only present এর উপর depend করে পরবর্তী state দিবে।

$\boxed{P(X_{t+1} = j \mid X_t = i)}$ → Markov property

Example:

Restaurant → Pizza, Burger, Hotdog

→ condition যখন pizza serve করবে তখন সেটাই করবে। এটা সব item এর জন্য প্রযোজ্য। আবার আজকে যা serve করছে তার উপর depend করে কালকে serve করবে।



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matrix multiplication এর মাধ্যমে কোন দিন কোনটা serve করবে পাওয়া যাবে।

$$\pi_0 A = [0.3 \quad 0 \quad 0.7] = \pi_1$$

$$\pi_1 A = \pi_2$$

$$\pi_2 A = \pi_3$$

. . . . .

. . . . .

. . . . .

একসময় converge value পাওয়া যাবে।
যেটা stationary state

* প্রত্যেক state থেকে প্রত্যেক state reachable হলে irresistable property. Direct না হলেও কোনো না কোনোভাবে reach করা যাবে।

* reach না করা গেলে converge করে।

* Value গুলো একটা পর্যায়ে converge করে। কিন্তু যদি converge না করে তাহলে aperiodic property. certain point পর repeat করবে।

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$$\begin{bmatrix} 0.2-\lambda & 0.3 & 0.5 \\ 0.6 & 0-\lambda & 0 \\ 0.2 & 0.7 & 0.5-\lambda \end{bmatrix} \begin{bmatrix} \pi(1) \\ \pi(2) \\ \pi(3) \end{bmatrix} = 0$$

$\downarrow$ stationary vector

$$\Rightarrow \begin{bmatrix} 0.8 & 0.3 & 0.5 \\ 0.6 & -1 & 0 \\ 0.2 & 0.7 & -0.5 \end{bmatrix} \begin{bmatrix} \pi(1) \\ \pi(2) \\ \pi(3) \end{bmatrix} = 0$$

$$\Rightarrow 0.8(\pi 1) + 0.3(\pi 2) + 0.5(\pi 3) = 0$$

$$\Rightarrow 0.2(\pi 1) + 0.7(\pi 2) - 0.5(\pi 3) = 0$$

$$\Rightarrow 0.6(\pi 1) - 0.2(\pi 2) = 0$$

$\rightarrow$ সবগুলো solve করলে infinite আসে but

$$\pi(1) + \pi(2) + \pi(3) = 1$$

$$\pi(1) = \begin{bmatrix} 0.3521 \\ 0.2113 \\ 0.4366 \end{bmatrix}$$

$\pi(2) =$

$\pi(3) =$

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$$A\vec{v} = \lambda \vec{v}$$

$\lambda \rightarrow$ Eigen value

$\vec{v} \rightarrow$ Eigen vector

* Vector এর value scale up করে কিন্তু value change হয়না। এটা stationary ~~property~~ distribution.

* Eigen value হবে scaling factor

* Eigen value 1 এর জন্য eigen vector calculate করলে stationary distribution পাওয়া যাবে।

$\det[A - \lambda I] = 0$

$\pi_0 A = \pi_1$

$\pi_1 A = \pi_2$

$$\begin{bmatrix} 0.2 & 0.3 & 0.5 \\ 0.6 & 0 & 0 \\ 0.2 & 0.7 & 0.5 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \\ \\ \\ \end{bmatrix}$$

$\rightarrow$ same value আসবে।

Transpose করা

Calculate eigen vector for eigen value $\lambda = 1$

$\downarrow$

Vector position & value change করে না।

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Stochastic Process

* state গুলোর মধ্যে transition হতে পারে।
* কিছু সময় পর পর state change হয়।

state :

Transition :

$X(t) \longrightarrow$ time এর সাপেক্ষে output state

Time $\longrightarrow$ discrete, continuous
space $\longrightarrow$ " , continuous

} এদের combination এ 4টি case হয়।

~~* Time conti~~

Discrete Time, discrete ~~state~~ space :

Discrete time Markov chain

Properties :

Memoryless property / Markov property :

Given that, present state is known the conditional probability of next state is independent of states priorr to present state.

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Queuing Theory ⟶ optimal state এ যাতে যেন কম time ~~এ~~ wait করতে হয়।

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Application

Multinomial Distribution:

$\left.\begin{array}{l} n>1 \\ K=2 \end{array}\right\}$ Binomial distribution

$\left.\begin{array}{l} n=1 \\ K=2 \end{array}\right\}$ Bernouli distribution

*trail multiple হয়, outcome ও ~~multinom~~ multiple হয়।

$\Sigma P = 1$

→ মতোগুলো event হওয়া possible সব গুলোর probability র

$$\frac{n!}{k_1!\, k_2! \cdots k_s!}\, p_1^{k_1}\, p_2^{k_2} \cdots p_s^{k_s}$$

success → x
failure → $n-x$

Example 6.15:

$$P = \frac{8!}{2!\,2!\,1!\,1!\,1!}\left(\frac{1}{6}\right)^{8}$$

8 → as fair and 8 times roll করবে