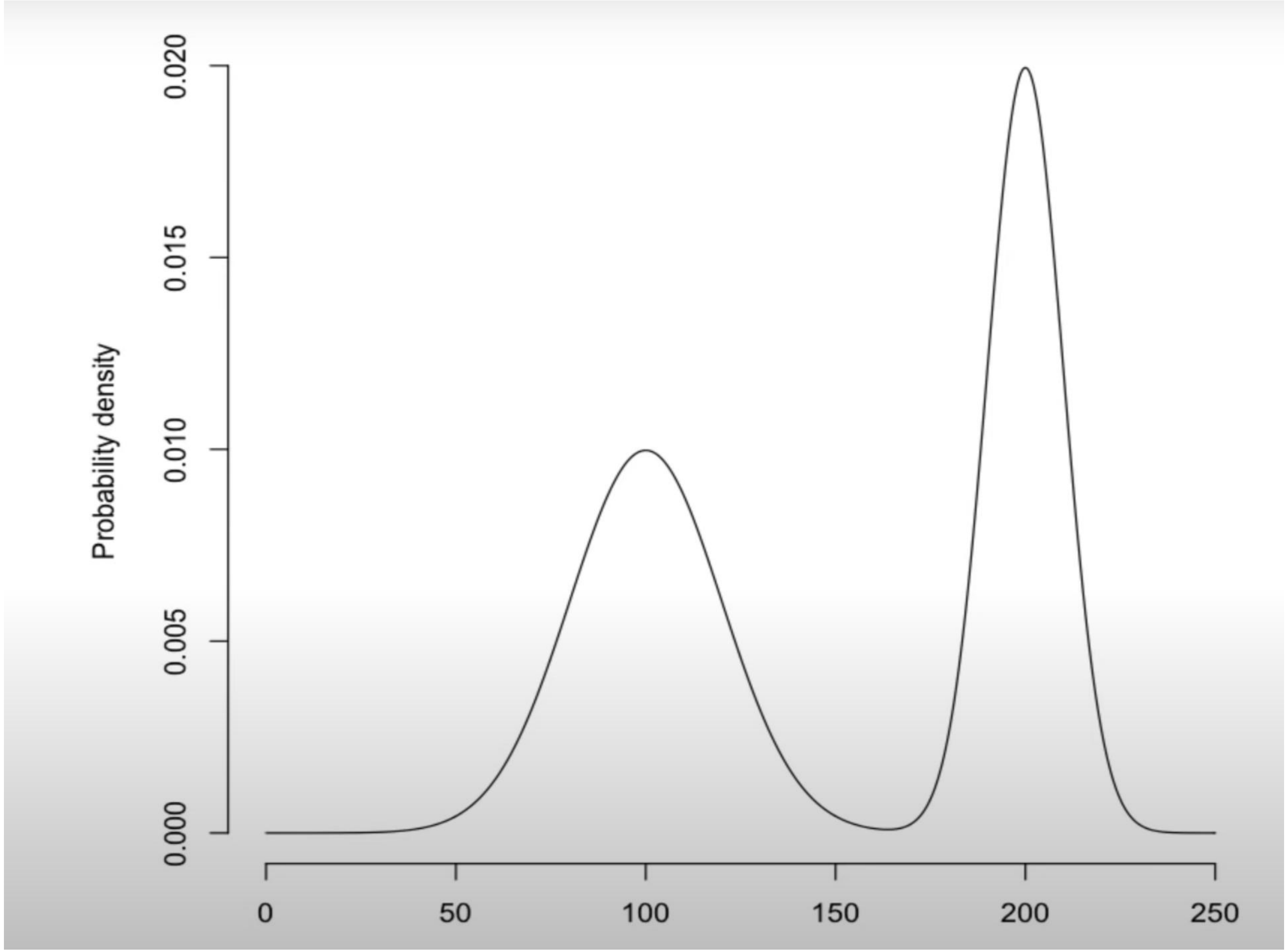
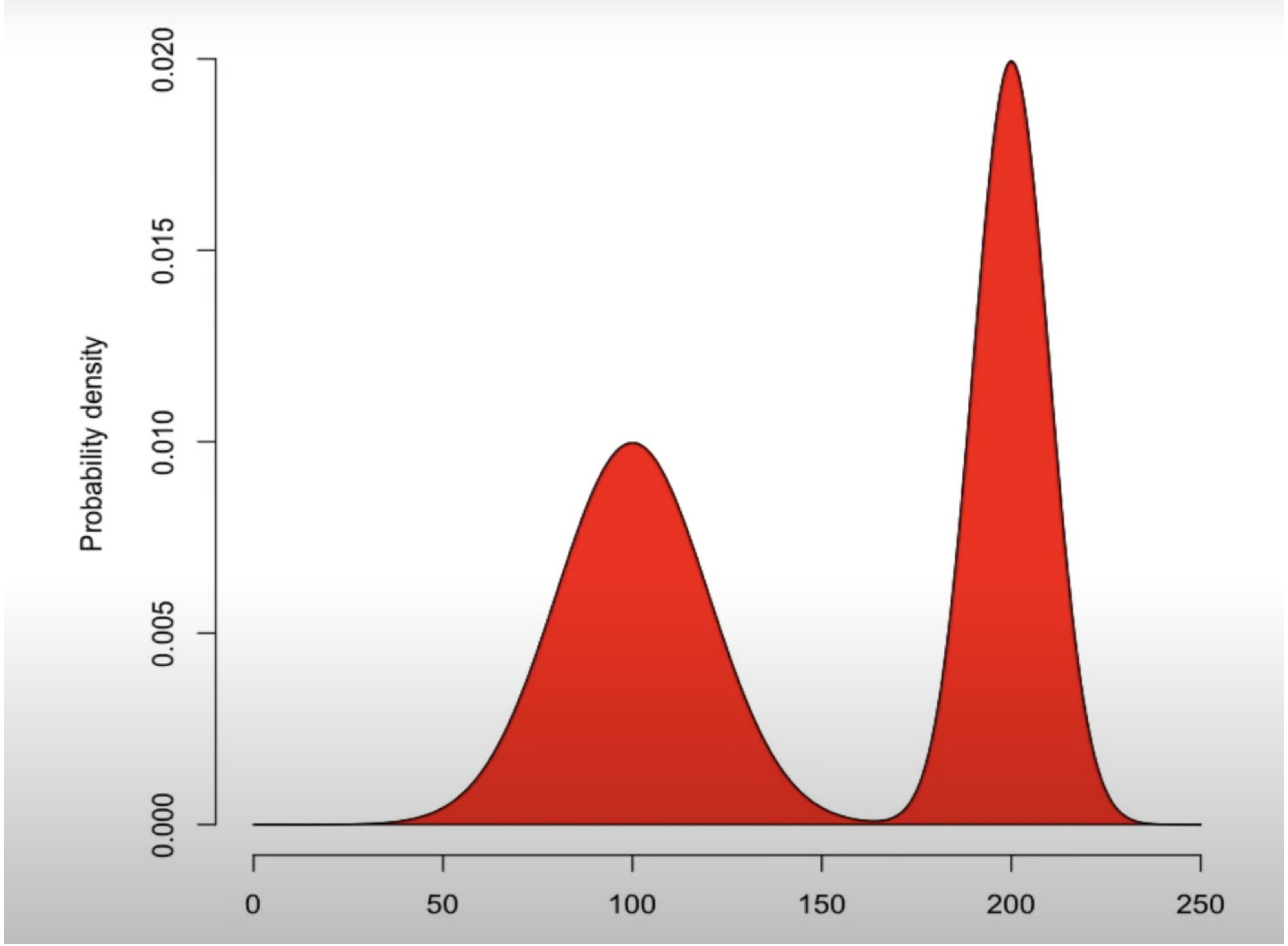
PDF and CDF

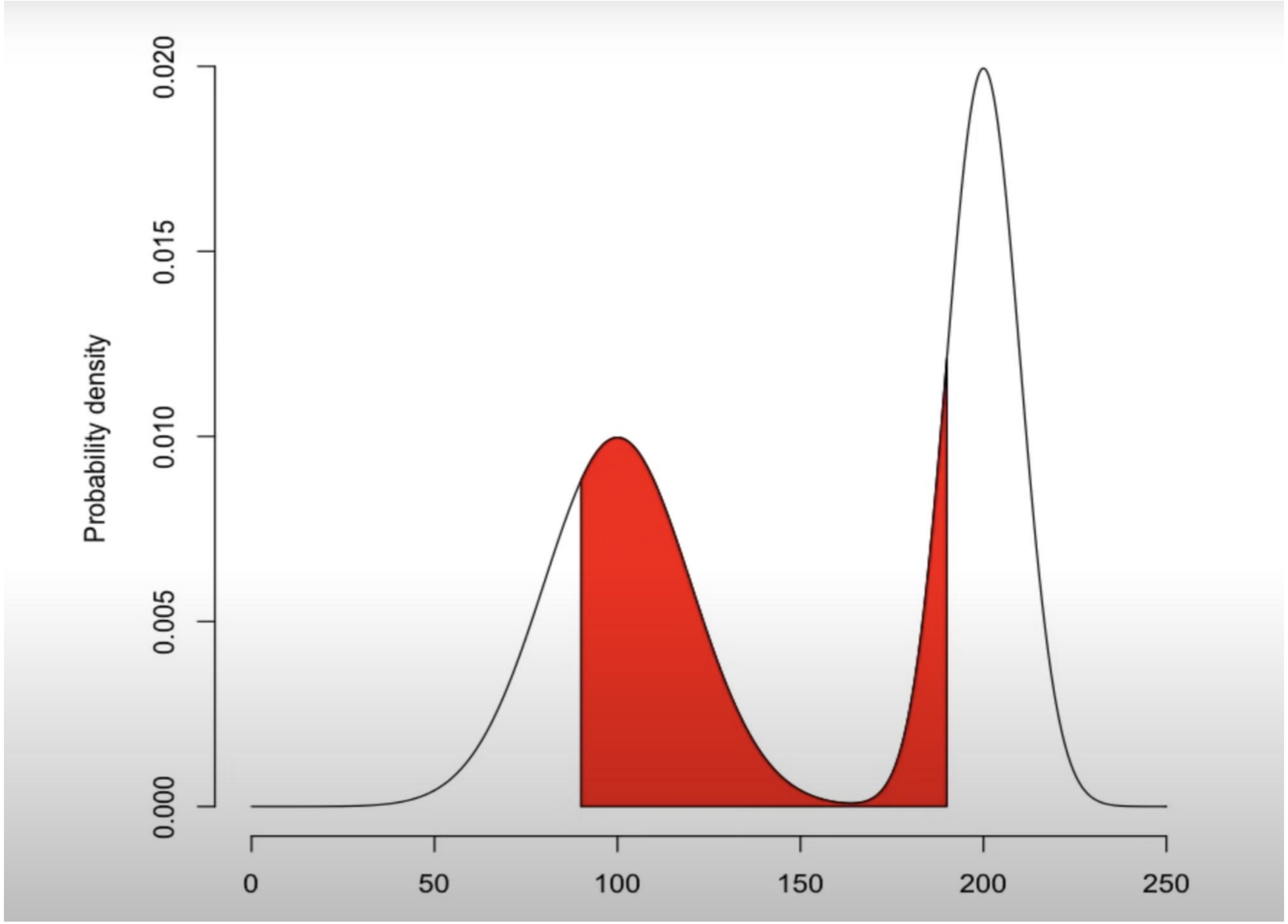

Probability density
0.000
0.005
0.010
0.015
0.020
0
50
100
150
200
250



Probability density
0.000
0.005
0.010
0.015
0.020
0
50
100
150
200
250

The probability that X is between a and b is the area under the curve between a and b.

$$P(a \le X \le b) = \int_a^b f(x)\,\mathrm{d}x$$



Probability density
0.000
0.005
0.010
0.015
0.020
0
50
100
150
200
250

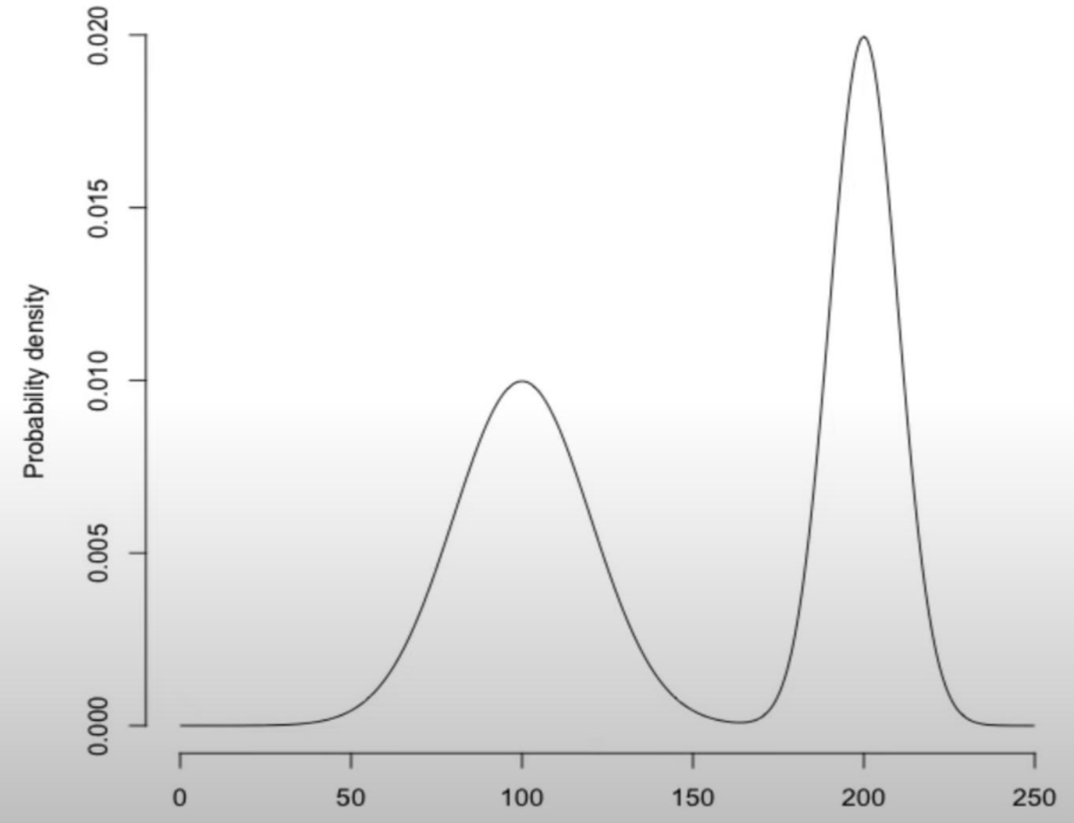
The probability density cannot be negative.

$$f(x) \geq 0 \text{ for all } x$$

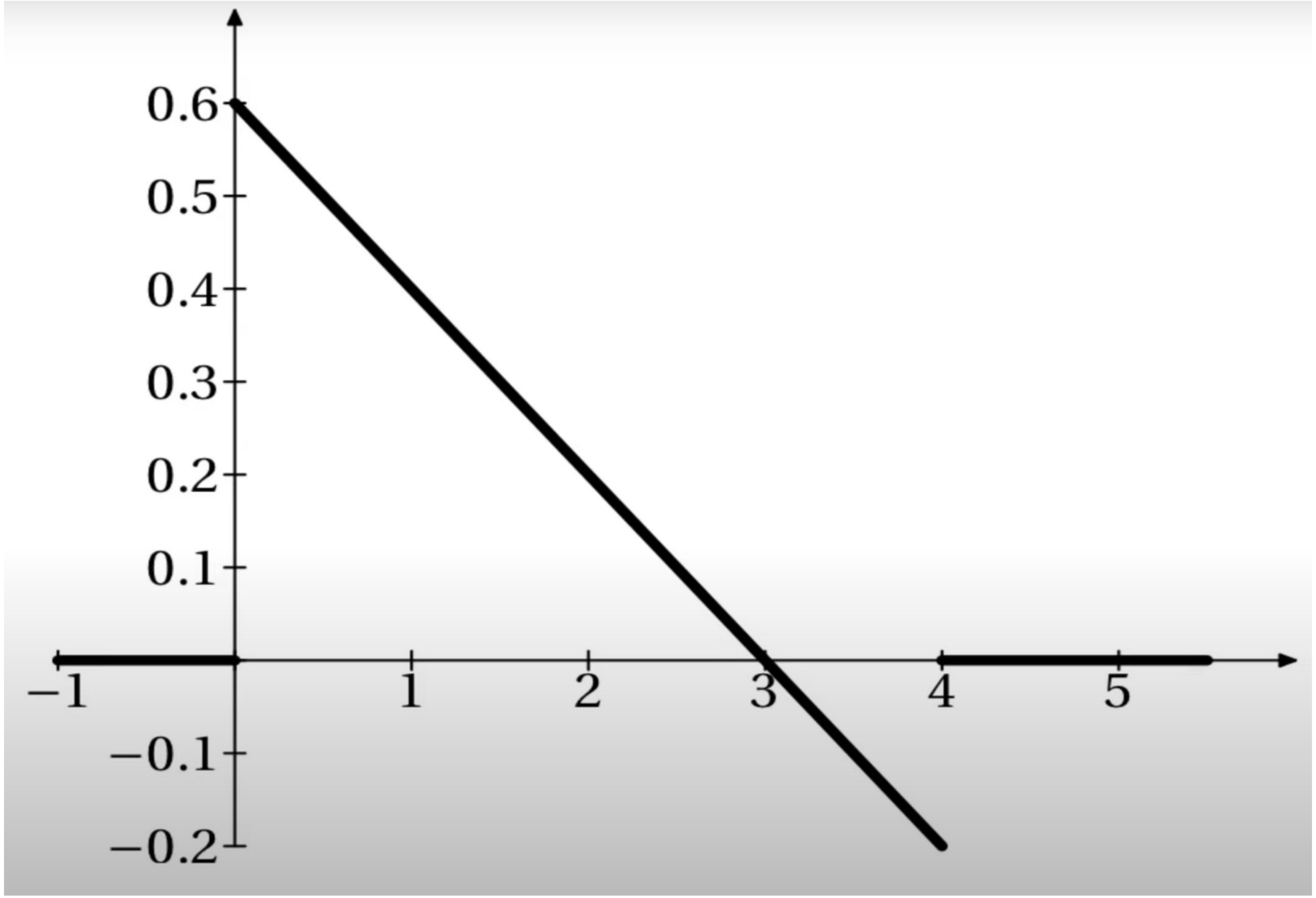
The total area under the curve must be 1.

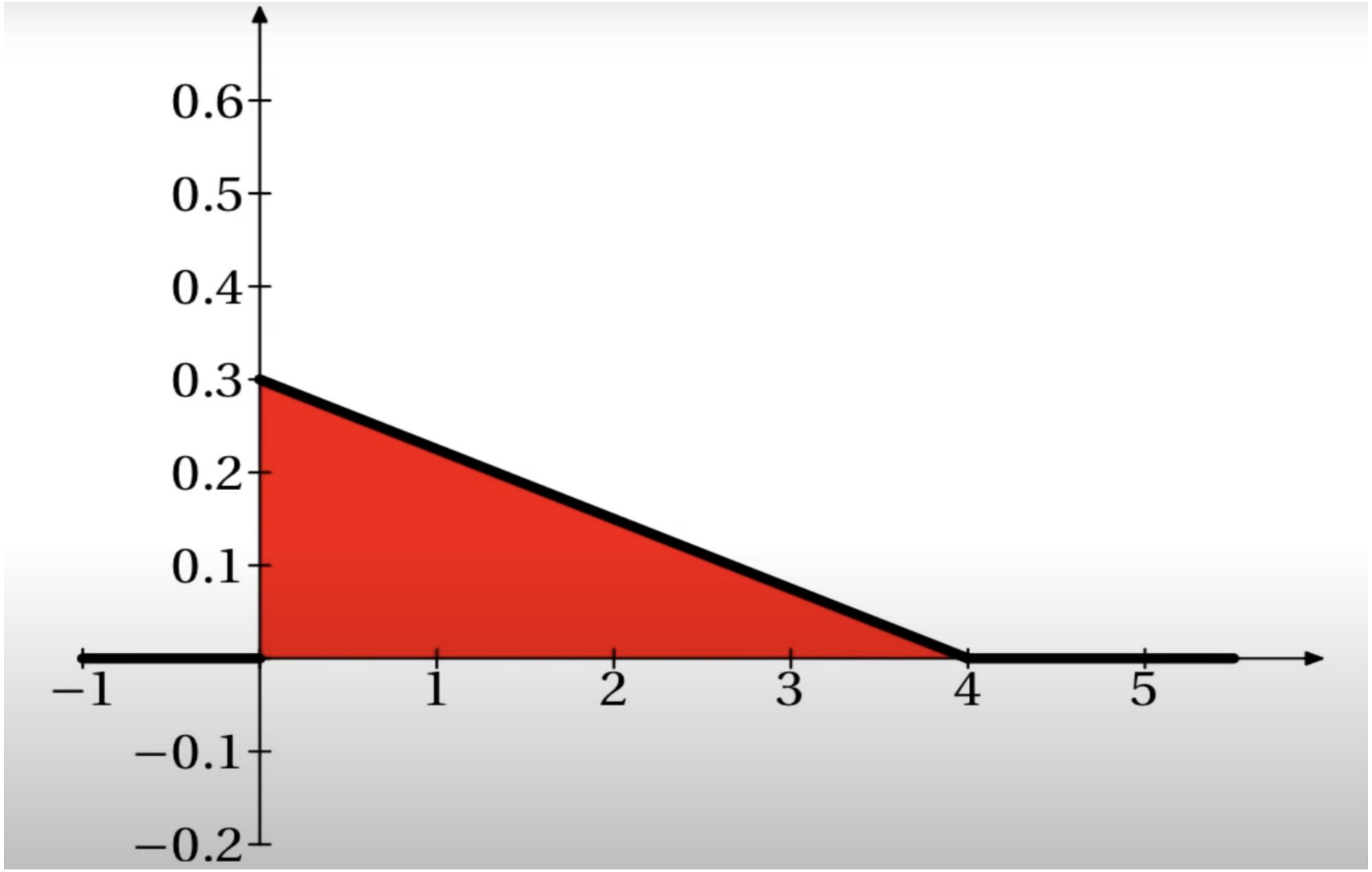
$$\int_{-\infty}^{\infty} f(x)\mathrm{d}x = 1$$

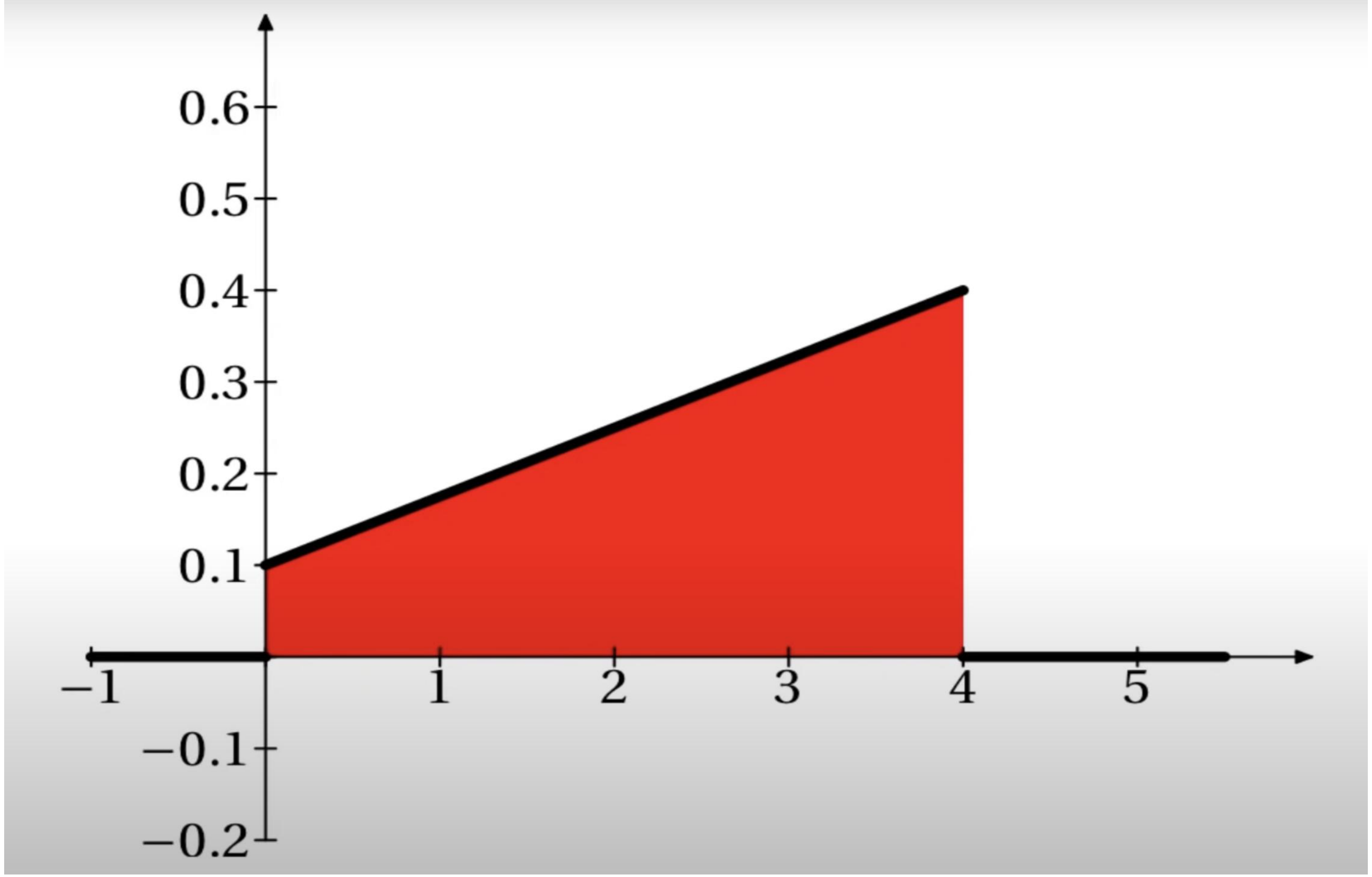
$$f(x) = \frac{1}{2}\left(\frac{1}{20\sqrt{2\pi}}e^{-\frac{(x-100)^2}{800}} + \frac{1}{10\sqrt{2\pi}}e^{-\frac{(x-200)^2}{200}}\right)$$



RECOGINZING PDF


0.6
0.5
0.4
0.3
0.2
0.1
−0.1
−0.2
−1
1
2
3
4
5


0.6
0.5
0.4
0.3
0.2
0.1
−0.1
−0.2
−1
1
2
3
4
5



0.6
0.5
0.4
0.3
0.2
0.1
−1
1
2
3
4
5
−0.1
−0.2

Find the value of k for which the following function would be a valid probability density function.

$$f(x) = \begin{cases} k(x^2+1) & 1 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

$$f(x) = \begin{cases} k(x^2+1) & 1 \le x \le 4 \\ 0 & \text{otherwise} \end{cases}$$

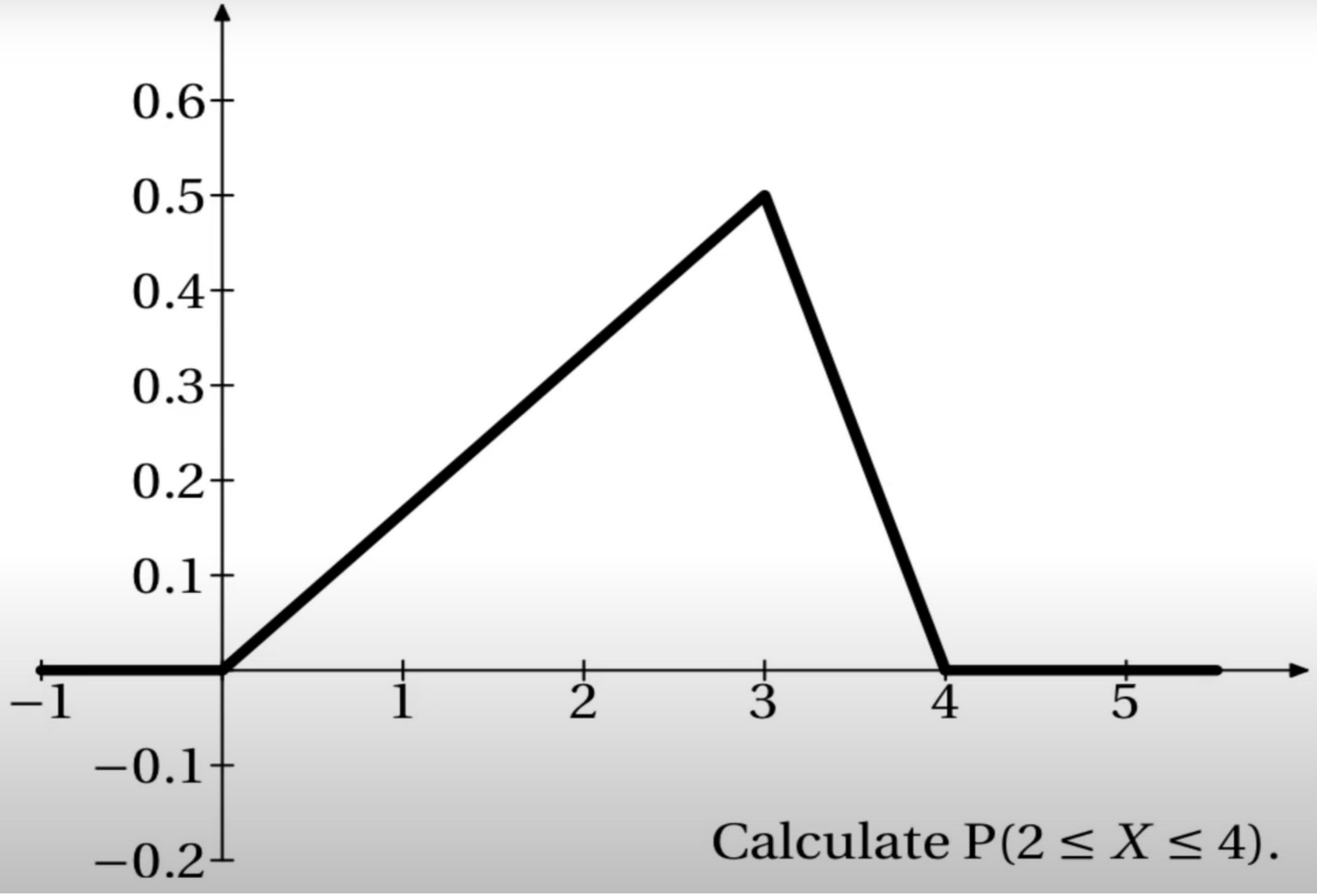
$$\begin{aligned}
\int_1^4 k(x^2+1)\,\mathrm{d}x &= \left[k\left(\tfrac{1}{3}x^3 + x\right)\right]_1^4 \\
&= k\left(\tfrac{1}{3} \times 4^3 + 4\right) - k\left(\tfrac{1}{3} \times 1^3 + 1\right) \\
&= k\left(\tfrac{64}{3} + 4\right) - k\left(\tfrac{1}{3} + 1\right) \\
&= k\left(\tfrac{63}{3} + 3\right) \\
&= 24k \\
24k &= 1 \\
k &= \tfrac{1}{24}
\end{aligned}$$

$$f(x) = \begin{cases} \frac{1}{24}(x^2+1) & 1 \le x \le 4 \\ 0 & \text{otherwise} \end{cases}$$

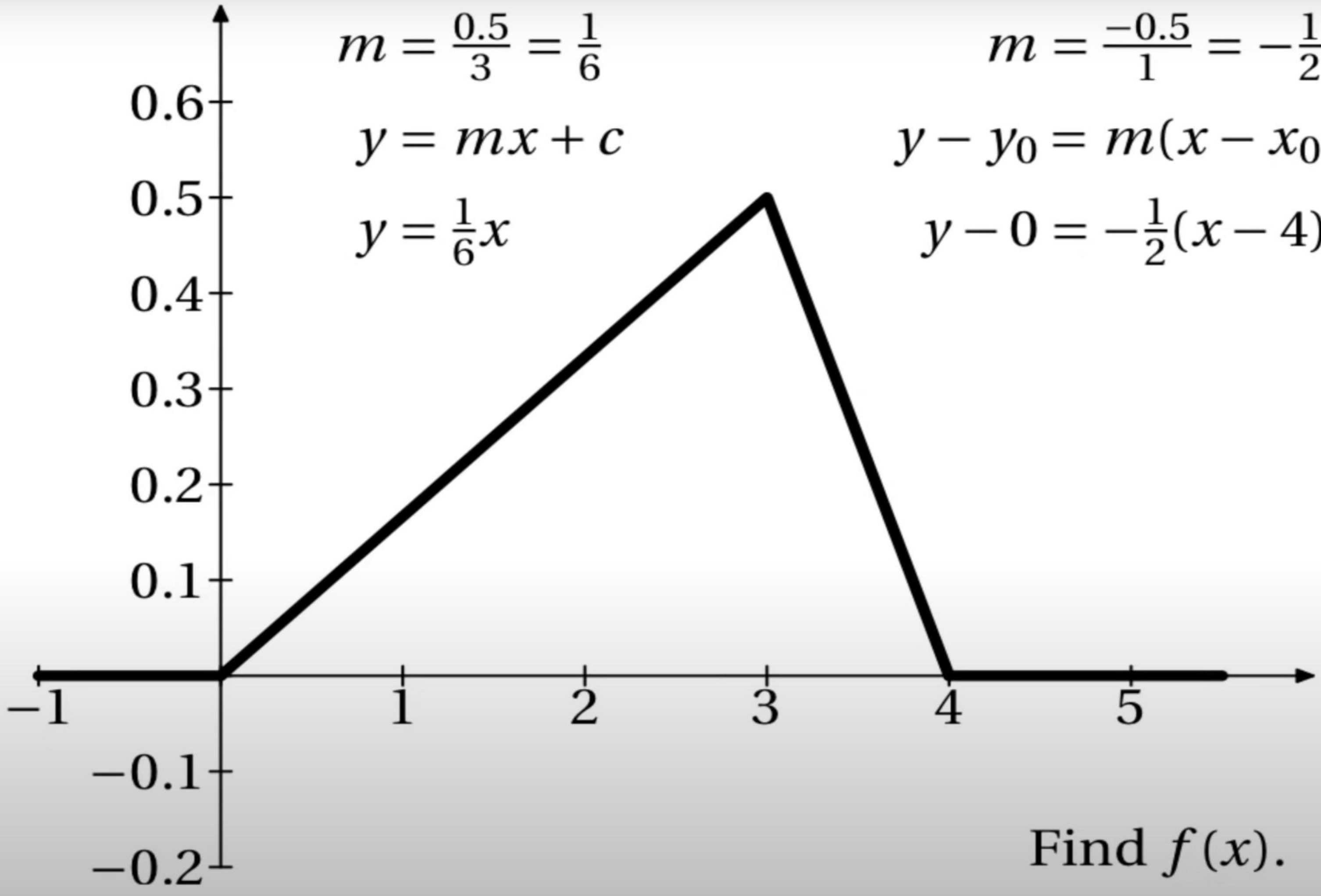
Calculate $\mathrm{P}(X = 3)$. $\underline{0}$

Calculate $\mathrm{P}(2 < X \le 3)$.

$$\begin{aligned} \int_2^3 \frac{1}{24}(x^2+1)\mathrm{d}x &= \left[\frac{1}{24}\left(\frac{1}{3}x^3 + x\right)\right]_2^3 \\ &= \frac{1}{24}\left(\frac{1}{3} \times 3^3 + 3\right) - \frac{1}{24}\left(\frac{1}{3} \times 2^3 + 2\right) \\ &= \frac{1}{24}(9+3) - \frac{1}{24}\left(\frac{8}{3} + 2\right) \\ &= \frac{11}{36} \end{aligned}$$



Calculate $P(2 \le X \le 4)$.



$$m = \frac{0.5}{3} = \frac{1}{6}$$

$$y = mx + c$$

$$y = \frac{1}{6}x$$

$$m = \frac{-0.5}{1} = -\frac{1}{2}$$

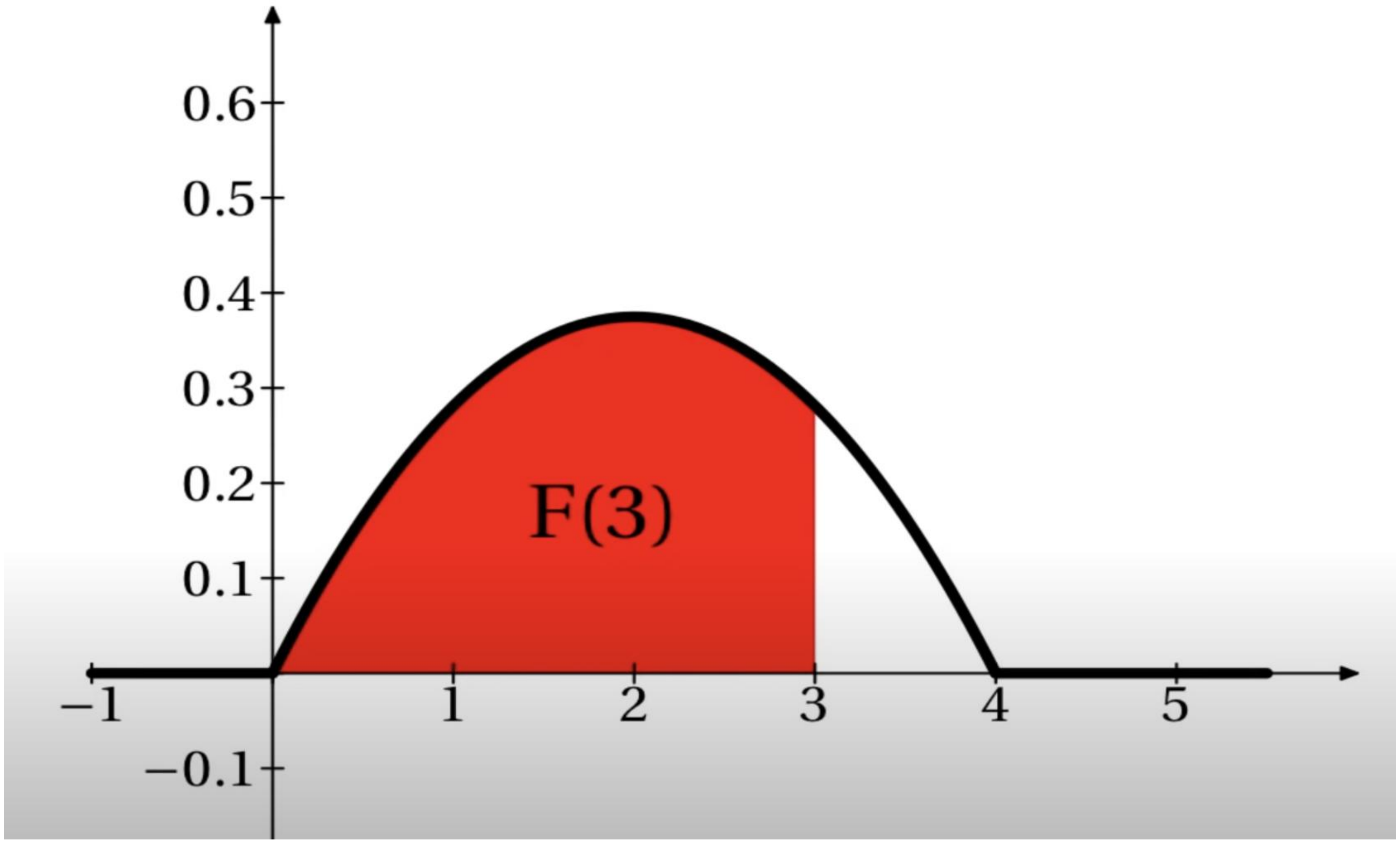
$$y - y_0 = m(x - x_0)$$

$$y - 0 = -\frac{1}{2}(x - 4)$$

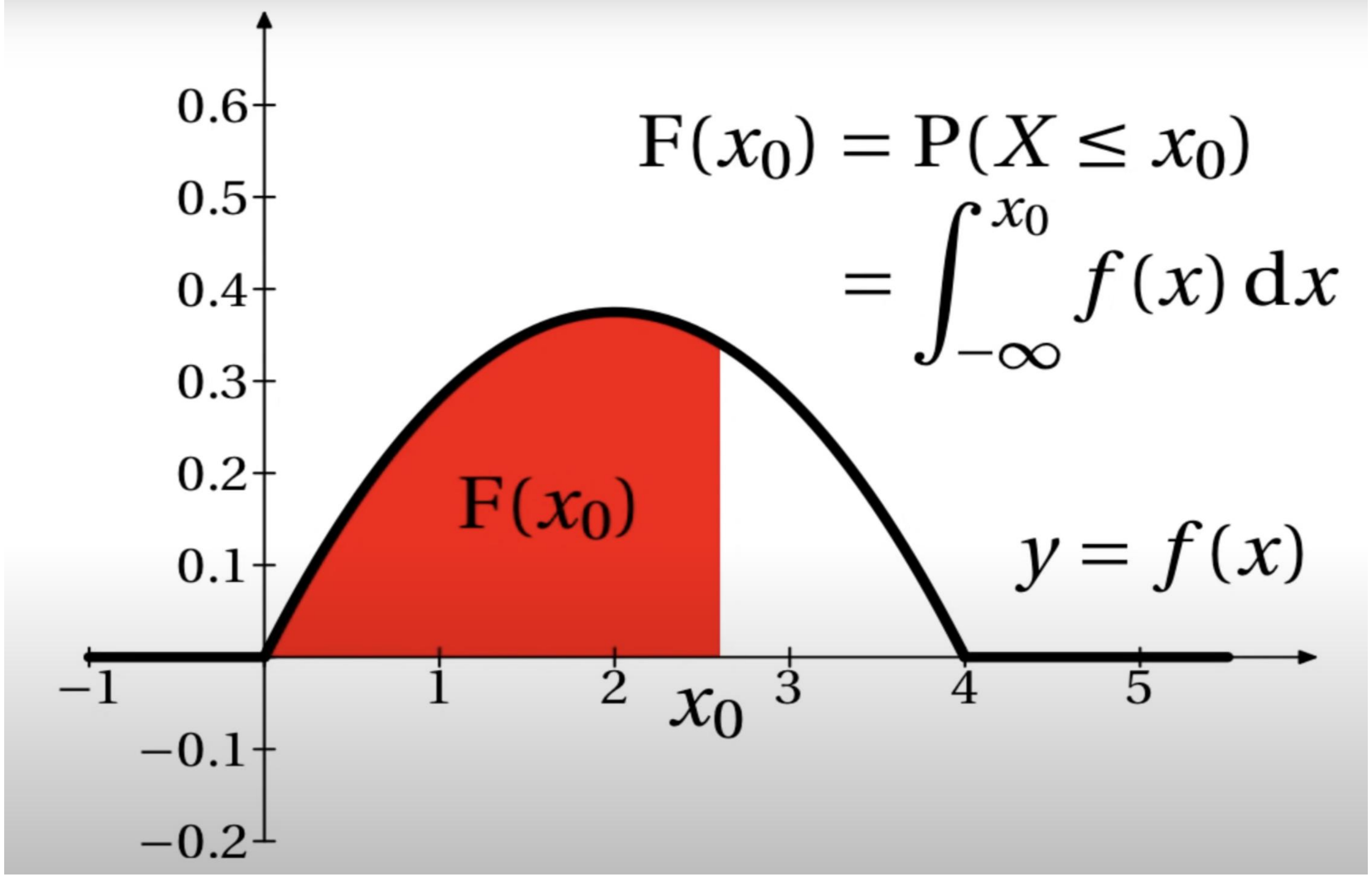
Find $f(x)$.

Cumulative Density Function

$$F(x_0) = P(X \leq x_0)$$



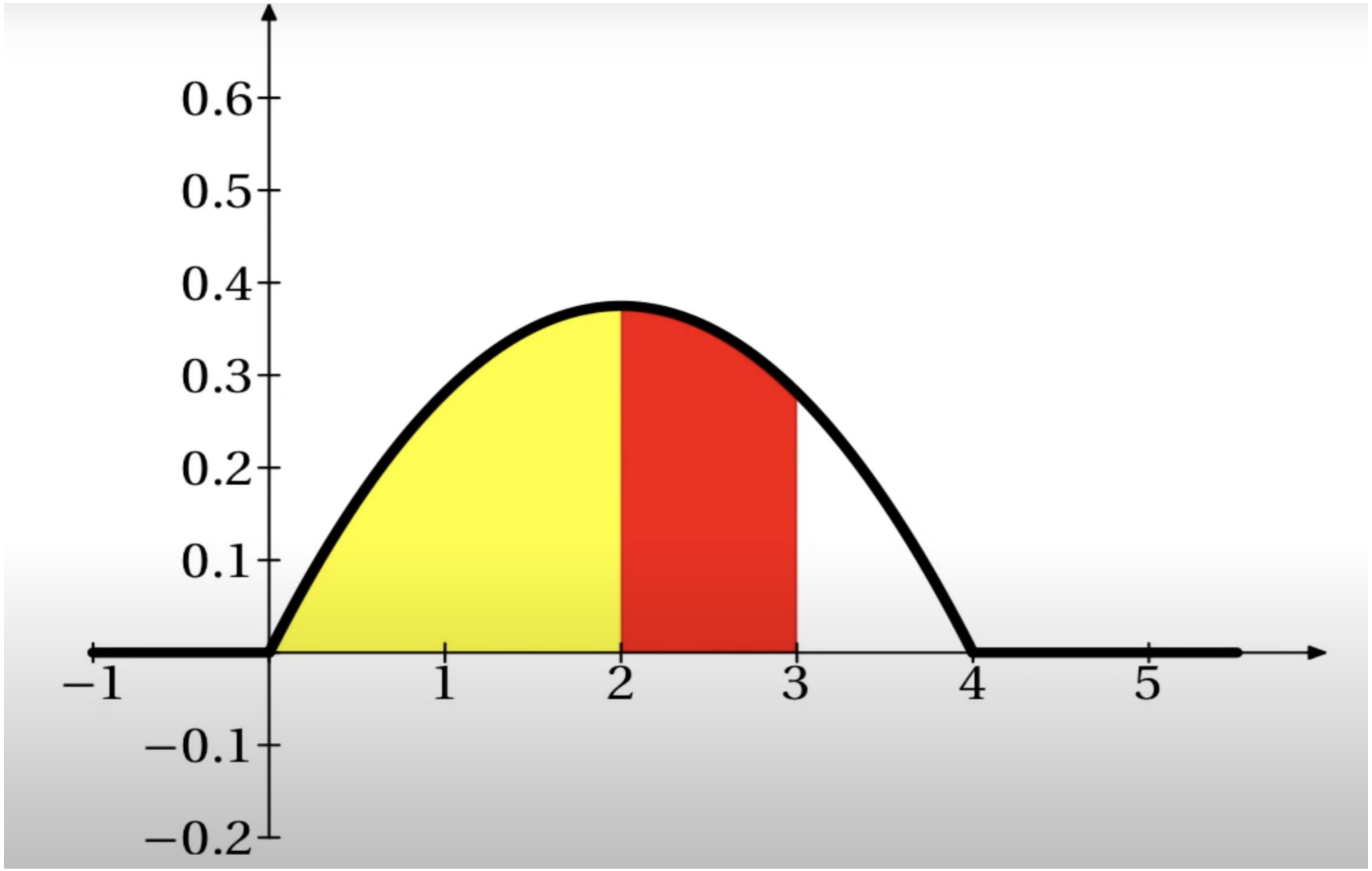
$$F(3) = P(X \leq 3)$$

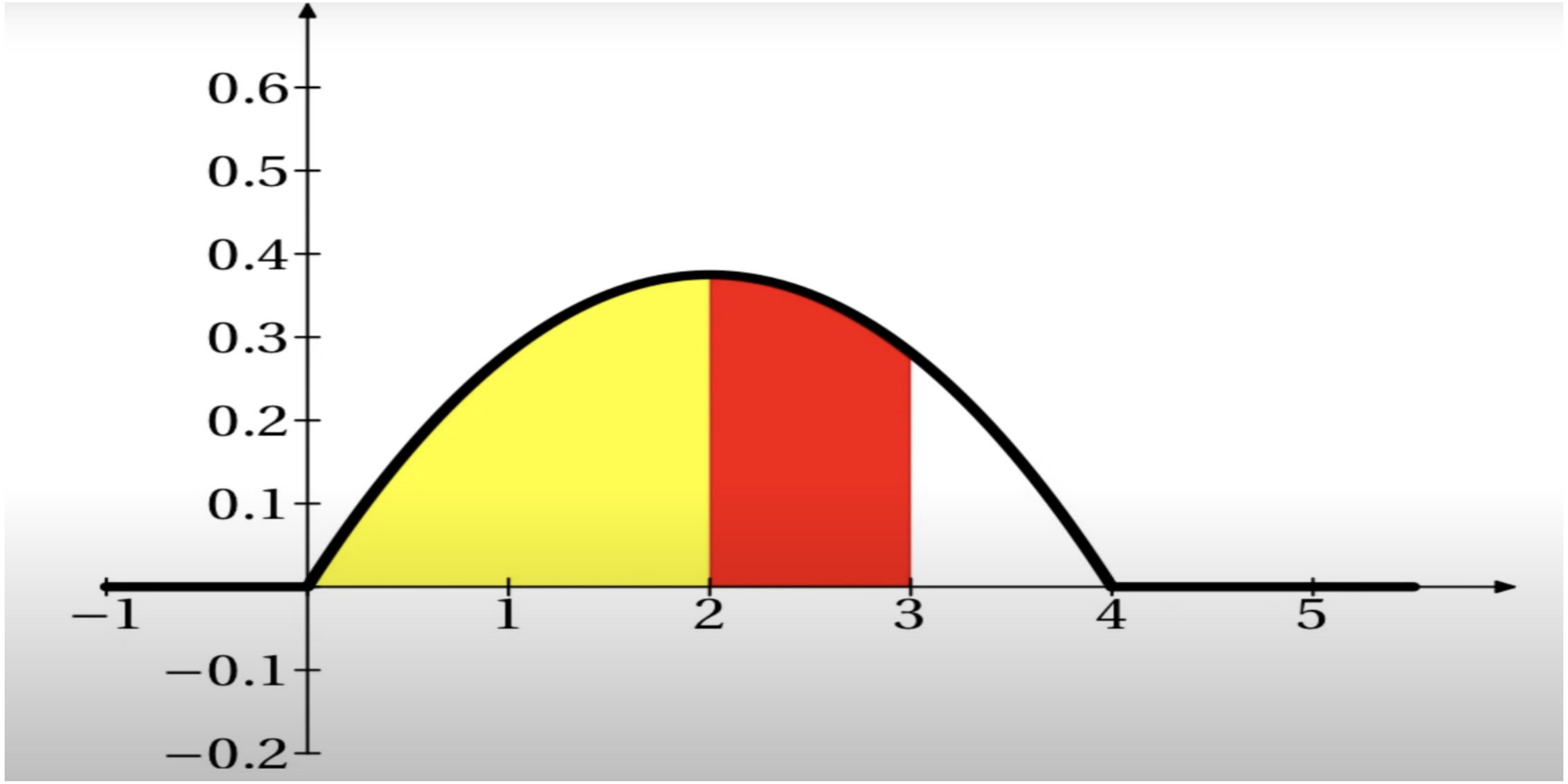

0.6
0.5
0.4
0.3
0.2
0.1
−0.1
−0.2
−1
1
2
3
4
5
x_0
$F(x_0)$
$F(x_0) = P(X \leq x_0)$
$= \int_{-\infty}^{x_0} f(x)\,dx$
$y = f(x)$

The cumulative distribution function for the random variable X is given by

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{32}\left(6x^2 - x^3\right) & 0 \leq x \leq 4 \\ 1 & x > 4 \end{cases}$$

Calculate $P(2 < X < 3)$.


0.6
0.5
0.4
0.3
0.2
0.1
−0.1
−0.2
−1
1
2
3
4
5

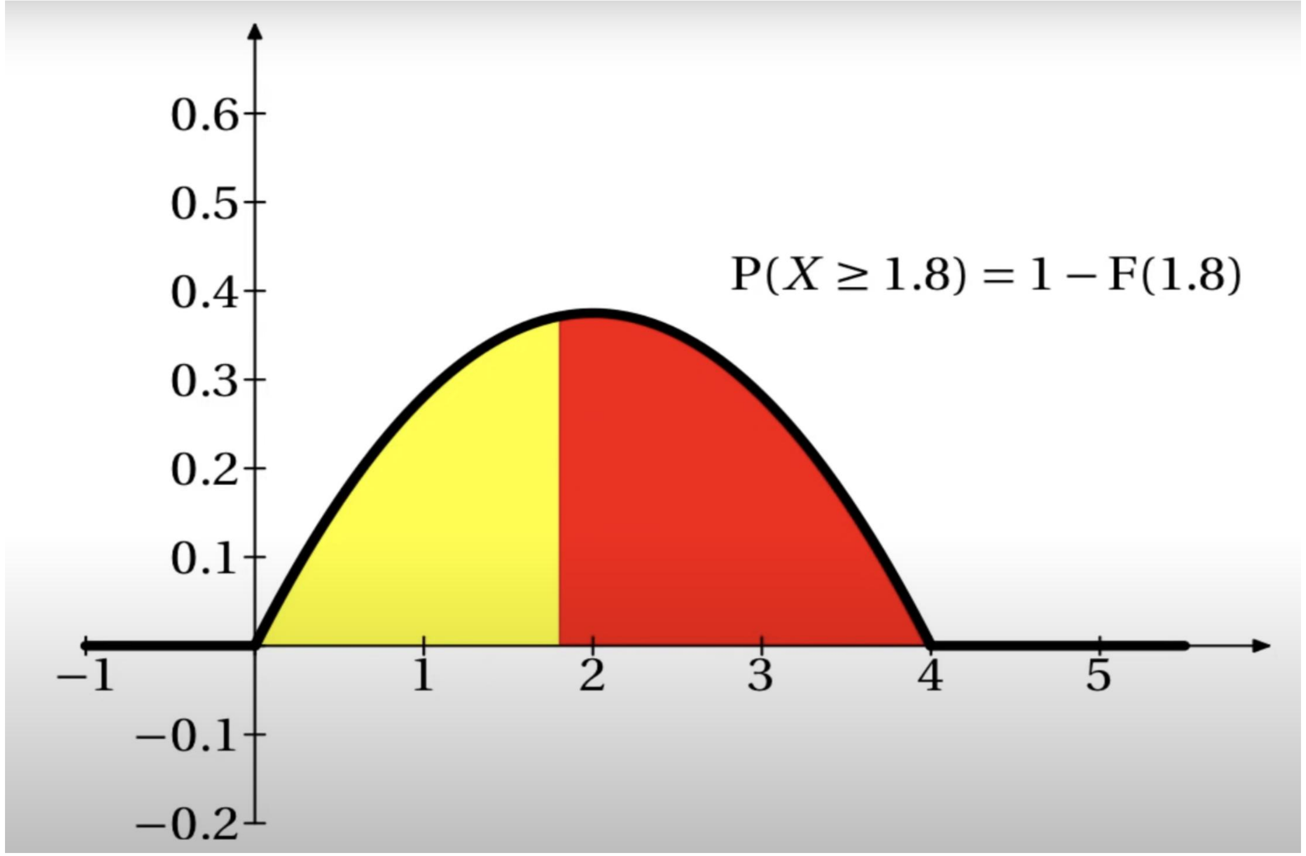


$$P(2 < X < 3) = F(3) - F(2)$$
$$= \frac{1}{32}(6 \times 3^2 - 3^3) - \frac{1}{32}(6 \times 2^2 - 2^3)$$
$$= \frac{11}{32}$$

The cumulative distribution function for the random variable X is given by

$$\mathrm{F}(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{32}\left(6x^2 - x^3\right) & 0 \le x \le 4 \\ 1 & x > 4 \end{cases}$$

Calculate $\mathrm{P}(X \ge 1.8)$.



$$P(X \geq 1.8) = 1 - F(1.8)$$
$$= 1 - \frac{1}{32}(6 \times 1.8^2 - 1.8^3)$$
$$= 0.57475$$

PDF TO CDF

The probability density function for the random variable X is given by

$$f(x) = \begin{cases} \frac{1}{8}x^2 & 0 \leq x \leq 2 \\ \frac{1}{8}x(4 - x) & 2 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

Find $\mathrm{F}(x)$.

$$f(x) = \begin{cases} \frac{1}{8}x^2 & 0 \le x \le 2 \\ \frac{1}{8}x(4-x) & 2 \le x \le 4 \\ 0 & \text{otherwise} \end{cases}$$

When $0 \le x \le 2$, $\mathrm{F}(x) = \int \frac{1}{8}x^2\,\mathrm{d}x = \frac{1}{24}x^3 + C$

But $\mathrm{F}(0) = 0$. Therefore, $C = 0$. So $\mathrm{F}(x) = \frac{1}{24}x^3$.

When $2 \le x \le 4$, $\mathrm{F}(x) = \int \frac{1}{8}x(4-x)\,\mathrm{d}x = \frac{1}{4}x^2 - \frac{1}{24}x^3 + D$

But $\mathrm{F}(2) = \frac{1}{24} \times 2^3 = \frac{1}{3}$. Therefore, $D = -\frac{1}{3}$.

So $\mathrm{F}(x) = \frac{1}{4}x^2 - \frac{1}{24}x^3 - \frac{1}{3} = \frac{1}{24}\left(6x^2 - x^3 - 8\right)$.

$$\mathrm{F}(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{24}x^3 & 0 \le x \le 2 \\ \frac{1}{24}\left(6x^2 - x^3 - 8\right) & 2 \le x \le 4 \\ 1 & x > 4 \end{cases}$$

CDF TO PDF

The cumulative distribution function for the random variable X is given by

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{24}x^3 & 0 \le x \le 2 \\ \frac{1}{24}\left(6x^2 - x^3 - 8\right) & 2 \le x \le 4 \\ 1 & x > 4 \end{cases}$$

Find $f(x)$.

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{24}x^3 & 0 \le x \le 2 \\ \frac{1}{24}\left(6x^2 - x^3 - 8\right) & 2 \le x \le 4 \\ 1 & x > 4 \end{cases}$$

$$\frac{d}{dx}\frac{1}{24}x^3 = \frac{1}{8}x^2$$

$$\frac{d}{dx}\frac{1}{24}\left(6x^2 - x^3 - 8\right) = \frac{1}{24}\left(12x - 3x^2\right) = \frac{1}{8}x(4 - x)$$

$$f(x) = \begin{cases} \frac{1}{8}x^2 & 0 \le x \le 2 \\ \frac{1}{8}x(4 - x) & 2 \le x \le 4 \\ 0 & \text{otherwise} \end{cases}$$

Mean and Variance of Continous Random Variable

$$\mu = \int_{-\infty}^{\infty} x f(x)\,\mathrm{d}x$$

$$\sigma^2 = \int_{-\infty}^{\infty} x^2 f(x)\,\mathrm{d}x - \mu^2$$

$$\mu = \int_{-\infty}^{\infty} x f(x)\,\mathrm{d}x$$

$$\sigma^2 = \int_{-\infty}^{\infty} x^2 f(x)\,\mathrm{d}x - \mu^2$$

The random variable X has probability density function given by

$$f(x) = \begin{cases} \frac{3}{10}(3x - x^2) & 0 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

Find the mean and variance of X.

$$f(x) = \begin{cases} \frac{3}{10}(3x - x^2) & 0 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

$$\begin{aligned} \mu &= \int_{-\infty}^{\infty} x f(x)\,\mathrm{d}x \\ &= \int_0^2 x \times \tfrac{3}{10}(3x - x^2)\,\mathrm{d}x \\ &= \int_0^2 \tfrac{9}{10}x^2 - \tfrac{3}{10}x^3\,\mathrm{d}x \\ &= \left[\tfrac{3}{10}x^3 - \tfrac{3}{40}x^4\right]_0^2 \\ &= \left(\tfrac{3}{10} \times 2^3 - \tfrac{3}{40} \times 2^4\right) - 0 \\ &= 1\tfrac{1}{5} \end{aligned}$$

$$
\begin{aligned}
\mathrm{E}(X^2) &= \int_{-\infty}^{\infty} x^2 f(x)\,\mathrm{d}x \\
&= \int_0^2 x^2 \times \tfrac{3}{10}(3x - x^2)\,\mathrm{d}x \\
&= \int_0^2 \tfrac{9}{10}x^3 - \tfrac{3}{10}x^4\,\mathrm{d}x \\
&= \left[\tfrac{9}{40}x^4 - \tfrac{3}{50}x^5\right]_0^2 \\
&= \left(\tfrac{9}{40} \times 2^4 - \tfrac{3}{50} \times 2^5\right) - 0 \\
&= \tfrac{42}{25}
\end{aligned}
$$

$$
\begin{aligned}
\sigma^2 &= \mathrm{E}(X^2) - \mu^2 \\
&= \frac{42}{25} - \left(\frac{6}{5}\right)^2 \\
&= \frac{6}{25}
\end{aligned}
$$

The probability density function for the random variable X is given by

$$f(x) = \begin{cases} \frac{1}{4}x & 0 \le x \le 2 \\ \frac{1}{4}(4 - x) & 2 \le x \le 4 \\ 0 & \text{otherwise} \end{cases}$$

Find the mean and standard deviation of X.

The probability density function for the random variable X is given by

$$f(x) = \begin{cases} \frac{1}{4}x & 0 \leq x \leq 2 \\ \frac{1}{4}(4 - x) & 2 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

Find the mean and standard deviation of X.

The probability density function for the random variable X is given by

$$f(x) = \begin{cases} \frac{1}{4}x & 0 \le x \le 2 \\ \frac{1}{4}(4 - x) & 2 \le x \le 4 \\ 0 & \text{otherwise} \end{cases}$$

Calculate $\mathrm{P}(X < \mu - \sigma)$.