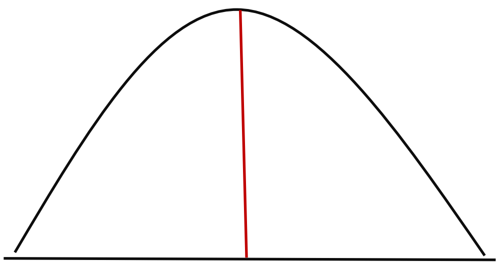
Chapter: Skewness, Moments and Kurtosis

The study of central tendency provides us the valuable information relating to the central value and measures of variation provides us the variability of the distribution. Unfortunately these measures fail to demonstrate how the data are arranged about central value of the distribution. The arrangement of data determine the characteristics of the distribution such as asymmetry and peakedness.

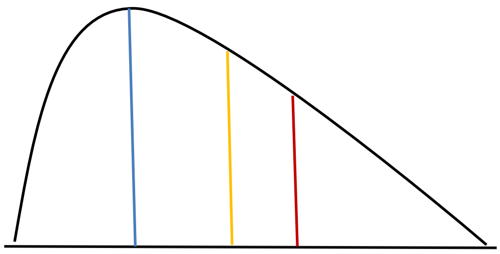
Skewness:

The term skewness refers to lack of symmetry or departure of symmetry. When a distribution is not symmetrical it is called a skewed distribution. The measures of skewness indicate the difference between the manner in which the observations are distributed in a particular distribution compared with a symmetrical distribution.

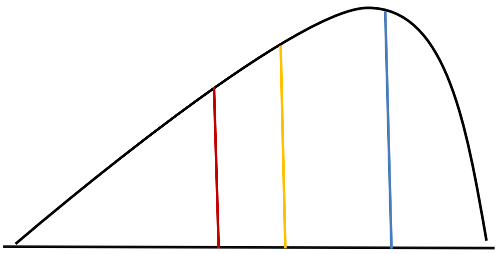
- Measures of variation tells us about the amount of the variation, Measures of skewness tells us about the direction of variation.



$\bar{x}$ = Me = Mo
(a) Symmetrical Distribution



$\bar{x}$ > Me > Mo
(b) Positively Skewed Distribution



$\bar{x}$ < Me < Mo
(c) Negatively Skewed Distribution

a) In a symmetrical distribution, the values of mean, median and mode are same.

b) In positively skewed distribution, mean is greater than mode and median lies somewhere between them.

c) In negatively skewed distribution, mode is greater than mean and median lies somewhere between them.

Measures of Skewness:

The parameter which is used to find the skewness about the data is called Karl Pearson's coefficient of skewness. It is donate by SK_p and defied by

$$SK_p = \frac{\text{Mean} - \text{Mode}}{\sigma}$$

- If $SK_p > 0$, the distribution positively skewed (Mean > Mode).
- If $SK_p < 0$, the distribution negatively skewed (Mean < Mode).

Another two parameter are used to calculate skewness. SK_B called Bowley's coefficient of skewnwss and SK_K called Kelly's coefficient of skewness. Defined as

$$SK_B = \frac{(Q_3 - Q_2) - (Q_2 - Q_1)}{Q_3 - Q_1} = \frac{Q_3 + Q_1 - 2\,Q_2}{Q_3 - Q_1}$$

- If $Q_3 - Q_2 = Q_2 - Q_1$, $SK_B = 0$ and the distribution is symmetrical.
- If $Q_3 - Q_2 > Q_2 - Q_1$, $SK_B > 0$ and the distribution is positively skewed.
- If $Q_3 - Q_2 < Q_2 - Q_1$, $SK_B < 0$ and the distribution is negatively skewed.

and

$$SK_K = \frac{P_{90} - 2\,P_{50} + P_{10}}{P_{90} - P_{10}} \quad \text{or} \quad SK_K = \frac{D_9 - 2\,D_5 + D_1}{D_9 - D_1}$$

Example: The following data relate to the profits of 1000 companies

Profit(Lakhs)	100-120	120-140	140-160	160-180	180-200	200-220	220-240
No of Companies	17	53	199	194	327	208	2

Calculate coefficient of skewness and comment on it's value.

Solution:

Profit	Mid value x_i	f_i	d_i	$f_i d_i$	$f_i d_i^2$
100-120	110	17	-3	-51	153
120-140	130	53	-2	-106	212
140-160	150	199	-1	-199	199
160-180	170	194	0	0	0
180-200	190	327	+1	+327	327
200-220	210	208	+2	+416	832
220-240	230	2	+3	+6	18
		N = 1000		$\sum f_i d_i = 393$	$\sum f_i d_i^2 = 1741$

Mean $\bar{x} = A + \frac{\sum f_i d_i}{N} \times h = 170 + \frac{393}{1000} \times 20 = 177.86$

Since highest frequency is 327 which lies in the class 180 – 200. Modal class is 180 – 200.

Mode = $L + \frac{\Delta_1}{\Delta_1 + \Delta_2} \times h = 180 + \frac{133}{133+119} \times 20 = 190.56$

Standard deviation $\sigma = \sqrt{\frac{\sum f_i d_i^2}{N} - \left(\frac{\sum f_i d_i}{N}\right)^2} \times h = \sqrt{\frac{1741}{1000} - \left(\frac{393}{1000}\right)^2} \times 20 = 25.2$

Coefficient of skewness $SK_p = \frac{\text{Mean}-\text{Mode}}{\sigma} = \frac{177.86-190.56}{25.2} = -0.504$

The mode is greater than the mean by an amount equal to about 50.4 percent of the value of standard deviation. It is a case of moderate negatively skewed distribution.

Example: An analysis of production rejects resulted in the following distribution:

No. of rejects	21-25	26-30	31-35	36-40	41-45	46-50	51-55
No of Companies	5	15	28	42	15	12	3

Calculate coefficient of skewness and comment on the result.

Solution:

No. of rejects	Mid value x_i	f_i	d_i	$f_i d_i$	$f_i d_i^2$
20.5-25.5	23	5	-3	-15	45
25.5-30.5	28	15	-2	-30	60
30.5-35.5	33	28	-1	-28	28
35.5-40.5	38	42	0	0	0
40.5-45.5	43	15	+1	+15	15
45.5-50.5	48	12	+2	+24	48
50.5-55.5	53	3	+3	+9	27
		N = 120		$\sum f_i d_i$ = -25	$\sum f_i d_i^2$ = 223

Mean $\bar{x} = A + \frac{\sum f_i d_i}{N} \times h = 38 + \frac{-25}{120} \times 5 = 36.96$

Since highest frequency is 42 which lies in the class 35.5-40.5. Modal class is 35.5-40.5.

Mode = $L + \frac{\Delta_1}{\Delta_1 + \Delta_2} \times h = 35.5 + \frac{14}{14+17} \times 5 = 37.21$

Standard deviation $\sigma = \sqrt{\frac{\sum f_i d_i^2}{N} - \left(\frac{\sum f_i d_i}{N}\right)^2} \times h = \sqrt{\frac{223}{120} - \left(\frac{-25}{120}\right)^2} \times 5 = 6.736$

Coefficient of skewness $SK_p = \frac{\text{Mean} - \text{Mode}}{\sigma} = \frac{36.96 - 37.21}{6.736} = -0.037$

The value of mean = 36.96 indicates that on an average, rejects per company were 37 in number. The value of standard deviation = 6.736 suggests that the variation in the data from the central value is approximately 7. Coefficient of skewness = -0.037 indicates that the distribution is slightly skewed to the left.

Example: An analysis of workers resulted in the following distribution:

Age	20-25	25-30	30-35	35-40	40-45	45-50	50-55
No of Employees	8	12	20	25	15	12	8

Calculate coefficient of skewness and comment on the result.

Solution:

Age	Mid value x_i	f_i	d_i	$f_i d_i$	$f_i d_i^2$
20-25	22.5	8	-3	-24	72
25-30	27.5	12	-2	-24	48
30-35	32.5	20	-1	-20	20
35-40	37.5	25	0	0	0
40-45	42.5	15	+1	+15	15
45-50	47.5	12	+2	+24	48
50-55	52.5	8	+3	+24	72
		N = 100		$\sum f_i d_i$ = -5	$\sum f_i d_i^2$ = 275

Mean $\bar{x} = A + \frac{\sum f_i d_i}{N} \times h = 37.5 + \frac{-5}{100} \times 5 = 37.25$

Since highest frequency is 25 which lies in the class 35-40. Modal class is 35-40.

Mode = $L + \frac{\Delta_1}{\Delta_1 + \Delta_2} \times h = 35 + \frac{5}{5+10} \times 5 = 36.67$

Standard deviation $\sigma = \sqrt{\frac{\sum f_i d_i^2}{N} - \left(\frac{\sum f_i d_i}{N}\right)^2} \times h = \sqrt{\frac{275}{100} - \left(\frac{-5}{100}\right)^2} \times 5 = 8.29$

Coefficient of skewness $SK_p = \frac{\text{Mean} - \text{Mode}}{\sigma} = \frac{37.25 - 36.67}{8.29} = 0.07$

Coefficient of skewness = 0.07 indicates that the distribution is positively skewed, that is mode of the distribution is less than mean.

For Practice

1. Calculate coefficient of skewness for the following distribution:

Marks	0-20	20-40	40-60	60-80	80-100
No of Students	18	22	30	20	10

ANS: $\bar{\mathbf{x}} = \mathbf{46.4}, \mathbf{Mode} = \mathbf{48.89}, \boldsymbol{\sigma} = \mathbf{24.56}, \mathbf{SK_p} = \mathbf{-0.101}$

2. An analysis of electricity consumption resulted in the following distribution:

Consumption (kw/h)	0-10	10-20	20-30	30-40	40-50
No of Users	6	25	36	20	13

Calculate coefficient of skewness and comment on the result.

ANS: $\bar{\mathbf{x}} = \mathbf{25.9}, \mathbf{Mode} = \mathbf{24.07}, \boldsymbol{\sigma} = \mathbf{10.963}, \mathbf{SK_p} = \mathbf{0.167}$

3. Calculate coefficient of skewness for the following distribution:

Class	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	11	22	30	35	21	11	6	5

ANS: $\bar{\mathbf{x}} = \mathbf{33.156}, \mathbf{Mode} = \mathbf{32.63}, \boldsymbol{\sigma} = \mathbf{17.08}, \mathbf{SK_p} = \mathbf{0.031}$

4. Calculate coefficient of skewness for the following distribution:

Class	4000-4200	4200-4400	4400-4600	4600-4800	4800-5000	5000-5200	5200-5400
Frequency	22	38	65	75	80	70	50

ANS: $\bar{\mathbf{x}} = \mathbf{4781.5}, \mathbf{Mode} = \mathbf{4866.67}, \boldsymbol{\sigma} = \mathbf{340.4}, \mathbf{SK_p} = \mathbf{-0.25}$

5. Calculate coefficient of skewness for the following distribution:

Wages (tk)	2000-2200	2200-2400	2400-2600	2600-2800	2800-3000	3000-3200	3200-3400	3400-3600
No of Workers	12	18	35	42	50	45	30	8

ANS: $\mathbf{SK_p} = \mathbf{-0.267}$

6. Calculate coefficient of skewness for the following distribution:

Scores	10-15	15-20	20-25	25-30	30-35	35-40	40-45	45-50	45-50	45-50
Frequency	2	8	6	12	7	6	4	3	1	1

ANS: $\bar{\mathbf{x}} = \mathbf{30.1}, \mathbf{Mode} = \mathbf{27.73}, \boldsymbol{\sigma} = \mathbf{10.45}, \mathbf{SK_p} = \mathbf{0.227}$

7. Calculate coefficient of skewness for the following distribution:

Overtime (hr)	10-15	15-20	20-25	25-30	30-35	35-40
No of Workers	11	20	35	20	8	6

ANS: $\bar{\mathbf{x}} = \mathbf{23.1}, \mathbf{Mode} = \mathbf{22.5}, \boldsymbol{\sigma} = \mathbf{6.4915}, \mathbf{SK_p} = \mathbf{0.0924}$

8. Calculate coefficient of skewness for the following distribution:

Wages (tk)	15-20	20-25	25-30	30-35	35-40	40-45
No of Workers	10	25	145	220	70	30

ANS: $\mathbf{SK_p} = \mathbf{-0.023}$

9. Calculate coefficient of skewness for the following distribution:

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
No of Students	5	6	11	21	35	30	22	11

ANS: $\mathbf{SK_p} = \mathbf{-0.026}$

10. Calculate coefficient of skewness for the following distribution:

Class Interval	130-134	135-139	140-144	145-149	150-154	155-159	160-164
Frequency	3	12	21	28	19	12	5

ANS: $\mathbf{SK_p} = \mathbf{-0.572}$

Moments:

A set of descriptive measures which can provide a unique characterization of a distribution and determine the distribution uniquely is called moments. Basically moments is the unification of all measures (central tendency, variation).

Moments about assumed mean (A):

Moments about assumed mean (A) is called **raw moments**. The rth raw moments about the value A is denoted by μ'_r. Defined by

For Ungrouped data $\mu'_r = \frac{\sum(x_i - A)^r}{N}$ **r = 1, 2, 3, 4**

For Grouped data $\mu'_r = \frac{\sum f_i(x_i - A)^r}{N}$ **r = 1, 2, 3, 4**

1st raw moments for grouped data is $\mu'_1 = \frac{\sum f_i(x_i - A)}{N}$

2nd raw moments for grouped data is $\mu'_2 = \frac{\sum f_i(x_i - A)^2}{N}$ etc.

Moments about mean ($\bar{x}$):

Moments about mean ($\bar{x}$) is called **central moments**. The rth central moments about the value $\bar{x}$ is denoted by μ_r. Defined by

For Ungrouped data $\mu_r = \frac{\sum(x_i - \bar{x})^r}{N}$ **r = 1, 2, 3, 4**

For Grouped data $\mu_r = \frac{\sum f_i(x_i - \bar{x})^r}{N}$ **r = 1, 2, 3, 4**

- 1st central moments for grouped data is $\mu_1 = \frac{\sum f_i(x_i - \bar{x})}{N}$. If $\bar{x} = 0$ then $\mu_1 = \frac{\sum f_i x_i}{N}$ which represents mean.
- 2nd central moments for grouped data is $\mu_2 = \frac{\sum f_i(x_i - \bar{x})^2}{N}$ which represents variance σ^2.
- Similarly 3rd central moments relates to skewness and 4th central moments relates to kurtosis.

Relation between central moments and raw moments:

We know $\mu_1 = \frac{\sum(x_i - \bar{x})}{N} = \frac{\sum x_i - \sum \bar{x}}{N}$

$$= \frac{\sum x_i}{N} - \frac{\sum \bar{x}}{N} = \bar{x} - \frac{N\bar{x}}{N} \qquad \because \sum C = NC$$

$$= \bar{x} - \bar{x} = 0$$

$$\mu_2 = \frac{\sum(x_i - \bar{x})^2}{N} = \frac{\sum\{(x_i - A) - (\bar{x} - A)\}^2}{N}$$

$$= \frac{\sum\{(x_i - A)^2 - 2(x_i - A)(\bar{x} - A) + (\bar{x} - A)^2\}}{N} \qquad \because (a-b)^2 = a^2 - 2ab + b^2$$

$$= \frac{\sum(x_i - A)^2}{N} - \frac{\sum 2(x_i - A)(\bar{x} - A)}{N} + \frac{\sum(\bar{x} - A)^2}{N}$$

$$= \frac{\sum(x_i - A)^2}{N} - 2(\bar{x} - A)\frac{\sum(x_i - A)}{N} + \frac{\sum(\bar{x} - A)^2}{N} \qquad \because \sum C\, x_i = C \sum x_i$$

$$= \mu_2' - 2\mu_1'\mu_1' + \frac{\sum(\mu_1')^2}{N}$$

$$\mu_2 = \mu_2' - {\mu_1'}^2$$

$$\because \mu_1' = \frac{\sum(x_i - A)}{N} = \frac{\sum x_i}{N} - \frac{\sum A}{N}$$

$$= \bar{x} - \frac{NA}{N} = \bar{x} - A$$

$$\mu_3 = \frac{\sum(x_i - \bar{x})^3}{N} = \frac{\sum\{(x_i - A) - (\bar{x} - A)\}^3}{N}$$

$$= \frac{\sum\{(x_i - A)^3 - 3(x_i - A)^2(\bar{x} - A) + 3(x_i - A)(\bar{x} - A)^2 - (\bar{x} - A)^3\}}{N}$$

$$\because (a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

$$= \frac{\sum(x_i - A)^3}{N} - 3(\bar{x} - A)\frac{\sum(x_i - A)^2}{N} + 3(\bar{x} - A)^2\frac{\sum(x_i - A)}{N} - \frac{\sum(\bar{x} - A)^3}{N}$$

$$= \mu_3' - 3\mu_1'\mu_2' + 3(\mu_1')^2\mu_1' - \frac{\sum(\mu_1')^3}{N}$$

$$\mu_3 = \mu_3' - 3\,\mu_2'\,\mu_1' + 2\,{\mu_1'}^3$$

$$\because \mu_1' = \frac{\sum(x_i - A)}{N} = \frac{\sum x_i}{N} - \frac{\sum A}{N}$$

$$= \bar{x} - \frac{NA}{N} = \bar{x} - A$$

$$\mu_4 = \frac{\sum(x_i - \bar{x})^4}{N} = \frac{\sum\{(x_i - A) - (\bar{x} - A)\}^4}{N}$$

$$= \frac{\sum\{(x_i - A)^4 - 4(x_i - A)^3(\bar{x} - A) + 6(x_i - A)^2(\bar{x} - A)^2 - 4(x_i - A)(\bar{x} - A)^3 + (\bar{x} - A)^4\}}{N}$$

$$\because (a-b)^4 = a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4$$

$$= \frac{\sum(x_i - A)^4}{N} - 4(\bar{x} - A)\frac{\sum(x_i - A)^3}{N} + 6(\bar{x} - A)^2\frac{\sum(x_i - A)^2}{N} - 4(\bar{x} - A)^3\frac{\sum(x_i - A)}{N} + \frac{\sum(\bar{x} - A)^4}{N}$$

$$= \mu_4' - 4\mu_1'\mu_3' + 6(\mu_1')^2\mu_2' - 4(\mu_1')^3\mu_1' - \frac{\sum(\mu_1')^4}{N}$$

$$\mu_4 = \mu_4' - 4\,\mu_3'\,\mu_1' + 6\,\mu_2'\,{\mu_1'}^2 - 3\,{\mu_1'}^4$$

$$\because \mu_1' = \frac{\sum(x_i - A)}{N} = \frac{\sum x_i}{N} - \frac{\sum A}{N}$$

$$= \bar{x} - \frac{NA}{N} = \bar{x} - A$$

Conversion of Raw moments (Grouped data):

We know $\quad d_i = \frac{x_i - A}{h} \Rightarrow x_i - A = d_i h$

1st raw moments for grouped data is $\quad \mu_1' = \frac{\sum f_i(x_i - A)}{N} = \frac{\sum f_i d_i h}{N} = \frac{\sum f_i d_i}{N} \times h$

2nd raw moments for grouped data is $\quad \mu_2' = \frac{\sum f_i(x_i - A)^2}{N} = \frac{\sum f_i {d_i}^2 h^2}{N} = \frac{\sum f_i {d_i}^2}{N} \times h^2$

3rd raw moments for grouped data is $\quad \mu_3' = \frac{\sum f_i(x_i - A)^3}{N} = \frac{\sum f_i {d_i}^3 h^3}{N} = \frac{\sum f_i {d_i}^3}{N} \times h^3$

4th raw moments for grouped data is $\quad \mu_4' = \frac{\sum f_i(x_i - A)^4}{N} = \frac{\sum f_i {d_i}^4 h^4}{N} = \frac{\sum f_i {d_i}^4}{N} \times h^4$

Raw Moments (About A)	**Central Moments (About $\bar{x}$)**
$\mu_1' = \frac{\sum f_i d_i}{N} \times h$	$\mu_1 = 0$
$\mu_2' = \frac{\sum f_i {d_i}^2}{N} \times h^2$	$\mu_2 = \mu_2' - {\mu_1'}^2$
$\mu_3' = \frac{\sum f_i {d_i}^3}{N} \times h^3$	$\mu_3 = \mu_3' - 3\,\mu_2'\,\mu_1' + 2\,{\mu_1'}^3$
$\mu_4' = \frac{\sum f_i {d_i}^4}{N} \times h^4$	$\mu_4 = \mu_4' - 4\,\mu_3'\,\mu_1' + 6\,\mu_2'\,{\mu_1'}^2 - 3\,{\mu_1'}^4$

Example: An analysis of companies resulted in the following distribution:

Profit(Lakhs)	10-20	20-30	30-40	40-50	50-60
No of Companies	18	20	30	22	10

Calculate the first four moments about assumed mean. Convert the result into moments about the mean.

Solution:

Profit (Lakhs)	Mid value x_i	f_i	d_i	$f_i d_i$	$f_i d_i^2$	$f_i d_i^3$	$f_i d_i^4$
10-20	15	18	-2	-36	72	-144	288
20-30	25	20	-1	-20	20	-20	20
30-40	35	30	0	0	0	0	0
40-50	45	22	+1	+22	22	+22	22
50-60	55	10	+2	+40	40	+80	160
		N = 100		$\sum f_i d_i$ = -14	$\sum f_i d_i^2$= 154	$\sum f_i d_i^3$= -62	$\sum f_i d_i^4$= 490

Moments about assumed mean:

$$\mu_1' = \frac{\sum f_i d_i}{N} \times h = \frac{-14}{100} \times 10 = -1.4$$

$$\mu_2' = \frac{\sum f_i d_i^2}{N} \times h^2 = \frac{154}{100} \times 10^2 = 154$$

$$\mu_3' = \frac{\sum f_i d_i^3}{N} \times h^3 = \frac{-62}{100} \times 10^3 = -620$$

$$\mu_4' = \frac{\sum f_i d_i^4}{N} \times h^4 = \frac{490}{100} \times 10^4 = 49000$$

Moments about mean:

$\mu_1 = 0$

$\mu_2 = \mu_2' - {\mu_1'}^2 = 154 - (-1.4)^2 = 152.04$

$\mu_3 = \mu_3' - 3\,\mu_2'\,\mu_1' + 2\,{\mu_1'}^3 = -620 - 3\,(154)(-1.4) + 2\,(-1.4)^3 = 21.312$

$\mu_4 = \mu_4' - 4\,\mu_3'\,\mu_1' + 6\,\mu_2'\,{\mu_1'}^2 - 3\,{\mu_1'}^4$

$\quad = 49000 - 4(-620)(-1.4) + 6(154)(-1.4)^2 - 3(-1.4)^4 = 47327.51$

Example: An analysis of companies resulted in the following distribution:

Profit(Lakhs)	70-90	90-110	110-130	130-150	150-170
No of Companies	8	11	18	9	4

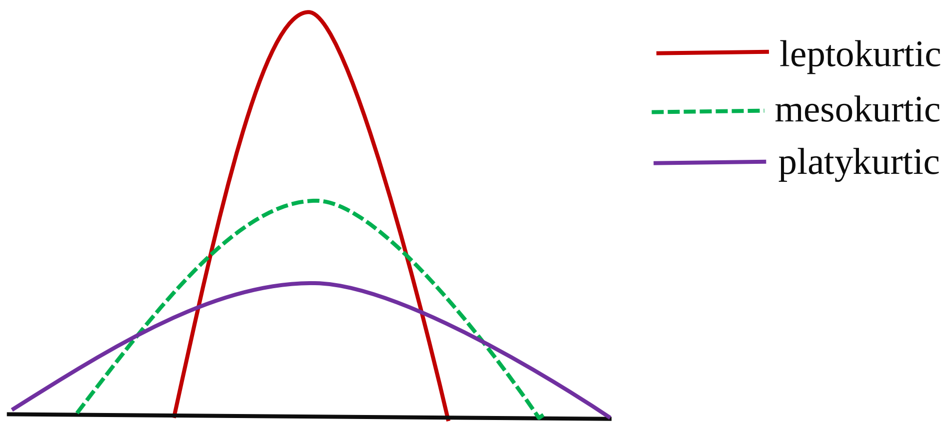
Calculate moments about the mean.

ANS: $\boldsymbol{\mu_1 = 0, \mu_2 = 528,\ \mu_3 = 960,\ \mu_4 = 642816}$

Kurtosis:

Kurtosis refers to the degree of flatness or peakedness in the region about the mode of a frequency curve. The degree of kurtosis measured the peakedness of a distribution relative to the normal curve.

- For the peakedness normal curve is called "mesokurtic".
- A curve more peaked than normal curve is called "leptokurtic".
- A curve less peaked than normal curve is called "platykurtic".



Measures of Skewness and Kurtosis using Moments:

❖ A relative measures of skewness denoted by

$$\beta_1 = \frac{{\mu_3}^2}{{\mu_2}^3}$$

Since β_1 is always positive, it will determine the magnitude of the skewness but cannot tell us about the direction of skewness. Instead of β_1, Karl Pearson suggested γ_1 to be used as a measure of skewness, where

$$\gamma_1 = \sqrt{\beta_1} = \frac{\mu_3}{{\mu_2}^{\frac{3}{2}}}$$

Obviously for symmetrical distribution $\gamma_1 = 0$, for positively and negatively skewed distribution $\gamma_1 < 0$ and $\gamma_1 > 0$ respectively.

❖ The degree of kurtosis is denoted by β_2. Defined by

$$\beta_2 = \frac{\mu_4}{{\mu_2}^2} \text{ or } \gamma_2 = \beta_2 - 3$$

If $\beta_2 > 3$, the distribution is called leptokurtic i.e. $\gamma_2 > 0$

If $\beta_2 = 3$, the distribution is called mesokurtic i.e. $\gamma_2 = 0$

If $\beta_2 < 3$, the distribution is called platykurtic i.e. $\gamma_2 < 0$

Example: An analysis of workers resulted in the following distribution:

Earnings (tk)	50-70	70-90	90-110	110-130	130-150	150-170	170-190
No of Employees	4	8	12	20	6	7	3

Calculate the first four moments about assumed mean. Convert the result into moments about the mean. Compute the value of γ_1 and γ_2 and comment on the result.

Solution:

Earnings (tk)	Mid value x_i	f_i	d_i	$f_i d_i$	$f_i {d_i}^2$	$f_i {d_i}^3$	$f_i {d_i}^4$
50-70	60	4	-3	-12	36	-108	324
70-90	80	8	-2	-16	32	-64	128
90-110	100	12	-1	-12	12	-12	12
110-130	120	20	0	0	0	0	0
130-150	140	6	+1	+6	6	+6	6
150-170	160	7	+2	+14	28	+56	112
170-190	180	3	+3	+9	27	+81	243
		N = 60		$\sum f_i d_i = -5$	$\sum f_i {d_i}^2 = 141$	$\sum f_i {d_i}^3 = -41$	$\sum f_i {d_i}^4 = 825$

Moments about assumed mean:

$$\mu_1' = \frac{\sum f_i d_i}{N} \times h = \frac{-5}{60} \times 20 = -3.67$$

$$\mu_2' = \frac{\sum f_i {d_i}^2}{N} \times h^2 = \frac{141}{60} \times 20^2 = 940$$

$$\mu_3' = \frac{\sum f_i {d_i}^3}{N} \times h^3 = \frac{-41}{60} \times 20^3 = -5466.67$$

$$\mu_4' = \frac{\sum f_i {d_i}^4}{N} \times h^4 = \frac{825}{60} \times 20^4 = 2200000$$

Moments about mean:

$\mu_1 = 0$

$\mu_2 = \mu_2' - {\mu_1'}^2 = 940 - (-3.67)^2 = 926.56$

$\mu_3 = \mu_3' - 3\,\mu_2'\,\mu_1' + 2\,{\mu_1'}^3 = -5466.67 - 3\,(940)(-3.67) + 2\,(-3.67)^3 = 4774.832$

$\mu_4 = \mu_4' - 4\,\mu_3'\,\mu_1' + 6\,\mu_2'\,{\mu_1'}^2 - 3\,{\mu_1'}^4$

$= 2200000 - 4(-5466.67)(-3.67) + 6(940)(-3.67)^2 - 3(-3.67)^4 = 2195107.3$

$$\gamma_1 = \sqrt{\beta_1} = \frac{\mu_3}{\mu_2^{\frac{3}{2}}} = 0.1693$$

$$\gamma_2 = \beta_2 - 3 = -0.44$$

The value of γ_1 indicates that the distribution is slightly skewed to the right i.e. it is not perfectly symmetrical. Since the value of γ_2 is less than zero, therefore the distribution is platykurtic.