

Probability Distribution

Binomial Distribution

- The binomial distribution is a probability distribution.
- It is used when an action is repeated and we are interested in the number of successes.
- It gives the probability for each number of successes.
- There is a formula for calculating these probabilities.

- The probability of success must be the same in each trial.
- The trials must be independent.

**What is the probability
of getting x successes
in n trials?**

What is the probability of winning 2 games out of 5?

Probability of winning a game = $\frac{1}{4}$

Number of ways

P(2 wins out of 5) = of winning 2 games \times P(SSFFF)
out of 5

$$P(SSFFF) = \left(\frac{1}{4}\right)^2 \times \left(\frac{3}{4}\right)^3$$

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Number of ways

$$\text{of winning 2 games out of 5} = 10$$

$$\text{P(2 wins out of 5)} = 10 \times \left(\frac{1}{4}\right)^2 \times \left(\frac{3}{4}\right)^3 = \frac{135}{512}$$

What is the probability of getting x successes in n trials?

Probability of success = p

Probability of failure = $1 - p$

Probability of x successes in a row
followed by $n - x$ failures in a row = $p^x \times (1 - p)^{n-x}$

Number of ways of getting
 x successes in n trials = $\binom{n}{x} = {}^nC_x$

$P(x \text{ successes out of } n) = \binom{n}{x} \times p^x \times (1 - p)^{n-x}$

n = number of trials

p = probability of success in one trial

X = number of successes

$$X \sim B(n, p)$$

$$P(X = x) = \frac{n!}{x!(n-x)!} \times p^x \times (1-p)^{(n-x)}$$

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If $X \sim B(6, \frac{2}{5})$, what is $P(X = 4)$?

$$P(X = 4) = {}^6C_4 \times \left(\frac{2}{5}\right)^4 \times \left(\frac{3}{5}\right)^2$$

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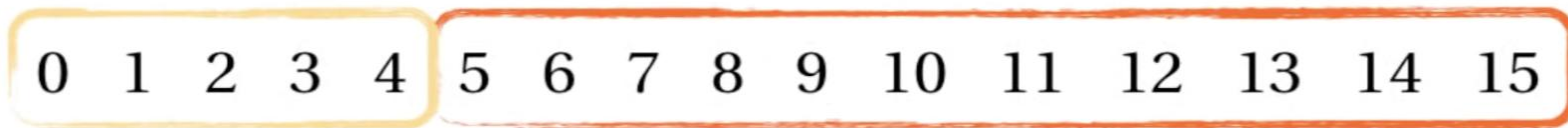
$$P(X = 4) = 15 \times \left(\frac{2}{5}\right)^4 \times \left(\frac{3}{5}\right)^2$$

Five dice are thrown. What is the probability of getting two multiples of 3?

$$X \sim \text{B}(12, 0.35)$$

$$P(X \leq 3)$$

If $X \sim B(15, 0.3)$, what is $P(X \geq 5)$?



$$P(X \geq 5) = 1 - P(X \leq 4)$$

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 $n \rightarrow \infty$, $p \rightarrow 0$, $\lambda = np$ stays
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We need to show

$$\binom{n}{x} p^x (1-p)^{n-x} \rightarrow \frac{\lambda^x e^{-\lambda}}{x!}$$

$$\binom{n}{x} p^x (1-p)^{n-x}$$

$$= \binom{n}{x} \left(\frac{\lambda}{n}\right)^x \left(1 - \frac{\lambda}{n}\right)^{n-x}$$

$$= \frac{n!}{x!(n-x)!} \left(\frac{\lambda}{n}\right)^x \left(1 - \frac{\lambda}{n}\right)^{n-x}$$

$$= \frac{\lambda^x}{x!} \frac{n!}{(n-x)! n^x} \left(1 - \frac{\lambda}{n}\right)^{n-x}$$

$$\frac{n!}{(n-x)!} \frac{1}{n^x}$$

$$= \frac{n(n-1)(n-2)\dots(n-x+1)(n-x)!}{(n-x)! n^x}$$

$$= \frac{n(n-1)(n-2)\dots(n-x+1)}{n^x}$$

$$= \frac{n}{n} \cdot \frac{n-1}{n} \cdot \frac{n-2}{n} \cdot \dots \cdot \frac{n-x+1}{n}$$

$$= 1 \left(1 - \frac{1}{n}\right) \left(1 - \frac{2}{n}\right) \dots \left(1 - \frac{x+1}{n}\right)$$

$$e^x = \lim_{n \rightarrow \infty} \left(1 + \frac{x}{n}\right)^n$$

$$\binom{n}{x} p^x (1-p)^{n-x}$$

$$= \frac{\lambda^x}{x!} \left(1 - \frac{1}{n}\right) \left(1 - \frac{2}{n}\right) \dots \left(1 - \frac{x+1}{n}\right) \left(1 - \frac{\lambda}{n}\right)^{n-x}$$

$$\frac{\lambda^x}{x!} \left(1 - \frac{1}{n}\right) \left(1 - \frac{2}{n}\right) \dots \left(1 - \frac{x+1}{n}\right) \left(1 - \frac{\lambda}{n}\right)^n \left(1 - \frac{\lambda}{n}\right)^{-x}$$

$$\lim_{n \rightarrow \infty} \binom{n}{x} p^x (1-p)^{n-x}$$

$$= \frac{\lambda^x}{x!}$$

$$\times \lim_{n \rightarrow \infty} \left(1 - \frac{1}{n}\right) \left(1 - \frac{2}{n}\right) \dots \left(1 - \frac{x+1}{n}\right)$$

$$\times \lim_{n \rightarrow \infty} \left(1 - \frac{\lambda}{n}\right)^n$$

$$\times \lim_{n \rightarrow \infty} \left(1 - \frac{\lambda}{n}\right)^{-x}$$

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