

Chapter: Measures of Central Tendency

Measures of central tendency shows the tendency of some central value around which data tends to converge. For further analysis of the tabular data, measures of central tendency represents the entire mass of data.

Objectives:

- To get one single value that describe the characteristics of the entire data.
- To easily compare the data.

Types:

Different types of central tendency are:

1. Arithmetic Mean
2. Median
3. Mode
4. Geometric Mean
5. Harmonic Mean

Arithmetic Mean:

The arithmetic mean, often simply referred to as mean, is the total of the values of a set of observations divided by their number of observations.

If $x_1, x_2, x_3, \dots, x_N$ represent the values of N items or observations, the arithmetic mean denoted by \bar{x} is defined by

$$\bar{x} = \frac{x_1 + x_2 + x_3 + \dots + x_N}{N} = \frac{\sum_{i=1}^N x_i}{N}$$

It's also written as $\bar{x} = \frac{\sum x_i}{N}$

In case of frequency distribution

$$\bar{x} = \frac{f_1 x_1 + f_2 x_2 + f_3 x_3 + \dots + f_N x_N}{f_1 + f_2 + f_3 + \dots + f_N} = \frac{\sum f_i x_i}{\sum f_i} = \frac{\sum f_i x_i}{N}$$

Where N is called total frequency.

Example: The monthly income of 10 employees working in a firm is as follows:

4487	4493	4502	4446	4475	4492	4572	4516	4468	4489
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Find the average monthly income.

Solution: The total income

$$\begin{aligned}\sum x_i &= 4487 + 4493 + 4502 + 4446 + 4475 + 4492 + 4572 + 4516 + 4468 + 4489 \\ &= 44,940\end{aligned}$$

$$\bar{x} = \frac{\sum x_i}{N} = \frac{44940}{10} = 4494$$

Hence the average monthly income is tk 4494

Example: Find the mean of the following data

Class	8	10	15	20
Frequency	5	8	8	4

Solution:

Class (x_i)	Frequency(f_i)	$f_i x_i$
8	5	40
10	8	80
15	8	120
20	4	80
	$\sum f_i = 25$	$\sum f_i x_i = 320$

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{\sum f_i x_i}{N} = \frac{320}{25} = 12.8$$

Calculating Mean Using Short-cut Method: The short-cut method is suitable for grouped data. The formula is

$$\bar{x} = A + \frac{\sum f_i d_i}{N} \times h$$

Where

h = The size of class interval.

A = The assumed mean. (It is the middle no of the mid values).

$d_i = \frac{x_i - A}{h}$ = The step deviation from A .

x_i = Mid values of each class.

N = The total frequency.

Example: Calculate mean for the following grouped data using short-cut method.

Class	0-10	10-20	20-30	30-40	40-50
frequency	7	8	20	10	5

Solution: Here $A = 25$ and $h = 10$

Class	Mid value x_i	Frequency f_i	$d_i = \frac{x_i - A}{h}$	$f_i d_i$
0-10	5	7	-2	-14
10-20	15	8	-1	-8
20-30	25 → A	20	0	0
30-40	35	10	+1	+10
40-50	45	5	+2	+10
		$N = 50$		$\sum f_i d_i = -2$

We know mean $\bar{x} = A + \frac{\sum f_i d_i}{N} \times h$

$$\bar{x} = 25 + \frac{-2}{50} \times 10 = 24.6$$

Example: Calculate mean for the following data representing the marks of statistics for 80 students in a class.

Marks	0-20	20-40	40-60	60-80	80-100	100-120	120-140
No of Student	4	26	22	10	9	6	3

Solution: Here $A = 70$ and $h = 20$

Marks	Mid value x_i	No of Student f_i	$d_i = \frac{x_i - A}{h}$	$f_i d_i$
0-20	10	4	-3	-12
20-40	30	26	-2	-52
40-60	50	22	-1	-22
60-80	70	10	0	0
80-100	90	9	+1	+9
100-120	110	6	+2	+12
120-140	130	3	+3	+9
		$N = 80$		$\sum f_i d_i = -56$

We know, Mean $\bar{x} = A + \frac{\sum f_i d_i}{N} \times h$

$$\bar{x} = 70 + \frac{-56}{80} \times 20 = 56$$

Example: Calculate the arithmetic mean of the frequency distribution given below

Height	130-134	135-139	140-144	145-149	150-154	155-159	160-164
No of Students	5	15	28	24	17	10	1

Solution: Here $A = 147$ and $h = 5$

Height	Mid value x_i	No of Students f_i	$d_i = \frac{x_i - A}{h}$	$f_i d_i$
129.5–134.5	132	5	–3	–15
134.5–139.5	137	15	–2	–30
139.5–144.5	142	28	–1	–28
144.5–149.5	147	24	0	0
149.5–154.5	152	17	+1	+17
154.5–159.5	157	10	+2	+20
159.5–164.5	162	1	+3	+3
		$N = 100$		$\sum f_i d_i = -33$

$$\text{Mean } \bar{x} = A + \frac{\sum f_i d_i}{N} \times h = 147 + \frac{-33}{100} \times 5 = 145.35$$

For Practice

1. Calculate the mean of the following data

Height(cm)	65	66	67	68	69	70	71	72	73
No of Plants	1	4	5	7	11	10	6	4	2

ANS: 69.18

2. Find the mean of the following data

Marks	No of Students
0-10	3
10-20	5
20-30	7
30-40	10
40-50	12
50-60	15
60-70	12
70-80	6
80-90	2
90-100	8

ANS: 51.75

Median:

The median is defined as the measure of middle value when set of data are arranged in ascending or descending order.

Calculation of Median (Ungrouped Data)

- First arrange them in ascending or descending order and count number of observation or items N.
- If number of observation N is odd, then $\frac{N+1}{2}$ th observation is median.
- If number of observation N is even, then median is the average of $\frac{N}{2}$ th and $\frac{N}{2} + 1$ th observation.

Example: The weights of 11 mothers in kg were recorded as follows:

47 44 42 41 58 52 55 39 40 43 61

Find the median.

Solution:

Given data in ascending order

39 40 41 42 43 44 47 52 55 58 61

Number of observation $N = 11$, which is odd number.

Median is $\frac{N+1}{2}$ th observation = $\frac{11+1}{2} = 6$ th observation.

6 th observation is 44. Therefore median is 44.

Example: Find the median of the following

20 18 22 27 25 12 15

ANS: 20

Example: The weights of 10 mothers in kg were recorded as follows:

47 44 42 41 58 55 39 40 43 61

Find the median.

Solution:

Given data in ascending order

39 40 41 42 43 44 47 55 58 61

Number of observation $N = 10$, which is even number.

Median is average of $\frac{N}{2} = \frac{10}{2} = 5$ th and $\frac{N}{2} + 1 = \frac{10}{2} + 1 = 6$ th observation.

Therefore median = $\frac{5 \text{ th observation} + 6 \text{ th observation}}{2} = \frac{43 + 44}{2} = 43.5$

Calculation of Median (Grouped Data)

For Grouped data, Median = $L + \frac{\frac{N}{2} - \text{p.c.f}}{f} \times h$

Where

h = The size of class interval.

L = Lower limit of median class. (The class where middle ($\frac{N}{2}$ th) observation lies.)

p. c. f = Preceding cumulative frequency of median class. (Cumulative frequency above median class)

f = Frequency of the median class.

Example: Calculate the median for the distribution of the weights of 150 students from the given below:

Weight	30-40	40-50	50-60	60-70	70-80	80-90
Frequency	18	37	45	27	15	8

Solution:

Weight	Frequency	Cumulative frequency
30-40	18	18
40-50	37	55 → p.c.f
L ← 50-60	45 → f	100
60-70	27	127
70-80	15	142
80-90	8	150
	N = 150	

55 – 100
observation

Median is $\frac{N}{2} = \frac{150}{2} = 75$ th observation. 75 th observation lies in class 50 – 60.

Median class is 50 – 60.

$$\text{Median} = L + \frac{\frac{N}{2} - \text{p.c.f}}{f} \times h = 50 + \frac{\frac{150}{2} - 55}{45} \times 10 = 54.44$$

Example: Following distribution gives the pattern of overtime done by 100 employee.

Calculate the median

Overtime	10-15	15-20	20-25	25-30	30-35	35-40
No of employee	11	20	35	20	8	6

Solution:

Overtime	No of employee	Cumulative frequency
10-15	11	11
15-20	20	31
20-25	35	66
25-30	20	86
30-35	8	94
35-40	6	100
	N = 100	

Median is $\frac{N}{2} = \frac{100}{2} = 50$ th observation. 50 th observation lies in class 20 – 25.

Median class is 20 – 25.

$$\text{Median} = L + \frac{\frac{N}{2} - \text{p.c.f}}{f} \times h = 20 + \frac{\frac{100}{2} - 31}{35} \times 5 = 22.714$$

Hence 50% of the workers doing overtime up to 22.714 hrs and the remaining 50% of the workers doing overtime more than 22.714 hrs.

Example: Calculate the median from the following distribution gives the profit of 125 companies:

Profit (crore)	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
No of Companies	4	12	24	36	20	16	8	3

Comment on your result.

Solution:

Profit (crore)	No of Companies	Cumulative frequency
0-10	4	4
10-20	12	16
20-30	24	40
30-40	36	76
40-50	20	96
50-60	16	112
60-70	8	120
70-80	3	125
	N = 125	

Median is $\frac{N}{2} = \frac{125}{2} = 62.5$ th observation. 62.5 th observation lies in class 30 – 40.

Median class is 30 – 40.

$$\text{Median} = L + \frac{\frac{N}{2} - \text{p.c.f}}{f} \times h = 30 + \frac{\frac{125}{2} - 40}{36} \times 10 = 36.25$$

Hence 50% of the companies have profits up to 36.5 crores and the remaining 50% of the companies have profits more than 36.5 crores.

Example: Calculate the median of the frequency distribution given below

Height	130-134	135-139	140-144	145-149	150-154	155-159	160-164
No of Students	5	15	28	24	17	10	1

Solution:

Height	No of Students f_i	Cumulative frequency
129.5–134.5	5	5
134.5–139.5	15	20
139.5–144.5	28	48
144.5–149.5	24	72
149.5–154.5	17	89
154.5–159.5	10	99
159.5–164.5	1	100
	$N = 100$	

Median is $\frac{N}{2} = \frac{100}{2} = 50$ th observation. 50 th observation lies in class 144.5–149.5.

Median class is 144.5–149.5.

$$\text{Median} = L + \frac{\frac{N}{2} - \text{p.c.f}}{f} \times h = 144.5 + \frac{\frac{100}{2} - 48}{24} \times 5 = 144.917$$

Example: Calculate the median from the following distribution

No of days absent	5	10	15	20	25	30	35	40	45
No of Students	29	195	241	117	52	10	6	3	2

Solution:

Class	No of Students	Cumulative frequency
0-5	29	29
5-10	195	224
10-15	241	465
15-20	117	582
20-25	52	634
25-30	10	644
30-35	6	650
35-40	3	653
40-45	2	655
	N = 655	

Median is $\frac{N}{2} = \frac{655}{2} = 327.5$ th observation. 327.5 th observation lies in class 10 - 15.

Median class is 10 - 15.

$$\text{Median} = L + \frac{\frac{N}{2} - \text{p.c.f}}{f} \times h = 10 + \frac{\frac{655}{2} - 224}{241} \times 5 = 12.15$$

For Practice

1. Calculate the median of the following

Marks	0-20	20-40	40-60	60-80	80-100	100-120	120-140
No of Students	4	26	22	10	9	6	3

ANS: 49.09

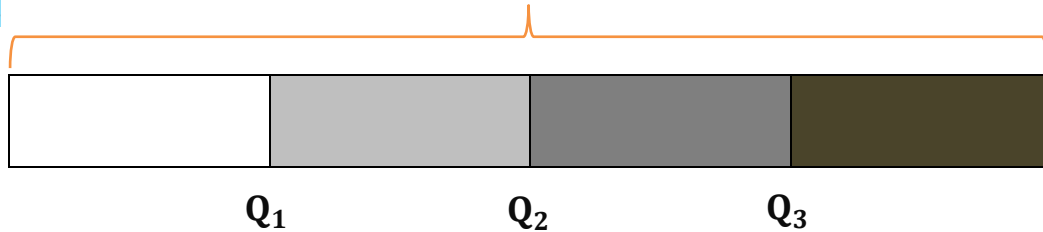
2. Find the median of the following data

Marks	No of Students
0-10	7
10-20	32
20-30	56
30-40	106
40-50	180
50-60	164
60-70	86
70-80	44

ANS: 47.58

Quartiles:

Quartiles are those values which divide the total frequency into four parts. We need three values to divide the whole frequency into four parts. That is why there are three quartile Q_1 denote first quartile, Q_2 second quartile, Q_3 third quartile. Q_2 is called the median of the frequency.



The quartiles are important in grading, rating, scoring , ranking etc.

Calculation of Quartiles (Grouped Data)

For Grouped data, quartiles $Q_i = L + \frac{\frac{i \times N}{4} - \text{p.c.f}}{f} \times h$ $i = 1, 2, 3$

Where

h = The size of class interval.

L = Lower limit of quartile class.

p. c. f = Preceding cumulative frequency of quartile class.

f = Frequency of the quartile class.

Quartile class identified by $Q_i = \frac{i \times N}{4}$ th observation.

Example: The profits earned by 100 companies are given below:

Profits (lakhs)	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
No of Companies	4	8	18	30	15	10	8	7

Calculate Q_1 , Median or Q_2 , Q_3 .

Solution:

Profits (lakhs)	No of Companies	Cumulative frequency
20-30	4	4
30-40	8	12
40-50	18	30
50-60	30	60
60-70	15	75
70-80	10	85
80-90	8	93
90-100	7	100
	N = 100	

The first quartile $Q_1 = \frac{1 \times 100}{4} = 25$ th observation. 25 th observation lies in 40 - 50.

Quartile class is 40 - 50.

We know $Q_i = L + \frac{\frac{i \times N}{4} - \text{p.c.f}}{f} \times h$

$$Q_1 = 40 + \frac{\frac{1 \times 100}{4} - 12}{18} \times 10 = 47.22$$

25% of the companies earn an annual profit of 47.22 lakhs or less.

Median or $Q_2 = \frac{2 \times 100}{4} = 50$ th observation. 50 th observation lies in 50 - 60.

Quartile class is 50 - 60.

We know $Q_i = L + \frac{\frac{i \times N}{4} - \text{p.c.f}}{f} \times h$

$$Q_2 = 50 + \frac{\frac{2 \times 100}{4} - 30}{30} \times 10 = 56.67$$

50% of the companies earn an annual profit of 56.67 lakhs or less.

The third quartile $Q_3 = \frac{3 \times 100}{4} = 75$ th observation. 75 th observation lies in 60 - 70.

Quartile class is 60 - 70.

We know $Q_i = L + \frac{\frac{i \times N}{4} - \text{p.c.f}}{f} \times h$

$$Q_3 = 60 + \frac{\frac{3 \times 100}{4} - 60}{15} \times 10 = 70$$

75% of the companies earn an annual profit of 70 lakhs or less.

Example: Following distribution gives the pattern of overtime done by 100 employee.

Calculate first quartile Q_1

Overtime	10-15	15-20	20-25	25-30	30-35	35-40
No of employee	11	20	35	20	8	6

ANS: 18.5

Deciles:

Deciles are those values which divide the total frequency into ten parts. We need nine values to divide the whole frequency into ten parts. Deciles are denoted by D_1, D_2, D_9 etc.

Calculation of Deciles (Grouped Data)

For Grouped data, deciles $D_i = L + \frac{\frac{i \times N}{10} - \text{p.c.f}}{f} \times h$ $i = 1 \text{ to } 9$

Deciles class identified by $D_i = \frac{i \times N}{10}$ th observation.

Percentiles:

Percentiles are those values which divide the total frequency into hundred parts. We need ninety nine values to divide the whole frequency into hundred parts. Percentiles are denoted by P_1, P_{22}, P_{99} etc.

Calculation of Percentiles (Grouped Data)

For Grouped data, percentiles $P_i = L + \frac{\frac{i \times N}{100} - \text{p.c.f}}{f} \times h$ $i = 1 \text{ to } 99$

Percentile class identified by $P_i = \frac{i \times N}{100}$ th observation.

Mode:

Mode is defined as the value which occurs the maximum number of times i.e. having the maximum frequency.

Calculation of Mode (Ungrouped Data)

Example: Six different observations

5 8 10 8 5 8

Find the mode.

Solution:

Since 8 has occurred maximum number of times, i.e. 3 times. So modal value is 8.

Example: Find the mode of the following

0, 1, 6, 7, 2, 3, 7, 6, 6, 2, 6, 0, 5, 6, 0

ANS: 6

Calculation of Mode (Grouped Data)

For Grouped data, $\text{Mode} = L + \frac{\Delta_1}{\Delta_1 + \Delta_2} \times h$

Where

h = The size of class interval.

L = Lower limit of modal class. (The class having maximum frequency.)

Δ_1 = Difference between the frequency of the modal class and the pre-modal class.

Δ_2 = Difference between the frequency of the modal class and the post-modal class.

Example: Calculate the mode for the distribution of the weights of 150 students from the given below:

Weight	30-40	40-50	50-60	60-70	70-80	80-90
Frequency	18	37	45	27	15	8

Solution:

Weight	Frequency	
30-40	18	
40-50	37	} $\Delta_1 = 45 - 37$
L ← 50-60	45	
60-70	27	} $\Delta_2 = 45 - 27$
70-80	15	
80-90	8	

Since highest frequency is 45 which lies in the class 50 – 60.

Modal class is 50 – 60.

$$\text{Mode} = L + \frac{\Delta_1}{\Delta_1 + \Delta_2} \times h = 50 + \frac{8}{8+18} \times 10 = 53.08$$

Example: Find the mode of the following data

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
No of Students	7	32	56	106	180	164	86	44

Solution:

Marks	No of Students
0-10	7
10-20	32
20-30	56
30-40	106
40-50	180
50-60	164
60-70	86
70-80	44

Since highest frequency is 180 which lies in the class 40 – 50.

Modal class is 40 – 50.

$$L = 40, \Delta_1 = 180 - 106 = 74, \Delta_2 = 180 - 164 = 16$$

$$\text{Mode} = L + \frac{\Delta_1}{\Delta_1 + \Delta_2} \times h = 40 + \frac{74}{74+16} \times 10 = 48.22$$

For Practice

1. Find mode of the following data relates to the sales of 100 companies:

Sales	58-60	60-62	62-64	64-66	66-68	68-70	70-72
No of Companies	12	18	25	30	10	3	2

ANS: 64.4

Geometric Mean (G.M.)

For Ungrouped data **G.M.** = A. L. $\left(\frac{\sum \log X}{N} \right)$

For Grouped data **G.M.** = A. L. $\left(\frac{\sum f \log X}{N} \right)$

A.L stands for Anti Log.

Harmonic Mean (H.M.)

For Ungrouped data **H.M.** = $\frac{N}{\sum \left(\frac{1}{X} \right)}$

For Grouped data **H.M.** = $\frac{N}{\sum \left(\frac{f}{X} \right)}$

Empirical Relation between Mean, Median, Mode

Mode = 3 Median – 2 Mean

Example: Calculate the median and mode of the frequency distribution given below.

Hence calculate the mean using empirical relation between them.

Weight	30-40	40-50	50-60	60-70	70-80	80-90
Frequency	18	37	45	27	15	8

Solution:

Weight	Frequency	Cumulative frequency
30-40	18	18
40-50	37	55
50-60	45	100
60-70	27	127
70-80	15	142
80-90	8	150
	N = 150	

Median:

Median is $\frac{N}{2} = \frac{150}{2} = 75$ th observation. 75 th observation lies in class 50 – 60. Median class is 50 – 60.

$$\text{Median} = L + \frac{\frac{N}{2} - \text{p.c.f}}{f} \times h = 50 + \frac{\frac{150}{2} - 55}{45} \times 10 = 54.44$$

Mode:

Since highest frequency is 45 which lies in the class 50 – 60. Modal class is 50 – 60.

$$\text{Mode} = L + \frac{\Delta_1}{\Delta_1 + \Delta_2} \times h = 50 + \frac{8}{8+18} \times 10 = 53.08$$

$$\text{Mode} = 3 \text{ Median} - 2 \text{ Mean}$$

$$\Rightarrow 2 \text{ Mean} = 3 \text{ Median} - \text{Mode}$$

$$\Rightarrow \text{Mean} = (3 \text{ Median} - \text{Mode})/2$$

$$\therefore \text{Mean} = \frac{3 \times 54.44 - 53.08}{2} = 55.12$$

Example: Calculate the arithmetic mean and median of the frequency distribution given below. Hence calculate the mode using empirical relation between them.

Marks	0-20	20-40	40-60	60-80	80-100	100-120	120-140
No of Student	4	26	22	10	9	6	3

Solution: Here $A = 70$ and $h = 20$

Marks	Mid value x_i	No of Student f_i	$d_i = \frac{x_i - A}{h}$	$f_i d_i$	Cumulative frequency
0-20	10	4	-3	-12	4
20-40	30	26	-2	-52	30
40-60	50	22	-1	-22	52
60-80	70	10	0	0	62
80-100	90	9	+1	+9	71
100-120	110	6	+2	+12	77
120-140	130	3	+3	+9	80
		$N = 80$		$\sum f_i d_i = -56$	

Mean:

$$\text{Mean } \bar{x} = A + \frac{\sum f_i d_i}{N} \times h = 70 + \frac{-56}{80} \times 20 = 56$$

Median:

Median is $\frac{N}{2} = \frac{80}{2} = 40$ th observation. 40 th observation lies in class 40–60. Median class is 40–60.

$$\text{Median} = L + \frac{\frac{N}{2} - \text{p.c.f}}{f} \times h = 40 + \frac{40 - 30}{22} \times 20 = 49.09$$

$$\therefore \text{Mode} = 3 \text{ Median} - 2 \text{ Mean} = (3 \times 49.09) - (2 \times 56) = 35.27$$

Example: Calculate the arithmetic mean and median of the frequency distribution given below. Hence calculate the mode using empirical relation between them.

Height	130-134	135-139	140-144	145-149	150-154	155-159	160-164
No of Students	5	15	28	24	17	10	1

Solution: Here $A = 147$ and $h = 5$

Height	Mid value x_i	No of Students f_i	$d_i = \frac{x_i - A}{h}$	$f_i d_i$	Cumulative frequency
129.5–134.5	132	5	–3	–15	5
134.5–139.5	137	15	–2	–30	20
139.5–144.5	142	28	–1	–28	48
144.5–149.5	147	24	0	0	72
149.5–154.5	152	17	+1	+17	89
154.5–159.5	157	10	+2	+20	99
159.5–164.5	162	1	+3	+3	100
		$N = 100$		$\sum f_i d_i = -33$	

Mean:

$$\text{Mean } \bar{x} = A + \frac{\sum f_i d_i}{N} \times h = 147 + \frac{-33}{100} \times 5 = 145.35$$

Median:

Median is $\frac{N}{2} = \frac{100}{2} = 50$ th observation. 50 th observation lies in class 144.5–149.5. Median class is 144.5–149.5.

$$\text{Median} = L + \frac{\frac{N}{2} - \text{p.c.f}}{f} \times h = 144.5 + \frac{\frac{100}{2} - 48}{24} \times 5 = 144.917$$

$$\begin{aligned}\therefore \text{Mode} &= 3 \text{ Median} - 2 \text{ Mean} = (3 \times 144.917) - (2 \times 145.35) \\ &= 144.051\end{aligned}$$

For Practice

1. Calculate the arithmetic mean and median of the frequency distribution given below.

Hence calculate the mode using empirical relation between them.:

Marks	20-29	30-39	40-49	50-59	60-69	70-79	80-89	90-99
No of Students	2	12	15	20	18	10	9	4

ANS: Mean = 58.5, Median = 57.5, Mode = 55.5

2. The median and mode of the following wage distribution are tk 33.5 and tk 34 respectively. However there frequencies are missing. Determine their values:

Wage	0-10	10-20	20-30	30-40	40-50	50-60	60-70	Total
Frequencies	4	16	?	?	?	6	4	230

ANS: 60, 100, 40