Motion in a Straight Line

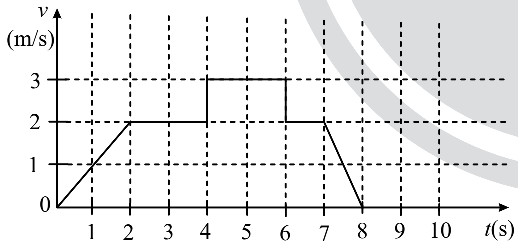
Single Correct Type Questions

1. A particle moves from the point $(2.0\hat{i} + 4.0\hat{j})\text{m}$, at $t = 0$ with an initial velocity $(5.0\hat{i} + 4.0\hat{j})\,\text{ms}^{-1}$. It is acted upon by a constant force which produces a constant acceleration $(4.0\hat{i} + 4.0\hat{j})\,\text{ms}^{-2}$. What is the distance of the particle from the origin at time 2s? **[11 Jan, 2019 (Shift-II)]**

(*a*) 15 m (*b*) $20\sqrt{2}$ m

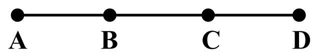
(*c*) 5 m (*d*) $10\sqrt{2}$ m

2. A particle starts from the origin at time $t = 0$ and moves along the positive x-axis. The graph of velocity with respect to time is shown in figure. What is the position of the particle at time $t = 5s$? **[10 Jan, 2019 (Shift-II)]**



(*a*) 10 m (*b*) 6 m (*c*) 3 m (*d*) 9 m

3. An object moves with speed v_1, v_2, and v_3 along a line segment AB, BC and CD respectively as shown in figure. Where AB = BC and AD = 3 AB, then average speed of the object will be: **[1 Feb, 2023 (Shift-I)]**



(*a*) $\dfrac{(v_1 + v_2 + v_3)}{3}$ (*b*) $\dfrac{v_1 v_2 v_3}{3(v_1 v_2 + v_2 v_3 + v_3 v_1)}$

(*c*) $\dfrac{3 v_1 v_2 v_3}{v_1 v_2 + v_2 v_3 + v_3 v_1}$ (*d*) $\dfrac{(v_1 + v_2 + v_3)}{3 v_1 v_2 v_3}$

4. A car travels a distance of 'x' with speed v_1 and then same distance 'x' with speed v_2 in the same direction. The average speed of the car is: **[25 Jan, 2023 (Shift-I)]**

(*a*) $\dfrac{v_1 v_2}{2(v_1 + v_2)}$

(*b*) $\dfrac{v_1 + v_2}{2}$

(*c*) $\dfrac{2x}{v_1 + v_2}$

(*d*) $\dfrac{2 v_1 v_2}{v_1 + v_2}$

5. The position of a particle related to time is given by $x = (5t^2 - 4t + 5)$m. The magnitude of velocity of the particle at $t = 2$s will be: **[15 April, 2023 (Shift-I)]**

(*a*) 10 ms^{-1} (*b*) 14 ms^{-1}

(*c*) 16 ms^{-1} (*d*) 06 ms^{-1}

6. The velocity of a particle is $v = v_0 + gt + Ft^2$. Its position is $x = 0$ at $t = 0$; then its displacement after time ($t = 1$) is: **[17 March, 2021 (Shift-II)]**

(*a*) $v_0 + g + F$

(*b*) $v_0 + \dfrac{g}{2} + \dfrac{F}{3}$

(*c*) $v_0 + 2g + 3F$

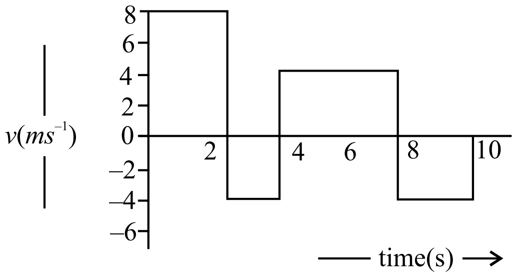
(*d*) $v_0 + \dfrac{g}{2} + F$

7. The position of a particle as a function of time ts, is given by $x(t) = at + bt^2 - ct^3$ where a, b and c are constants. When the particle attains zero acceleration, then its velocity will be: **[9 April, 2019 (Shift-II)]**

(*a*) $a + \dfrac{b^2}{4c}$ (*b*) $a + \dfrac{b^2}{c}$

(*c*) $a + \dfrac{b^2}{2c}$ (*d*) $a + \dfrac{b^2}{3c}$

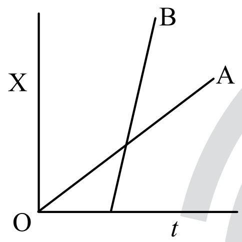
8. The velocity time graph of a body moving in a straight line is shown in figure.



The ratio of displacement to distance travelled by the body in time 0 to 10 s is **[24 Jan, 2023 (Shift-II)]**

(*a*) 1 : 1 (*b*) 1 : 4 (*c*) 1 : 2 (*d*) 1 : 3

9. The position-time graphs for two students *A* and *B* returning from the school to their homes are shown in figure:



(A) A lives closer to the school

(B) B lives closer to the school

(C) A takes lesser time to reach home

(D) A travels faster than B

(E) B travels faster than A **[10 April, 2023 (Shift-1)]**

Choose the correct answer from the options given below:

(*a*) (A) and (E) only (*b*) (B) and (E) only

(*c*) (A), (C) and (E) only (*d*) (A), (C) and (D) only

10. An engine of a train, moving with uniform acceleration, passes the signal - post with velocity u and the last compartment with velocity v. The velocity with which middle point of the train passes the signal post is :

[25 Feb, 2021 (Shift-I)]

(*a*) $\frac{u+v}{2}$

(*b*) $\sqrt{\frac{v^2+u^2}{2}}$

(*c*) $\frac{v-u}{2}$

(*d*) $\sqrt{\frac{v^2-u^2}{2}}$

11. In a car race on straight road, car *A* takes a time '*t*' less than car *B* at the finish and passes finishing point with a speed '*V*' more than that of car *B*. Both the cars start from rest and travel with constant acceleration a_1 and a_2 respectively. Then '*v*' is equal to **[9 Jan, 2019 (Shift-II)]**

(*a*) $\frac{2a_1a_1}{a_1+a_2}t$ (*b*) $\sqrt{2a_1a_2}t$

(*c*) $\sqrt{a_1a_2}\,t$ (*d*) $\frac{a_1+a_2}{2}t$

12. A ball is thrown vertically upward with an initial velocity of 150 m/s. The ratio of velocity after 3s and 5s is $\frac{x+1}{x}$.

The value of x is ________ . **[12 April, 2023 (Shift-1)]**

Take ($g = 10$ m/s^2).

(*a*) 6 (*b*) 5

(*c*) –5 (*d*) 10

13. A juggler throws ball vertically upwards with same initial velocity in air. When the first ball reaches its highest positions, he throws the next ball. Assuming the juggler throws n balls per second, the maximum height the balls can reach is **[29 July, 2022 (Shift-II)]**

(*a*) $g/2n$ (*b*) $2\,gn$

(*c*) g/n (*d*) $g/2n^2$

14. An object of mass 5 kg is thrown vertically upwards from the ground. The air resistance produces a constant retarding force of 10 N throughout the motion. The ratio of time of ascent to the time of descent will be equal to: [Use $g = 10$ ms^{-2}]. **[24 June, 2022 (Shift-II)]**

(*a*) 1 : 1 (*b*) $\sqrt{2}:\sqrt{3}$

(*c*) $\sqrt{3}:\sqrt{2}$ (*d*) 2 : 3

15. A balloon was moving upwards with a uniform velocity of 10 m/s. An object of finite mass is dropped from the balloon when it was at a height of 75m from the ground level. The height of the balloon from the ground when object strikes the ground was around: (takes the value of g as 10 m/s^2) **[25 July, 2021 (Shift-II)]**

(*a*) 250 m (*b*) 300 m

(*c*) 200 m (*d*) 125 m

16. A stone is dropped from the top of a building. When it crosses a point 5m below the top, another stone starts to fall from a point 25m below the top. Both stones reach the bottom of building simultaneously. The height of the building is: **[25 Feb, 2021 (Shift-II)]**

(*a*) 35 m (*b*) 45 m

(*c*) 50 m (*d*) 25 m

17. A ball is thrown up with a certain velocity so that it reaches a height 'h'. Find the ratio of the two different times of the ball reaching $h/3$ in both the directions:

[27 July, 2021 (Shift-I)]

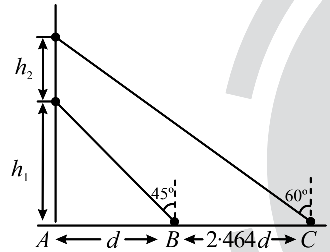
(*a*) $\dfrac{\sqrt{2}-1}{\sqrt{2}+1}$

(*b*) $\dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

(*c*) $\dfrac{\sqrt{3}-1}{\sqrt{3}+1}$

(*d*) $\dfrac{1}{3}$

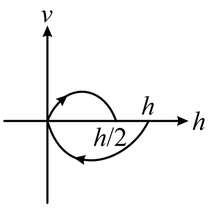
18. A balloon is moving up in air vertically above a point A on the ground. When it is at a height h_1, a girl standing at a distance d (point B) from A (see figure) sees it at an angle 45° with respect to the vertical. When the balloon climbs up a further height h_2, it is seen at an angle 60° with respect to the vertical if the girl moves further by a distance 2.464 d (point C). Then the height h_2 is (given tan 30° = 0.5774) **[5 Sep, 2020 (Shift-I)]**

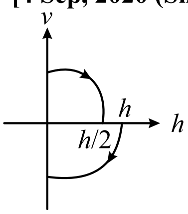


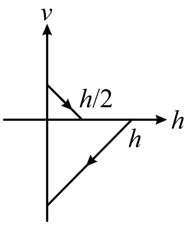
(*a*) d
(*b*) 0.732 d
(*c*) 1.464 d
(*d*) 0.464 d

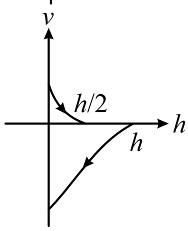
19. A tennis ball is released from a height h and after freely falling on a wooden floor it rebounds and reaches height $h/2$. The velocity versus height of the ball during its motion may be represented graphically by (graph are drawn schematically and on not to scale).

[4 Sep, 2020 (Shift-I)]

(*a*) 

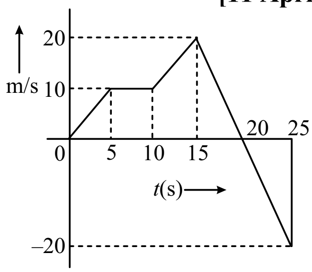
(*b*) 

(*c*) 

(*d*) 

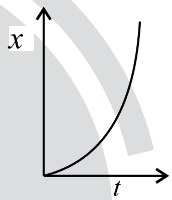
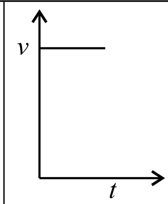
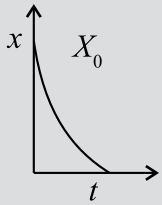
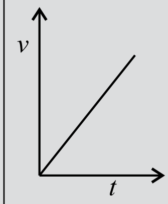
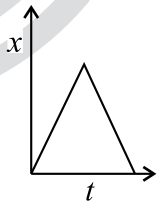
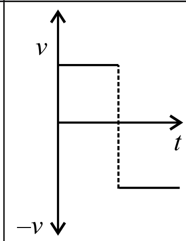
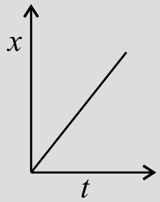
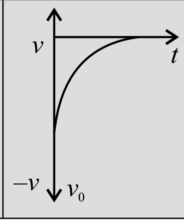
20. From the $v-t$ graph shown. the ratio of distance to displacement in 25 s of motion

[11 April, 2023 (Shift-I)]



(*a*) $\dfrac{3}{5}$
(*b*) $\dfrac{1}{2}$
(*c*) $\dfrac{5}{3}$
(*d*) 1

21. Match Column-I with Column-II:

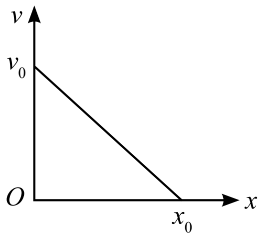
Column-I (x-t graphs)		Column-II (v-t graphs)	
A.		I.	
B.		II.	
C.		III.	
D.		IV.	

Choose the correct answer from the options given below:

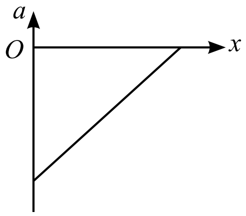
[30 Jan, 2023 (Shift-I)]

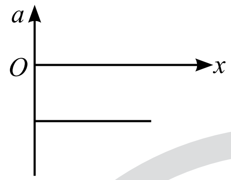
(*a*) A- II, B-IV, C-III, D-I
(*b*) A- I, B-II, C-III, D-IV
(*c*) A- II, B-III, C-IV, D-I
(*d*) A- I, B-III, C-IV, D-II

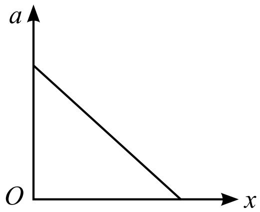
22. The velocity - displacement graph of a particle is shown in the figure. **[18 March, 2021 (Shift-II)]**

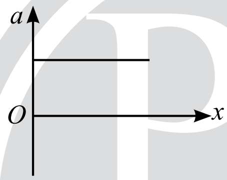


The acceleration - displacement graph of the same particle is represented by:

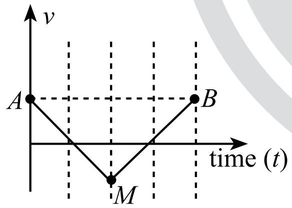
(*a*)


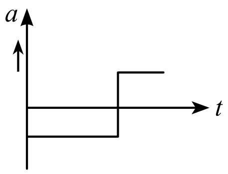
(*b*)


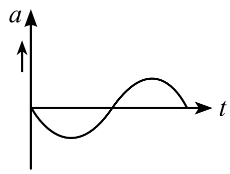
(*c*)


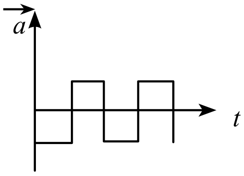
(*d*)


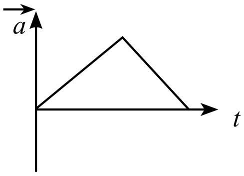
23. If the velocity-time graph has the shape AMB, what would be the shape of the corresponding acceleration-time graph? **[24 Feb, 2021 (Shift-I)]**



(*a*)


(*b*)


(*c*)


(*d*)


24. The relation between time t and distance x for a moving body is given as $t = mx^2 + nx$, where m and n are constants. The retardation of the motion is: (Where v stands for velocity) **[25 July, 2021 (Shift-II)]**

(*a*) $2n^2v^2$ (*b*) $2mnv^3$ (*c*) $2mv^3$ (*d*) $2nv^3$

Integer Type Questions

25. A ball is thrown vertically upwards with a velocity of 19.6 ms^{-1} from the top of a tower. The ball strikes the ground after 6 s. The height from the ground up to which the ball can rise will be $\left(\frac{k}{5}\right)m$. The value of k is _______

(use $g = 9.8$ m/s^2) **[28 July, 2022 (Shift-II)]**

26. A tennis ball is dropped on to the floor from a height of 9.8m. It rebounds to a height 5.0 m. Ball comes in contact with the floor for 0.2 s. The average acceleration during contact is _________ ms^{-2} [Given g = 10 ms^{-1}]

[29 Jan, 2023 (Shift-I)]

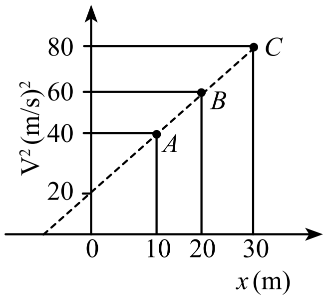
27. For a train engine moving with speed of 20 ms^{-1}. the driver must apply brakes at a distance of 500m before the station for the train to come to rest at the station. If the brakes were applied at half of this distance, the train engine would cross the station with speed $\sqrt{x}ms^{-1}$. The value of x is ________.

(Assuming same retardation is produced by brakes)

[1 Feb, 2023 (Shift-II)]

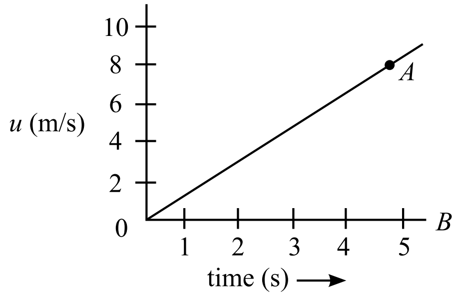
28. A particle is moving with constant acceleration 'a'. Following graph shows v^2 versus x (displacement) plot. The acceleration of the particle is _____m/s^2.

[31 Aug, 2021 (Shift-II)]



29. The speed versus time graph for a particle is shown in the figure. The distance travelled (in m) by the particle during the time interval $t = 0$ to $t = 5$ s will be __________.

[3 Sep, 2020 (Shift-II)]



30. The distance x covered by a particle in one dimensional motion varies with time t as $x^2 = at^2 + bt + c$. If the acceleration of the particle depends on x as x^{-n}, where n is an integer, the value of n is __________.

[9 Jan, 2020 (Shift-I)]

ANSWER KEY

1. (*b*)	**2.** (*d*)	**3.** (*c*)	**4.** (*d*)	**5.** (*c*)	**6.** (*b*)	**7.** (*d*)	**8.** (*d*)	**9.** (*a*)	**10.** (*b*)
11. (*c*)	**12.** (*b*)	**13.** (*d*)	**14.** (*b*)	**15.** (*d*)	**16.** (*b*)	**17.** (*b*)	**18.** (*a*)	**19.** (*b*)	**20.** (*c*)
21. (*a*)	**22.** (*a*)	**23.** (*a*)	**24.** (*c*)	**25.** [392]	**26.** [120]	**27.** [200]	**28.** [1]	**29.** [20]	**30.** [3]

EXPLANATIONS

1. (*b*) $\vec{s} = \left(5\hat{i}+4\hat{j}\right)2+\frac{1}{2}\left(4\hat{i}+4\hat{j}\right)4 \ = 10\hat{i}+8\hat{j}+8\hat{i}+8\hat{j}$

$\vec{r}_2 - \vec{r}_1 = 18\hat{i}+16\hat{j}$

$\vec{r}_2 = 20\hat{i}+20\hat{j}$

$|\vec{r}_f| = 20\sqrt{2}$

2. (*d*) $r_{t=5}$ = area

$= \left(\frac{1}{2}\times 2\times 2+2\times 2+3\times 1\right)$m

= (2 + 4 + 3) = 9 m.

3. (*c*) Given AB = x and BC = x

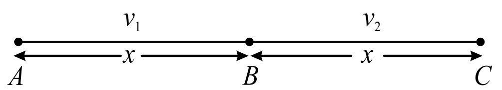
$\because$ AB + BC + CD = 3x

$\Rightarrow$ 2x + CD = 3x

$\therefore$ CD = x

Average speed $< v > = \dfrac{3x}{\frac{x}{v_1}+\frac{x}{v_2}+\frac{x}{v_3}} = \dfrac{3v_1v_2v_3}{v_2v_3+v_1v_3+v_1v_2}$

4. (*d*)



Average velocity $= \dfrac{\text{Total displacement of the car}}{\text{Total time}}$

$= \dfrac{x+x}{\frac{x}{v_1}+\frac{x}{v_2}}$

$= \dfrac{2v_1v_2}{v_1+v_2}$

5. (*c*) $x = 5t^2 - 4t + 5 \Rightarrow v = \dfrac{dx}{dt} = \dfrac{d}{dt}(5t^2-4t+5)$

$= 10t - 4$

At t = 2s and v = 16m/s

6. (*b*) $V = V_0 + gt + Ft^2$

$\dfrac{dx}{dt} = V_0 + gt + Ft^2 \Rightarrow \int_{x=0}^{x} dx = \int_{t=0}^{t=1}\left(V_0+gt+Ft^2\right)dt$

$x = \left[V_0 t + \dfrac{gt^2}{2} + \dfrac{Ft^3}{3}\right]_{t=0}^{t=1} \Rightarrow x = V_0 + \dfrac{g}{2} + \dfrac{F}{3}$

7. (*d*) $x = at + bt^2 - ct^3$

$v = \dfrac{dx}{dt} = a + 2bt - 3ct^2$

$a = \dfrac{dv}{dt} = 2b - 6ct = 0 \Rightarrow t = \dfrac{b}{3c}$

$v_{\left(at\, t=\frac{b}{3c}\right)} = a + 2b\left(\dfrac{b}{3c}\right) - 3c\left(\dfrac{b}{3c}\right)^2$

$= a + \dfrac{b^2}{3c}$

8. (*d*) Displacement of the body = 16 – 8 + 16 – 8 = 16m

Distance travelled by the body = Σ |area| = 48 m

$\Rightarrow \dfrac{\text{displacement}}{\text{Distance}} = \dfrac{1}{3}$

9. (*a*) As slope of B > Slope of A

$\therefore$ Speed of B > Speed of A

$\Rightarrow t_B < t_A$

10. (*b*) Let the length of train be l and its acceleration be a.

From kinematic equation.

$\Rightarrow al = \dfrac{v^2 - u^2}{2}$

Velocity when middle point crosses the post,

$V_m = \sqrt{u^2 + 2a\dfrac{l}{2}}. = \sqrt{u^2 + \dfrac{v^2-u^2}{2}} = \sqrt{\dfrac{u^2+v^2}{2}}$

11. (*c*) $\sqrt{\frac{2\ell}{a_2}}-\sqrt{\frac{2\ell}{a_1}}=t \Rightarrow \frac{\sqrt{2\ell}}{t}=\frac{\sqrt{a_1a_2}}{\sqrt{a_1}-\sqrt{a_2}}$

$$\sqrt{2a_1\ell}-\sqrt{2a_2\ell}=\nu \Rightarrow \frac{\sqrt{2\ell}}{v}=\frac{1}{\sqrt{a_1}-\sqrt{a_2}}$$

$$\Rightarrow \frac{\nu}{t}=\sqrt{a_1a_2} \Rightarrow \nu=\left(\sqrt{a_1a_2}\right)t$$

12. (*b*) Given, $u = 150$ m/s

From kinematic equations, $\vec{v}=\vec{u}+\vec{a}t$

$V = 150 - 10t$

At, $t = 3$ seconds

$V_1 = 150 - 30 = 120$

At, $t = 5$ seconds

$V_2 = 150 - 50 = 100$

$$\frac{120}{100}=\frac{x+1}{x} \Rightarrow x=5$$

13. (*d*) Time to reach at max height, $t=\frac{u}{g}$

No. of balls thrown in 1 sec = n.

So, time taken by each ball $=\frac{1}{n}$ sec

i.e. $\frac{u}{g}=\frac{1}{n} \Rightarrow u=\frac{g}{n}$

So, $h_{\max}=\frac{u^2}{2g}=\frac{g^2}{2gn^2}=\frac{g}{2n^2}$

14. (*b*) For time of ascent,

$$a=\frac{mg+10}{g}=\frac{5\times10+10}{10}=6\text{m}/\sec^2$$

From formula, $v = u + at$

$0 = u - 6t$

$$t_A=\frac{u}{6} \qquad ...(i)$$

$$h=ut+\frac{1}{2}at^2$$

$$=u\times\frac{u}{6}-\frac{1}{2}\times6\times\frac{u}{6}\times\frac{u}{6}$$

$$=\frac{u^2}{6}-\frac{u^2}{12}=\frac{u^2}{12} \qquad ...(ii)$$

For time of descent,

$$a=\frac{50-10}{10}=4\text{m}/\sec^2$$

Using formula, $h=ut+\frac{1}{2}at^2$

$$\Rightarrow \frac{u^2}{12}=0\times t+\frac{1}{2}\times4t^2 \qquad \text{[From equation (ii)]}$$

$$\Rightarrow \frac{u^2}{24}=t^2$$

$$t_B=\frac{u}{\sqrt{24}} \qquad ...(iii)$$

On dividing equation (*i*) with equation (*iii*)

$$\frac{t_A}{t_B}=\frac{u\times\sqrt{24}}{6\times u}=\frac{\sqrt{2}}{\sqrt{3}}$$

15. (*d*) Let time taken by the object to reach the ground be t.

Therefore, $s=ut+\frac{1}{2}at^2$

$$-75=10t-\frac{1}{2}gt^2$$

$5t^2 - 10t - 75 = 0$

$t = 5$ sec.

Distance covered by balloon in 5 sec,

$h = 10 \times 5 = 50$m

Total height of balloon = 75 + 50 = 125m

16. (*b*) 5m below, velocity of first stone, $v=\sqrt{2gs}=10m/s$

$$\therefore x+20=10t+\frac{1}{2}gt^2$$

For second stone,

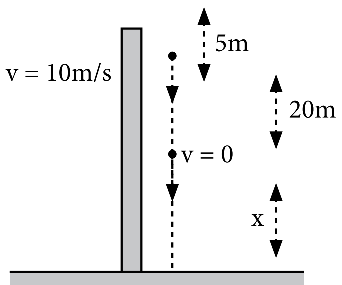
$$x=\frac{1}{2}gt^2$$

On solving these two equation, we get

$t = 2$sec

$$\therefore x=\frac{1}{2}\times10\times4=20\text{m}$$

$H = 20 + 5 = 45$m.



17. (*b*) Speed of ball to reach a height h, $u=\sqrt{2gh}$

Given, $S=\frac{h}{3}$

Using kinematic equation, $S=ut+\frac{1}{2}at^2$

$$\frac{h}{3}=\sqrt{2gh}t-\frac{1}{2}gt^2$$

$$0=t^2\left(\frac{g}{2}\right)-\sqrt{2gh}t+\frac{h}{3}$$

$$t=\frac{\sqrt{2gh}t\pm\sqrt{2gh-4\times\frac{g}{2}\times\frac{h}{3}}}{2\times\frac{g}{2}}$$

$$\frac{t_1}{t_2}=\frac{\sqrt{2gh}-\sqrt{\frac{4gh}{3}}}{\sqrt{2gh}+\sqrt{\frac{4gh}{3}}}$$

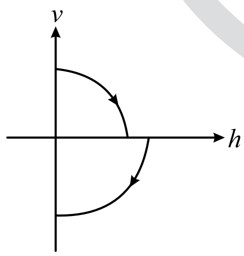
$$\frac{t_1}{t_2}=\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$$

18. (*a*) $\frac{h_1}{d}=\tan 45^\circ$

$$\frac{h_1+h_2}{d+2.464d}=\tan 30^\circ$$

$\Rightarrow h_2=d$

19. (*b*) At $H=h$, $v=0$



At $h=0$, $v=\sqrt{2gh}$

Also $a=-g$, throughout this motion

20. (*c*) Area under the graph from $t=0$ to $t=20$ sec

$$A_1=\frac{1}{2}\times5\times10+(5\times10)+\frac{1}{2}\times(10+20)\times5+\frac{1}{2}\times5\times20=200m^2$$

Area under the graph from $t=20$ to $t=25$ sec

$$A_2=\frac{1}{2}\times5\times(20)=50m^2$$

So distance covered $=A_1+A_2=(200+50)\text{m}=250$ m

Displacement $=A_1-A_2=(200-50)\text{m}=150$ m

21. (*a*) $\frac{dx}{dt}=$ slope ≥ 0 always increasing

$(A-\text{II})$

$\frac{dx}{dt}<0$; and at $t\to\infty\,\frac{dx}{dt}\to 0$

$(B-\text{IV})$

$\frac{dx}{dt}>0$ for first half $\frac{dx}{dt}<0$ for second half.

$(C-\text{III})$

$\frac{dx}{dt}=$ constant

$(D-\text{I})$

22. (*a*) The slope of the given v versus x graph is $m=-\frac{v_0}{x_0}$ and intercept is $c=+v_0$. Hence, varies with x as

$$v=-\left(\frac{v_0}{x_0}\right)x+v_0 \qquad \ldots(i)$$

Using equation (*i*) in equation (*ii*), we get

$$a=v\frac{dv}{dx}-\left(\frac{v_0}{x_0}\right)\left(-\frac{v_0}{x_0}x+v_0\right)\Rightarrow a=\left(\frac{v_0}{x_0}\right)^2x-\frac{v_0^2}{x_0}$$

Thus the graph of a versus x is a straight line having a positive slope $=\left(\frac{v_0}{x_0}\right)^2$ and negative intercept $=-\frac{v_0^2}{x_0}$.

23. (*a*) Slope of $v-t$ graph is constant but negative for part AM. Slope of $v-t$ graph is constant but positive for part MB. Hence, option (*a*) is the correct slope of the acceleration- time graph.

24. (*c*) Given, $t=mx^2+nx$

$$\frac{dt}{dx}=2mx+n\Rightarrow\frac{dx}{dt}=\frac{1}{2mx+n}$$

$$v=\frac{1}{2mx+n}$$

Retardation of the motion.

$$a=v\frac{dv}{dx}=v\frac{-2m}{(2mx+n)^2}=-2mv^3$$

25. [392] Applying, $s=ut+\frac{1}{2}at^2$

$$h=19.6\times2+\frac{1}{2}\times9.8\times4=58.8\text{m}$$

Height above tower

$$h'=\frac{u^2}{2g}=\frac{19.6\times19.6}{2\times9.8}=19.6$$

$\therefore H_{\text{(from ground)}} = 19.6 + 58.8 = 78.4\text{m}$

As, $\frac{k}{5} = 78.4 \Rightarrow k = 392$

26. [120] Velocity of the ball just before reaching the ground,

$v_i = \sqrt{2gh_i}$

$= 14\,m/s$

Velocity of the ball just after the rebound,

$v_f = \sqrt{2gh_f}$

$= 10\,m/s$

$\because \vec{a}_{Qvg} = \frac{\vec{v}_f - \vec{v}_i}{t}$

$\Rightarrow |\vec{a}_{avg}| = \left|\frac{\Delta\vec{v}}{\Delta t}\right| = 120\,m/s^2$

27.[200] $u = 20\,m/s, S_1 = 500\,m, v = 0$

Using third equation of motion

$0 = (20)^2 - 2a.500$

$\Rightarrow a = \frac{4}{10} m/s^2$

After brakes are applied

$u = 20\,m/s, S_2 = 250\,m, v = ?$

$v^2 = (20)^2 - 2a.250$

$= v = \sqrt{200}\,m/s$

$\therefore x = 200$

28. [1] As, $y = mx + c$

$v^2 = 2x + 20$

$v^2 = 2x + 20$

$\Rightarrow 2v\frac{dv}{dt} = 2v$

$\Rightarrow a = 1\text{ m/s}^2$

29. [20] Distance travelled = Area under the u-t graph

$\therefore \Delta S = \frac{1}{2} \times 5 \times 8 = 20$

30. [3] We have given, the distance covered by particle varies with time t, $x^2 = at^2 + 2bt + c$

$2xv = 2at + 2b$

$xv = at + b$

$v^2 + ax = a$

$ax = a - \left(\frac{at+b}{x}\right)^2$

$a = \frac{a\left(at^2 + 2bt + c\right) - (at+b)^2}{x^3}$

$a = \frac{ac - b^2}{x^3}; a \propto x^{-3}$

Hence, value of n is 3.

Vector and Calculus

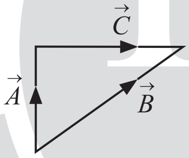
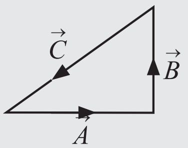
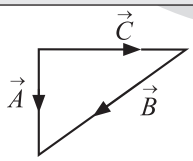
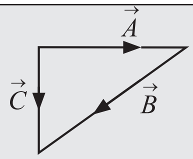
Single Correct Type Questions

1. When vector $\vec{A} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ is subtracted from vector $\vec{B}$, it gives a vector equal to $2\hat{j}$. Then the magnitude of vector $\vec{B}$ will be: **[11 April, 2023 (Shift-II)]**

(*a*) $\sqrt{13}$ (*b*) $\sqrt{33}$

(*c*) $\sqrt{6}$ (*d*) $\sqrt{5}$

2. Match List-I with List-II.

List-I		List-II	
A.	$\vec{C} - \vec{A} - \vec{B} = 0$	I.	
B.	$\vec{A} - \vec{C} - \vec{B} = 0$	II.	
C.	$\vec{B} - \vec{A} - \vec{C} = 0$	III.	
D.	$\vec{A} + \vec{B} = -\vec{C}$	IV.	

Choose the correct answer from the options given below: **[25 July, 2021 (Shift-I)]**

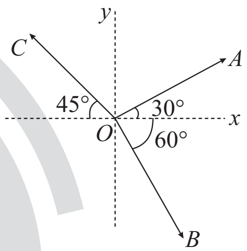
(*a*) A-IV, B-I, C-III, D-II

(*b*) A-IV, B-III, C-I, D-II

(*c*) A-I, B-IV, C-II, D-III

(*d*) A-III, B-II, C-IV, D-I

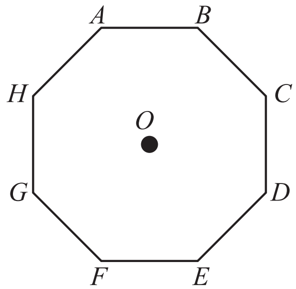
3. The magnitude of vectors $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC}$ in the given figure are equal. The direction of $\overrightarrow{OA}, + \overrightarrow{OB} - \overrightarrow{OC}$ with x-axis will be **[26 Aug, 2021 (Shift-I)]**



(*a*) $\tan^{-1}\dfrac{(\sqrt{3}-1+\sqrt{2})}{(1-\sqrt{3}+\sqrt{2})}$ (*b*) $\tan^{-1}\dfrac{(1+\sqrt{3}-\sqrt{2})}{(1-\sqrt{3}-\sqrt{2})}$

(*c*) $\tan^{-1}\dfrac{(1-\sqrt{3}-\sqrt{2})}{(1+\sqrt{3}+\sqrt{2})}$ (*d*) $\tan^{-1}\dfrac{(\sqrt{3}-1+\sqrt{2})}{(1+\sqrt{3}-\sqrt{2})}$

4. In an octagon $ABCDEFGH$ of equal side, what is the sum of $\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{AE} + \overrightarrow{AF} + \overrightarrow{AG} + \overrightarrow{AH}$, if, $\overrightarrow{AO} = 2\hat{i} + 3\hat{j} - 4\hat{k}$ **[25 Feb, 2021 (Shift-I)]**



(*a*) $16\hat{i} + 24\hat{j} - 32\hat{k}$

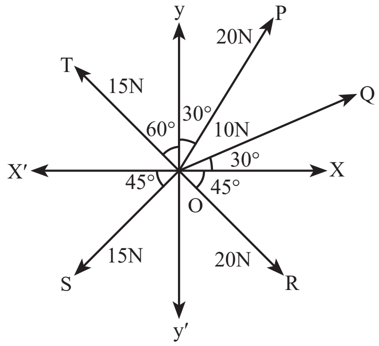
(*b*) $16\hat{i} - 24\hat{j} + 32\hat{k}$

(*c*) $-16\hat{i} - 24\hat{j} + 32\hat{k}$

(*d*) $16\hat{i} + 24\hat{j} - 32\hat{k}$

5. The resultant of these forces $\overrightarrow{OP}, \overrightarrow{OQ}, \overrightarrow{OR}, \overrightarrow{OS}$ and $\overrightarrow{OT}$ is approximately ...... N. **[27 Aug, 2021 (Shift-I)]**

[Take $\sqrt{3} = 1.7, \sqrt{2} = 1.4$ .Given $\hat{i}$ and $\hat{j}$ unit vectors along x, y axis]



(*a*) $9.25\hat{i} + 5\hat{j}$ (*b*) $2.5\hat{i} - 14.5\hat{j}$

(*c*) $-1.5\hat{i} - 15.5\hat{j}$ (*d*) $3\hat{i} + 15\hat{j}$

6. Two forces having magnitude A and $\frac{A}{2}$ are perpendicular to each other. The magnitude of their resultant is **[8 April, 2023 (Shift-I)]**

(*a*) $\frac{\sqrt{5}A}{4}$ (*b*) $\frac{5A}{2}$

(*c*) $\frac{\sqrt{5}A^2}{2}$ (*d*) $\frac{\sqrt{5}A}{2}$

7. A vector in x–y plane makes an angle of 30° with y-axis. The magnitude of y-component of vector is $2\sqrt{3}$. The magnitude of x-component of the vector will be: **[15 April, 2023 (Shift-I)]**

(*a*) $\frac{1}{\sqrt{3}}$ (*b*) 6

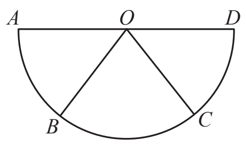
(*c*) $\sqrt{3}$ (*d*) 2

8. **Assertion A:** If A, B, C and D are four points on a semicircular arc with centre at 'O' such that $|\overrightarrow{AB}| = |\overrightarrow{BC}| = |\overrightarrow{CD}|$, then

$$\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} = 4\overrightarrow{AO} + \overrightarrow{OB} + \overrightarrow{OC}$$

Reason R: Polygon law of vector addition yields

$$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} = \overrightarrow{AD} = 2\overrightarrow{AO}$$



In the light of the above statements, choose the most appropriate answer from the options given below: **[27 July, 2021 (Shift-I)]**

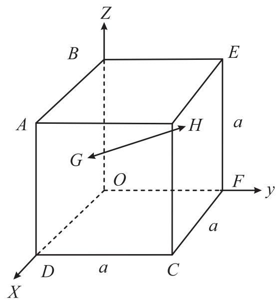
(*a*) **A** is not correct but **R** is correct.

(*b*) **A** is correct but **R** is not correct.

(*c*) Both **A** and **R** are correct and **R** is the correct explanation of **A**.

(*d*) Both **A** and **R** are correct but **R** is not the correct explanation of **A**.

9. In the cube of side 'a' shown in the figure, the vector from the central point of the face $ABOD$ to the central point of the face $BEFO$ will be **[10 Jan, 2019 (Shift-I)]**



(*a*) $\frac{1}{2}a(\hat{k} - \hat{i})$ (*b*) $\frac{1}{2}a(\hat{i} - \hat{k})$

(*c*) $\frac{1}{2}a(\hat{j} - \hat{i})$ (*d*) $\frac{1}{2}a(\hat{j} - \hat{k})$

10. If two vectors $\vec{P} = \hat{i} + 2m\hat{j} + m\hat{k}$ and $\vec{Q} = 4\hat{i} - 2\hat{j} + m\hat{k}$ are perpendicular to each other. Then, the value of m will be: **[24 Jan, 2023 (Shift-II)]**

(*a*) 1 (*b*) –1

(*c*) –3 (*d*) 2

11. Which of the following relations is true for two unit vector $\hat{A}$ and $\hat{B}$ making an angle to θ each other? **[25 June, 2022 (Shift-I)]**

(*a*) $|\hat{A} + \hat{B}| = |\hat{A} - \hat{B}| \tan\frac{\theta}{2}$

(*b*) $|\hat{A} - \hat{B}| = |\hat{A} + \hat{B}| \tan\frac{\theta}{2}$

(*c*) $|\hat{A} + \hat{B}| = |\hat{A} - \hat{B}| \cos\frac{\theta}{2}$

(*d*) $|\hat{A} - \hat{B}| = |\hat{A} + \hat{B}| \cos\frac{\theta}{2}$

12. Two vectors $\vec{A}$ and $\vec{B}$ have equal magnitudes. If magnitude of $\vec{A} + \vec{B}$ is equal to two times the magnitude of $\vec{A} - \vec{B}$, then the angle between $\vec{A}$ and $\vec{B}$ will be: **[29 June, 2022 (Shift-I)]**

(*a*) $\sin^{-1}\left(\frac{3}{5}\right)$ (*b*) $\sin^{-1}\left(\frac{1}{3}\right)$

(*c*) $\cos^{-1}\left(\frac{3}{5}\right)$ (*d*) $\cos^{-1}\left(\frac{1}{3}\right)$

13. If $\vec{A}$ and $\vec{B}$ are two vectors satisfying the relation $\vec{A}.\vec{B} = \left|\vec{A}\times\vec{B}\right|$. Then the value of $\left|\vec{A}-\vec{B}\right|$ will be:

[20 July, 2021 (Shift-I)]

(a) $\sqrt{A^2+B^2-\sqrt{2}AB}$

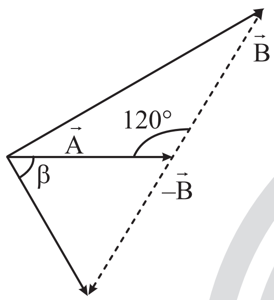
(b) $\sqrt{A^2+B^2}$

(c) $\sqrt{A^2+B^2+2AB}$

(d) $\sqrt{A^2+B^2+\sqrt{2}AB}$

14. The angle between vector $\left(\vec{A}\right)$ and $\left(\vec{A}-\vec{B}\right)$ is:

[26 Aug, 2021 (Shift-II)]



(a) $\tan^{-1}\left(\dfrac{\sqrt{3}B}{2A-B}\right)$

(b) $\tan^{-1}\left(\dfrac{B\cos\theta}{A-B\sin\theta}\right)$

(c) $\tan^{-1}\left(\dfrac{A}{0.7\,B}\right)$

(d) $\tan^{-1}\left(\dfrac{\frac{-B}{2}}{A-B\frac{\sqrt{3}}{2}}\right)$

15. What will be the projection of vector $\vec{A}=\hat{i}+\hat{j}+\hat{k}$ on vector $\vec{B}=\hat{i}+\hat{j}$? **[22 July, 2021 (Shift-II)]**

(a) $2(\hat{i}+\hat{j}+\hat{k})$ (b) $\sqrt{2}(\hat{i}+\hat{j})$

(c) $(\hat{i}+\hat{j})$ (d) $\sqrt{2}(\hat{i}+\hat{j}+\hat{k})$

16. Two vectors $\vec{A}$ and $\vec{B}$ have equal magnitudes. The magnitude of ($\vec{A}+\vec{B}$) is 'n' times the magnitude of ($\vec{A}-\vec{B}$). The angle between $\vec{A}$ and $\vec{B}$ is:

(a) $\cos^{-1}\left[\dfrac{n^2-1}{n^2+1}\right]$ (b) $\cos^{-1}\left[\dfrac{n-1}{n+1}\right]$

(c) $\sin^{-1}\left[\dfrac{n^2-1}{n^2+1}\right]$ (d) $\sin^{-1}\left[\dfrac{n-1}{n+1}\right]$

17. Two vectors $\vec{X}$ and $\vec{Y}$ have equal magnitude. The magnitude of $\left(\vec{X}-\vec{Y}\right)$ is n times the magnitude of $\left(\vec{X}+\vec{Y}\right)$. The angle between $\vec{X}$ and $\vec{Y}$ is:

[25 July, 2021 (Shift-II)]

(a) $\cos^{-1}\left(\dfrac{n^2+1}{n^2-1}\right)$ (b) $\cos^{-1}\left(\dfrac{-n^2-1}{n^2-1}\right)$

(c) $\cos^{-1}\left(\dfrac{n^2-1}{-n^2-1}\right)$ (d) $\cos^{-1}\left(\dfrac{n^2+1}{-n^2-1}\right)$

18. Two vectors $\vec{P}$ and $\vec{Q}$ have equal magnitudes. If the magnitude of $\vec{P}+\vec{Q}$ is n times the magnitude of $\vec{P}-\vec{Q}$, then angle between $\vec{P}$ and $\vec{Q}$ is:**[20 July, 2021 (Shift-II)]**

(a) $\sin^{-1}\left(\dfrac{n^2-1}{n^2+1}\right)$ (b) $\cos^{-1}\left(\dfrac{n^2-1}{n^2+1}\right)$

(c) $\sin^{-1}\left(\dfrac{n-1}{n+1}\right)$ (d) $\cos^{-1}\left(\dfrac{n-1}{n+1}\right)$

19. **Statement-I:** Two forces $(\vec{P}+\vec{Q})$ and $(\vec{P}-\vec{Q})$ where $\vec{P}\perp\vec{Q}$, when act at an angle θ_1 to each other, the magnitude of their resultant is $\sqrt{3\left(P^2+Q^2\right)}$, when they act at an angle θ_2, the magnitude of their resultant becomes $\sqrt{2\left(P^2+Q^2\right)}$. This is possible only when $\theta_1<\theta_2$.

[31 Aug, 2021 (Shift-II)]

Statement-II: In the situation given above. $\theta_1 = 60°$ and $\theta_2 = 90°$

In the light of the above statements, choose the most appropriate answer from the options given below:

(a) Both Statement-I and Statement-II are true

(b) Both Statement-I and Statement-II are false

(c) Statement-I is true but Statement-II is false

(d) Statement-I is false but Statement-II is true

20. Let $\left|\vec{A}_1\right|=3, \left|\vec{A}_2\right|=5$ and $\left|\vec{A}_1+\vec{A}_2\right|=5$. The value of $(2\vec{A}_1+3\vec{A}_2).(3\vec{A}_1-2\vec{A}_2)$ is: **[8 April, 2019 (Shift-II)]**

(a) –112.5 (b) –106.5

(c) –118.5 (d) –99.5

21. Two forces P and Q, of magnitude $2F$ and $3F$, respectively, are at an angle θ with each other. If the force Q is doubled, then their resultant also gets doubled. Then, the angle θ is: **[10 Jan, 2019 (Shift-II)]**

(a) 120° (b) 60°

(c) 90° (d) 30°

22. $\vec{A}$ is a vector quantity such that $|\vec{A}|$ = non-zero constant. Which of the following expression is true for $\vec{A}$?

[25 June, 2022 (Shift-I)]

(a) $\vec{A}.\vec{A} = 0$

(b) $\vec{A} \times \vec{A} < 0$

(c) $\vec{A} \times \vec{A} = 0$

(d) $\vec{A} \times \vec{A} > 0$

23. If force $\vec{F} = 3\hat{i} + 4\hat{j} - 2\hat{k}$ acts on a particle having position vector $2\hat{i} + \hat{j} + 2\hat{k}$ then, the torque about the origin will be: **[25 June, 2022 (Shift-I)]**

(a) $3\hat{i} + 4\hat{j} - 2\hat{k}$

(b) $-10\hat{i} + 10\hat{j} + 5\hat{k}$

(c) $10\hat{i} + 5\hat{j} - 10\hat{k}$

(b) $10\hat{i} + \hat{j} - 5\hat{k}$

24. If $t = \sqrt{x} + 4$, then $\left(\frac{dx}{dt}\right)_{t=4}$ is: **[29 July, 2022 (Shift-I)]**

(a) 4 (b) zero

(c) 8 (d) 16

Integer Type Questions

25. Vectors $a\hat{i} + b\hat{j} + \hat{k}$ and $2\hat{i} - 3\hat{j} + 4\hat{k}$ are perpendicular to each other when $3a + 2b = 7$, the ratio of a to b is $\frac{x}{2}$. The value of x is _________. **[24 Jan, 2023 (Shift-I)]**

26. If $\vec{A} = (2\hat{i} + 3\hat{j} - \hat{k})m$ and $\vec{B} = (\hat{i} + 2\hat{j} - 2\hat{k})m$. The magnitude of component of vector $\vec{A}$ along vector $\vec{B}$ will be ____________ m. **[26 July, 2022 (Shift-II)]**

27.If the projection of $2\hat{i} + 4\hat{j} - 2\hat{k}$ on $\hat{i} + 2\hat{j} + \alpha\hat{k}$ is zero. Then, the value of α will be ________.[28 July, 2022 (Shift-I)]

28. If $\vec{P} = 3\hat{i} + \sqrt{3}\hat{j} + 2\hat{k}$ and $\vec{Q} = 4\hat{i} + \sqrt{3}\hat{j} + 2.5\hat{k}$ then, the unit vector in the direction of $\vec{P} \times \vec{Q}$ is $\frac{1}{x}\left(\sqrt{3}\hat{i} + \hat{j} - 2\sqrt{3}\hat{k}\right)$. The value of x is **[25 Jan, 2023 (Shift-I)]**

29. If $\vec{P} \times \vec{Q} = \vec{Q} \times \vec{P}$ the angle between $\vec{P}$ and $\vec{Q}$ is θ (0° < θ < 360°). The value of 'θ' will be _______

[25 Feb, 2021 (Shift-II)]

30. A person of height 1.6 m is walking away from a lamp post of height 4m along a straight path on the flat ground. The lamp post and the person are always perpendicular to the ground. If the speed of the person is 60 cm s^{-1}, the speed of the tip of the person's shadow on the ground with respect to the person is ____________ cm s^{-1}.

[JEE Adv, 2023]

ANSWER KEY

1. (*b*)	**2.** (*b*)	**3.** (*c*)	**4.** (*a*)	**5.** (*a*)	**6.** (*d*)	**7.** (*d*)	**8.** (*d*)	**9.** (*c*)	**10.** (*d*)
11. (*b*)	**12.** (*c*)	**13.** (*a*)	**14.** (*a*)	**15.** (*c*)	**16.** (*a*)	**17.** (*c*)	**18.** (*b*)	**19.** (*a*)	**20.** (*c*)
21. (*a*)	**22.** (*c*)	**23.** (*b*)	**24.** (*b*)	**25.** [1]	**26.** [2]	**27.** [5]	**28.** [4]	**29.** [180]	**30.** [40]

EXPLANATIONS

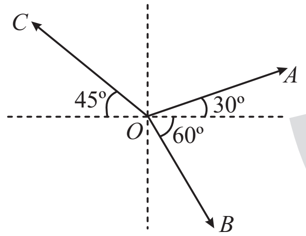
1. (*b*) Given, $\vec{B}-\vec{A}=2\hat{j}, \vec{B}=2\hat{j}+2\hat{i}+3\hat{j}+2\hat{k}$

$\therefore \vec{B}=2\hat{i}+5\hat{j}+2\hat{k}$

$|\vec{B}|=\sqrt{(2)^2+(5)^2+(2)^2}=\sqrt{33}$ (Bonus)

2. (*b*) Apply the triangle law of addition, to get answer.

3. (*c*)



According to diagram, let magnitude be equal to

$$\overrightarrow{OA}=\lambda\left[\cos 30^\circ\,\hat{i}+\sin 30\,\hat{j}\right]=\lambda\left[\frac{\sqrt{3}}{2}\hat{i}+\frac{1}{2}\hat{j}\right]$$

$$\overrightarrow{OB}=\lambda\left[\cos 60^\circ\,\hat{i}-\sin 60\,\hat{j}\right]=\lambda\left[\frac{1}{2}\hat{i}-\frac{\sqrt{3}}{2}\hat{j}\right]$$

$$\overrightarrow{OC}=\lambda\left[\cos 45^\circ(-\hat{i})+\sin 45\,\hat{j}\right]=\lambda\left[-\frac{1}{\sqrt{2}}\hat{i}+\frac{1}{\sqrt{2}}\hat{j}\right]$$

$\therefore \overrightarrow{OA}+\overrightarrow{OB}-\overrightarrow{OC}$

$$=\lambda\left[\left(\frac{\sqrt{3}+1}{2}+\frac{1}{\sqrt{2}}\right)\hat{i}+\left(\frac{1}{2}-\frac{\sqrt{3}}{2}-\frac{1}{\sqrt{2}}\right)\hat{j}\right]$$

$$\tan^{-1}\left[\frac{\frac{1}{2}-\frac{\sqrt{3}}{2}-\frac{1}{\sqrt{2}}}{\frac{\sqrt{3}}{2}+\frac{1}{2}+\frac{1}{\sqrt{2}}}\right]=\tan^{-1}\left[\frac{\sqrt{2}-\sqrt{6}-2}{\sqrt{6}+\sqrt{2}+2}\right]$$

$$=\tan^{-1}\left[\frac{1-\sqrt{3}-\sqrt{2}}{\sqrt{3}+1+\sqrt{2}}\right]$$

4. (*a*) $\overrightarrow{AB}+\overrightarrow{AH}=\overrightarrow{AO}$

$\overrightarrow{AC}+\overrightarrow{AG}=2\overrightarrow{AO}$

$\overrightarrow{AD}+\overrightarrow{AF}=3\overrightarrow{AO}$

$\overrightarrow{AE}=2\overrightarrow{AO}$

Adding all,

$\overrightarrow{AB}+\overrightarrow{AC}+\overrightarrow{AD}+\overrightarrow{AE}+\overrightarrow{AF}+\overrightarrow{AG}+\overrightarrow{AH}=8\overrightarrow{AO}$

$=16\hat{i}+24\hat{j}-32\hat{k}$

5. (*a*) The horizontal component of force,

$$\vec{F}_x=\left[10\times\frac{\sqrt{3}}{2}+20\times\frac{1}{2}+\frac{20}{\sqrt{2}}-\frac{15}{\sqrt{2}}-\frac{15\sqrt{3}}{2}\right]=9.25\hat{i}$$

The vertical component of force,

$$\vec{F}_y=\left[15\times\frac{1}{2}+20\times\frac{\sqrt{3}}{2}+10\times\frac{1}{2}-\frac{15}{\sqrt{2}}-\frac{20}{\sqrt{2}}\right]=5\hat{j}$$

$\therefore \vec{F}_R=\vec{F}_x+\hat{F}_y=9.25\hat{i}+5\hat{j}$

6. (*d*) $\vec{F}=\left(\vec{F}_1+\vec{F}_2\right)$

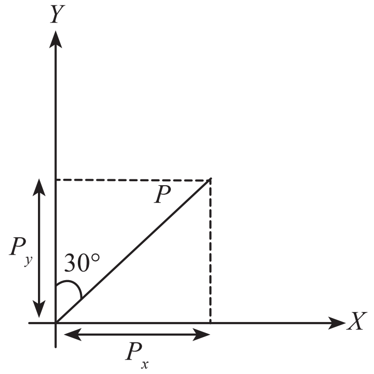
$\left|\vec{F_1}\right|=A, \left|\vec{F_2}\right|=\dfrac{A}{2}$

$\theta=90°$

$\therefore \left|\vec{F}\right|=\sqrt{F_1^2+F_2^2+2F_1F_2\cos\theta}$

$$|\vec{F}| = \sqrt{A^2 + \frac{A^2}{4}} = \frac{A\sqrt{5}}{2}$$

7. (*d*) Let the vector be P.



$P_y = P\cos 30° = 2\sqrt{3}$

$\Rightarrow P\frac{\sqrt{3}}{2} = 2\sqrt{3} \Rightarrow P = 4$

Now $P_x = P\sin 30^{\circ} = 4\times\frac{1}{2} = 2$

8. (*d*) Polygon law is applicable in both the cases given but the equation given in the reason is not useful in explaining the assertion.

9. (*c*) $G \equiv \left(\frac{a}{2}, 0, \frac{a}{2}\right), \quad H \equiv \left(0, \frac{a}{2}, \frac{a}{2}\right)$

$\overline{GH} = -\frac{a}{2}\hat{i} + \frac{a}{2}\hat{j} = \frac{a}{2}\left(\hat{j} - \hat{i}\right)$

10. (*d*) As $\vec{P}$ and $\vec{Q}$ are perpendicular their dot product must be zero.

$\vec{P}\cdot\vec{Q} = 0$

$\left(\hat{i} + 2m\hat{j} + m\hat{k}\right)\cdot\left(4\hat{i} - 2\hat{j} + m\hat{k}\right) = 0$

$\Rightarrow 4 - 4m + m^2 = 0$

$\Rightarrow m = 2$

11. (*b*) $|\vec{A}+\vec{B}| = \sqrt{|\vec{A}|^2 + |\vec{B}|^2 + 2|\vec{A}||\vec{B}|\cos\theta}$

$= \sqrt{1+1+2\cos\theta}$

$= 2\cos\theta/2$

$|\vec{A}-\vec{B}| = \sqrt{|\vec{A}|^2 + |\vec{B}|^2 - 2|\vec{A}||\vec{B}|\cos\theta}$

$= \sqrt{1-2\cos\theta} = 2\sin\theta/2$

$\frac{|\vec{A}+\vec{B}|}{|\vec{A}-\vec{B}|} = \cot\theta/2 \Rightarrow |\vec{A}-\vec{B}| = |\vec{A}+\vec{B}|\tan\frac{\theta}{2}$

12. (*c*) According to the question $|\vec{A}+\vec{B}| = 2|\vec{A}-\vec{B}|$ ... (*i*)

It is given that, $|\vec{A}| = |\vec{B}|$

Squaing equation (*i*) both side. $[\because A = B]$

$|\vec{A}+\vec{B}|^2 = (2|\vec{A}-\vec{B}|)^2$

$\Rightarrow A^2 + B^2 + 2\vec{A}\cdot\vec{B} = 4\left(A^2 + B^2 - 2\vec{A}\cdot\vec{B}\right)$

$\Rightarrow 2A^2 + 2A^2\cos\theta = 4(A^2 - B^2 - A^2\cos\theta)$

$\Rightarrow 2A^2 + 2A^2\cos\theta = 8A^2 - 8A^2\cos\theta$

$\Rightarrow 10A^2\cos\theta = 6A^2 \Rightarrow \cos\theta = \frac{3}{5}$

$\therefore \theta = \cos^{-1}(3/5)$

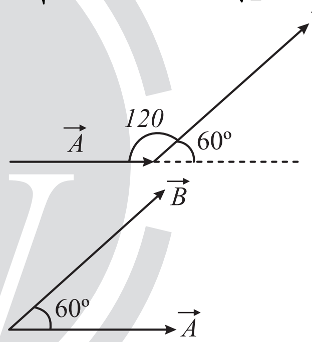
13. (*a*) Given that, $\vec{A}.\vec{B} = |\vec{A}\times\vec{B}| \Rightarrow AB\cos\theta = AB\sin\theta$

$\Rightarrow \theta = 45°$

$|\vec{A}-\vec{B}| = \sqrt{A^2 + B^2 - 2AB\cos\theta}$

$= \sqrt{A^2 + B^2 - 2AB\times\frac{1}{\sqrt{2}}} = \sqrt{A^2 + B^2 - \sqrt{2}AB}$

14. (*a*)



According to diagram,

Angle between $\vec{A}$ and $\vec{B}$, $\theta = 60°$

Angle between $\vec{A}$ and $-\vec{B}$, $\theta = 120°$

If angle between $\vec{A}$ and $\vec{A} - \vec{B}$ is α

then $\tan\alpha = \frac{|-\vec{B}|\sin\theta}{|\vec{A}| + |-\vec{B}|\cos\theta}$

$$= \frac{B\sin 120^{\circ}}{A + B\cos 120^{\circ}}$$

$$= \frac{B\frac{\sqrt{3}}{2}}{A - \frac{B}{2}}$$

$\Rightarrow \tan\alpha = \frac{\sqrt{3}B}{2A - B}$

Hence, the angle between vector $\vec{A}$ and $(\vec{A} - \vec{B})$ is

$$\tan\alpha = \frac{\sqrt{3}B}{2A-B}$$

15. (*c*) Component of $\vec{A}$ an $\vec{B}$

Projection of $\vec{A}$ on $\vec{B}$ $= \left(\vec{A}\,.\hat{B}\right)\hat{B}$

$$= \left[\left(\hat{i}+\hat{j}+\hat{k}\right).\frac{(\hat{i}+\hat{j})}{\sqrt{2}}\right]\frac{(\hat{i}+\hat{j})}{\sqrt{2}}$$

$$= \frac{1}{2}(1+1)(\hat{i}+\hat{j}) = \left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}\right)\frac{(\hat{i}+\hat{j})}{\sqrt{2}} = \hat{i}+\hat{j}$$

16. (*a*) $|\vec{A}+\vec{B}| = n|\vec{A}-\vec{B}|$

$\Rightarrow A^2 + B^2 + 2AB\cos\theta$

$= n^2(A^2 + B^2 - 2AB\cos\theta)$

$\Rightarrow \cos\theta(1+n^2) = \dfrac{2a^2(n^2-1)}{2a^2}$ $[A = B = a]$

$$\cos\theta = \frac{n^2-1}{n^2+1}$$

17. (*c*) Given,

$|\vec{x}| = |\vec{y}|$ and $|\vec{x}-\vec{y}| = n|\vec{x}+\vec{y}|$

$$x^2+y^2-2\vec{x}\cdot\vec{y} = n^2\left(x^2+y^2+2\vec{x}\cdot\vec{y}\right)$$

$$\left(1-n^2\right)\left(x^2+y^2\right) = \left(1+n^2\right)2\vec{x}\cdot\vec{y}$$

$$\left(1-n^2\right)\left(x^2+y^2\right) = \left(1+n^2\right)2xy\cos\theta$$

$$\cos\theta = \frac{1-n^2}{1+n^2} \qquad \therefore\ 2xy = x^2+y^2$$

$$\theta = \cos^{-1}\left(\frac{n^2-1}{-n^2-1}\right)$$

18. (*b*) It is given that, $\left|\vec{P}\right| = \left|\vec{Q}\right|$

Let us suppose, $\left|\vec{P}\right| = \left|\vec{Q}\right| = x$

Let the angle between $\vec{P}$ and $\vec{Q}$ is θ, so according to question, we can write

$$\left|\vec{P}+\vec{Q}\right| = n\times\left|\vec{P}-\vec{Q}\right| \Rightarrow \sqrt{P^2+Q^2+2PQ\cos\theta}$$

$$= n\times\sqrt{P^2+Q^2-2PQ\cos\theta}$$

$$\Rightarrow \sqrt{x^2+x^2+2x^2\cos\theta} = n\times\sqrt{x^2+x^2-2x^2\cos\theta}$$

$$\Rightarrow 2+2\cos\theta = 2n^2 - 2n^2\cos\theta \Rightarrow \cos\theta = \frac{n^2-1}{n^2+1}$$

$$\Rightarrow \theta = \cos^{-1}\frac{n^2-1}{n^2+1}$$

19. (*a*) According to question the given data is two forces $(\vec{P}+\vec{Q})$ and $(\vec{P}-\vec{Q})$

$$\vec{A} = \vec{P}+\vec{Q} \Rightarrow \vec{B} = \vec{P}-\vec{Q} \Rightarrow \vec{P}\perp\vec{Q}$$

$$\Rightarrow |\vec{A}| = |\vec{B}| = \sqrt{2\left(P^2+Q^2\right)(1+\cos\theta)}$$

$$\Rightarrow \text{ For } |\vec{A}+\vec{B}| = \sqrt{3\left(P^2+Q^2\right)} \Rightarrow \theta_1 = 60°$$

$$\Rightarrow \text{ For } |\vec{A}+\vec{B}| = \sqrt{2\left(P^2+Q^2\right)} \Rightarrow \theta_2 = 90°$$

According to solution Both Statement-I and Statement-II are true.

20. (*c*) $\left|\vec{A}_1\right| = 3$, $\left|\vec{A}_2\right| = 5$, $\left|\vec{A}_1+\vec{A}_2\right| = 5$

$$\left|\vec{A}_1+\vec{A}_2\right| = \sqrt{\left|\vec{A}_1\right|^2+\left|\vec{A}_2\right|^2+2\left|\vec{A}_1\right|\left|\vec{A}_2\right|\cos\theta}$$

$$5 = \sqrt{9+25+2\times3\times5\cos\theta}$$

$$\Rightarrow \cos\theta = -\frac{9}{2\times3\times5} = -\frac{3}{10}$$

$$(2\vec{A}_1+3\vec{A}_2).(3\vec{A}_1-2\vec{A}_2)$$

$$= 6|\vec{A}_1|^2 + 9\vec{A}_1.\vec{A}_2 - 4\vec{A}_1\vec{A}_2 - 6|\vec{A}_2|^2 = -118.5$$

21. (*a*) $2|\vec{P}+\vec{Q}| = |\vec{P}+2\vec{Q}|$

$\Rightarrow 13 + 12\cos\theta = 10 + 6\cos\theta$

$\cos = -\dfrac{1}{2}$

$\theta = 120°$.

22. (*c*) Given, $|A| \neq 0$

$$\vec{A}\times\vec{A} = |A||A|\sin\theta\hat{n}$$

$$= |A||A|\sin 0°\,\hat{n}$$

= 0[Since Angle between the vectors are zero degree]

$$\vec{A}\times\vec{A} = 0$$

23. (*b*) $\vec{\tau} = \vec{r}\times\vec{F}$

$$= \left(2\hat{i}+\hat{j}+2\hat{k}\right)\times\left(3\hat{i}+4\hat{j}-2\hat{k}\right) = \begin{vmatrix} \hat{i} & -\hat{j} & \hat{k} \\ 2 & 1 & 2 \\ 3 & 4 & -2 \end{vmatrix}$$

$$= \hat{i}(-2-8) - \hat{j}(-4-6) + \hat{k}(8-3)$$

$$\vec{\tau} = -10\hat{i} + 10\hat{j} + 5\hat{k}$$

24. (*b*) $t = \sqrt{x} + 4$, at $t = 4, \sqrt{x} = 0$

So, $\dfrac{dt}{dx} = \dfrac{1}{2\sqrt{x}} \quad \therefore \left(\dfrac{dx}{dt}\right)_{t=4} = (2\sqrt{x})_{t=4} = 0$

25. [1] For two perpendicular vectors

$\left(a\hat{i} + b\hat{j} + \hat{k}\right)\cdot\left(2\hat{i} - 3\hat{j} + 4\hat{k}\right) = 0$

$2a - 3b + 4 = 0$

On solving, $2a - 3b = -4$

Also given

$3a + 2b = 7$

We get $a = 1, b = 2$

$\dfrac{a}{b} = \dfrac{x}{2} \Rightarrow x = \dfrac{2a}{b} = \dfrac{2\times 1}{2}$

$\Rightarrow x = 1$

26. [2] The magnitude of the component of vector A along vector B will be

$= \dfrac{\vec{A}\cdot\vec{B}}{|\vec{B}|} = \dfrac{2+6-2}{3} = 2$

27. [5] $0 = (2\hat{i} + 4\hat{j} - 2\hat{k})\cdot\left(\dfrac{\hat{i} + 2\hat{j} + \alpha\hat{k}}{\sqrt{1+4+\alpha^2}}\right)$

$0 = 2 + 8 - 2\alpha$

$\alpha = 5$

28. [4] $\vec{P}\times\vec{Q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & \sqrt{3} & 2 \\ 4 & \sqrt{3} & 2.5 \end{vmatrix} = \sqrt{3}\dfrac{\hat{i}}{2} + \dfrac{\hat{j}}{2} - \sqrt{3}\,\hat{k}$

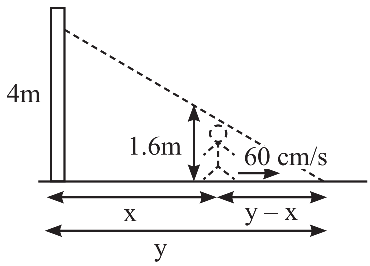
and $\Rightarrow \left|\vec{P}\times\vec{Q}\right| = \dfrac{1}{2} \Rightarrow \dfrac{\vec{P}\times\vec{Q}}{\left|\vec{P}\times\vec{Q}\right|} = \dfrac{1}{2}\left(\sqrt{3}\dfrac{\hat{i}}{2} + \dfrac{\hat{j}}{2} - \sqrt{3}\hat{k}\right)$

$= \dfrac{1}{4}\left(\sqrt{3}\hat{i} + \hat{j} - 2\sqrt{3}\hat{k}\right) \Rightarrow x = 4$

29. [180] If angle between them is 180° then

$\vec{P}\times\vec{Q} = \vec{q}\times\vec{P} = 0$

30. [40]



From the property of similar triangle,

$\dfrac{4}{y} = \dfrac{1.6}{y-x}$

$x = 0.6y$

$\dfrac{dx}{dt} = 0.6\times\dfrac{dy}{dt}$

$60 = 0.6\times\dfrac{dy}{dt}$

$\therefore \dfrac{dy}{dt} = 100\text{ cm/ s}$

Speed of tip of person's. Shadow w.r.t person = 100 – 60 = 40 cm/s

Motion in Plane

Single Correct Type Questions

1. The initial speed of a projectile fired from ground is u. At the highest point during its motion, the speed of projectile is $\frac{\sqrt{3}}{2}u$. The time of flight of the projectile is:

[31 Jan, 2023 (Shift-I)]

(*a*) $\frac{u}{2g}$ (*b*) $\frac{u}{g}$

(*c*) $\frac{2u}{g}$ (*d*) $\frac{\sqrt{3}u}{g}$

2. The trajectory of projectile, projected from the ground is given by $y = x - \frac{x^2}{20}$. Where x and y are measured in meter. The maximum height attained by the projectile will be. **[8 April, 2023 (Shift-II)]**

(*a*) 5 m (*b*) $10\sqrt{2}\,m$

(*c*) 200 m (*d*) 10 m

3. For a body projected at an angle with the horizontal from the ground, choose the correct statement.

[1 Feb, 2023 (Shift-II)]

(*a*) Gravitational potential energy is maximum at the highest point.

(*b*) The horizontal component of velocity is zero at highest point.

(*c*) The vertical component of momentum is maximum at the highest point.

(*d*) The kinetic energy (K.E.) is zero at the highest point of projectile motion.

4. A projectile is projected at 30° from horizontal with initial velocity 40 ms^{-1}. The velocity of the projectile at t = 2 s from the start will be:
(Given $g = 10\ m/s^2$) **[11 April, 2023 (Shift-II)]**

(*a*) $20\sqrt{3}\,ms^{-1}$

(*b*) $40\sqrt{3}\,ms^{-1}$

(*c*) 20 ms^{-1}

(*d*) Zero

5. Two projectiles are projected at 30° and 60° with the horizontal with the same speed. The ratio of the maximum height attained by the two projectiles respectively is:

[10 April, 2023 (Shift-II)]

(*a*) $2:\sqrt{3}$

(*b*) $\sqrt{3}:1$

(*c*) 1 : 3

(*d*) $1:\sqrt{3}$

6. The maximum vertical height to which a man can throw a ball is 136 m. The maximum horizontal distance upto which he can throw the same ball is

[24 Jan, 2023 (Shift-I)]

(*a*) 192 m

(*b*) 136 m

(*c*) 272 m

(*d*) 68 m

7. A projectile is launched at an angle 'α' with the horizontal with a velocity 20 ms. After 10 s, it inclination with horizontal is 'β'. The value of $\tan\beta$ will be: ($g = 10\ ms^{-2}$).

[27 June, 2022 (Shift-I)]

(*a*) $\tan\alpha + 5\sec\alpha$

(*b*) $\tan\alpha - 5\sec\alpha$

(*c*) $2\tan\alpha - 5\sec\alpha$

(*d*) $2\tan\alpha + 5\sec\alpha$

8. A body of mass 10 kg is projected at an angle of 45° with the horizontal. The trajectory of the body is observed to pass through a point (20, 10). If T is the time of flight, then its momentum vector, at time $t = \frac{T}{\sqrt{2}}$ is______

[Take $g = 10$ m/s^2] **[27 July, 2022 (Shift-II)]**

(*a*) $100\hat{i} + \left(100\sqrt{2} - 200\right)\hat{j}$

(*b*) $100\sqrt{2}\,\hat{i} + \left(100 - 200\sqrt{2}\right)\hat{j}$

(*c*) $100\hat{i} + \left(100 - 200\sqrt{2}\right)\hat{j}$

(*d*) $100\sqrt{2}\,\hat{i} + \left(100\sqrt{2} - 200\right)\hat{j}$

9. A projectile is projected with velocity of 25 m/s at an angle θ with the horizontal. After t seconds its inclination with horizontal becomes zero. If R represents horizontal range of the projectile, the value of θ will be:

[Use $g = 10$ m/s^2] **[24 June, 2022 (Shift-I)]**

(*a*) $\frac{1}{2}\sin^{-1}\left(\frac{5t^2}{4R}\right)$ (*b*) $\frac{1}{2}\sin^{-1}\left(\frac{4R}{5t^2}\right)$

(*c*) $\tan^{-1}\left(\frac{4t^2}{5R}\right)$ (*d*) $\cot^{-1}\left(\frac{R}{20t^2}\right)$

10. A person can throw a ball upto a maximum range of 100 m. How high above the ground he can the same ball?

[29 June, 2022 (Shift-II)]

(*a*) 25 m (*b*) 50 m

(*c*) 100 m (*d*) 200 m

11. A body is projected at $t = 0$ with a velocity 10 ms^{-1} at an angle of 60° with the horizontal. The radius of curvature of its trajectory at $t = 1$ *s* is R. Neglecting air resistance and taking acceleration due to gravity $g = 10$ ms^{-2} the radius of R is: **[11 Jan, 2019 (Shift-I)]**

(*a*) 10.3 m (*b*) 2.8 m

(*c*) 2.5 m (*d*) 5.1 m

12. A shell is fired from a fixed artillery gun with an initial speed u such that it hits the target on the ground at a distance R from it. If t_1 and t_2 are the values of the time taken by it to hit the target in two possible ways, the product t_1t_2 is: **[12 April, 2019 (Shift-I)]**

(*a*) R/g (*b*) $R/4g$

(*c*) $2R/g$ (*d*) $R/2g$

13. The trajectory of a projectile near the surface of the earth is given as $y = 2x - 9x^2$. If it were launched at an angle θ_0 with speed v_0 then (g =10 ms^{-2}) **[12 April, 2019 (Shift-I)]**

(*a*) $\theta_0 = \cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$ and $v_0 = \frac{3}{5}$ ms^{-1}

(*b*) $\theta_0 = \sin^{-1}\left(\frac{1}{\sqrt{5}}\right)$ and $v_0 = \frac{5}{3}$ ms^{-1}

(*c*) $\theta_0 = \sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$ and $v_0 = \frac{3}{5}$ ms^{-1}

(*d*) $\theta_0 = \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$ and $v_0 = \frac{5}{3}$ ms^{-1}

14. A stone is projected at angle 30° to the horizontal. The ratio of kinetic energy of the stone at point of projection to its kinetic energy at the highest point of flight will be

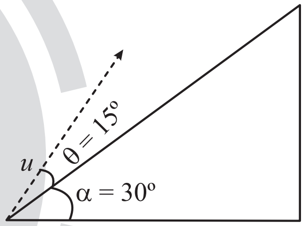
[29 Jan, 2023 (Shift-I)]

(*a*) 1 : 2 (*b*) 1 : 4

(*c*) 4 : 1 (*d*) 4 : 3

15. A plane is inclined at an angle $\alpha = 30°$ with a respect to the horizontal. A particle is projected with a speed $u = 2$ ms^{-1} from the base of the plane, making an angle $\theta = 15°$ with respect to the plane as shown in the figure. The distance from the base, at which the particle hits the plane is close to: **[10 April, 2019 (Shift-II)]**

(Take $g = 10$ ms^{-2})



(*a*) 14 cm (*b*) 20 cm

(*c*) 18 cm (*d*) 26 cm

16. A passenger sitting in a train A moving at 90 km/h observes another train B moving in the opposite direction for 8 s. If the velocity of the train B is 54 km/h, then length of train B is: **[13 April, 2023 (Shift-II)]**

(*a*) 80 m (*b*) 200 m

(*c*) 120 m (*d*) 320 m

17. Two buses P and Q start from a point at the same time and move in a straight line and their positions are represented by $X_p(t) = \alpha t + \beta t^2$ and $X_Q(t) = ft - t^2$. At what time, both the buses have same velocity? **[25 June, 2022 (Shift-II)]**

(*a*) $\frac{\alpha - f}{1+\beta}$

(*b*) $\frac{\alpha + f}{2(\beta - 1)}$

(*c*) $\frac{\alpha + f}{2(1+\beta)}$

(*d*) $\frac{f - \alpha}{2(1+\beta)}$

18. Train A and train B are running on parallel tracks in opposite directions with speeds of 36 km/hour and 72 km/hour, respectively, A person is walking in train A in the direction opposite to its motion with a speed of 1.8 km/hour. Speed (in ms^{-1}) of this person as observed from train B will be close to: (take the distance between the tracks as negligible) **[2 Sep, 2020 (Shift-I)]**

(*a*) 28.5 ms^{-1}
(*b*) 31.5 ms^{-1}
(*c*) 30.5 ms^{-1}
(*d*) 29.5 ms^{-1}

19. A particle is moving with a velocity $\vec{v} = K(y\hat{i} + x\hat{j})$, where K is a constant. The general equation for its path is: **[9 Jan, 2019 (Shift-I)]**

(*a*) $y = x^2$ + constant
(*b*) $y^2 = x$ + constant
(*c*) $y^2 = x^2$ + constant
(*d*) xy = constant

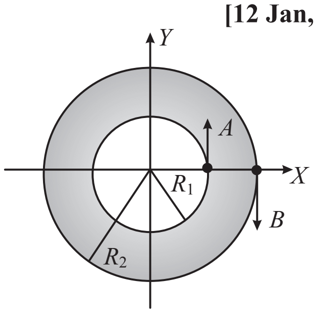
20. The stream of a river is flowing with a speed of 2km/h. A swimmer can swim at a speed of 4km/h. What should be the direction of the swimmer with respect to the flow of the river to cross the river straight? **[9 April, 2019 (Shift-I)]**

(*a*) 60°
(*b*) 150°
(*c*) 90°
(*d*) 120°

21. Two projectile thrown at 30° and 45° with the horizontal respectively, reach the maximum height in same time. The ratio of their initial velocities is **[26 July, 2022 (Shift-I)]**

(*a*) $1:\sqrt{2}$
(*b*) 2 : 1
(*c*) $\sqrt{2}:1$
(*d*) 1 : 2

22. Two particles A, B are moving on two concentric circles of radii R_1 and R_2 with equal angular speed ω. At $t = 0$, their positions and direction of motion are shown in the figure: **[12 Jan, 2019 (Shift-II)]**



The relative velocity $\overrightarrow{v_A} - \overrightarrow{v_B}$ at $t = \frac{\pi}{2\omega}$ is given by:

(*a*) $\omega(R_1 + R_2)\hat{i}$
(*b*) $-\omega(R_1 + R_2)\hat{i}$
(*c*) $\omega(R_2 - R_1)\hat{i}$
(*d*) $\omega(R_1 - R_2)\hat{i}$

Integer Type Questions

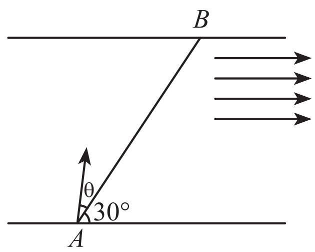
23. If the initial velocity in horizontal direction of a projectile is unit vector $\hat{i}$ and the equation of trajectory is $y = 5x(1 - x)$. The y component vector of the initial velocity is _________ $\hat{j}$.
(Take $g = 10$ m/s^2) **[26 July, 2022 (Shift-I)]**

24. An object is projected in the air with initial velocity u at an angle θ. The projectile motion is such that the horizontal range R, is maximum. Another object is projected in the air with a horizontal range half of the range of first object. The initial velocity remains same in both the case. The value of the angle of projection, at which the second object is projected, will be ____________ degree. **[29 July, 2022 (Shift-I)]**

25. If the initial velocity in horizontal direction of a projectile is unit vector $\hat{i}$ and the equation of trajectory is $y = 5x(1 - x)$. The y component vector of the initial velocity is ______ $\hat{j}$
(Take $g = 10$ m/s^2) **[JEE Adv, 2022]**

26. A fighter jet is flying horizontally at a certain with a speed of 200 ms^{-1}.When it passes directly overhead ad anti-aircraft gun, a bullet is fired from the gun, at an angle θ with the horizontal, to hit the jet. If the bullet speed is 400 m/s, the value of θ will be __________. **[26 June, 2022 (Shift-I)]**

27. A swimmer wants to cross a river from point A to point B. Line AB makes an angle of 30° with the flow of river. Magnitude of velocity of the swimmer is same as that of the river. The angle θ with the line AB should be _________ °, so that the swimmer reaches point B. **[27 July, 2021 (Shift-II)]**



28. A particle is moving along the x-axis with its coordinate with time 't' given by $x(t) = 10 + 8t - 3t^2$. Another particle is moving along the y-axis with its coordinate as a function of time given by $y(t) = 5 - 8t^3$. At $t = 1s$, the speed of the second particle as measured in the frame of the first particle is given as $\sqrt{v}$. Then v(in m/s) is

[8 Jan, 2020 (Shift-I)]

29. A ball of mass m is thrown vertically upward. Another ball of mass 2 m is thrown an angle θ with the vertical. Both the balls stay in air for the same period of time. The ratio of the heights attained by the two balls respectively is $\frac{1}{x}$. The value of x is __________. **[JEE Adv, 2022]**

30. An object is projected in the air with initial velocity u at an angle θ. The projectile motion is such that the horizontal range R, is maximum. Another object is projected in the air with a horizontal range half of the range of first object. The initial velocity remains same in both the case. The value of the angle of projection, at which the second object is projected, will be ________degree.(Mark the smallest angle possible) **[JEE Adv, 2022]**

ANSWER KEY

1. (*b*)	**2.** (*a*)	**3.** (*a*)	**4.** (*a*)	**5.** (*c*)	**6.** (*c*)	**7.** (*b*)	**8.** (*d*)	**9.** (*d*)	**10.** (*b*)
11. (*b*)	**12.** (*c*)	**13.** (*d*)	**14.** (*d*)	**15.** (*b*)	**16.** (*d*)	**17.** (*d*)	**18.** (*d*)	**19.** (*c*)	**20.** (*d*)
21. (*c*)	**22.** (*c*)	**23.** [5]	**24.** [15or75]	**25.** [5]	**26.** [60]	**27.** [30]	**28.** [580]	**29.** [1]	**30.** [15]

EXPLANATIONS

1. (*b*) Velocity of the projectile at the highest point $= u\cos\theta$

$$u\cos\theta = \frac{\sqrt{3}u}{2}$$

$\Rightarrow \theta = 30^\circ$

Time of flight $T = \dfrac{2u\sin 30^\circ}{g} = \dfrac{u}{g}$

2. (*a*) $y = x - \dfrac{x^2}{20}$

For maximum height,

$$\frac{dy}{dx} = 0 \Rightarrow \frac{d}{dx}\left(x - \frac{x^2}{20}\right) = 0$$

$$1 - \frac{2x}{20} = 0$$

$x = 10$

So, $y_{max} = 10 - \dfrac{100}{20} = 5m$

3. (*a*) At the highest point

Vertical component of the velocity, $v_y = 0$

Horizontal component of the velocity

$v_x = u_x = u\cos\theta$

Gravitational potential energy

$U_g = mgh$, it is maximum at H_{max}.

4. (*a*) Horizontal component of velocity, $v_x = u\cos 30^\circ$

$= 40 \times \dfrac{\sqrt{3}}{2} = 20\sqrt{3}$ m/s

Vertical component of velocity, $v_y = u\sin 30^\circ$

$= 40 \times \dfrac{1}{2} = 20$ m/s

At $t = 2$s, $v_x = u_x = 20\sqrt{3}$

$v_y = u_y - gt = 20 - 10 \times 2 = 0$

$\therefore\ v = \sqrt{v_x^2 + v_y^2} = 20\sqrt{3}ms^{-1}$

5. (*c*) Maximum height $H_{max} = \dfrac{u^2\sin^2\theta}{2g} \Rightarrow H_{max} \propto \sin^2\theta$

$$\frac{H_1}{H_2} = \frac{\sin^2\theta_1}{\sin^2\theta_2} = \frac{\sin^2 30^\circ}{\sin^2 60^\circ} = \frac{(1/2)^2}{(\sqrt{3}/2)^2} = \frac{1}{3}$$

6. (*c*) Maximum vertical height is given by

$$H_{max} = \frac{v^2}{2g} = 136m$$

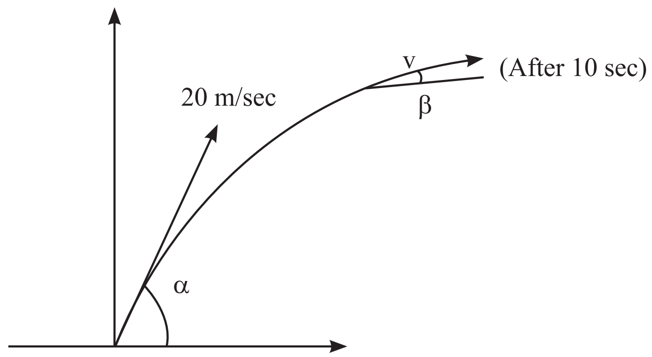
and maximum horizontal range is

$$R_{max} = \frac{v^2}{g} = 2H_{max}$$

$= 2(136)$

$\Rightarrow R_{max} = 272m$

7. (*b*) Through out the motion, the horizontal speed will remain constant, therefore,



Along *x*-axis,

$u_x = v_x = 20\cos\alpha$ ...(*i*)

Along *y*-axis

$u_y = 20\sin\alpha$,

Applying Kinematic eqn.

$v_y = u_y + a_y t$

$v_y = 20\sin\alpha - 10 \times 10$ ...(*ii*)

$$\tan\beta = \frac{v_y}{v_x} = \frac{20\sin\alpha - 100}{20\cos\alpha}$$

$\Rightarrow \tan\beta = \tan\alpha - 5\sec\alpha$

8. (*d*) $y = x - \dfrac{10x^2}{2u^2\left(\frac{1}{2}\right)}$

$$\Rightarrow 10 = 20 - \frac{(10)(400)}{u^2} \Rightarrow u = 20\,\text{m/s}$$

$$t = \frac{T}{\sqrt{2}} = \frac{(2)(20)\times\sin 45^\circ}{\sqrt{2}(10)} = 2\sqrt{s}$$

$$\vec{v} = u\cos\theta\hat{i} + (u\sin\theta - gt)\hat{j} = 10\sqrt{2}\,\hat{i} + \left(10\sqrt{2} - 10(2)\right)\hat{j}$$

Momentum $\vec{p} = M\vec{v} = 100\sqrt{2}\,\hat{i} + \left(100\sqrt{2} - 200\right)\hat{j}$

9. (*d*) $R = \dfrac{V^2(2\sin\theta\cos\theta)}{g}$

$$t = \frac{V\sin\theta}{g} \Rightarrow V = \frac{gt}{\sin\theta}$$

$$\Rightarrow R = \frac{g^2t^2}{\sin^2\theta}\cdot\frac{2\sin\theta\cos\theta}{g}$$

$$\tan\theta = \frac{2gt^2}{R} = \frac{20t^2}{R}$$

$$\cot\theta = \frac{R}{20t^2}$$

10. (*b*) According to question, we can write

$$R = \frac{u^2\sin 2q}{g}$$

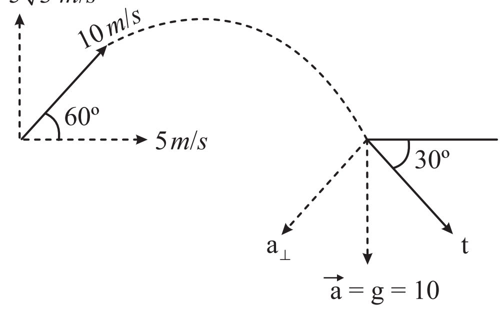
For maximum range, (R_{max})

$\theta = 45^\circ$.

$$R_{max} = \frac{u^2}{g} \Rightarrow u^2 = gR_{max}$$

$$\Rightarrow H_{max} = \frac{u^2}{2g} = \frac{gR_{max}}{2g} = \frac{R_{max}}{2} = \frac{100}{2} = 50\text{ m}$$

11. (*b*) $5\sqrt{3}\ m/s$



At $t = 1$

$u_x = 5,\ u_y = 5\sqrt{3}$

$v_y = 5\sqrt{3} - 10;\ v_x = 5$

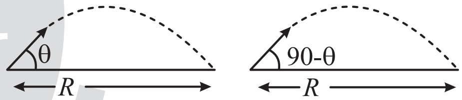
Angle made by velocity vector at $t = 1$ sec.

$$|\tan\alpha| = \left|\frac{v_y}{v_x}\right| = |2 - \sqrt{3}|$$

$\alpha = 15^\circ$

$$\therefore R = \frac{5^2 + \left(10 - 5\sqrt{3}\right)^2}{10\cos\alpha} = \frac{200 - 100\sqrt{3}}{10\times 0.965} = 2.8\text{ m}$$

12. (*c*) Range will be same for time t_1 and t_2, so angles of projection will be 'θ' and '90° – θ'



$$t_1 = \frac{2u\sin\theta}{g}\quad t_2 = \frac{2u\sin(90^\circ - \theta)}{g}$$

and $R = \dfrac{u^2\sin 2\theta}{g}$

$$t_1t_2 = \frac{4u^2\sin\theta\cos\theta}{g^2} = \frac{2}{g}\left[\frac{2u^2\sin\theta\cos\theta}{g}\right] = \frac{2R}{g}$$

13. (*d*) Equation of trajectory is given as

$y = 2x - 9x^2$ ... (*i*)

Comparing with equation:

$y = x\tan\theta - \dfrac{g}{2u^2\cos^2\theta}\cdot x^2$ ... (*ii*)

We get; $\tan\theta = 2$

$$\therefore\ \cos\theta = \frac{1}{\sqrt{5}}$$

Also, $\dfrac{g}{2u^2\cos^2\theta} = 9$

$$\Rightarrow \frac{10}{2\times 9\times\left(\frac{1}{\sqrt{5}}\right)^2} = u^2 \quad \Rightarrow u^2 = \frac{25}{9} \quad \Rightarrow u = \frac{5}{3} \text{ m/s}$$

14. (*d*) At the maximum height velocity = ucosθ = u cos30°

$$\frac{KE_{initial}}{KE_{top}} = \frac{\frac{1}{2}M(u)^2}{\frac{1}{2}M\left(u\cos 30^\circ\right)^2} = \frac{4}{3}$$

15. (*b*) $t = \frac{2\times 2\sin 15^\circ}{g\cos 30^\circ}$

$S = 2\cos 15^\circ \times t - \frac{1}{2}g\sin 30^\circ\, t^2$

Put values and solve: $S = 20$ cm

16. (*d*) $\boxed{B} \xrightarrow{V_B} \xleftarrow[V_A]{} \boxed{A}$

Velocity of train A, $V_A = 90$ km/h = – 2.5 m/s

$\because$ 1 km/h $= \frac{5}{18} m/s$

Velocity of train B, $V_B = 54$ km/h = 15 m/s

Velocity of train B w.r.t. train A

$\vec{V} = \vec{V}_B - \vec{V}_A = 15 - (-25)$

= 40 m/s

Time of crossing (t) $= \frac{\text{length of train } (l)}{\text{relative velocity} (V)}$

(8) $= \frac{l}{40} \Rightarrow l = 8 \times 40 = 320$ m

17. (*d*) $X_p(t) = \alpha t + \beta t^2$

$V_p(t) = \frac{dX_P(t)}{dt} = \alpha + 2\beta t$ ... (*i*)

$X_Q(t) = ft - t^2$

$V_Q(t) = \frac{dX_Q(t)}{dt} = f - 2t$ ... (*ii*)

According to the quastion, (*i*) = (*ii*)

$\alpha + 2\beta t = f - 2t \Rightarrow t(2\beta + 2) = f - \alpha$

$\therefore t = \frac{f-\alpha}{2(\beta+1)}$

18. (*d*) $V_A = 36$ km/hr = 10 m/s

$V_B = -72$ km/hr $= -20$ m/s

$V_{MA} = -1.8$ km/hr $= -0.5$ m/s

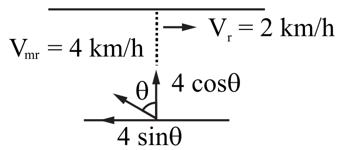
$V_{man, B} = V_{man, A} + V_{A, B}$

$= -0.5 + 10 - (-20) = 29.5$ m/s

19. (*c*) $\frac{dx}{dt} = y; \frac{dy}{dt} = x$

$\frac{dx}{dy} = \frac{y}{x} \Rightarrow y^2 = x^2 + c$

20. (*d*)



For swimmer to cross the river straight

$\Rightarrow 4\sin\theta = 2$

$\Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = 30^\circ$

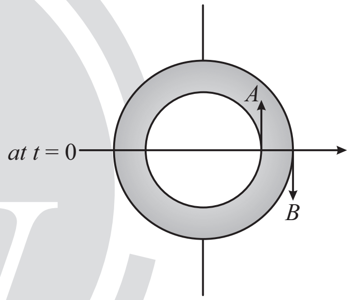
So, angle with direction of river flow = 90° + θ = 120°

21. (*c*) Time taken to reach maximum height

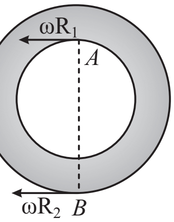
$t = \frac{u\sin\theta}{g} \quad \therefore \frac{u_1\sin\theta_1}{g} = \frac{u_2\sin\theta_2}{g}$

$\Rightarrow u_1\sin 30^\circ = u_2\sin 45^\circ \quad \Rightarrow \frac{u_1}{u_2} = \frac{1/\sqrt{2}}{1/2} = \frac{\sqrt{2}}{1}$

22. (*c*)



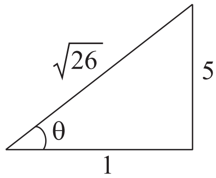
at $t = \frac{\pi}{2\omega}$



$\vec{v}_A - \vec{v}_B = -\omega R_1\hat{i} + \omega.R_2\hat{i} = \omega(R_2 - R_1)\hat{i}$

23. [5] Given, $u_x = \hat{i}, |u_x| = 1$

$y = x5(1-x) = x\tan\theta\left(1 - \frac{x}{R}\right)$



tanθ = 5, $R = 1$

$$\tan\theta = \frac{|4u_x|}{|4u_y|}$$

$$|4u_y| = |4u_x|\tan\theta = 1 \times 5 = 5 \text{ m/s}$$

24. [15or75] $R = \dfrac{u^2 \sin(2\times 45^\circ)}{g} = \dfrac{u^2}{g}$

$$\frac{R}{2} = \frac{u^2}{2g} = \frac{u^2\sin 2\theta}{g}$$

$$\Rightarrow \sin 2\theta = \frac{1}{2} \Rightarrow \theta = 15^\circ, 75^\circ$$

25. [5] $u_x = 1$

$$y = 5x(1-x)$$

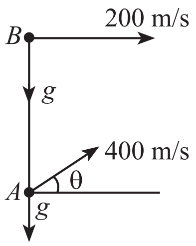
$$\frac{dy}{dt} = 5\frac{dx}{dt} - 10x\frac{dx}{dt}$$

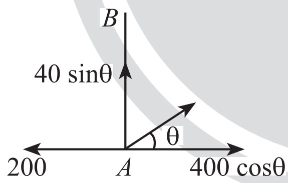
For initial y-component of velocity

$$u_y = \left(\frac{dy}{dt}\right)_{x=0} \Rightarrow 5(1) = 5$$

$$\vec{u}_y = 5\hat{j}$$

26. [60] If A hits B





Then relative velocity perpendicular to the line joining A to B will be zero.

$\Rightarrow 400\cos\theta = 200$

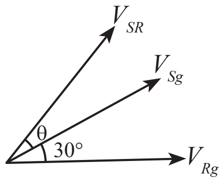
$$\cos\theta = \frac{1}{2}$$

$$\theta = 60^\circ$$

27. [30] Both velocity vector are of same magnitude. Therefore resultant would pass exactly midway through them.

So, $\theta = 30^\circ$

Hence, angle θ with the line AB is 30°



28. [580]

$X_1 = -3t^2 + 8t + 10$

$\vec{v}_1 = (-6t + 8)\hat{i} \;\; = 2\hat{i}$

$Y_2 = 5 - 8t^3$

$\vec{v}_2 = -24t^2\hat{j}$

$\sqrt{v} = |\vec{v}_2 - \vec{v}_1| = |-24\hat{j} - 2\hat{i}|$

$\sqrt{v} = \sqrt{24^2 + 2^2}$

$v = 580$ m/s

29. [1] Time of flight is same

$\Rightarrow$ Vertical component of velocity is same

$\Rightarrow H_{max}$ is same

30. [15] $R_{max} = \dfrac{u^2 \sin 2(45^\circ)}{g} = \dfrac{u^2}{g}$

$$\frac{R}{2} = \frac{u^2}{2g} = \frac{u^2 \sin 2\theta}{g}$$

$$\sin 2\theta = \frac{1}{2}$$

$2\theta = 30^\circ, 150^\circ$

$\theta = 15^\circ, 75^\circ$