Adaptive Iterative Quantum Amplitude for Value at Risk Estimation

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(Asymptotically) Free Qubits

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Contents

Defining Value at Risk (VaR)

The **Value at Risk (VaR)** of an investment indicates the maximum expected loss over a specified time period under normal market conditions, defined by a confidence level.

$$VaR_{1-\alpha} \equiv F_L^{-1}(\alpha)$$

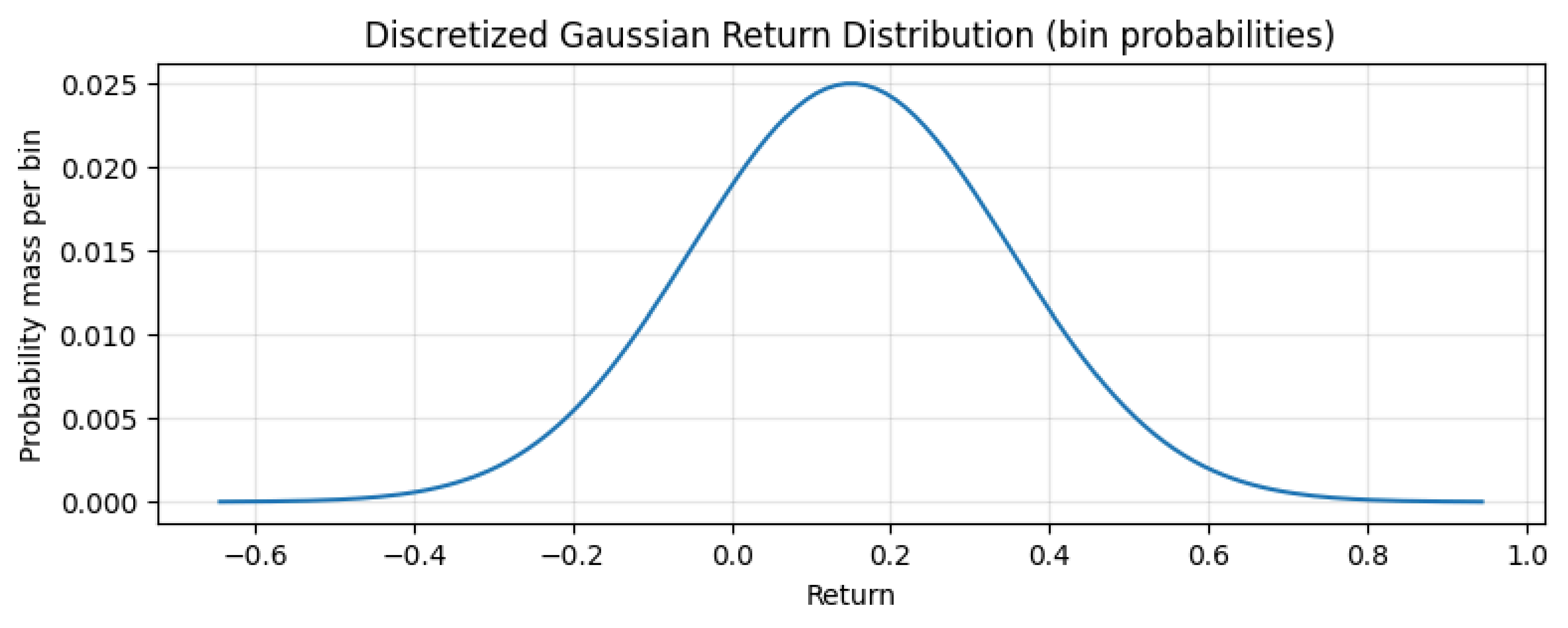
Time is typically measured in **years**. The return of an asset from time t to t+1 is calculated as:

$$R_t = (V_{t+1} - V_t) / V_t$$

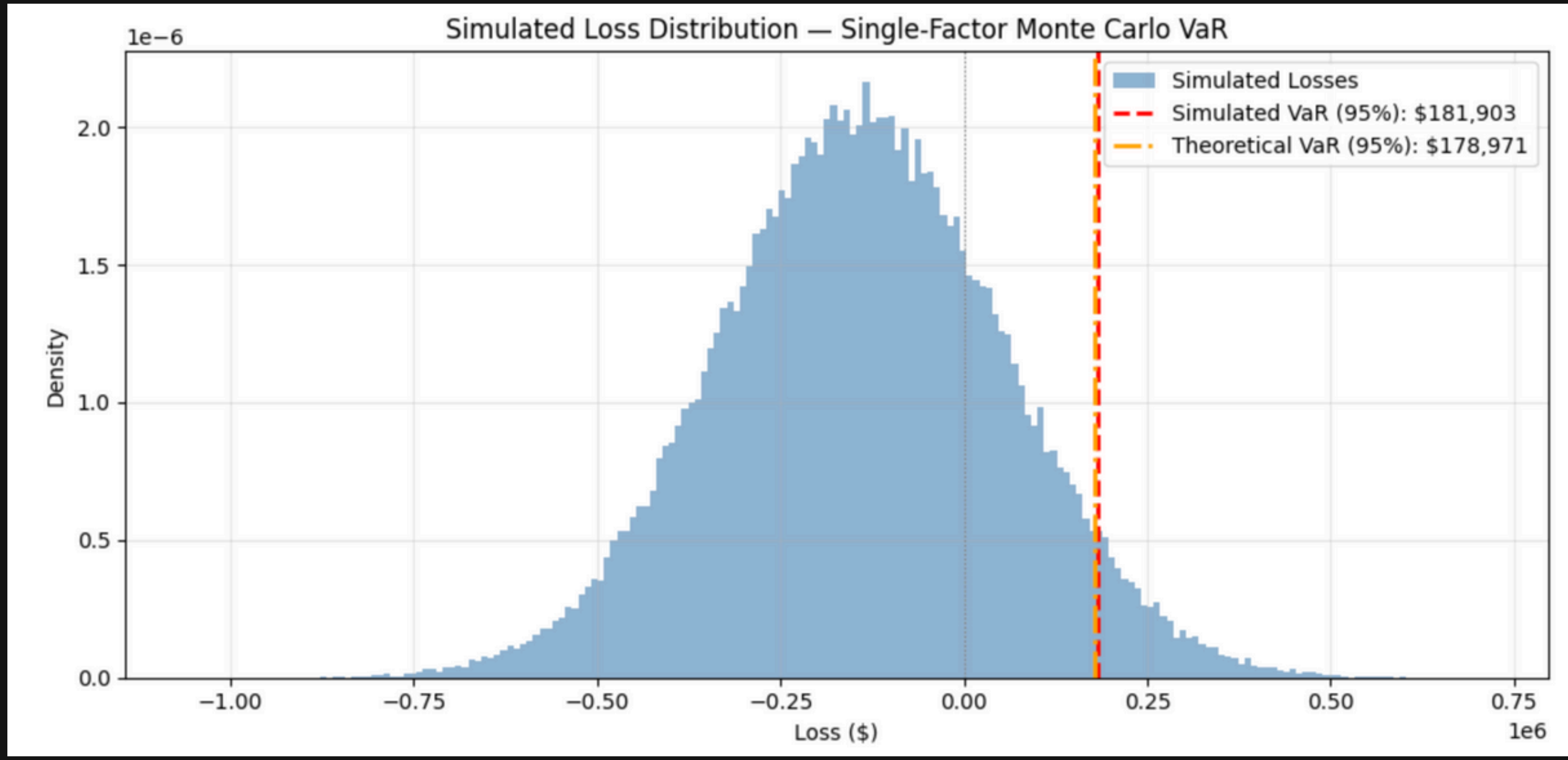
The asset return distribution represents the probability distribution of R_t, estimated using historical data, structural models, or simulations.

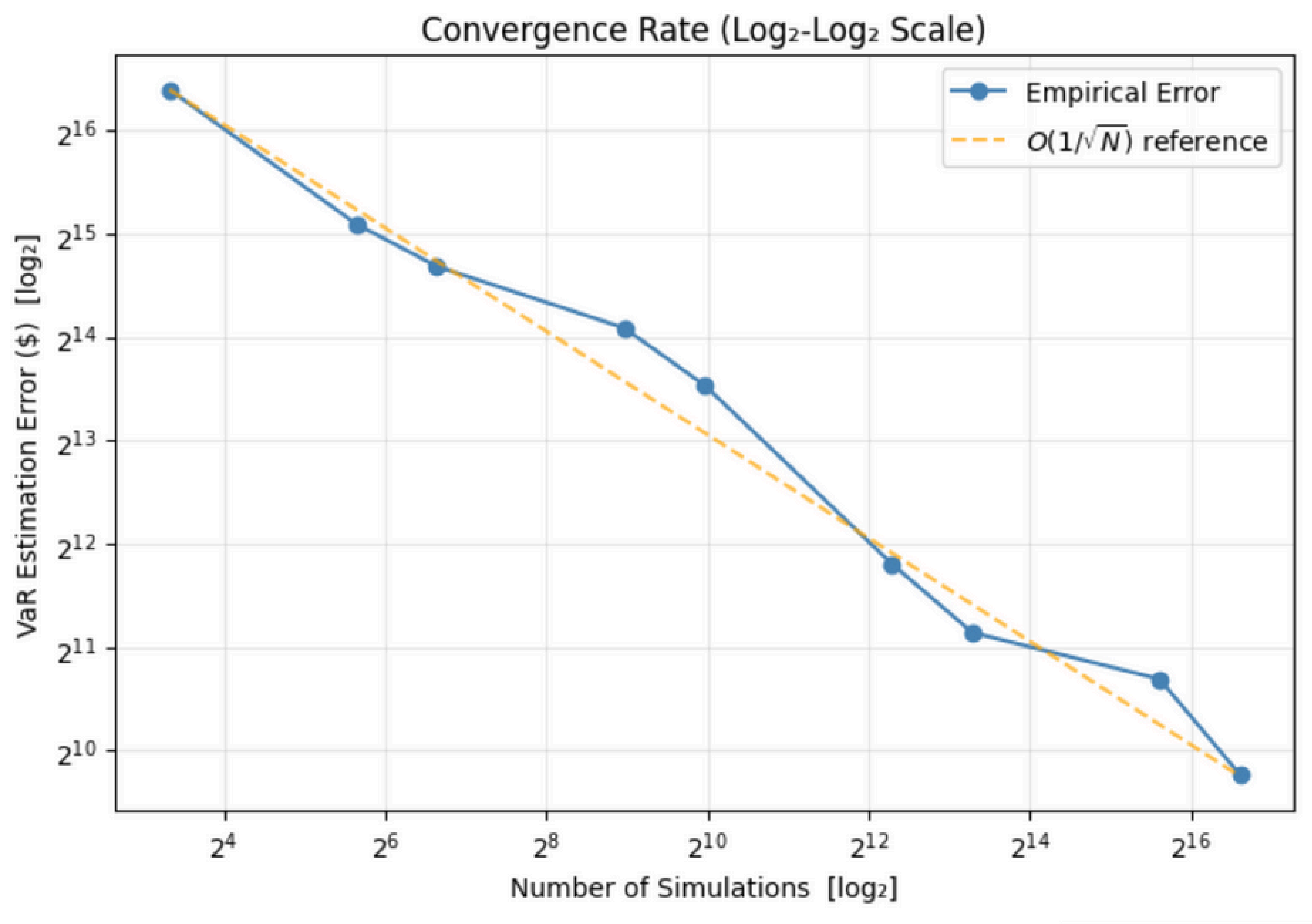
Return Distribution

- Discretize Gaussian distribution into **2^n bins** and use the **bin probabilities** as the target amplitude distribution
- Truncates after a few σ, leading to **modelling/discretization error**



Monte Carlo Estimation of VaR





Quantum Amplitude Estimation (QAE)

Step 1. Quantum State Preparation

$$R \sim \mathcal{N}(\mu, \sigma^2)$$

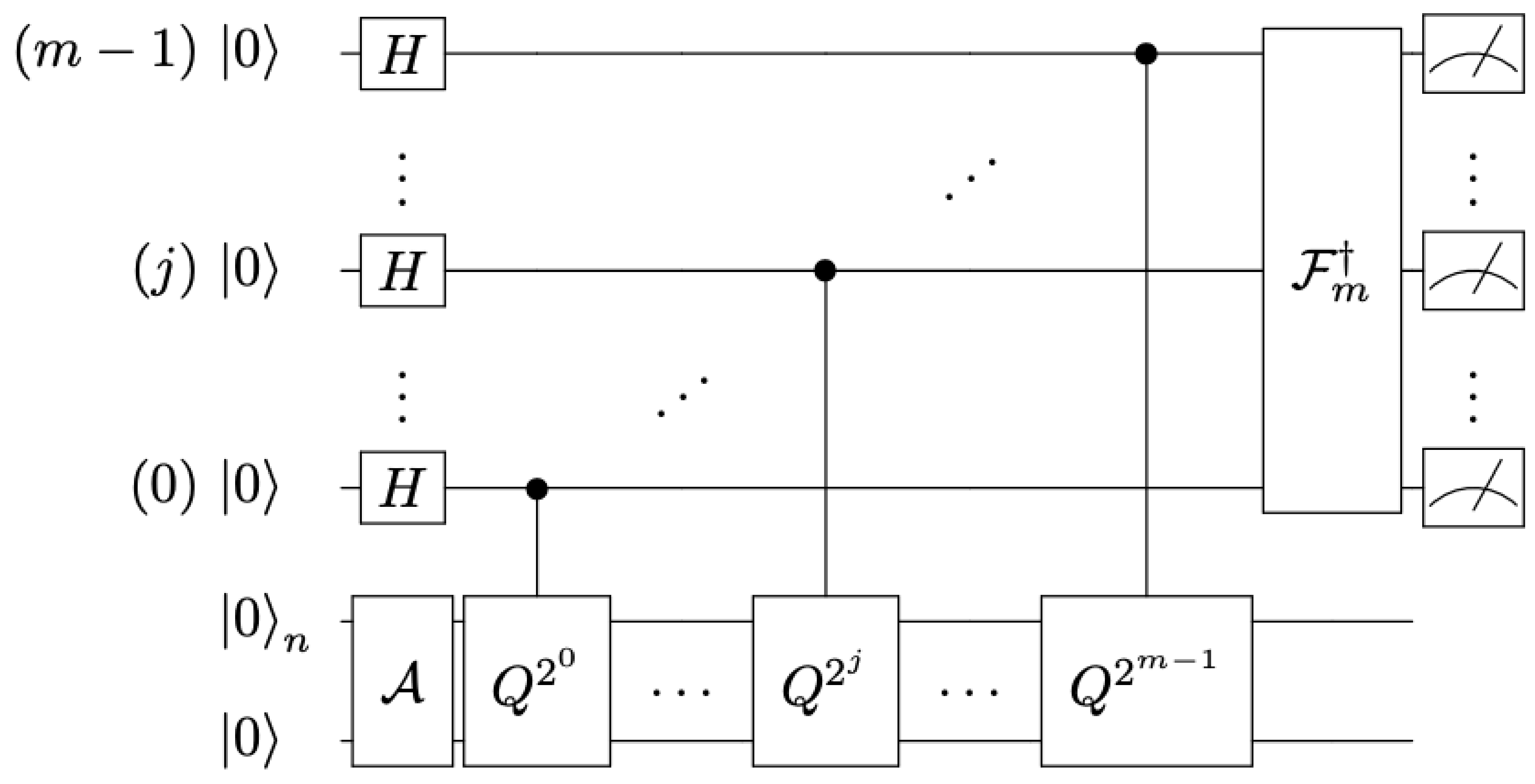
Discretized Normal Distribution

$$|\psi\rangle = \mathcal{A}\,|0\rangle_n\,|0\rangle = \sum_{i<\text{thres}}^{N} \sqrt{p_i}\,|i\rangle\,|0\rangle + \sum_{i\geq\text{thres}}^{N} \sqrt{p_i}\,|i\rangle\,|1\rangle$$

Encoding operator acting on indicator qubit

Quantum Amplitude Estimation (QAE)

Step 2. Quantum Phase Estimation of Grover Operator



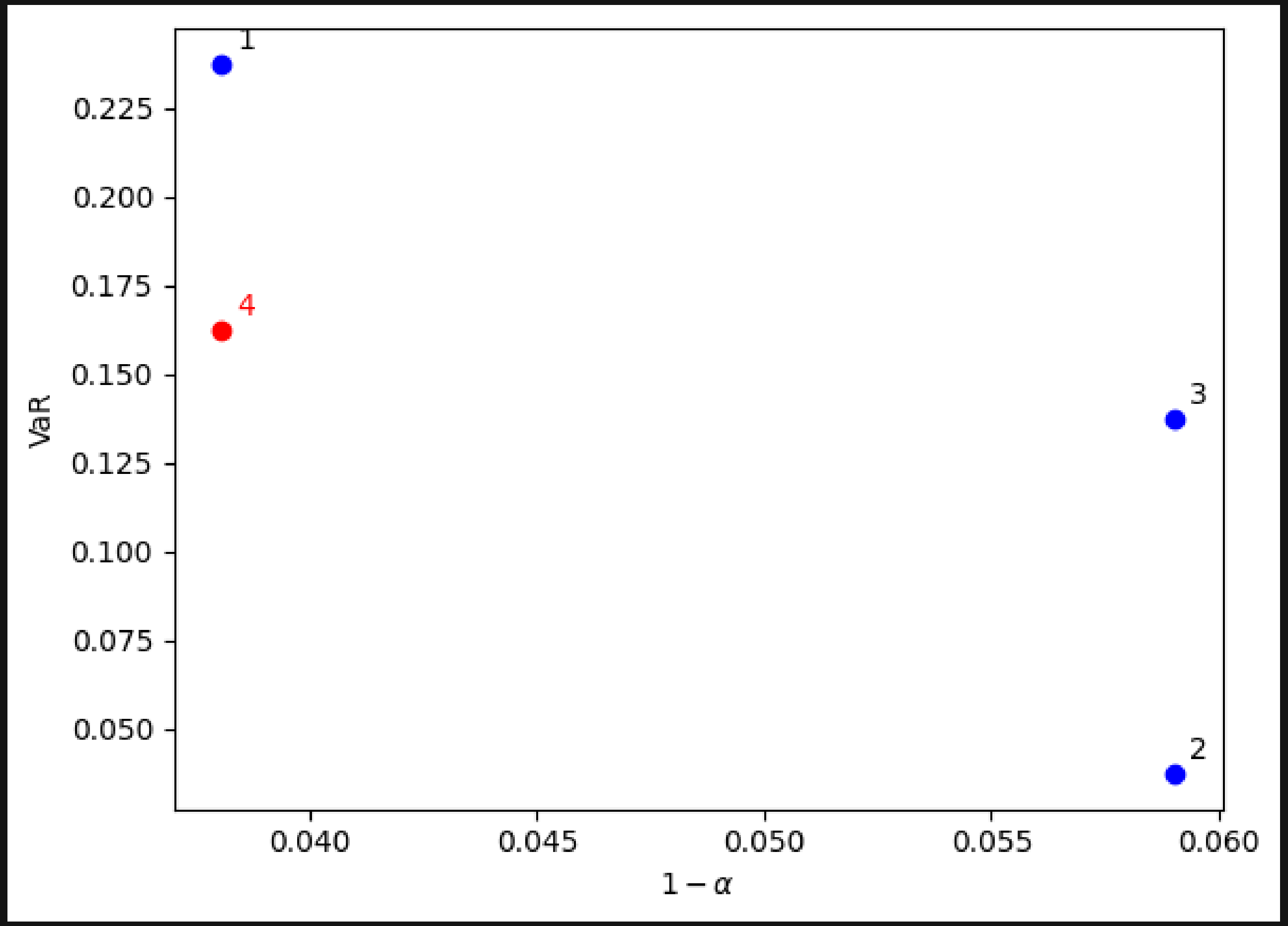
Brassard et. al. (2000), Grinko et. al. (2021)

$$\mathcal{Q} = \mathcal{A}\mathcal{S}_0\mathcal{A}^\dagger\mathcal{S}_{\psi_0}$$

Quantum Amplitude Estimation (QAE)

Step 3. Bisection Search

- After finding the estimated α, use **bisection search** to look for the VaR.
- Repeat until we reach our precision parameter

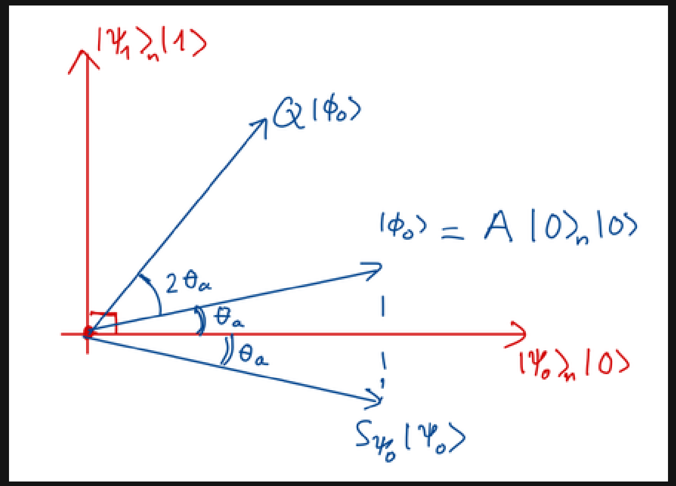


Simulation using QAE for each data (discritizing using n = 6 qubits).

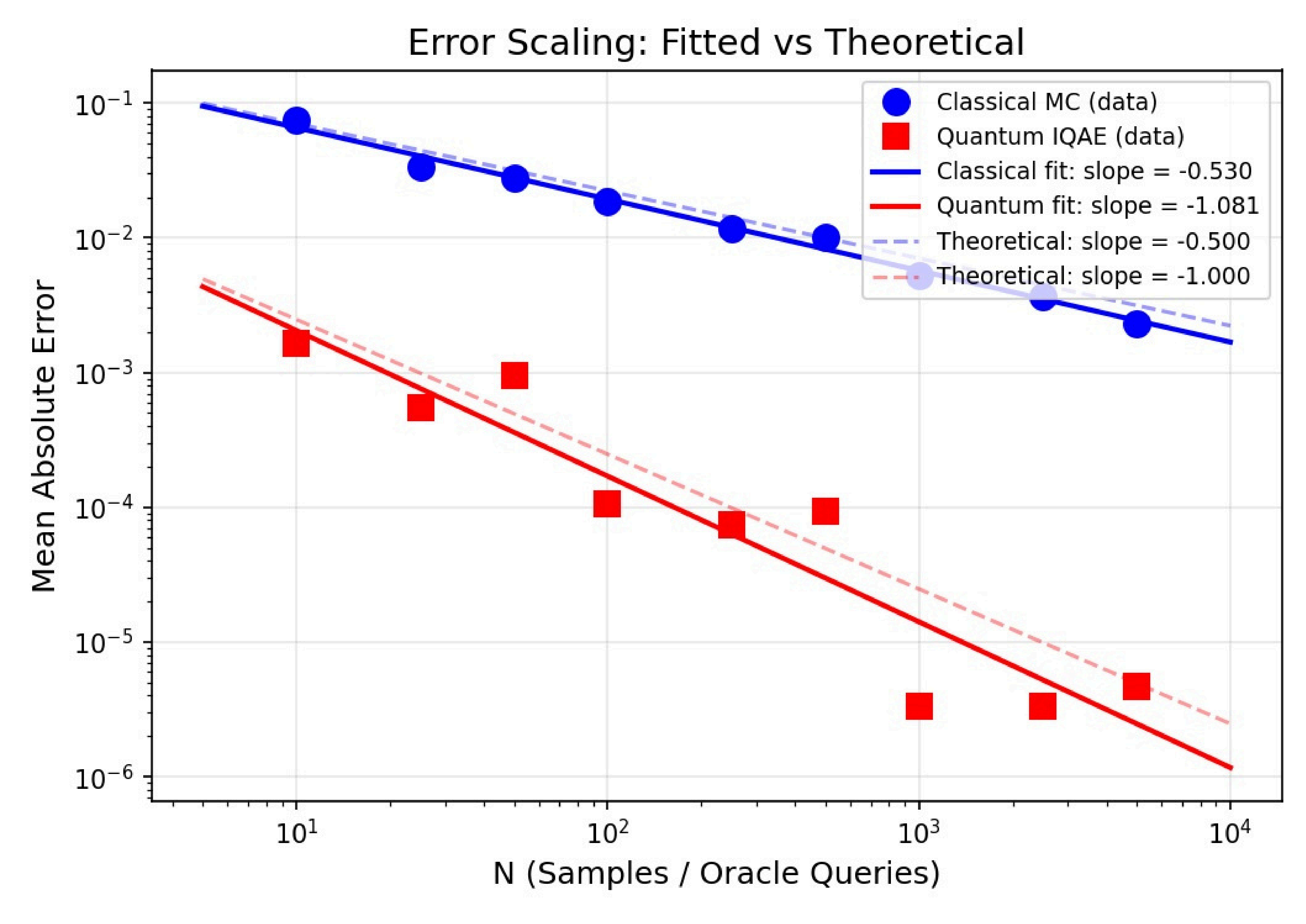
VaR = 16.25

13% off the theoretical value.

Iterative QAE (IQAE)



- No QPE, run a sequence of simpler circuits
- Update a confidence interval iteratively using the Grover operator until reaching the desired precision



Error Scaling Comparison

MC theoretical Error

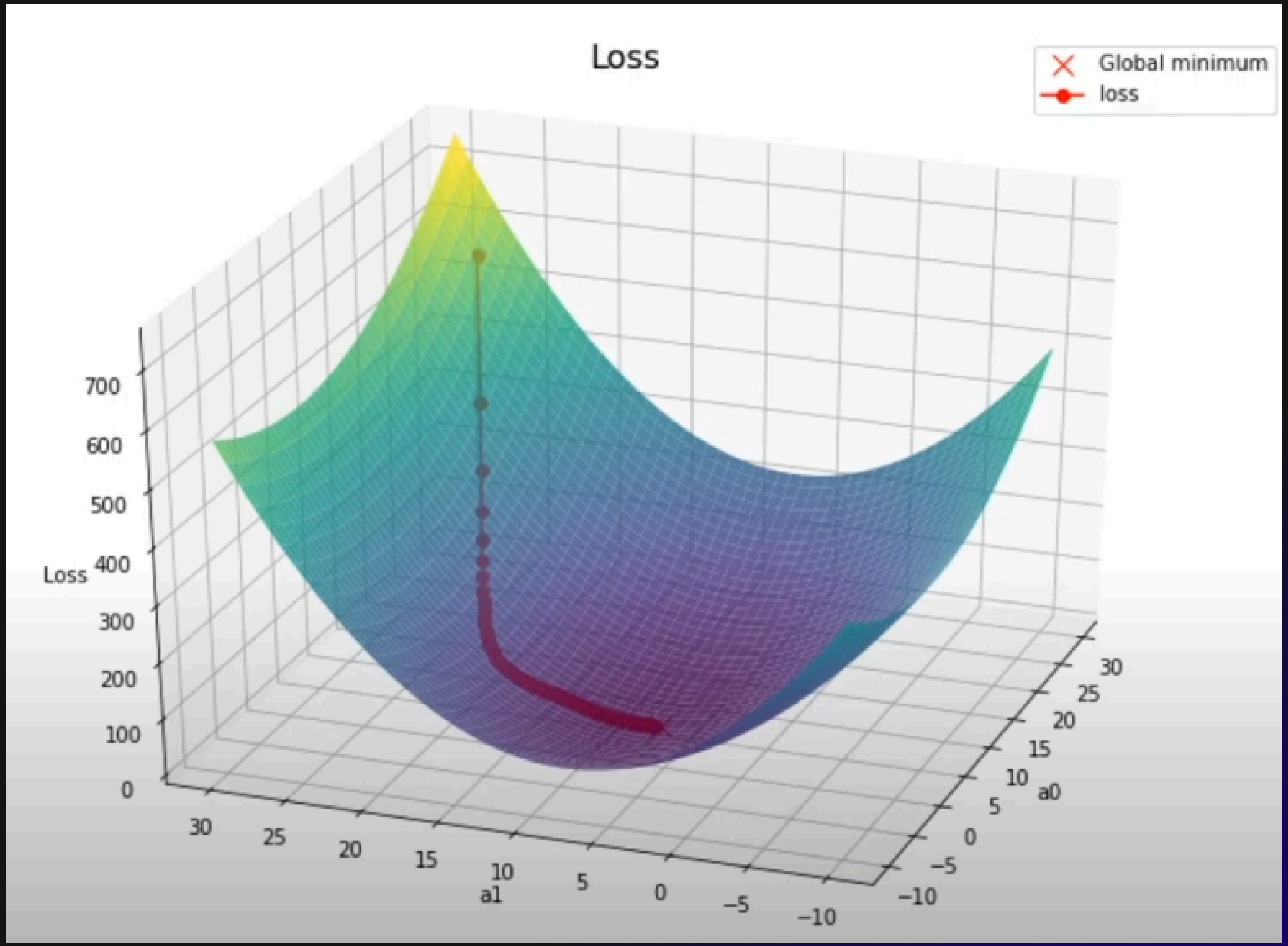
$$\frac{1}{\sqrt{N}} \approx \epsilon.$$

IQAE theoretical Error

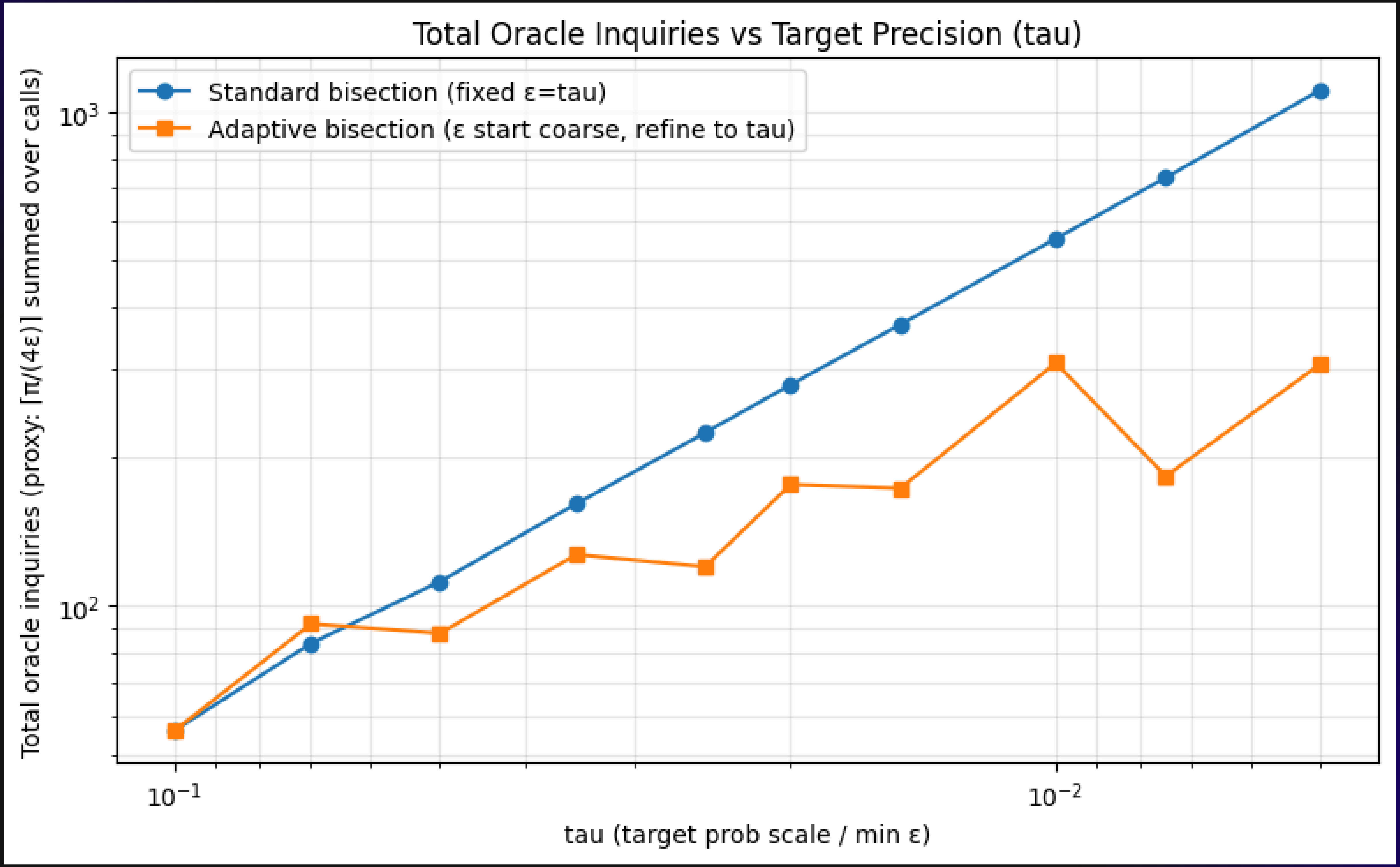
$$N_{\text{oracle}} < \frac{50}{\epsilon} \log\left(\frac{2}{\alpha} \log_2\left(\frac{\pi}{4\epsilon}\right)\right).$$

Adaptive IQAE

- Recall in IQAE: Update a confidence interval iteratively until you reach the desired precision
- Now, instead of a fixed precision, we take inspiration from **momentum gradient descent**



https://medium.com/@pumadd1227/adam-vs-sgd-what-are-the-optimizers-in-neural-network-and-when-do-we-use-238478a0eaea



Total Oracle Inquiries vs Target Precision (tau)
Standard bisection (fixed ε=tau)
Adaptive bisection (ε start coarse, refine to tau)
Total oracle inquiries (proxy: ⌈π/(4ε)⌉ summed over calls)
10^3
10^2
10^{-1}
10^{-2}
tau (target prob scale / min ε)

Conclusion

- Calculated modeling error by comparing discretized and continuous distribution
- Found VaR via QAE and bisection search
- Demonstrated $1/\epsilon^2$ for MC and $1/\epsilon$ for IQAE
- **Adaptive IQAE**
- Performed MC and IQAE on fat tail distribution (t-distribution)
  - found that for MC, it converges slower compared to normal distribution
  - found that for IQAE, even though N still scale with $1/\epsilon$, the linear fit is noisy (R^2=0.68)

Thank You!