Kepler's Laws & the Mass of Jupiter – V2.1 ASTR 1010

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Overview:
In this activity you will calculate the orbital periods of the Galilean moons. Using Isaac Newton's updated version of Kepler's Third Law of Planetary Motion for two bodies, you will also calculate the mass of Jupiter.

Objectives:
After completing this activity, students will be able to:
• Use Microsoft Excel to graph the orbital period of a celestial body.
• Compare the orbital periods of other moons in the Solar System to our Moon.
• Calculate the mass of Jupiter using orbital parameters of the Galilean moons.

Definitions
Here are some terms from lecture that we will be using today in lab:

• **Galilean Moons**: the four largest moons of Jupiter – Io, Europa, Ganymede, and Callisto. Observed by Galileo in late 1609/early 1610.

• **Period**: the length from one peak/trough to the next (or from any point to the next matching point) of a periodic function (see figure in Part 2).

• **Amplitude**: the height from the center line to the peak or trough of a periodic function. (see figure in Part 2).

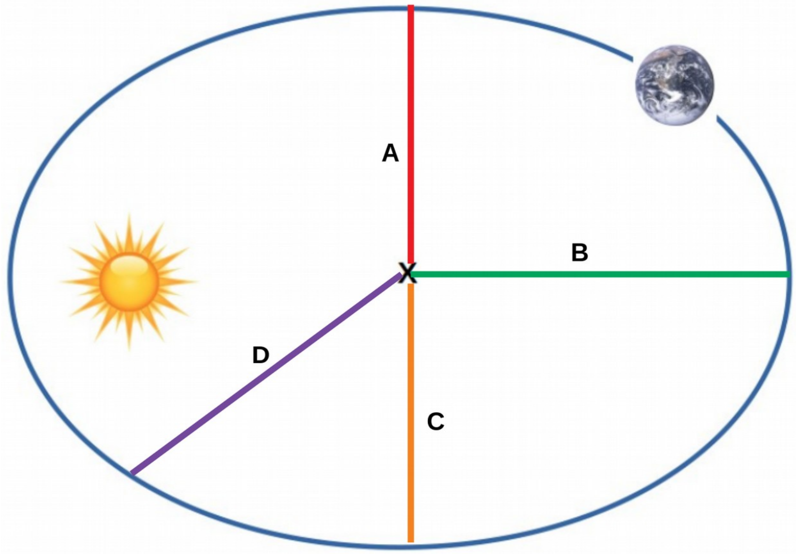
• **Kepler's First Law:** All planets orbit the Sun in an ellipse, and the Sun is at one foci.

• **Kepler's Second Law:** As a planet orbits the Sun, it sweeps out equal areas in equal time.

• **Kepler's Third Law:** $\mathbf{P^2 = a^3}$, where **P** is the orbital period in YEARS, **a** is the semi-major axis or radius of the orbit in AUs

Part 1: Reviewing the Three Laws

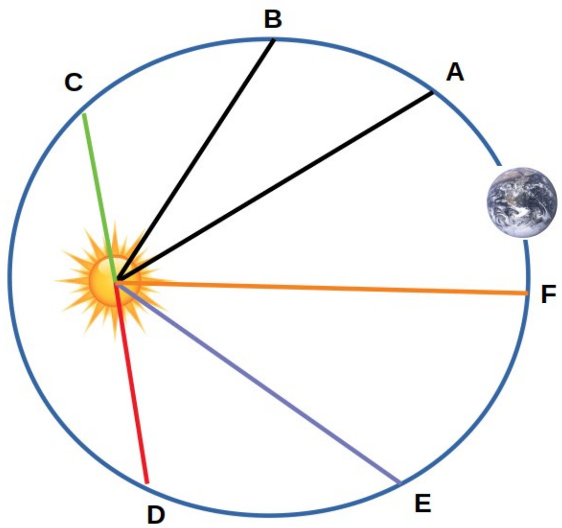
1)Consider the Figure below of an Earth-like planet orbiting a star. **X** is the center of the ellipse. Match each letter with the corresponding item. Note that some items may have multiple correct answers.



a)The semi-major axis: B, D

b)The semi-minor axis: A, C

2)Now consider the figure below. As the planet orbits the star, it takes it 2 months to go from A to B, 1 month to go from B to C, 2 months to go from C to D, 2.5 months to go from D to E, 3 months to go from E to F, and finally 3.5 months to go from F to A. Order the area swept from <u>largest to smallest</u>. For example, if A to B is greater than B to C, you would write: A-B > B-C. If A-B were equal to B-C you would then write: A-B = B-C. The areas are: A-B, B-C, C-D, D-E, E-F, F-A.

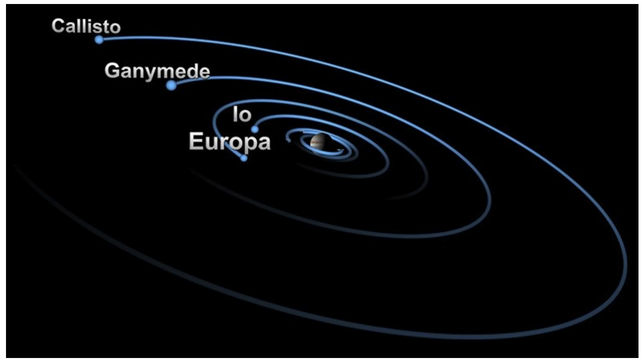


F-A > E-F > D-E > C-D, A-B > B-C

3)Neptune is about 30 AUs from the Sun. What is its period? Show your work, and leave your answer in years.

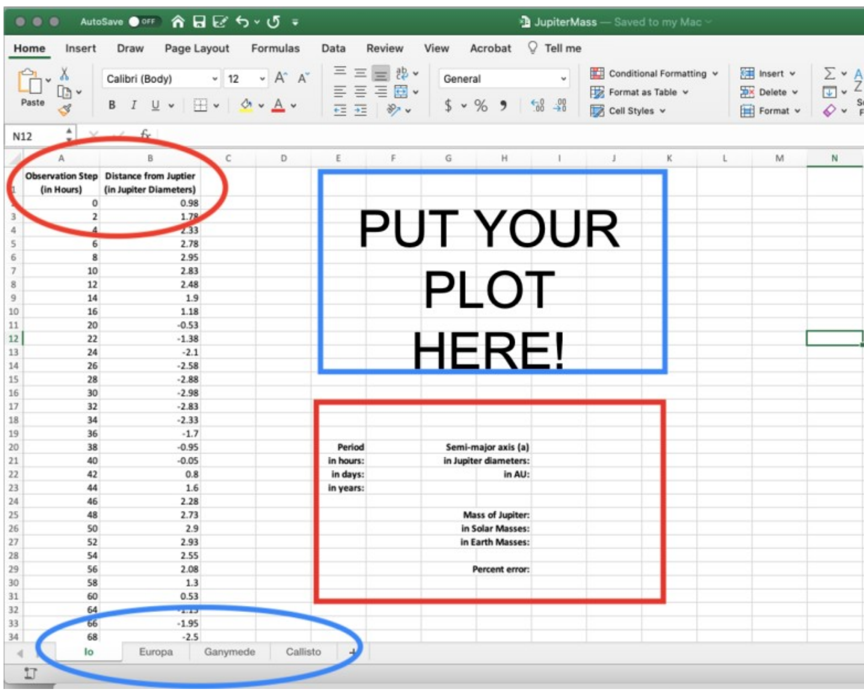
T^2 = a^3, T^2 = 30^3 —> T^2 = 27000 —> T = 164.32 years

Part 2: Graphing Orbital Periods of the Galilean Moons



On the left is a NASA collage of actual images that shows Jupiter and its moons scaled proportionally with respect to each other. Note how huge Jupiter's Great Red Spot is relative to the moons! On the right is an illustration that shows the orbits.

You have been provided with an Excel file that has measurements recorded for you. Here is a screen capture of the Excel file. Let's walk through the different parts:



• Red Circle: Data columns. This is the data you will use to create your plot.

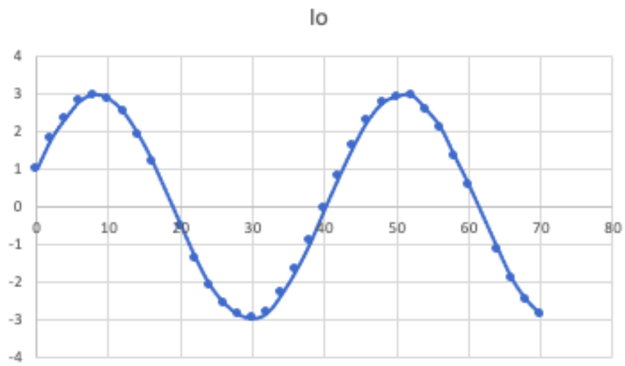
• Blue Square – Your plot, put it here!

• Blue Circle – Each moon has its own tab. You must plot and do calculations for the moons specified!

• Red Square – complete your calculations here. This is where your TA will be grading your calculations, so please make sure you place them in the correct spot!

To make the plot:
a. Select the data in columns A (x-axis, time) and B (y-axis, distance from Jupiter. +/- indicate if the moon is being observed on the left or right side of Jupiter) in your Excel file (minus the first row!).
b. On the menu bar, click on 'Insert', click on 'Scatter', 'Scatter with Smooth Lines and Markers'.
c. Place plot in requested area and change the title to the name of the moon you are analyzing.

To verify that you are on the right track, your plot for Io should look like this: →



If your plot for Io looks good, complete the plot for **Callisto**. Your x-axis will be in units of hours and your y-axis will be in units of Jupiter diameters (1 Jupiter diameter is about 140,000 km).

Before you start calculating, consider your plots, and the answer the following thought questions (no calculations need):

4)Looking at both of your plots, what do you notice about the differences in values on the x-axis. How do you think difference will influence the orbital periods?

The x-values representing hours indicate a moon ' s speed and orbital period. In this case, Io ' s range is around 420 hours shorter than Callisto's, indicating Io ' s shorter orbital

5)Looking again at both plots, now consider the y-axis. What orbital parameter do you think is influenced by the maximum/minimum y-values?

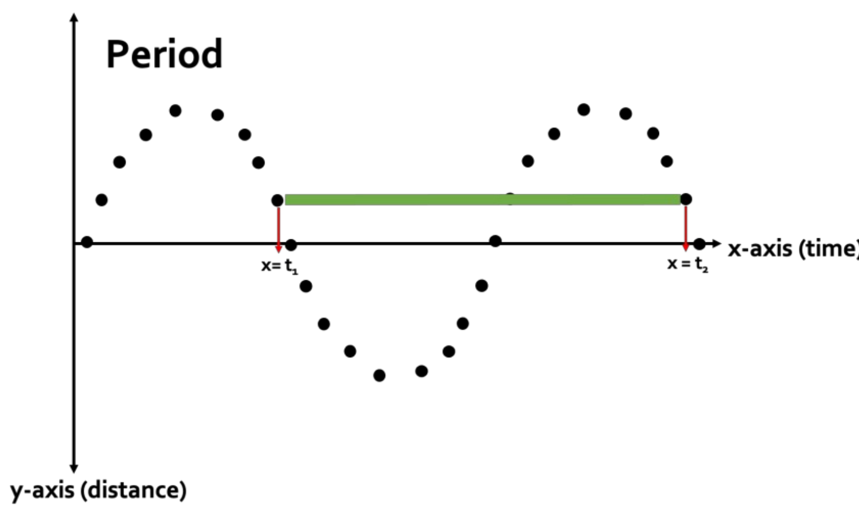
The y-values representing each moon ' s distance from Jupiter represent the semi-major axis. Callisto has a larger range, indicating that it orbits farther from Jupiter than Io.

6)Based on your answers for Questions 4 and 5, which moon do you think is closest to Jupiter, and which is farthest? How do you know?

Io is closer to Jupiter than Callisto because it has a smaller range of y-values (distance from Jupiter)

Part 3: Calculating Orbital Parameters

Now that your plots are ready, let us find the orbital parameters. First, we need the orbital period. The **period** is the length between repeating intervals. The figure on the right shows the period, indicated by the green line, starting at t_1 and ending at t_2 . Look at your plot. Find the x-values that have matching (or very nearly matching) y-values and record the x-values in t_1 and t_2 in your Excel sheet. Then, next to the cell called "in hours", calculate the period (in hours) using Equation 1:



$$P = t_2 - t_1 \text{ (Equation 1)}$$

To complete your final calculation, you need your period to be in years, so first convert hours to days by dividing your value in hours by 24 and record that next to the “in days” cell, and then divide your value in days by 365 to convert your orbital period into years. Record that value next to the “in years” cell. Calculate orbital periods for all of the moons.

7)Which of the two Galilean moons has the shortest period and which has the longest?

Io has the shortest period consisting of about 0.0048 years and Callisto has the longest consisting of 0.047 years

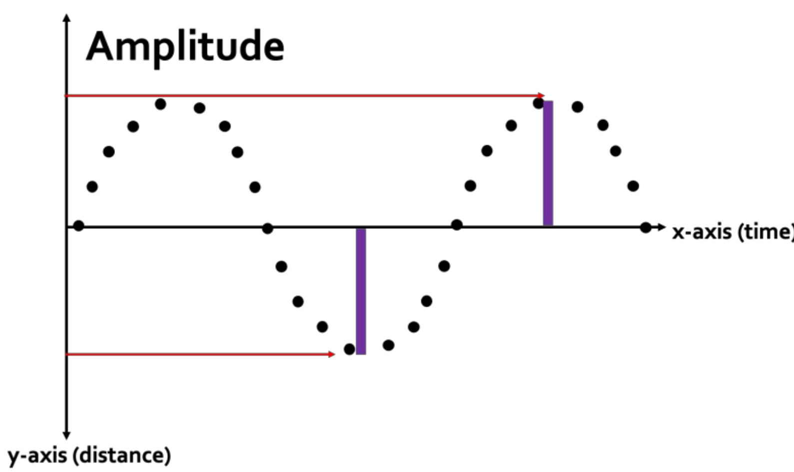
8)Now, we want to calculate how many times longer or shorter, the orbital period of the Galilean moons are compared to the Earth’s Moon. To do this, take the period of our Moon (P1) and divide it by the period you obtained for the Galilean moons (P2). Complete the table:

Moon	Period	P1 / P2
Earth’s Moon	27 days	N/A
Io	15.42857143	27 days/1.75 days
Callisto	1.588235294	27 days/17 days

9)Are the answer to Q8 surprising? Explain.

Yes, I ’ ve learned a lot of Jupiter having more than one moon throughout high school but never thought much about its orbital periods or how long/short it would be compared to our time with the Moon.

Next, you need to calculate your **amplitude**. Amplitude is the height from the center line to the peak or trough of a periodic function (but *not* from trough to peak). The figure on the right shows the amplitude for both the peak and trough. The easiest way to find the amplitude in your data is to look at the “Distance from Jupiter” column in your data and find the largest number (positive or negative). Record that value next to “in Jupiter diameters”.



→ If your amplitude value was negative,take the absolute value and make it positive.

In order to correctly calculate the mass of Jupiter, the semi-major axis value needs to be in AU instead of Jupiter diameters. Multiply your amplitude value by **9.545×10^{-4}** to convert from Jupiter diameters to AU. Calculate and convert the amplitudes for all of the moons in your Excel sheet.

10) Which of the two Galilean moons is closest to Jupiter and which is the farthest?

Io is closest around 0.0028 AU and Callisto is the farthest with 0.013 in AU

Part 4: Calculating the Mass of Jupiter

To calculate Jupiter's mass, we need to consider Newton's version of Kepler's 3rd law. In that equation, you must account for the masses and it is more complicated. Fear not! Fortunately, we are dealing with a large planet and a tiny moon. Therefore, we can assume that the moon has almost no relative mass to Jupiter. We will use this equation: $P^2(M_1 + M_2) = a^3$ where M_1 is Jupiter's mass and M_2 the mass of the moon, which we set to zero. This simplifies our equation to $M_1 \times P^2 = a^3$. Rearranging this, you will use Equation 2 to solve for the mass of Jupiter:

$$M_J = \frac{a^3}{P^2} \text{ (Equation 2)}$$

Where **a** is the semi-major axis in AU and **P** is the orbital period in **YEARS**. Go back to your spreadsheet and calculate Jupiter's mass with each of the moons in solar masses ($M_\odot$). Let's now compare Jupiter's mass to the mass of the Earth. The Earth has a mass of 3.0×10^{-6} $M_\odot$. To convert your Jupiter masses from solar mass to Earth mass ($M_\oplus$), divide your mass of Jupiter by the mass of Earth.

11) Calculate Jupiter's mass. Show your work.

Io: (0.00284441)^3/(0.0048)^2 = 9.988 x 10^-4 M. Callisto: (0.012647125)^3/(0.0434)^2 = 0.00107 M. Europa: (0.004514785)^3/(0.00959)^2 = 0.00100 M. Ganymede: (0.00715875)^3/(0.01)^2 = 0.00367 M.

Jupiter is sometimes referred to (incorrectly) as a failed star. The smallest known stars have a mass of around 10% of the Sun. Using your measurements, let's find out how many times more massive Jupiter would need to be in order to be a true star (this is covered in detail in ASTR 1020)!

12) Choose your favorite of the two Galilean moons, and then calculate how many times more massive would Jupiter need to be in order to be a star? Show your work.

(0.1 M)/0.00107 = 93.46. Jupiter would need to be around 93.46 times more massive in order to be a star.