Generative AI vs. Human Feedback: A Case Study on Formative Feedback Pedagogical Quality in STEM Higher Education
[Supplementary materials]

Task

Let x be a real number. Prove that if x^2 is irrational, then x is irrational using

a) a proof by contraposition

b) a proof by contradiction

Detail the two proofs in a way that shows that you understood the difference between a proof by contraposition and a proof by contradiction.

Task Correction

a) A proof by contraposition of an implication consists in showing that if x is rational, then x^2 is rational.

1. x is rational [Assumption]
2. Then we have x = a/b, where a and b\neq 0 are integers with no common factors. [Definition of a rational number]
3. Therefore, x^2 = a^2/b^2. [Squaring]
4. Hence, x^2 is rational. [Definition of a rational number]
5. By contraposition, if x^2 is irrational, then x is irrational.

b) A proof by contradiction of an implication consists in showing that assuming that [if x^2 is irrational, then x is irrational] is false leads to a contradiction.

1. x^2 is irrational. [Assumption]
2. x is rational [Assumption]
3. Then, $x = a/b$, where a and $b\neq 0$ are integers with no common factors. [Definition of a rational number]
4. Therefore, $x^2 = a^2/b^2$. [Squaring]
5. Hence, x^2 is rational. [Definition of a rational number]
6. Assuming that x is rational hence leads to a contradiction (line 1 \& line 5). We therefore conclude that if x^2 is irrational, then x is irrational.

AI Feedback Generation – Generic Prompt

You are an excellent instructor teaching a course called [COURSE NAME]. You gave the students the following assignment: [ASSIGNMENT]. The student submission was [STUDENT ANSWER]. The correct solution is [SOLUTION]. Please evaluate the student's answer and provide elaborated formative feedback. Please follow the following instructions: The feedback should be addressed directly to the student as is. It should be no more than [NUMBER] lines. Please provide one sentence of the overall evaluation at the end.

Example of Student Response

1. CONTRAPOSITION

If x^2 is rational, then x is rational.

1. x is rational
2. x = 2k, k is rational
3. x^2 = 4k^2 = 2*(2k^2). Let's say that k' = 2k^2
4. x^2 = 2k'
5. x = sqrt(2k) = sqrt(2)*k

but sqrt(2) isn't rational so k is not rational. It means that x is not rational, then x^2 is not rational either. Therefore x is irrational and so is x^2.

2. CONTRADICTION

By contradiction, we suppose that x^2 is rational. It means that there exists two integers, a and b such that x = a/b. We then suppose that a and b have no factors in common. Then we obtain x^2 = a^2 / b^2 it implies that x^2 * b^2 = a^2. Or, if b is even, then so is b^2 and so is a^2. Same if b is odd. So there is a contradiction and there can't be any positive integers a and b such that x^2 = a^2 / b^2, and therefore x is irrational.

Example of 2 Feedback Variants

Human feedback

In your contraposition attempt, your understanding of what constitutes a rational number appears flawed. A rational number is expressed as the quotient of two integers, not just as a product involving an integer. Your step "x = 2k, k is rational" doesn't necessarily ensure x is rational.

In the contradiction approach, the logic connecting the evenness of b to a^2 isn't sufficiently explained, leading to an incorrect conclusion. Both methods require a clearer understanding of rational numbers and a more rigorous application of the respective proof techniques.
Your assumptions are unfortunately also nor right. For the contraposition you don't want to prove that "If x^2 is rational, then x is rational." but the inverse that "If x is rational, then x^2 is rational.". In proposition you say that p --> q is equivalent to NOT q --> NOT p. So just proving that x rational implies that x^2 is also rational is enough to prove that x^2 irrational implies x irrational.
For the contradiction proof, your assumption is also not right. You don't want to prove that if x^2 irrational then x rational leads to a contradiction, but x^2 irrational AND x rational leads to a contradiction. The logic behind it is that what at the end you want to prove is that "x^2 irrational --> x irrational" and this is FALSE only if x^2 irrational is TRUE and x irrational is FALSE and thus by proving that x^2 irrational AND x rational is always FALSE then you prove that "x^2 irrational -- > x irrational" is always TRUE because it can never be FALSE. That is the proof by contradiction

AI-Generated Feedback

You've shown good understanding of how to approach proofs. However, in the contraposition proof, your steps 2 and 3 aren't clear - specifically your assumption that if x is rational then x=2k. Contraposition implies proving that if x is rational (i.e., x=a/b), then x^2 is rational. In the contradiction proof, while you made a good start by assuming x^2 as rational and deriving a contradiction for it, there's confusion when you mention "even" and "odd". Remember we're considering irrationality or rationality here and not parity. Keep going!

Feedback Evaluation Grading rubric – explanations of how the grading rubric is applied to the specific task

Effectiveness of the feedback	Category Explanation	In the context of our contradiction/contraposition task	Code
Task level Goals	Feedback reminds again the goals of the task	It reminds students that they must prove that ● "if x is rational, then x^2 is rational." ● Proof should be by contraposition/contradiction, respectively. ● Logical reasoning proof expected	**Task goals**
Task level Current state	It focuses on the submission's accuracy, completeness, and quality (e.g., corrections, annotations, or indications of errors and successes in the work submitted.) Helps the student understand how close they are to achieving the task's objectives and what aspects have been mastered.	Everything related to the current state of knowledge: ● What was correct/ incorrect in the answer ● Pointed out misconceptions/misunderstandings ● What was not clear from the answer ● What is missing from the answer ● What is in the answer but should not be there (some additional not relevant information) It can refer to the specifics of the task, general misunderstandings, or good knowledge of required concepts.	**Current state**
Task level Next steps	It helps students improve their future task performances and can suggest steps to correct errors /misunderstandings.	What should be done/improved/changed/ fixed in the solution referring to the specifics of the task, for example: ● x, x^2 ● definition of the rational number, GCD, and so on ● concrete steps in the task solution	**Task Next steps**
Specific strategy	Feedback provides specific strategies for solving the task or similar tasks (e.g., hints, tips)	It refers to strategies how to prove by contradiction or contraposition or any other logical proof, for example: ● The definition of contraposition as "p -> q" is equivalent to "not q -> not p". ● Names of the steps: e.g., "Premise," "Simplification."	**Strategy Next Steps**

		● Start with what you want to prove, continue with the assumptions and proof steps, and finally conclude your proof by addressing what should be proven.	
General learning strategy	Provides insights into how students can independently monitor and adjust their learning strategies. It focuses on developing students' ability to plan, reflect, self-assess their progress, and adjust as needed.	It refers to the learning strategies/ good practices students can use in generic learning scenarios. For example: ● Each step of your proof should follow from the previous one. ● Pay attention to the notation. ● Read your answer again before submission. ● Plan your time. ● Read lecture notes/books/ask for TAs help	**Self-Regulated Next Steps**
About self	Encouraging and positive comments		**Praise**

Examples of feedback evaluation per sentence:

Student solution	Feedback	Feature	Correctness	Readability
1. CONTRAPOSITION R(X) : X IS RATIONAL 1) R(X) / PREMISE 2) R(A/B) FOR SOME A, B, B!= 0 INTEGERS / EXISTENTIAL INSTANT 3) R(A^2) AND R(B^2) 4) X^2 = A^2 / B^2 5) R(X^2) 2. CONTRADICTION	You showed good understanding of contraposition,	Current state Praise		
	however for the future you need to explain at the beginning where you want to go with this proof.	Strategy Next Steps		
	You could start with something like "If we can prove that NOT q --> NOT p it always TRUE then by contraposition this will implie that p --> q is also always TRUE:".	Strategy Next Steps		
	And more important you need to had a conclusion to your proof and rethink "What am i trying to prove here and did I show that I proved this statement"	Strategy Next Steps		

	, here you need to prove that x^2 irrational --> x irrational	Task goals		
	but you only proved x rational --> x^2 irrational.	Current state		
	You need to show and write that NOT q --> NOT p is equivalent to p -->q and thus by contraposition you proved that x^2 irrational --> x.	Task Next steps		
	For future correction, try to do it some points are attributed to proof of comprehension of the concepts and not just application of these concepts.	Strategy Next Steps		Not readable
1. CONTRAPOSITION R(X) : X IS RATIONAL 1) R(X) / PREMISE 2) R(A/B) FOR SOME A, B, B!= 0 INTEGERS / EXISTENTIAL INSTANT 3) R(A^2) AND R(B^2) 4) X^2 = A^2 / B^2 5) R(X^2) 2. CONTRADICTION	You've started correctly by identifying the type of proofs.	Current state	Incorrect	
	But, you need more clarity and correctness in your steps for both methods.	Strategy Next Steps	Incorrect	
	For contraposition, ensure that in step 4 you express x^2 as $(a/b)^2$.	Task Next steps		
	In contradiction proof, make sure to assume that x is rational but x^2 is irrational and show how it leads to a contradiction.	Task Next steps		
	Keep practicing!	Praise		
CONTRAPOSITION to prove p->q we prove that NOT q-> NOT p 0) x is rational [Premise] 1) x = a/b [Simplification from 1.] 2) x POWER 2 = (a/b) POWER 2 [Simplification from 2.] 3) x POWER 2 = (a POWER 2) / (b POWER 2) [Simplification from 3.] 4) x POWER 2 is rational 5) x is rational -> x POWER 2 is rational	Great job!	Praise		
	Your understanding of proof by contraposition and contradiction is quite clear.	Task Next steps		
	Just a minor suggestion: when writing your proofs, avoid using phrases like "simplification from" as these might not convey the exact operation you performed.	Strategy Next Steps		

6) x POWER 2 is irrational -> x is irrational [Conclusion] 2. CONTRADICTION we show that NOT (p->q) leads to a contradiction NOT (p->) EQUIV p AND NOT q 0) x POWER 2 is irrational AND x is rational [Premise] 1) x POWER 2 is irrational AND x = a/b [Simplification from 0.] 2) x = a/b [Simplification from 1.] 3) x POWER 2 = (a/b) POWER 2 [Simplification from 2.] 4) x POWER 2 = (a POWER 2) / (b POWER 2) [Simplification from 3.] 5) x POWER 2 is rational [Simplification from 4.] 6) x POWER 2 is irrational [Simplification from 0.] 7) (x POWER 2 is rational) AND (x POWER 2 is irrational) [Conjunction from 5. and 6.] 8) False [Conclusion]"	Instead, use more precise terms like "definition of a rational number," or "squaring."	Task Next steps		
	Keep up the good work!	Praise		
"1. CONTRAPOSITION x is rational (Premise) so x can be expressed by a/b where a and b are both integers 3) so x^2 should be rational so x^2 = a^2/b^2 4) so x^2 is. rational since a^2 and b^2 is ratio 2. CONTRADICTION 1)X^2 IS IRRATIONAL(ASSUMPTION) 2)X IS RATIONAL(ASSUMPTION) 3) X= A/B. A AND B ARE INTEGERS 4) IF A IS AN INTEGER SO A^2 AND THE SAME FOR B AND B^2 6) X^2 IS A DIVISION OF TWO INTEGERS (A^2/B^2) 7) X^2 IS A RATIONAL NUMBER"	You've done a good job with the structure of both proofs.	Praise Current state		
	However, you need to make sure that each step of your proof follows logically from the previous one.	Self-Regulated Next Steps		
	In step 4 of your contraposition proof, it's unclear why x^2 being representable as a ratio confirms its rationality -	Current state		
	better specify this (using definition of rational number).	Task Next steps		
	Similarly in contradiction proof, mention explicitly that x^2 being rational contradicts our initial assumption thus proving original stat	Task Next steps		