

The sorting algorithms chosen:

- $O(n^2)$: Insertion sort
- $O(n \lg n)$: Heapsort.
- $O(n)$: Counting sort.

1.

a. INSERTION SORT

Best Case Input : Elements sorted in increasing order

The time complexity will be of the order of n .

Average Case Input: Elements in a random order.

The time complexity will be of the order of n^2 .

Worst Case Input : Elements sorted in nonincreasing order.

The time complexity will be of the order of n^2 .

b. HEAP SORT

Input for best case, worst case and average case doesn't matter since the algorithm is independent of type of input.

The time complexity will be of the order of $n \log n$.

c. COUNTING SORT

Input for best case, worst case and average case doesn't matter since the algorithm is independent of type of input.

The time complexity will be of the order of n .

2. Describe your experimental setup.

a. What kind of machine did you use?

A Laptop with basic configuration with eclipse IDE installed (2GB RAM)

b. What timing mechanism?

A timer built in method in java was used for timing mechanism

c. How many times did you repeat each experiment?

Each experiment is repeated 75 times (for each input of size n)

d. What times are reported?

Time taken only for sorting n elements.

e. How did you select the inputs you would use?

We are using randomized generator to generate inputs.

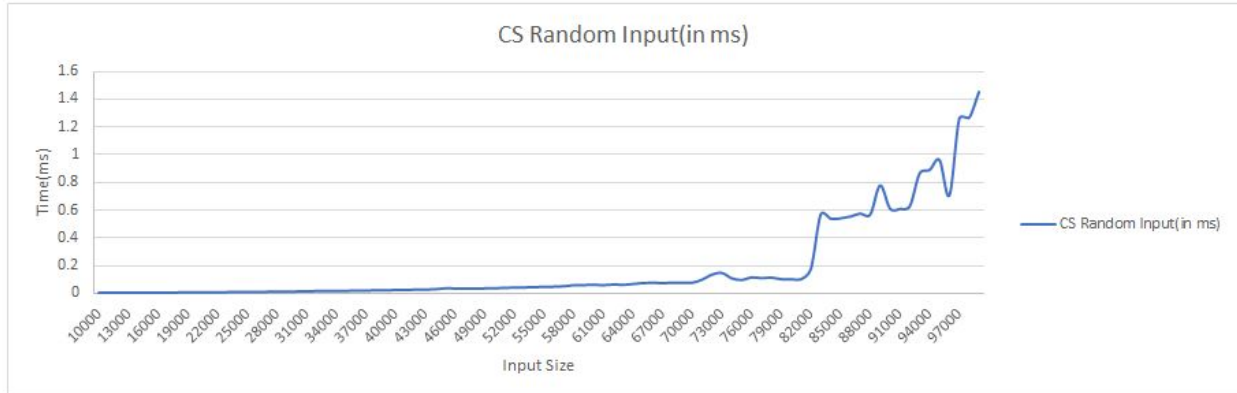
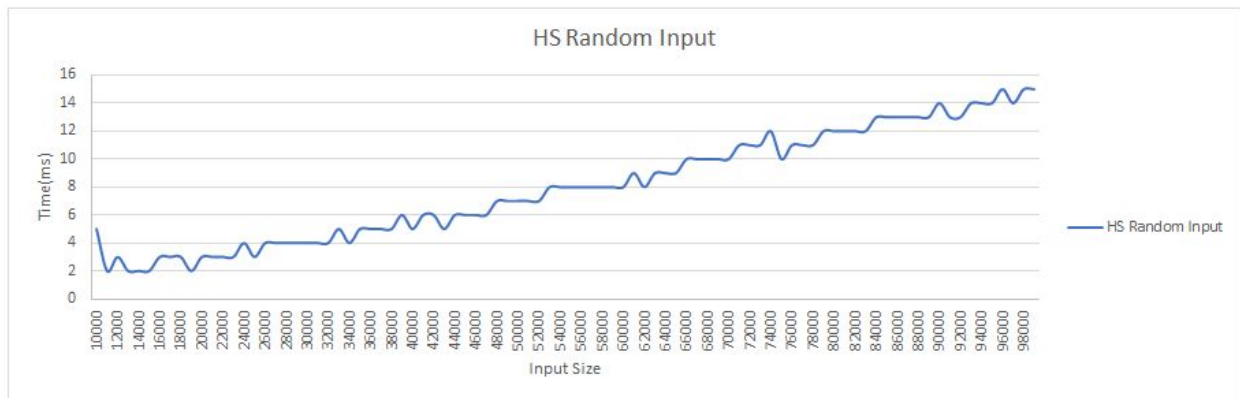
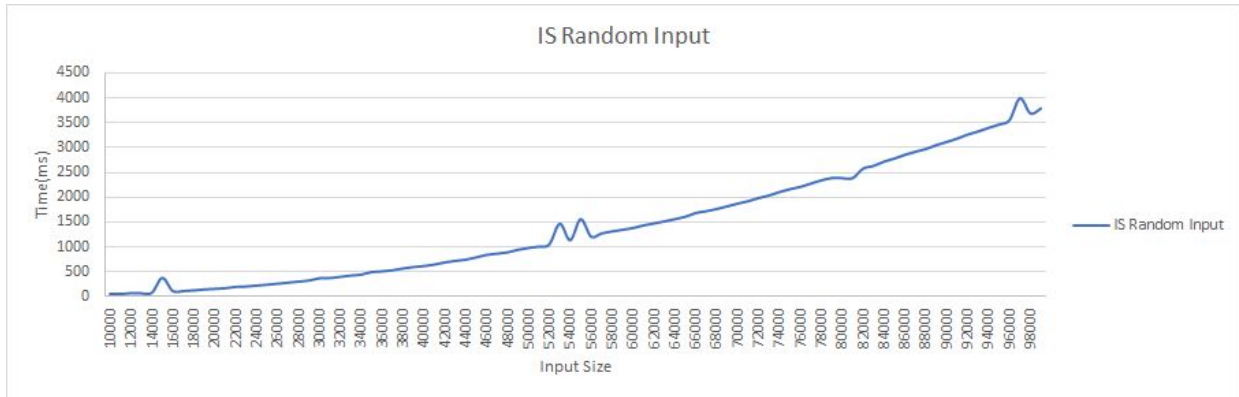
f. Do you use the same inputs for all sorting algorithms?

Yes. Since we are comparing each sorting algorithm. The time taken by each algorithm depends on the size of the input and sometimes type of the input.

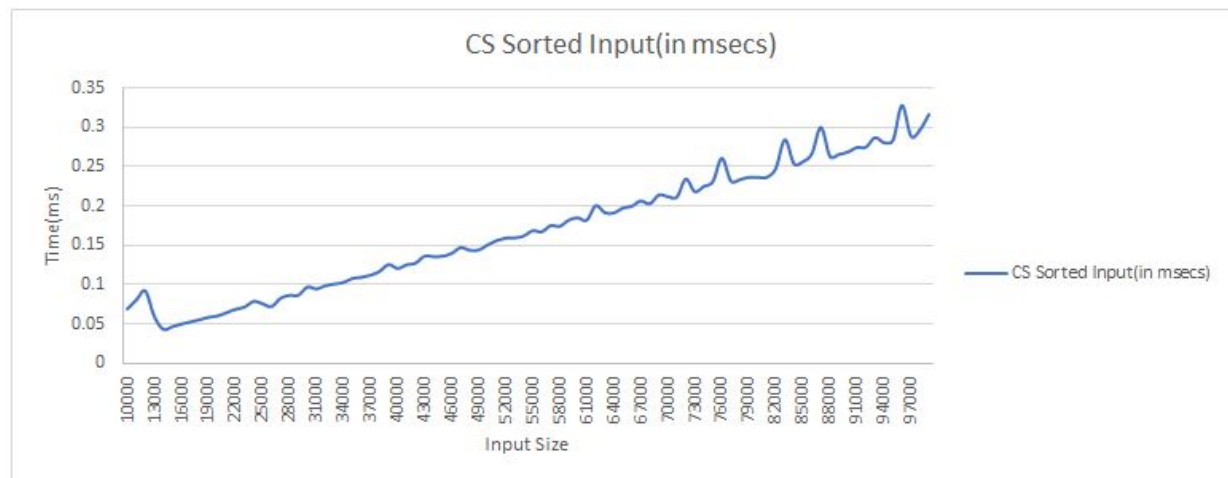
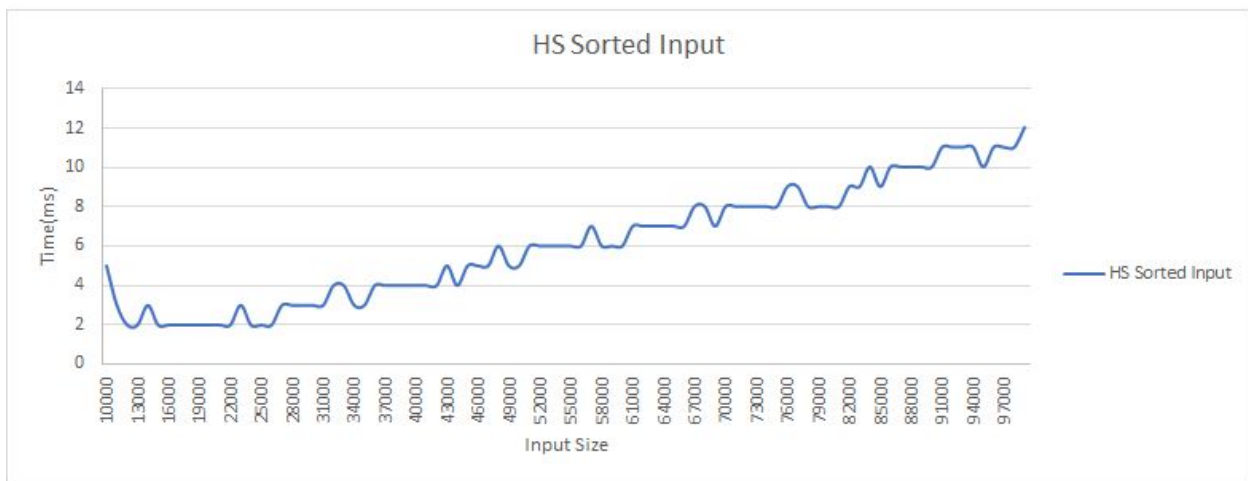
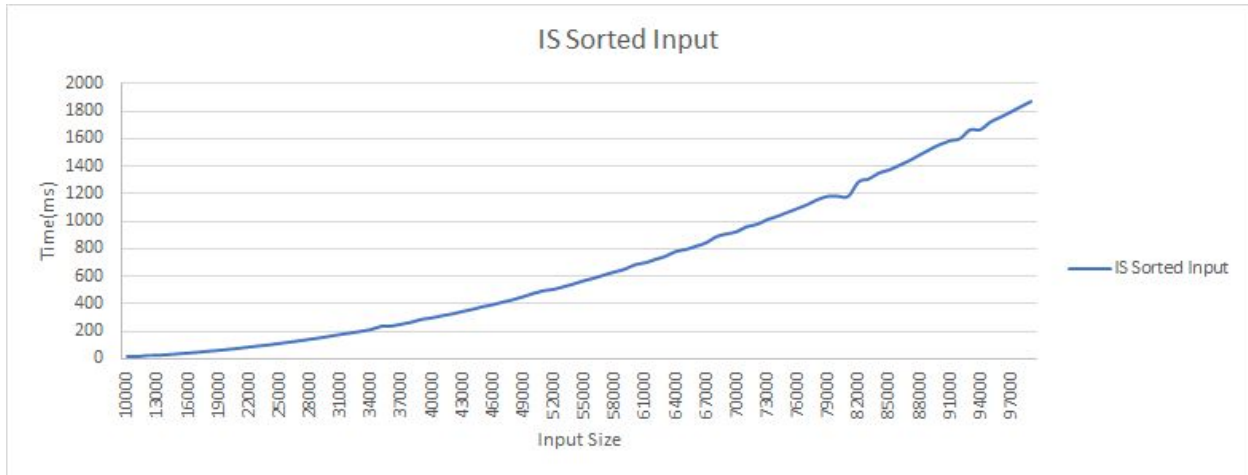
3.

In each of the cases, we can see that counting sort seems to perform the best taking the least time to sort. The next best sort appears to be heapsort which takes the next least amount of time. Finally, insertion sort takes the most amount of time.

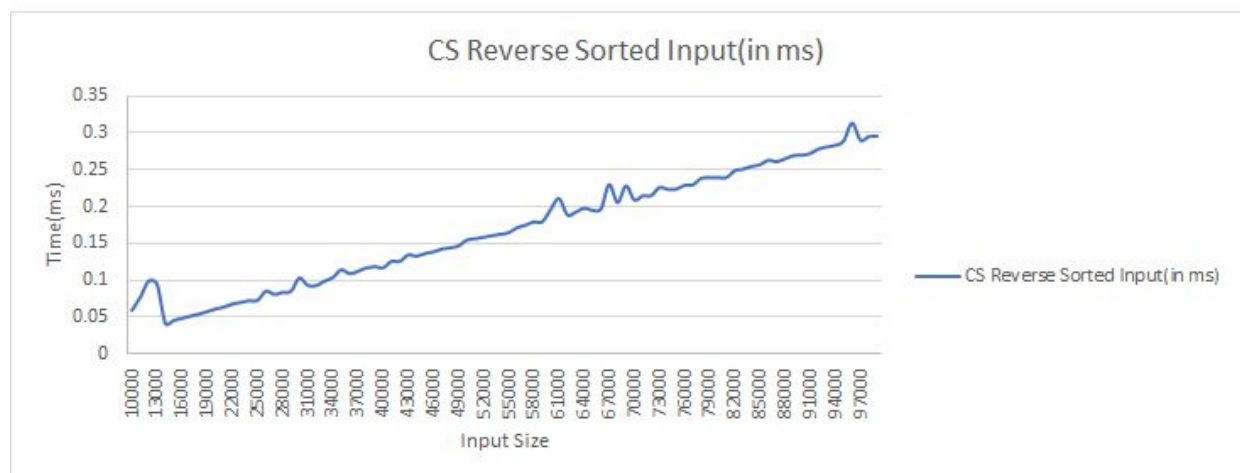
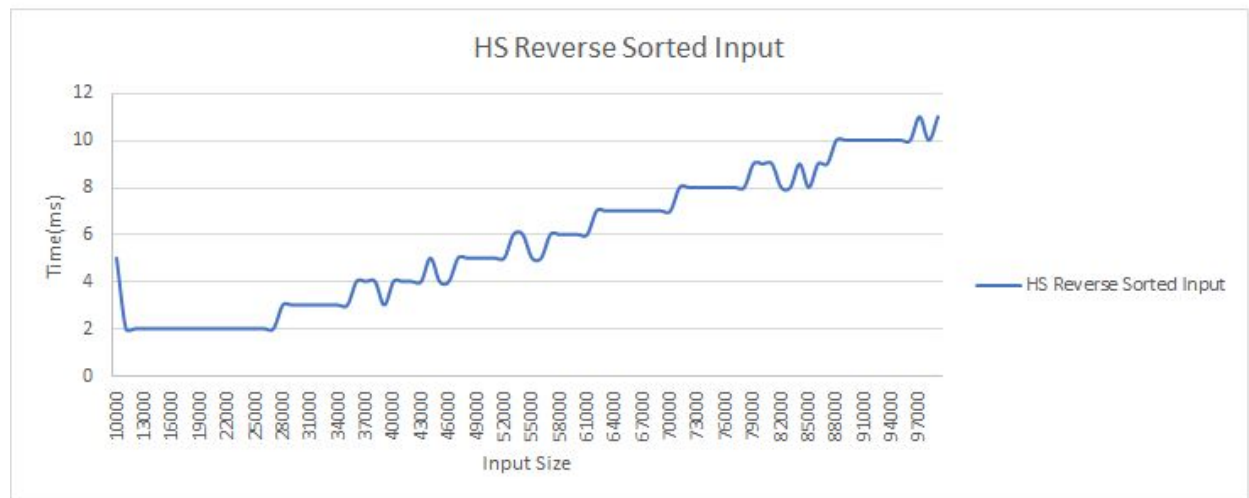
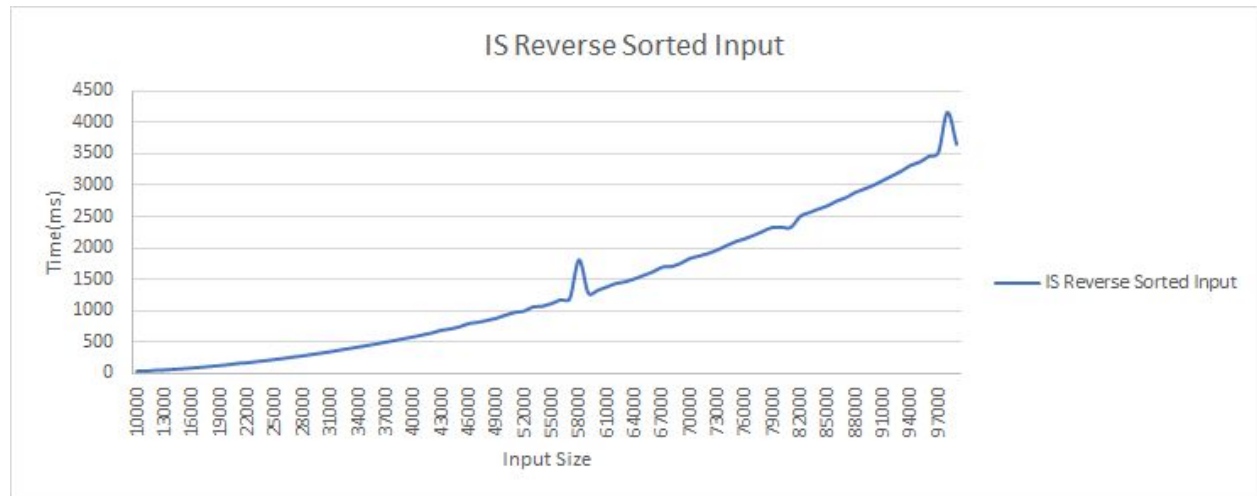
Graphs for Random Input i.e. Average Case



Graphs for Sorted Input i.e. Best Case



Graphs for Reverse Sorted Input i.e. Worst Case



4.

The analysis of algorithms comprises of finding the complexities of each algorithm to estimate the efficiency of that algorithm. The algorithm in their increasing order of complexity can be given as follows : $O(1)$, $O(\log n)$, $O(n)$, $O(n \log n)$, $O(n^2)$, $O(2^n)$, $O(n!)$.

For the calculation of n_0 :

We calculate n_0 of $O(n) = c \cdot f(n)$ (where $f(n)$ is n^2 , $n \log n$ or n depending on the sorting algorithm. So to get the value of c we use the formula $c = O(n)/f(n)$.

In our case $f(n)$ will be n^2 , $n \log n$ and n for Insertion sort, heap sort and counting sort respectively. We plot that constant calculated vs the input and find that value of input after which the experimental and theoretical results appear to match.

A. Insertion Sort

Input Size	IS Random Input	time/(n*n) (Random)	IS Sorted Input	time/n (Sorted)	IS Reverse Sorted Input	time/(n*n) (Reverse)
10000	51	0.00000051	25	0.00000025	43	0.00000043
11000	52	4.29752E-07	23	1.90083E-07	44	3.63636E-07
12000	70	4.86111E-07	31	2.15278E-07	57	3.95833E-07
13000	66	3.90533E-07	32	1.89349E-07	62	3.66864E-07
14000	75	3.82653E-07	37	1.88776E-07	72	3.67347E-07
15000	376	1.67111E-06	43	1.91111E-07	83	3.68889E-07
16000	111	4.33594E-07	48	1.875E-07	94	3.67188E-07
17000	111	3.84083E-07	55	1.90311E-07	106	3.66782E-07
18000	124	3.82716E-07	62	1.91358E-	120	3.7037E-07

				07		
19000	144	3.98892E-07	68	1.88366E-07	133	3.68421E-07
20000	154	0.000000385	76	0.00000019	148	0.00000037
21000	169	3.8322E-07	83	1.88209E-07	168	3.80952E-07
22000	196	4.04959E-07	92	1.90083E-07	178	3.67769E-07
23000	203	3.83743E-07	100	1.89036E-07	195	3.6862E-07
24000	222	3.85417E-07	108	1.875E-07	212	3.68056E-07
25000	239	3.824E-07	118	1.888E-07	231	3.696E-07
26000	259	3.83136E-07	128	1.89349E-07	249	3.68343E-07
27000	280	3.84088E-07	138	1.893E-07	269	3.68999E-07
28000	302	3.85204E-07	149	1.90051E-07	288	3.67347E-07
29000	322	3.82878E-07	159	1.89061E-07	310	3.68609E-07
30000	367	4.07778E-07	171	0.00000019	332	3.68889E-07
31000	372	3.87097E-07	183	1.90427E-07	354	3.68366E-07
32000	396	3.86719E-07	194	1.89453E-07	378	3.69141E-07
33000	422	3.87511E-07	206	1.89164E-07	403	3.70064E-07
34000	442	3.82353E-07	219	1.89446E-07	427	3.69377E-07

35000	493	4.02449E-07	242	1.97551E-07	452	3.6898E-07
36000	506	3.90432E-07	244	1.88272E-07	478	3.68827E-07
37000	529	3.86413E-07	258	1.88459E-07	506	3.69613E-07
38000	564	3.90582E-07	272	1.88366E-07	533	3.69114E-07
39000	594	3.90533E-07	293	1.92636E-07	562	3.69494E-07
40000	612	3.825E-07	303	1.89375E-07	589	3.68125E-07
41000	644	3.83105E-07	319	1.89768E-07	621	3.69423E-07
42000	686	3.88889E-07	332	1.88209E-07	650	3.68481E-07
43000	718	3.88318E-07	349	1.88751E-07	694	3.75338E-07
44000	741	3.82748E-07	365	1.88533E-07	714	3.68802E-07
45000	786	3.88148E-07	384	1.8963E-07	748	3.69383E-07
46000	838	3.9603E-07	399	1.88563E-07	801	3.78544E-07
47000	861	3.89769E-07	418	1.89226E-07	821	3.71661E-07
48000	890	3.86285E-07	435	1.88802E-07	850	3.68924E-07
49000	941	3.9192E-07	457	1.90337E-07	885	3.68596E-07
50000	976	3.904E-07	481	1.924E-07	935	0.000000374

51000	1005	3.8639E-07	500	1.92234E-07	979	3.76394E-07
52000	1048	3.87574E-07	510	1.88609E-07	997	3.68713E-07
53000	1464	5.21182E-07	530	1.88679E-07	1066	3.79494E-07
54000	1131	3.8786E-07	550	1.88615E-07	1075	3.68656E-07
55000	1555	5.1405E-07	573	1.89421E-07	1117	3.69256E-07
56000	1204	3.83929E-07	592	1.88776E-07	1180	3.76276E-07
57000	1270	3.9089E-07	615	1.89289E-07	1199	3.69037E-07
58000	1310	3.89417E-07	636	1.89061E-07	1814	5.39239E-07
59000	1342	3.85521E-07	656	1.88452E-07	1285	3.69147E-07
60000	1377	3.825E-07	688	1.91111E-07	1328	3.68889E-07
61000	1429	3.84037E-07	702	1.88659E-07	1382	3.71406E-07
62000	1470	3.82414E-07	727	1.89126E-07	1439	3.7435E-07
63000	1510	3.80448E-07	749	1.88713E-07	1465	3.69111E-07
64000	1556	3.79883E-07	784	1.91406E-07	1511	3.68896E-07
65000	1604	3.79645E-07	798	1.88876E-07	1567	3.70888E-07
66000	1679	3.85445E-07	822	1.88705E-	1621	3.7213E-07

				07		
67000	1715	3.82045E-07	847	1.88683E-07	1698	3.78258E-07
68000	1759	3.80407E-07	891	1.9269E-07	1708	3.69377E-07
69000	1812	3.80592E-07	911	1.91346E-07	1758	3.6925E-07
70000	1868	3.81224E-07	927	1.89184E-07	1835	3.7449E-07
71000	1916	3.80083E-07	964	1.91232E-07	1874	3.71752E-07
72000	1979	3.81752E-07	980	1.89043E-07	1914	3.69213E-07
73000	2030	3.80935E-07	1014	1.9028E-07	1969	3.69488E-07
74000	2099	3.83309E-07	1038	1.89554E-07	2040	3.72535E-07
75000	2157	3.83467E-07	1067	1.89689E-07	2104	3.74044E-07
76000	2204	3.81579E-07	1095	1.89578E-07	2151	3.72403E-07
77000	2269	3.82695E-07	1125	1.89745E-07	2206	3.72069E-07
78000	2335	3.83794E-07	1160	1.90664E-07	2271	3.73274E-07
79000	2379	3.81189E-07	1183	1.89553E-07	2331	3.73498E-07
80000	2379	3.71719E-07	1183	1.84844E-07	2331	3.64219E-07
81000	2379	3.62597E-07	1183	1.80308E-07	2331	3.55281E-07

82000	2570	3.82213E-07	1288	1.91553E-07	2503	3.72249E-07
83000	2625	3.81042E-07	1309	1.90013E-07	2566	3.72478E-07
84000	2711	3.84212E-07	1351	1.91468E-07	2623	3.7174E-07
85000	2773	3.83806E-07	1374	1.90173E-07	2677	3.70519E-07
86000	2848	3.85073E-07	1407	1.90238E-07	2751	3.71958E-07
87000	2910	3.84463E-07	1441	1.90382E-07	2805	3.70591E-07
88000	2964	3.82748E-07	1481	1.91245E-07	2887	3.72805E-07
89000	3042	3.84042E-07	1521	1.92021E-07	2942	3.71418E-07
90000	3106	3.83457E-07	1558	1.92346E-07	3003	3.70741E-07
91000	3173	3.83166E-07	1585	1.91402E-07	3079	3.71815E-07
92000	3255	3.8457E-07	1601	1.89154E-07	3151	3.72283E-07
93000	3317	3.83513E-07	1665	1.92508E-07	3227	3.73107E-07
94000	3389	3.83545E-07	1666	1.88547E-07	3318	3.75509E-07
95000	3456	3.82936E-07	1722	1.90803E-07	3370	3.73407E-07
96000	3544	3.84549E-07	1757	1.90647E-07	3463	3.7576E-07
97000	3983	4.23318E-07	1794	1.90669E-	3514	3.73472E-07

				07		
98000	3678	3.82965E-07	1833	1.90858E-07	4158	4.32945E-07
99000	3776	3.85267E-07	1870	1.90797E-07	3654	3.72819E-07

(Refer to graphs in 3)

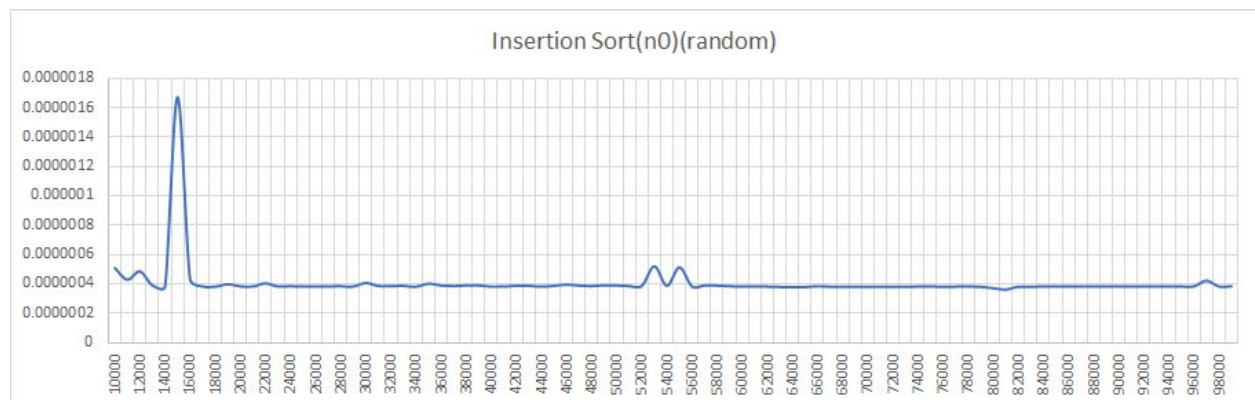
Analysis: On comparing these graphs with predicted theoretical times, we observe that the graphs do match except for certain values of n .

For the random graph, the generation of numbers is being done randomly and thus it might happen so that the random numbers generated may be partially sorted, completely sorted or even reverse sorted. This could explain the fluctuations in the graph for random numbers. Along with this, the processor is a factor as it could have put this process on a queue while performing other processes as seen at $n=57000$. But we can safely conclude that the overall behavior is similar to the theoretical analysis.

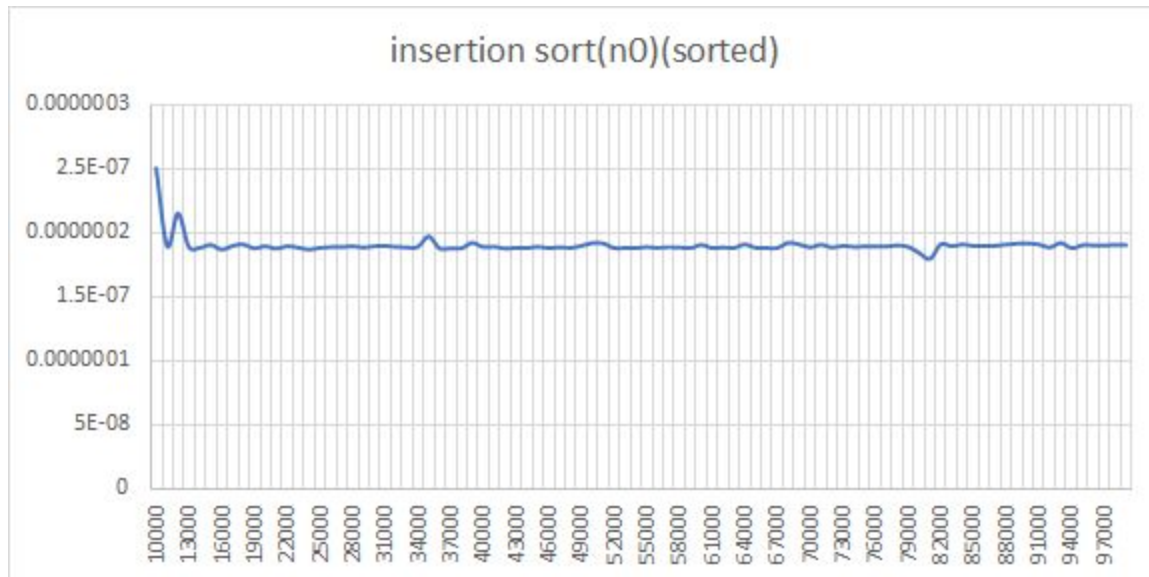
For the best case, the graph is nearly identical to the predicted theoretical times while for the worst case, keeping in mind the issue of processor queue, we can safely conclude that the overall behavior is similar to the theoretical analysis.

For calculating n_0 , we take the running time for a particular input size (n) and divide it by n^2 since the running time of insertion sort is $O(n^2)$. Here are the graphs of input vs the constant (time/n^2). In all these graphs the input is plotted in the x axis while the constant value is plotted in the y axis.

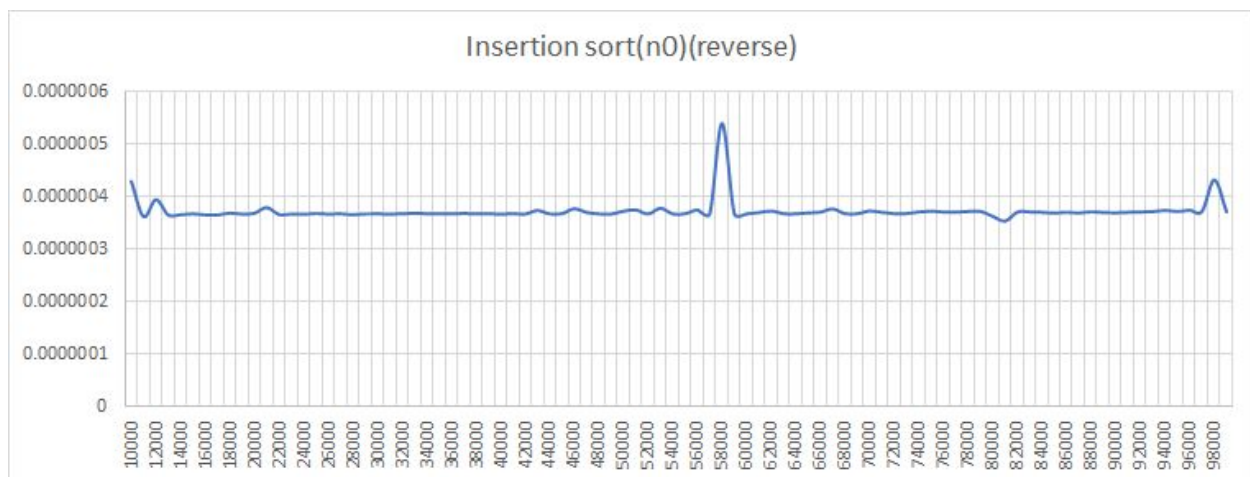
Random



Sorted



Reverse Sorted



From the graphs we can observe that after one value we are getting the value of c to be almost constant. We can safely conclude that after $n_0 = 60,000$ the value of $O(n)/f(n)$ is almost constant. After this point, the experimental readings are almost matching with the theoretical readings i.e. $O(n) = cf(n)$.

B. Heap Sort

Input Size	HS Random Input	time/nlogn(Random)	HS Sorted Input	time/nlogn(Sorted)	HS Reverse Sorted Input	time/nlogn(Reverse)
10000	5	3.76287E-05	5	3.76287E-05	5	3.76287E-05
11000	2	1.3543E-05	3	2.03146E-05	2	1.3543E-05
12000	3	1.84492E-05	2	1.22994E-05	2	1.22994E-05
13000	2	1.12574E-05	2	1.12574E-05	2	1.12574E-05
14000	2	1.03722E-05	3	1.55582E-05	2	1.03722E-05
15000	2	9.61122E-06	2	9.61122E-06	2	9.61122E-06
16000	3	1.34257E-05	2	8.95045E-06	2	8.95045E-06
17000	3	1.25573E-05	2	8.37152E-06	2	8.37152E-06
18000	3	1.17905E-05	2	7.86031E-06	2	7.86031E-06
19000	2	7.40575E-06	2	7.40575E-06	2	7.40575E-06
20000	3	1.04985E-05	2	6.99902E-06	2	6.99902E-06
21000	3	9.94958E-06	2	6.63306E-06	2	6.63306E-06
22000	3	9.45314E-06	2	6.3021E-06	2	6.3021E-06
23000	3	9.00212E-06	3	9.00212E-06	2	6.00141E-06
24000	4	1.14542E-05	2	5.72708E-06	2	5.72708E-06
25000	3	8.21375E-06	2	5.47584E-06	2	5.47584E-06
26000	4	1.04898E-05	2	5.24491E-06	2	5.24491E-06
27000	4	1.0064E-05	3	7.54796E-06	2	5.03198E-06
28000	4	9.67006E-06	3	7.25255E-06	3	7.25255E-06
29000	4	9.30472E-06	3	6.97854E-06	3	6.97854E-06
30000	4	8.96499E-06	3	6.72374E-06	3	6.72374E-06

31000	4	8.64829E-06	3	6.48621E-06	3	6.48621E-06
32000	4	8.35239E-06	4	8.35239E-06	3	6.26429E-06
33000	5	1.00942E-05	4	8.07533E-06	3	6.0565E-06
34000	4	7.81539E-06	3	5.86155E-06	3	5.86155E-06
35000	5	9.46383E-06	3	5.6783E-06	3	5.6783E-06
36000	5	9.17624E-06	4	7.34099E-06	4	7.34099E-06
37000	5	8.90498E-06	4	7.12398E-06	4	7.12398E-06
38000	5	8.64871E-06	4	6.91897E-06	4	6.91897E-06
39000	6	1.00875E-05	4	6.72499E-06	3	5.04374E-06
40000	5	8.1765E-06	4	6.5412E-06	4	6.5412E-06
41000	6	9.55024E-06	4	6.36682E-06	4	6.36682E-06
42000	6	9.30174E-06	4	6.20116E-06	4	6.20116E-06
43000	5	7.55449E-06	5	7.55449E-06	4	6.04359E-06
44000	6	8.84031E-06	4	5.89354E-06	5	7.36692E-06
45000	6	8.62573E-06	5	7.1881E-06	4	5.75048E-06
46000	6	8.42094E-06	5	7.01745E-06	4	5.61396E-06
47000	6	8.22529E-06	5	6.85441E-06	5	6.85441E-06
48000	7	9.3779E-06	6	8.0382E-06	5	6.6985E-06
49000	7	9.16897E-06	5	6.54927E-06	5	6.54927E-06
50000	7	8.96882E-06	5	6.4063E-06	5	6.4063E-06
51000	7	8.77689E-06	6	7.52305E-06	5	6.26921E-06
52000	7	8.59271E-06	6	7.36518E-06	5	6.13765E-06
53000	8	9.61809E-06	6	7.21356E-06	6	7.21356E-06
54000	8	9.42378E-06	6	7.06783E-06	6	7.06783E-06

55000	8	9.23688E-06	6	6.92766E-06	5	5.77305E-06
56000	8	9.05699E-06	6	6.79274E-06	5	5.66062E-06
57000	8	8.88371E-06	7	7.77325E-06	6	6.66278E-06
58000	8	8.7167E-06	6	6.53753E-06	6	6.53753E-06
59000	8	8.55563E-06	6	6.41672E-06	6	6.41672E-06
60000	8	8.40018E-06	6	6.30014E-06	6	6.30014E-06
61000	9	9.28134E-06	7	7.21882E-06	6	6.18756E-06
62000	8	8.10505E-06	7	7.09192E-06	7	7.09192E-06
63000	9	8.96046E-06	7	6.96924E-06	7	6.96924E-06
64000	9	8.8079E-06	7	6.85059E-06	7	6.85059E-06
65000	9	8.66026E-06	7	6.73576E-06	7	6.73576E-06
66000	10	9.46368E-06	7	6.62457E-06	7	6.62457E-06
67000	10	9.30981E-06	8	7.44785E-06	7	6.51687E-06
68000	10	9.16069E-06	8	7.32855E-06	7	6.41248E-06
69000	10	9.0161E-06	7	6.31127E-06	7	6.31127E-06
70000	10	8.87583E-06	8	7.10067E-06	7	6.21308E-06
71000	11	9.61368E-06	8	6.99177E-06	8	6.99177E-06
72000	11	9.4683E-06	8	6.88604E-06	8	6.88604E-06
73000	11	9.3271E-06	8	6.78334E-06	8	6.78334E-06
74000	12	1.00253E-05	8	6.68356E-06	8	6.68356E-06
75000	10	8.2332E-06	8	6.58656E-06	8	6.58656E-06
76000	11	8.92682E-06	9	7.30376E-06	8	6.49223E-06
77000	11	8.80065E-06	9	7.20053E-06	8	6.40047E-06
78000	11	8.67787E-06	8	6.31118E-06	8	6.31118E-06

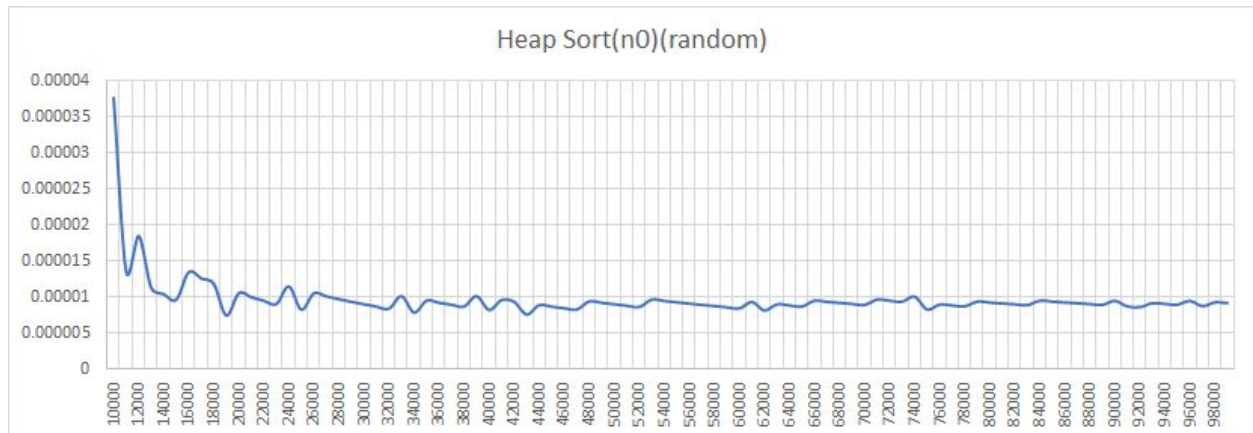
79000	12	9.33637E-06	8	6.22425E-06	9	7.00228E-06
80000	12	9.2094E-06	8	6.1396E-06	9	6.90705E-06
81000	12	9.0857E-06	8	6.05714E-06	9	6.81428E-06
82000	12	8.96517E-06	9	6.72388E-06	8	5.97678E-06
83000	12	8.84768E-06	9	6.63576E-06	8	5.89845E-06
84000	13	9.46087E-06	10	7.27759E-06	9	6.54983E-06
85000	13	9.33982E-06	9	6.46603E-06	8	5.74758E-06
86000	13	9.22171E-06	10	7.09363E-06	9	6.38426E-06
87000	13	9.10645E-06	10	7.00496E-06	9	6.30447E-06
88000	13	8.99393E-06	10	6.91841E-06	10	6.91841E-06
89000	13	8.88406E-06	10	6.83389E-06	10	6.83389E-06
90000	14	9.45188E-06	10	6.75134E-06	10	6.75134E-06
91000	13	8.67189E-06	11	7.33776E-06	10	6.67069E-06
92000	13	8.56943E-06	11	7.25106E-06	10	6.59187E-06
93000	14	9.12076E-06	11	7.16631E-06	10	6.51483E-06
94000	14	9.0153E-06	11	7.08345E-06	10	6.4395E-06
95000	14	8.91217E-06	10	6.36584E-06	10	6.36584E-06
96000	15	9.44066E-06	11	6.92315E-06	10	6.29377E-06
97000	14	8.71258E-06	11	6.8456E-06	11	6.8456E-06
98000	15	9.2314E-06	11	6.7697E-06	10	6.15427E-06
99000	15	9.13009E-06	12	7.30407E-06	11	6.6954E-06

(Refer to graphs in 3)

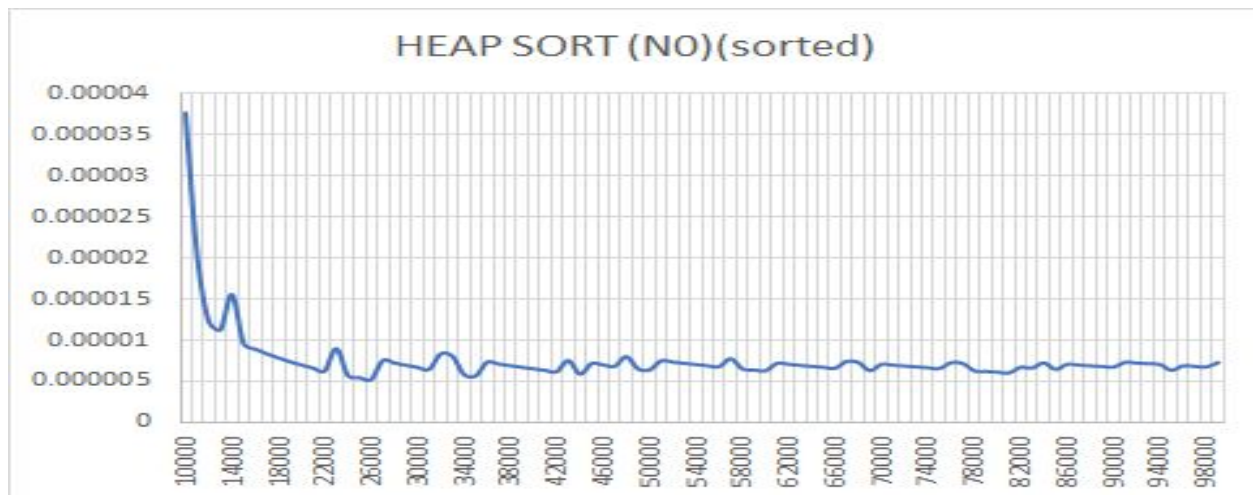
The running time for heap sort is given as $N \log N$ which leads to the graph being a parabola. As seen in all the graphs (Random input, Sorted Input, Reverse Sorted Input), the experimental results nearly match the predicted times. The fluctuations in the graph can be attributed to delays related to processor queue.

For calculating n_0 , we take the running time for a particular input size (n) and divide it by $n \log n$ since the running time of insertion sort is $O(n \log n)$. Here are the graphs of input vs the constant (time/ $n \log n$). In all these graphs the input is plotted in the x axis while the constant value is plotted in the y axis.

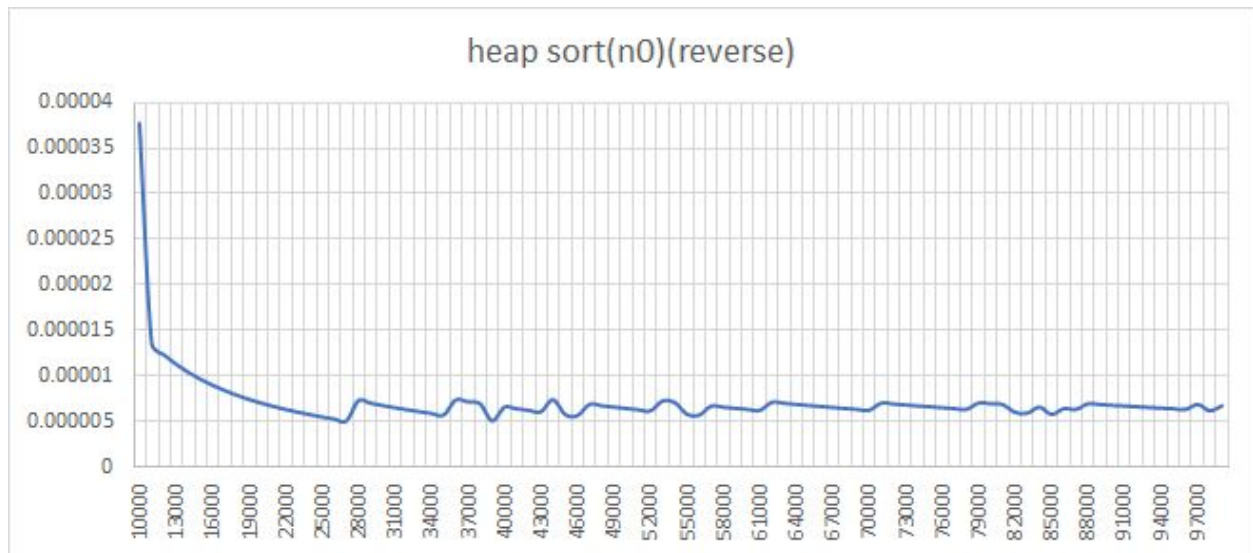
Random



Sorted



Reverse



From the graphs we can observe that after one value we are getting the value of c to be almost constant. We can safely conclude that after $n_0 = 60,000$ the value of $O(n)/f(n)$ is almost constant. After this point, the experimental readings are almost matching with the theoretical readings i.e. $O(n) = cf(n)$.

C. Counting Sort

Input Size	CS Random Input	time/n (Random)	CS Sorted Input	time/n (sorted)	CS Reverse Sorted Input	time/n (Reverse)
10000	0.001235	1.235E-07	0.068044	6.8044E-06	0.059677	5.9677E-06
11000	0.001319	1.19909E-07	0.079554	7.23218E-06	0.077501	7.04555E-06
12000	0.001847	1.53917E-07	0.091013	7.58442E-06	0.099567	8.29725E-06
13000	0.001891	1.45462E-07	0.058509	4.50069E-06	0.095803	7.36946E-06
14000	0.002452	1.75143E-07	0.042085	3.00607E-06	0.042085	3.00607E-06
15000	0.002897	1.93133E-07	0.045507	3.0338E-06	0.046191	3.0794E-06
16000	0.002897	1.81063E-07	0.048928	0.000003058	0.048928	0.000003058

17000	0.003484	2.04941E-07	0.051666	3.03918E-06	0.052008	3.05929E-06
18000	0.003959	2.19944E-07	0.054403	3.02239E-06	0.054745	3.04139E-06
19000	0.004374	2.30211E-07	0.057483	3.02542E-06	0.058166	3.06137E-06
20000	0.004841	2.4205E-07	0.059193	2.95965E-06	0.061588	3.0794E-06
21000	0.004987	2.37476E-07	0.063299	3.01424E-06	0.064325	3.0631E-06
22000	0.00568	2.58182E-07	0.067747	3.07941E-06	0.068431	3.1105E-06
23000	0.006191	2.69174E-07	0.070484	3.06452E-06	0.070484	3.06452E-06
24000	0.006645	2.76875E-07	0.077669	3.23621E-06	0.072879	3.03663E-06
25000	0.007424	2.9696E-07	0.074932	2.99728E-06	0.073221	2.92884E-06
26000	0.008226	3.16385E-07	0.07115	2.73654E-06	0.085881	3.30312E-06
27000	0.008856	0.000000328	0.081775	3.0287E-06	0.081433	3.01604E-06
28000	0.009452	3.37571E-07	0.085539	3.05496E-06	0.08417	3.00607E-06
29000	0.010394	3.58414E-07	0.085881	2.96141E-06	0.085881	2.96141E-06
30000	0.011274	3.758E-07	0.096146	3.20487E-06	0.103673	3.45577E-06
31000	0.01193	3.84839E-07	0.093751	3.02423E-06	0.093751	3.02423E-06
32000	0.014695	4.59219E-07	0.097857	3.05803E-06	0.093409	2.91903E-06

				6		6
33000	0.014257	4.3203E-07	0.09991	3.02758E-06	0.099567	3.01718E-06
34000	0.01493	4.39118E-07	0.101963	2.99891E-06	0.104357	3.06932E-06
35000	0.015941	4.55457E-07	0.107095	3.05986E-06	0.114964	3.28469E-06
36000	0.017307	4.8075E-07	0.108463	3.01286E-06	0.10949	3.04139E-06
37000	0.018034	4.87405E-07	0.111201	3.00543E-06	0.112912	3.05168E-06
38000	0.019118	5.03105E-07	0.11599	3.05237E-06	0.11736	3.08842E-06
39000	0.020202	0.000000518	0.124887	3.20223E-06	0.11907	3.05308E-06
40000	0.021919	5.47975E-07	0.119754	2.99385E-06	0.117701	2.94253E-06
41000	0.022727	5.54317E-07	0.124544	3.03766E-06	0.126255	3.07939E-06
42000	0.024262	5.77667E-07	0.126598	3.01424E-06	0.126255	3.00607E-06
43000	0.025255	5.87326E-07	0.135494	3.15102E-06	0.135151	3.14305E-06
44000	0.027136	6.16727E-07	0.134809	3.06384E-06	0.133441	3.03275E-06
45000	0.034509	7.66867E-07	0.135494	3.01098E-06	0.136862	3.04138E-06
46000	0.032841	7.13935E-07	0.138916	3.01991E-06	0.139258	3.02735E-06
47000	0.03261	6.9383E-07	0.146443	3.11581E-06	0.143021	0.000003043

48000	0.032401	6.75021E-07	0.143021	2.9796E-06	0.144732	3.01525E-06
49000	0.033698	6.87714E-07	0.143363	2.92578E-06	0.147127	3.00259E-06
50000	0.034673	6.9346E-07	0.149864	2.99728E-06	0.155339	3.10678E-06
51000	0.037289	7.31157E-07	0.155339	3.04586E-06	0.15705	3.07941E-06
52000	0.039502	7.59654E-07	0.158418	3.0465E-06	0.159103	3.05967E-06
53000	0.040243	7.59302E-07	0.15876	2.99547E-06	0.161156	3.04068E-06
54000	0.042735	7.91389E-07	0.161155	2.98435E-06	0.163208	3.02237E-06
55000	0.044181	8.03291E-07	0.167999	3.05453E-06	0.165261	3.00475E-06
56000	0.04619	8.24821E-07	0.16663	2.97554E-06	0.172104	3.07329E-06
57000	0.04872	8.54737E-07	0.174499	3.06139E-06	0.175526	3.0794E-06
58000	0.056814	9.79552E-07	0.173473	2.99091E-06	0.179974	0.000003103
59000	0.057537	9.75203E-07	0.181342	3.07359E-06	0.179974	3.05041E-06
60000	0.059309	9.88483E-07	0.184423	3.07372E-06	0.19674	0.000003279
61000	0.057283	9.39066E-07	0.181343	2.97284E-06	0.211452	3.46643E-06
62000	0.061782	9.96484E-07	0.199819	3.22289E-06	0.189554	3.05732E-06
63000	0.059447	9.43603E-07	0.190923	3.03052E-06	0.192976	3.06311E-06

				6		6
64000	0.065707	1.02667E-06	0.190581	2.97783E-06	0.19845	3.10078E-06
65000	0.072168	1.11028E-06	0.19674	3.02677E-06	0.195713	3.01097E-06
66000	0.074224	1.12461E-06	0.199135	3.0172E-06	0.197082	2.98609E-06
67000	0.071973	1.07422E-06	0.205978	3.0743E-06	0.230613	3.44199E-06
68000	0.074116	1.08994E-06	0.202214	2.97374E-06	0.206321	3.03413E-06
69000	0.075357	1.09213E-06	0.213506	3.09429E-06	0.228902	3.31742E-06
70000	0.07469	0.000001067	0.211453	3.02076E-06	0.209741	2.9963E-06
71000	0.096844	0.000001364	0.210769	2.96858E-06	0.215216	3.03121E-06
72000	0.133345	1.85201E-06	0.233692	3.24572E-06	0.215559	2.99388E-06
73000	0.145234	1.98951E-06	0.217611	2.98097E-06	0.226507	3.10284E-06
74000	0.106629	1.44093E-06	0.224454	3.03316E-06	0.224112	3.02854E-06
75000	0.094778	1.26371E-06	0.230271	3.07028E-06	0.224455	2.99273E-06
76000	0.113007	1.48693E-06	0.260381	3.42607E-06	0.229586	3.02087E-06
77000	0.107849	1.40064E-06	0.231298	3.00387E-06	0.230271	2.99053E-06
78000	0.111994	1.43582E-06	0.233009	2.98729E-06	0.239167	3.06624E-06

79000	0.100474	1.27182E-06	0.236088	2.98846E-06	0.240193	3.04042E-06
80000	0.100474	1.25593E-06	0.236088	2.9511E-06	0.240193	3.00241E-06
81000	0.100474	1.24042E-06	0.236088	2.91467E-06	0.240193	2.96535E-06
82000	0.174379	2.12657E-06	0.247379	3.01682E-06	0.249432	3.04185E-06
83000	0.569283	6.85883E-06	0.28399	3.42157E-06	0.251485	3.02994E-06
84000	0.537283	6.39623E-06	0.253538	3.01831E-06	0.255249	3.03868E-06
85000	0.539653	6.34886E-06	0.255933	3.01098E-06	0.257643	3.03109E-06
86000	0.55269	6.42663E-06	0.266198	3.09533E-06	0.263461	3.0635E-06
87000	0.573233	6.58889E-06	0.299729	3.44516E-06	0.26175	3.00862E-06
88000	0.567308	6.44668E-06	0.262776	2.98609E-06	0.265513	3.01719E-06
89000	0.77748	8.73573E-06	0.265171	2.97945E-06	0.269961	3.03327E-06
90000	0.609579	6.7731E-06	0.26825	2.98056E-06	0.270303	3.00337E-06
91000	0.606418	6.66393E-06	0.274067	3.01173E-06	0.272698	2.99668E-06
92000	0.627899	6.82499E-06	0.274409	2.98271E-06	0.278857	3.03105E-06
93000	0.864612	9.2969E-06	0.286727	3.08309E-06	0.281594	3.02789E-06
94000	0.888878	9.45615E-06	0.280226	2.98113E-06	0.28399	3.02117E-06

				6		6
95000	0.960393	1.01094E-05	0.282963	2.97856E-06	0.289806	3.05059E-06
96000	0.705184	7.34567E-06	0.327785	3.41443E-06	0.314099	3.27186E-06
97000	1.258664	1.29759E-05	0.288437	2.97358E-06	0.290832	2.99827E-06
98000	1.266468	1.29231E-05	0.296649	3.02703E-06	0.295622	3.01655E-06
99000	1.452244	1.46691E-05	0.316152	3.19345E-06	0.296307	0.000002993

(Refer to graphs in 3)

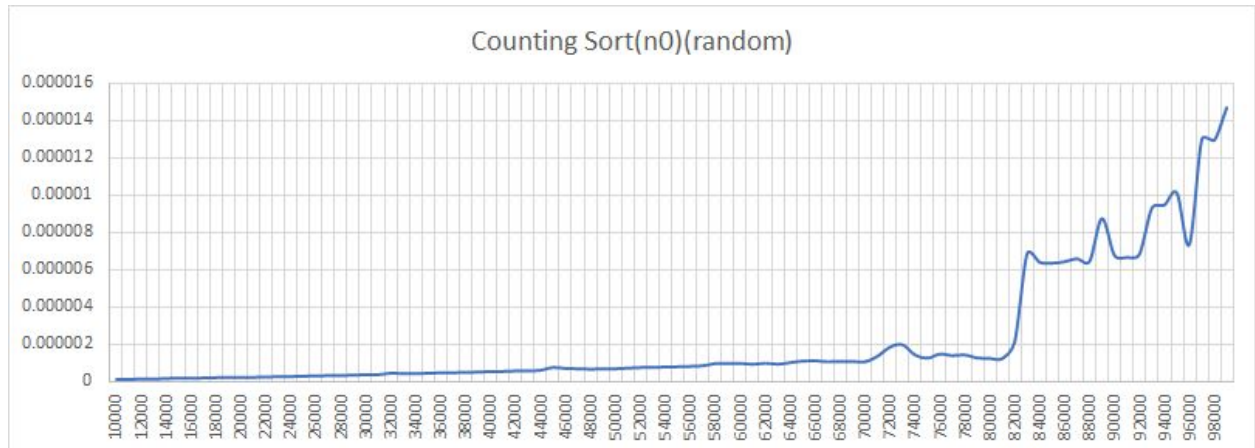
Analysis: On comparing these graphs with predicted theoretical times, we observe that the graphs do match except for certain values of n .

For the random graph, the generation of numbers is being done randomly and thus it might happen so that the random numbers generated may be partially sorted, completely sorted or even reverse sorted. This could explain the fluctuations in the graph for random numbers. Along with this, the processor is a factor as it could have put this process on a queue while performing other processes as seen at $n=84000$. But we can safely conclude that the overall behavior is leading to be similar to the theoretical analysis.

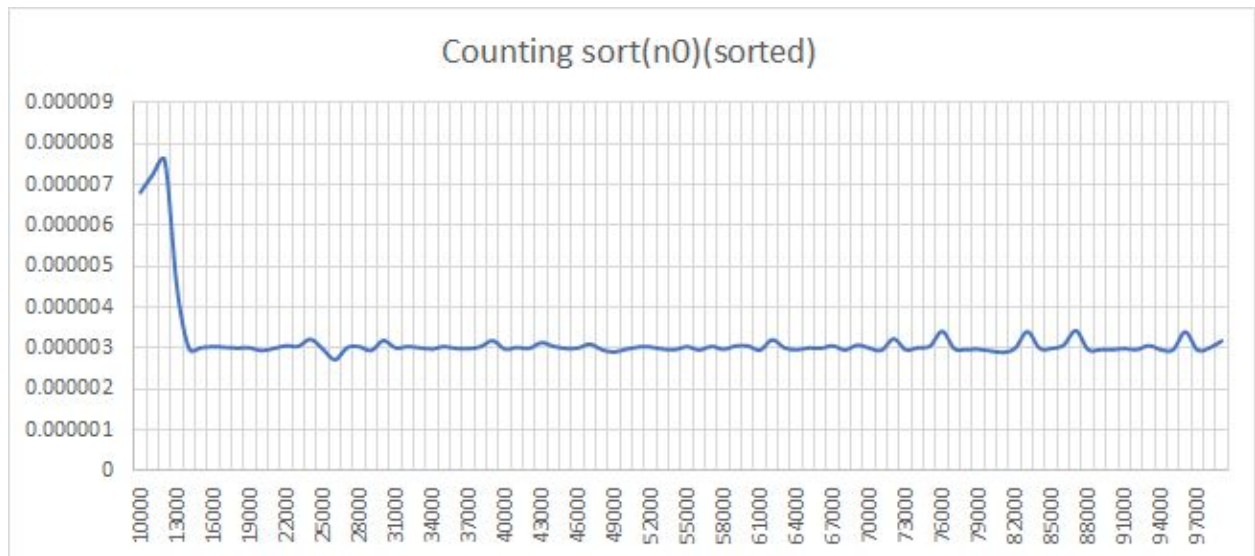
For the best case and the worst case, the graphs are nearly identical to the predicted theoretical times, keeping in mind the issue of processor queue. Hence, we can safely conclude that the overall behavior is similar to the theoretical analysis.

For calculating n_0 , we take the running time for a particular input size (n) and divide it by n since the running time of insertion sort is $O(n)$. Here are the graphs of input vs the constant (time/n). In all these graphs the input is plotted in the x axis while the constant value is plotted in the y axis.

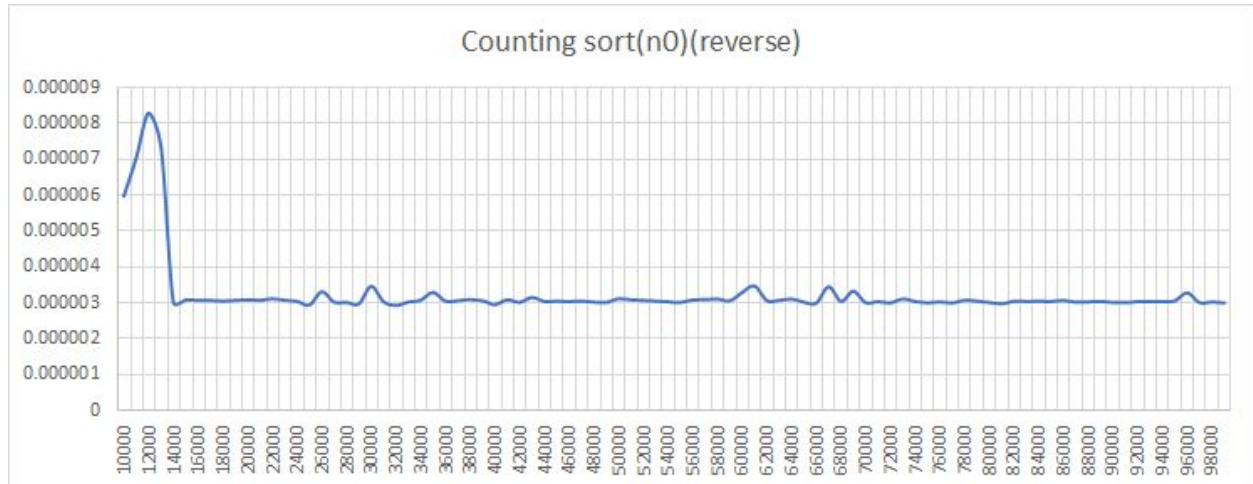
Random



Sorted



Reverse Sorted



From the graphs we can observe that after one value we are getting the value of c to be almost constant. We can safely conclude that after $n_0 = 72,000$ the value of $O(n)/f(n)$ is almost constant. After this point, the experimental readings are almost matching with the theoretical readings i.e. $O(n) = cf(n)$.

For the random input graph, we see a rise in the graph because the inputs are randomly generated and it might also happen that the processor might have put this process in queue and could be scheduling another process or task at that instant so for some values of n we can see that the time is large. This could probably explain some abnormal readings.