Opinion Dynamics & Pricing Behaviour in Financial Markets

MSc Dissertation Presentation

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Introduction

- Nobel Laureate Robert Shiller, in *Narrative Economics* (2019), argues that stories spread like epidemics through social interactions and can shape economic behaviour as strongly as, and sometimes more than, fundamentals.

- The backbone of my thesis is to implement this idea into a computational model of financial markets.

- The objective is to study how narratives evolve by analysing how trader opinions propagate through markets, influence behaviour, and how social structures together with external signals shape prices and outcomes.

Methodology

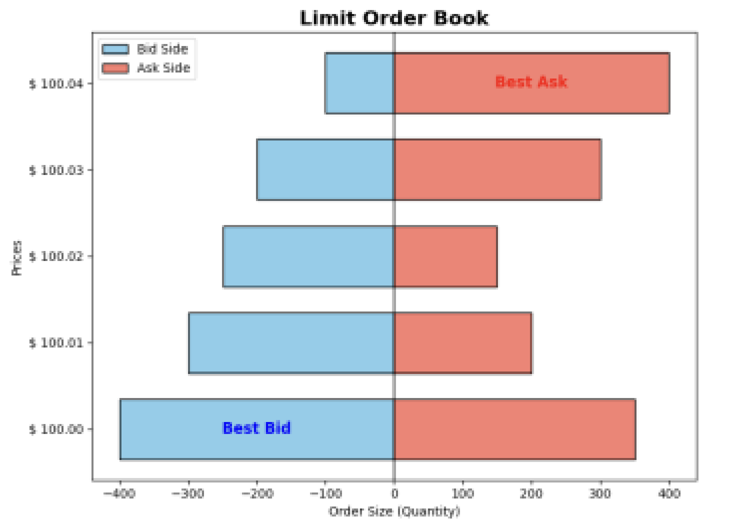
- Using the Bristol Stock Exchange (BSE) simulation, 60 automated traders interact through a limit order book, adapting strategies in response to both market conditions and peer influence.
- An O-PRDE trader model integrates opinion dynamics into adaptive decision-making, so traders adjust not only to profitability but also to evolving beliefs shaped by others.
- Opinion dynamics are captured through the Relative Agreement–Relative Disagreement (RA–RD) framework, which models both convergence and polarisation of opinions.
- Social network structures are introduced through Watts-Strogatz (small-world) and Klemm-Eguíluz (scale-free) networks, reflecting clustering, shortcuts, and influential hubs found in real markets.
- Finally, dynamic locality extends earlier models by allowing traders to vary how much weight they place on social interactions versus external signals in real time.

Bristol Stock Exchange

- BSE is a minimal simulation of a financial market built around a limit order book, developed by Prof. Dave Cliff as a research and teaching tool.

- It replicates the continuous double auction (CDA) mechanism, where buyers and sellers place competing orders that are matched in real time.

- While not co-evolutionary, BSE supports strategic adaptation when populated with adaptive agents (e.g., PRDE, O-PRDE), making it a platform for studying evolving trading behaviour.



Limitations:

- Simplifies trading dynamics by restricting traders to one active order, assuming zero latency, and granting perfect information access.

Adaptive Traders: PRDE & O-PRDE

- PRDE (Parameterized-Response Differential Evolution) trader, introduced by Prof. Dave Cliff, it is an adaptive trader algorithm in the Bristol Stock Exchange (BSE).

$$S = \{s_1, s_2, \ldots, s_{NP}\}, \quad s_i \in [-1, 1]$$

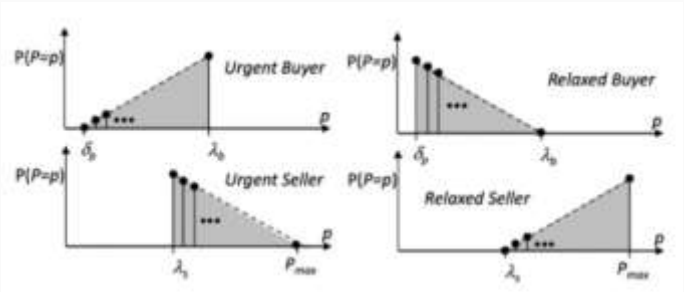
- Strategy parameter s, that controls quote urgency.

- Strategies are evaluated on profitability (profit per second), and the differential evolution optimiser selects and refines them over time.

$$pps_s = \frac{\text{Total Profit from Trades using} s}{\text{Evaluation Time}}$$

$$s_{\text{new}} = s_{r1} + F \cdot (s_{r2} - s_{r3})$$

$$\text{if} pps_{\text{new}} > pps_{s_0}, \quad \text{then} s_0 \leftarrow s_{\text{new}}$$

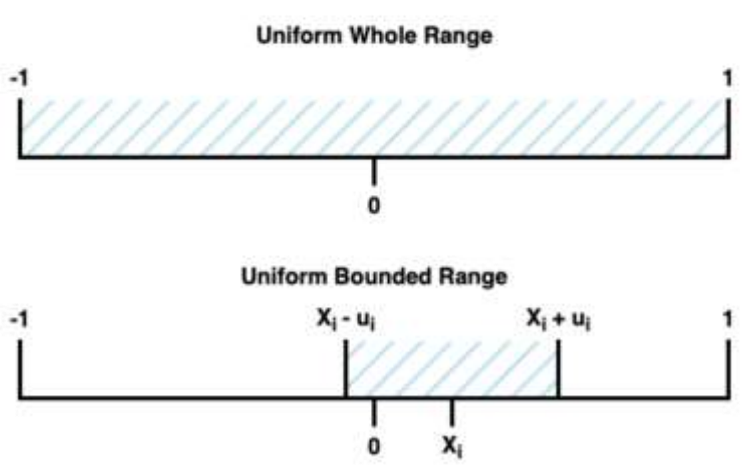


- This makes PRDE a co-evolutionary trader type, since strategies evolve dynamically in response to market conditions.

Adaptive Traders: PRDE & O-PRDE

- O-PRDE, introduced by Abraham Salih, is a variant of PRDE in which each trader is assigned an opinion and an uncertainty, together defining a range from which new strategies are drawn.

$$B_i(t) = \left[\max(-1,\ X_i(t) - u_i(t)), min(1,\ X_i(t) + u_i(t))\right]$$



- In O-PRDE, the strategy window Bi(t) is centred on the trader’s opinion xi(t) and scaled by uncertainty ui(t) so higher uncertainty promotes wider exploration while lower uncertainty keeps proposals near the current opinion.

- This contrasts with the traditional PRDE model, where strategies are always drawn uniformly from the fixed range [−1,1].

Relative Agreement Model

- The Relative Agreement (RA) model was introduced by Deffuant et al. and represents each trader with an opinion x and an uncertainty u, where influence occurs only when their uncertainty intervals overlap.

$$h_{ij} = \min(x_i + u_i,\, x_j + u_j) - \max(x_i - u_i,\, x_j - u_j)$$

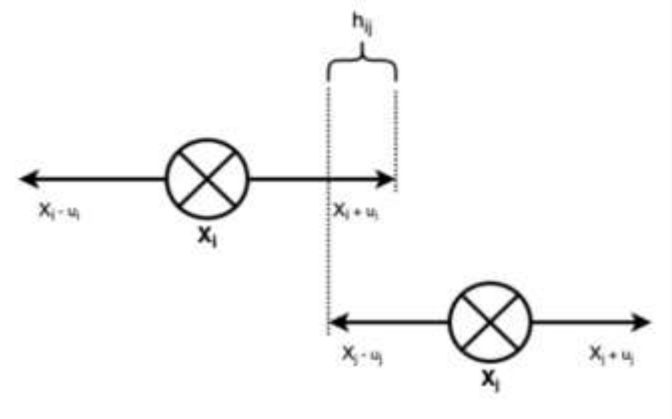
$$RA = \frac{h_{ij}}{u_i} - 1$$

A higher RA value indicates that most of agent i's confidence range overlaps with agent j's, making agent i more likely to adjust toward j's opinion.

Now, when hij>ui, agent i is influenced by agent j, leading to updates in both opinion and uncertainty.

$$x_i = x_i + \mu \left(\frac{h_{ij}}{u_i} - 1 \right) (x_j - x_i)$$

$$u_i = u_i + \mu \left(\frac{h_{ij}}{u_i} - 1 \right) (u_j - u_i)$$

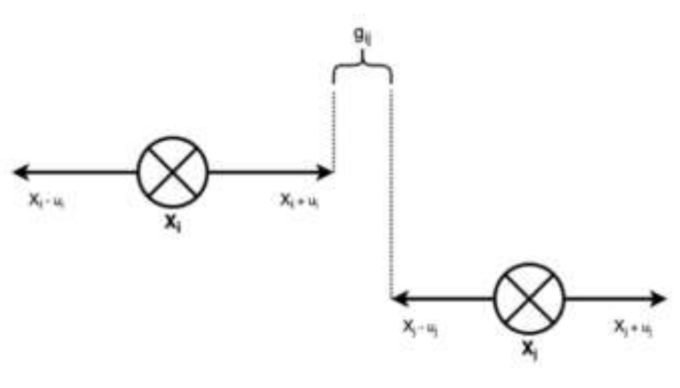


Relative Disagreement Model

- The Relative Disagreement (RD) model, introduced by Meadows and Cliff extends RA, where disagreement occurs if their opinion intervals do not overlap (hij≤0) and is applied with probability λ.

$$g_{ij} = \min(x_j + u_j,\, x_i + u_i) - \max(x_j - u_j,\, x_i - u_i)$$

$$RD = \frac{|g_{ij}|}{u_i} - 1$$



A high RD value indicates that agent j's opinion lies outside agent i's acceptable range, causing agent i to shift further away and increase their distance.

With probability λ, RD occurs, and agent i's opinion and uncertainty are updated based on agent j.

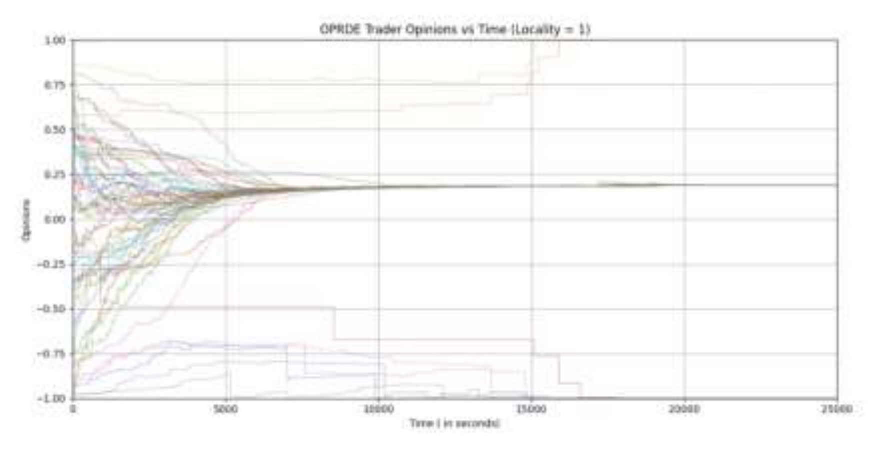
$$x_i = x_i - \mu \left(\frac{|g_{ij}|}{u_i} - 1 \right) (x_j - x_i)$$

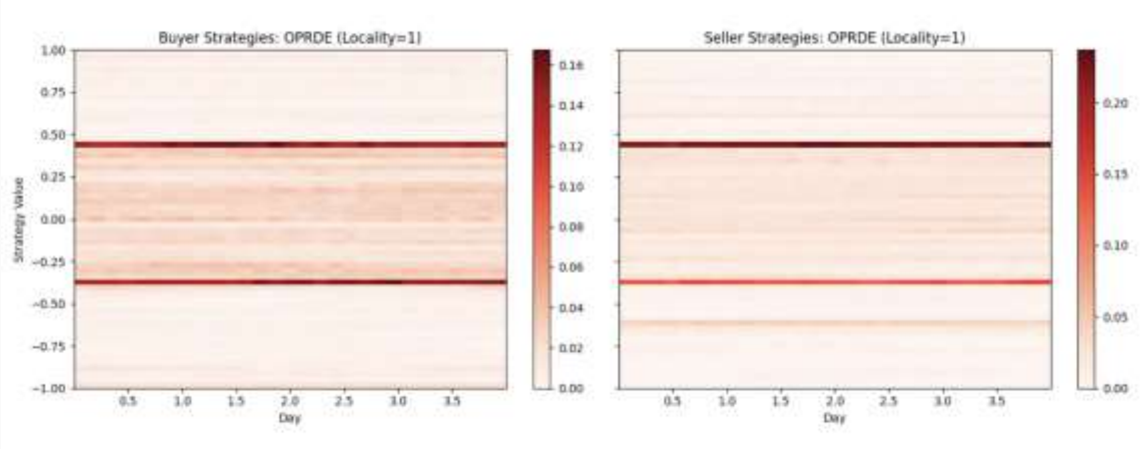
$$u_i = u_i - \mu \left(\frac{|g_{ij}|}{u_i} - 1 \right) (u_j - u_i)$$

Social Network Topologies

Fully Connected Graph

- For the baseline O-PRDE results, a fully connected graph was implemented, i.e., every trader could interact with every other trader, allowing opinions to spread freely without network constraints.



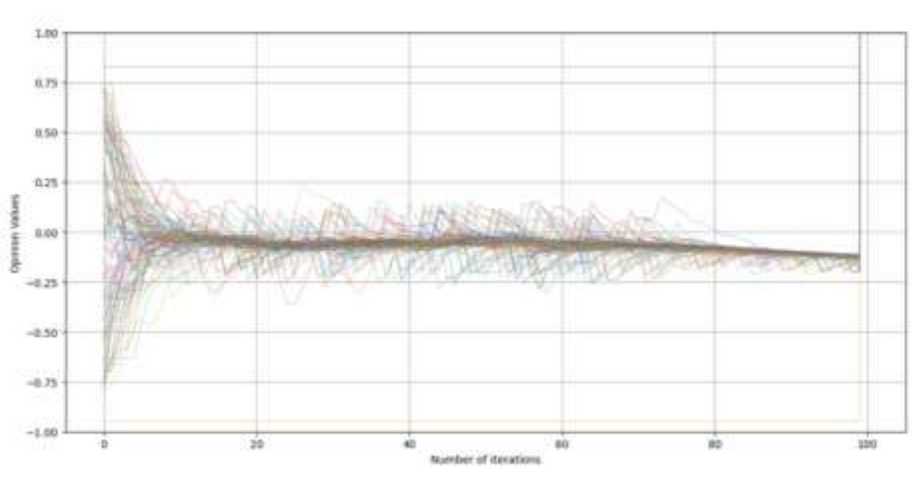


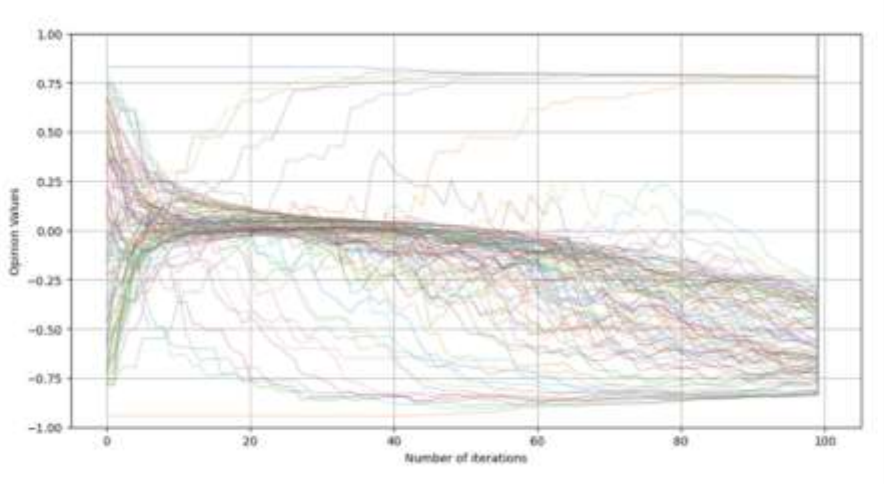
- The locality parameter was fixed at 1, ensuring that trader opinions were shaped solely by peer interactions without influence from external market signals.
- A consensus formed around $x \approx 0.2$, leading to stable opinion and strategy clusters, while persistent outliers with low uncertainty resisted convergence and maintained divergent strategies.

Social Network Topologies

Watts-Strogatz small-world network

- The Watts-Strogatz (WS) small-world network model, was designed to overcome the limitations of traditional random graph models.
- The WS model begins with a regular ring lattice in which each node is connected to its K nearest neighbours.
- With probability β, some links are rewired to random nodes, creating shortcuts that reduce path lengths while preserving clustering, thereby reproducing the “small-world” effect.

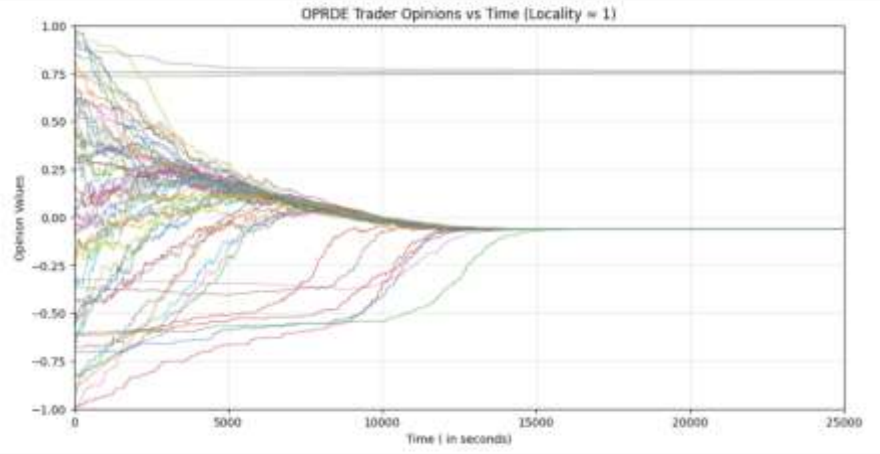


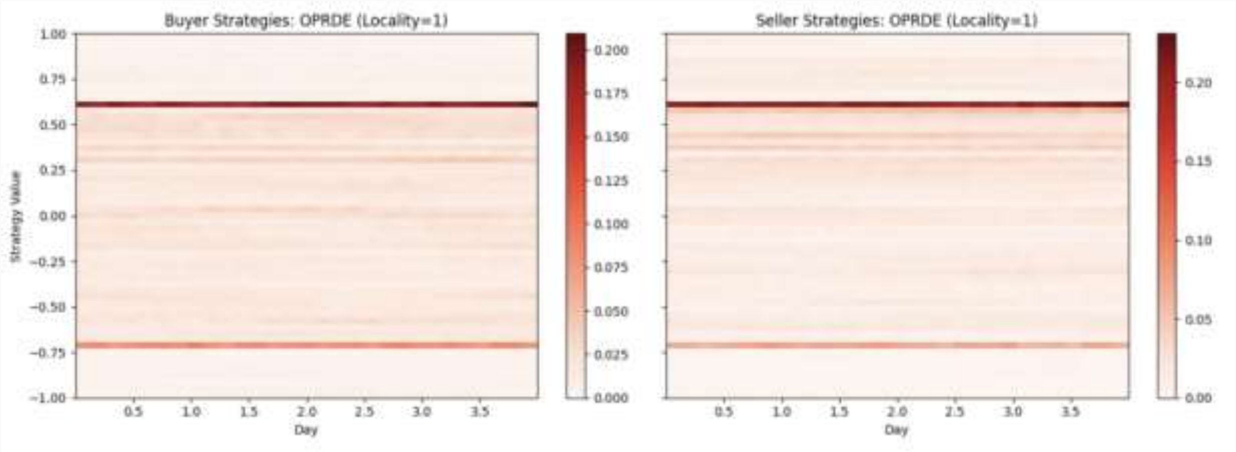


RHS (WS): higher clustering sustains local groups and slows convergence. LHS (fully connected): opinions converge smoothly to consensus.

Social Network Topologies

O-PRDE Traders in a Watts-Strogatz Small-World Network



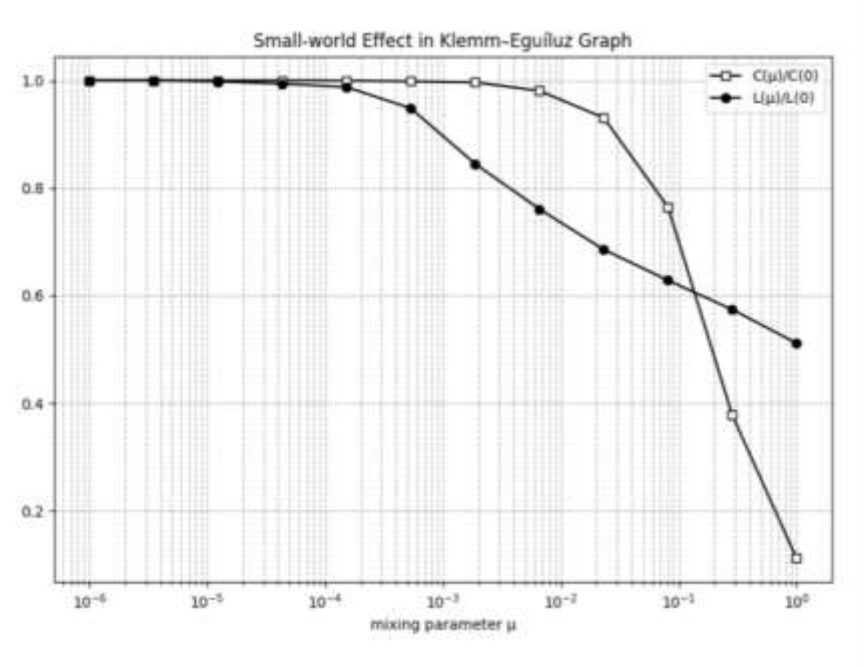


- In the WS network, clustering slows and fragments convergence compared to the fully connected case.
- Buyer strategies cluster around higher positive values, urgent bidding, as opinions converged with a slight bullish drift.
- Seller strategies concentrate around moderately negative values, relaxed selling.

Social Network Topologies

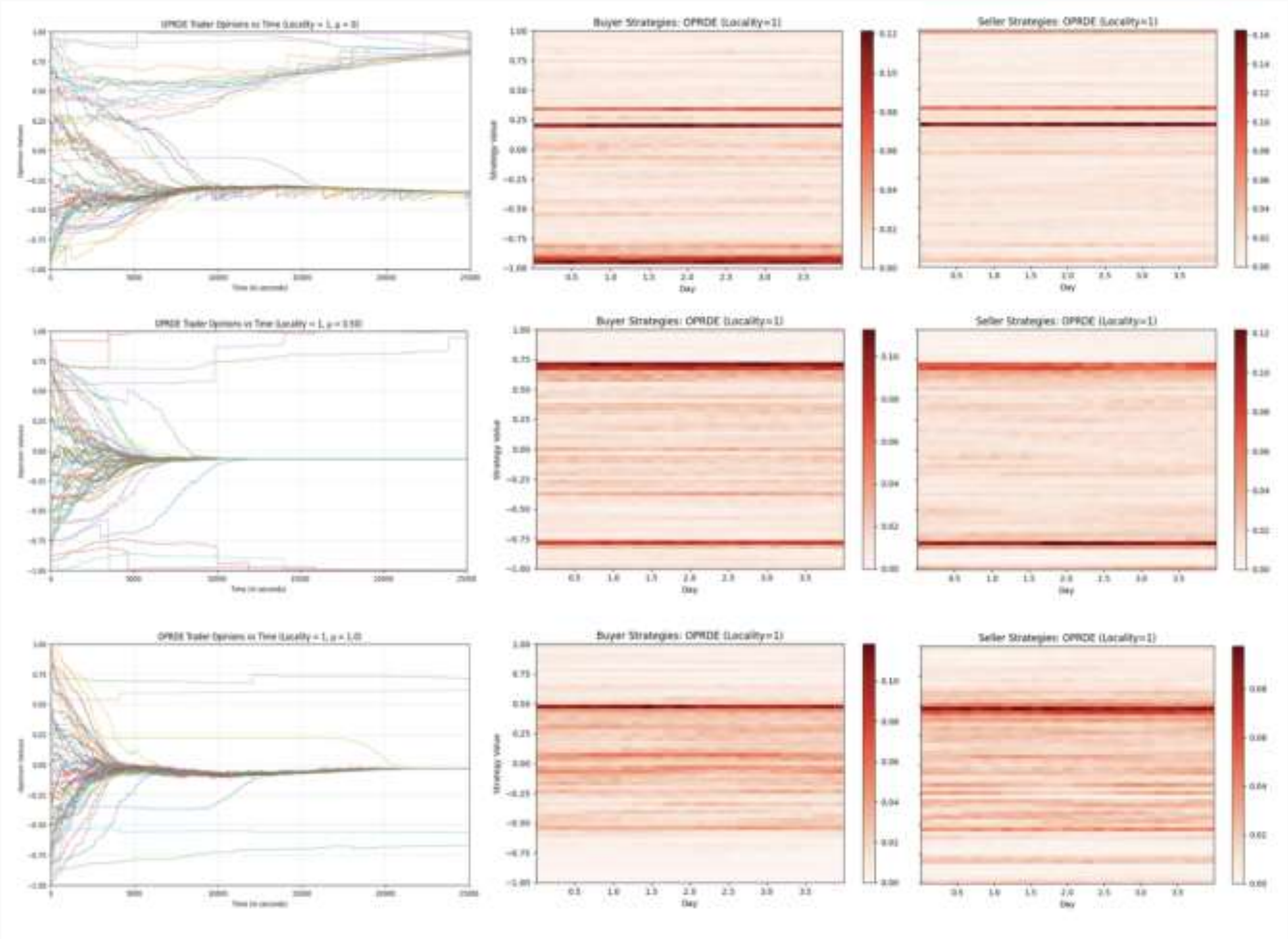
Klemm-Egúiluz scale-free network

- The scale-free Klemm-Eguíluz (KE) graph was proposed by Víctor Klemm and Víctor M. Eguíluz in 2002, and it argues that real-world networks exhibit both scale-free degree distributions and high clustering, reflecting market-like communication with tight groups, short paths, and influential hubs.



- The small-world effect in the (KE) network was examined by varying the mixing parameter μ on a logarithmic scale from 10^{-6} *to* 10^{0}, using *N=500* nodes and *m=5* links per new node.
- At very low μ values (10^{-6} *to* 10^{-3}), the network remains highly clustered but the average path length is large due to predominantly local connections.
- As μ increases slightly, a small fraction of random links dramatically reduces the average path length while preserving high clustering, demonstrating the small-world effect.
- At higher μ values ($> 10^{-1}$), clustering drops sharply, and the network structure approaches that of a random graph with short paths but low clustering.

O-PRDE Traders in a KE Scale-Free Network



Trader opinion trajectories and corresponding buyer/seller strategy distributions under a Klemm-Egúiluz scale-free network. With μ varied over {0, 0.5, 1.0}.

- At *μ=0*, the KE network is highly clustered, leading to fragmented local cliques, slow convergence, and polarised buyer/seller strategies concentrated at extreme values.
- As *μ* increases (0.5 - 1.0), shortcut links reduce path lengths, promote cross-clique mixing, weaken extremes, and drive opinions and strategies toward central consensus.

Dynamic O-PRDE

External Market Signals

- Traders are also influenced by external market signals, captured through the LOB, via the micro-price adjustment $\Delta m(t)$, which reflects shifts in overall market sentiment.

$$\Delta m(t) = P_\mu(t) - P_m(t) \qquad P_m(t) = \frac{P_a(t) + P_b(t)}{2}$$

$$P_\mu(t) = \frac{P_a(t)\,Q_b(t) + P_b(t)\,Q_a(t)}{Q_a(t) + Q_b(t)}$$

$P_m and\ P_\mu\ and\ define\ the\ mid\ and\ micro - price.$
$P_a(t)\ and\ P_b(t)\ represent\ the\ best\ ask\ and\ bid.\ Whereas, Q_a(t)\ and\ Q_b(t)\ denote\ the\ standing\ volumes\ at\ those\ prices.$

- To map external market signals into the opinion dynamics model, Nowicka transformed $\Delta m(t)$ into the opinion range [−1,1] using a nonlinear cubic function.

$$x_{i,\mathrm{external}} = x_i + \frac{|x_i|}{10}(\Delta m)^3$$

- This external opinion was then combined with the network opinion through a linear weighting mechanism m∈[0,1], known as the locality variable (or malleability).

$$x_i = (m_i)x_{i,network} + (1 - m_i)x_{i,external}$$

Dynamic O-PRDE

- Nowicka's cubic mapping made external opinion updates overly sensitive to short-term fluctuations in $\Delta m(t)$, causing traders to follow noise rather than genuine trends.

- Abraham Salih introduced a hyperbolic tangent mapping, which smooths out small fluctuations and responds strongly only to persistent changes.

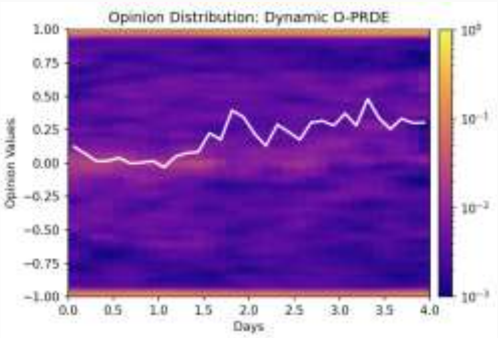
$$x_{\text{external}} = \frac{\sum_{t-n}^{t} \tanh(\alpha\,\Delta m)}{n}$$

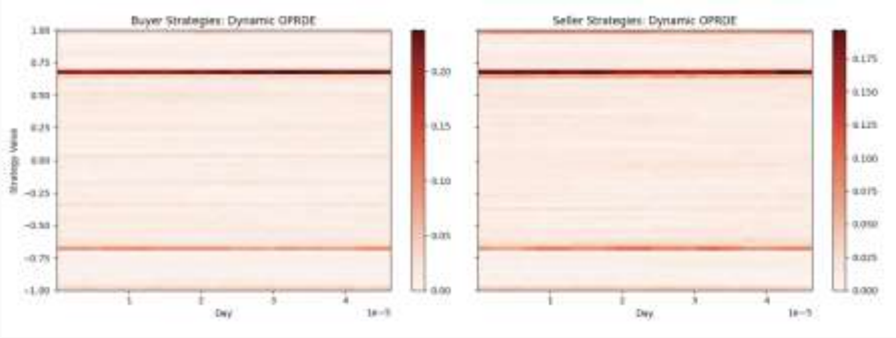
- The locality variable was extended to adjust in real time with market sentiment, so traders relied more on external signals in stable conditions and less during volatile phases

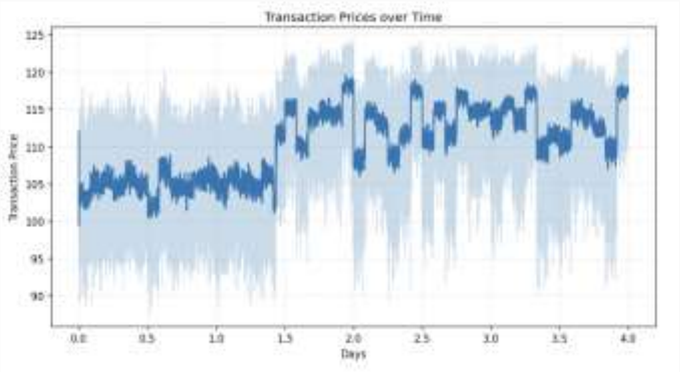
$$n_{external} = (x_{external})^2 \left(\frac{1}{\sigma + 1}\right) + 0.1$$

Dynamic O-PRDE

- The Dynamic O-PRDE model extends the baseline by making locality dynamic, with stationary market conditions defined by fixed supply (75) and demand (125), ensuring no external shocks.





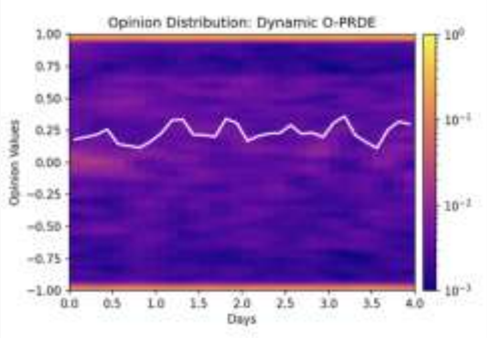


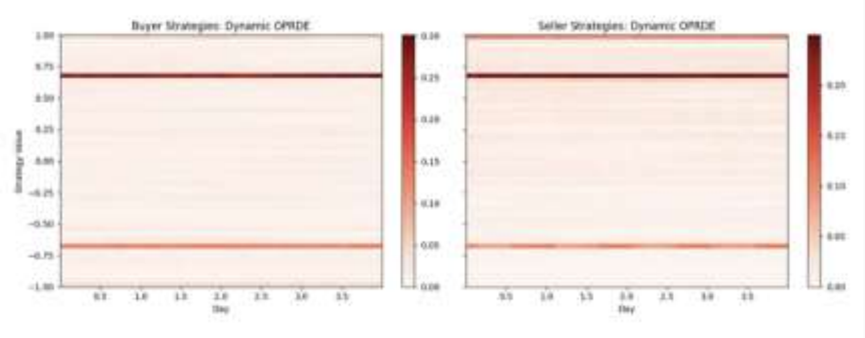
- This setup produced a gradual upward drift in trader sentiment. As the market turned moderately bullish, buyers adopted more urgent bidding strategies while sellers became more relaxed, reflecting greater optimism.

Dynamic O-PRDE

Exogenous Supply-Demand Shock

- Building on the stationary baseline results, the experiment was extended by introducing an external disturbance. The demand-supply schedules shifted from (75,125) to (60,110), generating an exogenous market shock and a price drop.





- Following the shock, trader sentiment remained mildly positive; buyers bid more aggressively while sellers stayed relaxed, reflecting optimism and adjustment to a lower equilibrium rather than a collapse in confidence.

Conclusion & Proposed Future Work

Conclusion

- Network topologies: Watts–Strogatz sustained fragmented clusters with slower convergence, while Klemm–Eguíluz showed hubs driving faster consensus but persistent extremists.

- Dynamic malleability: Enabled shifts between social and external influence, leading to sentiment stabilising at modestly positive levels even under shocks.

- Key finding: Results support Shiller's narrative economics, showing markets shaped not only by fundamentals but also by evolving narratives.

Future Work

- Future work could explore allowing the mixing parameter μ in the Klemm-Eguíluz model to vary dynamically during a simulation, enabling the network to transition between clustered and scale-free structures.

- This would better reflect real-world markets, where communication patterns evolve from local clustering to hub dominance (as in the housing bubble), offering deeper insight into how network changes shape the spread and persistence of narratives.