Electric Field and Potential due to a Uniformly Charged Sphere

Tanya Chhabra

September 28, 2024

Abstract

This report investigates the electric field and potential due to a uniformly charged spherical shell. Three numerical methods—Riemann Sum, Trapezoidal Rule, and Simpson's Rule—are compared with the analytical solution. The error between numerical and analytical solutions is analyzed, and the effectiveness of each method is discussed

Table of Contents

1 Introduction

This report presents a computational analysis of the electric field due to a charge spherical shell of radius R. We implement various numerical integration methods and compare them to Python library, Scipy's quad function. The methods include:

1. Trapezoidal Rule
2. Simpson Rule
3. Riemann Sum

2 Theoretical Background

he electric field is a conservative vector field, meaning its curl is zero. As a result, it can be expressed as the negative gradient of a scalar potential, denoted by V. Where,

$$V(r) = -\int_{O}^{r} E \cdot dl \tag{1}$$

Here O is a reference point, which is conventionally taken to be ∞ in the cases where the charge distribution is not going to infinity. We take the case for the potential due to a charged spherical shell of radius R, inside and outside the sphere.

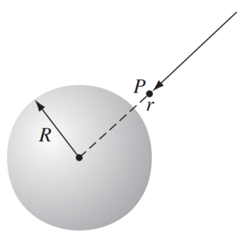

Figure 1: Spherical shell of radius R

From Gauss's Law, the electric field due to the sphere is:

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r} \tag{2}$$

The potential for any point r, where, $r > R$

$$V(r) = \frac{1}{4\pi\epsilon_0}\frac{q}{r} \tag{3}$$

The potential inside the sphere $r < R$ is given by,

$$V(r) = \frac{1}{4\pi\epsilon_0}\frac{q}{R} \tag{4}$$

3 Computational Methods

We approximated the integral $\int E(r)\dot{d}r$ to calculate the electric potential, where E is the electric field at a distance r due to the spherical shell, given by equation 2. Numerical integration techniques such as the Riemann Sum, Trapezoidal Rule, and Simpson's Rule offer varying degrees of accuracy and computational efficiency. In this report, we apply each method to compute the electric potential and compare their performances against an analytical solution

3.1 Riemann Sum

The Riemann Sum approximates the integral by summing the rectangles:

$$V(r) = -h\sum_{i=0}^{n-1} E(r+ih) \tag{5}$$

Where, $h = (b-a)/n$, a is the lower limit, b is the upper limit and n is the number of intervals.

3.2 Trapezoidal Rule

The Trapezoidal Rule approximates the integral using trapezoids:

$$V(r) = \frac{h}{2}[E(a) + E(b) + 2\sum_{i=1}^{n} E(r+ih)] \tag{6}$$

3.3 Simpson's Rule

Simpson's rule approximates the integral using parabolic arcs,

$$V(r) = -\frac{h}{3}[E(a) + E(b) + 4\sum_{i_{odd}=1}^{n-1} E(r+ih) + 2\sum_{i_{even}=1}^{n-2} E(r+ih)] \tag{7}$$

4 Results and Solutions

4.1 Example Solutions

The electric field taken due to a uniformly charged spherical shell is The potential obtained

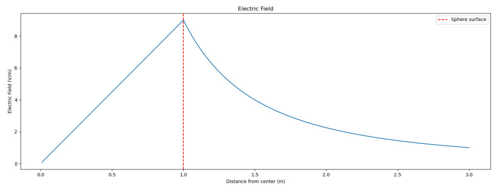

Figure 2: Plot for electric field due to a uniformly charged spherical shell

by various integration methods are:

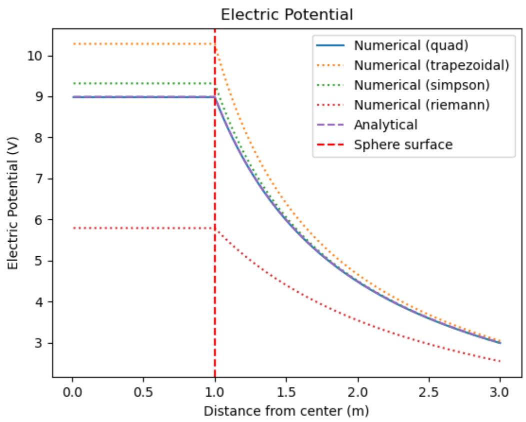

Figure 3: Comparison of various numerical methods with the analytical solution for potential in a Uniformly charged spherical shell

Figure 3 shows the comparison of numerical methods with the analytical solution for the potential. The Trapezoidal and Simpson methods closely follow the analytical solution, while the Riemann Sum deviates significantly from it.

4.2 Error Analysis

To quantify the performance of each method, we calucualted the relative error between the numerical and analytical solution. The relative error is defined as:

$$\text{Relative Error} = \frac{|V_{\text{num}} - V_{\text{exact}}|}{|V_{\text{exact}}|} \tag{8}$$

The maximum error obtained by each method, when step size is of the order of 10^{-3} is:

1. Numerical: $3.00e-03$

2. Trapezoidal: $1.44e-01$

3. Simpson: $3.63e-02$

4. Riemann: $3.56e-01$

5 Conclusion

This study demonstrates the effectiveness of various numerical methods for calculating the Potential of a Spherical Shell with a uniform surface charge. The Simpson's method for integration provides the best convergence to the analytical calculation.

6 References

1. Introduction to electrodynamics by David J. Griffiths

2. Scipy library