

August 18, 2015

Launchpad

Recursion++

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Let learn how to
think
recursively

What to look for:

Given any problem look for following:

- Is there a simple case which can be done by inspection?
- Can it be broken into similar but smaller/simpler subproblems?

Example: coin change problem

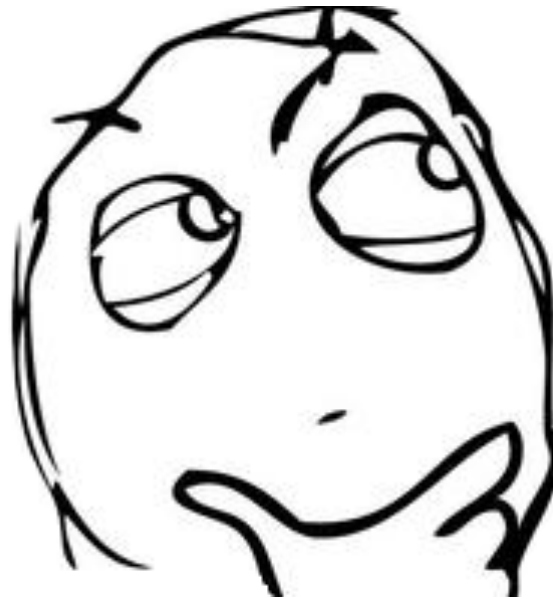
Problem: Given a value SUM and infinite supply of N types of coins. What is the minimum number of coins required to make up the given SUM. Output -1 if it is not possible.

Example:

SUM : 6

Coins : 1,2,3

Is there a simple case which can
be done by inspection?



Is there a simple case which can be done by inspection?

- What if I don't have any coin?
 - Answer would be -1.
- What if sum is negative?
 - Answer would be -1.
- What if sum is zero
 - Answer would be zero



Can it be broken into similar but smaller subproblems?



Can it be broken into similar but smaller subproblems?

1. Give your tool/function a name.

`find_min_coins()`

2. write what it need

`find_min_coins(all coin types, sum)`

3. Write in comments above what it would return.

// given various types of coins and a number this

// tool will compute minimum number of coins

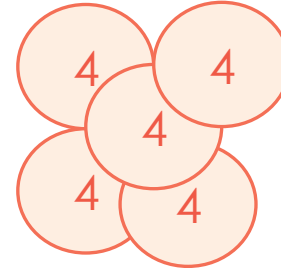
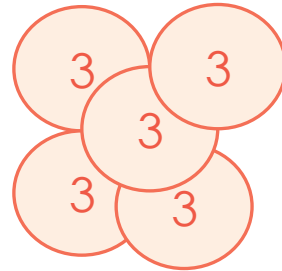
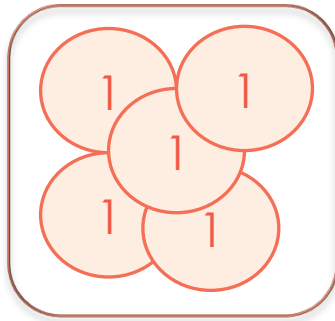
// needed to make that number

`min_coins find_min_coins(all coin types, sum)`

Literally write these three steps(in notebook/ comments) until you get comfortable in recursion, these steps will reinforce something you need for next step. 😊



Can it be broken into similar but smaller subproblems?

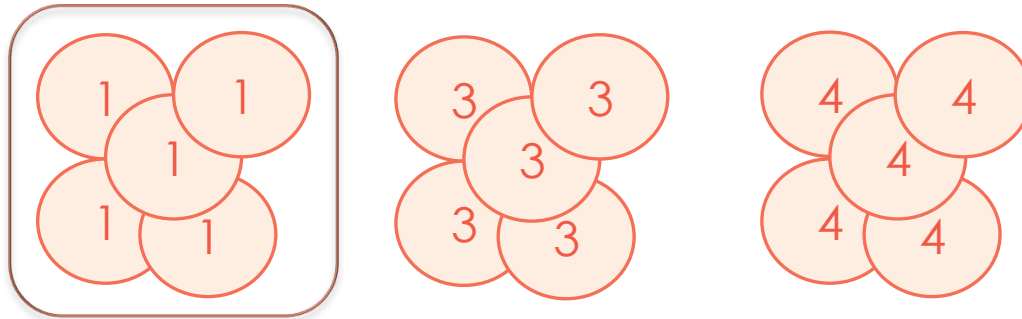


Lets just look at the first type of coin we have, think the **basics**

Either we will use the coins of this type OR we will skip this type.



Can it be broken into similar but smaller subproblems?



Option 1: Lets choose coin of first type to include in our solution.

We have picked one coin, so sum would be reduced by value of that coin.

$$\text{SUM} = \text{SUM} - V_1 \quad (6 - 1 = 5)$$

Lets look at the what problem we have left with:

*given all types of coins and a number **(SUM - V₁)** compute minimum number of coins needed to make that number. (sounds familiar 😊)*

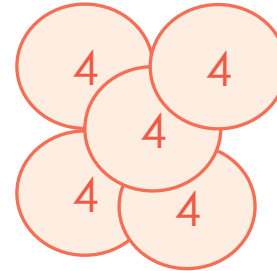
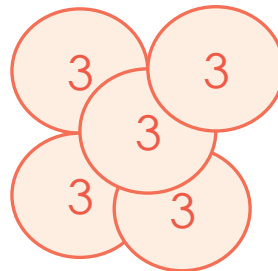
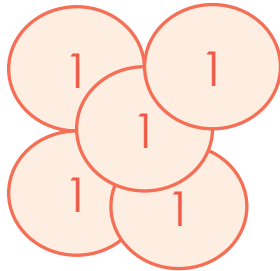
*Is this similar – **YES***

*Is this smaller/simpler – **YES**(the SUM needed is less now)*

find_min_coins(all coin types, **sum-V₁**)



Can it be broken into similar but smaller subproblems?



Option 2: Lets not pick coin of first type.

We have not picked any coin, so sum would be same.

Lets look at the what problem we have left with:

given **fewer** types of coins and a number compute minimum number of coins needed to make that number. *(sounds familiar 😊)*

*Is this similar – **YES***

*Is this smaller/simpler – **YES**(the types of coins are lesser now)*

find_min_coins(all coin types – first coin type, sum)

Hence solution would be:

```
on_including = find_min_coins(all_coins, SUM-V1)  
on_excluding = find_min_coins(all_coins – first coin, SUM)
```

```
If on_including = -1:
```

```
    return on_excluding
```

```
If on_excluding = -1:
```

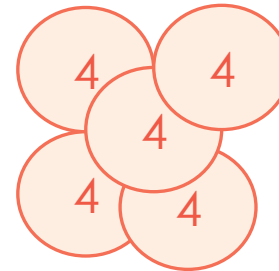
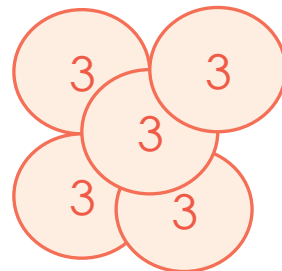
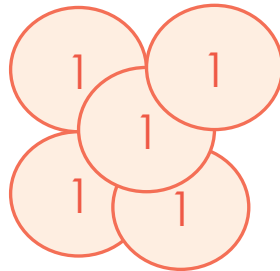
```
    return on_including + 1
```

```
return min(on_including+1, on_excluding)
```

Can it be broken into similar but
smaller subproblems?
-- another approach



Can it be broken into similar but smaller subproblems?

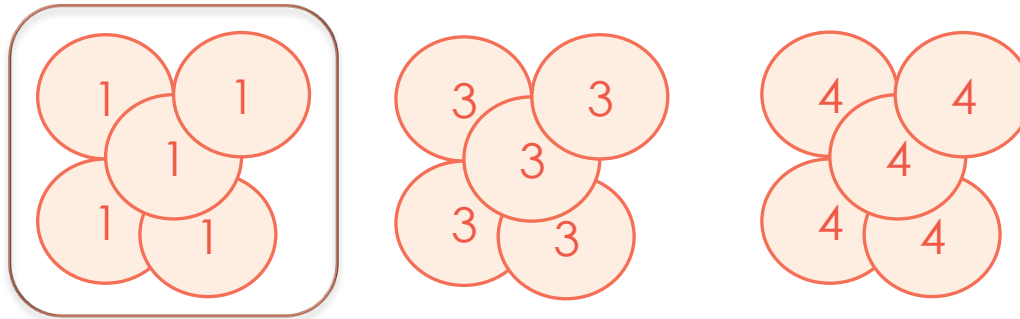


Lets just try to find what
can we choose for the
first coin

*Either we will choose first
coin or second coin or
third coin or*



Can it be broken into similar but smaller subproblems?



Option 2: Lets choose the coin of first type

We have picked one coin, so sum would be reduced by value of that coin.

$$\text{SUM} = \text{SUM} - V_1 \quad (6 - 1 = 5)$$

Lets look at the what problem we have left with:

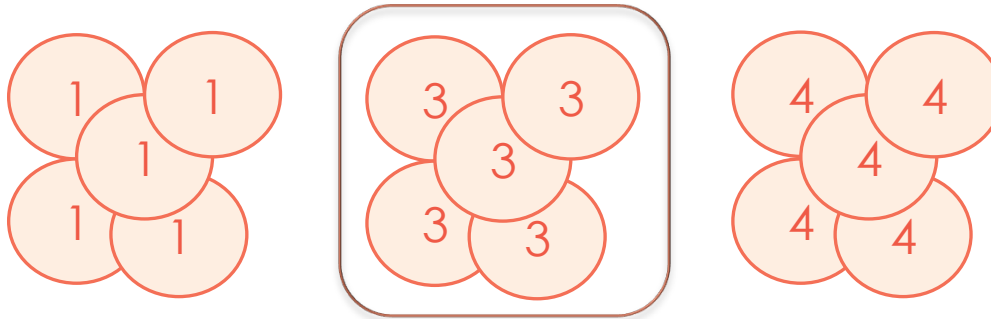
*given all types of coins and a number **(SUM - V₁)** compute minimum number of coins needed to make that number. (sounds familiar 😊)*

*Is this similar – **YES***

*Is this smaller/simpler – **YES** (the SUM needed is less now)*

find_min_coins(all coin types, **sum-V₁**)

Can it be broken into similar but smaller subproblems?



Option 1: Lets choose the coin of second type

We have picked one coin, so sum would be reduced by value of that coin.

$$\text{SUM} = \text{SUM} - V_2 \quad (6 - 3 = 3)$$

Lets look at the what problem we have left with:

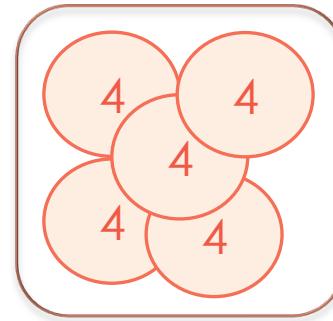
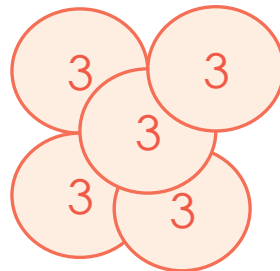
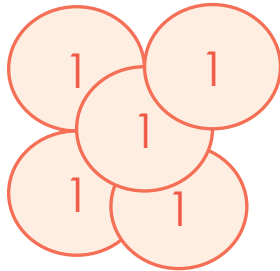
*given all types of coins and a number **(SUM - V₂)** compute minimum number of coins needed to make that number. (sounds familiar 😊)*

*Is this similar – **YES***

*Is this smaller/simpler – **YES** (the SUM needed is less now)*

find_min_coins(all coin types, **sum-V₂**)

Can it be broken into similar but smaller subproblems?



Option 3: Lets choose the coin of third type

We have picked one coin, so sum would be reduced by value of that coin.

$$\text{SUM} = \text{SUM} - V_3 \quad (6 - 4 = 2)$$

Lets look at the what problem we have left with:

*given all types of coins and a number **(SUM - V₃)** compute minimum number of coins needed to make that number. (sounds familiar 😊)*

*Is this similar – **YES***

*Is this smaller/simpler – **YES** (the SUM needed is less now)*

find_min_coins(all coin types, **sum-V₃**)

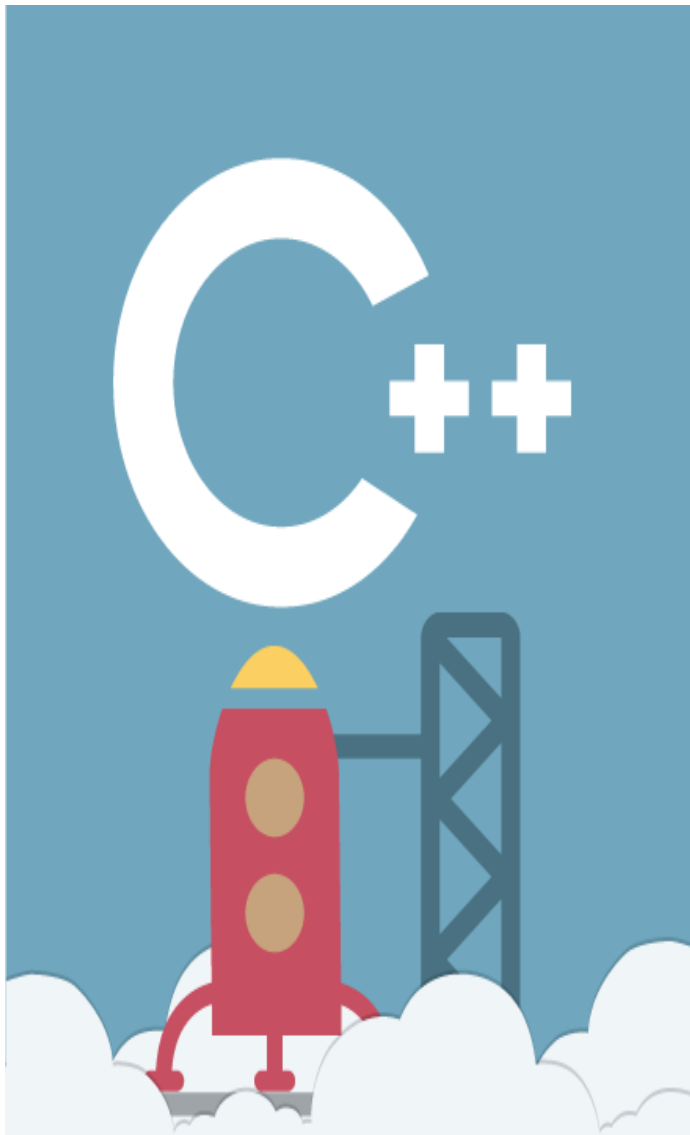
Hence solution would be:

`min_coins = infinity`

For (each coin type i):

`on_inc = find_min_coins(all_coins, SUM- V_i) + 1`

`min_coins = min(min_coins, on_inc)`



Thank You!

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