Projectile Motion

Purpose

To determine the launch angle which gives the maximum collision height for a projectile fired at a wall, given the distance to the wall and the initial speed of the projectile.

Pre-Lab Exercises

1. Read this lab handout carefully.
2. Prepare for your pre-lab quiz by doing the derivation for Equation (1) based on the following tips.
   (a) Given the initial conditions in Figure 1, write the equation for the horizontal displacement as a function of time.
   (b) Consider the moment t_{hit} when $x = D$ and solve for t_{hit}.
   (c) Given the initial conditions in Figure 1, write the equation for the vertical displacement as a function of time, then substitute t_{hit} for t. This gives the height of the ball $y(t_{hit})$ at the moment it hits the wall.
   (d) To determine the launch angle which gives the maximum value of $y(t_{hit})$, you will need to take the derivative of $y(t_{hit})$ with respect to θ. To do this you will need the derivative of $\tan\theta$ and the derivative of $1/cos^2(\theta)$.
   (e) Set the derivative of $y(t_{hit})$ with respect to θ equal to zero and solve for θ. This value of θ (the one that makes the derivative zero) can be called $\theta_{y_{max}}$.

Introduction

In this lab we will be using a projectile launcher to launch a plastic ball against the classroom wall, with the objective of maximizing the collision height. Figure 1 shows the initial conditions of the experiment as well as the coordinate system we will be using in this lab.

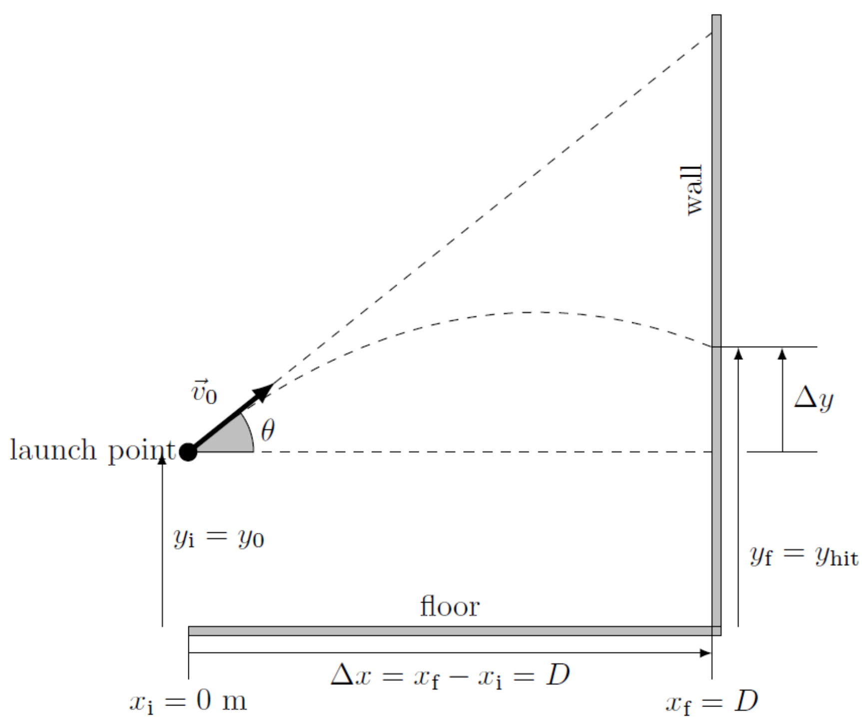

Figure 1: Trajectory diagram

Take the initial speed of the projectile to be v_0, the horizontal distance to the wall to be D, and use the accepted value for the acceleration due to gravity of $9.806\ 65\ m/s^2$ for g. It can be shown using calculus that the launch angle θ which gives the maximum collision height is:

$$\tan \theta_{y_{max}} = \frac{v_0^2}{gD}$$

Eq. 1

Procedure

Determining the initial velocity v_0:

- Measure the distance between the two photogate "eyes" Δd using a ruler. Make sure you also record the uncertainty in this measurement $\delta\Delta d$.
- Set the projectile launcher to a $0°$ angle, insert the plastic ball, and cock it to the "long range" setting using the loading bar. Turn on the Smart Timer and set it to measure TIME with TWO GATES. When you are ready to fire the ball, press [3] on the Smart Timer (make sure not to trigger the photogates after you have turned them on). Launch the ball and record the time Δt it takes for the ball to pass between the photogates. Repeat this measurement for a total of 5 times.
- Use the average of these times $\overline{\Delta t}$ and the photogate separation Δd to calculate your initial velocity v_0.

Measuring the horizontal distance D:

- Set the projectile launcher to a $45°$ angle. Using either a 2-meter stick or a measuring tape, determine the horizontal distance from the muzzle of the projector launcher to the wall. Make sure to estimate the uncertainty δD in the horizontal distance.

Calculating the theoretical value for $\theta_{y_{max}}$:

- Using your value for the initial velocity v_0 and your horizontal distance D, calculate the theoretical value of $\theta_{y_{max}}$ (the launch angle that will maximize the projectile's impact height on the wall).

Preparing for height measurements:

- Adjust the angle on your projectile launcher to the theoretical value of $\theta_{y_{max}}$.
- Launch your ball at the wall and determine the impact position on the wall.
- Place a piece of blank white paper on the wall covering the impact position, with most of the paper extending below the area of the impact position. Cover the white paper with a piece of carbon paper so impact positions will be marked for later launches of the ball. Make sure you are able to lift the carbon paper between trials so you can record the launch angle associated with each impact position.

Measurement trials:

- Launch the ball at five different angles, five times at each angle (for a total of 25 launches). The angles you should use are your theoretical $\theta_{y_{max}}$ as well as angles approximately $3°$ and $6°$ both greater and less than your theoretical $\theta_{y_{max}}$.
  - *Example: for a theoretical $\theta_{y_{max}} = 43°$, you would use the launch angles:* $37°$, $40°$, $43°$, $46°$, $49°$
- Record the vertical positions on the wall for all marked impact points and be sure to identify the launch angle associated with each vertical measurement.
- For each launch angle, determine the average height $\bar{y}$ of the impact positions.

Error Analysis

$\delta\overline{\Delta t}$ — Find the standard uncertainty in the mean as outlined in the *Treatment of Data* document.

δv_0 — Use the Power-Product rule, as well as the uncertainties δd and $\delta\overline{\Delta t}$, to find v_0.

$\delta\, \theta_{y_{max}}$ — Use the "Brute Force" method for determining the uncertainty in the theoretical value of $\theta_{y_{max}}$. You should be able to express $\theta_{y_{max}}$ as a function of v_0 and D first, then determine the uncertainty.

$\delta\bar{y}$ — Separately for each of the five angles, find the standard uncertainty in the mean of $\bar{y}$.

Graphing and Analysis

Using the Python plotting template *specific to this lab,* make a plot of $\bar{y}$ v. θ for the five angles tested. Instead of a linear fit, this data set will be fit with a second order polynomial.

The fit equation will be in the form $\bar{y} = A\theta^2 + B\theta + C$.

To determine the experimental angle that maximizes the impact position, you will need to find the maximum of the fit equation. In case you haven't learned to maximize a function in your calculus courses yet, here are the basic steps:

1. Determine the derivative of the fit equation $\frac{\delta\bar{y}}{d\theta}$.
2. Set the derivative equal to zero (because maxima and minima have slopes of magnitude zero).
3. Substitute $\theta = \theta_{y_{max}}$ and solve for $\theta_{y_{max}}$.

After you know your experimental value for $\theta_{y_{max}}$, you have a couple more things to consider:

- Estimate the uncertainty in the experimental angle $\delta\theta_{y_{max}}$ using the smallest angular division on the launcher.
- Determine whether your experimental value for $\theta_{y_{max}} \pm \delta\theta_{y_{max}}$ agrees with your theoretical value for $\theta_{y_{max}} \pm \delta\theta_{y_{max}}$.

Final Considerations

Prepare for the post-lab quiz by making sure you have your experimental results $\theta_{y_{max}} \pm \delta\theta_{y_{max}}$. Compare these results to your theoretical value for $\theta_{y_{max}} \pm \delta\theta_{y_{max}}$. State whether the results agree or disagree.

Make sure you have submitted the GitHub link for your plot, and have verified that the link you provided will take your instructor to a **displayed** plot (not just the code), prior to our next lab meeting.