HW set2:

1. Specify condition on f, and solve the initial boundary value problem of heat equations

(1)
$$\begin{cases} u_t = u_{xx} + t\cos x, & x \in [0,1], \ t > 0. \\ u_x(0,t) = u_x(1,t) = 0, \\ u(x,0) = f(x) \end{cases}$$

by the method of separation of variables.

2. Solve the boundary value problem of the Laplace equation in disc:

(2)
$$\begin{cases} \Delta u = u_{xx} + u_{yy}, & x^2 + y^2 < R^2, \\ u(R\cos\theta, R\sin\theta) = f(\theta) \end{cases}$$

by the method of separation of variables (in polar coordinates), in the form

$$u(r,\theta) = \int_0^{2\pi} G(r,\theta,R,\phi)f(\phi)d\phi \ .$$

3. Solve the initial boundary value problem of the equation

(3)
$$\begin{cases} \partial_t^2 u = u_{xx} + u_{yy}, & (x,y) \in [0,1] \times [0,\pi], \\ u(0,y,t) = u(1,y,t) = u_y(x,0,t) = u_y(x,\pi,t) = 0 \\ u(x,y,0) = f(x,y), u(x,y,0) = 0 \ . \end{cases}$$