Project II: Boolean Algebra of Yin Sets in $\mathbb{R}^3$

due 2019 DEC 25, 13:15

In this project, you are supposed to develop the mathematical theory of Boolean algebra on three-dimensional (3D) Yin sets, design a set of algorithms corresponding to your theory, write a `C++` package implementing these algorithms, and validate your final product by thorough unit tests and acceptance tests.

Theorem 1, as well as anything on the notes, can be taken as given.

Theorem 1. Every orientable compact surface is homeomorphic either to a sphere or a connected sum of tori.

Theorem 2. For any Yin set $\mathcal{Y}$ in $\mathbb{R}^3$, its boundary $\partial\mathcal{Y}$ is homeomorphic to the gluing of a collection of orientable compact surfaces along $L \subset \partial\mathcal{Y}$ homeomorphic to some one-dimensional cell complex.

The mathmatical problems are as follows:

(A) Prove Theorem 2.

(B) Give a constructive definition of your 3D Boolean algebra of Yin sets

$$(\mathbb{Y}, \cup^{\perp\perp}, \cap, ^{\perp}, \emptyset, \mathbb{R}^3)$$

in terms of the orientable compact surfaces in Theorem 2.

(C) Prove the correctness of your Boolean algebra in (B).

Yin sets can be represented in a way similar to that in project IV. In other words, the boundary of a Yin set is approximated by a two-dimensional simplicial complex. In your `C++` package, you may follow the same data format as in project IV. Just as in project I, your implementation of the Boolean algebra should take an uncertainty parameter $\epsilon > 0$ and be consistent to the convention that any two points with their distance less than ϵ are considered as the same point.

You should provide enough details in your design document so that the grader can follow your logic. You are also required to perform thorough tests and organize them in a single `cpp` file so that the grader can easily reproduce your test results. The use of `GNU make` is strongly recommended.