Chapter 1

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1.1 Problem

Problem 1.1. Use Frobenius method to find the complete asymptotic series expansion for the 2nd-order *modified bessel Differential Equation of order* ν:

$$y'' + \frac{1}{x}y' \mp \left(1 \pm \frac{\nu^2}{x^2}\right) y = 0$$

near $x = 0$. How many independent solutions can be found as a Frobenius series?

Hint: Disscuss different root scenarios of the indicial polynomial

$$P(\alpha) = \alpha^2 - \nu^2$$

Solution.

Problem 1.2. Identify the drastic change in the behavior of the solution to the ODE

$$\varepsilon y'' + \left(x^2 - \frac{1}{4}\right) y' - e^{2x-1} y = 0, 0 < x < 1$$

with $y(0) = 2$ and $y(1) = 3$ with the method of matched asymptotic expansions. Find the leading order, composite expansion of the exact solution.

Solution.

Problem 1.3. Derive the leading order asymptotic behavior of the solution to the ODE

$$y'' + k^2(\varepsilon t)y = 0, 0 < t$$

where $\varepsilon \ll 1$ and

$$y(0) = a, y'(0) = b.$$

Try solving with the method of multiple scales.

1.2 Bibliofraphy Review

$$\sup \inf \leftarrow \cos \in$$