# Chapter 1

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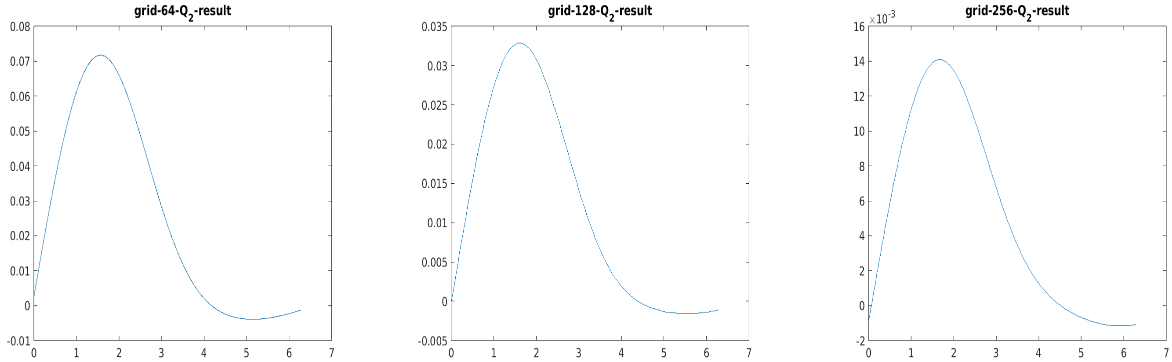
## 1.1 Program1

计算初值为 $cos(x)$ 的对流扩散方程 $u_t = -u_x + 0.1u_{xx}$ 在时间 $\pi$ 后的结果。

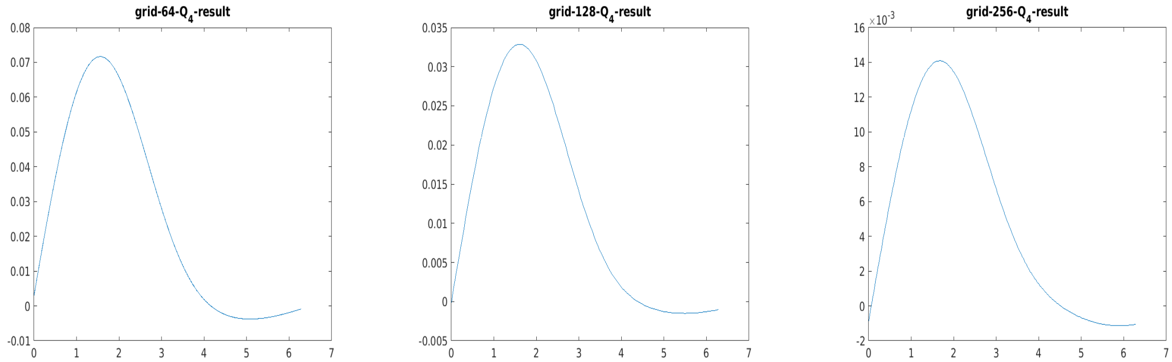
以 512 网格的 $Q_2$ 方法结果作为精确解, 有如下误差结果，不难发现 $Q_4$ 方法在误差上没有明显优势

图 1.1: initial function 1




(a) $64grid$ (b) $128grid$ (c) $256gird$

图 1.2: initial function 2




(a) $Q_2$ (b) $Q_4$ (c) $Q_6$

## 1.2 BDF method stability domain

**Exercise 1.1.** Try to display the stability domain for the BDF methods for $3 \le p \le 6$.
**Solution.**

- $p = 3$. the characteristic equation is

$$(1 - \frac{6}{11}\mu)z^3 - \frac{18}{11}z^2 + \frac{9}{11}z - \frac{2}{11} = 0.$$

- $p = 4$. the characteristic equation is

$$(1 - \frac{12}{25}\mu)z^4 - \frac{48}{25}z^3 + \frac{36}{25}z^2 - \frac{16}{25}z + \frac{3}{25} = 0.$$

- $p = 5$. the characteristic equation is

$$(1 - \frac{60}{137}\mu)z^5 - \frac{300}{137}z^4 + \frac{300}{137}z^3 - \frac{200}{137}z^2 + \frac{75}{137}z - \frac{12}{137} = 0.$$

- $p = 6$. the characteristic equation is

$$(1 - \frac{60}{147}\mu)z^6 - \frac{360}{147}z^5 + \frac{450}{147}z^4 - \frac{400}{147}z^3 + \frac{225}{147}z^2 - \frac{72}{147}z + \frac{10}{147} = 0.$$

  don't know how to plot, only find wiki picture;

图 1.3:


## 1.3 2.2.1

**Exercise 1.2.** Derive the stability condition for Simpson' s rule (2.2.10) when used for the equation $u_t = u_x$ with $Q_4$ .
**Solution.** the characteristic equation is

$$(1 - \frac{\mu}{3})z^2 - \frac{4\mu}{3}z - (1 + \frac{\mu}{3}) = 0.$$

with two roots be

$$|z_{1,2}| = |\frac{2\mu \pm \sqrt{3\mu^2 + 9}}{3 - \mu}| \le 1.$$

have $|2\mu + \sqrt{3\mu^2+9}| \leq |3-\mu|$ and $|2\mu - \sqrt{3\mu^2+9}| \leq |3-\mu|$ get $Re\ \mu = 0$, and $|Im\ \mu| \leq \sqrt{3}$.

By $Q_4 = i\lambda \sin(\xi)(1 - \frac{2}{3}\sin^2\frac{\xi}{2})$, induce

$$|\lambda \sin(\xi)(\frac{2}{3} - \frac{1}{3}\cos\xi)| \leq \sqrt{3}.$$

Solve it $\frac{k}{h} = \lambda \leq \sqrt[4]{27}(\sqrt{6} - \sqrt{2})$