

Chapter 1

Homework 12235005 谭焱

1.1 3.2.1

Exercise 1.1. Consider the differential equation

$$\frac{\partial u}{\partial t} = \sum_{j=0}^4 a_j \frac{\partial^j u}{\partial x^j}$$

Derive the condition for stability corresponding to Eq. (3.2.3). Is it true that the problem is always stable if $\operatorname{Re} a_4 < 0$?

Solution. Like Eq. (3.2.3) we are going to prove there exists a Real α such that

$$\operatorname{Re} \kappa < \alpha, \quad \kappa := a_4 \omega^4 - i a_3 \omega^3 - a_2 \omega^2 + i a_1 \omega + a_0, \text{ For all Real } \omega.$$

If $\operatorname{Re} a_4 < 0$, there exists a Real W , such that $\forall |\omega| > W, \operatorname{Re} \kappa < 0$.

So Take $\alpha = |a_4 W^4| + |a_3 W^3| + |a_2 W^2| + |a_1 W| + |a_0|$ will satisfy $\operatorname{Re} \kappa < \alpha$.

Which means the problem is always stable.