Chapter 1

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Exercise 1.1. 设 $\alpha : I \to X$ 一条道路 $\alpha(0) = x_0$,$e_{x_0} : I \to X$ 表示 x_0 处常道路作连接

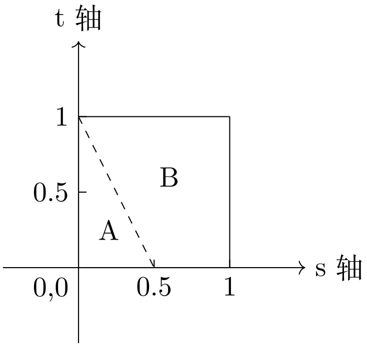
证明: $e_{x_0} * \alpha \simeq_p \alpha.$

Solution.

$$e_{x_0} * \alpha = \begin{cases} x_0 & 0 \le s \le \frac{1}{2} \\ \alpha(2s-1) & \frac{1}{2} < s \le 1. \end{cases} \tag{1.1}$$

Define

$$H(s,t) = \begin{cases} F(s,t) = x_0 & (s,t) \in A \\ G(s,t) = \alpha(\frac{2s+t-1}{1+t}) & (s,t) \in B. \end{cases} \tag{1.2}$$



在上图中

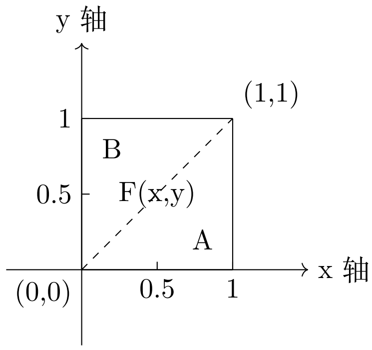
$$H(s,1) = \alpha(s) \qquad H(s,0) = e_{x_0} * \alpha(s)$$
$$H(0,t) = F(0,t) = x_0 \qquad H(1,t) = G(1,t) = \alpha(1) = x_1.$$

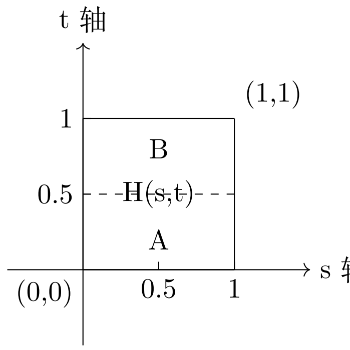
因为在 A, B 的公共边界 $2s + t - 1 = 0$ 上 $F(s,t) = x_0 = \alpha(0) = G(s,t)$ 由粘合引理 $H(s,t)$ 连续，所以 $e_{x_0} * \alpha \simeq_p \alpha.$

Exercise 1.2. 设 $F : I \times I \to X$ 连续，令 $F(0,t) = \gamma(t), F(s,1) = \beta(s), F(1,t) = \delta(t), F(s,0) = \alpha(s),$

证明: $\alpha \simeq_p (\gamma * \beta) * \delta^{-1}.$

Solution. $F(s,t)$ 在 $I \times I$ 上的图像如下





Define

$$H(s,t) = \begin{cases} S(s,t) = F(2s - 2st, 2st) & (s,t) \in \\ G(s,t) = F(1 - 2t + 2st, 2s - 1 + 2t - 2st) & (s,t) \in \end{cases} \tag{1.3}$$

容易验证

$$H(s,1) = \gamma * \beta \qquad H(s,0) = \alpha * \delta$$
$$H(0,t) = F(0,0) = \alpha(0) = \gamma(0)$$
$$H(1,t) = F(1,1) = \delta(1) = \beta(1).$$

容易看到图 2 中的虚线映射到图一的虚线上，因此由 F 的连续性得 H 连续，即 $\alpha * \delta \simeq_p \gamma * \beta$. 又因为 $*$ 构成群，即 $\alpha * \delta \simeq_p \gamma * \beta \Longleftrightarrow \alpha \simeq_p \gamma * \beta * \delta^{-1}.$