Chapter 1

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1.1 3.8.1

Exercise 1.1. Let $B(x,t)$ be a Hermitian matrix and let $C(x,t)$ be a skew-Hermitian matrix. Prove that the system

$$u_t = (Bu)_x + Bu_x + Cu$$

is energy conserving that is

$$\|u(\cdot,t)\| = \|u(\cdot,0)\|$$

Solution. consider semibounded and B is Hermitian, C is skew-Hermitian.

$$\begin{aligned}
(Pu,u)+(u,Pu) &= ((Bu)_x,u)+(Bu_x,u)+(Cu,u)+(u,(Bu)_x)+(u,Bu_x)+(u,Cu)\\
&= -(Bu,u_x)+(Bu_x,u)-(u_x,Bu)+(u,Bu_x)+(C+C^*)(u,u)\\
&= -(Bu,u_x)+(Bu_x,u)-(Bu_x,u)+(Bu,u_x)\\
&= 0*\|u\|
\end{aligned}$$

0 so we have $\|u(\cdot,t)\| \le e^{0t}\|f(x)\| = \|u(\cdot,0)\|$. And change system sign, revert $u(\cdot,t)$ to $u(\cdot,0)$ have same condition. so it's conserving.

1.2 3.8.3

Exercise 1.2. Consider the linearized one-dimensional Euler equations, where $U=0$ and R is a constant. Prove that the system represents two "sound-waves" moving with the velocities $\pm a(R)$.

Solution. By (3.7.5), take $u=U=\rho_x=v_x=0$ the system is

$$\begin{aligned}
v_t &= Vv_y + \frac{a^2(R)}{R}\rho_y\\
\rho_t &= Rv_y + V\rho_y
\end{aligned}$$

Solve this get

$$\begin{aligned}
v &= \pm a(R)(y+(\pm a(R)+V)t)\\
\rho &= R(y+(\pm a(R)+V)t)
\end{aligned}$$