Numerical Analysis: Homework #3

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Programming Assignments

A

See the design document.

B

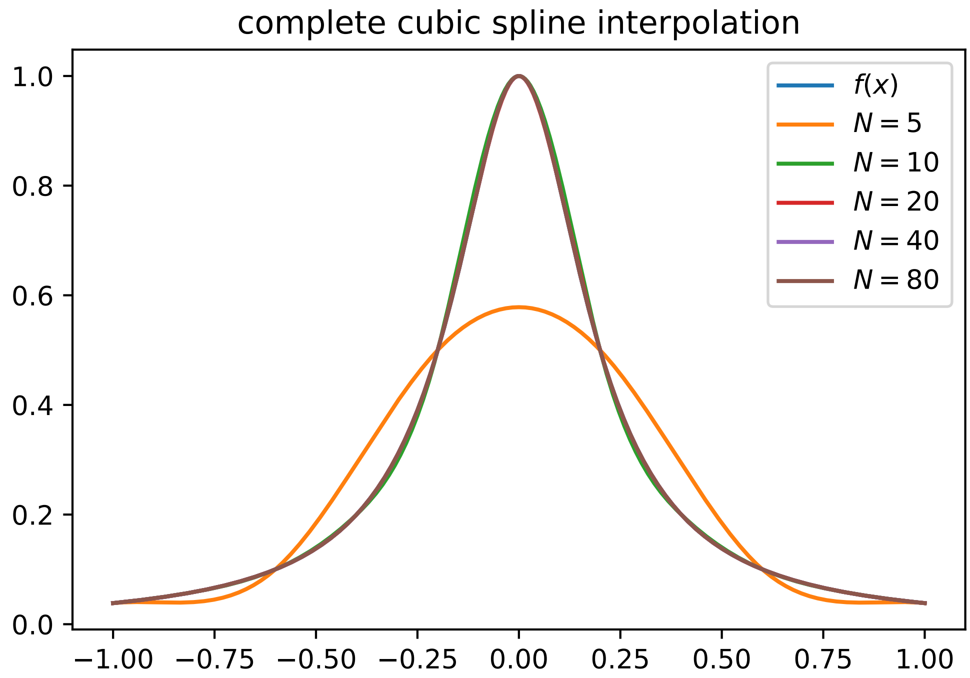
The max-norm of the errors are listed in the following table.

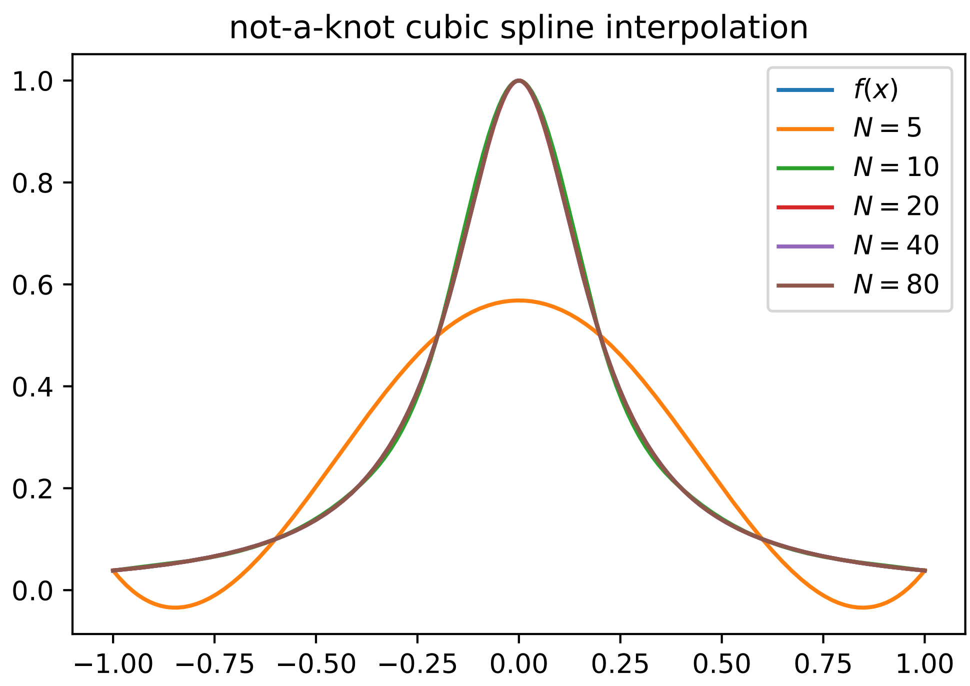
N	complete	not-a-knot
5	4.21705×10^{-1}	4.31538×10^{-1}
10	2.05289×10^{-2}	2.05334×10^{-2}
20	3.16894×10^{-3}	3.16894×10^{-3}
40	2.75356×10^{-4}	2.75356×10^{-4}
80	1.60900×10^{-5}	1.60900×10^{-5}

The rates of convergence are estimated as follows.

complete	not-a-knot
3.483	3.489

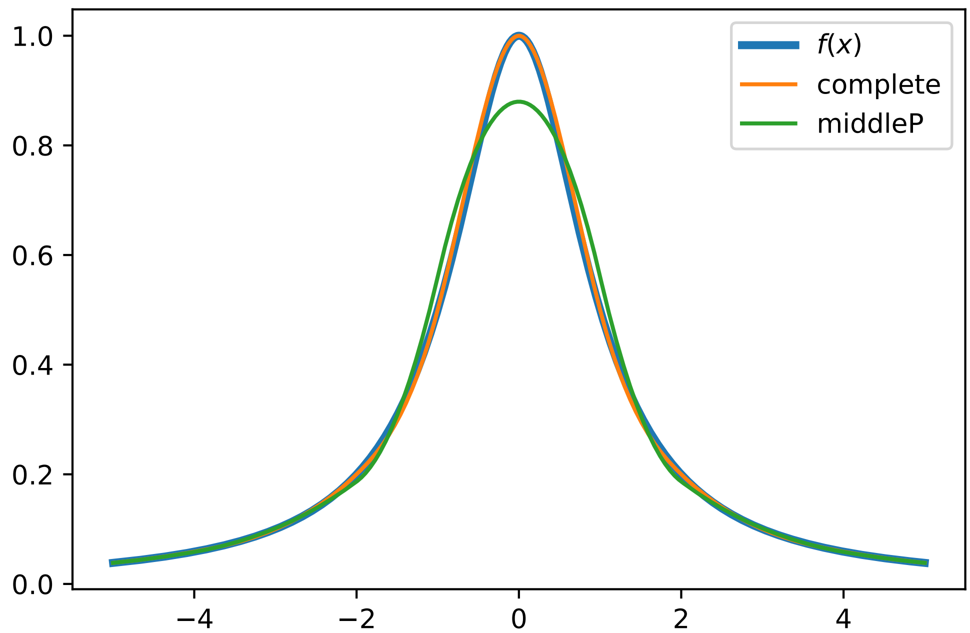
The interpolations are plotted.


complete cubic spline interpolation
1.0
0.8
0.6
0.4
0.2
0.0
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00
$f(x)$
$N = 5$
$N = 10$
$N = 20$
$N = 40$
$N = 80$


not-a-knot cubic spline interpolation
1.0
0.8
0.6
0.4
0.2
0.0
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00
$f(x)$
$N = 5$
$N = 10$
$N = 20$
$N = 40$
$N = 80$

C

The following is the plot.



D

The errors are listed in the following table.

x	E_{complete}	E_{middleP}
-3.50	6.6957×10^{-4}	0.0000×10^{0}
-3.00	0.0000×10^{0}	1.4184×10^{-3}
-0.50	2.0529×10^{-2}	1.1102×10^{-16}
0.00	1.1102×10^{-16}	1.2024×10^{-1}
0.50	2.0529×10^{-2}	1.1102×10^{-16}
3.00	0.0000×10^{0}	1.4184×10^{-3}
3.50	6.6957×10^{-4}	0.0000×10^{0}

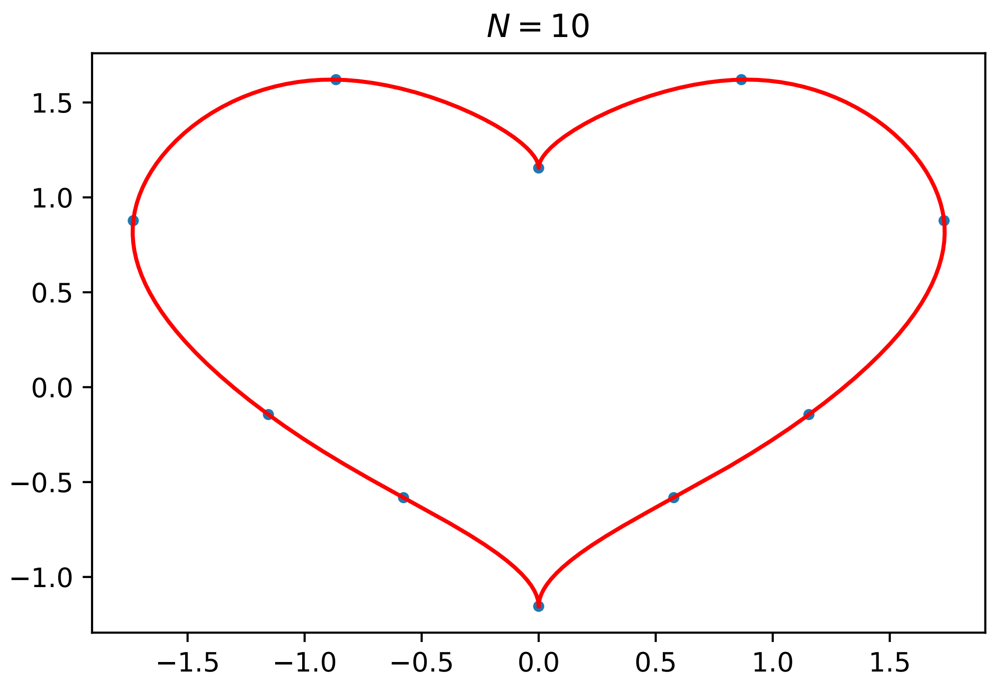
Some errors are close to the machine precision, because by the interpolation conditions, the spline should match exactly with the exact function. Therefore, mathematically the error is 0, and computationally the error should be close to machine precision.

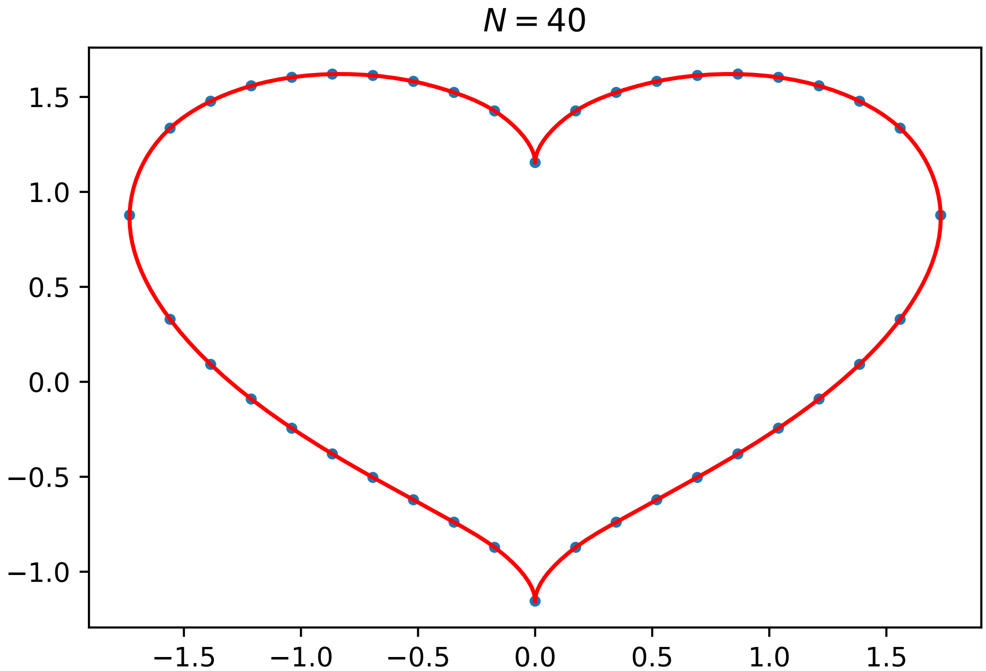
The accuracies of the two splines differ at different points. In general, the complete B-spline is more accurate at approximating points, i.e., points not in the interpolation conditions.

E

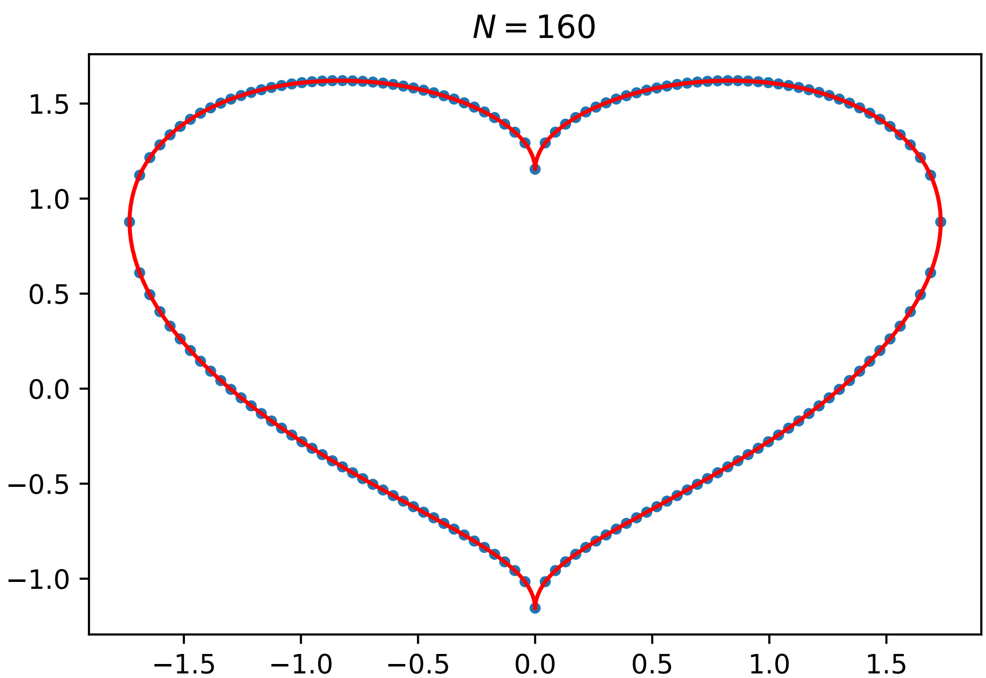
First, to get an understanding of the curve, I picked some points on it and scatter-plotted them. I found that the **critical points** are those with $x = 0$ and $x = \pm\sqrt{3}$. I also decided to **use two pieces of spline** to plot the curve, one for the left half and one for the right half. They are symmetric so practically I only need to calculate the right half. By examing the property of the curve when $x \to 0^+$, I found that locally the curve looks like a vertical line, which means that $x'(t_0) = 0$. So I think we should pose constraints on the first derivative of the spline at endpoints, which means the **complete spline condition** is appropriate.

The three plots are listed below, with blue points indicating the sample points and the red curve is our spline.





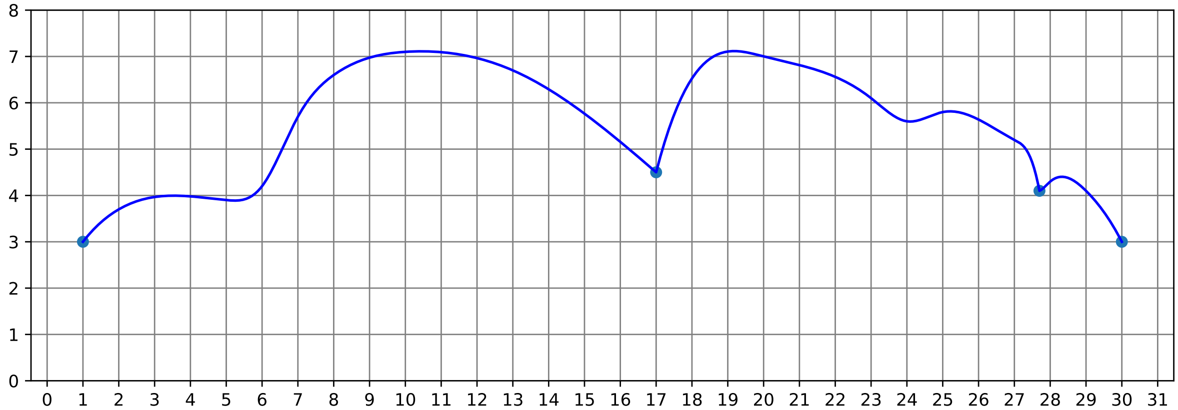
$N = 40$
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5



$N = 160$
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5

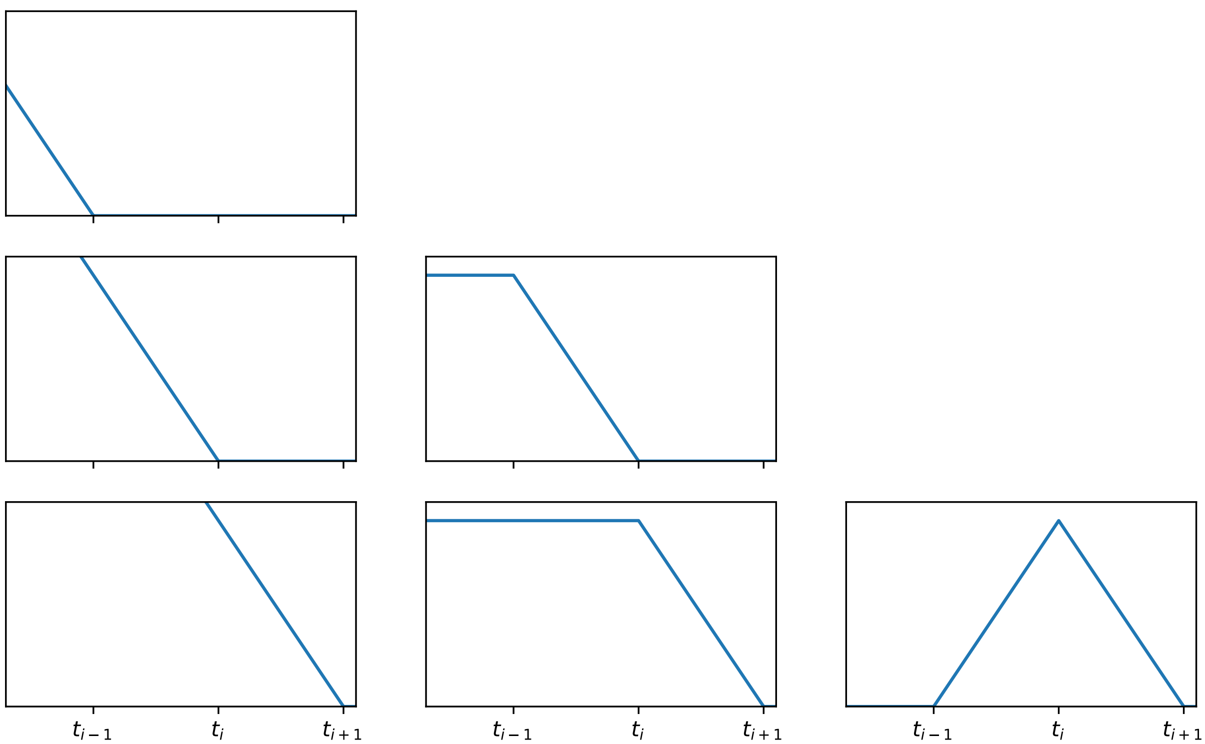
F

My construction of the three splines are plotted below.

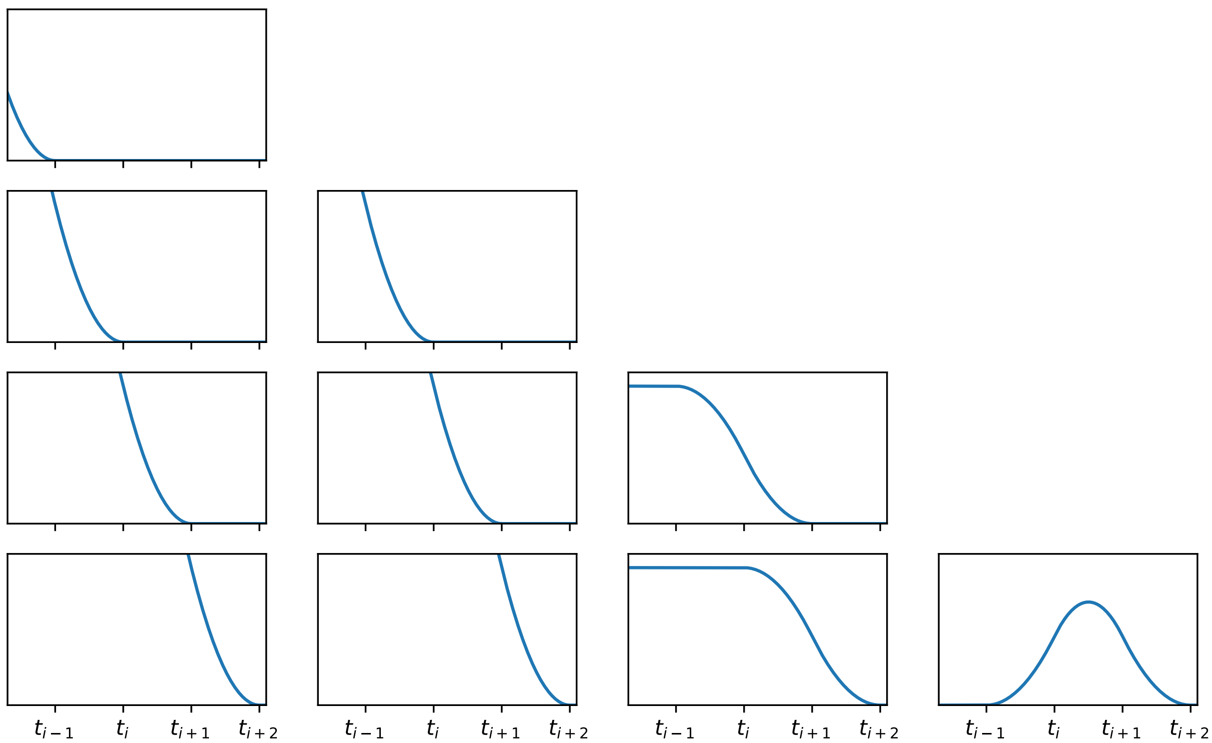


G

$n = 1$



$n = 2$



$n = 3$