International Macro and Trade Assignment 2

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Urban Theory. In Henderson (1974) there are a continuum of equilibria. In Behrens, Duranton, Robert-Nicoud (2014), Proposition 4 says there is a unique talent-homogeneous equilibrium. Is Proposition 4 correct? If it is correct, very clearly explain why talent heterogeneity makes the equilibrium unique when the homogeneous-worker case in Henderson (1974) yields multiple equilibria. If it is incorrect, identify the error and state the correct claim.

Proposition 4 of the paper says that the talent homogeneous equilibrium is unique and is such that:

$$L^*(t_c) = \left(\frac{1+\gamma}{1+\varepsilon}\xi t_c^{1+a}\right)^{\frac{1}{\gamma-\varepsilon}} \qquad \text{where} \qquad \xi \equiv \frac{(\varepsilon\sigma)^{1+\varepsilon}S^{1+a}}{\gamma\theta}$$

In their proof of this proposition, they plug in equations from the model until they have the equation:

$$\gamma\theta L(t_c)^{\varepsilon}\left[\frac{\xi t_c^{1+a} - L(t_c)^{\gamma-\varepsilon}}{L(t_c)}\mathrm{d}L(t_c) + \frac{1+a}{1+\varepsilon}\xi t_c^{a}\ \mathrm{d}t_c\right] = 0$$

Rearranging this to look like a usual ODE, we get

$$\begin{aligned} &= \frac{\xi t_c^{1+a} - L(t_c)^{\gamma-\varepsilon}}{L(t_c)} dL(t_c) = -\frac{1+a}{1+\varepsilon}\xi t_c^{a} dt_c \\ &= \frac{dL(t_c)}{dt} = -\frac{1+a}{1+\varepsilon}\frac{\xi t_c^{a} L(t_c)}{\xi t_c^{1+a} - L(t_c)^{\gamma-\varepsilon}} \end{aligned}$$

Plugging this into Wolfram Alpha, we get a solution of the form:

$$\xi t^{a+1} L(t_c)^{1+\varepsilon} - \frac{(1+\varepsilon)L(t_c)^{\gamma+1}}{\gamma+1} = c_1,$$

where $c_1 \in$ is a constant. Clearly this does not align with their guess of $L(t_c)$ from the paper. Their solution only works if we set $c_1 = 0$. Then, we can rearrange so that:

$$\begin{aligned} \xi t^{a+1} L(t_c)^{1+\varepsilon} &= \frac{(1+\varepsilon)L(t_c)^{\gamma+1}}{\gamma+1} \\ L(t_c)^{\varepsilon-\gamma} = L(t_c)^{\varepsilon-\gamma} &= \frac{1+\varepsilon}{(\gamma+1)\xi t^{a+1}} \\ L(t_c) &= \left(\frac{1+\varepsilon}{\gamma+1}\right)^{\frac{1}{\varepsilon-\gamma}}\left(\frac{1}{\xi t^{a+1}}\right)^{\frac{1}{\varepsilon-\gamma}} \\ &= \left(\underbrace{\frac{\gamma+1}{1+\varepsilon}}_{=z}\xi t^{a+1}\right)^{\frac{1}{\gamma-\varepsilon}} \end{aligned}$$

Clearly there are a couple of issues here. One is that their guess for the solution to the ODE is wrong unless we use the fact that $c_1 = 0$, which requires the initial value constraint that $L(0) = 0$ (i.e. the population of talent $t = 0$ in any city is always 0). Though this seems to be a relatively innocuous assumption, they do specify that talent is on a continuum lying between $[\underline{t}, \bar{t}]$. If $\underline{t} \neq 0$, it is unclear that their assumed initial value makes sense. The second issue is that given their guess for $L(t_c)$, they provide the wrong equation for z in the paper. As seen from the derivation above, $z = \frac{\gamma+1}{1+\varepsilon}$ if we use the correct solution to the ODE.

All of this is to say that proposition 4 is wrong. There is only a unique equilibrium if we are able to pin down a specific initial value (which would then give a specific value for c_1). The unique equilibrium presented in the proof of proposition 4 only works for $L(0) = 0$. But one could imagine that since $\underline{t}$ is arbitrary, different values of $\underline{t}$ (and different labor curves) might result in different initial values, which means that multiple equilibria may arise depending on the skill distribution and the share of the people with $t = 0$ in the city.

> **Trade and Urban Theory**. Look at Behrens Robert-Nicoud's "Agglomeration Theory with Heterogeneous Agents" chapter in the Handbook of Urban and Regional Economics. On page 204, they propose a theory of metropolitan specialization in which different cities are home to different industries. What is their prediction? How can this be investigated empirically? What is the role of comparative advantage? Is comparative advantage sufficient for the existence of a specialized equilibrium?

The theory of specialization proposed by Behrens and Robert-Nicoud predicts that cities will end up specializing in sectors where they have a comparative advantage. This occurs because local governments want to choose the sector that maximizes utility, so they end up choosing those where they have a comparative advantage. There are two sources of advantage: usual Ricardian comparative advantage where relative technologies play a role. There is an additional element of comparative advantage coming from the positive assortative matching between industries and cities (i.e. complementarities between agglomeration and and comparative advantage).

Comparative advantage is not sufficient for the existence of a specialized equilibrium in the Behrens and Nicoud model. Unlike Davis and Dingel, their model relies on the fact that migration into cities is limited (i.e. local governments can set hard limits on city size and use this as a choice variable in their optimization problem). Additionally, the model requires that there are no Jacbos' externalities (i.e. spillovers generated from a different industry). If there are such externalities, then there is advantage to diversifying in each city.

Empirics

It is hard to test the model as is since there is not great data on the technologies of cities. However, Davis and Dingel (2020)[1] set up a model of city-based comparative advantage where a prediction of the model is that larger cities have comparative advantage in high-skill sectors. To test this, Davis and Dingel propose a pairwise comparisons of cities ordered by their population sizes. Additionally, they use a linear regression of the form $\ln f(\nu, c) = \alpha_\nu + \beta_\nu \ln L(c) + \varepsilon_{\nu,c}$ to test log supermodularity. Here f is the skill distribution, ν is the skill level ω or the sector index σ, α_ν are fixed effects, and $L(c)$ is a city's population. Log supermodularity requires that $\beta_\nu \geq \beta_{\nu'} \iff \nu \geq \nu'$. They implement these tests using data on city and sector skill distributions from the census and sectoral employment from the Occupational Employment statistics. The paper generally finds evidence supporting log-supermodularity and showing that there is a pattern of comparative advantage across cities. [2]

[1] Davis, D. R. and Dingel, J. I. The comparative advantage of cities. Journal of International Economics.

[2] As a note, Davis and Dingel (2020) have a different model than that proposed by Behrens and Robert-Nicoud. They assume the same continuum of skills as the Behrens model, but also include multiple sectors and intra-city heterogeneity so that there is overlap in the skill distributions between cities. The paper also allows for more flexibility by assuming that valuations of sites

within cities may vary by income (this is different than the Behrens assumption that in equilibrium, individuals are indifferent between locations with a city). The increased flexibility is more realistic because if there was indifference between locations within a city, we would expect to see strict sorting between cities rather than the distribution of skills we actually observe in the data.