International Macro and Trade Assignment 1

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> Before writing your solver function, write a function that plots the $A(z)$ and $B(z; L*/L)$ functions that appear in equations (1) and (10') of DFS to produce a version of DFS Figure 1.

Passing the a and b vectors to the plotter function and setting $L = L^* = 1$ gives the following figure.

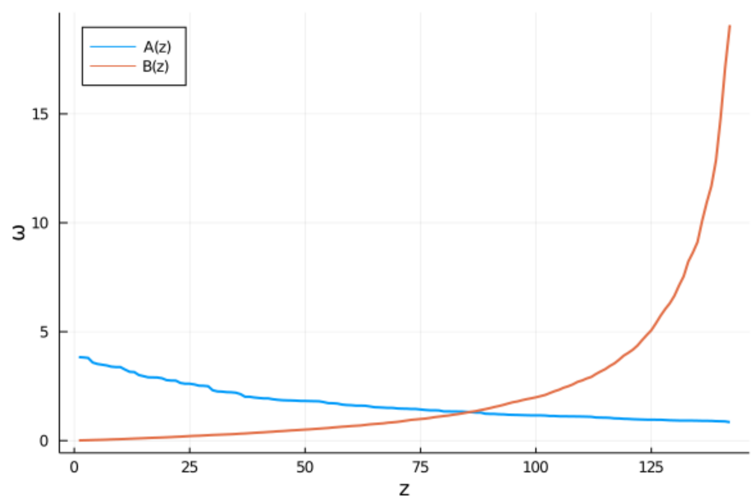

Figure 1: Intersection of $A(z)$ and $B(z; L)$ Schedules

> Write a function that solves a discrete approximation of Dornbusch, Fischer Samuelson (AER 1997)

See the attached code for the details of this function. It verifies that:

- a is $N \times 2$, has $N > 2$ rows, and has non-negative elements
- $A(z)$ is monotone decreasing
- The dimensions of a and b do not match
- b must be strictly positive and sum to 1
- Scalar $g \in (0, 1]$

Following the notation of DFS, I denote z^* as the intersection of $A(z)g$ and $\bar{\omega}$. Goods in the range $z \leq z^*$ are exported by the home country. I denote $\bar{z}$ as the intersection of $A(z)/g$ and $\bar{\omega}$. Goods in the range $z \geq \bar{z}$ are imported by the home country. Below I report the output of the solver function for $L = L^* = 1$ and two different trade costs: $g = 1$ and $g = 0.9$.

Table 1: DFS Solver Function Outputs

	z^*	$\bar{z}$	$\bar{\omega}$
$g = 1$	86	86	1.325
$g = 0.9$	72	94	1.297

In class, we said that Home's gains from trade can be written as an integral that depends on $\tilde{z}$, $b(z), a(z), a^*(z)$, and ω. Similarly for Foreign. Write a function that computes Home welfare and Foreign welfare in equilibrium as a function of a, b, L, and g.

Normalize home wages, $w = 1$, and calculate welfare as follows:

$$\text{Home Autarky: } \ln(U^a/L) = -\int_0^1 b(z)\ln[a(z)]dz$$

$$\text{Foreign Autarky: } \ln(U^{a*}/L) = \ln(w^*) - \int_0^1 b(z)\ln\left[w^* a^*(z)\right] dx$$

$$\text{Home Trade: } \ln\big(U^t/L\big) = -\int_0^{\bar{z}} b(z)\ln[a(z)]dz - \int_{\bar{z}}^{l} b(z)\ln\left[\frac{w^* a^*(z)}{g}\right] dz$$

$$\text{Foreign Trade: } \ln\big(U^{t^*}/L\big) = \ln(w^*) - \int_0^{z^*} b(z)\ln\left[\frac{a(z)}{g}\right] dz - \int_{z^*}^{1} b(z)\ln\left[w^* a^*(z)\right] dz$$

Using these equations, I write a function that calculates home and foreign welfare as well as gains from trade. Gains from trade are calculated as the difference between welfare in the trade equilibirum minus welfare in autarky. The table below reports the results for $g = 1$ and $g = 0.9$.

Table 2: Welfare Calculations

	$g = 1.0$	$g = 0.9$
Home Autarky	0.421	0.421
Home Trade	0.542	0.478
Home GFT	0.121	0.057
Foreign Autarky	0.000	0.000
Foreign Trade	0.254	0.211
Foreign GFT	0.254	0.211

What is the relationship between the volume of trade and each country's gains from trade in this model? Use your solver to produce an example of different equilibria (with the same L, L^*, and $g < 1$) that exhibit the same volume of trade and different gains from trade. If you also hold fixed the b schedule, can you produce such an example? Why or why not? What can be said about the magnitude of the gains from trade in this model if we observe the equilibrium volume of trade and do not observe autarky prices?

The volume of trade is defined as the sum of imports and exports from the home country. Since $b(z)$ defines the share of income spent on good z in the home country, imports are equal to income share multiplied by labor income (with w normalized to 1). Similarly, exports from the home country are equal to the income share multiplied by foreign labor income, w^*L^*. This sum of imports and exports is then divided by g to account for the goods lost in transit. We can write this as:

$$\text{Volume} \; = \frac{1}{g}\left(\underbrace{\int_0^{z^*} b(z)w^*L^*dz}_{\text{exports}} + \underbrace{\int_{\bar{z}}^{1} b(z)Ldz}_{\text{imports}}\right) \tag{1}$$

It is possible to change the $A(z)$ and $b(z)$ schedules so that the volume of trade remains the same while GFT changes. I start with the baseline model setting $L = L^* = 1$, $g = 0.9$, and using the a, b vectors given in the supplementary files.

I create a comparison model by changing parameters. First, I change the $A(z)$ schedule by assuming there is uniform technical progress for the home country (i.e. home unit labor requirements $a(z)$ decrease across the board). Next, I try to find a new $b(z)$ schedule that results in the same original volume of trade. I do this by defining a cutpoint $p \in 1, \ldots, 149$ and multiplying elements of the b vector below index p by some constant, c. I then renormalize the vector so that it sums to 1 and is a valid $b(z)$ schedule. A constant with value $c < 1$ would shift the $b(z)$ distribution towards foreign goods so that $b(z)$ has higher values for higher z.

Creating a grid of cutpoint values from 1 to 149 and constant values between 0 and 0.2,[1] I search for the combination of parameters (p, c) that results in the same trade volume as the baseline model. The grid search returned a cutpoint of $p = 143$ and a constant of $c = 0.076$, which heavily shifts income shares towards goods in which the foreign country has the competitive advantage.

Table 3: Models with Similar Volumes but Different GFTs

Parameter	Baseline	New
z^*	72.0	116.0
$\bar{z}$	94.0	142.0
$\bar{\omega}$	1.297	1.374
GFT Home	0.057	0.079
GFT Foreign	0.211	0.431
Volume	0.831	0.831

If we hold the b schedule fixed, we will not be able to find two economies with the same trade volumes and different GFTs. In the previous exercise, the uniform rise in home productivity led to more goods being exported by the home country (higher z^*).[2] This increased the value of the exports term in Equation 1. But by shifting the b schedule to favor foreign goods, the relative weight of the exports term in Equation 1 decreases. As a result, the volume of trade remains the same. If we hold b fixed, we cannot make this offsetting adjustment.

If we only observe the equilibrium volume of trade but no autarky prices, we cannot say much about the gains from trade. As shown earlier, we can construct A and b schedules such that we have the same trade volume but different GFT.

[1] I started with a larger grid search range and a larger step size, but narrowed the range in order to decrease computational time while increasing precision.

[2] Note that $\bar{\omega}$ also rises, which would lower w^* and the exports term.