PART 1
Understanding the Relationship between the Quantity, Volume, Temperature, and Pressure of Gas Particles in order to Derive the Combined and Ideal Gas Laws

Introduction: Technical and Historical Background + Usage of Kelvin + Hypothesis and Research Question *
* The rest of the Introduction is in "Gas Laws Part III – Ideal"
Previously, in our IS9 course, we developed our understanding of Newtonian Dynamics in the macroworld. One example can be our knowledge of forces. Forces are a push or a pull in a certain direction. Though, one special case of this is Frictional Force, because it's neither a push or a pull. Friction is a force that opposes motion, and it generates loss in the form of Thermal, Light, and Sound Energy. This can be seen when we pull a rock against a sidewalk. We usually notice that the surface the rock was pulled against became warmer, the action pulling the rock against the ground caused noise, and sometimes even light such as sparks. As observed in the rock and sidewalk, a certain force is applied against the motion over a certain distance to generate loss. This can be justified by the equation $W = F \bullet d$, where the Work is the loss done by friction over the distance it was pulled. Scientists such as Robert Hooke decided to incorporate our knowledge of these energies to the microworld. So, Robert Hooke used the concept of energies to describe the motion of atoms. Though, he noticed that if there are energies for moving objects such as a rock, there should also be energy for objects that are not in motion, which we later called Potential Energy. One form of this, Elastic Potential Energy, can be seen in springs because whenever we compress or stretch springs, they always return to their original form. In springs, a force is applied to the spring to compress it over a certain distance, which was our definition of Work/Energy. Now, after he developed the concept of Elastic Potential Energy, he created the idea of elastic and inelastic collisions between gas particles in the microworld; in elastic collisions all the Kinetic Energy is conserved and in inelastic collisions, Kinetic Energy is not conserved. Kinetic Energy is the energy of an object in motion, therefore if an elastic collision resembles no change in Kinetic Energy, the motion of the particles before the collision must be equal to the motion of the particle in a different direction. To study and predict the motion of these particles, we created an experiment on the Kinetic Molecular Theory, which is a set of laws that only apply to ideal gasses.

When light bulbs were first made, they contained a vacuum between the filament and the shell between the light bulb to prevent the light bulb from heating up immensely. Though, this caused light bulbs to then blacken and fuse quickly. This occurred because the Tungsten atom in the filament would sublimate due to the heat and light energy produced and turn into gas, which would quickly travel through the vacuum onto the outer shell of the bulb, causing the bulb to blacken. To fix this issue, Langmuir decided to fill the light bulb with Argon gas instead of keeping it vacuum based on his knowledge of the Kinetic Molecular Theory (KMT) and gas laws. These laws focused on the movement of molecules in the microworld. After filling the light bulb with gas, the issue was fixed and all the light bulbs had a greater life span.

In the 17th century, Evangelista Torricelli created the first Mercury barometer. This barometer was in the shape of the candy cane, and the mercury was filled into the longer leg with air trapped into the smaller leg. After the creation of this barometer, it was observed that the tube never fully filled with Mercury and the smaller leg always remained empty. Later in the 18th century, Robet Hooke and Robert Boyle decided to run some experiments to test the reason why the barometer never filled up with Mercury. To test this, they decided to add more Mercury to the barometer, though they observed that the air in the smaller leg continued to push the Mercury back, thus causing the barometer to never fully fill up. Therefore, they concluded that the air had weight and force, causing air pressure. In the 18th century, they branched off of this idea to conclude that air must be made of tiny particles floating in an invisible substance to have mass.

Later, in 1738, Daniel Bernoulli created an experiment to find the relationship between the motion of particles, temperature, and pressure. In this experiment, he had a piston in a glass tube filled with glass, and he would heat up the gas inside the glass tube to observe the movable piston. As he increased the temperature of the gas, he observed that the pistol would move in such a way that provides more room for the gas. Therefore, he concluded that when the gas is heated up, the particles move faster causing the pressure exerted by the particles to increase. This theory was widely not accepted compared to the Caloric theory, which stated that a heat substance engulfed gas molecules causing them to repel forcefully, which causes air pressure. However, this theory was eventually rejected after knowledge of the Kinetic Molecular Theory.

Advancements in 1857 occurred when Clausius incorporated energy into the Kinetic Molecular Theory. Since it was previously observed that as the temperature of the gas increases, the speed of the molecules increases, Clausius proposed that the average Kinetic Energy of the molecules must be proportional to the temperature increase of the gas. This makes sense because Kinetic Energy is the measure of the amount of energy possessed by an object in motion, therefore the faster the molecules, the greater the Kinetic Energy. With his knowledge of Kinetic Energy, he was able to calculate one Oxygen Atom to be 461 m/s. Though, his idea was not widely accepted because people observed that odors in the air take a longer time to reach from one end of a room to another, even though the speed suggested that it would take less than a second for odor to spread. Therefore, he came up with a couple of ideas that portray the movement of gas molecules that apply to ideal gasses, which were some of the assumptions made in the Kinetic Molecular Theory. He had previously conjectured that the molecules move in a linear pattern, though after he realized it takes longer for odor to travel, he predicted that there must be obstacles in a molecule's path, such as other molecules. He theorized that these molecules are big enough to collide and bounce off of one another, causing a random pattern rather than a linear pattern. Additionally, since the molecules move so fast, he assumed that there must be several collisions per second for these molecules. He also assumes that the temperature does not change despite the fact that the velocities change because the collisions were elastic and all kinetic energy was conserved. His theories helped Langmuir fix his light bulb problem by adding Argon, which caused the Tungsten atoms to constantly collide with Argon atoms, hence taking longer time for the light bulb to become darker.

The three main assumptions Clausius made were applied to ideal gasses, gasses that follow these assumptions. The first assumption was that the total volume of the molecules are negligible compared to the actual volume of the container they are in. The second assumption he made was that the molecules were not attracted to one another, they only interacted when they collided. The last main assumption he made was that the collisions were perfectly elastic, and no energy was lost into surroundings. He built off of these assumptions to create the five main assumptions for ideal gasses, which were known as RAVED. Firstly, the molecules had random motion. Secondly, there is no attraction between molecules, and intermolecular forces are negligible. Thirdly, the volume of the molecules were much less than the total volume of the container they were in. Fourthly, all the collisions between the particles were elastic and no energy was lost into the surroundings. Lastly, the duration of the collisions were very short compared to the time travelled between the collisions. He has also made a connection that the Kinetic Energy of the particles were directly proportional to the temperature of the gas.

To test our knowledge of ideal gasses and Kinetic Molecular Theory, the research question was asked, "What are the relationships between the number of particles, volume, pressure, and temperature in ideal gasses and which laws can be derived from our knowledge of these relationships?" Though before we come up with a way to answer this question, we needed to define our variables to conduct this experiment. One variable is the number of particles (n). Other variables are the pressure, which is the amount of force (N) exerted over a certain area (m^2). Pressure can be measured in N/m^2, though in this experiment, the pressure is the force exerted on the container by the gas, and we will be using the units kilo-pascals (kPa), which is 1,000 pascals, to measure this. The volume, which is the amount of space inside the container, would be measured in nanometers cubed (nm^3), and the temperature of the gas is the amount of kinetic energy the gas particles possess, which is measured in Kelvin. Based off of our conjectures in described in Part II of this lab, we hypothesized that "If we demonstrate our understanding of the behavior of gas particles in an ideal state, then the number of particles should be directly proportional to pressure, the temperature should be directly proportional to pressure, the volume should be inversely proportional to pressure, the temperature should be directly proportional to volume, and the Ideal gas law, PV=nRT, and Combined gas law, $P_1V_1/T_1=P_2V_2/T_2$ can be derived from these relationships." To justify this, we performed Ideal and Combined gas law derivation in Part III, as well as our linear/inverse regressions in Part II (Figures 5, 6, 7, and 8). This can also be supported by Boyle's, Gay-Lussac's, and Charles' Law described in Part III.

In the experiment we are conducting, we will be using Kelvin instead of Celsius for various reasons. Though, one main reason is to represent direct proportionality between the Pressure and Temperature. Direct proportionality means that if one of the factors doubles or quadruples, then the other factor should also double or quadruple respectively. Based on this definition of a direct relationship, if the temperature is at zero, then the pressure must be zero kPa, symbolizing no movement. Since there is still movement between atoms at zero degrees Celsius because that is not the coldest temperature and there can be negative degrees Celsius, we know that if the temperature in Celsius doubles, then the pressure would increase but not necessarily double. This means that the temperature in Celsius is not directly

proportional to the pressure. Therefore, to represent a temperature where there is no movement, we use the unit Kelvin. At zero Kelvin, which is about –273.15 degrees Celsius, there is theoretically no movement, symbolizing no pressure or 0 kPa of pressure. This would cause the pressure to double as the temperature of the gas in Kelvin doubles. Therefore, temperature in Kelvin represents direct proportionality to pressure, which is the reason we used Kelvin in this lab to represent the temperature of the gas. This can be justified by the graphs in Figure 0. In a direct relationship, the graphs should pass through the origin. In our Pressure (kPa) vs. Temperature (Celsius) Graph, the graph does not pass through the origin, though in our Pressure (kPa) vs. Temperature (Kelvin) graph, the graph passes through the origin. This is why we use Kelvin to measure temperature instead of Celsius.

Figure 0: Pressure (kPa) vs. Temperature (Celsius) Graph vs. Pressure (kPa) vs. Temperature (Kelvin) Graph
Pressure (kPa) vs. Temperature (Celsius)
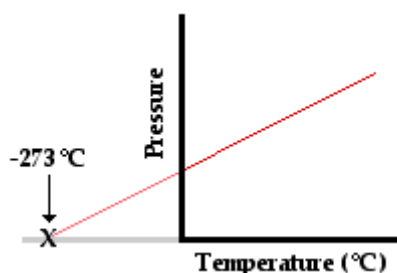
**Pressure (kPa) vs. Temperature (K)**
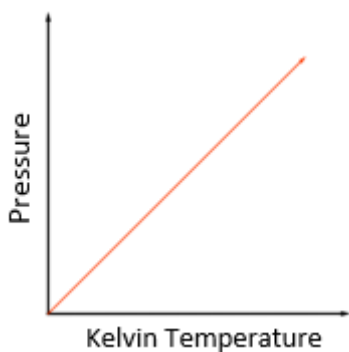
PART 2
Conjectures
Before we conducted our experiment, we made a few conjectures about the relationship between the number of particles, pressure, volume, and temperature. Our first conjecture is that if the number of particles increases, then the pressure of the gas exerted on the container increases. This makes sense because as the number of particles increases, there are more collisions against the wall of the container, therefore the pressure exerted on the container would increase and there is the direct relationship. Secondly, we conjectured that as the temperature increases, then the pressure increases because as the temperature increases, the speed of the particles increases causing more collisions against the walls of the container. This causes the pressure on the container to increase and this is also a direct relationship.

Thirdly, as the volume of the container the gas is in increases, the pressure decreases. This occurs because as the volume increases, the particles can spread out more and it would take longer for these particles to collide against the walls, therefore the pressure would decrease. This portrays an inverse relationship. The last conjecture we made was that as the volume increases, the temperature increases, portraying a direct relationship. We conjecture this because as the temperature increases of the gas, the gas expands, having a greater volume.

Experimental Method

We used the PHET Simulation (Figure 9) to test the relationship between number of gas particles, pressure, temperature, and volume. In our first experiment (a), we changed the number of gas particles, which was our independent variable, to see the effect on pressure, which was the dependent variable. To do this, we used the pump PHET simulation to increase the number of the same type of particles by intervals of 50 particles (about one pump), and we recorded the pressure reading when the particle count was at 0 particles, 50 particles, 100 particles, 150 particles, 200 particles, and 250 particles. During this trial, we kept the temperature, type of particle, and the volume constant at 300 K, Heavy Particles, and 1000 nm^3 respectively. The control of this experiment was the trial where we had 0 particles. We recorded the pressure readings in Table 1 below.

Figure 9: Layout of PHET Gas Laws Simulation
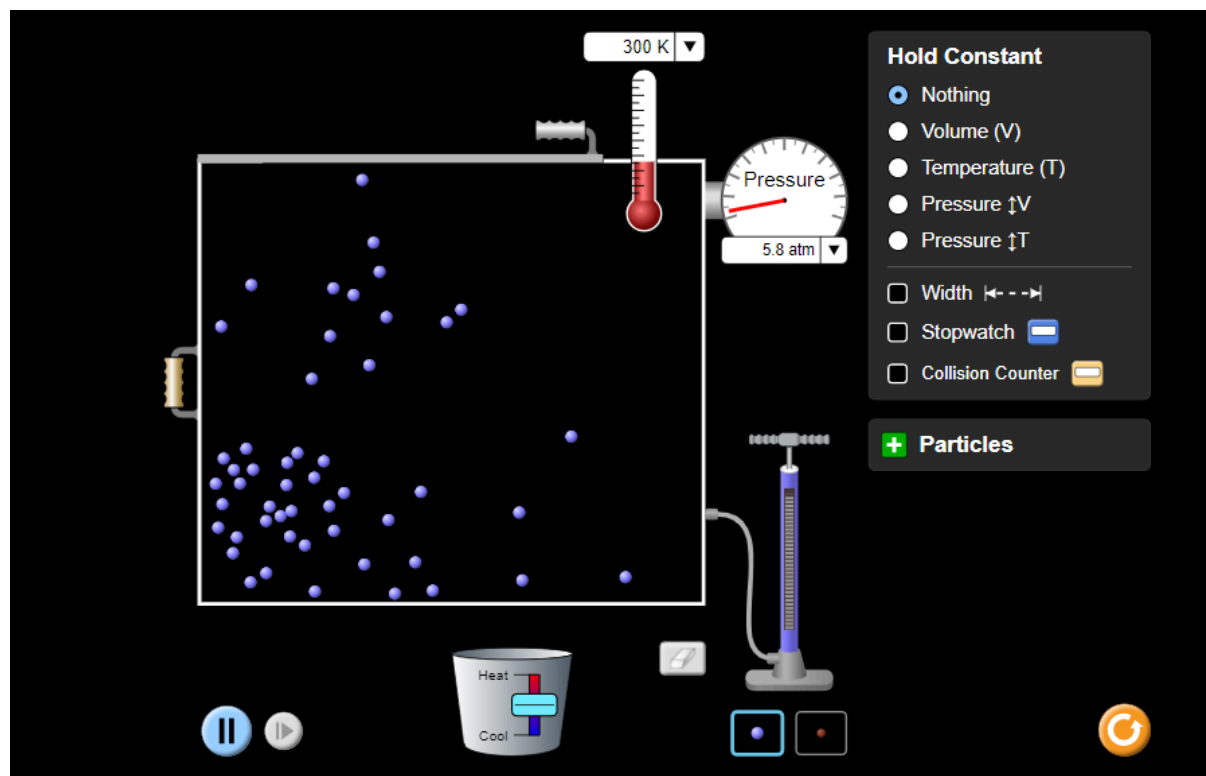
 In our second experiment (b), we changed the temperature of the gas, which was our independent variable, to see the effect on pressure, which was the dependent variable. To do this, we used the

heating and cooling slider in the PHET simulation to change the temperature of the gas, and we recorded the pressure reading at temperature readings of 0 K, 100 K, 300 K, 600 K, and 900 K. During this trial, we kept the number of particles, type of particle, and the volume constant at 50 particles, Heavy Particles, and 1000 nm^3 respectively. The control of this experiment was the trial where the temperature of the particles were 0 K. We recorded the pressure readings in Table 2 below.

In our third experiment (c), we changed the volume of the container, which was our independent variable, to see the effect on pressure, which was the dependent variable. To do this, we used the volume slider in the PHET simulation to change the volume of the container, and we recorded the pressure reading at 500 nm^3, 750 nm^3, 1000 nm^3, 1200 nm^3, and 1500 nm^3. During this trial, we kept the number of particles, type of particle, and the temperature constant at 50 particles, Heavy particles, and 300 K respectively. The constant for this trial was a volume reading of 500 nm^3. We recorded the pressure readings in Table 3 below.

In our last experiment (d), we changed the temperature of the gas, which was our independent variable, to see the effect on volume, which was the dependent variable. To do this, we used the heating and cooling slider in the PHET simulation to change the temperature of the gas, and we recorded the volume reading at 316 K, 350 K, 425 K, 445 K, and 450 K. During this trial, we kept the number of particles, type of particle, and the pressure constant at 50 particles, Heavy particles, and 592 kPa respectively. This was our only experiment in which we did not have an elaborate control. We recorded the volume readings in Table 4 below.

After we recorded our data, we plotted these into Desmos Graphing Calculator (Figure 10). The graphs for experiments a, b, c, and d and represented in Figures 1, 2, 3, and 4 respectively. In Figures 1, 3, and 4, we observed a straight line with an increasing slope, and in Figure 4, we observed an inverse curve with the y-values, which is the pressure, decreasing. Therefore, for experiments a, b, and d, we performed linear regressions in the form $y_1 \sim mx_1 + b$. The line regressions for experiment a is represented in Figure 5, experiment b in Figure 6, and experiment d in Figure 8. From this, we were able to get our m values, which is the slope of the linear lines, and b values, which is the y-intercept. In all the trials, we predict that the b value should be close to 0 considering that all the graphs should pass through the origin as described in Part I. Though, experiment c was different than the rest of the experiments because pressure decreased as volume increased, portraying an inverse relationship. Therefore we performed an inverse regression in the form $y_1 \sim a/x_1$, where our values are the constant of proportionality between the volume and pressure. This regression is represented in Figure 7.

Figure 10: Layout of Desmos Graphing Calculator
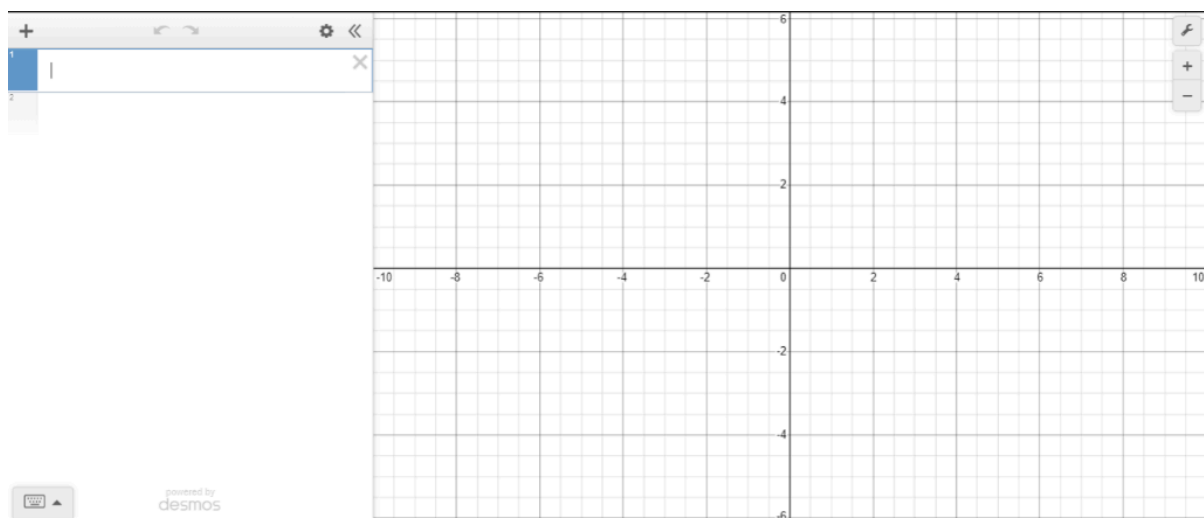
Data

Table 1: Data for Pressure vs. Number of Particles

	Independent Variable	Dependent Variable
Trial	Number of particles	Pressure (kPa)
1	0	0
2	50	592
3	100	1183
4	150	1775
5	200	2366
6	250	2958

Table 2: Data for Pressure vs. Temperature

	Independent Variable	Dependent Variable
Trial	Temperature (K)	Pressure (kPa)
1	0	0
2	100	198
3	300	592
4	600	1186

05	900	1774

Table 3: Data for Pressure vs. Volume

	Independent Variable	Dependent Variable
Trial	Volume (nm^3) *	Pressure (kPa)
1	500	1183
2	750	786
3	1000	592
4	1200	473
5	1500	394

* To find the volume, we changed the width but we assumed the height and depth of the container was 10 nm.

Table 4: Data for Volume vs. Temperature

	Independent Variable	Dependent Variable
Trial	Temperature (K)	Volume (nm^3)
1	316	1050
2	350	1170
3	425	1420
4	445	1480
5	450	1500

Figure 1: Pressure vs. Number of Particles Graph

x_1	y_1
50	592
100	1183
150	1775
200	2366
250	2958
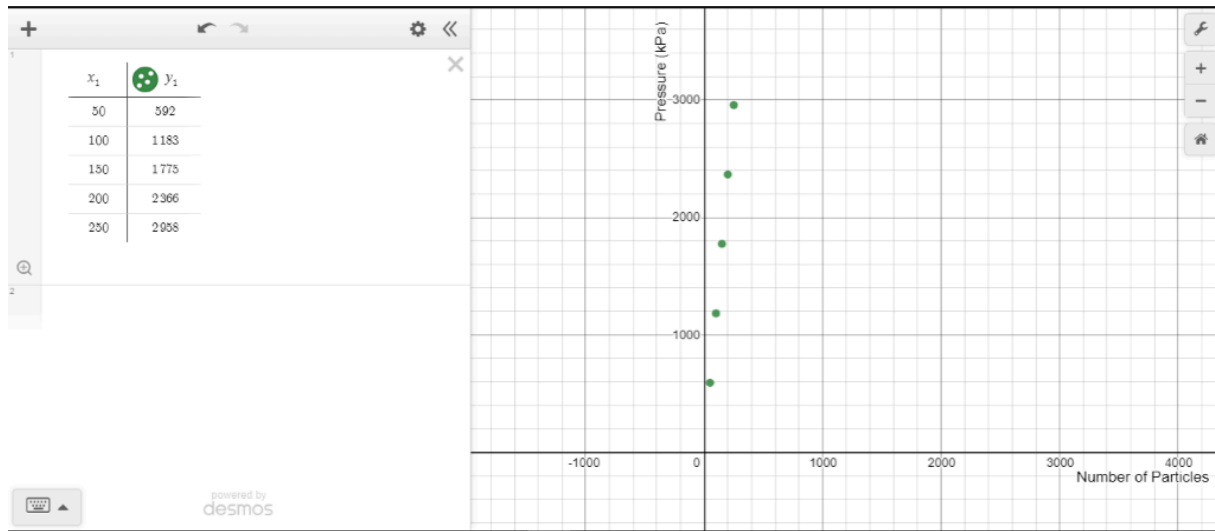
Figure 2: Pressure vs. Temperature Graph

x_1	y_1
300	592
600	1186
900	1774
100	198
1	1
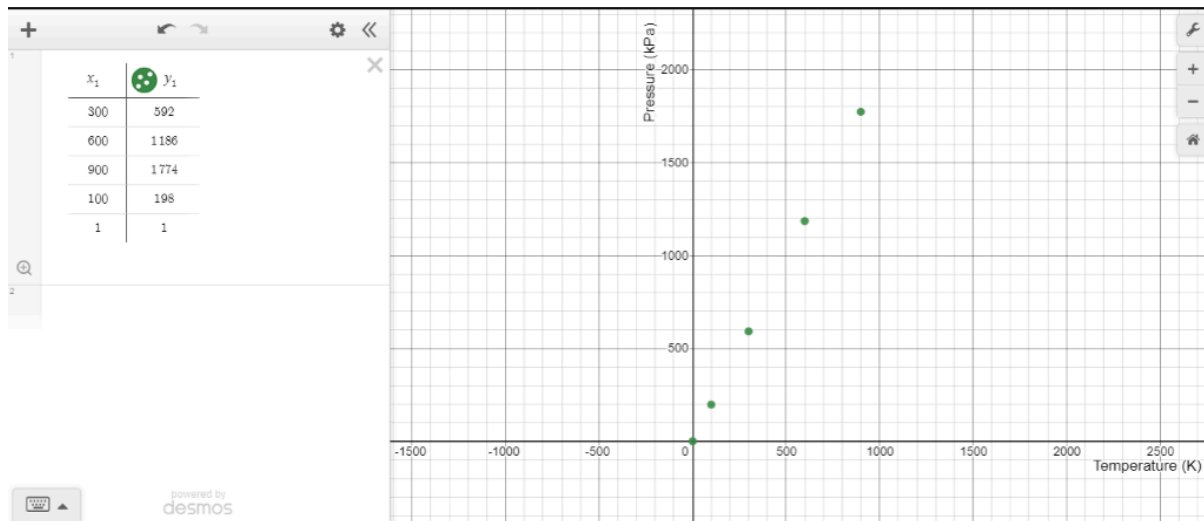
Figure 3: Pressure vs. Volume Graph

x_1	y_1
500	1183
750	786
1000	592
1200	473
1500	394
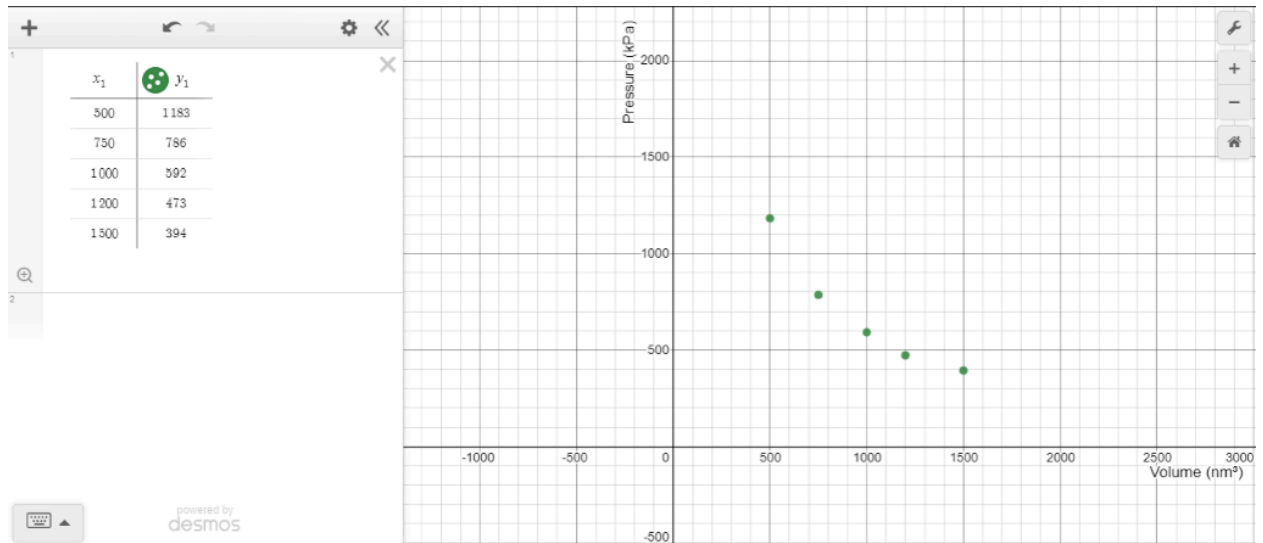
Figure 4: Volume vs. Temperature Graph

x_1	y_1
316	1050
350	1170
425	1420
445	1480
450	1500
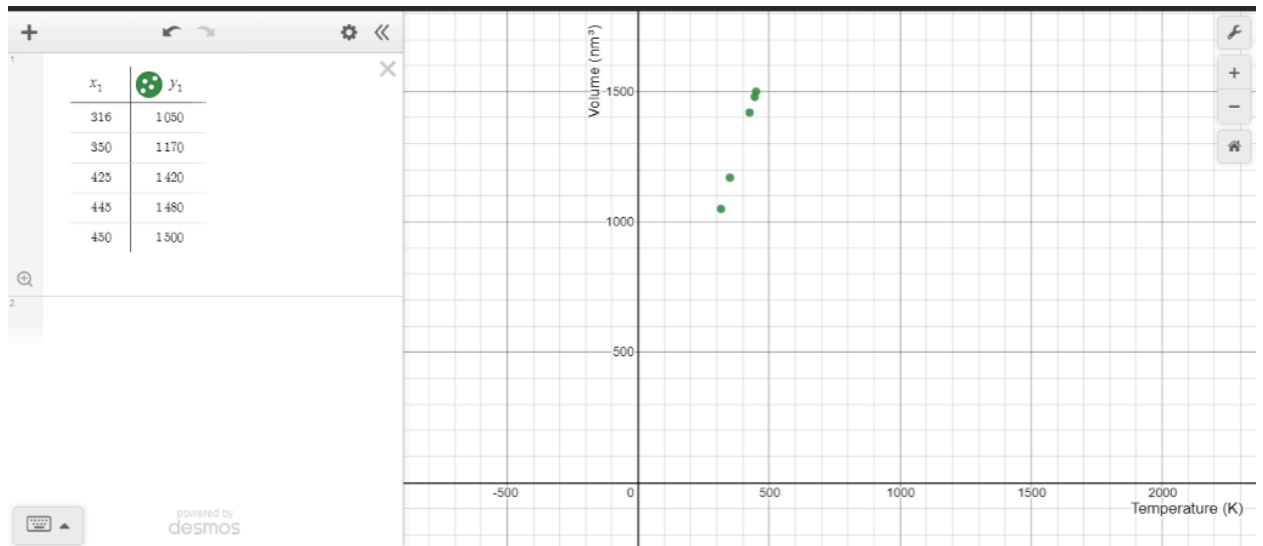
Results: Regression Graphs

Figure 5: Pressure vs. Number of Particles Graph
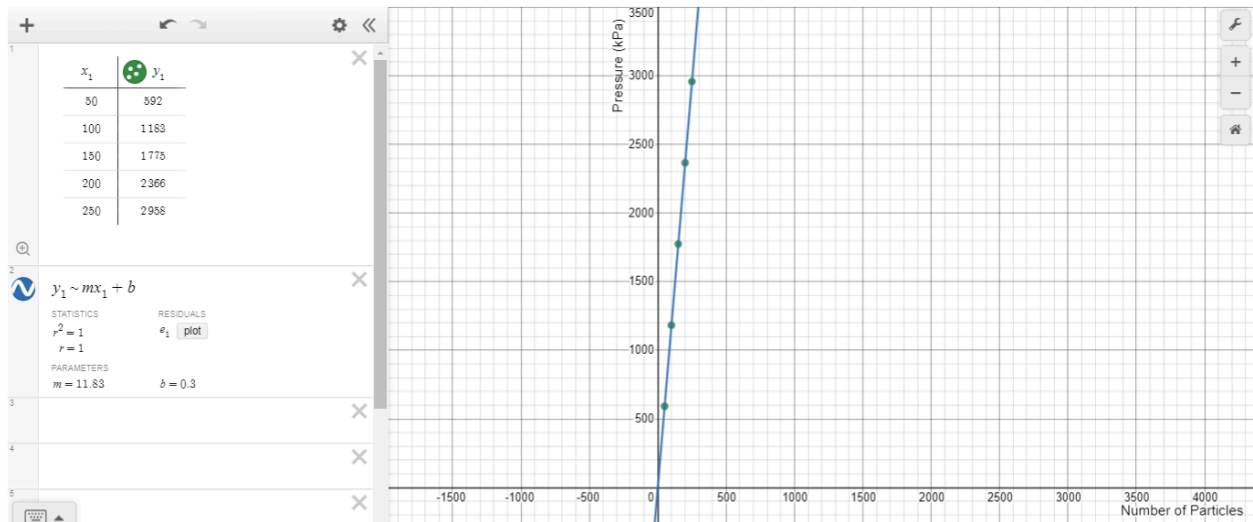
Figure 6: Pressure vs. Temperature Graph

Table:

x_1	y_1
50	592
100	1183
150	1775
200	2366
250	2958

$y_1 \sim mx_1 + b$

STATISTICS — RESIDUALS — e_1 plot
$r^2 = 1$
$r = 1$
PARAMETERS
$m = 11.83$ $b = 0.3$

(Axis labels: Pressure (kPa) vs. Number of Particles)

Figure 7: Pressure vs. Volume Graph

Table:

x_1	y_1
300	592
600	1186
900	1774
100	198
1	1

$y_1 \sim mx_1 + b$

STATISTICS — RESIDUALS — e_1 plot
$r^2 = 1$
$r = 1$
PARAMETERS
$m = 1.97257$ $b = 0.228875$

(Axis labels: Pressure (kPa) vs. Temperature (K))

Table:

x_1	y_1
500	1183
750	786
1000	592
1200	473
1500	394

$y_1 \sim \dfrac{a}{x_1}$

☐ Log Mode
STATISTICS — RESIDUALS — e_1 plot
$R^2 = 0.9991$
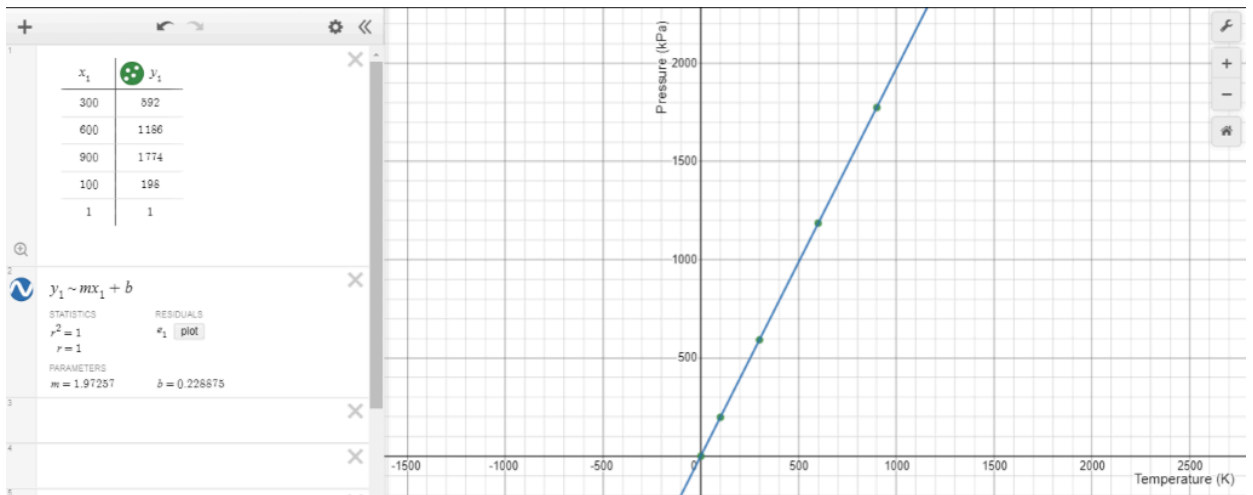PARAMETERS
$a = 588989$
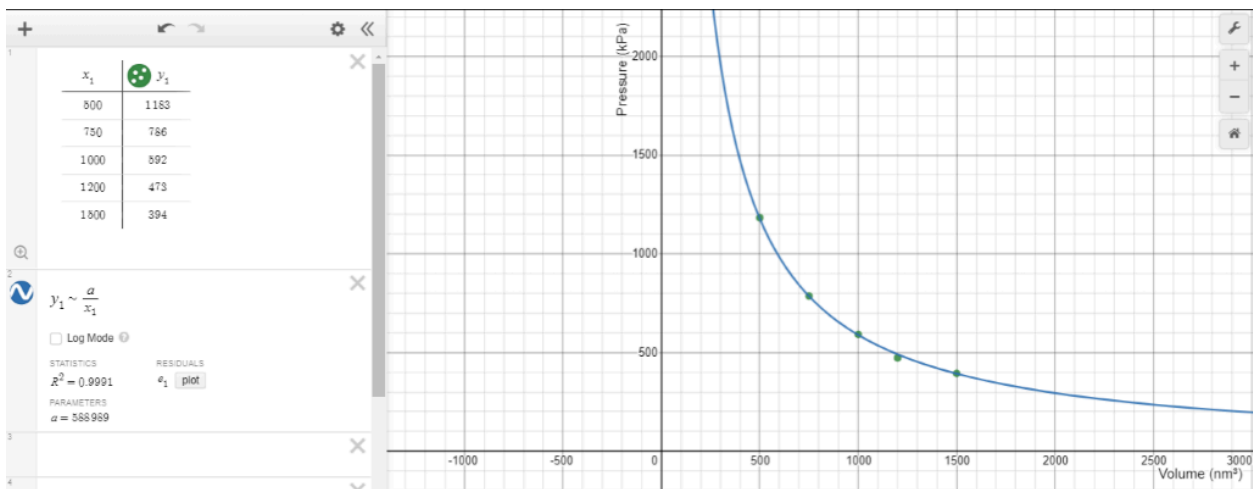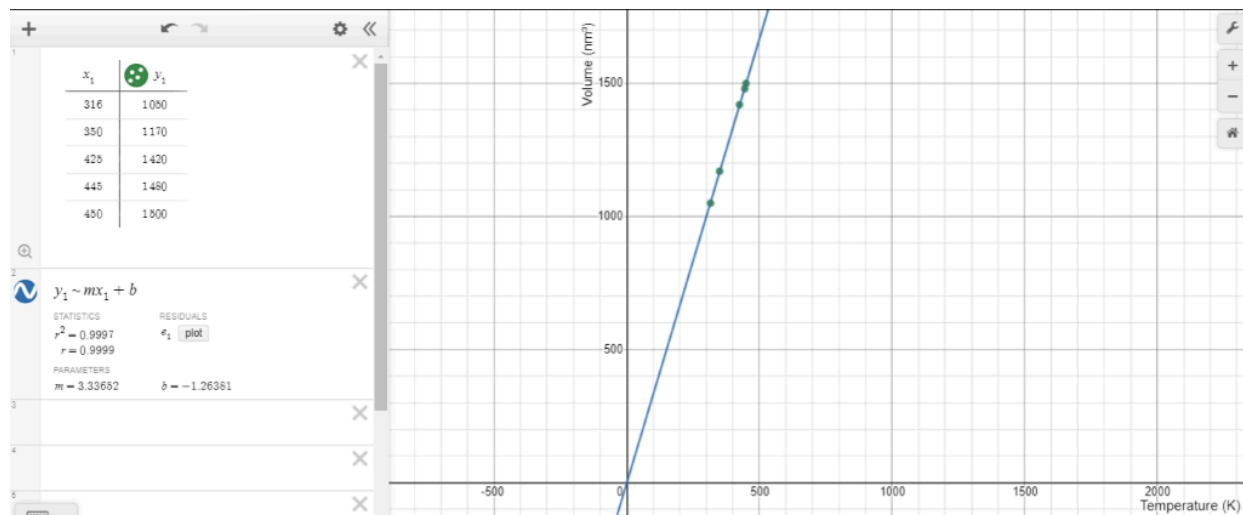
(Axis labels: Pressure (kPa) vs. Volume (nm³))

Figure 8: Volume vs. Temperature Graph

PART 3

Introduction: Full Derivation + Laws

In order to derive the Ideal and Combined gas laws the express the relationships between the number of particles (n), Volume (V), Pressure (P), and Temperature (T), we need to know some background on the four basic laws. The four basic laws are Boyle's Law, Charles' Law, Gay-Lussac's Law, and Avogadro's Law. Firstly, Boyle's Law states that Volume is inversely proportional to Pressure ($V \propto 1/P$). Secondly, Charles' Law states that Volume is directly proportional to Temperature ($V \propto T$). Thirdly, Gay-Lussac's Law states that Pressure is directly proportional to Temperature ($P \propto T$). Lastly, Avogadro's Law states that the number of particles is directly proportional to Volume ($n \propto V$). These laws accurately resemble what we conjectured in Part II of this lab. For our purposes, the constant of proportionalities for these statements is the ideal gas constant, which is represented by the letter R.

To derive the Combined Gas Law, we created a joint proportionality between Boyle's Law and Charles' Law. Since Volume is inversely proportional to Pressure and directly proportional to Temperature, we can combine these two by multiplying the right side of the "$\propto$" symbol in these two laws. This is done because Volume is related to both factors (represented by the "and"), and the Volume is the only thing locate on the left side of the "$\propto$", therefore we are easily able to combine them. After we combined these two, we result in the join proportionality statement that states Volume and Pressure is Directly Proportional to Temperature ($PV \propto T$). This can be converted into an equation by adding the constant of proportionality, R (PV=RT). After we rearrange the equation to solve for R, we get R=PV/T. Though, since the ideal gas constant is universal, we can equal the "R" values to form the Combined Gas Law,

$P_1V_1/T_1 = P_2V_2/T_2$. The work for this derivation is shown in Figure 11.

Figure 11: Combined Gas Law Derivation

$$V \propto \frac{1}{P}$$

$$V \propto T$$

$$V \propto \frac{1}{P} \cdot T$$

$$V \propto \frac{T}{P}$$

$$PV \propto T$$

$$PV = RT$$

$$\frac{P1V1}{T1} = R = \frac{P2V2}{T2}$$

$$\frac{P1V1}{T1} = \frac{P2V2}{T2}$$

Then, to derive the Ideal Gas Law, we use our already created join proportionality statement that states Volume and Pressure is Directly Proportional to Temperature (PV∝T). From here, we can add Avogadro's Law (n∝V). Since Volume is already on the left side of the "∝" symbol, in order to depict direct proportionality, the number of particles must be on the right side of the "∝". This can also be justified by the fact that we would need to multiply the variable on the opposite side of the proportionality symbol with Volume on it to create a joint proportionality statement. After we combine these three laws, we result in a joint proportionality statement saying that the Pressure and Volume are directly proportional to the number of particles and Temperature of the gas (PV∝nT). Then, we would convert this statement into an equation by adding the ideal gas constant proportionally, R. Therefore, we are able to generate the Ideal Gas Law, which is PV=nRT. The work for the Ideal Gas Law derivation is shown in Figure 12. These laws are supposed to portray the behavior for gasses under the ideal condition, also known as ideal gasses.

Figure 12: Ideal Gas Law Derivation

$$V \propto \frac{1}{P}$$

$$V \propto T$$

$$V \propto n$$

$$V \propto n \cdot \frac{1}{P} \cdot T$$

$$V \propto \frac{nT}{P}$$

$$PV \propto nT$$

$$PV = nRT$$

Citations

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