ECON 7710 TA Session

Midterm Review

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Sep 2023

Outline

Midterm 2018 Q1

Powerball is an American lottery game offered by 44 states, the District of Columbia, Puerto Rico and the US Virgin Islands. Powerball tickets have 6 numbers on them from 1 to 99 each. While the numbers on each ticket are randomly generated, each combination of numbers is unique, i.e. no two tickets share the same combination. The winning combination is generated by randomly drawing 6 numbers from 1 to 99 with replacement. Find the probability that a ticket that you bought today wins powerball given the total number of tickets sold by today's drawing is 90,674,762.

Midterm 2019 Q5

A gambler rolls two dice. Find the expectation of the sum of the outcomes, given that the outcomes of the dice are different.

Midterm 2017 Q4

n-dimensional random vector $(X_1, ..., X_n)$ such that $0 \leq X_i \leq 1$ has density

$$f(x_1, ..., x_n) = \begin{cases} 1 + \prod_{i=1}^{n}(x_i - \frac{1}{2}), & \text{if } 0 \leq x_i \leq 1, \\ 0, & \text{otherwise} \end{cases}$$

a Are $X_1, ..., X_n$ independent?

b Take a subset of k variables $(2 \leq k \leq n-1)$ from $X_1, ..., X_n$. Could the variables in this subset be independent?

Midterm 2021 Q4

Random variables X and Y are independent and identically distributed such that

$$P(X = k) = P(Y = k) = 1/N \quad \text{for } k = 1, 2, ..., N$$

Find distribution of random variable:

$$Z = max\{X, Y\} - min\{X, Y\}$$

Midterm 2022 Q2

Suppose that X and Y are absolutely continuous random variables. Can random variables X and $Z = X + Y$ be independent? If your answer is yes. prove a necessary and sufficient conditions under which this is possible. If your answer is "no", prove that formally.

Core Jun 2018 Q2

Consider the non-negative function of two arguments:

$$G(x, y) = \begin{cases} 0, & \text{if } x \leq 0 \text{ or } y \leq 0 \\ min\{1, max\{x, y\}\}, & \text{otherwise.} \end{cases}$$

Formally show whether this function is a joint cumulative distribution function.

Midterm 2020 Q3

Suppose that random variable Z is such that $a \leq Z \leq b$ and $E[Z] = 0$. Show that:

$$E[e^{tZ}] \leq e^{t^2(b-a)^2/8}$$

Hint: It may be useful to explore convexity of function e^{tZ} for each Z to produce deterministic bound $e^{tZ} \leq \frac{Z-a}{b-a}e^{tb} + \frac{b-Z}{b-a}e^{ta}$

Midterm 2020 Q4

Prove that for any random variable Z for which the moment generating function is well-defined

$$P(Z > \epsilon) \leq e^{-t\epsilon} E[e^{tZ}]. \quad (**)$$

Using inequality$(**)$ in combination with the result in Question 3,

$$E[e^{tZ}] \leq e^{t^2(b-a)^2/8}$$

evaluate the bound for deviation probability for the sample mean $\bar{Z}$ constructed from the sample of i.i.d. random variables $Z_1, ..., Z_n$ such that $a \leq Z_i \leq b$ and $E[Z_i] = 0$:

$$P(\bar{Z} > \epsilon)$$

i.e. bound this probability from above as a function of ϵ, n, a, b. Discuss the difference between this bound and the bound that can be obtained using the Chbychev's inequality.

Hint: Note that the property of characteristic function of the sum of independent random variables also applies to the moment generating function. The bound can then be computed by minimizing the expression you obtain over t.