Econ 7040: Assignment #1
Spring 2024

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Due Monday, January 29, 2024
Instructions: Type all answers in LaTeX

This assignment asks you to use the simple constant-real-interest rate permanent income model to derive equilibrium time paths for consumption, saving, and asset accumulation under alternative exogenous processes for income.

The basic model is the consumption function

$$C_t = (1-\beta)\sum_{j=0}^{\infty}\beta^j E_t Y_{t+j} + rA_{t-1} \tag{1}$$

where $r = 1/\beta - 1$, Y_t is income, and A_{t-1} is assets at the beginning of period t.

The household's flow budget constraint is

$$C_t + A_t = Y_t + (1+r)A_{t-1} \tag{2}$$

For questions 1–3, assume that income obeys the process

$$Y_t = Y_1 + \nu_t \tag{3}$$

$$\nu_t = \phi\nu_{t-1} + \varepsilon_t, \quad E_t\varepsilon_{t+1} = 0, \quad 0 \le \phi \le 1 \tag{4}$$

1. Derive an analytical expression for C_t as a function of (Y_1, ν_t, A_{t-1}).

2. Define saving, S_t, and assets at t, A_t, as

   $$S_t = Y_t - C_t + rA_{t-1}$$
   $$A_t = S_t + A_{t-1}$$

   Derive an analytical expression for S_t.

3. What are the marginal propensities to consume and save out of permanent income, transitory income, and assets (A_{t-1})? Explain each result that you report.

   For the parameter settings $r = 0.02, Y_1 = 10$ and various settings for ϕ, plot the paths of $\{C_t, Y_t, S_t, A_t\}$ when $\varepsilon_t = 1$ over the periods $t, t+1, \ldots, 20$. Assume $A_{t-1} = 0$.

   (a) $\phi = 0.0$
   (b) $\phi = 0.9$
   (c) $\phi = 1.0$

4. Now assume the path of $\{Y_t\}$ is known and compute perfect foresight paths for two different thought experiments for income. Imagine the economy starts in period 0 and $A_{-1} = 0$, so households start with no assets. Use $r = 0.02$. Report paths for $\{C_t, S_t, A_t\}$ for $t = 0, 1, 2, \ldots, 10$. Explain each result you report.

(a) A known temporary increase in income from $Y_1 = 10$ to $Y_2 = 12$.

$$Y_t = \begin{cases} Y_1, & \text{for } t = 0, 1 \\ Y_2, & \text{for } t = 2, 3, 4 \\ Y_1, & \text{for } t = 5, 6, \ldots \end{cases}$$

(b) A known permanent increase in income from $Y_1 = 10$ to $Y_2 = 12$.

$$Y_t = \begin{cases} Y_1, & \text{for } t = 0, 1 \\ Y_2, & \text{for } t = 2, 3, \ldots \end{cases}$$