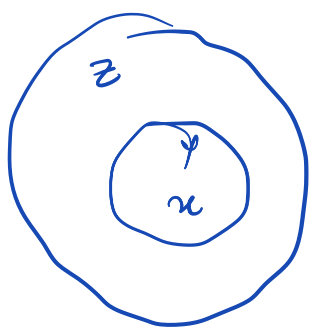
1. The UVA Economics Department is looking to hire a new professor next year. For simplicity, assume there are only 3 new Ph.D. graduates on the market, named $\mathcal{X} = \{Ann, Bob, Charlie\}$. It may be that only some (and not all) of the candidates apply. For any potential set of applicants $Y \subset \mathcal{X}$, the department has a choice rule $C(Y)$ that determines who will be given an offer. The following condition, known as Sen's α, is a (partial) alternative to WARP. Let Y, Z be two sets of potential applicants.

   - *Sen's α:* If $x \in Y$, $Y \subseteq Z$, and $x \in C(Z)$, then $x \in C(Y)$.

   (a) Translate this condition into words. (It may be helpful, though not necessary, to draw a Venn diagram.)

(a) $X = \{A, B, C\}$



If x is chosen from set Z & x is also present in Y, which is a subset of Z, then x is also chosen from set Y.

(b) $A \succ B \succ C$

Let

$B \in Y$	$A \in X$
$C \in Y$	$Y \in X$

$C(Y) = B$ but $C(X) = A$

and $Y \in Y$

$\therefore$ Violates Sen's α.

(c) Let $Y = \{A, C\}$ & $Z = \{A, B, C\}$ be 2 candidate pools.

and the ordering be $A > B > C$.

$\Rightarrow V_A > V_B > V_C$.

Threshold be $\bar{V}$

① when $\bar{V} > V$

$C(Y) = C(Z) = C$ → Sen α ✓

② when $\bar{V} < V$

$C(Y) = C(Z) = A$ → Sen's α holds

③ When $\bar{V} < V$ & only person exceeds $\bar{V}$

$C(Y) = C(Z) = A$ → Sen's α holds

④ When $\bar{V} < V$ & 2 people exceed $\bar{V}$

$C(Y) = C(Z) = A$ → Sen's α holds

(d) WARP $\longrightarrow$ Sen's α

WARP holds $\Rightarrow$ $x, y \in A \cap B$

$x \in C(B)$

$y \in C(A)$

$\Rightarrow x \in C(B)$

If $A \subseteq B$ $\Rightarrow$ the above become

Sen's α.

2. Prove the following proposition:

> A preference relation, $\succeq$, can be represented by a utility function only if it is rational.

Note :-

1, A preference relation is rational if it is:-

1. Complete: $\forall\, x, y \in X$, either $x \succsim y$ or $y \succsim x$ (or both)
2. Transitive: $\forall\, x, y, z \in X$, if $x \succsim y$ & $y \succsim z$, then $x \succsim z$.

2, A preference relation $\succsim$ is represented by a utility f^n $u: X \to R$ if $\forall\, x, y$

$$x \succsim y \iff u(x) \geq u(y)$$

Proof by contradiction:-

Assume $\succsim$ is not transitive & thus irrational,
Then, suppose the pref are such:-

$a > b$
$b > c$
$c > a$

$\Rightarrow u(a) > u(b)$
$\Rightarrow u(b) > u(c)$
$\Rightarrow u(c) > u(a)$

Proposition 1.B.2: A preference relation $\succsim$ can be represented by a utility function only if it is rational.

Proof: To prove this proposition, we show that if there is a utility function that represents preferences $\succsim$, then $\succsim$ must be complete and transitive.

Completeness. Because $u(\cdot)$ is a real-valued function defined on X, it must be that for any $x, y \in X$, either $u(x) \geq u(y)$ or $u(y) \geq u(x)$. But because $u(\cdot)$ is a utility function representing $\succsim$, this implies either that $x \succsim y$ or that $y \succsim x$ (recall Definition 1.B.2). Hence, $\succsim$ must be complete.

Transitivity. Suppose that $x \succsim y$ and $y \succsim z$. Because $u(\cdot)$ represents $\succsim$, we must have $u(x) \geq u(y)$ and $u(y) \geq u(z)$. Therefore, $u(x) \geq u(z)$. Because $u(\cdot)$ represents $\succsim$, this implies $x \succsim z$. Thus, we have shown that $x \succsim y$ and $y \succsim z$ imply $x \succsim z$, and so transitivity is established. ■

At the same time, one might wonder, can *any* rational preference relation $\succsim$ be described by some utility function? It turns out that, in general, the answer is no. An example where it is not possible to do so will be discussed in Section 3.G. One case in which we can always represent a rational preference relation with a utility function arises when X is finite (see Exercise 1.B.5). More interesting utility representation results (e.g., for sets of alternatives that are not finite) will be presented in later chapters.

3. Let $x(p, m)$ be an agent's Marshallian demand function. Prove that if the agent's preferences are convex, then $x(p, m)$ is convex, and further, if the agent's preferences are strictly convex, then $x(p, m)$ is a singleton. (Note: you may assume a continuous utility representation $u(x)$ exists, but do not assume it is differentiable. Let $x', x' \in x(p, m)$, and define $\bar{x} = \alpha x + (1-\alpha)x'$. To prove the first part, it is sufficient to show the following: (i) $\bar{x}$ is feasible at (p, m) and (ii) $\bar{x}$ gives at least as much utility as x, x'. Use the definition of quasiconcavity that says $u(\bar{x}) > \min\{u(x), u(x')\}$.)

To prove :—

1) If agent's preferences are convex $\rightarrow$ $x(p,m)$ is convex
2) If agent's pref are strictly convex $\rightarrow$ $x(p,m)$ is a singleton

$\succsim$ convex

$\Rightarrow$ $u(\cdot)$ is quasiconcave

- Let x & $x' \in x(p,m)$ s.t. $x \neq x'$

$\Rightarrow$ $u(x) = u(x') = u^*$

Let $\bar{x} = \alpha x + (1-\alpha)x'$

$\Rightarrow$ $u(\bar{x}) \geq u^*$ as $u(\cdot)$ is quasiconcave.

$\Rightarrow$ $\bar{x}$ gives at least as much utility as x, x'

As $p.x \leq m$ & $p.x' \leq m$

$\Rightarrow$ $p.\bar{x} = p[\alpha x + (1-\alpha)x']$

$\Rightarrow$ $p.\bar{x} = p[\alpha x + x' - \alpha x'] = px' + p\alpha(x-x')$

$\Rightarrow p.\bar{x} \leq m$

$\Rightarrow \bar{x}$ is feasible at (p, m)

- $\succsim$ is strictly convex

$\Rightarrow u(\cdot)$ is strictly quasiconcave.

As earlier, $u(x) = u(x') = u^*$ and,
$\bar{x} = \alpha x + (1-\alpha)x'$

By strict quasiconcavity, $u(\bar{x}) > u^*$.

$\Rightarrow \bar{x}$ gives strictly more utility than x & x'.

Further as above $\bar{x}$ is a feasible pt at (p, m).

As $u(\bar{x}) > u^*$; x' & $x \notin x(p, m)$

$\Rightarrow x''$ is the only pt. in $x(p, m)$

$\Rightarrow x(p, m)$ is a singleton.

4. MWG 3.D.4 (Do not assume differentiability for part (a) - it is actually easier to show directly, without FOCs; for part (c), you may assume that η is differentiable and strictly increasing)

3.D.4[B] Let $(-\infty, \infty) \times \mathbb{R}_+^{L-1}$ denote the consumption set, and assume that preferences are strictly convex and quasilinear. Normalize $p_1 = 1$.

(a) Show that the Walrasian demand functions for goods $2, \ldots, L$ are independent of wealth. What does this imply about the wealth effect (see Section 2.E) of demand for good 1?

(b) Argue that the indirect utility function can be written in the form $v(p, w) = w + \phi(p)$ for some function $\phi(\cdot)$.

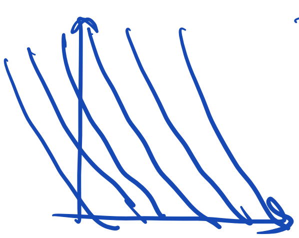
(c) Suppose, for simplicity, that $L = 2$, and write the consumer's utility function as $u(x_1, x_2) = x_1 + \eta(x_2)$. Now, however, let the consumption set be $\mathbb{R}_+^2$ so that there is a nonnegativity constraint on consumption of the numeraire x_1. Fix prices p, and examine how the consumer's Walrasian demand changes as wealth w varies. When is the nonnegativity constraint on the numeraire irrelevant?

- Consumption set : $(-\infty, \infty) \times R_+^{L-1}$
- $\succsim$: strictly convex & quasilinear
- $P_1 = 1$

(a) WTS : $x_2 \ldots x_L$ independent of wealth.

Acc. to Walras law,

$$p_1 x_1 + p_2 x_2 + \ldots + p_L x_L = w$$



x_1

Now, $x_1(p_1, w)$

and if $x_1 \sim x_2 \sim \ldots \sim x_L$

then; $(x + \alpha e_1) \sim (x_2 + \alpha e_2) \ldots \sim (x_L + \alpha e_L)$

for $e_1 = (1, 0, 0 \ldots) \in R^L$

$$x + \alpha e_1 = x(p, w + \alpha)$$

$\vdots$

$$x_L + \alpha e_L = x(p, w + \alpha)$$

$p_1 = 1$

$p \cdot (x + \alpha e_1) \leq \alpha w$

$\Rightarrow$ $x + \alpha e_1$ is affordable at $(p, w + \alpha e_1)$

Let $y \in \mathbb{R}^L_+$ & $p \cdot y \leq w + \alpha$

$\Rightarrow p \cdot (y - \alpha e) \leq w$

$\Rightarrow \quad x \succsim y - \alpha e_1$

$\Rightarrow \quad x + \alpha e_1 \succsim y$

$\Rightarrow \quad x + \alpha e_1 = x(p, w + \alpha)$

(b) $v(p, w) = w + \phi(p)$ for some f^n $\phi(\cdot)$

Let $\phi(p) = u(x(p, 0))$

We know,

$x(p, w) = x(p, 0) + we_1$ &,

$u(x) = x_1 + \tilde{u}(x_2 \ldots x_L)$

$$v(p, w) = u(x(p, w))$$
$$= x_1(p, w) + \tilde{u}(x_2(p, w) \ldots x_L(p, w))$$
$$= w + x_1(p, 0) + \tilde{u}(x_2(p, 0) \ldots x_L(p, 0))$$
$$= w + u(x(p, 0))$$
$$= w + \phi(p)$$

(c) ✓✓

(c) Suppose, for simplicity, that $L = 2$, and write the consumer's utility function as $u(x_1, x_2) = x_1 + \eta(x_2)$. Now, however, let the consumption set be $\mathbb{R}^2_+$ so that there is a nonnegativity constraint on consumption of the numeraire x_1. Fix prices p, and examine how the consumer's Walrasian demand changes as wealth w varies. When is the nonnegativity constraint on the numeraire irrelevant?

$L = 2$

$u(x_1, x_2) = x_1 + \eta(x_2)$

The non-negativity constraint is binding iff

$p_2 x_2(p, 0) > w$

As $p_1 = 1$, $x_2(p, 0) = \eta'^{-1}(p_2)$

$\to p_2 (\eta')^{-1}(p_2) > w$

$\Rightarrow x(p, w) = (0, w/p_2)$

As w changes, cons^m of first good is unchanged but the cons^m of second good changes at $1/p_2$ rate with w until the non negativity constraint binds.

/

5. MWG 2.D.1, 2.D.2,

2.D.1^A A consumer lives for two periods, denoted 1 and 2, and consumes a single consumption good in each period. His wealth when born is $w > 0$. What is his (lifetime) Walrasian budget set?

2.D.2^A A consumer consumes one consumption good x and hours of leisure h. The price of the consumption good is p, and the consumer can work at a wage rate of $s = 1$. What is the consumer's Walrasian budget set?

2.D.1 Let price of the single cons^n good in pd 1 be p_1 & in pd2 be p_2.
Then; walrasian budget set :—

$$p_1 x + p_2 x \leq w$$

2.D.2 $$px = s(24 - h) + w$$

where w is any wealth cons^m may have

6. Consider the utility function

$$u(x_1, x_2) = (x_1 - a_1)^{b_1}(x_2 - a_2)^{b_2},$$

where $b_1, b_2 > 0, b_1 + b_2 < 1$, and $a_1, a_2 > 0$. Find:

(a) the Marshallian demands,

(b) the expenditure function,

(c) the indirect utility function, and

(d) the money metric indirect utility function.

(e) Confirm the identities $e(p, v(p, m)) = m$ and $v(p, e(p, \bar{u})) = \bar{u}$.

(f) Assume you are given the expenditure function and indirect utility function from parts (b) and (c), but have no other knowledge of the consumer (in particular, you do not know the direct utility function $u(x_1, x_2)$). Calculate calculate the consumer's Hicksian demand using only this information. Then, calculate the consumer's Marshallian demand *without using Roy's identity* (Roy's identity is of course a valid way to calculate the Marshallian demand given knowledge of $v(p, m)$, but this question is asking you to do this in another way).

(a) $u(x_1, x_2) = (x_1 - a_1)^{b_1}(x_2 - a_2)^{b_2}$

(a) $\mathcal{L} = b_1 \log(x_1 - a_1) + b_2 \log(x_2 - a_2) + \lambda (w - p_1 x_1 + p_2 x_2)$

$\Rightarrow \dfrac{p_1}{p_2} = \dfrac{b_1 (x_2 - a_2)}{b_2 (x_1 - a_2)}$

From BC

$$x_1(p, w) = \frac{w - p_2 a_2 + p_1 \frac{b_2}{b_1} a_1}{p_1(1 + b_2/b_1)}$$

$$x_2 = \frac{w - p_1 a_1 + p_2 \frac{b_1}{b_2} a_2}{p_2 (1 + b_1 / b_2)}$$

(b) $\bar{u} = b_1 \log(x_1 - a_1) + b_2 \log(x_2 - a_2)$

$$\log(x_1 - a_1) = \frac{\bar{u} - b_2 \log\left(\frac{p_1}{p_2} \frac{b_2}{b_1}\right)}{b_1 + b_2}$$

$$x_1 = \left[e^{\bar{u}} e^{k_1(p)}\right]^{1/b_1+b_2} + a_1$$

$$x_2 = \left[e^{\bar{u}} e^{k_2(p)}\right]^{1/b_1+b_2} + a_2$$

$$e(p, u) = e^{\bar{u}/b_1+b_2}\left[p_1 \left(\frac{p_2 b_1}{p_1 b_2}\right)^{b_2/b_1+b_2} + \right.$$

$$\left. + p_2 \left(\frac{p_1}{p_2} \frac{b_2}{b_1}\right)^{b_1/b_1+b_2}\right] + pa$$

(c) $$V(p,w) = u(x^*)$$

$$= b_1 \log\left(\frac{w - p_2 a_2 + p_1 \frac{b_2}{b_1} a_1}{p_1(1+b_2/b_1)}\right) +$$

$$+ b_2 \log\left(\frac{w - p_1 a_1 + p_2 \frac{b_1}{b_2} a_2}{p_2(1+b_1/b_2)}\right)$$

(d) $$\mu(p,q,w) = e(p, v(q,w))$$

$$= p_1\left[\left(e^{v(q,w)} e^{k_1(q)}\right)^{1/b_1+b_2} + a_1\right] +$$

$$+ p_2\left[\left(e^{v(q,w)} e^{k_2(q)}\right)^{1/b_1+b_2} + a_2\right]$$

(e) $e(p, v(p,m)) = m$

$$e(p \cdot u) = p_1 e^{v/b_1+b_2}\left[\frac{p_2}{p_1}\frac{b_1}{b_2}\right]^{b_2/b_1+b_2}$$

$$+ p_2 e^{v/b_1+b_2}\left[\frac{p_1}{p_2}\frac{b_2}{b_1}\right]^{b_1/b_1+b_2} + p \cdot a \cdot e^{v/(b_1+b_2)}$$

$$= m$$

$$v(p, w) = b_1 \log\left(\frac{e(p, \bar{u}) - p_2 a_2 + p_1 {}^{b_2}/b_1 a_1}{p_1 (1 + b_2/b_1)}\right)$$

$$+ b_2 \log\left(\frac{e(p, \bar{u}) - p_1 a_1 + p_2 \, b_2/b_1 \, a_1}{p_2 (1 + b_1/b_2)}\right)$$

$$= \bar{u}$$

(f) $h_1 = \left[e^{\bar{u}} e^{k_1(p)}\right]^{1/b_1+b_2} e a_1$

$h_2 = \left[e^{\bar{u}} e^{k_2(p)}\right]^{1/b_1+b_2} e a_2$

$$k_1(p) = -b_2 \log\left(\frac{p_1}{p_2} \frac{b_2}{b_1}\right)$$

$$k_2(p) = -b_1 \log\left(\frac{p_2}{p_1} \frac{b_1}{b_2}\right)$$

7. Consider the indirect utility function,

$$v(p,m) = mp_1^{-1/2}p_2^{-1/2}.$$

Find expressions for the expenditure function, the Hicksian and Marshallian demands, and the (direct) utility function.

$$v(p,m) = mp_1^{-1/2}p_2^{-1/2} = \frac{m}{\sqrt{p_1 p_2}}$$

* For fixed price vector $\bar{p}$, $e(\bar{p},\cdot)$ & $v(\bar{p},\cdot)$ are inverses to one another.

- Expenditure Function

$$v\sqrt{p_1 p_2} = m$$

$$e(p,u) = v\sqrt{p_1 p_2}$$

- Marshallian Demand

$$x_1(p,m) = -\frac{\partial v(p,m)/\partial p_1}{\partial v(p,m)/\partial m}$$

$$= +\frac{\frac{1}{2} m p_1^{-3/2} \cdot p_2^{-1/2}}{p_1^{-1/2} \cdot p_2^{-1/2}} = \frac{1m}{2p_1}$$

$$x_2(p,m) = \frac{\frac{1}{2} m p_1^{-1/2} \cdot p_2^{-3/2}}{p_1^{-1/2} p_2^{-1/2}} = \frac{m}{2p_2}$$

$\frac{1}{2} - 1$

- Hicksian Demand

$$h_1(p,u) = \frac{\partial e(p,u)}{\partial p_1}$$

$$= \frac{1}{2} v \left(\frac{p_2}{p_1}\right)^{1/2} = \frac{\bar{u}}{2}\left(\frac{p_2}{p_1}\right)^{1/2}$$

$$h_2(p,u) = \frac{\partial e(p,u)}{\partial p_2}$$

$$= \frac{1}{2} v \left(\frac{p_1}{p_2}\right)^{1/2} = \frac{\bar{u}}{2}\left(\frac{p_1}{p_2}\right)^{1/2}$$

- Direct Utility function

$$h_1 h_2 = \frac{u^2}{4}$$

$$\Rightarrow \quad u = 2\sqrt{h_1 h_2} = 2\sqrt{x_1 x_2}$$

8. Show that Roy's identity is a direct consequence of the duality identities (those that relate the expenditure function with the indirect utility function and the Marshallian demand function with the Hicksian demand function) and Shepard's lemma. (Hint: Start with the identity $v(p, e(p, \bar{u})) = \bar{u}$ and differentiate this equation with respect to p_i.)

Roy's Identity :—

$$x_e(\bar{p}, \bar{w}) = \frac{-\partial v(\bar{p}, \bar{w}) / \partial p_e}{\partial v(\bar{p}, \bar{w}) / \partial w}$$

We know,

$$v(p, e(p, \bar{u})) = \bar{u}$$

Differentiating wrt p

$$\frac{\partial v(p, e(p, \bar{u}))}{\partial p} + \frac{\partial v(p, e(p, u))}{\partial e} \cdot \underbrace{\frac{\partial e(p, u)}{\partial p}}_{h(p,u)} = 0$$

$$\rightarrow h(p, u) = -\frac{\partial v(p, e(p, \bar{u})) / \partial p}{\partial v(p, e(p, u)) / \partial e}$$

from the duality results,

$$\Rightarrow \quad x(p,w) = -\frac{\partial v(p,w)/\partial p}{\partial v(p,w)/\partial w}$$

$\square$

9. MWG 3.E.1

Assume that $u(\cdot)$ is differentiable. Show that the first order conditions for the EMP are

$$p \geq \lambda \nabla u(x^*) \quad \& $$

$$x^* \cdot [p - \lambda \nabla u(x^*)] = 0 \quad \text{for some } \lambda \geq 0.$$

Compare this with the FOC of the UMP.

EMP :—

$$\max \quad p \cdot x. \quad \text{s.t.} \quad u(x) \geq u \quad \& \quad x \geq 0$$

$$\mathcal{L} = p \cdot x + \lambda (u - u(x)) + \mu(-x)$$

FOC $\quad \frac{\partial \mathcal{L}}{\partial x} : \quad p - \lambda u'(x^*) - \mu = 0$

$$p = \lambda \nabla u(x^*) + \mu$$

$$\Rightarrow p \geq \lambda \nabla u(x^*)$$

Also $\quad \mu = p - \lambda \nabla u(x^*)$

From K-T conditions we know,
for $\lambda > 0$
$\mu x = 0$

$$\rightarrow [p - \lambda \nabla u(x^*)] x^* = 0$$

This is the same as FOC for UMP.

10. MWG 3.G.4, parts (a)-(c). The wording of this problem is a bit unclear. For part (a), you need to show that if u is additively separable, and $g(x)$ is a *linear* monotonic transformation (i.e., $g(x) = ax + b$ for some constants $a > 0$ and b), then, the transformed utility $v(x) = g(u(x))$ has an additively separable representation (i.e., we can write $v(x) = \sum_{i=1}^{N} g_i(x_i)$ for some single-variable functions $g_i(\cdot)$). For part (b), partition the consumption space $X = Y \times Z$, where $Y = \mathbb{R}^m$ and $Z = \mathbb{R}^{n-m}$. Then, consider consumption bundles $(a, c), (a, d), (b, c)$ and (b, d), where $a, b \in Y$ and $c, d \in Z$. The question is asking you to prove that $(a, c) \succeq (b, c)$ if and only if $(a, d) \succeq (b, d)$.[2] For part (c), you just need to show that there are no inferior goods (i.e., the wealth effects for all goods are nonnegative).

3.G.4[B] A utility function $u(x)$ is *additively separable* if it has the form $u(x) = \sum_\ell u_\ell(x_\ell)$.

(a) Show that additive separability is a cardinal property that is preserved only under linear transformations of the utility function.

(b) Show that the induced ordering on any group of commodities is independent of whatever fixed values we attach to the remaining ones. It turns out that this ordinal property is not only necessary but also sufficient for the existence of an additive separable representation. [You should *not* attempt a proof. This is very hard. See Debreu (1960)].

(c) Show that the Walrasian and Hicksian demand functions generated by an additively separable utility function admit no inferior goods if the functions $u_\ell(\cdot)$ are strictly concave. (You can assume differentiability and interiority to answer this question.)

(d) (Harder) Suppose that all $u_\ell(\cdot)$ are identical and twice differentiable. Let $\hat{u}(\cdot) = u_\ell(\cdot)$. Show that if $-[t\hat{u}''(t)/\hat{u}'(t)] < 1$ for all t, then the Walrasian demand $x(p, w)$ has the so-called *gross substitute property*, i.e., $\partial x_\ell(p, w)/\partial p_k > 0$ for all ℓ and $k \neq \ell$.

$u(x)$ is additively separable if
$u(x) = \Sigma_\ell u_\ell(x_\ell)$

(a) Let $u(x)$ be additively separable & $g(x)$ be a linear monotonic transformation

$$v(x) = g(u(x))$$
$$= a\,u(x) + b$$
$$= a\,(\Sigma_\ell u_\ell(x_\ell)) + b$$
$$= \Sigma_\ell \left[a\,u_\ell(x_\ell) + \frac{b}{\ell} \right]$$

$$= \Sigma_\ell\, h\left(u_\ell(x_\ell)\right) \qquad \square$$

(b) Partition the cons^m base: $X = Y \times Z$ $\quad Y = R^m$, $Z = R^{n-m}$

Cons^m bundles:

$(a,c), (a,d), (b,c)$ & (b,d)

Prove $(a,c) \succsim (b,c)$ iff $(a,d) \succsim (b,d)$

Let $(a,c) \succsim (b,c)$

$\Rightarrow a \succsim b$

$\Rightarrow (a,d) \succsim (b,d)$ (WARP)

If $(a,d) \succsim (b,d)$

$\Rightarrow a \succsim b$

$\Rightarrow (a,c) \succsim (b,c)$ (WARP)

(c) $\max\ u(x)$ s.t $\Sigma p_i x_i \leq w$

$$\mathcal{L} = \sum_{i=1}^{n} u(x_i) + \lambda\left(w - \sum_{i=1}^{n} p_i x_i\right)$$

$$\frac{\partial \mathcal{L}}{\partial x_i} = u'(x_i) = \lambda p_i$$

$$\Rightarrow \frac{u'(x_i)}{p_i} = \frac{u'(x_j)}{p_j}$$

$$\Rightarrow \quad \frac{u'(x_i(p,w))}{p_i} = \frac{u'(x_j(p,w))}{p_j}$$

$$\frac{\partial}{\partial w}: \quad \frac{u''\frac{\partial x_i(p,w)}{dw}}{p_i} = \frac{u''\frac{dx_j(p,w)}{dw}}{p_j}$$

If w increases, consn^m of atleast one group will increase (walras law).

As wealth effect is linearly related b/w goods as per their price ratios,

$\frac{dx_i}{dw} > 0 \quad \forall i$.

The results are same for Hicksian dd functions as the FOCs are the same.