Econ 7010 - Microeconomics I
University of Virginia
Fall 2023

Problem Set 3

Due Monday, October 16th. Upload your submissions to Gradescope by 11:59pm.

1. MWG 5.C.8

2. A firm has an input requirement set of $V(y) = \{x : ax_1 + bx_2 + cx_3 \geq e^y - 1\}$. Find the firm's cost function.

3. Consider a firm with cost function $c(w, y) = (y(\sqrt{w_1} + \sqrt{w_2}))^2$. Find the firm's production function $f(x)$.

4. Examine the parametric form of a candidate cost function $c(w, y) = y(a_1 w_1 + a_2 w_2 + b w_3^{\frac{1}{2}} w_4^{\frac{1}{2}})$. For what set of parameter values can this be a cost function? (explain). Now assume that these parameter restrictions hold. Find the form of the production function that generates this form of cost function.

5. Define $C(q) = c(w, q)$ to be the cost function of a firm holding the factor prices w fixed, and define the average cost function $AC(q) = C(q)/q$ for $q > 0$.

   (a) Prove that if $AC(q)$ is minimized at $\bar{q}$ and if $C'(\bar{q})$ exists, then $AC(\bar{q}) = C'(\bar{q})$.

   (b) Prove that if $AC(q)$ is minimized at $\bar{q}$, then

   $$\frac{C(\bar{q}) - C(\bar{q} - \epsilon)}{\epsilon} \leq AC(\bar{q}) \leq \frac{C(\bar{q} + \epsilon) - C(\bar{q})}{\epsilon} \text{ for all } \epsilon \in [0, \bar{q}]$$

   Using a well-labeled picture, explain the implications of these inequalities as $\epsilon \to 0$ for the case where $C(q)$ is not differentiable at $\bar{q}$.

6. MWG 5.C.11

7. Consider a firm that uses labor (L) and capital (K) to produce a single output according to a production function $f(L, K)$. You may assume that f is differentiable, strictly increasing, strictly concave, and satisfies: $f(L, 0) = f(0, K) = 0$ for all L, K.

   The standard profit-maximization problem for a neoclassical firm is

   $$\max_{L,K \geq 0} pf(L, K) - wL - rK,$$

   where p is the output price and w and r are input prices. In this problem, we focus on the short-run, where the level of capital is fixed at some $\bar{K} > 0$, and you may ignore corner solutions.

   (a) Show that if the output price p rises, the neoclassical firm will increase its use of labor.

   Consider instead a *labor-managed firm,* which, rather than paying a fixed market wage w, takes the production surplus and distributes it equally among the workers; that is, the "wage" each worker receives is $W = \frac{pf(L,K)-rK}{L}$, and the goal of the firm is to maximize W:

   $$\max_{L,K \geq 0} \frac{pf(L, K) - rK}{L}.$$

   Continue to assume that capital is fixed at $\bar{K}$.

   (b) How will a labor-managed firm respond to an increase in p? Prove your answer.

   (c) Consider an increase in the price of capital, r. How does the output of each firm—neoclassical and labor-managed—respond?

   (d) Consider the following microfoundation for the labor-managed firm's problem: there is a worker with utility function $u(W)$, where W is the compensation remunerated to the worker per unit of labor (i.e., the "wage"). You may assume that $u' > 0$, $u'' < 0$, and $u'(0) = \infty$. The objective of the firm is to maximize the worker's utility, subject to the constraint that the firm is profitable (i.e., revenues cover both the total wages paid as well as the fixed cost $r\bar{K}$). Formulate this as a constrained optimization problem. Show that its solution leads to the same optimal choice of labor as you found in part (b).

8. Sally runs a lemonade stand, and, as she is the only lemonade stand in the neighborhood, she is a monopoly. She faces an inverse demand curve of $p(y, \lambda)$, where y denotes the liters of lemonade and λ is a parameter that denotes the outside temperature, which can shift demand. Sally has cost function $c(y, \gamma)$, where γ is a parameter that can shift the cost function. Assume that all functions are twice continuously differentiable, and that $p_\lambda(y, \lambda) > 0$ and $c_\gamma(y, \gamma) > 0$.

   (a) Write down the monopolist's problem, and take the first order condition. Provide conditions on the derivatives of p and c that guarantee that the first-order condition is sufficient for an optimum. Provide a second set of conditions that guarantee that the first order condition is sufficient for the optimum to be unique. Your conditions should be on p and c independently, and the second set of conditions should be stronger than the first.

   Assume that the conditions from part (a) hold.

   (b) Derive expressions showing how the optimal output of the monopolist responds to changes in λ and γ (consider each separately). Provide sufficient conditions under which the monopolist's output is increasing in λ. Answer the same for γ.

   (c) Consider the special case where y, λ, and γ are separable, i.e., the inverse demand function can be written as $p(y, \lambda) = r(y) + s(\lambda)$ and the cost function can be written $c(y, \gamma) = a(y) + b(\gamma)$ for some (differentiable) functions $r(\cdot), s(\cdot), a(\cdot)$, and $b(\cdot)$. Answer part (b) in this case. Explain the intuition behind the conditions you derived. (If you have done part (b) correctly, this part should not take you very long.)