Econ 7010 - Microeconomics I
University of Virginia
Fall 2023

Problem Set 5
Due Friday, November 17th

1. MWG 2.E.2, 2.E.7, 2.F.17

2. A consumer has utility function

$$u(c, x) = \min\{c, x\},$$

where c is consumption and x is leisure. The consumer has T units of time and revives wage w for each unit of time she works. Wages are the only source of income. Use L to denote the number of units of time spent working, so that $x = T - L$. The price of consumption is p.

(a) Find the Marshallian demands for consumption and leisure.

(b) Find the indirect utility function and the expenditure function.

(c) Draw a graph of the Marshallian labor supply curve. Labor supply is said to be *backward bending* if there is a point at which an increase in w leads to a decline in L. Does this occur here?

(d) A politician says that "a proposed tax on income will lead to ruin! If we tax income, no one will work and we'll lose our cherished way of life." Is the politician right in the context of the model?

3. Darrell consumes two goods, beer and Stata packages, at prices p_b and p_s respectively. Let $h_b(p, \bar{u})$ be Darrell's Hicksian demand for beer for utility level $\bar{u}$ and $x_b(p, m)$ be his Marshallian demand for beer at prices p and wealth m. Show that beer and Stata packages are substitutes, i.e., show that $\partial h_b(p, \bar{u})/\partial p_s \geq 0$. (Hint: you may want to consult your notes on cost minimization for a firm.) Are beer and Stata packages also gross substitutes?

4. (Core June 2000) Assume that an individual has an expenditure function

$$e\left(p_1, p_2, u\right) = 2u\sqrt{p_1 p_2} - 100p_1$$

over the set of values (p_1, p_2, u) such that this function is non-decreasing in p_1.

Suppose that in the absence of government action this person has a Walrasian budget set with $p_1 = \$1$, $p_2 = \$1$, and $w = \$800$. Now suppose that the government offers this person a cash payment of \$1700 on the condition that the person abstains from consumption of the first good.

(a) Will the person accept this grant? Explain.

(b) If the participation in this program were mandatory, what would be the equivalent variation measure of the net benefit of the program to this person? Show your work.

5. Maggie has a utility function $u(x_1, x_2)$, but rather than a monetary income m, she is given an endowment $E = (E_1, E_2)$ of goods 1 and 2, respectively. She maximizes utility subject to her budget constraint.

   (a) If the prices of the goods are p_1 and p_2, what is her budget constraint, written in terms of the endowment (E_1, E_2)? Write down Maggie's utility maximization problem.

   (b) Draw Maggie's budget set, highlighting the endowment point E. Show what happens to the budget set if the price of p_1 rises.

   (c) Now suppose that you know that Maggie's demand for good 1 exceeds her endowment of good 1. What can you say about her choice of x_2?

   (d) Write out Maggie's expenditure minimization problem (be careful: the objective will now contain endowments). Show that the Hicksian demands do not depend on E, but the expenditure function does.

   (e) Denote the expenditure function as $e(p, \bar{u}, E)$. What is the analogue of Shepard's Lemma in this setting? That is, find an expression for $\frac{\partial e(p,\bar{u},E)}{\partial p_i}$ in terms of the Hicksian demand and the endowments E.

   (f) Derive the Slutsky equation in this setting. (The derivation is similar to what we showed in class for the case of no endowments, but the final result will include the endowments.)

   (g) If $\frac{p_1}{p_2}$ increases is Maggie better off or worse off? You can argue this graphically if you would like.

6. Consider a consumer with wealth m who consumes two goods, x_1 and x_2. Normalize the price $p_2 = 1$. Let $x_1(p, m)$ be the Marshallian demand, and $h_1(p, v(\bar{p}, m))$ be the corresponding Hicksian demand when the required utility level is $v(\bar{p}, m)$ for some fixed $\bar{p}$. Assume that good 1 is an inferior good.

   (a) On a graph with demand for good 1 on the horizontal axis and price p_1 on the vertical axis, draw a sketch showing the relationship between the Marshallian and Hicksian demands. The key features of the sketch are the points where the two curves cross and the relative slopes of the two curves.

   (b) Assume that the price of good 1 falls from p_1^0 to p_1^1 and all other prices remain fixed. Show algebraically that the equivalent variation for this price change is less than or equal to the compensating variation. (Hint: what is the relationship between the Hicksian demand at the old utility u^0 and the new utility u^1 for an inferior good? How do these demand curves relate to EV and CV?)

7. Consider the candidate indirect utility function

$$v(p_1, p_2, m) = p_1^a p_2^b m^c$$

   where $a = b = -\frac{1}{2}, c = 1$. Find expressions for:

   (a) the expenditure function,

   (b) the Hicksian and Marshallian demands,

   (c) the direct utility function, and

   (d) CV and EV for a price change from (p_1, p_2) to (q_1, q_2).

8. MWG 3.D.7, 3.G.15 (for question 3.D.7, you may assume that preferences can be represented by a differentiable, monotonic, and concave utility function u)

9. Consider two consumers with utility functions $u_A(x_1, x_2) = x_1 + \alpha \ln(x_2)$ and $u_B(x_1, x_2) = x_1 + \beta\sqrt{x_2}$. Assume that the numeraire good, x_1, can take on any value in the set $(-\infty, \infty)$ (i.e., ignore the nonnegativity constraint on x_1). You may normalize $p_1 = 1$, and let the price of good 2 be p_2. The consumers have incomes m_A and m_B respectively.

   (a) Derive the individual demand functions for each consumer for each good.

(b) What is the aggregate demand function?

(c) Can you write the aggregate demand functions only as a function of aggregate wealth $\bar{m} = m_A + m_B$? Do you need any additional restrictions on the parameters of the utility functions for this to hold?

(d) Do the consumers have preferences of the Gorman form? Prove your answer (i.e., if yes, find the functions $a_i(p)$ and $b(p)$).

10. [From the January 2009 core exam] Consider an economy with a continuum of consumers, indexed by $a \in [0, 1]$, and two goods. Consumer a's utility function over the two goods is $u_a(x_1, x_2) = ax_1 + (1 - a)x_2$. All consumers are endowed with the same income m.

    (a) Derive aggregate demand $X_1(\vec{p}, m)$ and $X_2(\vec{p}, m)$ (where $\vec{p} = (p_1, p_2)$).

    (b) Suppose that these aggregate demands can be interpreted as the bundle chosen by a single representative agent maximizing some function $U(X_1, X_2)$ subject to income m. Determine whether the representative agent's preferences are homothetic. Derive this agent's elasticity of substitution between the two goods and compare to an individual consumer.

    (c) Now suppose that there are only two consumer types. Fraction $\lambda \geq 0$ of consumers have preferences $a = 1$ and fraction $1 - \lambda$ have preferences $a = 0$. Are there any values of λ for which there is a representative agent whose utility function has a constant elasticity of substitution equal to 1?