Econ 7010 - Microeconomics I
University of Virginia

Problem Set 2

Due Monday, October 2nd by 11:59 pm. Submit your solutions on Gradescope via Collab.

1. Mr. Jefferson, the owner of Hoos Incorporated, is worried about the quality of the manager he has hired to oversee production, and has hired you as a consultant. He is afraid the manager may not be acting in the firm's best interest when procuring inputs for production. View the firm as producing one aggregate output, y, using one aggregate input, x, with associated prices p and w, respectively. Last quarter, when $(p, w) = (1, 1)$, you observed the manager using two units of input to produce one unit of output. This quarter, the prices have risen to $(p', w') = (3, 4)$. What observable data (input-output pairs) would cause you to recommend to Mr. Jefferson that the manager be fired for not maximizing the profits of the firm? Sketch all such data points on a graph.

2. (Varian 2.6) Let $\{(p^1, y^1), \ldots, (p^T, y^T)\}$ be a (finite) set of observed choices of a firm that satisfy WAPM, and define $Y^I = conv_{fd}(y^1, \ldots, y^T)$ (the convex hull of the data points augmented by free disposal) and $Y^O = \{y \in \mathbb{R}^n : p^t \leq \pi(p^t) \text{ for all } t = 1, \ldots, T\}$ (these are just the inner bounds and outer bounds defined in class). Let $\pi^I(p)$ and $\pi^O(p)$ be the profit functions associated with Y^I and Y^O, respectively, and $\pi(p)$ be the profit function associated with the true technology Y. Prove that $\pi^O(p) \geq \pi(p) \geq \pi^I(p)$ for all p.

3. Prove that the profit function $\pi\,(p)$ is a convex function (do **not** assume any type of differentiability for this problem).

4. MWG 5.C.13 (Hint: write down the constrained optimization problem *carefully* and then use the Envelope Theorem)

5. Suppose a firm's production function is given by $f(x_1, x_2) = \min\{2x_1 + x_2, x_1 + 2x_2\}$.

(a) Sketch the associated isoquant map.

(b) Find the firm's conditional factor demand functions $x_1(w, y)$ and $x_2(w, y)$. (Remember, the conditional factor demand is the demand for inputs *conditional* on wanting to produce some target level of output y in the cheapest way possible. Your answer should be a function only of the input prices w and the target output y; the output price p should *not* appear in your answers.)

6. Consider a firm with a single-output production function $f(x_1, x_2) = \ln(1 + \min\{x_1, x_2\})$. Let the output price be p and the input price vector be $w = (w_1, w_2)$. Assume throughout that $p > w_1, w_2$.

   (a) Write the corresponding cost-minimization problem and solve for the conditional factor demands $x_1^*(w, y)$, $x_2^*(w, y)$ and cost function $c(w, y)$.

   (b) Using the cost function derived in part (a), write the firm's profit maximization problem (where the only choice variable should be the choice of output y). Solve this problem for the optimal input demands $x_1(p, w)$ and $x_2(p, w)$ and the profit function $\pi(p, w)$.

   Let $\bar{x}_1 = x_1(\bar{p}, \bar{w})$ and $\bar{x}_2 = x_2(\bar{p}, \bar{w})$ be the optimal input demands when the prices are $(\bar{p}, \bar{w})$. Suppose that in the short run, x_1 is fixed at $\bar{x}_1$, but x_2 is flexible.

   (c) Write the short-run profit maximization problem and find $x_2^{SR}(p, w)$, the short-run input demand function for x_2. (N.B.: watch out for kink points.)

   (d) Consider a small change in prices from $(\bar{p}, \bar{w}_1, \bar{w}_2)$ to $(\bar{p}, \bar{w}_1, \bar{w}_2 + \epsilon)$, for some small $\epsilon > 0$. What is the change in demand for x_2 in the short run? What is the change in demand for x_2 in the long run? Does the Le Chatelier principle hold?

7. Derive the cost function for the following technologies:

   (a) $f(x_1, x_2, ..., x_n) = \min\{a_1x_1, a_2x_2, ...a_nx_n\}$.

   (b) $f(x_1, x_2, ..., x_n) = \sum_{i=1}^{n} a_ix_i$.

   (c) $f(x_1, x_2) = \min\{a_1x_1, a_2x_2 - k\}$, where $k > 0$ is a constant. (There is a little problem with this formulation. Think about what that could be and how to fix it.)

(d) $f(x_1, x_2) = [x_1^\rho + x_2^\rho]^{\frac{1}{\rho}}$, where $\rho \leq 1$.

8. Let $c(w, y)$ and $x(w, y)$ be the cost function and conditional factor demands for a firm with a many input, single output production technology described by production function $f(x)$ that exhibits constant returns to scale.

   (a) Prove that if $c(\cdot)$ and $x(\cdot)$ both exist (i.e., the cost minimization problem has a solution), they are homogeneous of degree 1 in y.

   (b) Prove that if the production function $f(x)$ is concave, then cost function $c(w, y)$ is convex in y. What is the economic interpretation of this result?