IC → Combine feasibility, govt budget etc. w/o prices.

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Today :-

[Ramsey Problem with cons^m & labor income tax]

TDCE

Represent HH $\sum_{t=0}^{\infty} \beta^t u(c_t, n_t) \to \max$

Representative firm $y_t \leq F(k_t, n_t)$

Govt. has to finance $\{g_t\}_{t=0}^{\infty}$

Can use $\{\tau_{ct}, \tau_{nt}\}_{t=0}^{\infty}$ only.

GBC $\sum_{t=0}^{\infty} p_t g_t = \sum_{t=0}^{\infty} p_t c_t \tau_{ct} + w_t n_t \tau_{nt}$

Representative → Could mean here aggregate or a single HH where everyone has the same utility f^{n}s.

(Clarify when is which).

BC for HH:

$$\sum_{t=0}^{\infty} p_t(1+\tau_{ct})c_t + p_t k_{t+1} = \sum_{t=0}^{\infty} w_t n_t (1-\tau_{nt}) + r_t k_t + p_t(1-\delta)k_t \quad (\lambda)$$

Assume sol^n is interior / why?

$$(c_t): \beta^t u'_c(t) = \lambda p_t (1+\tau_{ct})$$

$$(n_t): \beta^t u'_n(t) = \lambda w_t (1-\tau_{nt})$$

$$(k_{t+1}): -p_t + p_{t+1}(1-\delta) + r_{t+1} = 0$$

$$\Rightarrow \frac{u'_c(t)}{\beta u'_c(t+1)} = \frac{p_t}{p_{t+1}} \frac{(1+\tau_{ct})}{(1+\tau_{ct+1})}$$

$$\frac{u'_n(t)}{u'_c(t)} = \frac{w_t}{p_t} \frac{(1-\tau_{nt})}{1+\tau_{ct}}$$

$$\frac{p_t}{p_{t+1}} = \frac{r_{t+1}}{p_{t+1}} + 1-\delta$$

$$= \frac{u'_c(t)\,(1+\tau_{ct+1})}{\beta u'_c(t+1)\,(1+\tau_{ct})}$$

Euler Eq^n:—

$$\frac{u'_c(t)}{\beta u'_c(t+1)} \frac{(1+\tau_{ct+1})}{1+\tau_{ct}} = \frac{r_{t+1}}{p_{t+1}} + 1-\delta$$

2) Derive IC for the economy

(Use BC for HH & get rid of prices)

Note that $w_t(1-\tau_{nt}) = \frac{u'_n(t)}{u'_c(t)} p_t(1+\tau_{ct})$

&

$$\frac{u'_c(t)}{\beta u'_c(t+1)} = \frac{p_t\,(1+\tau_{ct})}{p_{t+1}\,(1+\tau_{ct+1})}$$

$$\Rightarrow \frac{\beta^t u'_c(t)}{u'_c(0)} \cdot \frac{(1+\tau_{c0})}{1+\tau_{ct}} = \frac{p_t}{p_0}$$

How?

$$\Rightarrow \quad (1+\tau_{ct})\, p_t = \frac{p_0 \; u'_c(t)\, \beta^t}{u'_c(0)} \; (1+\tau_{c0})$$

BC for HH :—

$$\sum_{t=0}^{\infty} p_t (1+\tau_{ct}) c_t + p_t k_{t+1} = \sum_{t=0}^{\infty} w_t n_t (1-\tau_{nt}) + r_t k_t + p_t (1-\delta) k_t \quad (\lambda)$$

$$\Rightarrow \sum_{t=0}^{\infty} p_0 \beta^t \frac{u'_c(t)}{u'_c(0)} (1+\tau_{c0}) c_t = \sum \frac{u'_n(t)}{u'_c(t)} \underbrace{p_t (1+\tau_{ct})}_{\frac{p_0 \beta^t u'_c(t)}{u'_c(0)} (1+\tau_{c0})} n_t + r_t k_t + p_t (1-\delta) k_t - p_t k_{t+1}$$

$$\sum_{t=0}^{\infty} p_0 \beta^t \frac{1}{u'_c(0)} (1+\tau_{c0}) \left[u'_c(t) c_t - u'_n(t) n_t \right]$$

$$= \sum_{t=0}^{\infty} r_t k_t + p_t (1-\delta) k_t - p_t k_{t+1}$$

$$= \sum_{t=0}^{\infty} \underbrace{r_{t+1} k_{t+1} + p_{t+1} (1-\delta) k_{t+1} - p_t k_{t+1}}_{k_{t+1}(r_{t+1} + (1-\delta) p_{t+1} - p_t) \rightarrow 0 \text{ b/c of FOC}} + r_0 k_0 + p_0 (1-\delta) k_0$$

IC: $$\sum_{t=0}^{\infty} \beta^t \left[u'_c(t) c_t - u'_n(t) n_t \right] = \frac{u'_c(0)}{p_0 (1+\tau_{c0})} \left(r_0 k_0 + p_0 (1-\delta) k_0 \right)$$

Q Is it optimal to have $\tau_{ct} \to 0$
$t \to \infty$

What about τ_{nt}

Does Chamley Judd thing hold here?

→ We formulate Ramsey Problem to answer this.

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t, n_t) \to \max$$

$$\text{s.t. } c_t + g_t + k_{t+1} - (1-\delta)k_t = F(k_t, n_t) \quad [\lambda_t]$$

$$\sum_{t=0}^{\infty} \beta^t [u_c'(t) c_t - u_n'(t) n_t] = A \quad [\mu]$$

Is it always in this form? why else we calculate it then?

FOCs :-

$$(c_t): \beta^t u_c'(t) - \lambda_t - \mu[\beta^t(u_{cc}''(t)c_t + u_c'(t) - u_{nc}''(t)n_t)] = 0$$

$$(n_t): \beta^t u_n'(t) - \lambda_t F_n'(t) - \mu[\beta^t(u_{nc}''(t)c_t - u_{nn}''(t)n_t - u_n'(t))] = 0$$

$$(k_{t+1}): -\lambda_t + \lambda_{t+1}(F_k'(t+1) + 1 - \delta) = 0$$

$$\lambda_t = \beta^t[u_c'(t) - \mu(u_{cc}''(t)c_t + u_c'(t) - u_{nc}''(t)n_t)]$$

$$= \frac{1}{F_n'(t)} \beta^t [u_n'(t) - \mu(u_{nc}''(t)c_t - u_{nn}''(t)n_t - u_n'(t))]$$

$$\frac{\lambda_t}{\lambda_{t+1}} = F_k'(t+1) + 1 - \delta$$

$$\frac{u'_c(t) - \mu\left(u''_{cc}(t)c_t + u'_c(t) - u''_{nc}(t)\, n_t\right)}{\beta\left(u'_c(t+1) - \mu\left(u''_{cc}(t+1)c_{t+1} + u'_c(t+1) - u''_{nc}(t+1)n_{t+1}\right)\right)}$$

$$= f'_k(t+1) + 1 - \delta$$

Euler equation in RP

Suppose $c_t^{RP} \longrightarrow c_\infty^{RP}$

$n_t^{RP} \longrightarrow n_\infty^{RP}$

$k_t^{RP} \longrightarrow k_\infty^{RP}$

$$\Rightarrow \quad \frac{1}{\beta} = F'_k(\lim k, \lim n) + 1 - \delta$$

Euler Eq^n in TDCE:

$$\frac{u'_c(t)}{\beta u'_c(t+1)} = \frac{1 + \tau_{ct}}{1 + \tau_{ct+1}}\left(F'_k(t+1) + 1 - \delta\right)$$

$$\tau_{ct} \to \tau_c < \infty$$

Ramsey

Also,
$$F'_n(t) = \frac{u'_n(t) - \mu\left(u''_{nc}(t)c_t - u''_{nm}(t)n_t - u'_n(t)\right)}{u'_c(t) - \mu\left(u''_{cc}(t)c_t + u'_c(t) - u''_{nc}(t)n_t\right)}$$

And
$$\frac{u'_n(t)}{u'_c(t)}\,\frac{(1+\tau_{ct})}{(1-\tau_{nt})} = F'_n(t)$$

TDCE

If $\mu = 0$

$\Rightarrow$ It is optimal to

$$1 + \tau_{ct} = 1 - \tau_{nt}$$

$$\Leftrightarrow \boxed{\tau_{ct} = -\tau_{nt}}$$

Q. What is the difference b/w TDCE & Ramsey Problem?