ECON7020: MACROECONOMIC THEORY

FALL 2023

Problem Set 3. Due date: before class on October 24.

Problem 1

Consider an economy in which the representative consumer lives forever. There is a good in each period that can be consumed or saved as capital as well as labor. The consumer's utility function is

$$\sum_{t=0}^{\infty} \beta^t \left(\log c_t + \gamma \log(1 - l_t)\right).$$

Here $\beta \in (0,1)$ and $\gamma > 0$. The consumer is endowed with 1 unit of time each period and with $\bar{k}_0$ units of capital at time 0. The feasibility constraint for this economy is

$$c_t + k_{t+1} \leq \theta k_t^{\alpha} l_t^{1-\alpha},$$

with $\alpha \in (0,1)$ and $\theta > 0$.

(a) Write down the Euler equations and the transversality condition for the problem of maximizing the representative consumer's utility subject to feasibility conditions.

(b) Formulate the problem of maximizing the representative consumer's utility subject to feasibility conditions as a dynamic programming problem. Write down the appropriate Bellman's equation.

(c) Guess that the value function has the form $a_0 + a_1 \log k$. Guess that in the solution to the dynamic programming problem the optimal labor supply $l(k)$ is constant. Solve for this constant l. Solve the dynamic programming problem.

(d) Show that the policy functions from the solution to the dynamic programming problem in parts (b) and (c) satisfy the Euler equations and transversality condition in part (a).

(e) Define a Sequential Markets equilibrium for this economy. Explain carefully how to use the solution to the dynamic programming problem in part (c) to calculate the sequential markets equilibrium.

Problem 2

Consider an economy in discrete time $t = 0, 1, \ldots$ with a representative consumer. The utility of that consumer is

$$\sum_{t=0}^{\infty} \beta^t \log c_t,$$

where $\beta \in (0, 1)$. There is also a firm with the following production technology

$$F(k_t) = A k_t^{\alpha},$$

where $A > 0$ and $\alpha \in (0, 1)$. Capital depreciates at a rate $\delta \in [0, 1]$, and initial level of capital k_0 is given.

(a) Write down the Bellman equation.

(b) Find the steady-state, i.e. an allocation where $k_t = k_{t+1} = k_{ss}$. What is the level of consumption at the steady-state?

(c) Let $\beta = 0.9$, $\delta = 0.1$, $A = 1$ and $\alpha = 0.3$. Using Excel or any other software of your choosing, plot the first 10 iterations of the value function iteration algorithm. That is, starting with any function $v_0(k)$, plot 10 lines $\{v_j(k)\}_{j=1}^{10}$ on the same graph. Briefly describe your results.

(d) Suppose $k_0 = 0.5 \times k_{ss}$. Starting from k_0, iteratively apply the decision rule obtained from the last iteration in (c) and construct (and plot) a time-series of capital stocks for 10 time periods, $\{k_t\}_{t=1}^{10}$.

(e) Repeat (d) for $k_0 = 2 \times k_{ss}$. Describe in words your results.

Problem 3

Consider the competitive equilibrium of an economy with two types of agents with an equal mass of each. The utility function of type i is given by $\sum_{t=0}^{\infty} \beta_i^t \frac{c_{it}^{1-\sigma}}{1-\sigma}$, where $\beta_1 > \beta_2$. Assume that initial endowments of period 0 capital stock are equal among agent types: $k_{1,0} = k_{2,0}$. There is a representative firm with access to technology $F(k_t) = Ak_t$, with $A > 0$.

Do the two agent types consume the same amount in period 0? If not, who consumes more? Prove your claim.

Hint: Derive the Euler equation for consumer i, and use it to express consumption of agent i in period t, $c_{i,t}$, as a function of consumption in period 0, $c_{i,0}$. Subsequently, invoke the budget constraint of agent i, and use the first-order condition of agent's problem with respect to $k_{i,t+1}$ to simplify the right-hand side of the budget constraint by expressing $\sum_{t=0}^{\infty} p_t c_{i,t}$ only in terms of parameters and $k_{i,0}$.