Oct 26

Today's Plan :—

1. Ramsay Problem

2. Chamley - Judd result

3. TDCE is not PO

TDCE allocation (given policy $\{\tau_{kt}, \tau_{nt}, g_t\}_{t=0}^{\infty}$) is characterized by :—

1) Normalize $p_0 = 1$: $p_t = \dfrac{\beta^t u'_c(t)}{u'_c(0)}$

2) $\dfrac{u'_n(t)}{u'_c(t)} = f'_n(t)(1-\tau_{nt})$

3) $u'_c(t) = \beta u'_c(t+1)[f'_k(t+1) + 1 - \delta]$

4) $f'_k = \dfrac{r_t}{p_t}$

5) $f'_n = \dfrac{w_t}{p_t}$

6) $n_t = n_t^f$

7) $k_t = k_t^f$

8) $c_t + g_t + x_t = F(k_t, n_t)$

9) $\sum_{t=0}^{\infty} [p_t c_t + p_t x_t] = \sum_{t=0}^{\infty} (r_t k_t (1-\tau_{kt}) + w_t n_t (1-\tau_{nt}))$

10) Law of motion for k_t

$k_{t+1} = (1-\delta)k_t + x_t$

Implementability Constraint

(Shrinking the 10 conditions in one constraint)

BC:

$$\sum_{t=0}^{\infty} p_t c_t + p_t (k_{t+1} - (1-\delta)k_t) = \sum_{t=0}^{\infty} (1-\tau_{nt}) w_t n_t + (1-\tau_{kt}) r_t k_t$$

$$= \underbrace{\sum_{t=0}^{\infty} p_t c_t}_{} = \sum_{t=0}^{\infty} (1-\tau_{nt}) w_t n_t + (1-\tau_{k0}) r_0 k_0 + (1-\delta) k_0 p_0 +$$

$$+ \sum_{t=0}^{\infty} k_{t+1} [\underbrace{-p_t + p_{t+1}(1-\delta) + (1-\tau_{kt+1}) r_{t+1}}_{=0 \text{ from FOC (2 classes back)}}]$$

Cond 1

$$p_t = \frac{\beta^t u'_c(t)}{u'_c(0)}$$

From cond 2 & 3 :-

$$(1-\tau_{nt}) w_t = \frac{u'_n(t)}{u'_c(t) \cancel{F'_n(t)}} \cancel{F'_n}(t) p_t$$

$$= \frac{u'_n(t)}{u'_c(t)} \beta^t \frac{u'_c(t)}{u'_c(0)}$$

$$= \beta^t \frac{u'_n(t)}{u'_c(0)}$$

IC :-

$$\sum_{t=0}^{\infty} \beta^t \frac{u'_c(t)}{u'_c(0)} c_t - \beta^t \frac{u'_n(t)}{u'_c(0)} n_t = (1-\tau_{k0}) F'_k(0) p_0 k_0 + (1-\delta) k_0 p_0$$

$$\Longleftrightarrow \sum_{t=0}^{\infty} \beta^t (u'_c(t) c_t - u'_n(t) n_t) = u'_c(0) [1-\tau_{k0} F'_k(0) p_0 k_0 + (1-\delta) k_0 p_0]$$

TDCE allocation (given the FP) has to satisfy:—

1) feasibility constraint : $c_t + x_t + g_t = f(k_t, n_t)$

2) law of motion : $k_{t+1} = (1-\delta) k_t + x_t$

3) Implementability

$$\sum_{t=0}^{\infty} \beta^t \left(u'_c(t) c_t - u'_n(t) n_t\right) = \mathcal{A}$$

This gives us a set of allocation. For any of these you can find a fiscal policy for which govt. budget is satisfied.

Conversely,

If an allocation $\{c_t, k_{t+1}, x_t, \ell_t, n_t\}_{t=0}^{\infty}$ satisfies 1, 2, 3 $\Rightarrow$ $\exists$ $\{p_t, w_t, r_t\}_{t=0}^{\infty}$ and $\{\tau_{kt}, \tau_{nt}\}_{t=0}^{\infty}$ which

TDCE where govt. raises enough to finance $\{g_t\}_{t=0}^{\infty}$

* Remark : g is given. The govt does not decide g.

Ramsey Problem:—

$$\sum_{t=0}^{\infty} \beta^t u(c_t, 1-n_t) \longrightarrow \max_{\{c_t, l_t, n_t, x_t, k_{t+1}\}_{t=0}^{\infty}}$$

s.t. $c_t + x_t + g_t = F(k_t, n_t)$

$k_{t+1} = (1-\delta)k_t + x_t$] → same as SP

$\sum_{t=0}^{\infty} \beta^t (u'_c(t)c_t - u'_n(t)n_t) = A$] → new

(As we have a new constraint now the sol^n to this may not be the same as SP's / PO).

Note:— The government spending does not go in consumer's utility f^n. It's all like govt is throwing away money in the ocean. (That's so strange!)

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t u(c_t, 1-n_t) + \lambda\left[A - \sum_{t=0}^{\infty} \beta^t (u'_c(t)c_t - u'_n(t)n_t)\right] +$$

$$+ \sum_{t=0}^{\infty} \gamma_t \left[F(k_t, n_t) - c_t - x_t - g_t\right] + \sum_{t=0}^{\infty} \eta_t \left[(1-\delta)k_t + x_t - k_{t+1}\right]$$

$$\mathcal{L} = \lambda A + \sum_{t=0}^{\infty} \beta^t B(c_t, n_t, \lambda) + \sum_{t=0}^{\infty} \gamma_t \left[F(k_t, n_t) - c_t - x_t - g_t\right] + \sum_{t=0}^{\infty} \eta_t \left[(1-\delta)k_t + x_t - k_{t+1}\right]$$

where

$$= \sum_{t=0}^{\infty} \beta^t \Big[\underbrace{u(c_t, 1-n_t) - \lambda(u'_c(t)c_t - u'_n(t)n_t)}_{B(c_t, n_t, \lambda)}\Big]$$

FOC :-

$c: \quad \beta^t B'_c(t) = \gamma_t$

$n: \quad \beta^t B'_n(t) = \gamma_t F'_n(t)$

$x: \quad -\gamma_t + \eta_t = 0$

$k_{t+1}: \quad -\eta_t + \eta_{t+1}(1-\delta) + \gamma_{t+1}(F'_k(t+1)) = 0$

$$\frac{B'_n(t)}{B'_c(t)} = F'_n(t)$$

$$\frac{\eta_t}{\eta_{t+1}} = F'_k(t+1) + 1 - \delta$$

$$\frac{B'_c(t)}{\beta B'_c(t+1)} = F'_k(t+1) + 1 - \delta$$

Assumption :-

$c_t^{RP} \longrightarrow c_\infty^{RP}$
$x_t^{RP} \longrightarrow x_\infty^{RP}$
$n_t^{RP} \longrightarrow n_\infty^{RP}$
$l_t^{RP} \longrightarrow l_\infty^{RP}$

(Convergence to steady state)

a constant level

There is a path to the steady state.

(But it does not mean this is the only unique steady state).

As $t \to \infty$

$$\frac{B'_c(\infty)}{B'_n(\infty)} = F'_n(\infty)$$

$$\frac{1}{\beta} = 1 - \delta + F'_k(\infty)$$

TDCE Euler eq (given FP $\{\tau_{nt}, \tau_{kt}, g_t\}$)

$$u'_c(t) = \beta u'_c(t+1)\left[(1-\tau_{kt+1}) F'_k(t+1) + 1 - \delta\right]$$

As $t \to \infty$

$$\frac{u'_c(\infty)}{\beta u'_c(\infty)} = (1 - \tau_{k\infty}) F'_k(\infty) + 1 - \delta$$

$$\Leftrightarrow \boxed{\frac{1}{\beta} = (1 - \tau_{k\infty}) F'_k(\infty) + 1 - \delta}$$

$$\Rightarrow \boxed{\tau_{k\infty} = 0}$$

Concepts.

1. Euler Equation