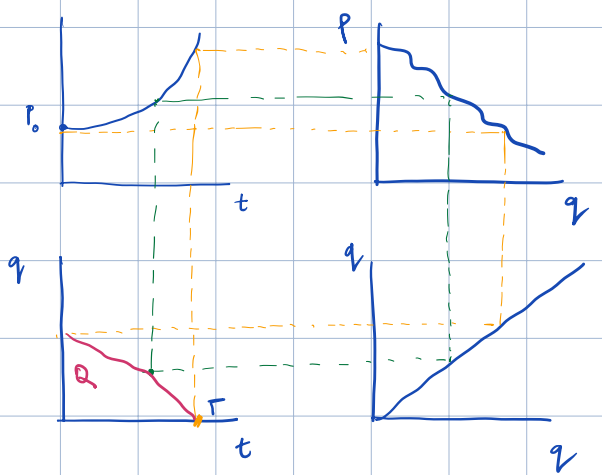
January 25, II

Hotelling's Rule.

For pricing of nature resources.

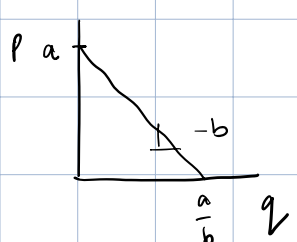
$\frac{\dot{P}}{P} = r$; $\dot{P} = \frac{dP}{dt}$, P = prices.

$P(t) = P_0 e^{rt}$



- At T oil runs out.

$P = a - bq$ *



$Q = \int_0^T q(t) \cdot dt$

$P_0 e^{rT} = a$

$T = \frac{1}{r} \log \frac{a}{P_0}$

Rearranging *, $q = \frac{1}{b}(a - P)$

$$Q = \frac{1}{b}\int_0^T (a - P_0 e^{rt})\,dt = \frac{1}{b}\left[at - \frac{P_0}{r}e^{rt}\right]_0^T$$

$$Q = \frac{1}{b}\left[aT - \frac{P_0}{r}e^{rT} + \frac{P_0}{r}\right]$$

$$= \frac{1}{b}\left[\frac{a}{r}\ln\frac{a}{P_0} - \frac{a}{r} + \frac{P_0}{r}\right]$$

$$\frac{dQ}{dP_0} = \frac{1}{b}\left[\frac{-a}{rP_0} + \frac{1}{r}\right] \quad (?) \quad < 0$$

* Trees :

$s(t) \rightarrow$ Size of the tree. Always increasing? (In cubic meter)

$P_w \rightarrow$ fixed ($/cubic meter)

$\Pi = P_w s(t) e^{-rt}$ → use this for PV in continuous time

(Present value of the chopping down the one tree I have)

$\max_t \ln P_w + \ln s(t) - rt$ ($\ln P_w$ is a constant, so we can drop it)

FOC:

$$\frac{s'(t)}{s(t)} = r$$

This is like the Hotelling's rule

* Read about log concavity

If the tree is growing faster than r → chop it down later & vice-versa.

(*) Allowing for replanting :-

$$\max_t \pi = P_w s(t) \left[e^{-rt} + e^{-2rt} + e^{-3rt} + \dots \right]$$

($e^{-rt} = x$)

$$\pi = P_w s \left[x + x^2 + x^3 + \dots \right] \quad *$$

$$x\pi = P_w s \left[x^2 + x^3 + \dots \right] \quad **$$

$$\pi = P_w s \frac{x}{1-x} \qquad (* - **)$$

$e^{-rt} = x$.

$-rt = \log x$.

$t = -\frac{\log x}{r}$

$$\ln \pi = \ln P_w + \ln s - rt - \ln(1-x)$$

$$\frac{d \ln \pi}{dt} = \frac{s'}{s} - r - \frac{rx}{1-x} = 0 \qquad (?)$$

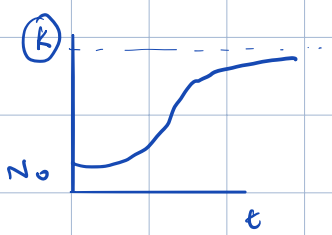
↳ chop down earlier from last problem. shows the O.C. of new trees for the future generations.

* Fish

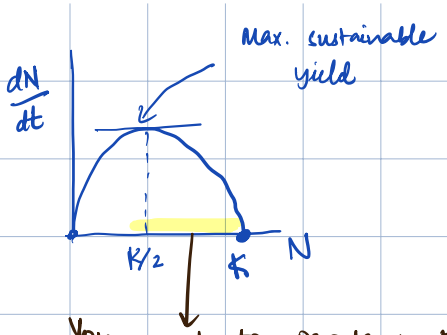
$N(t)$ → size of the fisheries

$$N(t) = \frac{K}{1 + \frac{K - N_0}{N_0} e^{-gt}}$$

g_t → exogeneous growth rate of the population.



$$\frac{dN}{dt} = gN\left(1 - \frac{N}{K}\right)$$



You want to operate on this part.

MC = MB