# Final Exam - Econ 7010

November 21, 2022

**Instructions:** Answer each question in your bluebook, or on your own paper that is stapled and *neatly organized.* If you are using loose paper, make sure to staple the pages together before turning them in.

There are three questions worth 25, 30, and 20 points respectively. Your answers to all questions must be fully justified to receive credit. If you cannot answer a question completely, a well-labeled diagram and/or clear explanation of the intuition behind the solution process can obtain partial credit. The **clarity and brevity** of yours answers is important: I should not have to (and will not) "search" for your answers to any question, and extraneous and/or irrelevant information will lose points.

You have 75 minutes to complete the exam. The total number of points is 75. Budget your time wisely; you may find later questions easier than earlier ones.

Good luck!

## Question 1 (25 points)

Recall the following definitions:

- A good is **regular** if $x_i(p, m)$ is decreasing in $p_i$. A good is **Giffen** if $x_i(p, m)$ is increasing in $p_i$.
- A good is **normal** if $x_i(p, m)$ is increasing in $m$. A good is **inferior** if $x_i(p, m)$ is decreasing in $m$.
- Good $i$ is a **substitute for good** $k$ if $h_i(p, u)$ is increasing in $p_k$. Good $i$ is a **complement for good** $k$ if $h_i(p, u)$ is decreasing in $p_k$.
- Good $i$ is a **gross substitute for good** $j$ if $x_i(p, m)$ is increasing in $p_k$. Good $i$ is a **gross compliment for good** $k$ if $x_i(p, m)$ is decreasing in $p_k$.

Also, recall the Slutsky equation:

$$\frac{\partial x_i(p,m)}{\partial p_k} = \frac{\partial h_i(p, v(p,m))}{\partial p_k} - \frac{\partial x_i(p,m)}{\partial m} x_k(p,m)$$

In answering each of the questions below, use the Slutsky equation to prove your answer.

(a) (5 points) Explain the Slutsky equation in words.

(b) (5 points) Fill in the blank with one of the above terms: A normal good must be ___________.

(c) (5 points) Fill in the blank with one of the above terms: If good $i$ is a gross complement for good $k$ and good $i$ is inferior, then good $i$ is a ___________ for good $k$.

(d) (5 points) Assume that good $i$ is a substitute for good $k$. Is good $k$ also a substitute for good $i$?

(e) (5 points) Assume that good $i$ is a gross substitute for good $k$. Is good $k$ also a gross substitute for good $i$?

## Answer

**Part (a)**

The Slutsky equation is

$$\frac{\partial x_i(p,m)}{\partial p_k} = \frac{\partial h_i(p, v(p,m))}{\partial p_k} - \frac{\partial x_i(p,m)}{\partial m} x_k(p,m)$$

The LHS is the total effect. On the RHS, the first term is the substitution effect, and the second term is the income effect.

**Part (b)**

A normal good has $\partial x_i/\partial m \geq 0$. Since $\partial h_i/\partial p_i \leq 0$, by the Slutsky equation, both terms are negative, and so $\partial x_i/\partial p_i \leq 0$, i.e., a normal good must be **regular**.

**Part (c)**

If good $i$ is a gross complement for good $k$, then $\partial x_i/\partial p_k \leq 0$, and if good $i$ is inferior, then $\partial x_i/\partial m \leq 0$. Rearranging the Slutsky equation, we have

$$\frac{\partial h_i(p, v(p,m))}{\partial p_k} = -\frac{\partial x_i(p,m)}{\partial p_k} - \frac{\partial x_i(p,m)}{\partial m} x_k(p,m).$$

Both terms on the RHS are positive, and so $\partial h_i/\partial p_k \geq 0$, i.e., $i$ is a **complement** for $k$.

**Part (d)**

Yes. By the symmetry of the substitution matrix, $\partial h_i/\partial p_k = \partial h_k/\partial p_i$. Since $i$ is a substitute for $k$, $\partial h_i/\partial p_k = \partial h_k/\partial p_i \geq 0$, and so $k$ is also a substitute for $i$.

**Part (e)**

No. We have

$$\frac{\partial x_i(p,m)}{\partial p_k} = \frac{\partial h_i(p,v(p,m))}{\partial p_k} - \frac{\partial x_i(p,m)}{\partial m} x_k(p,m) \geq 0.$$

However, flipping $i$ and $k$, we have

$$\frac{\partial x_k(p,m)}{\partial p_i} = \frac{\partial h_k(p,v(p,m))}{\partial p_i} - \frac{\partial x_k(p,m)}{\partial m} x_i(p,m).$$

While the substitution terms are symmetric, in general the income effect terms can be different:

$$\frac{\partial x_i(p,m)}{\partial m} x_k(p,m) \neq \frac{\partial x_k(p,m)}{\partial m} x_i(p,m).$$

So, knowing that $\partial x_i/\partial p_k \geq 0$ does not allow us to conclude that $\partial x_k/\partial p_i \geq 0$.

## Question 2 (30 points)

Consider a consumer with the following Marshallian demand functions and indirect utility function:

$$x_1(p,m) = \frac{m}{p_1 + 5p_2} \qquad x_2(p,m) = \frac{5m}{p_1 + 5p_2} \qquad v(p,m) = \frac{5m}{p_1 + 5p_2}$$

(a) (6 points) Find the consumer's Hicksian demand functions (hint: first find the expenditure function using the duality identities).

(b) (6 points) Consider a price change from $(p_1, p_2) = (1,1)$ to $(p_1', p_2') = (5,1)$. Let the consumer have wealth $m = 60$. Decompose the change in demand for good 1 into an income effect and a substitution effect.

(c) (6 points) Calculate the equivalent variation for the price change from (b).

(d) (6 points) Calculate the compensating variation for the price change from (b).

(e) (6 points) Draw a sketch with $p_1$ on the vertical axis and demand for good 1 on the horizontal axis. On your sketch, include both the Hicksian and Marshallian demands, and use your sketch to graphically illustrate both CV and EV, as well as Consumer Surplus (CS).

## Answer

**Part (a)**

To find the Hicksian demands, we can use the duality identities. First, find the expenditure function $v(p, e(p,u)) = u$:

$$\frac{5e(p,u)}{p_1 + 5p_2} = u \implies e(p,u) = \frac{u}{5}(p_1 + 5p_2)$$

Then, using Shepard's Lemma, we have

$$h_1(p,u) = \frac{\partial e}{\partial p_1} = u/5 \qquad h_2(p,u) = \frac{\partial e}{\partial p_2} = u$$

**Part (b)**

Before the price change, $x_1 = 10$ and utility is $u_1 = 50$, while after $x_1' = 6$ and $u_1' = 30$. To achieve utility of 50 at the new prices, we need to solve:

$$50 = \frac{5m'}{5+5(1)} \implies m' = 100$$

At $(p_1', p_2') = (5, 1)$ and $m' = 100$, the consumer's demand for good 1 is $x_1'' = 10$. This implies that the substitution effect is 0, and the total effect is equal to the income effect: $IE = TE = -4$.

**Part (c)**

The equivalent variation is $e(p, u') - e(p, u)$. Using the equation for the expenditure function from (a) and $u = 50$ and $u' = 30$ from (b), we see this becomes

$$e(p, u') - e(p, u) = (30/5) \times (1 + 5) - (50/5) \times (1 + 5) = -24$$

**Part (d)**

The compensating variation is $e(p', u') - e(p', u)$. This becomes

$$e(p', u') - e(p', u) = 60 - 50/5 \times (5 + 5) = -40$$

**Part (e)**

This is just like the graph from lecture, except notice that the Hicksian demand curves should be vertical (they don't depend on $p_1$). EV is the area to the left of the Hicksian demand at $u' = 30$, while CV is the area to the left of the Hicksian demand at $u = 50$.

## Question 3 (20 points)

Consider a farmer who must decide the quantity of wheat to produce next year, $q$. The market price that he can sell his wheat for is $p$ per unit. There is also probability $\theta$ that there will be a drought next year. If there is not a drought, then producing $q$ costs $c(q)$, where $c(q)$ is strictly increasing and strictly convex. If there is a drought, then the farmer must invest in additional water resources, and his total cost to produce $q$ is $c(q) + rq$, where $r > 0$ is the per-unit cost of additional water. The farmer must make his production decision before the realization of uncertainty regarding the weather.

(a) (5 points) Assume that the farmer wants to maximize expected profit, and let $q^*$ denote the farmer's optimal choice. Write down his objective function. Write an equation that characterizes $q^*$, and interpret.

(b) (5 points) Find an equation for $dq^*/d\theta$, and interpret.

(c) (5 points) Assume instead that the farmer is risk-averse, and has a utility function over profits $u(\cdot)$ that is increasing and strictly concave. His goal is to maximize the *expected utility* of profit. Let $q^{**}$ denote the optimal solution to this problem. Write down the farmer's objective. Write an equation that characterizes $q^{**}$, and interpret.

(d) (5 points) Which is greater, $q^*$ or $q^{**}$? Both justify your answer mathematically, and provide intuition.

## Answer

**Part (a)**

The firm wants to maximize

$$\max_q \theta(pq - c(q) - rq) + (1 - \theta)(pq - c(q)) = pq - c(q) - \theta rq$$

The FOC is

$$c'(q^*) = p - \theta r$$

This has the standard interpretation that marginal benefit equals (expected) marginal cost.

**Part (b)**

From the implicit function theorem:

$$\frac{dq^*}{d\theta} = -\frac{r}{c''(q^*)}.$$

Since $c'' > 0$, we have $dq^*/d\theta > 0$: if the probability of a failure goes up, the firm chooses to produce less ex-ante.

**Part (c)**

Now, the maximization problem is

$$\max_q \theta u(pq - c(q) - rq) + (1 - \theta)u(pq - c(q)).$$

The FOC in this case is

$$\theta u'(pq^{**} - c(q^{**}) - rq^{**}) \times (p - c'(q^{**}) - r) + (1 - \theta)u'(pq^{**} - c(q^{**})) \times (p - c'(q^{**})) = 0.$$

**Part (d)**

Evaluate the LHS of part (b) at the optimal $q^*$ from part (a) to get:

$$\begin{aligned}
&\theta u'(pq^* - c(q^*) - rq^*) \times (p - (p - \theta r) - r) + (1 - \theta)u'(pq^* - c(q^*)) \times (p - (p - \theta r)) \\
=&r\theta(\theta - 1) \times [u'(pq^* - c(q^*) - rq^*) - u'(pq^* - c(q^*)] \\
<&0
\end{aligned}$$

where the inequality follows because $\theta < 1$ and $u'$ is decreasing, which implies the term in brackets is positive. Therefore, $q^{**} < q^*$.