Econ 7040: Assignment #3
Spring 2024

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Due Monday, February 26, 2024
Instructions: Type all answers in LaTeX

Consider a perfect foresight, constant endowment economy in which the government seeks to optimally finance a given sequence of government purchases, $\{g_t\}$, using a mix of costly inflation, π_t, and taxes, τ_t. Treat $\{g_t\}$ as the source of uncertainty, with purchases a stationary stochastic process. The government's objective function is

$$-\frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t\left[\tau_t^2+\theta(\nu_t-1)^2\right] \tag{1}$$

where $\nu_t \equiv \frac{1}{\pi_t}$ is the inverse of the gross inflation rate. The bliss point is zero taxes and constant prices.

The economy's aggregate resource constraint is $c_t + g_t = y$. We imagine that individuals in the economy have preferences $u(c_t) = c_t$, so they are risk-neutral; hence, the stochastic discount factor in (2) is constant.

The government maximizes (1) subject to a bond-pricing condition and a government budget identity[1]

$$\text{bond pricing:} \qquad 1 = \beta E_t\left(\frac{\nu_{t+1}}{Q_t}\right) \tag{2}$$

$$\text{budget identity:} \qquad Q_t b_t = b_{t-1}\nu_t + g_t - \tau_t \tag{3}$$

where $b_t = B_t/P_t$ is the nominal bond stock deflated by the price level and Q_t is the price of the bond portfolio. All bonds mature in one period. $b_{-1} \geq 0$ is given.

Pay careful attention to where the expectations operator does *and does not* belong. This difference is central to the questions posed.

1. Describe *in words* what the objective function implies about social welfare and why (2) is a constraint on the government's problem.

2. Write down the lagrangian to maximize (1) subject to (2) and (3), letting μ_t be the multiplier on (2) and λ_t be the multiplier on (3).

3. Derive the first order conditions with respect to $\{\tau_t, \nu_t, Q_t, b_t, \mu_t, \lambda_t\}$, for $t = 0, 1, 2, \ldots$, solve for the multipliers, and substitute them into the first-order condition for ν_t. You should get an expression that involves $(\nu_t, \tau_{t-1}, \tau_t, b_{t-1})$. Notice that for $t = 0$ the expression is different from in subsequent periods, $t \geq 1$. Explain why they are different and what the difference implies for optimal policies at $t = 0$ and $t \geq 1$.

4. Consider the special case in which inflation becomes infinitely costly, $\theta \to \infty$. Derive

[1] Government policies must also satisfy a transversality condition for government bonds.

(a) the optimal rate of ν_t

(b) the optimal evolution of taxes, τ_t

(c) an expression for the optimal setting of the present value of taxes

5. Now consider the opposite extreme in which inflation is costless, so $\theta = 0$. **For this question, assume government purchases obey $g_t = \bar{g}$.** Derive

   (a) the optimal evolution of taxes, τ_t

   (b) an expression for optimal ν_t

   (c) explain how an increase in $\bar{g}$ is financed

* Surprise increase in the price level.

Non-surprise inflation would get priced in

$$\theta_t = \beta E_t v_{t+1} \qquad (2)$$

$$\theta_t b_t + \tau_t = \underbrace{b_{t-1} v_t}_{\text{This is the surprise inflation}} + g_t \qquad (3)$$

FOCs:—

$$\theta(v_0 - 1) = -\tau_0 b_{-1} \qquad (4)$$

$$\theta(v_t - 1) = (\tau_{t-1} - \tau_t) b_{t-1}, \quad t \geq 1 \qquad (5)$$

What is μ_{-1}? undefined or zero

$\mu_{-1} = 0$, then get (4)

Optimal policy fcts map state to policy variables (this is like D.P.)

Reoptimized at $t = 1$:

(4) applies at $t = 1$

(5) applies at $t \geq 2$

Optimal plans are time-inconsistent (i.e. if you permit reoptimization at a later date, policy rules are different.

* Time inconsistency happens because of govt's incentives.

* Optimal plans are chosen at $t=0$ create history dependence $\{\tau_t, r_t$ depend on $\tau_{t-1}\}$

* Time consistent policies are not history dependent.

Endogeneous state variables give time create persistence in time series.

* Commitment means at $t=0$, you make one policy for $t=\infty$. Allows govts. to choose current policy as well as expectations of future policy.

→ commitment allows for policy surprises.

Because g is stochastic. Govt. commits to a policy function given expectation of g.

→ Commitment is the benchmark policy because it maximizes social welfare.

* Much of the optimal monetary policy literature concludes that $\pi_t = \pi^*$ is optimal. (strict infl. targeting)

  ⇒ $\theta \to \infty$ this holds

  → π is very costly

  ⇒ everything is financed by taxes, tax distortions are not costly.

* Brings out the r/p b/w monetary & fiscal policy. (Leeper has focused very much on this. His dissatisfaction that taxes are considered benign (usually a lump-sum tax.)).

Q: How do we compare what is more distortionary?