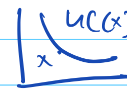
Oct 11, 2023

From conditions on preferences to conditions on $u(\cdot)$:—

1) $\succsim$ convex $\Rightarrow \exists$ a quasiconcave $u(\cdot)$

Proof : $\succsim$ convex means
$UC(x) = \{y,\ y \succsim x\}$ is convex



But $y \succsim x$ is equivalent to $u(y) \geq u(x)$
$UC(x) = \{y,\ u(y) \geq u(x)\}$ is convex.

This is (one) def^n of quasiconcave $u(\cdot)$.

Also true $\succsim$ strictly convex $\Rightarrow \exists$ a $u(\cdot)$ strictly quasiconcave. (Not equivalent to concave)

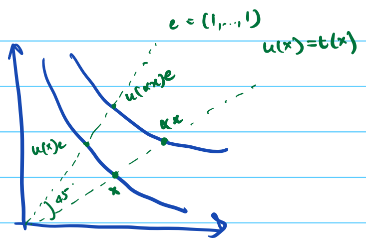
Not true: convex $\succsim \;\Rightarrow \exists$ $u(\cdot)$ concave

2) $\succsim$ homothetic (& monotone)
(homotheticity $\Rightarrow$ monotonocity)

$\Rightarrow \exists$ an hd-1 f^n $u(\cdot)$ that represents $\succsim$

Proof :—

Let $\succsim$ be homothetic & monotone



$u(x)e \sim x$ & $u(\alpha x)e \sim \alpha x$

$\alpha u(x)e \sim \alpha x$ (homotheticity)

$\overbrace{u(\alpha x)e}^{n} \sim \overbrace{\alpha u(x)e}^{m}$ (transitivity)

$u(\alpha x) = \alpha u(x)$ (monotonicity)

Classic homothetic utility : cobb douglas

$$u(x) = x_1^{\alpha_1} x_2^{\alpha_2} \cdots\cdots x_n^{\alpha_n}$$

It is wlog to assume $\alpha_1 + \ldots + \alpha_n = 1$

why? say $\alpha_1 + \ldots + \alpha_n = k$

Consider

$$\tilde{u}(x) = (u(x))^{1/k}$$

u & $\tilde{u}$ represent same preferences, & $\tilde{u}(x) = x_1, \ldots \ldots x_n$

$$\left(\frac{\alpha_1}{k} + \ldots + \frac{\alpha_n}{k}\right) = \frac{1}{k}(\alpha_1 + \ldots + \alpha_n) = 1$$

* CD is a special case of more general class :- Constant Elasticity of substitution (CES)

$$u(x) = [\alpha_1 x_1^{\delta} + \ldots\ldots + \alpha_n x_n^{\delta}]^{1/\delta}$$

Can check this is hd-1

special cases

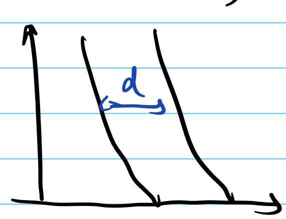
$\rho = 1$

$\rho \to 0$

$\rho \to -\infty$

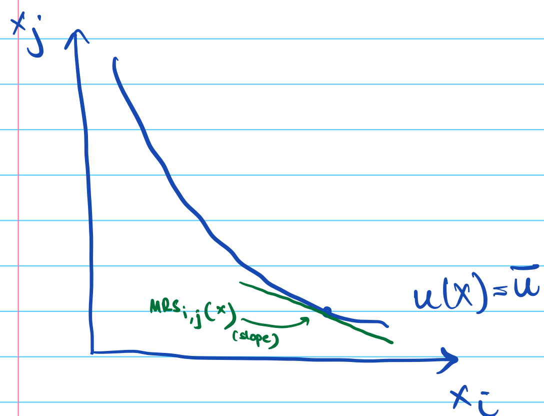
(3) $\succsim$ Quasilinear (in x_1) $\Rightarrow \exists$ a $u(\cdot)$ of the form

$$u(x_1, \ldots, x_n) = x_1 + v(x_2, \ldots, x_n)$$



(Income Effect)

Marginal Rate of Substitution



$$u(x) = \bar{u}$$

$$\frac{\partial u}{\partial x_i} dx_i + \frac{\partial u}{\partial x_j} dx_j = d\bar{u} = 0$$

(along the IC)

$$\Rightarrow \frac{dx_j}{dx_i} = \frac{-\partial u/\partial x_i}{\partial u/\partial x_j}$$

$$MRS_{ij}(x) = \frac{\partial u/\partial x_i}{\partial u/\partial x_j}$$

Budget sets :—

Consumption space $X = \mathbb{R}^n_+$
Standard (Walrasian) budget set :

$$B_{p,m} = \{ x \in X : p.x \leq m \}$$

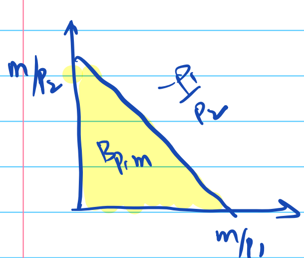
prices ↙ ↘ Income

$p=(p_1,\dots,p_n)$

In 2-D :

$$\{ x \in \mathbb{R}^2 : P_1 x_1 + P_2 x_2 \leq m \}$$

$$x_2 \leq \frac{m}{P_2} - \frac{P_1}{P_2} x_1$$



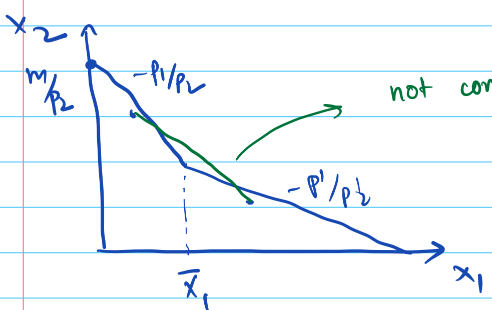
Walrasian budget set is convex.

Non-convex budget sets come up all the time.

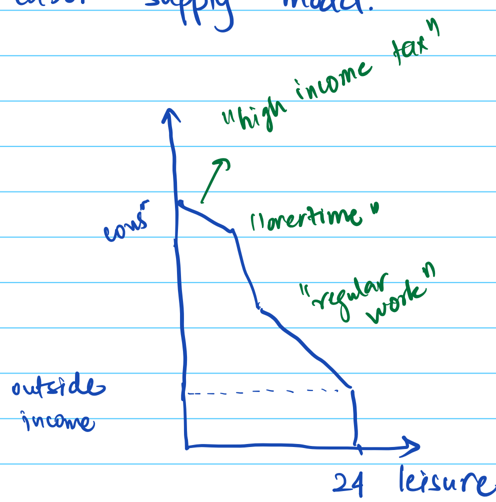
- Progressive taxation
- Welfare payments
- Non linear pricing

Non-linear pricing.

Discount on x_1 if you purchase $\geq \bar{x}_1$ units.



Labor supply model.



"Classic" Utility Maximization Problem (UMP)

$$\max_{\substack{x_i \geq 0 \\ p.x \leq m}} u(x)$$

Solution functions:

$$x(p,m) = \arg\max_{\substack{x_i \geq 0 \\ p.x \leq m}} u(x)$$ (Marshallian) Demand Correspondence

$$v(p,m) = \max_{\substack{x_i \geq 0 \\ p.x \leq m}} u(x)$$ Indirect utility function

1st question: existence?

Theorem: (i) If $p \gg 0$, & $u(\cdot)$ is continuous, then the ump has a solution.

(ii) If in addition $\succsim$ are locally unsatiated then $p.x^* = m \;\; \forall \; x^* \in x(p,m)$ (Walras Law)

Proof of (i):

$p \gg 0$ ensures $B_{p,m}$ is compact (closed & bounded)
Continuous f^n on compact sets have a maximum.

Proof of (ii)

Assume $p.x^* < m$
Then x^* is in the interior of $B_{p,m}$.
By LNS, $\exists$ a $y \in B_{p,m}$ s.t. $u(y) > u(x^*)$
This contradicts that x^* is optimal.