Midterm Exam - Econ 7010

October 17, 2022

Instructions: Answer each question in your bluebook, or on your own paper that is stapled and *neatly organized.* If you are using loose paper, make sure to staple the pages together before turning them in.

The **clarity and brevity** of yours answers is important: I should not have to (and will not) “search” for your answers to any question, and extraneous and/or irrelevant information will lose points.

There are two questions with varying numbers of parts. Your answers to all questions must be fully justified to receive credit. If you cannot answer a question completely, a well-labeled diagram and/or clear explanation of the intuition behind the solution process can obtain partial credit.

You have 75 minutes to complete the exam. The total number of points is 75. Budget your time wisely; you may find later questions easier than earlier ones.

Good luck!

Question 1 (40 points)

A firm uses labor L and capital K to produce goods according to the production function $f(L, K)$ to maximize profits:

$$\max_{L,K} pf(L, K) - wL - rK$$

where w is price of labor (wage) and r is the price (or rental rate) of capital. Assume that $f_L, f_K > 0$, $f_{LL}, f_{KK} < 0$, and, for now, $f_{KL} = f_{LK} < 0$, where subscripts denote partial derivatives. You may assume that all solutions are interior.

Suppose that the government enacts a law that increases the wage w.

(a) (6 points) Suppose capital is a manufacturing plant and so K is fixed at $\bar{K}$. How does L change in response to the wage increase? Derive an equation for dL^*/dw, and determine its sign.

(b) (10 points) Let $L^{SR*}(w)$ denote the firm's optimal labor demand in the short-run (when capital is fixed at $\bar{K}$) and $L^{LR*}(w)$ and $K^{LR*}(w)$ denote the firm's optimal demand for labor in the long-run, when L and K are allowed to vary. Prove that

$$\left| \frac{dL^{LR*}(w)}{dw}\Big|_{w=\bar{w}} \right| \geq \left| \frac{dL^{SR*}(w)}{dw}\Big|_{w=\bar{w}} \right|$$

where $\bar{w}$ is such that $K^{LR*}(\bar{w}) = \bar{K}$.

(c) (6 points) Suppose instead that labor laws restrict firing, so that now L is fixed at $\bar{L}$, and instead now only capital is variable. How does K change in response to the wage increase? Derive an equation for dK^*/dw, and determine its sign.

Assume now that both L and K are variable.

(d) (6 points) Write out one more assumption on the second-order derivatives of f that suffices to satisfy the second-order conditions of the maximization problem.

(e) (8 points) Assume the condition you found in (d) holds. How do L and K respond in response to the wage increase? Derive equations for both dL^*/dw and dK^*/dw.

(f) (4 points) How would your answers to all parts change if $f_{KL} = f_{LK} > 0$?

Question 2 is on the next page.

Question 2 (35 points)

(i) (10 points) Consider a single-output production function $f(x)$ and let $Y = \{(-x, y) : x \geq 0 \text{ and } y \leq f(x)\}$ be the associated production set. Prove that the following two statements are equivalent:

(1) $y \in Y$ implies $\alpha y \in Y$ for all $\alpha \in [0, 1]$

(2) $f(tx) \leq tf(x)$ for all $t \geq 1$.

(ii) (25 points total, 5 for each part) For each of the following production sets and/or production functions, determine whether they have non-increasing returns to scale, non-decreasing returns to scale, both (i.e., constant returns to scale), or neither.

(a) $f(x) = x_1^{1/4} x_2^{1/4} x_3^{1/4}$

(b) $Y = \{(y_1, y_2) \in \mathbb{R}^2 : y_1, y_2 \leq 0 \text{ or } y_2 \leq -y_1 - 3\}$ (sketching a picture might be helpful, though for full credit you must prove your answer formally)

(c) $f(x) = \max\{\min\{x_1, x_2 - k\}, 0\}$ for $k > 0$.

(d) $Y = \{(y_1, y_2, y_3) \in \mathbb{R} \times (-\infty, 0] \times \{0, -1\} : -y_1 \geq y_3\sqrt{-y_2}\}$

(e) $Y = \{(-x_1, -x_2, y_1, y_2) \in \mathbb{R}^4 : x_1 \geq 0, x_2 \geq 0, \frac{y_1^2 + y_2^2}{2} \leq x_1^\alpha x_2^\beta\}$. For this problem, determine values of α, β such that the function exhibits non-decreasing returns to scale.