Nov 14, 2023

Today :—

- Global Dynamics in NGM
- Intro to NGM with uncertainty

last class :—

Fiscal Policy in Model w/ Human Capital

[EEK] $$\left(\frac{c_{t+1}}{c_t}\right)^{\sigma} = \beta\left[1-\delta_k + (1-\tau_{kt})F'_k(t+1)\right]$$

[EEH] $$\left(\frac{c_{t+1}}{c_t}\right)^{\sigma} = \beta\left[1-\delta_h + (1-\tau_{ht})F'_h(t+1)\right]$$

Case 1:

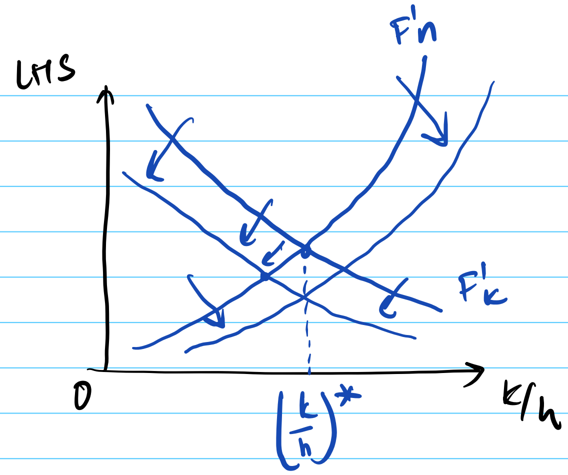
$\delta_k = \delta_h$, $\tau_{ht} = \tau_{kt} = \tau$

$k = \phi h$, growth rate $r^c_{t,t+1} \downarrow$ as $\tau \uparrow$

Case 2:

$\delta_k = \delta_h$, $\tau_{kt} = \tau_{kt+1} = \tau_k$
$\tau_{ht} = \tau_{ht+1} = \tau_h$

$$(1-\tau_k)F'_k = (1-\tau_h)F'_h$$



Effect on $\left(\frac{k}{h}\right)^*$ is ambiguous

rog of cons^m will fall if tax increases.

$\boxed{\text{Global Dynamics in NGM}}$

$$v(k) = \max_{0 \le k' \le f(k)} \left[u\left(f(k) - k'\right) + \beta v(k')\right]$$

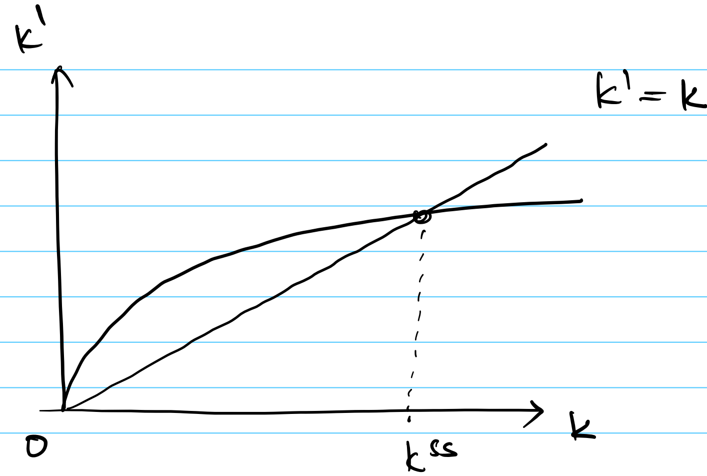
k_0 is given

Remark :

If $u(.)$ is strictly increasing & strictly concave,

If f is strictly increasing & strictly quasi-concave

$\Rightarrow$ $v(k)$ is strictly increasing & strictly concave.



$F(0, \bar{n}) = 0$

To ask derivative of $u(1-n)$?

it will have a negative sign while doing the derivatives.

FOC:

wrt k' :

$$-u'(f(k) - k') + \beta V'_k(k') = 0$$

$$u'(f(k) - g(k)) = \beta V'(g(k))$$

Env

$f(x, \alpha) \longrightarrow \max_x$

$f'_x = 0$

Given α, the optimal x that would maximize f

$$V(\alpha) = f(x^*(\alpha), \alpha)$$

$$\frac{\partial V}{\partial \alpha} = \underbrace{f'_x}_{} \cdot \frac{\partial x^*}{\partial \alpha} + \frac{\partial f}{\partial \alpha}$$

0 from foc as already at the optimal

$$\boxed{\frac{\partial V}{\partial \alpha} = \frac{\partial f}{\partial \alpha}}$$

Bellman:

$$v(k) = \max_{k'} [u(f(k) - k') + \beta v(k')]$$

from Envelope theorem :-

$$= u'(f(k) - k') f'(k)$$

$$v(k) = u[f(k) - g(k)) + \beta v(g(k))$$

$$v'(k) = u'(f(k) - g(k))(f'(k) - g'(k))$$
$$+ \beta v'(g(k)) \cdot g'(k)$$

Recall FOC:

$$u'(f(k) - g(k)) = \beta v'(g(k))$$

$$v'(k) = u'(f(k) - g(k))(f'(k) - g'(k)) +$$
$$+ u'(f(k) - g(k)) g'(k)$$
$$= u'(f(k) - g(k)) f'(k)$$

Theorem:

Let $T(z)$ be a strictly concave function.
(i.e. $T'' < 0$)

$\Rightarrow$ $[T'(z) - T'(z')](z - z') \leq 0$
with $=$ iff $z = z'$

Proof:

$z < z' \Rightarrow T'(z) > T'(z')$
$z > z' \Rightarrow T'(z) < T'(z')$
$z = z' \Rightarrow =$

$[v'(k) - v'(g(k))](k - g(k)) \leq 0$ with $=$ if $k = g(k)$

From: $v'(k) = u'(f(k) - g(k)) \cdot f'(k)$ (envelope)
$v'(g(k)) = \frac{1}{\beta} u'(f(k) - g(k))$ (FOC)

$[u'(f(k) - g(k)) f'(k) - \frac{1}{\beta} u'(f(k) - g(k))](k - g(k)) \leq 0$

$= \left[f'(k) - \frac{1}{\beta}\right](k - g(k)) \leq 0$

If $f'(k) = \frac{1}{\beta} \longrightarrow k_{ss}$

At steady state $k_{ss} = g(k_{ss})$ (It's an iff statement)

If $f'_k > \frac{1}{\beta} \Rightarrow k < g(k)$

$$\Rightarrow k < g(k) < g(k_{ss}) = k_{ss}$$

$$g(k) \in (k, k_{ss})$$

If $f'_k < \frac{1}{\beta} \Rightarrow k > k_{ss}$

$$k > g(k) > g(k_{ss}) = k_{ss}$$

$$g(k) \in (k_{ss}, k)$$

$\Rightarrow$ The steady state will be globally stable.

* Criticism of NGM:

→ Temporary growth.

→ very short pd. of growth

} Both violates the data.