Econ 7040: Assignment #1
Permanent Income Hypothesis
Spring 2024
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Answer Key

1. Plugging the income process $Y_t = Y_1 + \nu_t$ into the consumption function:

$$C_t = (1 - \beta) \sum_{j=0}^{\infty} \beta^j \mathbb{E}_t[Y_{t+j}] + rA_{t-1}$$

we get:

$$C_t = (1 - \beta) \sum_{j=0}^{\infty} \beta^j \mathbb{E}_t[Y_1 + \nu_{t+j}] + rA_{t-1}$$

$$= Y_1 + (1 - \beta) \sum_{j=0}^{\infty} \beta^j \mathbb{E}_t[\nu_{t+j}] + rA_{t-1}$$

Then use the fact that $\nu_t = \phi\nu_{t-1} + \varepsilon_t$ with $\mathbb{E}_t[\varepsilon_{t+j}] = 0$, $\forall j \geq 1$ to write:

$$\mathbb{E}_t[\nu_{t+j}] = \phi^j \nu_t$$

Combining this with the above we obtain:

$$C_t = Y_1 + (1 - \beta) \sum_{j=0}^{\infty} (\phi\beta)^j \nu_t + rA_{t-1}$$

which simplifies to

$$C_t = Y_1 + \frac{1 - \beta}{1 - \phi\beta} v_t + rA_{t-1} \tag{1}$$

2. Combining the definition of savings $S_t = Y_t - C_t + rA_{t-1}$ with $Y_t = Y_1 + \nu_t$ and our result in (1), we can write:

$$S_t = Y_1 + \nu_t - Y_1 - \frac{1 - \beta}{1 - \phi\beta} v_t - rA_{t-1} + rA_{t-1}$$

Cancelling out terms and simplifying the above reduces to:

$$S_t = \beta \frac{1 - \phi}{1 - \phi\beta} \nu_t \tag{2}$$

3. Since $Y_t = Y_1 + \nu_t$ we can interpret Y_1 as the permanent component and ν_t as the transitory component. Then, we can use our results for equilibrium consumption and savings (1) and (2) to compute marginal propensities to consume/save with respect to permanent income, transitory income, and assets:

$$\frac{dC_t}{dY_1} = 1 \quad ; \quad \frac{dS_t}{dY_1} = 0$$

$$\frac{dC_t}{d\nu_t} = \frac{1-\beta}{1-\phi\beta} \quad ; \quad \frac{dS_t}{d\nu_t} = \beta\frac{1-\phi}{1-\phi\beta}$$

$$\frac{dC_t}{dA_{t-1}} = r \quad ; \quad \frac{dS_t}{dA_{t-1}} = 0$$

The general interpretation here is that agents want to smooth their consumption across time.

When the increase in income is known to be permanent, agents will permanently shift up their consumption and leave their savings unchanged. They fully smooth consumption by consuming all the extra income every period and leaving saving and asset holdings unchanged.

When the increase is temporary, agents will increase their consumption by less than one-to-one and save the rest of the higher income to smooth out the path of future consumption. Higher saving increases asset holdings, the return from which financing permanently higher consumption. How much of the increase in income agents consume/save will depend on how impatient they are (i.e. β) and on how persistent the transitory shock is expected to be (i.e. ϕ).

The third row of derivatives reflects the fact that more financial wealth increases consumption but not savings as it is akin to a permanent increase in income. Agents with higher financial wealth will have higher future income permanently since they will have a higher stream of interest payments forever. They can therefore permanently increase their consumption without changing their savings pattern.

Plots for the paths of $\{C_t, Y_t, S_t, A_t\}$ after a shock $\varepsilon_t = 1$ are shown in figure 1 below.

4. The path of equilibrium C_t, S_t, A_t are shown in figure 2 below. First notice that in both cases the agent increases his consumption when **news** of higher income arrive (i.e. today), not when income actually increases. This occurs because the agent wants to smooth consumption over time. They increase consumption today in anticipation to the future increase so that the path of consumption is as smooth as possible.

For part a), the agent realizes he will have a temporarily higher income and borrows today against this higher future income. Before the increase he uses the borrowed resources to increase his consumption. When income is high, he saves most of his higher income to pay off the debt and accumulate assets in order to sustain higher consumption in the future using the higher interest payments from higher asset holdings when income falls back to normal.

For part b), since the agent knows the increase will be permanent he borrows heavily today against this higher future income to significantly increase his consumption today.

When the increase arrives, he consumes a large fraction of the increase in his income and only saves a small share. He saves just enough to be able to pay for the interest on the debt he accumulated in the beginning, but never pays off that debt.
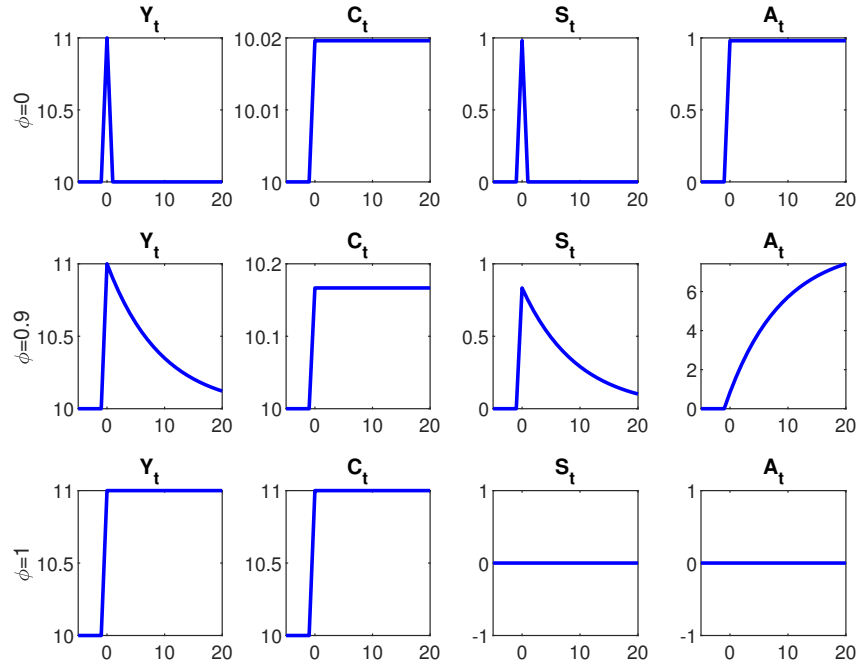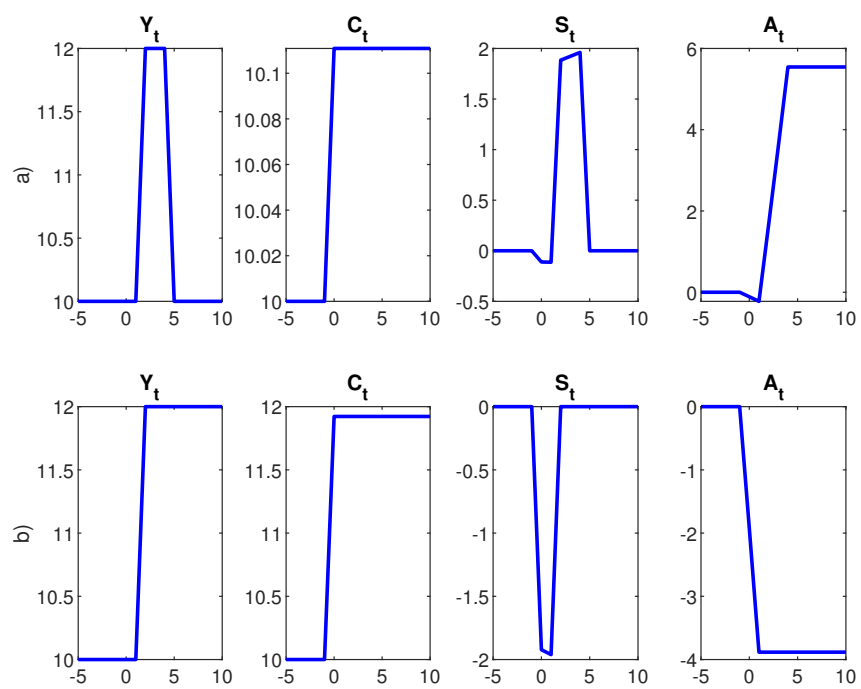

Figure 1: Plots for question 3.

Figure 2: Plots for question 4.