RECURSIVE FORMULATION

$f(k) = F(k, 1) + (1-\delta)\,k$

Output.

dep. capital

$$\sum_{t=0}^{\infty} \beta^{t}\, u\big(f(k_t) - k_{t+1}\big) \longrightarrow \max_{\{k_{t+1}\}_{t=0}^{\infty}}$$

$$\text{s.t.} \quad f(k_t) \geq k_{t+1} \geq 0$$

k_0 is given.

VALUE FUNCTION :-

Present Pd

Future pd.

$$v(k_0) = \max_{0 \leq k_1 \leq f(k_0)} \Big[u\big(f(\underline{k_0}) - \underline{k_1}\big) + \beta\, v(k_1) \Big]$$

(given)

Policy / Choice variable.

BELLMAN EQUATION / VALUE F^n ITERATION:-

① Guess $v_0(k)$ / k is given ②

(what do we guess? The form of $v f^n$).

② Iterate. / Update guess.

$$v_{j+1}(k) = \max_{0 \leq k' \leq f(k)} \left[u(f(k) - k') + \beta v_j(k') \right]$$

③ If $\| v_{j+1}(k) - v_j(k) \| < \varepsilon$,

then $v_{j+1}(k)$ is the solution.

Example:

$$u = \log(c)$$
$$\delta = 1$$
$$y_t = \theta k_t^{\alpha}$$

[This is the guess!]

$$V_k = \max_{0 \le k' \le \theta k^{\alpha}} [\log(c) + \beta . V(k')]$$

s.t. $c + k' = \theta k^{\alpha}$ | k_0 is given

$$\mathcal{L} = \log(c) + \beta(a_0 + a_1 \log k) + \lambda[\theta k^{\alpha} - c - k']$$

$$\frac{1}{c} = \frac{\beta a_1}{k'}$$

$$\frac{1}{\theta k^{\alpha} - k'} = \frac{\beta a_1}{k_1}$$

$$k' = \beta a_1 \theta k^{\alpha} - \beta a_1 k'$$

$$k' = \frac{\beta a_1}{1 + \beta a_1} . \theta k^{\alpha}$$

$$c = \theta k^{\alpha}\left(\frac{1}{1 + \beta a_1}\right)$$

Here we just found c, k, a, → so what?
This is not ~~recursive~~ bellman iteration