

Oct 31, 2023

Final Exam  
Dec 9  
2-5 pm

Today's topics:-

- TDCS is NOT always PO (Pareto optimal)
- Some examples when it is

Example 1

$$\{g_t\}_{t=0}^{\infty} \quad r_{ct} = r_{kt}, \quad r_{nt} = r_{xt} = 0$$

$$\text{Govt B.C.:} \quad \sum_{t=0}^{\infty} p_t g_t = \sum_{t=0}^{\infty} T_t$$

Claim: TDCS is PO in this case

$$\text{TDCS} \quad \sum_{t=0}^{\infty} \beta^t u(c_t, n_t) \rightarrow \max$$

$$\text{s.t.} \quad \sum_{t=0}^{\infty} p_t c_t + p_t (k_{t+1} - (1-\delta)k_t) = \sum_{t=0}^{\infty} r_t k_t + w_t n_t + T_t$$

FOC:

$$(c_t): \beta^t u'_c(t) = \lambda p_t$$

$$(n_t): \beta^t u'_n(t) = \lambda w_t$$

$$(k_{t+1}): -p_t + r_{t+1} + (1-\delta)p_{t+1} = 0$$

$$\frac{p_t}{p_{t+1}} = \frac{r_{t+1}}{p_{t+1}} + 1 - \delta$$

$$\Rightarrow \left\{ \begin{array}{l} \frac{u'_c(t)}{\beta u'_c(t+1)} = \frac{r_{t+1}}{p_{t+1}} + 1 - \delta \\ \frac{u'_n(t)}{\beta u'_c(t+1)} = \frac{w_t}{p_t} \\ c_t + x_t + g_t = F(k_t, n_t) \end{array} \right\}$$

TDCS equilibrium  
conditions:

Consider SPP :-

$$\sum_{t=0}^{\infty} p^t U(c_t, n_t) \rightarrow \max_{\{c_t, n_t\}_{t=0}^{\infty}}$$

$$\text{s.t. } c_t + x_t = \hat{F}(k_t, n_t), \quad \hat{F}(k_t, n_t) = F(k_t, n_t) - g_t$$
$$k_{t+1} = (1-\delta)k_t + x_t$$

$k_0$  is given

FOC for SPP: -

$$(c_t) \quad p^t U'_c(t) = \lambda_t$$

$$(n_t) \quad p^t U'_n(t) = \lambda_t \hat{F}'_n(t)$$

$$(k_{t+1}) \quad -\lambda_t + \lambda_{t+1} [\hat{F}'_k(t+1) + 1-\delta] = 0$$

$$\frac{U'_c(t)}{p U'_c(t+1)} = \hat{F}'_k(t+1) + 1-\delta$$

$$\frac{U'_n(t)}{U'_c(t)} = \hat{F}'_n(t)$$

This is the same as for TDCS ( $g_t$  is a constant).

$\Rightarrow$  TDCS is PO in this case (lump sum taxes & transfers)

### Example 2:

$$\tau_{kt} = \tau_{nt} = \tau_t \quad \forall t$$

$$\tau_{ct} = \tau_{xt} = 0 \quad \forall t$$

$$\tau_t = 0 \quad \forall t$$

$$\text{Govt. B.C.: } p_t g_t = \tau_t r_t k_t + \tau_t w_t n_t \quad \forall t \quad (\text{B.C. is cleared every period} \rightarrow \text{stricter cond}^n \text{ than ex 1})$$

$$\sum p_t^t u(c_t, n_t) \rightarrow \max$$

$$\sum_{t=0}^{\infty} p_t c_t + p_t (k_{t+1} - (1-\delta)k_t) = \sum_{t=0}^{\infty} (r_t k_t (1-\tau_t) + w_t n_t (1-\tau_t))$$

FOC: -

$$(c_t): p^t u'_c(t) = \lambda p_t$$

$$(n_t): p^t u'_n(t) = \lambda w_t (1-\tau_t)$$

$$(k_{t+1}): -p_t + p_{t+1} (1-\delta) + r_{t+1} (1-\tau_{t+1}) = 0$$

$$\frac{p_t}{p_{t+1}} = \frac{r_{t+1} (1-\tau_{t+1}) + 1-\delta}{p_{t+1}}$$

$$\Rightarrow \left[ \begin{array}{l} \frac{u'_c(t)}{p u'_c(t+1)} = \frac{r_{t+1} (1-\tau_{t+1}) + 1-\delta}{p_{t+1}} \\ \frac{u'_n(t)}{u'_c(t)} = \frac{w_t (1-\tau_t)}{p_t} \\ c_t + x_t + g_t = f(k_t, n_t) \end{array} \right]$$

Characterizes the equilibrium

where does this come from?

SPP

$$\sum_{t=0}^{\infty} \beta^t u(c_t, n_t) \rightarrow \max$$

$$\text{s.t. } c_t + x_t = (1 - \tau_t) \cdot f(k_t, n_t) \quad (\text{Redefine prod}^n \text{ technology})$$

$$0 \leq n_t + l_t \leq 1$$

$k_0$  is given

FOC

$$\beta^t u'_c(t) = \lambda_t$$

$$\beta^t u'_n(t) = \lambda_t (1 - \tau_t) F'_n(t)$$

$$-\lambda_t + \lambda_{t+1} [(1 - \tau_{t+1}) F'_k(t+1) + 1 - \delta] = 0$$

$$\frac{u'_c(t)}{\beta u'_c(t+1)} = (1 - \tau_{t+1}) F'_k(t+1) + 1 - \delta$$

$$\frac{u'_n(t)}{u'_c(t)} = (1 - \tau_t) F'_n(t)$$

Equilibrium conditions are the same as before.

Now we have to show that feasibility constraint in CE & DCE is the same.

$$c_t + x_t + g_t = f(k_t, n_t) \rightarrow \text{FC in CE}$$

$$p_t c_t + p_t x_t + p_t g_t = p_t f(k_t, n_t)$$

$$= p_t c_t + p_t x_t + \tau_t (r_t k_t + w_t n_t) = p_t f(k_t, n_t)$$

Under CPS:-

$$p_t c_t + p_t x_t = (1 - \tau_t) p_t F(k_t, n_t) + \tau_t (p_t F(k_t, n_t) - w_t n_t - r_t k_t)$$

Under CPS, this is 0.  
There is no profit

$$\Leftrightarrow p_t c_t + p_t x_t = (1 - \tau_t) p_t F$$

$$\Leftrightarrow c_t + x_t = (1 - \tau_t) F \quad \text{--- FC in SPP}$$

- Add some  $t_c$  to this, & you'll see that the cond<sup>n</sup> are not the same.

Example 3:

Same world as example 2

$$\tau_{nt} = \tau_{kt} = \tau_t = 0.2$$

Suppose  $\tau \uparrow$  to 30%. (unexpected change so that agents do not have a chance to change their behavior prior to tax change.)

Suppose labor supply inelastic  $u'_n = 0$

EE:

$$u'_c(t) = \beta u'_c(t+1) [F'_k(t+1) (1 - \tau_{k,t+1}) + 1 - \delta]$$

steady-state  $k_{0.2}^*$

$$k_{t+1} = (1 - \delta) k_t + x_t \Rightarrow x_{0.2}^* = \delta k_{0.2}^*$$

↓  
They are same in SS.

From feasibility constraint;

$$C_t = F(k_t) - k_{t+1} + (1-\delta)k_t \Rightarrow C_{0.2}^* = F(k_{0.2}^*) - \delta k_{0.2}^*$$

From EE;  $(u'_c(t) = u'_c(t+1))$  in SS :-  
 $1 = \beta [(1-0.2) F'_k(k_{0.2}^*) + 1-\delta]$

Assume  $F(k_t) = Ak_t^\alpha$ ,  $\alpha = 1/3$

$$F' = \frac{1 - (1-\delta)}{\beta}$$

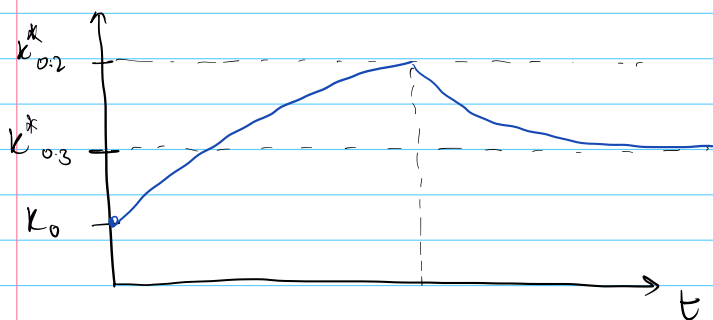
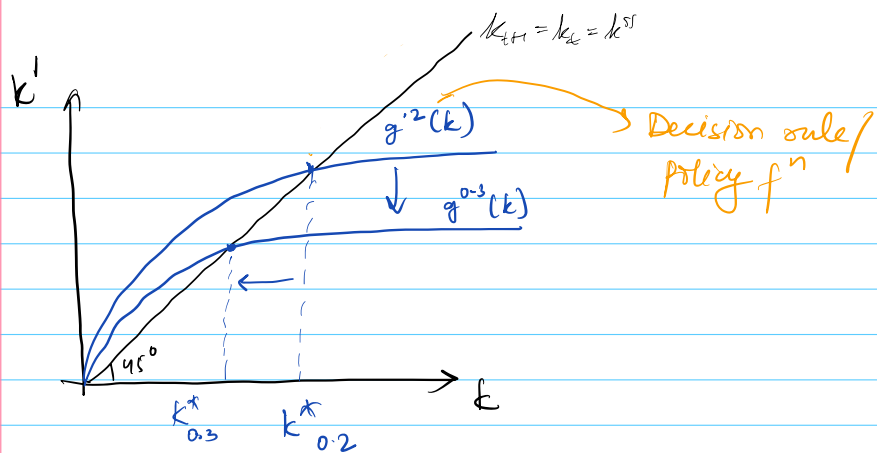
$$F' = (Ak^\alpha)' = A\alpha k^{\alpha-1}$$

$$\Rightarrow A\alpha (k_{0.2}^*)^{\alpha-1} = \frac{1 - (1-\delta)}{\beta}$$

$$\Rightarrow k_{0.2}^* = \left[ \frac{\text{const}}{(1-0.2)} \right]^{1/\alpha-1}$$

*This is negative*  
 *$\rightarrow k \downarrow$  &  $\beta \uparrow$*

$$\Rightarrow \frac{k_{0.3}^*}{k_{0.2}^*} = \frac{\text{const} (1-0.2)^{1/\alpha-1}}{(1-0.3)^{1/\alpha-1} \cdot \text{const}}$$
$$= \left( \frac{1-0.2}{1-0.3} \right)^{1/\alpha-1} \approx 0.82$$



⊗ Why CRS  $\rightarrow$  no profits?

$$f(x) = 2x$$

$$py - wx$$

$$pf(x) - wx$$

$$foc = 2p = w$$

$$p \cdot (2x) - (2p)w = 0 \quad (\text{zero profit})$$

