Midterm Exam - Econ 7010

November 2, 2021

Exam begins: 9:30 am

Exam ends: 10:45 am

Instructions: Answer each question in your bluebook, or on your own paper that is stapled and *neatly organized.* The **clarity and brevity** of yours answers is important: I should not have to (and will not) "search" for your answers to any question, and extraneous and/or irrelevant information will lose points.

Your answers to all questions must be fully justified to receive credit. If you cannot answer a question completely, a well-labeled diagram and/or clear explanation of the intuition behind the solution process can obtain partial credit.

You have 75 minutes to complete the exam. The total number of points is 75. Budget your time wisely; you may find later questions easier than earlier ones.

Good luck!

Question 1 (40 points)

A firm uses labor L and capital K to produce goods according to a production function $f(L, K) = L^{1/4}K^{1/2}$. Let the output price be p. The input prices are w and r for labor L and capital K, respectively. The firm's goal is to maximize the profit (objective) function

$$\tilde{\pi}(L, K, p, w, r) = pf(L, K) - wL - rK.$$

We write $\tilde{\pi}$ to distinguish this profit function (the firm's objective function), from the value function of the maximization problem, denoted π:

$$\pi(p, w, r) = \tilde{\pi}(L^*(p, w, r), K^*(p, w, r), p, w, r),$$

where $L^*(p, w, r)$ and $K^*(p, w, r)$ are the firm's optimal demands for capital and labor.

(a) (10 points) Solve the firm's optimization problem, and find the factor demands $L^*(p, w, r)$ and $K^*(p, w, r)$.

(b) (5 points) What is the marginal effect of a rise in w on the firm's profits? What is the marginal effect of a rise in r on the firm's profits?

Say that the firm, rather than trying to maximize pure profits, also cares about employment. The firm's objective function is now

$$g(\tilde{\pi}, L, \alpha)$$

where $\tilde{\pi}$ is the same profit function from above, L is the hours of labor, and $\alpha \in [0, 1]$ is a parameter that corresponds to the weight the firm puts on profits (relative to labor): $\alpha = 0$ corresponds to no weight on profits, and $\alpha = 1$ corresponds to full weight on profits. You may assume that g is strictly increasing in both $\tilde{\pi}$ and L. We will write

$$\begin{aligned} G(p, w, r, \alpha) &= \max_{L,K \geq 0} g(\tilde{\pi}, L, \alpha) \\ &= g(\tilde{\pi}(L^*, K^*, p, w, r), L^*, \alpha) \end{aligned}$$

for the value function corresponding to this optimization problem, where $L^* = L^*(p, w, r, \alpha)$ and $K^* = K^*(p, w, r, \alpha)$ are the optimal factor demands. Suppose that an econometrician friend gathers data, and estimates the following equation for G:

$$G(p, w, r, \alpha) = \frac{1}{64} \frac{\alpha^2 p^4}{r^2(\alpha w + \alpha - 1)}$$

(c) (8 points) One of the Econ 7010 homework questions is to find this firm's optimal choices, $L^*(p, w, r, \alpha)$ and $K^*(p, w, r, \alpha)$. A classmate of yours claims that he can do this very easily by simply applying Hotelling's Lemma as follows:

$$L^*(p, w, r, \alpha) = -\frac{\partial G}{\partial w} \qquad K^*(p, w, r, \alpha) = -\frac{\partial G}{\partial r}.$$

Is he correct? If not, can you provide a "modified" version of Hotelling's Lemma that will allow you to recover L^* and K^*? (In your answer, let $g_{\tilde{\pi}}(\tilde{\pi}, L, \alpha) = \partial g / \partial \tilde{\pi}$ denote the derivative of small g with respect to its first argument; you can leave this term in your final expressions, but otherwise simplify as much as possible.)

(Question 1 continues on next page.)

(d) (8 points) Find the optimal capital-labor ratio (K^*/L^*) for this firm in terms of the prices and α. Simplify as much as possible.

(e) (3 points) Calculate the derivative of the capital-labor ratio with respect to α. Interpret.

(f) (6 points) Suppose that the function g takes the specific form:

$$g(\tilde{\pi}, L, \alpha) = \alpha\tilde{\pi} + (1-\alpha)L.$$

Suppose also that the firm optimally chooses $L^* = 4$ at the prices $(p, w, r) = (4, 1, 3)$. Show how you could use this information to determine α, the weight that is placed on employment. (That is, provide an equation that implicitly characterizes what α must be; you need not solve for the actual value of α, but it should be the only variable left in your equation.)

Question 2 (35 points)

Consider a firm that uses 2 inputs x_1 and x_2. Denote the input prices by (w_1, w_2). Say you observe the following cost and input data for this firm (assume that each produces the same output y):

Prices (w_1, w_2)	Input choice (x_1, x_2)
$w' = (1, 4)$	$x' = (4, 2)$
$w'' = (4, 1)$	$x'' = (3, 5)$

(a) (7 points) Is this data consistent with the Weak Axiom of Cost Minimization? Prove or disprove.

(b) (7 points) Assume that the firm has a production technology that is convex and satisfies free disposal. If the answer to the previous part is yes, draw a clear, well-labeled sketch of the inner and outer bounds to the true input requirement set. If the answer to the previous part is no, then draw your own sketch of 2 data points that would satisfy WACM, and again, draw the inner and outer bounds. (In the latter case, you need not provide actual numbers or calculations; just a sketch of what the points should look like is sufficient.)

(c) (7 points) Now, assume that the production technology is **not convex** (but still assume it does satisfy free disposal). On a new picture, sketch one possibility for what the true production set could be.

(d) (7 points) Redraw your sketch from part (c). One of your empirical friends goes out and collects "infinite data" (i.e., he observes $c(w, y)$ for all prices w), and tries to use it to recover the true production technology. Add to your sketch what she would infer the firm's production technology to be. Will she recover the true production set? Explain.

(e) (7 points) Now, say you observe a third data point: at prices $(3, w_2''')$, the firm chose $x''' = (9, 1)$. For what values of w_2''' will WACM continue to hold?