Econ 7710
Assignment 7

The due date is Friday, December 1st

1. Using a sample of n two-dimensional observations $\{X_i\}_{i=1}^n$ construct a test with significance level α for the null hypothesis that this sample is generated by a uniformly distributed variable on $[0,1]\times[0,1]$ against the alternative hypothesis that the sample is generated by a uniformly distributed random variable on $[h,\, h+1]\times[h,\, h+1]$ for $h \geq 0$.

   (a) Is the size of your test always equal to its significance level?

   (b) Plot the graph of the power of your test as a function of h.

2. Consider the classical simple regression model

$$Y_i = \beta_0 + \beta_1 X_i + U_i$$

   in which

$$E\left(U_i|X_i\right) = 0 \qquad \text{and} \qquad Var\left(U_i|X_i\right) = \sigma^2, Var\left(X_i\right) = \sigma_x^2$$

   Let $(X_i, Y_i, i = 1, \ldots, n)$ be i.i.d. Instead of regressing y_i on a constant and x_i, you regress x_i on a constant and y_i:

$$\min_{\hat{\alpha}_0, \hat{\alpha}_1} \sum_{i=1}^{n} \left(x_i - \hat{\alpha}_0 - \hat{\alpha}_1 y_i\right)^2$$

   Is $1/\hat{\alpha}_1$ a consistent estimator of β_0? If not what does it converge to?