Problem 1

Consider an overlapping generations economy in which the representative consumer in generation t, $t = 1, 2, \ldots$ has preferences over the consumption of the single good in each of the two periods of life given by the utility function:

$$u(c_t^t, c_{t+1}^t) = \log c_t^t + \beta \log c_{t+1}^t.$$

This consumer is endowed with quantities of labor $(l_t^t, l_{t+1}^t) = (l_1, l_2)$. In addition, there is a generation 0 whose representative consumer lives only in period 1 and has the utility function

$$u^0(c_1^0) = \log c_1^0,$$

and the endowment of $l_1^0 = l_2$ units of labor and $k_1^0 = \bar{k}_1$ units of capital. This consumer also has an endowment of fiat money m, which can be positive, negative or zero. The production function is $f(k_t, l_t) = \theta k_t^\alpha l_t^{1-\alpha}$ and capital depreciates at the rate $\delta \in [0, 1]$.

(a) Define the Sequential Markets equilibrium for this environment.

(b) Define the Arrow-Debreu equilibrium for this environment.

(c) Suppose now that consumers live for four, rather than 2, periods. Specify an environment analogous to that above by specifying utility functions, endowments, and a production technology. Define a Sequential Markets equilibrium for this economy.

Problem 2

Consider an infinite-horizon ecnonomy with an equal measure of two types of agents. The agents' preferenes are of the form:

$$\sum_{t=0}^{\infty} \beta_i^t \log c_{it},$$

where $0 < \beta_1 < \beta_2 < 1$. The households are endowed with 1 unit of labor each. Type 1 households hold the entire initial stock of capital. The aggregate resource constraint is

$$c_t + k_{t+1} = k_t^\alpha l_t^{1-\alpha} + (1-\delta)k_t,$$

where c_t denotes period t aggregate consumption, k_t the capital stock, l_t labor, $0 < \alpha < 1$ a parameter and $\delta \in (0,1)$ the depreciation rate.

(a) Set up and define an Arrow-Debreu competitive equilibrium.

(b) What is $\lim_{t\to\infty} \frac{c_{1t}}{c_{2t}}$ in such equilibrium?

(c) To what level does the aggregate capital stock converge?