Inequalities

* Markov Inequality:-

An upperbound to the pr that the realization of a r.v. exceeds a given threshold.

$$P(X \geq c) \leq \frac{E[X]}{c}$$

Proof :

$$1_{\{x \geq c\}} + 1_{\{x < c\}} = 1$$

$$E[X] = E[X.1]$$

$$= E[X.(1_{\{x \geq c\}} + 1_{\{x < c\}})]$$

$$E[X] = E[X 1_{\{x \geq c\}}] + E[X 1_{\{x < c\}}] \geq E[X 1_{\{x \geq c\}}]$$

Note; $c.1_{\{X \geq c\}} \leq X.1_{\{x \geq c\}}$

$$c.1_{\{x \geq c\}} \leq X.1_{\{x \geq c\}}$$

$$\Rightarrow E[c.1_{\{x \geq c\}}] \leq E[X.1_{\{x \geq c\}}]$$

$$E[c.1_{\{x \geq c\}}] = c E[1_{\{x \geq c\}}]$$

$$= c P(X \geq c)$$

$$\Rightarrow c P(X \geq c) \leq E[X.1_{\{x \geq c\}}]$$

Putting inequalities together

$$E[X] \geq E[X \mathbb{1}\{X \geq c\}]$$

$$E[X \mathbb{1}\{X \geq c\}] \geq cP(X \geq c)$$

$$\Rightarrow \quad \frac{E[X]}{c} \geq \frac{\cancel{c}P(X \geq c)}{\cancel{c}}$$

* Chebyshev's Inequality

An upper bound to the probability that the absolute deviation of a r.v. from its mean will exceed a given threshold.

$$P(|X-\mu| \geq k) \leq \frac{\sigma^2}{k^2}$$

* ## Jensen's Inequality

let X be r.v.

(•) let $g: \mathbb{R} \to \mathbb{R}$ be a convex function s.t

$$Y = g(X)$$

$$E[g(X)] \geq g(E[X])$$

(•) if f^n g is concave then :-

$$E[g(X)] \leq g(E[X])$$

Ex

$$E[X] = 1$$

$$E[\ln(X)] < \ln(E[X]) = \ln(1) = 0$$