Econ7020: Macroeconomic Theory

Fall 2023

Problem Set 4. Due date: before class on November 14.

Problem 1

Consider an economy in which the representative consumer lives forever. There is a good in each period that can be consumed or saved as capital. The consumer's utility function is

$$\sum_{t=0}^{\infty} \beta^t u(c_t, l_t),$$

where c_t is consumption in period t, and l_t is leisure in period t.

The consumer is endowed with 1 unit of time endowment in each period, some of which can be consumed as leisure l_t, and some of which is supplied as labor n_t. The consumer is also endowed with k_0 units of capital in period 0. Feasibility constraint for this economy is:

$$c_t + k_{t+1} - (1-\delta)k_t = F(k_t, n_t).$$

1. Formulate the problem of maximizing the representative consumer's utility subject to feasibility constraint as a dynamic programming problem. Write down the appropriate Bellman equation. Make appropriate assumptions on the parameters β and δ and on functions u and F that guarantee a solution to the Bellman equation. Cite any results that you need, but do not prove anything (however, make sure to show in detail that assumptions of any theorems that you apply are satisfied).

2. Define an Arrow-Debreu equilibrium for this economy. Suppose you have solved the dynamic programming problem in part (1) under the assumptions you made. Explain carefully how to calculate the ADE.

Problem 2

Suppose that an infinitely lived government has to finance a fixed stream of expenditures $\{g_t\}_{t=0}^{\infty}$ and can only use consumption taxes for this purpose. Assume that the representative consumer has the utility function

$$\sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\sigma}}{1-\sigma} + v(l_t) \right],$$

where c_t is consumption in period t and l_t is leisure in period t. Assume that $\sigma \geq 0$ and v is an increasing function. Also assume that the production function $F(k,n)$ satisfies all the standard assumptions (including CRS), that the representative household has an initial endowment of capital stock k_0, and time endowment is $\bar{n}_t = 1$. The law of motion for capital stock is $k_{t+1} = (1-\delta)k_t + x_t$.

Set up the Ramsey problem for this economy, and show that the optimal policy is to set the consumption tax at a constant rate (i.e., $\tau_{ct}^{RP} = \tau_{ct+1}^{RP}\ \forall t$).

Problem 3

1. Give a complete definition of a CE in a model with taxation (i.e., a TDCE) in an infinite horizon, representative agent world when firms pay taxes which are proportional to revenue for their sales at a rate τ. Assume there are no other taxes in the system.

2. In there an equivalent way to set up the model (i.e., the allocations are the same in both systems) so that households pay all the taxes? If so, what taxes are needed and how are they related to τ? Show your work.

Problem 4

Consider an infinite horizon, single sector economy with 2 types of infinitely lived consumers. Assume that there are equal numbers of the two types of consumers, $N_1 = N_2$. Suppose that in each period, a lump sum tax is levied on each type of consumer, $T_{it}, i = 1, 2$. Assume that

$T_{1t} = -T_{2t}$ for all t.

1. Define a CE in this setting.
2. Define a PO allocation in this setting.
3. Prove that the allocation arising in the CE you defined in part (1) is PO. Make any assumptions you need to use clear.
4. Is the assumption $T_{1t} = -T_{2t}$ for all t necessary for the result in part (3)? If not, what is? Briefly discuss.