Final Exam - Econ 7010

November 21, 2022

Instructions: Answer each question in your bluebook, or on your own paper that is stapled and *neatly organized.* If you are using loose paper, make sure to staple the pages together before turning them in.

There are three questions worth 25, 30, and 20 points respectively. Your answers to all questions must be fully justified to receive credit. If you cannot answer a question completely, a well-labeled diagram and/or clear explanation of the intuition behind the solution process can obtain partial credit. The **clarity and brevity** of yours answers is important: I should not have to (and will not) "search" for your answers to any question, and extraneous and/or irrelevant information will lose points.

You have 75 minutes to complete the exam. The total number of points is 75. Budget your time wisely; you may find later questions easier than earlier ones.

Good luck!

Question 1 (25 points)

Recall the following definitions:

- A good is **regular** if $x_i(p,m)$ is decreasing in p_i. A good is **Giffen** if $x_i(p,m)$ is increasing in p_i.
- A good is **normal** if $x_i(p,m)$ is increasing in m. A good is **inferior** if $x_i(p,m)$ is decreasing in m.
- Good i is a **substitute for good** k if $h_i(p,u)$ is increasing in p_k. Good i is a **complement for good** k if $h_i(p,u)$ is decreasing in p_k.
- Good i is a **gross substitute for good** k if $x_i(p,m)$ is increasing in p_k. Good i is a **gross compliment for good** k if $x_i(p,m)$ is decreasing in p_k.

Also, recall the Slutsky equation:

$$\frac{\partial x_i(p,m)}{\partial p_k} = \frac{\partial h_i(p,v(p,m))}{\partial p_k} - \frac{\partial x_i(p,m)}{\partial m} x_k(p,m)$$

In answering each of the questions below, use the Slutsky equation to prove your answer.

(a) (5 points) Explain the Slutsky equation in words.

(b) (5 points) Fill in the blank with one of the above terms: A normal good must be ___________.

(c) (5 points) Fill in the blank with one of the above terms: If good i is a gross complement for good k and good i is inferior, then good i is a ___________ for good k.

(d) (5 points) Assume that good i is a substitute for good k. Is good k also a substitute for good i?

(e) (5 points) Assume that good i is a gross substitute for good k. Is good k also a gross substitute for good i?

Question 2 (30 points)

Consider a consumer with the following Marshallian demand functions and indirect utility function:

$$x_1(p,m) = \frac{m}{p_1 + 5p_2} \qquad x_2(p,m) = \frac{5m}{p_1 + 5p_2} \qquad v(p,m) = \frac{5m}{p_1 + 5p_2}$$

(a) (6 points) Find the consumer's Hicksian demand functions (hint: first find the expenditure function using the duality identities).

(b) (6 points) Consider a price change from $(p_1, p_2) = (1,1)$ to $(p_1', p_2') = (5,1)$. Let the consumer have wealth $m = 60$. Decompose the change in demand for good 1 into an income effect and a substitution effect.

(c) (6 points) Calculate the equivalent variation for the price change from (b).

(d) (6 points) Calculate the compensating variation for the price change from (b).

(e) (6 points) Draw a sketch with p_1 on the vertical axis and demand for good 1 on the horizontal axis. On your sketch, include both the Hicksian and Marshallian demands, and use your sketch to graphically illustrate both CV and EV, as well as Consumer Surplus (CS).

Question 3 (20 points)

Consider a farmer who must decide the quantity of wheat to produce next year, q. The market price that he can sell his wheat for is p per unit. There is also probability θ that there will be a drought next year. If there is not a drought, then producing q costs $c(q)$, where $c(q)$ is strictly increasing and strictly convex. If there is a drought, then the farmer must invest in additional water resources, and his total cost to produce q is $c(q) + rq$, where $r > 0$ is the cost of additional water. The farmer must make his production decision before the realization of uncertainty regarding the weather.

(a) (5 points) Assume that the farmer wants to maximize expected profit, and let q^* denote the farmer's optimal choice. Write down his objective function. Write an equation that characterizes q^*, and interpret.

(b) (5 points) Find an equation for $dq^*/d\theta$, and interpret.

(c) (5 points) Assume instead that the farmer is risk-averse, and has a utility function over profits $u(\cdot)$ that is increasing and strictly concave. His goal is to maximize the *expected utility* of profit. Let q^{**} denote the optimal solution to this problem. Write down the farmer's objective. Write an equation that characterizes q^{**}, and interpret.

(d) (5 points) Which is greater, q^* or q^{**}? Both justify your answer mathematically, and provide intuition.