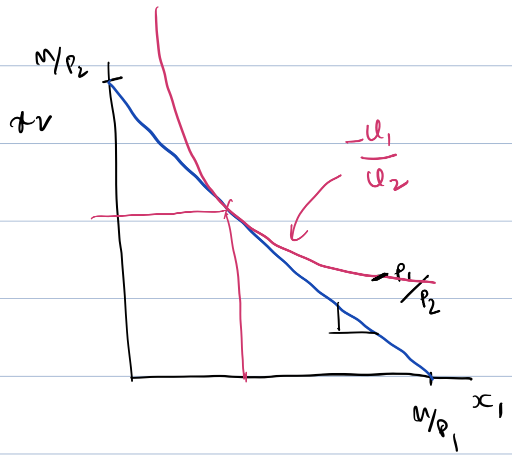
Jan 18, 2023



Slope : Rise over Run

Implicit Function Theorem
Used in MRS.

$$\cancel{d}u_i = u_1 dx_1 + u_2 dx_2$$
0

$$\frac{dx_2}{dx_1} = -\frac{u_1}{u_2}$$

Alt. using the Implicit f^n theorem

$$\bar{u}_i - u_i(x_{1i}, x_{2i}) = 0$$

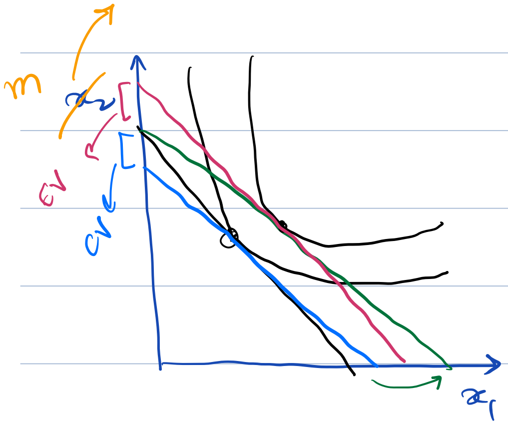
Q. EV, CV, Income Effects, Quasilinear Utility.
What happens $\frac{u_1}{u_2} = \frac{P_1}{P_2}$ (when P_2 is normalized?)

Jan 19.

Bank for Buck Condition: -

$$\frac{U_1}{P_1} = \frac{U_2}{P_2}$$

x_2 consider as a numeraire. $(P_2 = 1)$

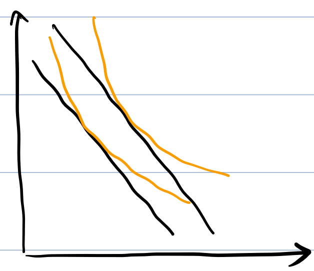


B.C. moves to right as $P_1 \downarrow$

EV : what can we give her

CV : what we can take away

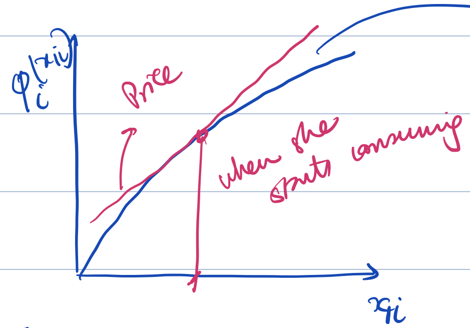
(*) when EV = CV ⇒ it is true when ICs are vertically parallel to each other → Quasilinear preferences.



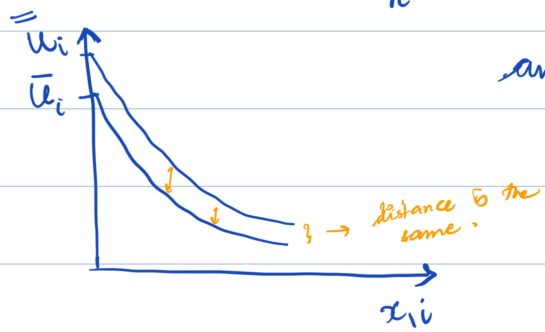
$u_i = \phi_i(x_i) + m_i$ (This is a quasilinear utility)

decreasing MU

ϕ_i is a concave f^n which means MU is decreasing.



an IC: $m_i = \bar{u}_i - \phi_i(x_i)$



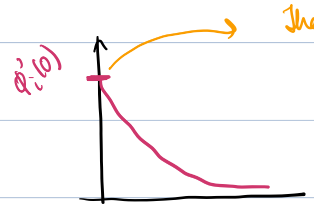
(To study this!)

By making this assumption, we have no income effects.

$$\frac{u_1}{P_1} = \frac{\cancel{u_2}\,1}{\cancel{P_2}\,1}$$ why?

$\phi_i'(x_{i1}) = P_1 \Rightarrow$ Demand function (when not corner pt. will do that later)

The pt. at which she starts consuming the price is too high presume



Example: $\phi(x) = \sqrt{x}$

$$\phi'(x) = \frac{1}{2\sqrt{x}} = P.$$

: Price as a f^n of qty (Inverse Dd curve).

Flip it.

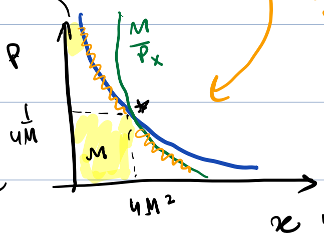
Demand Curve $\frac{1}{4P^2} = x$

Just plot this.

So this has no income effect.

does not touch the intercept. \$/unit P

(Infinite MU for the first unit).



orange → The actual dd line.

!! (*) Is this the pt. where all M is on x_1 or not?

Dd for numeraire : $x_2 = m - x$.

$px = M$ (green line)

$$\frac{1}{4p^2} = \frac{M}{p} \Rightarrow \frac{1}{4M} = P$$

(Cross verifying that calculus gives us the same results :—

Lagrangian :—

[True Budget Constraint]

$$\max_{\substack{\{x,\, m\} \\ \geq 0 \;\; \geq 0}} \phi(x) + m \quad \text{s.t. } \overset{\mu_2}{x \geq 0}, \; \overset{\mu_1}{m \geq 0}, \; m + px \leq M$$

$$\mathcal{L} = \phi(x) + m - \underbrace{\lambda(m + px - M)}_{0} + \underbrace{\mu_1 m}_{0} + \underbrace{\mu_2 x}_{0} \text{ (csc)}$$

$$\frac{d\mathcal{L}}{dx} = \frac{d\mathcal{L}}{dm} = 0$$

$x > 0$, $m > 0$ (if $\phi(x)$ is a concave f^n $\Rightarrow$ x is a 'good' good.

$\Rightarrow \mu_1 = 0$, $\mu_2 = 0$

Is this an assumption?

$$\phi'' - \lambda p = 0$$

$$\lambda = 1$$

$\max_{\{m, x_1\}} \quad \phi(x_1) + m \quad$ st $\quad px_1 + m \leq M \quad m \geq 0 \quad x_1 \geq 0$

non-neg

w/ $\phi(x_1) = \sqrt{x_1}$

stationary pts of $\mathcal{L}$; constraints of form $f(x) \leq 0$

$$\mathcal{L} = \phi(x_1) + m - \lambda(px_1 + m - M) + \mu_1 m + \mu_2 x_1$$

x_1: $\phi'(x_1) - \lambda p + \mu_2 = 0 \qquad \mu_1 m = 0 = \mu_2 x_1 \qquad$ (CS)

m: $1 - \lambda + \mu_1 = 0 \qquad \mu_1, \mu_2 \geq 0$

max^n: $px_1 + m = M$ since both goods

Cases: $m=0 \quad x_1 = 0$ not a max

$\underline{m, x_1 > 0}$; so $\mu_1 = \mu_2 = 0$ hence $\lambda = 1$ [unit MUM]

& so $\phi'(x_1) = p$ i.e. $\frac{1}{2\sqrt{x_1}} = p$ we had

{& no longer feasible
{when $px_1 \geq M$
or $\frac{1}{4p} \geq M$ $\quad *$

?

$\underline{x_1 = 0 \quad m > 0}$

$\mu_1 = 0, \ \lambda = 1.$

$\underbrace{\phi'(0)}_{\to\infty} - p + \underbrace{\mu_2}_{\geq 0} = 0$ not possible

$\underline{m = 0 \quad x > 0}$

b.c.: $x_1 = M/p$ & we know from $*$ this ok for $\frac{1}{4p} \geq M$

we can also get this condition from multipliers

from m eqn: $1 - \lambda + \mu_1 = 0$ or $\mu_1 = \lambda - 1 \geq 0$

from x_1: $\phi'(x_1) = \lambda p$ OR $\frac{1}{2\sqrt{x_1}\,p} = \lambda$. Insert $x_1 = M/p$

so that $\frac{1}{2\sqrt{M/p}\,p} - 1 \geq 0$ OR $\frac{1}{4M} \geq p$ as above.

Because both goods are good, income is exhausted here; and because the marginal utility of good 1 (x) is infinite around x=0 in this example, there can only be positive consumption of x at any solution with finite p. However, consumption of the second good can be zero if income isn't large enough.

As was pointed out, it would be a bit odd to refer to m as numeraire in such cases! And indeed, we will henceforth assume that consumers always have enough income to choose m>0. So the particular corner solution m=0 in the handout will not concern us henceforth for our quasi-linear analysis.

All we want to carry forward for our analysis is that the consumer i will choose such that

$\phi_i'(x)=p$ if $\phi_i'(0)>p$ and $x=0$ otherwise