Oct 31, 2023

Final Exam
Dec 9
2-5 pm

Today's topics :-

→ TDCE is NOT always PO (pareto optimal)

→ Some examples when it is

Example 1

$\{g_t\}_{t=0}^{\infty}$ $\quad \tau_{ct} = \tau_{kt}, \; \tau_{nt} = \tau_{xt} = 0$

Govt B.C. : $\sum_{t=0}^{\infty} p_t g_t = \sum_{t=0}^{\infty} T_t$

Claim : TDCE is PO in this case

TDCE

$$\sum_{t=0}^{\infty} \beta_t \, u(c_t, n_t) \longrightarrow \max$$

$$\text{s.t.} \quad \sum_{t=0}^{\infty} p_t c_t + p_t\left(k_{t+1} - (1-\delta) k_t\right) = \sum_{t=0}^{\infty} r_t k_t + w_t n_t + T_t$$

why?

FOC:

$$(c_t): \; \beta^t u_c'(t) = \lambda p_t$$

$$(n_t): \; \beta^t u_n'(t) = \lambda n_t$$

$$(k_{t+1}): \; -p_t + r_{t+1} + (1-\delta) p_{t+1} = 0$$

$$\frac{p_t}{p_{t+1}} = \frac{r_{t+1}}{p_{t+1}} + 1 - \delta$$

$$\Rightarrow \left[\begin{aligned} &\frac{u_c'(t)}{\beta u_c'(t+1)} = \frac{r_{t+1}}{p_{t+1}} + 1-\delta \\ &\frac{u_n'(t)}{\beta u_c'(t+1)} = \frac{w_t}{p_t} \quad f_N'(t) \\ &c_t + x_t + g_t = F(k_t, n_t) \end{aligned}\right]$$

$F_k'(t+1)$

TDCE equilibrium conditions :

Consider SPP :—

$$\sum_{t=0}^{\infty} \beta^t u(c_t, n_t) \longrightarrow \max_{\{c_t, n_t\}_{t=0}^{\infty}}$$

s.t. $c_t + x_t = \hat{F}(k_t, n_t)$, $\hat{F}(k_t, n_t) = F(k_t, n_t) - g_t$

$k_{t+1} = (1-\delta)k_t + x_t$

k_0 is given

FOC for SPP :—

(c_t) $\beta^t u'_c(t) = \lambda_t$

(n_t) $\beta^t u'_n(t) = \lambda_t \hat{F}'_n(t)$

(k_{t+1}) $-\lambda_t + \lambda_{t+1}[\hat{F}'_k(t+1) + 1 - \delta] = 0$

$$\frac{u'_c(t)}{\beta u'_c(t+1)} = \hat{F}_k(t+1) + 1 - \delta$$

$$\frac{u'_n(t)}{u'_c(t)} = \hat{F}_n(t)$$

This is the same as for TDCE (g_t is a constant).

$\Rightarrow$ TDCE is PO in this case (lump sum taxes & transfers)

Example 2:

$\tau_{kt} = \tau_{nt} = \tau_t \quad \forall\ t$

$\tau_{ct} = \tau_{xt} = 0 \quad \forall\ t$

$T_t = 0 \quad \forall\ t$

Govt. B.C. : $\quad p_t g_t = \tau_t r_t k_t + \tau_t w_t n_t \quad \forall\ t$ (B.C. is cleared every pd $\Rightarrow$ stricter condn than ex 1)

$$\sum \beta^t u(c_t, n_t) \longrightarrow \max$$

$$\sum_{t=0}^{\infty} p_t c_t + p_t(k_{t+1} - (1-\delta)k_t) = \sum_{t=0}^{\infty}(r_t k_t(1-\tau_t) + w_t n_t(1-\tau_t))$$

FOC: –

(c_t): $\beta^t u'_c(t) = \lambda p_t$

(n_t): $\beta^t u'_n(t) = \lambda w_t(1-\tau_t)$

(k_{t+1}): $-p_t + p_{t+1}(1-\delta) + r_{t+1}(1-\tau_{t+1}) = 0$

$$\frac{p_t}{p_{t+1}} = \frac{r_{t+1}}{p_{t+1}}(1-\tau_{t+1}) + 1 - \delta$$

$$\Rightarrow \left[\begin{aligned} \frac{u'_c(t)}{\beta u'_c(t+1)} &= \frac{r_{t+1}}{p_{t+1}}(1-\tau_{t+1}) + 1-\delta \\ \frac{u'_n(t)}{u'_c(t)} &= \frac{w_t}{p_t}(1-\tau_t) \\ c_t + x_t + g_t &= f(k_t, n_t) \end{aligned}\right]$$

Characterizes the equilibrium

where does this come from?

SPP

$$\sum_{t=0}^{\infty} \beta^t u(c_t, n_t) \to \max$$

s.t. $c_t + x_t = (1-\tau_t) \cdot f(k_t, n_t)$ (Redefine prodⁿ technology)

$0 \leq n_t + l_t \leq 1$

k_0 is given

FOC

$$\beta^t u'_c(t) = \lambda_t$$

$$\beta^t u'_n(t) = \lambda_t (1-\tau_t) F'_n(t)$$

$$-\lambda_t + \lambda_{t+1} \left[(1-\tau_{t+1}) F_k(t+1) + 1-\delta \right] = 0$$

$$\boxed{\frac{u'_c(t)}{\beta u'_c(t+1)} = (1-\tau_{t+1}) F'_k(t+1) + 1-\delta \qquad \frac{u'_n(t)}{u'_c(t)} = (1-\tau_t) F'_n(t)}$$

Equilibrium conditions is the same as before.

Now we have to show that feasibility constraint in CE & TDCE is the same.

$c_t + x_t + g_t = f(k_t, n_t) \to$ FC in CE

$$p_t c_t + p_t x_t + p_t g_t = p_t F(k_t, n_t)$$

$$= \quad p_t c_t + p_t x_t + \tau_t (r_t k_t + w_t n_t) = p_t F(k_t, n_t)$$

Under CRS :-

$$p_t c_t + p_t x_t = (1-\tau_t)\, p_t F(k_t, n_t) + \tau_t \big(p_t F(k_t, n_t) - w_t n_t - r_t k_t\big)$$

Under CRS, this is 0.

There is no profit

$$\Leftrightarrow \quad p_t c_t + p_t x_t = (1-\tau_t) p_t F$$

$$\Leftrightarrow \quad c_t + x_t = (1-\tau_t) F \quad \text{— FC in SPP}$$

- Add some t_c to this, & you'll see that the cond^n are not the same.

$\boxed{\text{Example 3}}$:

Same world as example 2

$\tau_{n_t} = \tau_{k_t} = \tau_t = 0.2$

Suppose $\tau \uparrow$ to 30%. (unexpected change so that agents do not have a chance to change their behavior prior to tax change)

Suppose labor supply inelastic $u'_n = 0$

EE:

$$u'_c(t) = \beta u'_c(t+1) \left[F'_k(t+1)(1-\tau_{t+1}) + 1 - \delta\right]$$

steady-state $k^*_{0.2}$

$$k_{t+1} = (1-\delta) k_t + x_t \Rightarrow x^*_{0.2} = \delta k^*_{0.2}$$

They are same in SS.

From feasibility constraint;

$$C_t = F(k_t) - k_{t+1} + (1-\delta)k_t \Rightarrow c^*_{0.2} = F(k^*_{0.2}) - \delta k^*_{0.2}$$

From EE; $\left(u'_c(t) = u'_c(t+1)\right)$ in SS :—

$$1 = \beta\left[(1-0.2)F'_k(k^*_{0.2}) + 1 - \delta\right]$$

Assume $F(k_t) = Ak_t^\alpha$, $\alpha = 1/3$

$$F' = \frac{\frac{1}{\beta} - (1-\delta)}{1-0.2}$$

$$F' = \left(Ak^\alpha\right)' = A\alpha k^{\alpha-1}$$

$$\Rightarrow A\alpha\left(k^*_{0.2}\right)^{\alpha-1} = \frac{\frac{1}{\beta} - (1-\delta)}{1-0.2}$$

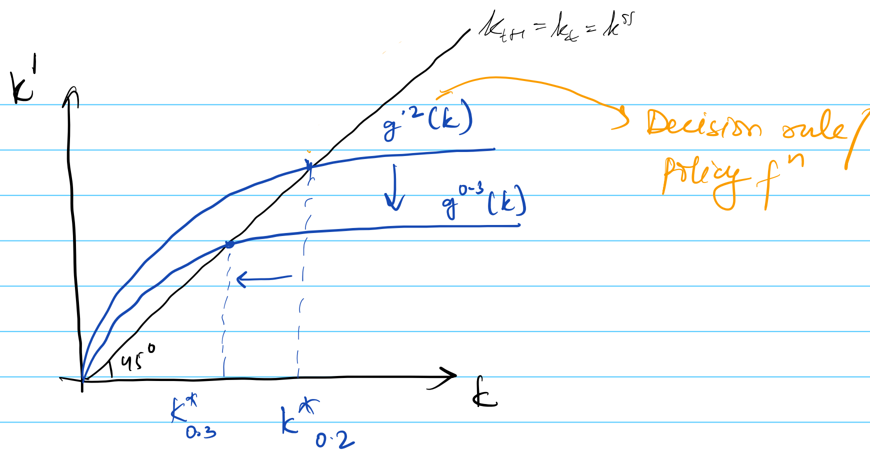
$$\Rightarrow k^*_{0.2} = \left[\frac{\text{const}}{(1-0.2)}\right]^{1/\alpha-1}$$

This is negative

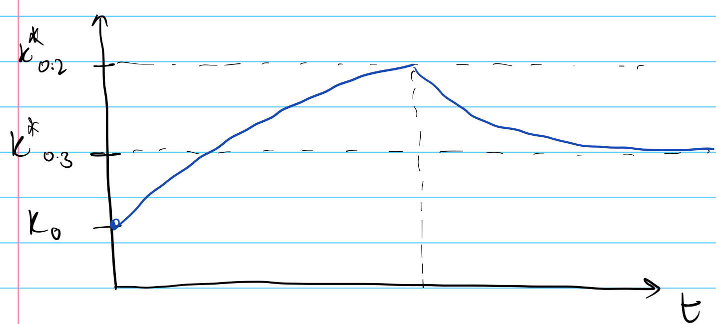
$k \downarrow$ & $c \uparrow$

$$\Rightarrow \frac{k^*_{0.3}}{k^*_{0.2}} = \frac{\cancel{\text{const}}\,(1-0.2)^{1/\alpha-1}}{(1-0.3)^{1/\alpha-1}\cdot\cancel{\text{const}}}$$

$$= \left(\frac{1-0.2}{1-0.3}\right)^{1/\alpha-1} \simeq 0.82$$



k'
k
$k_{t+1} = k_t = k^{ss}$
$g^{0.2}(k)$
$g^{0.3}(k)$
45°
$k^*_{0.3}$
$k^*_{0.2}$
Decision rule/
Policy f^n



$k^*_{0.2}$
$k^*_{0.3}$
k_0
t

Why CRS → no profits?

$f(x) = 2x$

$py - wx$

$pf(x) - wx$

foc = $2p = w$

$p\cdot(2x) - (2p)w = 0$ (zero profit)