Oct 24, 2023.

Today's plan :-

→ Tax - distorted CE (TDCE)
→ Ramsey Problem

Notation

$\{g_t\}_{t=0}^{\infty}$ — sequence of govt. expenditures

- τ_{ct} — Consumption Tax
- τ_{xt} — Investment Tax
- τ_{nt} — Labor Income Tax
- τ_{kt} — Capital Income Tax
- T_i — lump-sum tax $(T_t < 0)$ or transfer $(T_t > 0)$

TBO's.

Defn : A tax distorted CE (TDCE) given the fiscal policy $\{g_t, \tau_{ct}, \tau_{nt}, \tau_{xt}, \tau_{kt}, T_t\}_{t=0}^{\infty}$ is the price system $\{p_{ct}, p_{xt}, w_t, r_t\}_{t=0}^{\infty}$ allocation for the consumer $\{c_t, x_t, n_t, l_t, k_t\}_{t=0}^{\infty}$, and allocations for firms $\{y_t, k_t^f, n_t^f\}_{t=0}^{\infty}$ s.t :-

1) Given the prices, HH solves :-

$$\sum_{t=0}^{\infty} \beta^t u(c_t, l_t) \rightarrow \max_{\{c_t, l_t, n_t, k_t, x_t\}_{t=0}^{\infty}}$$

Investment

s.t.

3) Markets clear

$$n_t = n_t^f$$

$$k_t = k_t^f$$

$$c_t + x_t + g_t = F(k_t, n_t) \longrightarrow$$ (As c_t, x_t are produced with the same tech; the prices of all of them would be the same).

4) Government BC:

$$\sum_{t=0}^{\infty} (p_t g_t + T_t) = \sum_{t=0}^{\infty} \left(\tau_{ct} p_{ct} c_t + \tau_{xt} x_t p_{xt} + \tau_{nt} w_t n_t + \tau_{kt} k_t r_t \right)$$

* Govt. BC is redundant

HH BC:

$$\sum_{t=0}^{\infty} \left[p_t(1+\tau_{ct}) c_t + p_t(1+\tau_{xt}) x_t \right] = \sum_{t=0}^{\infty} \left[r_t k_t (1-\tau_{kt}) + w_t n_t (1-\tau_{nt}) + T_t \right]$$

$$\sum_{t=0}^{\infty} \left[p_t c_t \tau_{ct} + p_t \tau_{xt} x_t + \tau_{kt} r_t k_t + w_t n_t \tau_{nt} \right] = \sum_{t=0}^{\infty} \left[r_t k_t + w_t n_t - p_t c_t - p_t x_t + T_t \right]$$

Assume $F(\cdot)$ is CRS $\Rightarrow$ 0 profits.

$$p_t y_t = w_t n_t + r_t k_t$$

$$\Rightarrow p_t(c_t + x_t + g_t) = w_t n_t + r_t k_t \quad \text{(feasibility constraint)}$$

$$\sum_{t=0}^{\infty} \left[p_t c_t \tau_{ct} + p_t \tau_{xt} x_t + \tau_{kt} r_t k_t + w_t n_t \tau_{nt} \right] = \sum_{t=0}^{\infty} \left[\underbrace{r_t k_t + w_t n_t - p_t c_t - p_t x_t}_{p_t g_t} + T_t \right]$$

GBC

Exercise :– relax CRS assumption & show the same holds.

RAMSAY PROBLEM :–

$\{g_t\}$ is given,

$\tau_{ct} = \tau_{xt} = 0$
$T_t = 0$

Optimal sequence of $\{\tau_{kt}, \tau_{nt}\}$?

$$\sum_{t=0}^{\infty} \beta^t u(c_t, n_t) \longrightarrow \max$$

s.t. $\{c_t(\tau), k_t(\tau), n_t(\tau), l_t(\tau), x_t(\tau)\}$ is TDCE

where $\sum_{t=0}^{\infty} p_t g_t = \sum_{t=0}^{\infty} [r_t k_t \tau_{kt} + w_t n_t \tau_{nt}]$

B.C.

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t u(c_t, l_t) + \lambda \Big[\sum_{t=0}^{\infty} \big(r_t k_t (1-\tau_{kt}) + (1-\tau_{nt}) w_t n_t \big) - \sum_{t=0}^{\infty} [p_t c_t + p_t x_t] \Big] + \sum_{t=0}^{\infty} \beta^t \mu_t [(1-\delta) k_t + x_t - k_{t+1}]$$

↳ Law of motion of capital

FOC :

(c_t) :

Firm's Problem

TDCE allocation (given policy $\{\tau_{kt}, \tau_{nt}, g_t\}_{t=0}^{\infty}$) is characterized by :-

1) Normalize $p_0 = 1$: $p_t = \dfrac{\beta^t u'_c(t)}{u'_c(0)}$

2) $$\frac{u'_n(t)}{u'_c(t)} = f'_n(t)(1-\tau_{nt})$$

3) $$u'_c(t) = \beta\, u'_c(t+1)\left[f'_k(t+1) + 1 - \delta\right]$$

4) $$f'_k = \frac{r_t}{p_t}$$

5) $$f'_n = \frac{w_t}{p_t}$$

6) $$n_t = n_t^f$$

7) $k_t = k_t^{f}$

8) $c_t + g_t + x_t = F(k_t, n_t)$

9) $\sum_{t=0}^{\infty} [p_t c_t + p_t x_t] = \sum_{t=0}^{\infty} \left(r_t k_t (1 - \tau_{kt}) + w_t n_t (1 - \tau_{nt}) \right)$

10) Law of motion for k_t

$$k_{t+1} = (1-\delta) k_t + x_t$$