Econ 7710
Assignment 4

The due date for this assignment is Friday October 20th

1. Suppose that $X \sim N(0,1)$ and $Y \sim N(0,1)$ and X and Y are independent. Find the distribution of random variable $Z = X/Y$ and determine which moments of this random variable exist.

2. Suppose that $\{X_n\}_{n=1}^{\infty}$ is such that $X_n \xrightarrow{d} X$, where $X \sim N(0,1)$. Suppose that $Y_n = X_n$ for all $n \geq 1$.

   (a) Find the distribution limit of Y_n

   (b) Consider the distribution limit of Y_n, $Y_n \xrightarrow{d} Y$. Prove or disprove that $X_n + Y_n \xrightarrow{d} X + Y$. Comment your findings.

3. The *median* of the distribution of random variable X is the number $q_{.5}$ that solves

$$\inf_q \left\{ P(X \leq q) \geq \frac{1}{2} \right\}$$

   Suppose that for the sequence of random variables X_n there exists a numeric sequence a_n such that $X_n - a_n \xrightarrow{p} 0$. Let $q_{.5}^n$ be the median of the distribution of X_n.

   (a) Prove that $\lim_{n\to\infty} (q_{0.5}^n - a_n) = 0$.

   (b) Prove or disprove that $\lim_{n\to\infty} (E[X_n] - a_n) = 0$

4. Suppose that $X_1, X_2, \ldots$ is a sequence of independent and identically distributed random variables and $X_n \xrightarrow{p} X$. Prove that X has a degenerate distribution.