Individual agent
↓
Single decision problem

Micro (Wed Aug 23)

1. Firm → Varian.
2. Consumer Theory → MWG
3. Choice under uncertainty.

---

① Firm Theory :-

Assumptions :-

1. Firms are price takers
2. Tech is exogenous
3. Firms maximize profits.

Technology :-

- n commodities, $y = (y_1, \ldots, y_n)$ $(y \in \mathbb{R}^n)$
  - $y_i < 0$, i is an input
  - $y_i > 0$, i is an output.
- Production Set : $Y \subseteq \mathbb{R}^n$ — what the firm can do.

$n = 3$

$y = (-2, 1, -3)$

Firm's function : 2 units of som as input; 1 Output; 3 units of something as input

Properties of Production sets: —

1. $Y \neq \phi$
2. Y is closed
3. No free lunch. If $y \in Y$ and $\underbrace{y \geq 0}_{\text{there are no inputs.}}$, then $\underbrace{y = 0}_{\text{then you can't make anything.}}$

The following are sometimes assumed: —

4. Shutdown is possible: $0 \in Y$
5. Free disposal: If $y \in Y$ and $y' \leq y$, then $y' \in Y$.
(You can take more input, and not lead to increased output, say you get rid of it.)
6. Irreversibility: $y \in Y$, $y \neq 0$, then $-y \notin Y$
You can't take the output to make the input.

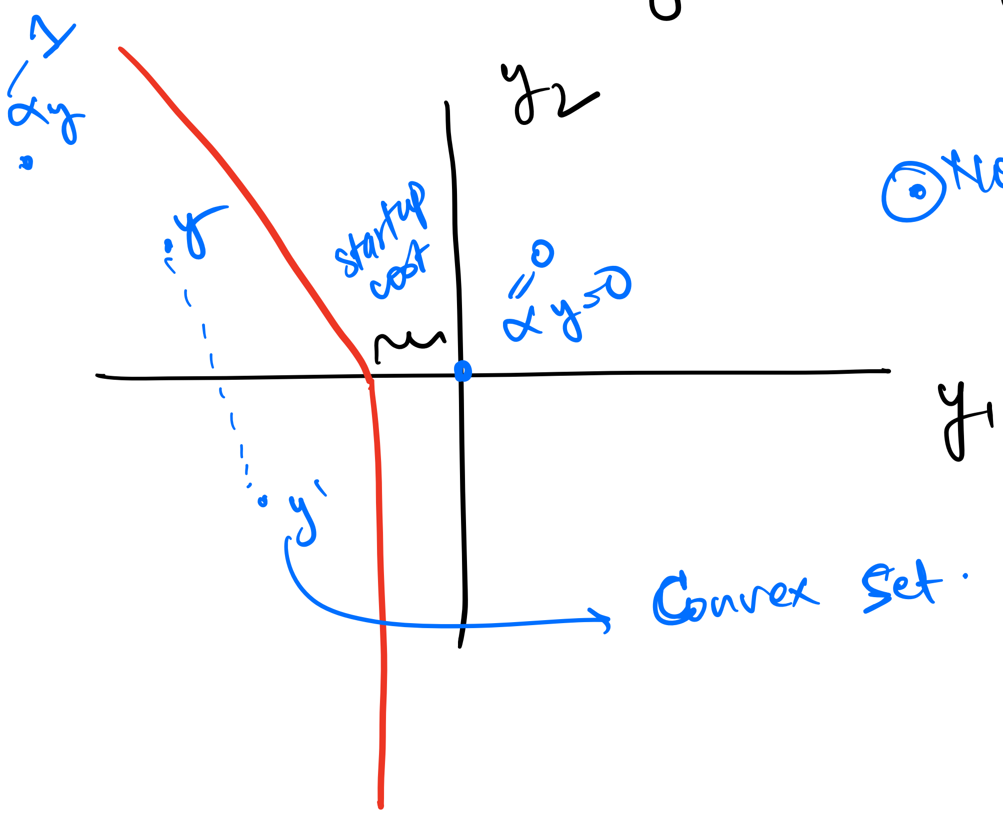
Returns to scale & Convexity

Y has:

- nonincreasing returns to scale if
  $y \in Y \Rightarrow \alpha y \in Y \quad \forall \quad \alpha \in [0, 1]$
- non decreasing returns to scale if.
  $y \in Y \Rightarrow \alpha y \in Y \quad \forall \quad \alpha \geq 1$
- Constant returns to scale
  $y \in Y \Rightarrow \alpha y \in Y \quad \forall \quad \alpha \geq 0$

Y is convex. if:

$$y, y' \in Y \Rightarrow \alpha y + (1-\alpha)y' \in Y \quad \forall \alpha \in [0,1]$$



⊙ Non decreasing returns to scale.

Why do we "usually" assume non increasing returns?

- Modelling only inputs under our control

Theorem

There is a constant returns production set $Y' \subseteq \mathbb{R}^{n+1}$ s.t. $Y = \{y \in \mathbb{R}^n : (y,-1) \in Y'\}$

Proof Define $Y' = \{y' = \mathbb{R}^{n+1} : y' = \alpha(y,-1)$ for some $y \in Y$ & $\alpha \geqslant 0\}$

Check that Y' in constant returns.

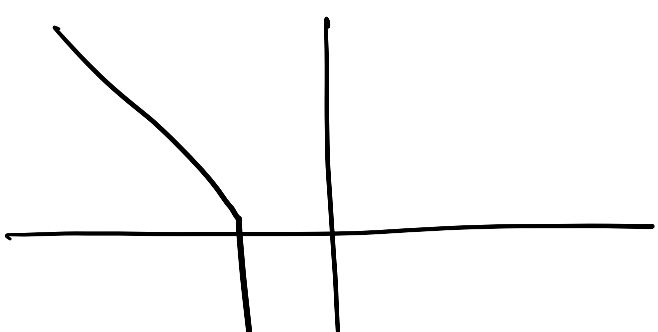
Efficiency :-

Technological efficiency:

A production plan y is technologically efficient if there does not exist $y' \in Y$ s.t. $\underbrace{y' > y}$

↓

$y_i' \geq y_1$ for all i & $y_i' > y_i$ for some i.



{ efficient pts are going to be on the } boundary.

Aug 28 (second class)

Transformation frontier.

firm, production set Y

The transformation function $T \; \mathbb{R}^n \to \mathbb{R}$

$T(y) < 0 \iff y$ is inefficient

$T(y) = 0 \iff y$ is efficient

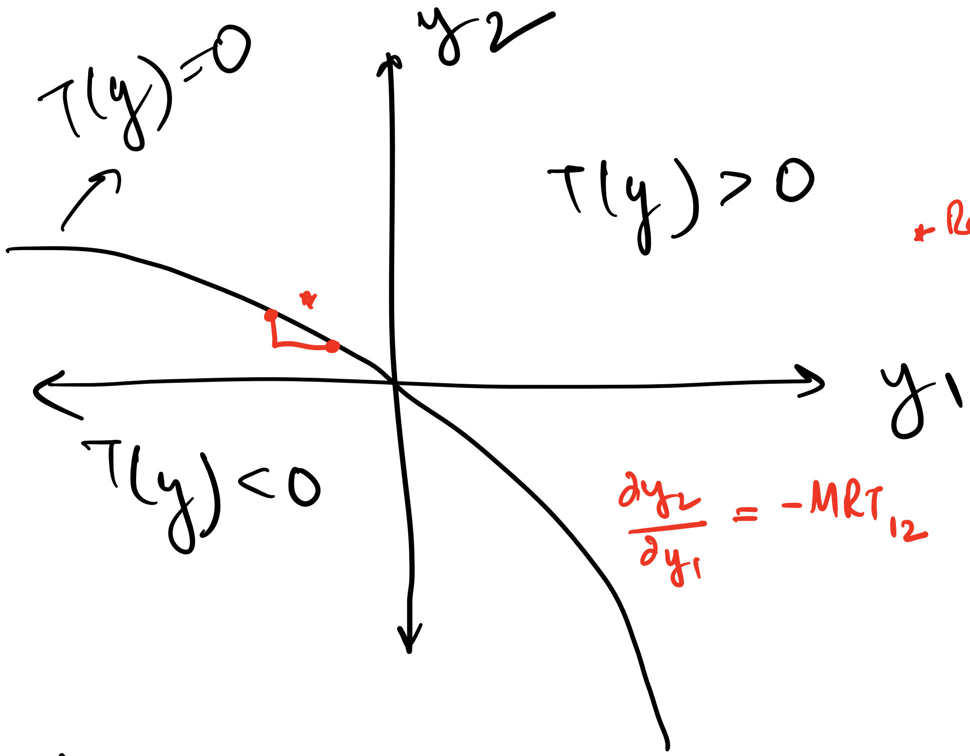
$T(y) > 0 \iff y$ is infeasible.

$\Rightarrow Y = \{y \in \mathbb{R}^n : T(y) \leq 0\}$

~~To check~~

Production frontier

$\{y \in \mathbb{R}^n : T(y) = 0\}$



* Rate of substitution.

Lets fix $\bar{y}_3, \dots \bar{y}_n$. Consider $y_1, y_2, \dots$

Define a function $y_2(y_1)$ by

$$T(y_1, \underbrace{y_2(y_1)}_{\text{implicit } f^n \text{ of } y_1}, \underbrace{\bar{y}_3, \dots \bar{y}_n}_{\text{constants.}}) = 0$$

Pick some y_1, what y_2 gives me $T(y)=0$

Let's differentiate both sides wrt y_1

$$\frac{\partial T}{\partial y_1} + \frac{\partial T}{\partial y_2} \cdot \frac{\partial y_2}{\partial y_1} = 0$$

slope in the picture

$$\frac{\partial y_2}{\partial y_1} = \frac{-\partial T/\partial y_1}{\partial T/\partial y_2}$$

Works for any 2 goods

$$\frac{\partial y_k}{\partial y_j} = -\left.\frac{\partial T/\partial y_j}{\partial T/\partial y_k}\right|_{y=\bar{y}} = -MRT_{jk}(\bar{y})$$

(Marginal Rate of Transformation)

My Ques:

Special Case: Many inputs, one output.

Inputs $(x_1 \ldots, x_m) \geq 0$

Output: $y \geq 0$

Production function: $f(x)$

Production set: $Y = \{(\underbrace{-x_1, -x_2, \ldots, -x_m}_{m}, y):$ Inputs.

$x_i \geq 0 \; \forall i$ & $\quad n = m+1$.

$y \leq f(x_1, \ldots x_m)\}$

$T(z) = y - f(x), \; z = (-x, y)$

Efficient frontier is

$T(z) = 0$

$\Rightarrow y - f(x) = 0$

$\rightarrow y = f(x)$

Example

Cobb-Douglas

- 2 inputs, capital (x_K) + labor (x_L)
- Output y
- $f(x_K, x_L) = x_K^{\alpha} x_L^{\beta}$, $\alpha + \beta = 1$
- $Y = \{(-x_K, -x_L, y) : x_K, x_L \geq 0,\ y \leq x_K^{\alpha} x_L^{\beta}\}$
- $T = (-x_K, -x_L, y) = y - x_K^{\alpha} x_L^{\beta}$

MRT between capital + output.

$$MRT_{(-x_K), y} = \frac{\partial T / \partial(-x_K)}{\partial T / \partial y}$$

$$= - \frac{\partial T / \partial x_K}{\partial T / \partial y}$$

$$= - \frac{- \alpha x_K^{\alpha - 1} x_L^{\beta}}{1}$$

$$= \alpha \left(\frac{x_L}{x_K} \right)^{\beta}$$

Marginal product of capital, MPK.

$$MPK = f_{x_K}(x_K, x_L) \rightarrow$$ from undergrad.

$$= \partial f / \partial x_K$$

$$= \alpha x_K^{\alpha-1} x_L^{\beta}$$

$$= \alpha \left(\frac{x_L}{x_K}\right)^{\beta}$$

The method now is more a general way of finding sol^ns.

MRT for capital & labor

$$MRT_{(-x_K),(-x_L)} = \frac{\partial T / \partial(-x_K)}{\partial T / \partial(-x_L)}$$

$$= \frac{\partial T / \partial x_K}{\partial T / \partial x_L}$$

$$= \frac{-\alpha x_K^{\alpha-1} x_L^{\beta}}{-\beta x_K^{\alpha} x_L^{\beta-1}}$$

$$= \frac{\alpha}{\beta}\left(\frac{x_L}{x_K}\right)$$

$$\frac{d(-x_L)}{d(-x_K)} = -MRT_{(-x_K,\,-x_L)}$$

$$\frac{dx_L}{dx_K} = -\frac{\alpha}{\beta}\frac{x_L}{x_K}$$

---

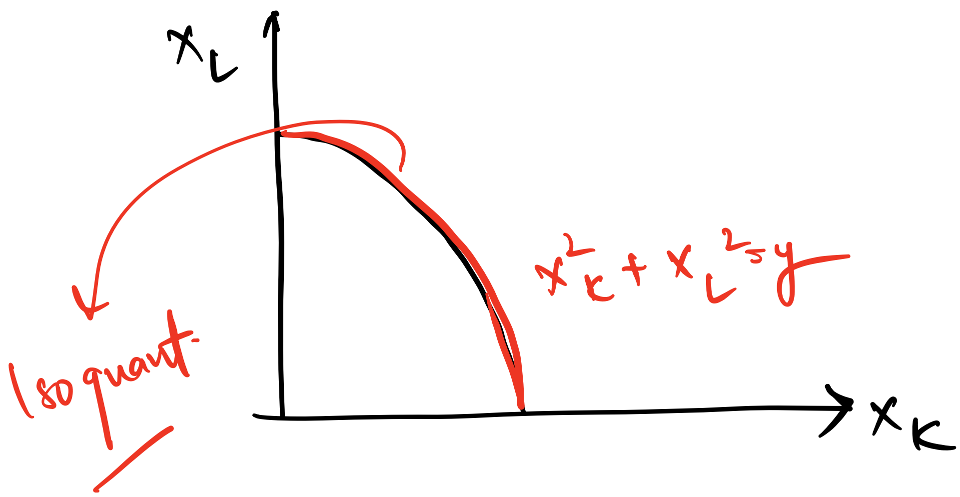
single output, fix y

$V(y) = \{x \in \mathbb{R}^n : f(x) \geq y\} \rightarrow$ Input requirement set.

$Q(y) = \{x \in \mathbb{R}^n : f(x) = y\} \rightarrow$ Isoquant.

Example:

$f(x_K, x_L) = x_K^2 + x_L^2 = y$



This is all fine as the inputs here are considered positive as we are only working with inputs.

With a single output, we can define the marginal rate of technical substitution between two inputs

$$MRTS_{x_k, x_l} = \frac{\partial f / \partial x_k}{\partial f / \partial x_l}$$

$$\frac{\partial x_l}{\partial x_k} = -MRTS_{x_k, x_l} = \frac{-f_{x_k}}{f_{x_l}}$$

In the example

$$\frac{dx_l}{dx_k} = \frac{-\partial x_k}{\partial x_l} = -\frac{x_k}{x_l}$$

This is a funny example because the slope is increasing (MRTS is usually decreasing). This is because $V(y)$ is not convex.

Theorem: If Y is convex, then $V(y)$ is also convex.

$(P \Rightarrow Q \iff \text{not } Q \Rightarrow \text{not } P)$

Proof $(-x, y) \in Y$

$(-x', y) \in Y$

Y convex $\Rightarrow$ $\underbrace{t(-x, y) + (1-t)(-x', y)}_{\text{linear combination.}} \in Y$

$\Rightarrow (-tx - (1-t)x', \; ty + (1-t)y) \in Y$

$\Rightarrow (-tx - (1-t)x', y) \in Y$

$\Rightarrow (-x'', y) \in Y$ where $x'' = tx + (1-t)x'$

$\Rightarrow x'' \in V(y)$

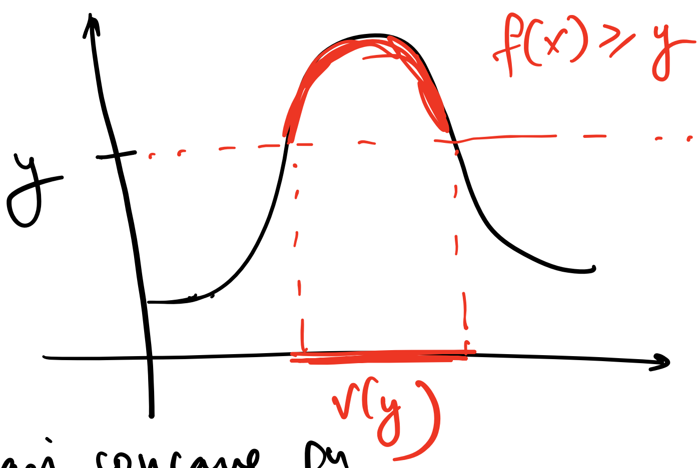
$\Rightarrow V(y)$ is convex.

<u>Theorem</u>: $V(y)$ is convex iff $f(x)$ is quasiconcave.

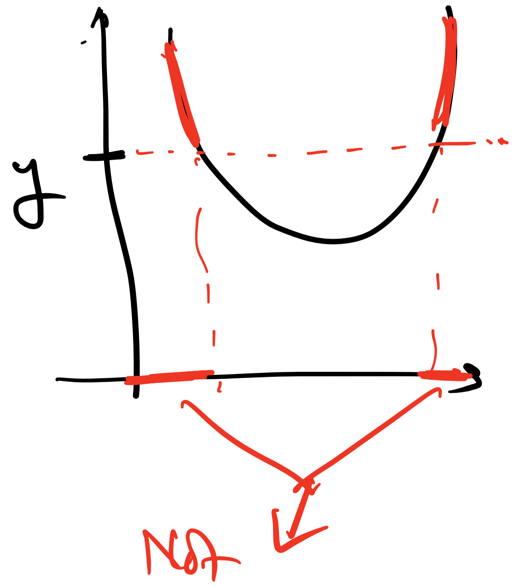
<u>Proof</u>: $V(y) = \{x \in \mathbb{R}^n : f(x) \geq y\}$ upper contour set of f.

(This more of a def^n)

Definition of a quasi concave f^n that has convex upper contour sets.



Quasi concave f^n.



$\boxed{\text{Concave f}^n \text{ are quasi concave f}^n.}$

Homogeneity + Homotheticity

A function $f: \mathbb{R}^n \to \mathbb{R}$ is <u>homogeneous of degree k</u>

if $\quad f(tx) = t^k f(x) \;\; \forall\, t = 0$

$k = 0 \;:\; f(tx) = f(x)$

$k = 1 \;:\; f(tx) = t f(x)$

Cobb-Douglas :

$$f(x_K, x_L) = x_K^{\alpha} x_L^{\beta} \qquad \alpha + \beta = 1.$$

$$f(tx_K, tx_L) = (tx_K)^{\alpha} (tx_L)^{\beta}$$

$$= t^{\alpha+\beta} x_K^{\alpha} x_L^{\beta} \xrightarrow{\text{when } \alpha+\beta=1.}$$

$$= t\, x_K^{\alpha} x_L^{\beta} = t f(x)$$

What about MRTS?

$$MRTS = \frac{\alpha}{\beta}\frac{x_L}{x_K}$$

$$= \frac{\alpha}{\beta}\frac{(tx_L)}{(tx_K)} = \frac{\alpha}{\beta}\frac{x_L}{x_K}$$

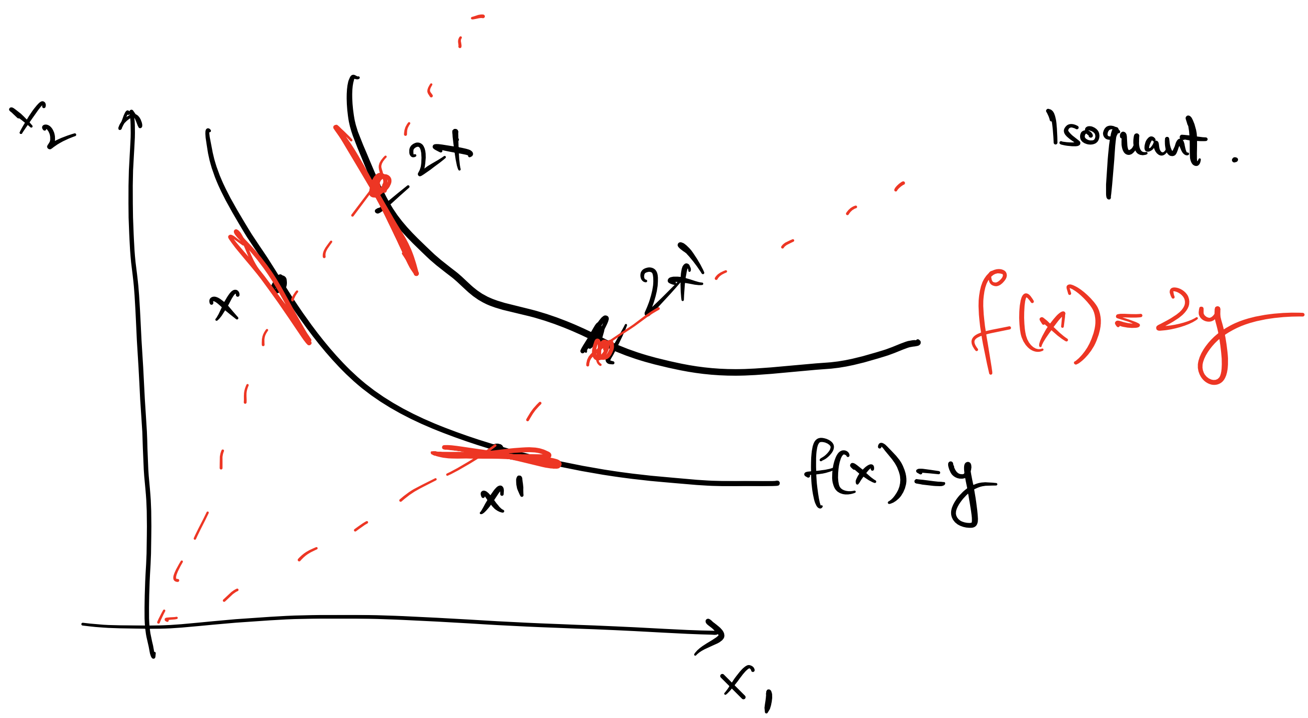
MRTS is homogeneous of degree 0.

Theorem: If f is hd-k, then the partial derivative f_{x_i} is hd-$(k-1)$.

<u>Proof</u>: $f(tx) = t^k f(x)$

$t f_{x_i}(tx) = t^k f_{x_i}(x)$

$f_{x_i}(tx) = t^{k-1} f_{x_i}(x)$



Say f is hd-1

$f(2x) = 2f(x) = 2y$

The slopes would be the same too.

$\Rightarrow$ Every isoquant is just a blown up version of the first one.