ECONOMETRICS I
ECON GR5411

Lecture 5 – Linear Regression Model I

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Least Squares Regression:

OLS objective function: minimize sum of squared errors

$$\min_b S(b) = (y - Xb)'(y - Xb)$$

$$= y'y - y'Xb - b'X'y + b'X'Xb$$

$$= y'y - 2y'Xb + b'X'Xb$$

The first order conditions:

$$\frac{\partial S(b)}{\partial b} = -2X'y + 2X'Xb$$

Setting $\frac{\partial S(b)}{\partial b} = 0$ and solving for b, gives us the estimator that minimizes $S(b)$ (must check the second order conditions to make sure we are minimizing and not maximizing)

Least Squares Regression:

$$\frac{\partial S(b)}{\partial b} = -2X'y + 2X'Xb = 0$$

$$2X'Xb = 2X'y$$
$$X'Xb = X'y$$

This is known as normal equations. Hence

$$b = (X'X)^{-1}X'y$$

As long as $X'X$ is non-singular

$$\begin{pmatrix} = X'X \text{ has full rank} \\ = \text{the inverse of } X'X \text{ exists} \\ = \text{the columns of } X'X \text{ are linearly independent} \end{pmatrix}$$

The solution that satisfies the FOC

$$\hat{\beta} = (X'X)^{-1}X'y$$

Least Squares Regression:

Verifying SOC

$$\frac{\partial^2 S(b)}{\partial b \partial b'} = 2X'X$$

$X'X$ must be a positive definite matrix.

Least Squares Regression:

So Normal equations are solved uniquely for b
and by pre-multiplying both sides of them by $(X'X)^{-1}$

$$\hat{\beta} = (X'X)^{-1}X'y$$

Viewed as a function of the sample (y, X), b called (ordinary) least squares **estimator**. For a given sample (y, X), the value of this function is the OLS **estimate**.

Two terms are used almost interchangeably.

Vector and Matrix Notation Match:

Simple regression model:

Vector version:

$$y_i = \boldsymbol{x}_i'\beta + \varepsilon_i$$

Matrix version:

$$y = X\beta + \varepsilon$$

Matching:

$$\sum_{i=1}^{n} \boldsymbol{x}_i \boldsymbol{x}_i' = X'X$$

$$\sum_{i=1}^{n} \boldsymbol{x}_i y_i = X'y$$