

**Econ 7710**  
**Assignment 3**

**The due date for this assignment is Friday September 29th**

1. Consider density function

$$f(x_1, x_2) = \begin{cases} \frac{1}{4}(1 + x_1x_2), & \text{if } (x_1, x_2) \in [-1, 1] \times [-1, 1] \\ 0, & \text{if } (x_1, x_2) \notin [-1, 1] \times [-1, 1]. \end{cases}$$

Find density function  $g(x_1, x_2)$  which is not equal to  $f(x_1, x_2)$  almost everywhere on  $[-1, 1] \times [-1, 1]$  but has the same marginal densities.

2. For the discrete random variable  $X$  with probability mass function

$$P(X = 2^n) = \frac{1}{e n!}, \quad n = 0, 1, 2, \dots$$

- (a) Prove that this random variable has moments of all orders and find them
  - (b) Find the characteristic function of this random variable
  - (c) Find the moment generating function of this random variable.
3. Suppose that  $X_1$  and  $X_2$  are independent  $N(0, 1)$  random variables. Two-dimensional normal random variable  $\mathbf{Y}$  is constructed as

$$\mathbf{Y} = (Y_1, Y_2) = \begin{cases} (X_1, |X_2|), & \text{if } X_1 \geq 0 \\ (X_1, -|X_2|), & \text{if } X_1 < 0. \end{cases}$$

- Find marginal distributions of  $(Y_1, Y_2)$
  - Are  $(Y_1, Y_2)$  jointly normally distributed? Prove or disprove.
4. Prove that

$$\phi(t) = \begin{cases} 1 - t^2, & |t| < 1, \\ 0, & |t| \geq 1, \end{cases}$$

is not a characteristic function.