Capital Overaccumulation in OLG

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This note explains the derivation on page 15 of "Overlapping Generations Models," which demonstrates how a social planner can achieve a Pareto-improved allocation.

When $k^* > k^{GR}$, reduced saving can increase consumption for every generation. Note that in steady state

$$\begin{aligned} f(k^*) - (1+n)k^* &= c^{y*} + (1+n)^{-1}c^{o*} \\ &\equiv c^* \end{aligned}$$

where the first line is just the aggregate resource constraint and the second defines c^*.

The impact of a change in k^* on c^* is

$$\frac{\partial c^*}{\partial k^*} = f'(k^*) - (1+n) \tag{1}$$

k^{GR} is defined as $f'(k^{GR}) = 1 + n$.

If $k^* > k^{GR}$, then $\partial c^* / \partial k^* < 0$. That is, reducing saving can increase total consumption for everyone. This is the case of dynamic inefficiency, which arises from overaccumulation of capital. Dynamic inefficiency implies that $r^* < n$.

Suppose we start from a steady state at T with $k^* > k^{GR}$. Consider the variation: change next period's capital stock by $-\Delta k$, where $\Delta k > 0$, and from then on we immediately move to a new steady state. This is clearly feasible.

Then consumption levels change as follows:

$$\begin{aligned} \Delta c_T &= (1+n)\Delta k > 0 \\ \Delta c_t &= -[f'(k^* - \Delta k) - (1+n)]\Delta k, \text{ for all } t > T \end{aligned}$$

The first expression is the direct increase in consumption due to decreased saving. The second expression comes from perturbing (1) by Δk.

In addition, since $k^* > k^{GR}$, for small enough Δk, $f'(k^* - \Delta k) - (1+n) < 0$ (because at k^*, $r^* < n$.) This implies that $\Delta c_t > 0$ for all $t \geq T$.

The increase in consumption for each generation can be allocated equally during the two periods of their lives, necessarily increasing the utility of all generations.

This explains the cryptic derivation on page 15.

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