TA session

Fri sep 8

From the core.

$$Y = (K+L)\ln(K+L) - K\ln K - L\ln L$$

1. $f(tK, tL) = t(K+L)\ln(t(K+L)) - tK\ln tK$

$- tL\ln tL$

$= tf(K, L)$ CRS.

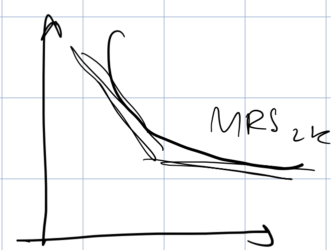
2. $MP_L = \ln(K+L) + (K+L)\frac{1}{K+L} - \ln L - 1$

$$= \ln\left(\frac{K+L}{L}\right)$$

$MP_L(tK, tL) = \ln\left(\frac{tK + tL}{tL}\right) = MP_L(K, L)$

HD-0.

(c) $f(K,L) = \bar{Y}$



For the isoquant to be convex,

$MRS_{L,K}$

MRS_{LK} has to be increasing in $K/L = k$

$$MRS_{LK} = \frac{\ln\left(\frac{K+L}{L}\right)}{\ln\left(\frac{K+L}{K}\right)} = \frac{\ln(k+1)}{\ln(1+k^{-1})}$$

$$\frac{\partial MRS_{L,K}}{\partial k} = \frac{\frac{1}{k+1}\ln(1+k^{-1}) + \frac{k^{-2}}{1+k^{2}}\ln(1+k)}{[\ln(1+k^{-1})]^2} > 0$$

Isoquants are convex.

(d) $\frac{1}{\varepsilon} = \frac{d MRS_{L,K}}{k} \cdot \frac{k}{MRS_{L,K}}$

$=$ When plugging the above, $\Rightarrow$ no, can't be a CES.

$\sigma = \frac{d \ln\left(\frac{x_2}{x_1}\right)}{d \ln\left(MRS_{12}\right)} = -\frac{d \ln\left(\frac{x_1}{x_2}\right)}{d \ln\left(MRS_{12}\right)}$

Check out wikipedia.

(e) $\frac{\partial f / \partial x_1}{\partial f / \partial x_j} = \frac{\ln\left(\sum_{k=1}^{n} \frac{x_k}{x_1}\right)}{\ln\left(\sum_{k=1}^{n} \frac{x_k}{x_j}\right)}$

$$H = \begin{pmatrix} f_{KK} & f_{KL} \\ f_{LK} & f_{LL} \end{pmatrix}$$

Concave → Hessian Matrix is negative semidefinite

$\det(H) \geq 0$

$f_{KK} < 0$