HW4_ECON7030_Seongin Hong
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#1 $P = a - bQ$, $C(q) = F + \alpha q + \frac{1}{2}\beta q^2$

(a) Since $(\alpha, \beta, a, b, F) = (1, 2, 10, 1, \frac{1}{64})$, then

$P = 10 - Q$, $C(q) = \frac{1}{64} + q + q^2$

At the long-run equilibrium, firms take market price as given, $\pi = 0$, & $P = MC$.

Since firms are identical, total output $Q = n \cdot q$.

Then, $10 - nq = 1 + 2q$. $\Rightarrow q = \frac{9}{2+n}$

And $\pi = (1+2q)q - \frac{1}{64} - q - q^2 = \cancel{q} + 2q^2 - \frac{1}{64} - \cancel{q} - q^2 = q^2 - \frac{1}{64} = 0$.

Hence, $q^{LR} = \frac{1}{8}$, then $n^{LR} = 70$, $p^{LR} = 10 - 70 \cdot \frac{1}{8} = \frac{10}{8} = \frac{5}{4}$.

$Q^{LR} = \frac{70}{8} = \frac{35}{4} = 8.75$.

$(p^{LR}, q^{LR}, Q^{LR}, n^{LR}) = (5/4, 1/8, \frac{35}{4}, 70)$

(b) Since n firm made a cartel, then cartel works as monopolist in the market and produce same amount of output and profits. Still $Q = n \cdot q$.

Cartel's joint profit maximization problem is

$$\max_Q P(Q) \cdot Q - C(Q) = \max_q P(nq) \cdot nq - nC(q)$$

$$= \max_q (a - bnq)nq - n(F + \alpha q + \tfrac{1}{2}\beta q^2).$$

FOC $an - 2bn^2 q - \alpha n - \beta n q = 0$.

$$q^* = \frac{n(a-\alpha)}{2bn^2 + \beta n} = \frac{a-\alpha}{2bn+\beta} \Rightarrow Q^* = n \cdot q = \frac{n(a-\alpha)}{2bn+\beta}$$

c) Since we need to find n^* that makes profit of firms in cartel zero.

$$\pi = p \cdot q - c(q) = (10 - n q^*) q^* - F - q^* - (q^*)^2$$

$$= \left(10 - \frac{9n}{2n+2}\right)\frac{9}{2n+2} - \frac{1}{64} - \frac{9}{2n+2} - \frac{9^2}{(2n+2)^2} = 0.$$

$$\Leftrightarrow \frac{99n+180}{(2n+2)^2} - \frac{1}{64}\cdot\frac{(2n+2)^2}{(2n+2)^2} - \frac{18n+18}{(2n+2)^2} - \frac{81}{(2n+2)^2} = 0$$

$$\Leftrightarrow 99n+180 - \frac{1}{4^3}4(n+1)^2 - 18n - 18 - 81 = 0$$

$$\Leftrightarrow 81n + 81 - \frac{1}{16}(n+1)^2 = 0.$$

$$\Leftrightarrow n^2+2n+1 - 1296n - 1296 = n^2 - 1294n - 1295 = 0.$$

$$\therefore n^* = 1295.$$

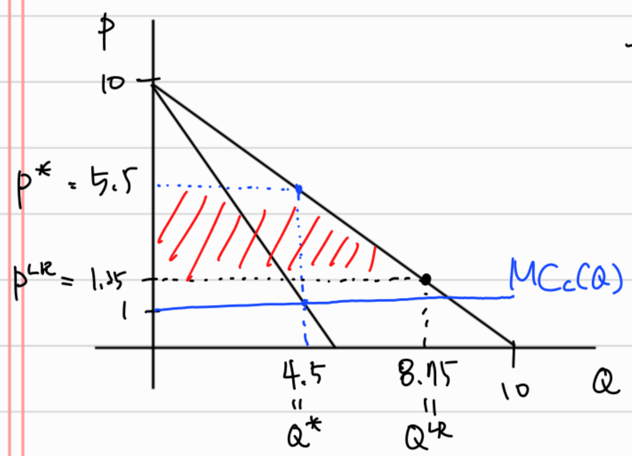
Then, $q^* = \frac{9}{2(1295)+2} = \frac{9}{2592} = \frac{1}{288}$

$$p^* = 10 - n^* q^* = 10 - \frac{1295}{288} = \frac{1585}{288} \approx 5.5$$

$$Q^* = n^* q^* = \frac{1295}{288} \approx 4.5.$$

$$\Rightarrow (p^*, q^*, Q^*, n^*) = (5.5, 1/288, 4.5, 1295)$$

(d)



TC of cartel $= TC_c = n\left(\frac{1}{64} + q + q^2\right)$

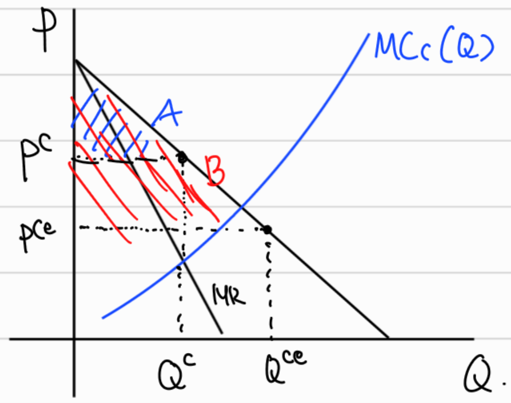
$$= \frac{n}{64} + nq + nq^2$$

$$= \frac{1}{n}\left(\frac{n^2}{64} + nQ + Q^2\right)$$

$$MC_c(Q) = 1 + \frac{2}{n}Q$$

$\Rightarrow$ Red Colored area is DWL.

(e)



In the long run competitive equilibrium, producers take price as given and then producers whose $MC = p^{ce}$, and $\pi = 0$ at long run enter market and produce. Since at p^{ce} we know total market demand Q^{ce}, then we can find out total number of producers n^{ce}.

On the other hand, if there is cartel in the market, producers want to join cartel since it is hard for a producer outside cartel to survive since he or she doesn't have market power. If cartel allows free entry, producers are going to join until $\pi = 0$. So, individual producers have same profit whether they are in cartel or at long run equilibrium. However, cartel works as a monopolist in the market, it sets market price above its marginal cost, So there must be positive profit without any other costs. But with fixed cost in the cost function

, that fixed cost can absorb all the profits under cartel.
If fixed cost is small enough to make producers enter market more than long run competitive equilibrium, the market under cartel with free entry can have lower consumer surplus ($A < B$), higher market price ($p^c > p^{ce}$) and more producers than competitive equilibrium.

#2 First, we need to find split point $\hat{x}$. For the consumer at $\hat{x}$, buying a good from firm 1 is indifferent from buying a good from firm 2. That is,

$$p_1 + t|\hat{x} - x_1| = p_2 + t|\hat{x} - x_2|$$

$$\Leftrightarrow p_1 + t(\hat{x} - x_1) = p_2 + t(x_2 - \hat{x})$$

$$\Rightarrow \hat{x} = \frac{p_2 - p_1}{2t} + \frac{x_1 + x_2}{2}.$$

Since there is no production cost,

$$\pi_1 = p_1 \cdot \hat{x} \quad , \quad \pi_2 = p_2 \cdot (1 - \hat{x})$$

FOC

$$\frac{d\pi_1}{dp_1} = \frac{p_2 - 2p_1}{2t} + \frac{x_1 + x_2}{2} = 0. \Rightarrow p_1 = \frac{1}{2}\left(p_2 + \frac{t(x_1 + x_2)}{2}\right)$$

$$\frac{\partial \pi_2}{\partial p_2} = \frac{p_1 - 2p_2}{2t} + 1 - \frac{x_1 + x_2}{2} = 0 \Rightarrow p_2 = \frac{1}{2}\left(p_1 + \frac{t(2 - x_1 - x_2)}{2}\right)$$

Since firm 2 commits to choosing price and revealing it to firm 1 before firm 1's decision, firm 2 must take firm 1 reaction into account when it decides its own price p_2. Therefore,

$$p_2 = \frac{1}{2}\left(p_1 + \frac{t(2 - x_1 - x_2)}{2}\right) = \frac{1}{2}\left(\frac{1}{2}p_2 + \cancel{\frac{t(x_1 + x_2)}{2}} + \frac{t(2 - \cancel{x_1} - \cancel{x_2})}{2}\right)$$

$$= \frac{1}{4}p_2 + \frac{1}{2}t. \Rightarrow p_2 = \frac{2}{3}t.$$

Then, after firm 2's decision, p_1 decides price based on p_2

$$p_1 = \frac{1}{2}\left(\frac{2}{3}t + \frac{t(x_1 + x_2)}{2}\right) = \frac{1}{2}\left(\frac{2}{3}t + \frac{t}{2}\right) = \frac{1}{3}t + \frac{1}{4}t = \frac{7}{12}t.$$

$$\therefore (p_1, p_2) = \left(\frac{7}{12}t, \frac{8}{12}t\right)$$

Therefore, $\hat{x} = \frac{\frac{1}{12}t}{2t} + \frac{1}{2} = \frac{13}{24} > \frac{1}{2}$

Since firm 2 chooses its price before firm 1 and should show it to firm 1, firm 2 should try to hold firm 1 at specific price level. To hold firm 1, firm 2 must think about firm 1's profit maximization problem, and internalize firm 1's profit maximization condition into its profit maximization condition. But firm 1 doesn't have to think about 2's strategy since it will be given when firm 1 choose its price, i.e, decision process is not simultaneous. Therefore, firm 2 suggests higher price to firm 1 and then firm 1 choose its price lower than 2's one, Moreover,

$$\pi_1 = p_1 \cdot \hat{x} = \frac{11}{12}t \cdot \frac{13}{24} = \frac{91}{288}t$$

$$\pi_2 = p_2 \cdot \hat{x} = \frac{8}{12}t \cdot \frac{11}{24} = \frac{88}{288}t.$$

> firm 1 gets more profit compared to firm 2.

#3

(a) It's Cournot oligopoly with n firm, zero marginal cost, and linear demand $p = 1 - Q$.

Nash equilibrium profit $\pi_N = \max_{q_i} (1 - q_i - (n-1)q_j) q_i$

$\underline{FOC}$: $1 - (n-1)q_j + 2q_i = 0 \Rightarrow q_N = \frac{1}{n+1}$, $p_N = \frac{1}{n+1}$ $\pi_N = \frac{1}{(n+1)^2}$

Collusion profit $\pi_C \Rightarrow \max_Q (1-Q)Q$, $\underline{FOC}$: $1 - 2Q = 0$ $Q = \frac{1}{2}$

$\Rightarrow q_C = \frac{1}{2n}$, $p_C = \frac{1}{2}$, $\pi_C = \frac{1}{4n}$

Defecting profit $\pi_D \Rightarrow \max_{q_i} \left(1 - (n-1)\frac{1}{2n} - q_i\right) q_i$

$FOC = \frac{n+1}{2n} - 2q_i = 0 \Rightarrow q_D = \frac{n+1}{4n}$, $p_D = \frac{n+1}{4n}$, $\pi_D = \left(\frac{n+1}{4n}\right)^2$

Incentive to collude

$$\Rightarrow \frac{\pi_C}{1-\delta} > \pi_D + \frac{\delta \pi_N}{1-\delta} \iff \delta_C > \frac{\pi_D - \pi_C}{\pi_D - \pi_N} = \frac{\left(\frac{n+1}{4n}\right)^2 - \frac{1}{4n}}{\left(\frac{n+1}{4n}\right)^2 - \left(\frac{1}{n+1}\right)^2}$$

$$= \frac{\frac{\cancel{n^2 - 2n + 1}}{\cancel{(4n)^2}}}{\frac{\cancel{(n^2-2n+1)}(n^2+6n+1)}{\cancel{(4n(n+1))^2}}} = \frac{n^2+2n+1}{n^2+6n+1}$$

(b) Since $\delta_C = \frac{(n+1)^2}{n^2+6n+1}$ is function of n, then $\lim_{n \to \infty} \delta_C = 1$, Also,

$$\frac{\partial \delta_C}{\partial n} = \frac{2(n+1)(n^2+6n+1) - (n+1)^2(2n+6)}{(n^2+6n+1)^2} = \frac{(n+1)(2n^2+12n+2 - 2n^2 - 8n - 6)}{(n^2+6n+1)^2}$$

$$= \frac{4(n+1)(n-1)}{(n^2+6n+1)^2}.$$

Therefore, δ_C decreases till $n=1$ and increases when $n > 1$

(c) Suppose that δ lies below its critical value, i.e, $\delta < \delta_c$.

$$\frac{\pi_c}{1-\delta} < \pi_D + \frac{\delta \pi_N}{1-\delta},$$

To make collusion attractive, Making defecting profit less attractive is one possible strategy. For example, if the collusion agreement includes that every period members of collusion share production facilities and producing outside of collusion high level facility investment fixed cost that makes $\frac{\pi_c}{1-\delta} > \pi_D + \frac{\delta \pi_N}{1-\delta} - F$.
Then, full collusion can be sustained.

(d) Let Collusion included $k(<n)$ firms. then $(n-k)$ firms plays nash strategy.
Thus, collusion's profit max problem is

$$\max \ (1-(n-k)q_j - Q)Q \quad \cdots ①$$

individual's profit max problem is

$$\max \ (1-(n-k-1)q_j - Q - q_i)q_i \quad \cdots ②$$

$$\rightarrow \text{FOC} \quad 1-(n-k)q_j - 2Q = 0 \qquad Q^k = \frac{1-(n-k)q_j}{2}$$

Then, ② turns into

$$\max \ \left(1-(n-k-1)q_j - \frac{1-(n-k)q_j}{2} - q_i\right)q_i$$

FOC

$$1-(n-k-1)q_j - \frac{1}{2} + \frac{(n-k)}{2}q_j - 2q_i = 0$$

$$\Leftrightarrow \frac{1}{2} + \frac{n-k-2n+2k+2}{2}q_j = 2q_i \qquad \Leftrightarrow \frac{1}{2} + \frac{2-n+k}{2}q_i = 2q_i$$

$$\Leftrightarrow \frac{1}{2} = \frac{n-k-2+4}{2}q_i \Rightarrow q_i^* = \frac{1}{n-k+2}$$

$$q_k^* = \frac{\frac{n-k+2-n+k}{k(n-k+2)}}{2} = \frac{1}{k(n-k+2)}$$

$$P = 1 - \frac{n-k}{n-k+2} - \frac{1}{n-k+2}$$

$$= \frac{n-k+2-n+k-1}{n-k+2} = \frac{1}{n-k+2}$$

$$\therefore P \cdot q_k = \frac{1}{k(n-k+2)^2}, \qquad P \cdot q_i = \frac{1}{(n-k+2)^2}$$

So if $k > 1$, $P \cdot q_k < P \cdot q_i$, no incentive to collude.
Hence, there will be no collusion with $k < n$

(e) Since this strategy collude to produce an agg. quantity not equal to monopoly quantity, $\pi_c'(Q') < \pi_c(Q^*)$ in (a)

Defecting profit max problem is

$$\max \left(1-\frac{n-1}{n}Q'-q_i\right) q_i = 0.$$

FOC : $1-\frac{n-1}{n}Q'-2q_i = 0 \Rightarrow \frac{n-(n-1)Q'}{2n} = q_d'$

$$p_d' = \frac{n-(n-1)Q'}{2n} \qquad \pi_d' = \frac{(n-(n-1)Q')^2}{4n^2}$$

$$\pi_c' = (1-Q')\cdot\frac{Q'}{n} = \frac{(1-Q')Q'}{n}$$

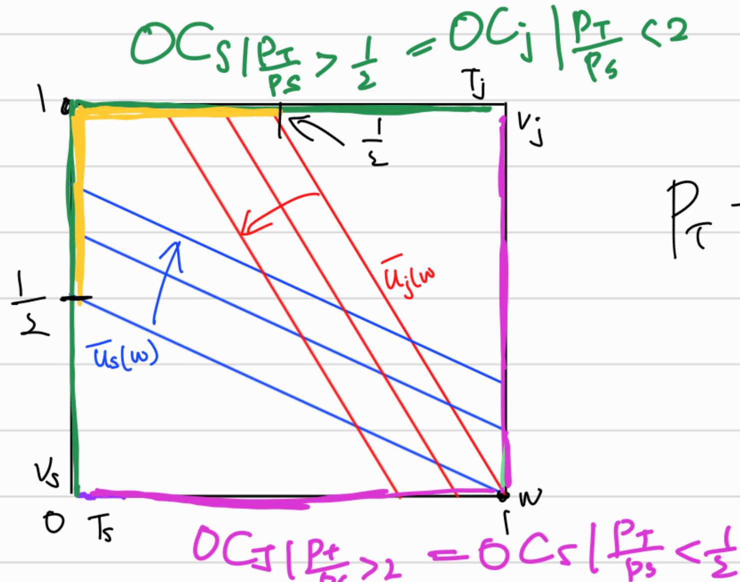
$$\pi_d - \pi_c = \frac{n^2-2n(n-1)Q'+Q'^2-4nQ'+4nQ'^2}{4n^2} = \frac{n^2-2n(n+1)Q'+5nQ'^2}{4n^2}$$

$$\pi_d - \pi_N = \frac{(n^2-2n(n-1)Q'+Q'^2)(n+1)^2-4n^2}{(4n(n+1))^2}$$

Therefore, if we can find Q' s.t $\frac{\pi_d-\pi_c}{\pi_d-\pi_N} < \delta$, then we can maintain collusion.

#4

a)



$P_T T_s + P_V \cdot V_s = P_T$

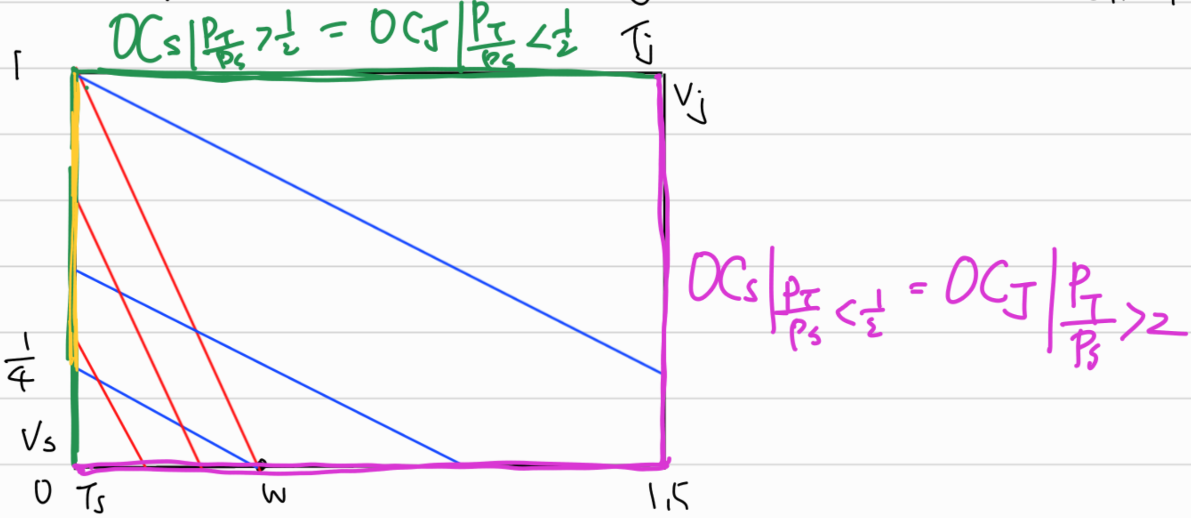
For Smith, offer curve is $\begin{cases} T_s=1, V_s=0 \text{ line if } P_T/P_V < \frac{1}{2} \\ V_s=1, T_s=0 \text{ line if } P_T/P_V > \frac{1}{2} \end{cases}$

On the other hand,
for Jefferson, offer curve is $\begin{cases} T_j=1, V_j=0 \text{ line if } P_T/P_V < 2 \\ V_j=1, T_j=0 \text{ line if } P_T/P_V > 2 \end{cases}$

The area where offer curves cross is $(T_s=0, V_s=1)$ line
$(=(T_j=1, U_j=0))$

Also, Pareto optimal locus is $(T_s=0\ V_s=1)$ line, and
Gains from trade lens is $(T_s=0, V_s=1) \cap (1 \le T_s \le \frac{1}{2}, \frac{1}{2} \le V_s \le 1)$
Since contract occurs as long as $\frac{1}{2} < \frac{P_T}{P_V} < 2$, there are multiple competitive equilibria in this situation, which is the line at left upper corner of box, i.e, gains from trade lens. = Contract curve

(b)



For both of them, utility structures are not changed.
Therefore, offer curves are same to (a)

For Smith, offer curve is $\begin{cases} T_S=1, V_S=0 \text{ line if } P_T/P_V < \frac{1}{2} \\ V_S=1, T_S=0 \text{ line if } P_T/P_V > \frac{1}{2} \end{cases}$

for Jefferson, offer curve is $\begin{cases} T_j=1, V_j=0 \text{ line if } P_T/P_V < 2 \\ V_j=1, T_j=0 \text{ line if } P_T/P_T > 2 \end{cases}$

Therefore,
the area where offer curves cross is $(T_S=0, V_S=1)$ line.
Also, like (a). Pareto optimal locus is $(T_S=0, U_S=1)$ line.
But, Gains from trade lens is line s.t $T_S=0$, $\frac{1}{4} \leq V_S \leq 1$.
Again,
Since contract occurs as long as $\frac{1}{2} < \frac{P_T}{P_V} < 2$, there are multiple competitive equilibria in this situation, i.e,
Contract curve = gains from trade lens = competitive equilibria

(c) In (b), Jefferson has more endowment for both tea and tobacco. Compared to Smith. But through trade with Smith, Jefferson can get more utility. It means even trade with counterpart who has less endowment can bring improvement in utility.

#5.

(a) i) retailer's profit max : $\max (1-q)q \Rightarrow \underline{FOC}$ $1-2q=0$.
$\Rightarrow$ MR of retailer : $p=1-2q$ $\Rightarrow$ demand curve for manufacturer

manufacturer's profit max : $\max (1-2q)q \Rightarrow FOC: 1-4q=0$.

$\Rightarrow q^*=\frac{1}{4}$, $p_m = 1-\frac{1}{2} = \frac{1}{2}$. $p_r = 1-\frac{1}{4} = \frac{3}{4}$.

$\Rightarrow P_1 = \frac{1}{2}, P_2 = \frac{1}{4}$

$\Rightarrow \pi_m = \frac{1}{8}$, $\pi_R = \frac{3}{4}\cdot\frac{1}{4} - \frac{1}{2}\cdot\frac{1}{4} = \frac{1}{16}$

ii) If they cooperate, joint profit max : $\max (1-q)q \Rightarrow FOC : 1-2q=0$.

$q^* = \frac{1}{2}$, $P_m=0$, $P_R=\frac{1}{2}$. $\Rightarrow \pi = \frac{1}{4}$

Since joint profit $\frac{1}{4} > \Sigma$ individual profit $= \frac{1}{8}+\frac{1}{16} = \frac{3}{16}$, both firms can be better off if they distributing their profits like. $(\pi_m, \pi_R) = \left(\frac{1}{8}+\alpha, \frac{1}{16}+\left(\frac{1}{16}-\alpha\right)\right)$
$0<\alpha<\frac{1}{16}$

(b) Since the final good's market is competitive market. let good's price p be given. then. manufacturer's π max problem is

$$\max\ p\cdot Q - p_1 x_1 - p_2 x_2$$

Since production function is $Q = \min\{x_1, x_2\}$, $Q = x_1 = x_2$.

then. $\max\ p\cdot Q - p_1 x_1 - p_2 x_1 = (p - p_1 - p_2) x_1$.

FOC $\quad p - p_1 - p_2 = 0. \iff p = p_1 + p_2$

Then, $Q = 1 - p = 1 - p_1 - p_2$.

Since, firm 1 choose its price first, then,

for firm 2, $\max\limits_{p_2}\ p_2(1 - p_1 - p_2) \rightarrow$ FOC : $1 - p_1 - 2p_2 = 0 \Rightarrow p_2 = \frac{1-p_1}{2}$

Thus, firm 1. $\max\limits_{p_1}\ p_1\left(1 - p_1 - \frac{1-p_1}{2}\right) = p_1\left(\frac{1-p_1}{2}\right) \rightarrow$ FOC: $\frac{1}{2} - p_1 = 0$. $\therefore p_1 = \frac{1}{2}$

$p_2 = \frac{1}{4}$, $\Rightarrow Q = 1 - \frac{1}{4} - \frac{1}{2} = \frac{1}{4}$ $\quad \pi_1 = \frac{1}{8}$, $\pi_2 = \frac{1}{16}$

Since, firm 1 and 2 know manufacturer's input demand is determined by their price. so each firm optimizes its own price. It's pretty similar to Bertrand competition because of the manufacturers production function,

(ii) If firms cooperate, they set prices that maximizes their joint profit.

$$\max_{p_1, p_2}\ (1 - p_1 - p_2)p_1 + (1 - p_1 - p_2)p_2 = (1 - p_1 - p_2)(p_1 + p_2)$$
$$= (1 - p_j)p_j \quad , p_j = p_1 + p_2$$

FOC : $1 - 2p_j = 0$, $p_j = \frac{1}{2}$, $Q = 1 - p_j = \frac{1}{2}$

$\Rightarrow$ joint profit $= \frac{1}{4} > \frac{1}{8} + \frac{1}{16}$ = individual profit without cooperation,

By redistributing joint profit, both firms can better off.

(c) Simultaneous Bertrand competition. Since firm 1 & 2 know each other's reaction function s.t $p_1 = \frac{1-p_2}{2}$, $p_2 = \frac{1-p_1}{2}$

they both take these into account when they set their prices.

Then $\quad p_1 = \frac{1}{2} - \frac{1}{2}\left(\frac{1}{2} - \frac{p_1}{2}\right) = \frac{1}{2} + \frac{1}{4}p_1 - \frac{1}{4} \Rightarrow \frac{3}{4}p_1 = \frac{1}{4}$ $\quad p_1 = \frac{1}{3}$, $p_2 = \frac{1}{3}$

$Q = \frac{1}{3}$. $\quad \pi_1 = \frac{1}{9} = \pi_2$

(d) Manufacturer's π max problem : $\left(p - \sum_{i=1}^{n} p_i\right) x_1$

FOC : $\quad p = \sum_{i=1}^{n} p_i$

i) Since they do not cooperate, nth firm set its price

s.t $\max \left(1-(p_1+p_2+\cdots+p_{n-1})-p_n\right)\cdot p_n.$

FOC $\quad 1-(p_1+p_2+\cdots+p_{n-1})-2p_n=0. \quad \Rightarrow p_n=\frac{1-(p_1+\cdots+p_{n-1})}{2}.$

Then (n-1)th firm set price based on nth reaction function.

$$\max \left(1-(p_1+p_2+\cdots+p_{n-2})-\frac{1-(p_1+\cdots+p_{n-1})}{2}-p_{n-1}\right)p_{n-1}$$

$$=\left[\frac{1}{2}\left(1-(p_1+\cdots+p_{n-2})\right)-\frac{1}{2}p_{n-1}\right]p_{n-1}$$

FOC : $\frac{1}{2}\left(1-(p_1+\cdots+p_{n-2})\right)-p_{n-1}=0 \Rightarrow p_{n-1}=\frac{1-(p_1+\cdots+p_{n-2})}{2}$

(n-2)th firm's price $\Rightarrow p_{n-2}=\frac{1-(p_1+\cdots+p_{n-3})}{2}$,

Then. for each firm $j\geq 2$, $p_j=\frac{1-\sum_{i}^{j-1}p_i}{2} \Rightarrow \frac{1-p_1}{2}+\frac{\sum_{i=2}^{j-1}p_i}{2}$

$\Rightarrow p_2=\left(\frac{1-p_1}{2}\right).\quad p_3=\frac{1-p_1}{2}-\frac{1}{2}p_2=\left(\frac{1-p_1}{2}\right)-\frac{1-p_1}{4}=(1-p_1)\frac{1}{4}$

$p_4=\left(\frac{1-p_1}{2}\right)-\frac{1}{2}p_2-\frac{1}{2}p_3=\frac{1}{8}(1-p_1)$

$\Rightarrow p_n=(1-p_1)\sum_{i=1}^{n-1}\left(\frac{1}{2}\right)^i=(1-p_1)\frac{1}{2^{n-1}}$

$\Rightarrow p_2+\cdots+p_n=(1-p_1)\left(1-\frac{1}{2^{n-1}}\right)$

Thus, for firm 1, $\max \left(1-p_1-(p_2+\cdots+p_n)\right)p_1=\left((1-p_1)-(1-p_1)\left(1-\frac{1}{2^{n-1}}\right)\right)p_1$

$=\left((1-p_1)\frac{1}{2^{n-1}}\right)p_1$

FOC : $\frac{1}{2^{n-1}}-\frac{1}{2^{n-2}}p_1=0. \Rightarrow p_1=\frac{1}{2},\ p_2=\frac{1}{2^2},\ p_3=\frac{1}{2^3},\cdots,p_n=\frac{1}{2^n}$

$Q=\frac{1}{2^n},\quad \pi_i=\left(\frac{1}{2}\right)^i\cdot\left(\frac{1}{2^n}\right)=\frac{1}{2^{n+i}}$

ii) maximizing joint profit, $\max (1-P_J)P_J$, $P_J=\sum_{i=1}^{n}P_i$

$\Rightarrow 1-2P_J=0. \Rightarrow P_J=\frac{1}{2}.\ Q_J=\frac{1}{2}.\ \pi_J=\frac{1}{4}$

By redistributing π_J, all firms can be better off.

(e) Since firm i's reaction function is

$p_i=\frac{1-\sum_{-i}p_j}{2}$, by using intuition in (a),

$p_i=\frac{1}{n+1},\quad Q=\frac{n}{n+1},\quad \pi_i=\frac{1}{(n+1)^2}$