Nov 21, 2023

Today's agenda :-

1) Kaldor Facts
2) Solow- Swan Model
3) Growth accounting

Kaldor stylized facts:-

1) Y grows at roughly constant rate.
So does $\frac{Y}{N}$.

2) $\frac{K}{Y}$ is roughly constant.

3) Total labor hours grow slower than K.
4) Rate of return on K is roughly constant (it does'nt seem to grow)

Solow - Swan Model

Technology : $Y_t = F(K_t, X_t, N_t)$

X_t : "productivity" of labor
N_t : Number of people

Assumptions:

- $F(\cdot)$ is CRS
- $F(0, XN) = F(K, 0) = 0$
- Inada conditions:

$F'_K(K, XN) \xrightarrow[K \to 0]{} \infty$

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- $X_{t+1} = (1+x)X_t$
  $N_{t+1} = (1+n)N_t$

- Feasibility constraint:
  $$C_t + I_t = F(K_t, X_t N_t)$$

- Law of motion for capital: $K_{t+1} = (1-\delta)K_t + I_t$

- <u>Consumer</u>: saves s of Y, consumes $(1-s)$
  $C_t = (1-s)Y_t$
  $S_t = sY_t$

  {inherently assumed homothetic preferences C_t's remains constant.

* $S_t = sY_t = Y_t - (1-s)Y_t$

$= F(K_t, X_t N_t) - C_t = I_t$

- Rewriting the law of motion of capital:

$$K_{t+1} = (1-\delta)K_t + I_t$$
$$= (1-\delta)K_t + sF(K_t, X_t N_t)$$

$$\frac{K_{t+1}}{X_t N_t} = (1-\delta)\frac{K_t}{X_t N_t} + s\frac{F(K_t, X_t N_t)}{X_t N_t}$$

Denote $k_t = \frac{K_t}{X_t N_t} \Rightarrow (1+n)(1+x)k_{t+1}$

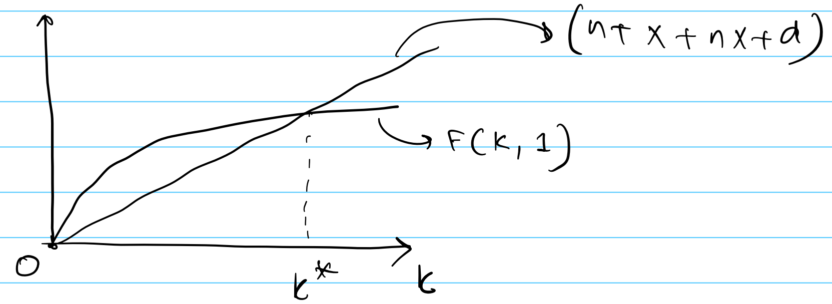
$$k_{t+1} = \frac{K_{t+1}}{X_{t+1}N_{t+1}} = \frac{K_{t+1}}{X_t N_t (1+n)(1+x)}$$

$$\Rightarrow \frac{K_{t+1}}{X_t N_t} = k_{t+1}(1+n)(1+x)$$

$$(1+n)(1+x)k_{t+1} = (1-\delta)k_t + sF(k_t, 1)$$

- Look at BGP: [Bellman Growth Path: LR equilibrium where growth rates grow at a constant rate)
  k doesn't grow, aggregates

$$(n+x+nx+\delta)k = sF(k,1)$$



Growth rate at BGP:-

- k doesnot grow
- K_t will grow at a rate $(1+n)(1+x)$
- Y_t will grow at a rate $(1+n)(1+x)$

$$K_{t+1} = (1-\delta)K_t + sY_t$$

- $\frac{Y_t}{N_t}$ will grow at a rate $(1+x)$

- $\frac{K_t}{Y_t}$ is a constant.

Imagine a firm now :—

$$\pi_t = \max_{K_t, N_t} \left[F(K_t, N_t X_t) - w_t N_t X_t - r_t K_t \right]$$

$$r_t = F'_K(K_t, N_t X_t)$$

$$\iff r_t = F'_K(k_t, 1)$$ (Can divide as F'_K is HD-0 & F_K is CRS)

We don't need to divide r_t by $N_t X_t$. Why?

$$\Rightarrow \{r_t\} \text{ is flat on BGP.}$$

$$w_t = F'_{NX}(K_t, X_t N_t) = F'_{NX}(k_t, 1)$$

$$\Rightarrow \{w_t\} \text{ is constant on BGP.}$$

wage rate on labor

$$W_T = w_t X_t \Rightarrow \{W_t\} \text{ grows at a rate } (1+x)$$

Application to Growth Accounting

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$$

A_t - productivity

L_t - # of working hours

$$\frac{Y_t}{N_t} = A_t \left(\frac{K_t}{N_t}\right)^{\alpha} \left(\frac{L_t}{N_t}\right)^{1-\alpha}$$

$$\frac{Y_t}{N_t} = A_t \left(\frac{K_t}{Y_t N_t}\right)^{\alpha} \left(\frac{L_t}{N_t Y_t}\right)^{1-\alpha} Y_t$$

$$\iff \frac{Y_t}{N_t} = A_t \left(\frac{K}{Y_t}\right)^{\alpha} \left(\frac{L_t}{N_t}\right)^{1-\alpha} \left(\frac{Y_t}{N_t}\right)^{\alpha}$$

Taylor Expansion

$$\Leftrightarrow \quad \frac{Y_t}{N_t} = A_t^{\frac{1}{1-\alpha}} \left(\frac{K_t}{Y_t}\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{L_t}{N_t}\right)$$

how?

$$\log\left(\frac{Y_t}{N_t}\right) = \frac{1}{1-\alpha}\log A_t + \frac{\alpha}{1-\alpha}\log\left(\frac{K_t}{Y_t}\right) + \log\left(\frac{L_t}{N_t}\right)$$

LHS:-

$$\log\left[\frac{(Y_{t+1}/N_{t+1})}{(Y_t/N_t)}\right] = \log\left(1 + g_{\frac{Y}{N}}\right) \approx g_{\frac{Y}{N}}$$

[$\log(1+x) \approx x$ if x is small (taylor expansion)]

$$g_{\frac{Y}{N}} \approx \underbrace{\frac{1}{1-\alpha} g_A}_{} + \frac{\alpha}{1-\alpha} g_{\frac{K}{Y}} + g_{\frac{L}{N}}$$

Most of the growth comes from this unaccounted A. Modern macro theory ~~wa~~ works on finding ~~what~~ what is this A?