Nov 16, 2023

Today:-

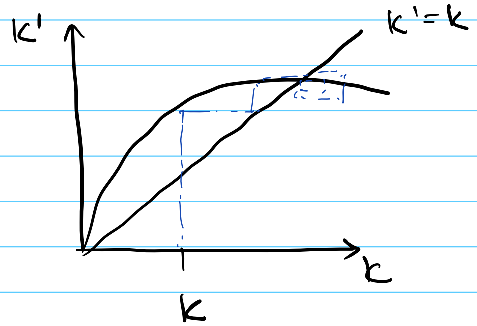
- Adding uncertainty to NGM
- ADE in a model w/ uncertainty
- RBC model

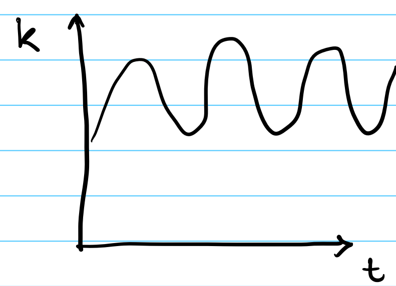
Why uncertainty?

→ stuff grows with wiggles

Two approaches

1) Deterministic complex dynamics





Generally, this model does not do a good job explaning the data. The capital oscillates, something we don't see in real world.

Aggregate Shocks

→ RBC = NGM + Aggregate productivity shocks

- Bz cycle fluctuations
- More complicated models collapse into others
- There are other shocks as well → 2008 fin crisis

Notation

- s_t – state of the economy in t
- $s_t \in \{High, Low\}$

- Def: $s^t = \{s_t, s_{t-1}, s_{t-2}, \ldots, s_0\}$ is a history of shocks at time t.
  $S^t \rightarrow$ set of all histories upto time t (all feasible)
  Ex $t=2$ (assuming starting at time 1)
  $S^2 = \{HH, LL, HL, LH\}$

- Let $\pi(s^t)$ – unconditional probability of history s^t.
  Routinely assume $\pi(s^0) = 1$

- Let $\tau < t$, $\pi(s^t | s^\tau)$ – conditional probability of s^t given s^τ happened.

Deterministic Case: Allocations are functions of time.
Uncertainty Case: Allocations are functions of histories.
Ex: s^t & $\hat{s}^t$ are 2 histories. In general,
$$c_t^i(s^t) \neq c_t^i(\hat{s}_t)$$

* Exchange Economy with uncertainty:
I – number of agents
$\{y_t^i(s^t)\}_t^{s^t}$ – sequences of endowments

An allocation is $\{c_t^i(s^t)\}_t^{s^t}$
$c_t^i : S^t \rightarrow \mathbb{R}_+$

* Arrow-Debreu Equilibrium.

they come at pd 0 to trade, based on the prices in pd. 0. (set by artificial auctioneer) st mkts clear as in vanilla A-D.

$q_t^0(s^t)$: price of one unit of consumption good at time t after sequence s^t.

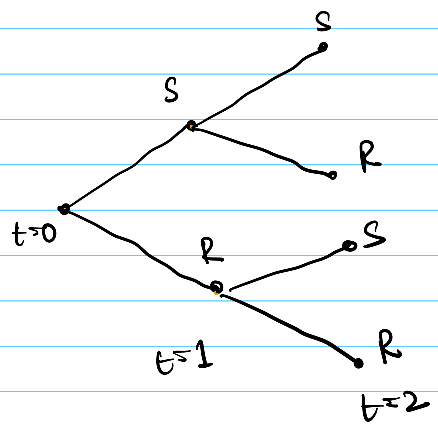
<u>Problem of agent i</u>

expected value at time 0.

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t^i(s^t)) \to \max_{\{c_t^i(s^t)\}_t^{s^t}}$$

$$\sum_{t=0}^{\infty} \sum_{s^t \in S^t} q_t^0(s^t)\, c_t^i(s^t) \le \sum_{t=0}^{\infty} \sum_{s^t \in S^t} q_t^0(s^t)\, y_t^i(s^t) \qquad (?)$$

$$c_t^i(s^t) \ge 0 \quad \forall t \quad \forall s^t$$



If U is vNM,

then $\sum_{t=0}^{\infty} \sum_{s^t \in S^t} \pi(s^t) \beta^t u(c_t^i(s^t))$

Def: An ADE for this environment is the price system $\{q_t^0(s^t)\}_t^{s^t}$ and allocations $\{c_t^i(s^t)\}_t^{s^t}$, such that:

1) Given $\{q_t^0(s^t)\}_t^{s^t}$, $\{c_t^i(s^t)\}_t^{s^t}$ solves the HH's problem;

2) Mkt clearing: $\sum_{i=1}^{I} c_t^i(s^t) = \sum_{i=1}^{I} y_t^i(s^t) \quad \forall t \ \forall s^t$

(Were the mkts supposed to clear in every pd in vanilla AD?)

yes!

Is this because we know probabilities?
How is this uncertai
(when we already know probab

Remarks:

1) The A-D equilibrium is PO. (Do at home)

2) Kenneth Arrow: same allocation will arise if mkts open every time & agents trade Arrow securities.

A (simple) RBC model

A SPP:

state today (source of uncertainty)

histories so far

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t(s^t), n(s^t))$$

$$c_t(s^t) + k_{t+1}(s^t) - (1-\delta)k_t(s^{t-1}) \leq A s_t F(k_t(s^{t-1}), n_t(s^t))$$

k_0, s_0 – given

$0 \leq n_t(s^t) \leq 1$

Simplifying Assumptions:

1) labor supply inelastic $u'_n(\cdot) = 0$

2) Production Technology is $A s_t k_t(s^{t-1})$

3) $\delta = 1$

$$P(k_0, s_0): \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t(s^t)) \rightarrow \max_{\{c_t(s^t), k_{t+1}(s^t)\}_t^{s^t}}$$

$$\text{s.t. } c_t(s^t) + k_{t+1}(s^t) = A s_t k_t(s^{t-1})$$

$$c_t(s^t) \geq 0, \; k_{t+1}(s^t) \geq 0 \quad \forall t \quad \forall s^t$$

let's assume $U(c_t) = \dfrac{c_t^{1-\sigma}}{1-\sigma}$ $\quad \sigma \rightarrow$ risk aversion

With this $u(\cdot)$, $V(\cdot)$ is homogeneous of degree $(1-\sigma)$

$$U(\lambda k_0, s_0) = \lambda^{1-\sigma} V(k_0, s_0)$$

$$\text{if } \{\tilde{c}^*(\lambda k_0, s_0), \tilde{k}^*(\lambda k_0, s_0)\} = \lambda\{\tilde{c}^*(k_0, s_0), \tilde{k}^*(k_0, s_0)\}$$

depends on prev. pd.

- Def: A stochastic process $s_0, s_1, s_2, \ldots$ is first-order markov if $P(s_{t+1} \mid s_t, s_{t-1}, \ldots, s_0) = P(s_{t+1} \mid s_t)$

Ex: AR(1) [Auto-regressive] is Markov of degree 1

$$s_{t+1} = \rho s_t + \varepsilon_{t+1}, \quad \varepsilon_{t+1} \overset{\text{i.i.d}}{\sim} N(0, \sigma^2)$$

I.I.D processes are Markov of degree 0.

Termo 3 | Stochastic Process

Decentralized

If $\{s_t\}$ is Markov of degree 1, — because of Markov of degree 1.

$$v(k,s) = \max_{c,k'} \left(\frac{c^{1-\sigma}}{1-\sigma} + \beta\, \mathbb{E}[v(k',s') \mid s] \right)$$

s.t. $c + k' = Ask$

k_0, s_0 given

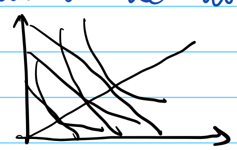
Invoke homogeneity of v:

$$v(k,s) = \max_{c,k' \geq 0} \left[\frac{c^{1-\sigma}}{1-\sigma} + \beta\, \mathbb{E}\left[(k')^{1-\sigma} v(1,s') \mid s\right] \right]$$

$c + k' = Ask$

$$= \max_{c,k' \geq 0} \left[\frac{c^{1-\sigma}}{1-\sigma} + (k')^{1-\sigma} \beta\, \mathbb{E}(v(1,s') \mid s) \right]$$

Preferences are homothetic as the above expression is hd $1-\sigma$



$\Rightarrow$ $c = \varphi Ask$

$k' = (1-\varphi) Ask$

φ - ?

$$\mathcal{L} = \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) \frac{c_t(s^t)^{1-\sigma}}{1-\sigma} + \sum_{t=0}^{\infty} \sum_{s^t} \lambda_t(s^t) \left[A s_t k_t(s^{t-1}) - c_t(s^t) - k_{t+1}(s^t) \right]$$

FOCs:

$(c_t(s^t))$: $\beta^t \pi(s^t) c_t(s^t)^{-\sigma} = \lambda_t(s^t)$

$(k_{t+1}(s^t))$: $-\lambda_t(s^t) + \sum_{s_{t+1}} \lambda_{t+1}(s^t, s_{t+1}) [A s_{t+1}] = 0$

$$\Rightarrow \beta^t \pi(s^t) c_t(s^t)^{-\sigma} = \sum_{s_{t+1}} \beta^{t+1} \pi(s^t, s_{t+1}) c_{t+1}^{-\sigma}(s^t, s^{t+1}) A s_{t+1}$$

$$\Rightarrow c_t(s^t)^{-\sigma} = \underbrace{\sum_{s_{t+1}} \pi(s_{t+1} \mid s^t)\, c_{t+1}^{-\sigma}(s^t, s_{t+1}) A s_{t+1}}_{\mathbb{E}\left[A s_{t+1} c_{t+1}^{-\sigma}(s^{t+1}) \mid s^t\right]}$$

$$\Rightarrow 1 = \beta\, \mathbb{E}\left[A s_{t+1} \left(\frac{c_t(s^t)}{c_{t+1}(s^{t+1})} \right)^{\sigma} \,\middle|\, s^t \right]$$

Euler Eqn.

What is φ

Recall $c = \varphi A s k$

$$\Rightarrow 1 = \beta \mathbb{E}\left[A s_{t+1} \left(\frac{\cancel{\varphi A} s_t k_t (s^{t-1})}{\cancel{\varphi A} s_{t+1} k_{t+1} (s^t)} \right)^{\sigma} \Big| s^t \right]$$

$$\Rightarrow 1 = \beta \mathbb{E}\left[A s_{t+1} \left(\frac{\cancel{s_t k_t (s^{t-1})}}{s_{t+1} (1-\varphi) A \cancel{s_t k_t (s^{t-1})}} \right)^{\sigma} \Big| s^t \right]$$

$$\Rightarrow 1 = \beta \mathbb{E}\left[(A s_{t+1})^{1-\sigma} (1-\varphi)^{-\sigma} \mid s^t \right]$$

$$\Rightarrow (1-\varphi)^{\sigma} = \beta \mathbb{E}\left[(A s_{t+1})^{1-\sigma} \mid s^t \right]$$

Assume $\{s_t\}_{t=0}^{\infty}$ is i.i.d

$$\Rightarrow (1-\varphi) = \left[\mathbb{E}(A s)^{1-\sigma} \cdot \beta \right]^{1/\sigma}$$

Growth of c_t:

$$\gamma^c_{t,t+1} = \frac{c_{t+1}}{c_t} = \frac{\varphi A s_{t+1} k_{t+1}}{\varphi A s_t k_t}$$

$$= \frac{s_{t+1} (1-\varphi) A s_t k_t}{s_t k_t} = (1-\varphi) A s_{t+1}$$

$$= \mathbb{E}[\gamma^c_{t,t+1}] = (1-\varphi) A \mathbb{E} s_{t+1}$$

Effect of uncertainty:

Case 1: $s_t = 1 \quad \forall t$

Case 2: $\mathbb{E}[s_t] = 1, \quad V(s_t) > 0$

$1 - \varphi$

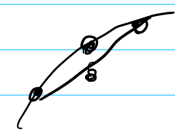
Might have something missing here.

Case I: $\sigma \in [0, 1]$ $\qquad$ $1-\varphi \rightarrow$ goes to investment

$$(1-\varphi)^{\sigma} = \mathbb{E}(As)^{1-\sigma}$$ derive euler

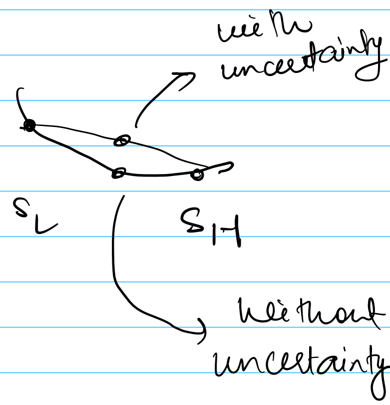
$\mathbb{V}(s) = 0 \qquad \mathbb{E}s = 1 \qquad \sigma \in [0, 1]$

$\mathbb{V}(s) > 0 \qquad \mathbb{E}(s) = 1$



Case 2: If $\sigma > 1$:

$s^{1-\sigma} \rightarrow$ convex f^n



With σ, the effect of uncertainty would be different.