Oct 17, 2023

Today: -

→ A simple first-cut model of aggregate time-series
→ Simplifying the model.

A simple model of aggregate time series.

Households: Make consumption, labor supply, investment decisions.
indexed by $i = 1, \ldots, I$

Firms that produce consumption goods
use capital & labor
indexed by $j_c = 1, \ldots, J_c$

$$c_{jt} \leq F_t^j(k_{ct}^j, n_{ct}^j) \quad \forall\ j \in \{1, \ldots, J_c\}$$

Firms that produce investment good
use capital & labor
indexed by $j_x = 1, \ldots, J_x$

$$x_{jt} \leq F_t^j(k_{xt}^j, n_{xt}^j) \quad \forall\ j \in \{1, \ldots, J_x\}$$

* k in both are stuff like machines.

$\left[\begin{array}{l}\vec{\ell} \to \text{time endowment} \\ \ell \to \text{leisure}\end{array}\right]$

Households

→ $u_i(\vec{c}, \vec{\ell})$, where $\vec{c} = \{c_t\}_{t=0}^{\infty}$, $\vec{\ell} = \{\ell_t\}_{t=0}^{\infty}$

→ u_i is strictly increasing, strictly concave

→ endowments : k_0^i, $\{\bar{n}_t^i\}_{t=0}^{\infty}$

→ ownership structure $\underbrace{\{\theta_{ij}\}_{j=1}^{J_c}}$, $\{\theta_{ij}\}_{j=1}^{J_x}$

share of profit of HH i in firm J_c.

CE Def: A CE in this environment is $\{p_{ct}, p_{xt}, w_t, r_t\}_{t=0}^{\infty}$,

$\{c_t^i, n_t^i, \ell_t^i, x_t^i, k_t^i\}_{t=0}^{\infty}$ — allocations for HHs,

allocation for cons^m good firms - $\{c_t^j, k_{ct}^j, n_{ct}^j\}_{t=0}^{\infty}$ $\forall$ $j \in \{1, \dots, J_c\}$

& allocation for investment firms $\{x_t^j, k_{xt}^j, n_{xt}^j\}_{t=0}^{\infty}$, s.t. :-

1) Given prices, allocations for HH solve:

$$u_i(\vec{c}, \vec{\ell}) \to \max_{c, \ell, n, x, k}$$

$$\text{s.t. } \sum_{t=0}^{\infty} (p_{ct} c_t^i + p_{xt} x_t^i) \le \sum_{t=0}^{\infty} (r_t k_t^i + w_t n_t^i) + \Pi_i$$

$$k_{t+1}^i = (1-\delta) k_t^i + x_t^i$$

$$0 \le n_t^i + \ell_t^i \le \bar{n}_t^i$$

+ non-negativity.

2) Given prices, cons. good firm solve

$$\sum_{t=0}^{\infty} \left(p_{ct} c_t^j - w_t n_{ct}^j - r_t k_{ct}^j\right) \to \max_{\{c^j, n^j, k^j\}_{t=0}^{\infty}}$$

$$\text{s.t.} \quad c_t^j \leq F_t^j\left(k_{ct}^j, n_{ct}^j\right)$$

(Firms don't own any k, rented from HHs).

3) Investment firms

$$\sum_{t=0}^{\infty} \left(p_{xt}, x_t^j - w_t n_{xt}^j - r_t k_{xt}^j\right) \to \max_{\{x_t^j, n_{xt}^j, k_{xt}^j\}}$$

$$\text{s.t.} \quad x_t^j \leq f_{xt}^j\left(k_{xt}^j, n_{xt}^j\right)$$

Profits: —

$$\Pi_i = \sum_{jc=1}^{Jc} \theta_{ijc}^{c} \sum_{t=0}^{\infty} \left(p_{ct} \cdot c_t^{jc} - w_t n_{ct}^{jc} - r_t k_{ct}^{jc}\right) +$$

$$+ \sum_{jx=1}^{Jx} \theta_{ijx}^{x} \sum_{t=0}^{\infty} \left(p_{xt} x_t^{jx} - w_t n_{xt}^{jx} - r_t k_{xt}^{jx}\right)$$

3) Markets clear

$$\sum_{i=1}^{I} c_t^i = \sum_{jc=1}^{Jc} c_t^{jc}$$

Simplifying firm side:—

1) Assume CRS. $\Rightarrow$ Profits are zero

$$\forall \lambda > 0, \; F(\lambda k, \lambda n) = \lambda F(k, n)$$

$$F'_k \cdot k + F'_n \cdot n = F(k, n)$$

$$= \; rk + wn = F(k, n)$$

2) Assume representative technology in each sector.

$$\forall \; jc, \; jc' \quad F_{ct}^{jc} = F_{ct}^{jc'} = F_{ct}$$

$$\forall \; jx, \; jx' \quad F_{xt}^{jx} = F_{xt}^{jx'} = F_{xt}$$

3) Collapse two sectors into one

$F_{ct} = F_{xt}$ (Potatoes Economy)

New market clearing: $\sum_{i=1}^{I} (c_t^i + x_t^i) = f_t(k_t, n_t)$

Question for Vlad: when can we not make this assumption? what things to keep in mind?

Simplifying the consumer side:—

1) Representative agent ($u_i = u_j$, $k_0^i = k_0^j$)

2) Homothetic Aggregation (need stronger assumptions on $\succsim$, no need to have the same endowments).

Representative Agent:

Remark: strict concavity needed if we want to go with this making same prefs (?)

$$\sum_{i=1}^{I} (c_t^i + x_t^i) = F_t(k_t, n_t)$$

$$I.(c_t^1 + x_t^1) = F_t(k_t, n_t)$$

$$\Rightarrow c_t^1 + x_t^1 = \frac{1}{I} F(k_t, n_t)$$

$$c_t^1 + x_t^1 = F\left(\frac{k_t}{I}, \frac{n_t}{I}\right)$$
$$= F(k_t^1, n_t^1)$$

(Writing cons problem as one person's).

Side Note: —

SPP: $u(\vec{c}_1, \tilde{\ell}_1) \to \max$

$$\text{s.t. } c_t' + x_t' = F(k_t', n_t')$$

$$k_{t+1}' = (1-\delta)k_t' + x_t', \quad 0 \leq n_t' + \ell_t' \leq \bar{n}_t' \quad \forall t$$

k_0' given

Homothetic Aggregation

Def: Preferences are homothetic if $\forall x, y$ $\quad U(x) = U(y) \Leftrightarrow u(\lambda x) = u(\lambda y)$; $\lambda > 0$

Alt; $\quad x \sim y \Leftrightarrow (\lambda x) \sim (\lambda y)$

Thm: If u is homogenous of any degree, then $\succsim$ it represents are homothetic.

Proof: U – homogenous of degree η

$$u(\lambda x, \lambda y) = \lambda^{\eta} u(x, y)$$

Take 2 pts. (x_1, y_1) & (x_2, y_2)

Let $u(x_1, y_1) = u(x_2, y_2)$

$$u(\lambda x_1, \lambda y_1) = \lambda^{\eta} u(x_1, y_1) = \lambda^{n} u(x_2, y_2)$$

$$= u(\lambda x_2, \lambda y_2) \qquad \square$$

We are doing this to understand how to ~~s~~ simplify models.