a constant-returns-to-scale production technology, competitive markets, and a complete set of flat-rate taxes.

Throughout the chapter we maintain the assumption that the government can commit to future tax rates.

NOTE: This book uses sequential markets, which I prefer, but Vladimir defaults to time-0 for tax setups

16.2. A nonstochastic economy

An infinitely lived representative household likes consumption, leisure streams $\{c_t, \ell_t\}_{t=0}^{\infty}$ that give higher values of

$$\sum_{t=0}^{\infty} \beta^t u\left(c_t, \ell_t\right), \ \beta \in (0,1) \tag{16.2.1}$$

where u is increasing, strictly concave, and three times continuously differentiable in consumption c and leisure ℓ. The household is endowed with one unit of time that can be used for leisure ℓ_t and labor n_t:

$$\ell_t + n_t = 1. \tag{16.2.2}$$

The single good is produced with labor n_t and capital k_t. Output can be consumed by the household, used by the government, or used to augment the capital stock. The technology is

$$c_t + g_t + k_{t+1} = F\left(k_t, n_t\right) + (1-\delta)\, k_t, \tag{16.2.3}$$

where $\delta \in (0,1)$ is the rate at which capital depreciates and $\{g_t\}_{t=0}^{\infty}$ is an exogenous sequence of government purchases. We assume a standard concave production function $F(k,n)$ that exhibits constant returns to scale. By Euler's theorem on homogeneous functions, linear homogeneity of F implies

$$F\left(k,n\right) = F_k k + F_n n. \tag{16.2.4}$$

Let u_c be the derivative of $u(c_t, \ell_t)$ with respect to consumption; u_ℓ is the derivative with respect to ℓ. We use $u_c(t)$ and $F_k(t)$ and so on to denote the time t values of the indicated objects, evaluated at an allocation to be understood from the context.

16.2.1. Government

The government finances its stream of purchases $\{g_t\}_{t=0}^{\infty}$ by levying flat-rate, time-varying taxes on earnings from capital at rate τ_t^k and earnings from labor at rate τ_t^n. The government can also trade one-period bonds, sequential trading of which suffices to accomplish any intertemporal trade in a world without uncertainty. Let b_t be government indebtedness to the private sector, denominated in time t-goods, maturing at the beginning of period t. The government's budget constraint is

$$g_t = \tau_t^k r_t k_t + \tau_t^n w_t n_t + \frac{b_{t+1}}{R_t} - b_t, \tag{16.2.5}$$

where r_t and w_t are the market-determined rental rate of capital and the wage rate for labor, respectively, denominated in units of time t goods, and R_t is the gross rate of return on one-period bonds held from t to $t+1$. Interest earnings on bonds are assumed to be tax exempt; this assumption is innocuous for bond exchanges between the government and the private sector.

16.2.2. Household

A representative household chooses $\{c_t, n_t, k_{t+1}, b_{t+1}\}_{t=0}^{\infty}$ to maximize expression (16.2.1) subject to the following sequence of budget constraints:

$$c_t + k_{t+1} + \frac{b_{t+1}}{R_t} = (1 - \tau_t^n) w_t n_t + \left(1 - \tau_t^k\right) r_t k_t + (1 - \delta) k_t + b_t, \tag{16.2.6}$$

for $t \geq 0$. With $\beta^t \lambda_t$ as the Lagrange multiplier on the time t budget constraint, the first-order conditions are

$$c_t\colon \quad u_c(t) = \lambda_t, \tag{16.2.7}$$

$$n_t\colon \quad u_\ell(t) = \lambda_t (1 - \tau_t^n) w_t, \tag{16.2.8}$$

$$k_{t+1}\colon \quad \lambda_t = \beta \lambda_{t+1} \left[\left(1 - \tau_{t+1}^k\right) r_{t+1} + 1 - \delta\right], \tag{16.2.9}$$

$$b_{t+1}\colon \quad \lambda_t \frac{1}{R_t} = \beta \lambda_{t+1}. \tag{16.2.10}$$

Substituting equation (16.2.7) into equations (16.2.8) and (16.2.9), we obtain

$$u_\ell(t) = u_c(t)(1 - \tau_t^n) w_t, \tag{16.2.11a}$$

$$u_c(t) = \beta u_c(t+1) \left[\left(1 - \tau_{t+1}^k\right) r_{t+1} + 1 - \delta\right]. \tag{16.2.11b}$$

Moreover, equations (16.2.9) and (16.2.10) imply

$$R_t = \left(1-\tau_{t+1}^k\right) r_{t+1} + 1 - \delta, \tag{16.2.12}$$

which is a condition not involving any quantities that the household is free to adjust. Because only one financial asset is needed to accomplish all intertemporal trades in a world without uncertainty, condition (16.2.12) constitutes a no-arbitrage condition for trades in capital and bonds that ensures that these two assets have the same rate of return. This no-arbitrage condition can be obtained by consolidating two consecutive budget constraints; constraint (16.2.6) and its counterpart for time $t+1$ can be merged by eliminating the common quantity b_{t+1} to get

$$\begin{aligned} c_t + \frac{c_{t+1}}{R_t} + \frac{k_{t+2}}{R_t} + \frac{b_{t+2}}{R_t R_{t+1}} &= \left(1-\tau_t^n\right) w_t n_t \\ &+ \frac{\left(1-\tau_{t+1}^n\right) w_{t+1} n_{t+1}}{R_t} + \left[\frac{\left(1-\tau_{t+1}^k\right) r_{t+1} + 1 - \delta}{R_t} - 1\right] k_{t+1} \\ &+ \left(1-\tau_t^k\right) r_t k_t + (1-\delta) k_t + b_t, \end{aligned} \tag{16.2.13}$$

where the left side is the use of funds and the right side measures the resources at the household's disposal. If the term multiplying k_{t+1} is not zero, the household can make its budget set unbounded either by buying an arbitrarily large k_{t+1} when $(1-\tau_{t+1}^k) r_{t+1} + 1 - \delta > R_t$, or, in the opposite case, by selling capital short to achieve an arbitrarily large negative k_{t+1}. In such arbitrage transactions, the household would finance purchases of capital or invest the proceeds from short sales in the bond market between periods t and $t+1$. Thus, to ensure the existence of a competitive equilibrium with bounded budget sets, condition (16.2.12) must hold.

If we continue the process of recursively using successive budget constraints to eliminate successive b_{t+j} terms, begun in equation (16.2.13), we arrive at the household's present-value budget constraint,

$$\begin{aligned} \sum_{t=0}^{\infty} \left(\prod_{i=0}^{t-1} R_i^{-1}\right) c_t &= \sum_{t=0}^{\infty} \left(\prod_{i=0}^{t-1} R_i^{-1}\right) \left(1-\tau_t^n\right) w_t n_t \\ &+ \left[\left(1-\tau_0^k\right) r_0 + 1 - \delta\right] k_0 + b_0, \end{aligned} \tag{16.2.14}$$

where we have imposed the transversality conditions

$$\lim_{T\to\infty}\left(\prod_{i=0}^{T-1}R_i^{-1}\right)k_{T+1} \;=\; 0, \tag{16.2.15}$$

$$\lim_{T\to\infty}\left(\prod_{i=0}^{T-1}R_i^{-1}\right)\frac{b_{T+1}}{R_T} \;=\; 0. \tag{16.2.16}$$

As discussed in chapter 13, the household would not like to violate these transversality conditions by choosing k_{t+1} or b_{t+1} to be larger, because alternative feasible allocations with higher consumption in finite time would yield higher lifetime utility. A consumption/savings plan that made either expression negative would not be possible because the household would not find anybody willing to be on the lending side of the implied transactions.

16.2.3. Firms

In each period, the representative firm takes (r_t, w_t) as given, rents capital and labor from households, and maximizes profits,

$$\Pi = F\left(k_t, n_t\right) - r_t k_t - w_t n_t. \tag{16.2.17}$$

The first-order conditions for this problem are

$$r_t = F_k\left(t\right), \tag{16.2.18a}$$

$$w_t = F_n\left(t\right). \tag{16.2.18b}$$

In words, inputs should be employed until the marginal product of the last unit is equal to its rental price. With constant returns to scale, we get the standard result that pure profits are zero and the size of an individual firm is indeterminate.

An alternative way of establishing the equilibrium conditions for the rental price of capital and the wage rate for labor is to substitute equation (16.2.4) into equation (16.2.17) to get

$$\Pi = \left[F_k\left(t\right) - r_t\right]k_t + \left[F_n\left(t\right) - w_t\right]n_t.$$

If the firm's profits are to be nonnegative and finite, the terms multiplying k_t and n_t must be zero; that is, condition (16.2.18) must hold. These conditions imply that in any equilibrium, $\Pi = 0$.

16.3. The Ramsey problem

We shall use symbols without subscripts to denote the one-sided infinite sequence for the corresponding variable, e.g., $c \equiv \{c_t\}_{t=0}^{\infty}$.

DEFINITION: A *feasible allocation* is a sequence (k, c, ℓ, g) that satisfies equation (16.2.3).

DEFINITION: A *price system* is a 3-tuple of nonnegative bounded sequences (w, r, R).

DEFINITION: A *government policy* is a 4-tuple of sequences (g, τ^k, τ^n, b).

DEFINITION: A *competitive equilibrium* is a feasible allocation, a price system, and a government policy such that (a) given the price system and the government policy, the allocation solves both the firm's problem and the household's problem; and (b) given the allocation and the price system, the government policy satisfies the sequence of government budget constraints (16.2.5).

There are many competitive equilibria, indexed by different government policies. This multiplicity motivates the Ramsey problem.

DEFINITION: Given k_0 and b_0, the *Ramsey problem* is to choose a competitive equilibrium that maximizes expression (16.2.1).

To make the Ramsey problem interesting, we always impose a restriction on τ_0^k, for example, by taking it as given at a small number, say, 0. This approach rules out taxing the initial capital stock via a so-called capital levy that would constitute a lump-sum tax, since k_0 is in fixed supply.[2]

[2] According to our assumption on the technology in equation (16.2.3), capital is reversible and can be transformed back into the consumption good. Thus, the capital stock is a fixed factor for only one period at a time, so τ_0^k is the only tax that we need to restrict to ensure an interesting Ramsey problem.

16.4. Zero capital tax

Following Chamley (1986), we formulate the Ramsey problem as if the government chooses the after-tax rental rate of capital $\tilde{r}_t$, and the after-tax wage rate $\tilde{w}_t$:

$$\begin{aligned}\tilde{r}_t &\equiv \left(1-\tau_t^k\right) r_t,\\ \tilde{w}_t &\equiv \left(1-\tau_t^n\right) w_t.\end{aligned}$$

Using equations (16.2.18) and (16.2.4), Chamley expresses government tax revenues as

$$\begin{aligned}\tau_t^k r_t k_t + \tau_t^n w_t n_t &= \left(r_t - \tilde{r}_t\right) k_t + \left(w_t - \tilde{w}_t\right) n_t\\ &= F_k\left(t\right) k_t + F_n\left(t\right) n_t - \tilde{r}_t k_t - \tilde{w}_t n_t\\ &= F\left(k_t, n_t\right) - \tilde{r}_t k_t - \tilde{w}_t n_t.\end{aligned}$$

Substituting this expression into equation (16.2.5) consolidates the firm's first-order conditions with the government's budget constraint. The government's policy choice is also constrained by the aggregate resource constraint (16.2.3) and the household's first-order conditions (16.2.11). To solve the Ramsey problem, form a Lagrangian

$$\begin{aligned}L = \sum_{t=0}^{\infty} \beta^t \Big\{ & u\left(c_t, 1-n_t\right)\\ & + \Psi_t \left[F\left(k_t, n_t\right) - \tilde{r}_t k_t - \tilde{w}_t n_t + \frac{b_{t+1}}{R_t} - b_t - g_t\right]\\ & + \theta_t \left[F\left(k_t, n_t\right) + \left(1-\delta\right) k_t - c_t - g_t - k_{t+1}\right]\\ & + \mu_{1t}\left[u_\ell\left(t\right) - u_c\left(t\right)\tilde{w}_t\right]\\ & + \mu_{2t}\left[u_c\left(t\right) - \beta u_c\left(t+1\right)\left(\tilde{r}_{t+1} + 1 - \delta\right)\right]\Big\},\end{aligned} \tag{16.4.1}$$

where $R_t = \tilde{r}_{t+1} + 1 - \delta$, as given by equation (16.2.12). Note that the household's budget constraint is not explicitly included because it is redundant when the government satisfies its budget constraint and the resource constraint holds.

The first-order condition for maximizing the Lagrangian (16.4.1) with respect to k_{t+1} is

$$\theta_t = \beta\left\{\Psi_{t+1}\left[F_k\left(t+1\right) - \tilde{r}_{t+1}\right] + \theta_{t+1}\left[F_k\left(t+1\right) + 1 - \delta\right]\right\}. \tag{16.4.2}$$

The equation has a straightforward interpretation. A marginal increment of capital investment in period t increases the quantity of available goods at time $t+1$ by the amount $[F_k(t+1)+1-\delta]$, which has a social marginal value θ_{t+1}. In addition, there is an increase in tax revenues equal to $[F_k(t+1)-\tilde{r}_{t+1}]$, which enables the government to reduce its debt or other taxes by the same amount. The reduction of the "excess burden" equals $\Psi_{t+1}[F_k(t+1)-\tilde{r}_{t+1}]$. The sum of these two effects in period $t+1$ is discounted by the discount factor β and set equal to the social marginal value of the initial investment good in period t, which is given by θ_t.

Suppose that government expenditures stay constant after some period T, and assume that the solution to the Ramsey problem converges to a steady state; that is, all endogenous variables remain constant. Using equation (16.2.18a), the steady-state version of equation (16.4.2) is

$$\theta = \beta\left[\Psi\left(r-\tilde{r}\right)+\theta\left(r+1-\delta\right)\right]. \tag{16.4.3}$$

Now with a constant consumption stream, the steady-state version of the household's optimality condition for the choice of capital in equation (16.2.11b) is

$$1 = \beta\left(\tilde{r}+1-\delta\right). \tag{16.4.4}$$

A substitution of equation (16.4.4) into equation (16.4.3) yields

$$\left(\theta+\Psi\right)\left(r-\tilde{r}\right) = 0. \tag{16.4.5}$$

Since the marginal social value of goods θ is strictly positive and the marginal social value of reducing government debt or taxes Ψ is nonnegative, it follows that r must be equal to $\tilde{r}$, so that $\tau^k = 0$. This analysis establishes the following celebrated result, versions of which were attained by Chamley (1986) and Judd (1985b).

PROPOSITION 1: If there exists a steady-state Ramsey allocation, the associated limiting tax rate on capital is zero.

It is important to keep in mind that the zero tax on capital result pertains only to the limiting steady state. Our analysis is silent about how much capital is taxed in the transition period.

16.5. Primal approach to the Ramsey problem

In the formulation of the Ramsey problem in expression (16.4.1), Chamley reduced a pair of taxes (τ_t^k, τ_t^n) and a pair of prices (r_t, w_t) to just one pair of numbers $(\tilde{r}_t, \tilde{w}_t)$ by utilizing the firm's first-order conditions and equilibrium outcomes in factor markets. In a similar spirit, we will now eliminate all prices and taxes so that the government can be thought of as directly choosing a feasible allocation, subject to constraints that ensure the existence of prices and taxes such that the chosen allocation is consistent with the optimization behavior of households and firms. This primal approach to the Ramsey problem, as opposed to the dual approach in which tax rates are viewed as governmental decision variables, is used in Lucas and Stokey's (1983) analysis of an economy without capital. Here we will follow the setup of Jones, Manuelli, and Rossi (1997).

It is useful to compare our primal approach to the Ramsey problem with the formulation in (16.4.1). Following the derivations in section 16.2.2, the constraints associated with Lagrange multipliers Ψ_t in (16.4.1) can be replaced with a single present-value budget constraint for either the government or the representative household. (One of them is redundant, since we are also imposing the aggregate resource constraint.) The problem simplifies nicely if we choose the present-value budget constraint of the household (16.2.14), in which future capital stocks have been eliminated with the use of no-arbitrage conditions. For convenience, we repeat the household's present-value budget constraint (16.2.14) here in the form:

$$\sum_{t=0}^{\infty} q_t^0 c_t = \sum_{t=0}^{\infty} q_t^0 \left(1 - \tau_t^n\right) w_t n_t + \left[\left(1 - \tau_0^k\right) r_0 + 1 - \delta\right] k_0 + b_0 \, . \tag{16.5.1}$$

In equation (16.5.1), q_t^0 is the Arrow-Debreu price

$$q_t^0 = \prod_{i=0}^{t-1} R_i^{-1}, \qquad \forall t \geq 1; \tag{16.5.2}$$

with the numeraire $q_0^0 = 1$. Second, we use two constraints in expression (16.4.1) to replace prices q_t^0 and $(1 - \tau_t^n) w_t$ in equation (16.5.1) with the household's marginal rates of substitution.

A stepwise summary of the primal approach is:

1. Obtain the first-order conditions of the household's and the firm's problems, as well as any arbitrage pricing conditions. Solve these conditions for $\{q_t^0, r_t, w_t, \tau_t^k,\ \tau_t^n\}_{t=0}^\infty$ as functions of the allocation $\{c_t, n_t, k_{t+1}\}_{t=0}^\infty$.

2. Substitute these expressions for taxes and prices in terms of the allocation into the household's present-value budget constraint. This is an intertemporal constraint involving only the allocation.

3. Solve for the Ramsey allocation by maximizing expression (16.2.1) subject to equation (16.2.3) and the "implementability condition" derived in step 2.

4. After the Ramsey allocation is solved, use the formulas from step 1 to find taxes and prices.

16.5.1. Constructing the Ramsey plan

We now carry out the steps outlined in the preceding list of instructions.

Step 1. Let λ be a Lagrange multiplier on the household's budget constraint (16.5.1). The first-order conditions for the household's problem are

$$\begin{aligned} c_t&: \quad \beta^t u_c(t) - \lambda q_t^0 = 0, \\ n_t&: \quad -\beta^t u_\ell(t) + \lambda q_t^0 (1-\tau_t^n) w_t = 0. \end{aligned}$$

With the numeraire $q_0^0 = 1$, these conditions imply

$$q_t^0 = \beta^t \frac{u_c(t)}{u_c(0)}, \tag{16.5.3a}$$

$$(1-\tau_t^n) w_t = \frac{u_\ell(t)}{u_c(t)}. \tag{16.5.3b}$$

As before, we can derive the arbitrage condition (16.2.12), which now reads

$$\frac{q_t^0}{q_{t+1}^0} = \left(1-\tau_{t+1}^k\right) r_{t+1} + 1 - \delta. \tag{16.5.4}$$

Profit maximization and factor market equilibrium imply equations (16.2.18).

Step 2. Substitute equations (16.5.3) and $r_0 = F_k(0)$ into equation (16.5.1), so that we can write the household's budget constraint as

$$\sum_{t=0}^{\infty} \beta^t \left[u_c(t)\, c_t - u_\ell(t)\, n_t\right] - A = 0, \tag{16.5.5}$$

where A is given by

$$A = A\left(c_0, n_0, \tau_0^k, b_0\right) = u_c(0) \left\{ \left[\left(1 - \tau_0^k\right) F_k(0) + 1 - \delta\right] k_0 + b_0 \right\}. \tag{16.5.6}$$

Step 3. The Ramsey problem is to choose an allocation to maximize expression (16.2.1) subject to equation (16.5.5) and the feasibility constraint (16.2.3). As before, we proceed by assuming that government expenditures are small enough that the problem has a convex constraint set and that we can approach it using Lagrangian methods. In particular, let Φ be a Lagrange multiplier on equation (16.5.5) and define

$$V(c_t, n_t, \Phi) = u(c_t, 1 - n_t) + \Phi\left[u_c(t)\, c_t - u_\ell(t)\, n_t\right]. \tag{16.5.7}$$

Then form the Lagrangian

$$\begin{aligned} J = \sum_{t=0}^{\infty} \beta^t & \left\{ V(c_t, n_t, \Phi) + \theta_t \left[F(k_t, n_t) + (1-\delta)\, k_t \right.\right. \\ & \left.\left. - c_t - g_t - k_{t+1}\right]\right\} - \Phi A, \end{aligned} \tag{16.5.8}$$

where $\{\theta_t\}_{t=0}^{\infty}$ is a sequence of Lagrange multipliers on the sequence of feasibility conditions (16.2.3). For given k_0 and b_0, we fix τ_0^k and maximize J with respect to $\{c_t, n_t, k_{t+1}\}_{t=0}^{\infty}$. First-order conditions for this problem are[3]

$$\begin{aligned} c_t\!: &\quad V_c(t) = \theta_t, \quad t \geq 1 \\ n_t\!: &\quad V_n(t) = -\theta_t F_n(t), \quad t \geq 1 \\ k_{t+1}\!: &\quad \theta_t = \beta\theta_{t+1}\left[F_k(t+1) + 1 - \delta\right], \quad t \geq 0 \\ c_0\!: &\quad V_c(0) = \theta_0 + \Phi A_c, \\ n_0\!: &\quad V_n(0) = -\theta_0 F_n(0) + \Phi A_n. \end{aligned}$$

[3] Comparing the first-order condition for k_{t+1} to the earlier one in equation (16.4.2), obtained under Chamley's alternative formulation of the Ramsey problem, note that the Lagrange multiplier θ_t is different across formulations. Specifically, the present specification of the objective function V subsumes parts of the household's present-value budget constraint. To bring out this difference, a more informative notation would be to write $V_j(t, \Phi)$ for $j = c, n$ rather than just $V_j(t)$.

These conditions become

$$V_c(t) = \beta V_c(t+1)\left[F_k(t+1) + 1 - \delta\right], \quad t \geq 1 \tag{16.5.9a}$$

$$V_n(t) = -V_c(t) F_n(t), \quad t \geq 1 \tag{16.5.9b}$$

$$V_c(0) - \Phi A_c = \beta V_c(1)\left[F_k(1) + 1 - \delta\right], \tag{16.5.9c}$$

$$V_n(0) = \left[\Phi A_c - V_c(0)\right] F_n(0) + \Phi A_n. \tag{16.5.9d}$$

To these we add equations (16.2.3) and (16.5.5), which we repeat here for convenience:

$$c_t + g_t + k_{t+1} = F(k_t, n_t) + (1-\delta) k_t, \quad t \geq 0 \tag{16.5.10a}$$

$$\sum_{t=0}^{\infty} \beta^t \left[u_c(t) c_t - u_\ell(t) n_t\right] - A = 0. \tag{16.5.10b}$$

We seek an allocation $\{c_t, n_t, k_{t+1}\}_{t=0}^{\infty}$, and a multiplier Φ that satisfies the system of difference equations formed by equations (16.5.9)–(16.5.10).[4]

Step 4: After an allocation has been found, obtain q_t^0 from equation (16.5.3a), r_t from equation (16.2.18a), w_t from equation (16.2.18b), τ_t^n from equation (16.5.3b), and finally τ_t^k from equation (16.5.4).

[4] This system of nonlinear equations can be solved iteratively. First, fix Φ, and solve equations (16.5.9) and (16.5.10a) for an allocation. Then check the implementability condition (16.5.10b), and increase or decrease Φ depending on whether the budget is in deficit or surplus. Note that the multiplier Φ is nonnegative because we are facing the constraint that the left-hand side of equation (16.5.10b) is *greater* than or equal to zero. That is, we are constrained by the equilibrium outcome that households fully exhaust their incomes and, hence, are not free to choose households' expenditures strictly less than their incomes.

16.5.2. Revisiting a zero capital tax

Consider the special case in which there is a $T \geq 0$ for which $g_t = g$ for all $t \geq T$. Assume that there exists a solution to the Ramsey problem and that it converges to a time-invariant allocation, so that c, n, and k are constant after some time. Then because $V_c(t)$ converges to a constant, the stationary version of equation (16.5.9a) implies

$$1 = \beta \left(F_k + 1 - \delta \right). \tag{16.5.11}$$

Now because c_t is constant in the limit, equation (16.5.3a) implies that $\left(q_t^0 / q_{t+1}^0 \right) \to \beta^{-1}$ as $t \to \infty$. Then the no-arbitrage condition for capital (16.5.4) becomes

$$1 = \beta \left[\left(1 - \tau^k \right) F_k + 1 - \delta \right]. \tag{16.5.12}$$

Equalities (16.5.11) and (16.5.12) imply that $\tau^k = 0$.

16.6. Taxation of initial capital

Thus far, we have set τ_0^k at zero (or some other small fixed number). Now suppose that the government is free to choose τ_0^k. The derivative of J in equation (16.5.8) with respect to τ_0^k is

$$\frac{\partial J}{\partial \tau_0^k} = \Phi u_c(0) F_k(0) k_0, \tag{16.6.1}$$

which is strictly positive for all τ_0^k as long as $\Phi > 0$. The nonnegative Lagrange multiplier Φ measures the utility costs of raising government revenues through distorting taxes. Without distortionary taxation, a competitive equilibrium would attain the first-best outcome for the representative household, and Φ would be equal to zero, so that the household's (or equivalently, by Walras' Law, the government's) present-value budget constraint would not constrain the Ramsey planner beyond the technology constraints (16.2.3). In contrast, when the government has to use some of the tax rates $\{\tau_t^n, \tau_{t+1}^k\}_{t=0}^{\infty}$, the multiplier Φ is strictly positive and reflects the welfare cost of the distorted margins, implicit in the present-value budget constraint (16.5.10b), that govern the household's optimization behavior.

By raising τ_0^k and thereby increasing the revenues from lump-sum taxation of k_0, the government reduces its need to rely on future distortionary taxation, and hence the value of Φ falls. In fact, the ultimate implication of condition (16.6.1) is that the government should set τ_0^k high enough to drive Φ down to zero. In other words, the government should raise *all* revenues through a time 0 capital levy, then lend the proceeds to the private sector and finance government expenditures by using the interest from the loan; this would enable the government to set $\tau_t^n = 0$ for all $t \geq 0$ and $\tau_t^k = 0$ for all $t \geq 1$.

16.7. Nonzero capital tax due to incomplete taxation

The result that the limiting capital tax should be zero hinges on a complete set of flat-rate taxes. The consequences of incomplete taxation are illustrated by Correia (1996), who introduces an additional production factor z_t in fixed supply $z_t = Z$ that cannot be taxed, $\tau_t^z = 0$.

The new production function $F(k_t, n_t, z_t)$ exhibits constant returns to scale in all of its inputs. Profit maximization implies that the rental price of the new factor equals its marginal product:

$$p_t^z = F_z\left(t\right).$$

The only change to the household's present-value budget constraint (16.5.1) is that a stream of revenues is added to the right side:

$$\sum_{t=0}^{\infty} q_t^0 p_t^z Z.$$

Following our scheme of constructing the Ramsey plan, step 2 yields the following implementability condition:

$$\sum_{t=0}^{\infty} \beta^t \left\{ u_c\left(t\right) \left[c_t - F_z\left(t\right) Z\right] - u_\ell\left(t\right) n_t \right\} - A = 0, \tag{16.7.1}$$

where A remains defined by equation (16.5.6). In step 3 we formulate

$$\begin{aligned} V\left(c_t, n_t, k_t, \Phi\right) &= u\left(c_t, 1 - n_t\right) \\ &\quad + \Phi \left\{ u_c\left(t\right) \left[c_t - F_z\left(t\right) Z\right] - u_\ell\left(t\right) n_t \right\}. \end{aligned} \tag{16.7.2}$$

In contrast to equation (16.5.7), k_t enters now as an argument in V because of the presence of the marginal product of the factor Z (but we have chosen to suppress the quantity Z itself, since it is in fixed supply).

Except for these changes of the functions F and V, the Lagrangian of the Ramsey problem is the same as equation (16.5.8). The first-order condition with respect to k_{t+1} is

$$\theta_t = \beta V_k\,(t+1) + \beta\theta_{t+1}\left[F_k\,(t+1) + 1 - \delta\right]. \tag{16.7.3}$$

Assuming the existence of a steady state, the stationary version of equation (16.7.3) becomes

$$1 = \beta\,(F_k + 1 - \delta) + \beta\frac{V_k}{\theta}. \tag{16.7.4}$$

Condition (16.7.4) and the no-arbitrage condition for capital (16.5.12) imply an optimal value for τ^k:

$$\tau^k = \frac{-V_k}{\theta F_k} = \frac{\Phi u_c Z}{\theta F_k} F_{zk}.$$

As discussed earlier, in a second-best solution with distortionary taxation, $\Phi > 0$, so the limiting tax rate on capital is zero only if $F_{zk} = 0$. Moreover, the sign of τ^k depends on the direction of the effect of capital on the marginal product of the untaxed factor Z. If k and Z are complements, the limiting capital tax is positive, and it is negative in the case where the two factors are substitutes.

Other examples of a nonzero limiting capital tax are presented by Stiglitz (1987) and Jones, Manuelli, and Rossi (1997), who assume that two types of labor must be taxed at the same tax rate. Once again, the incompleteness of the tax system makes the optimal capital tax depend on how capital affects the marginal products of the other factors.