Econ 7010 - Microeconomics I
University of Virginia
Fall 2023

Problem Set 6

This problem set is optional, but it is highly recommended that you complete it as preparation for the exam.

1. MWG 2.F.3

2. Clarence is a well known legal scholar who worries about his professional image. He has hired you to make sure that he does not generate a trail of consumption evidence that would suggest that he is irrational. He chooses bundle (1,1) when prices are (3,6), but he could have afforded bundle (a,b). Prices move to (4,4). Find the set of choices (a,b) at prices (4,4) that will maintain his image (i.e. the set of bundles that satisfy GARP).

3. In class, we showed that WARP does not imply GARP by giving an example of price-consumption data $\{(p^t, x^t)\}_{t=1}^{T}$ that satisfy WARP, but do not satisfy GARP. Construct an example of data points such that GARP is satisfied and yet WARP is not, thereby proving that GARP does not imply WARP either.

4. MWG 6.B.4, 6.B.7

5. A Heisman Trophy candidate quarterback has his final game of the season on Saturday. He believes that three states of the world are equally likely:

   A. The game is rained out.

   B. The game is played and he passes for 500 yards.

   C. The game is played and he breaks his leg.

   If B happens, he'll be able to sign an NFL contract worth $D + x$ dollars. If C happens, teams will be spooked, and he'll only be able to sign for $D - x$ dollars. If A happens, the teams learn nothing new, and he'll sign for D dollars. Suppose that the quarterback has decreasing absolute risk aversion and total wealth equal to w_0 plus his football income.

(a) Suppose that he is given the chance to sign a contract today, before finding out what will happen on Saturday. Let d be the smallest contract offer that he would accept today. How does d vary with w_0? You can appeal to results in the textbook, but be rigorous.

(b) Let's change the situation slightly. Suppose that w_0 is income that he could earn by going to his part-time job on Saturday. He can earn this money if the game is rained out, but not if he has to play. d is defined just as in (a). Once again, determine how d varies with w_0. Compare to your result in (a) and explain.

6. A risk-averse bakery owner faces an uncertain price $\tilde{p}$ for her bread. Her production function for bread is $y = x^a$, where $a \in (0,1)$ and x is labor. The wage rate is $w = 1$ with certainty. The bakery owner has CRRA utility $u(w) = \frac{1}{1-\sigma} w^{1-\sigma}$ over her wealth, which comes entirely from bakery profits. (Assume $\sigma \in [0, \infty)$.)

   (a) Suppose that the price of bread is equally likely to be p_1 or p_2 (with $p_1 < p_2$). The owner must make her production decision before the price uncertainty is resolved. Let x_0 be her optimal choice of x if she is risk neutral ($\sigma = 0$) and x_σ her optimal choice if she is strictly risk averse ($\sigma > 0$). Determine whether $x_0 > x_\sigma$ or *vice versa*. Provide some brief intuition for your result.

   (b) Now suppose that $\tilde{p}$ is distributed according to cdf $F(p)$ and, unlike (a), suppose that the owner gets to observe the realized value of p before making her production decision. Suppose that prior to all of this, the chairman of the Fed proposes a policy that will stabilize prices, replacing the uncertain $\tilde{p}$ with its expectation $\bar{p} = E_F(\tilde{p})$. For which values of σ, if any, would the bakery owner oppose this policy? Explain.

7. Suppose that an agent with CARA utility is offered Game 1: a gamble over the outcome of one toss of a loaded coin: Heads, he wins X, Tails, she loses X. (Heads has probability p.) Alternatively, suppose she is offered Game 2: the chance to play Game 1 100 times (that is, 100 independent coin flips), earning $\pm X$ each time. Let her initial wealth be denoted w_0. Are there conditions under which she is willing to play

Game 1 but not Game 2? Or willing to play Game 2 but not Game 1? Prove your answer.

8. Suppose we have a prize space in dollars of $X = \{1, 2, 3, 4, 5\}$.

   (a) Suppose a risk-averse expected utility maximizer is comparing the following two gambles: $p = (1/5, 1/5, 1/5, 1/5, 1/5)$ and $q = (2/5, 0, 1/5, 0, 2/5)$. Can you say unambiguously which she would prefer?

   (b) Suppose a risk-averse expected utility maximizer is is comparing $p' = (1/5, 1/5, 1/5, 1/5, 1/5)$ with $q' = (2/5, 0, 0, 1/5, /2/5)$. Can you say unambiguously which she would prefer?

   (c) Consider a decision maker with utility function $u(x) = x - ax^2$, defined on $x \in [1, 5]$, where $0 < a < 1/5$.

      i. Calculate the decision maker's coefficient of absolute risk aversion and coefficient of relative risk aversion. Does she have decreasing/increasing/constant absolute and/or relative risk aversion?

      ii. Show that for this Bernoulli utility function u, the corresponding expected utility function $U(F)$ depends on only the mean and variance of F; that is, show that we can write $U(F) = V(\mu_F, \sigma_F^2)$, where μ_F and σ_F^2 are the mean and variance of F, respectively (such utility functions are sometimes called *mean-variance utility*; it is a helpful simplification in many models, because it means we do not need to specify the entire distribution F to be able to calculate an agent's (expected) utility, but only need to know the mean and variance of F).

      iii. How will this decision maker rank p vs. q and p' vs. q'?

9. Mom would burst with pride if her two kids (creatively named 1 and 2) became astronauts. The probability that kid i becomes an astronaut is $p_i = \theta_i x_i$, where θ_i is kid i's natural talent, and x_i is how much time Mom spends sticking glow-in-the-dark stars to i's bedroom ceiling. (These probabilities are independent across kids.) Mom has a time constraint on her effort: $x_1, x_2 \geq 0$ and $x_1 + x_2 \leq 1$.[1] Her Bernoulli

[1] She can also also choose to spend some of her time with neither kid and just drink Chardonnay. However, this does not help either kid become an astronaut, and hence does not enter her utility function.

utility over the number of astronaut children is $u()$. You can assume that the talent levels satisfy $0 < \theta_1 < \theta_2 < 1$.

(a) Suppose that all that Mom cares about is raising at least one astronaut. That is, $u(2) = u(1) > u(0)$. Determine her optimal allocation of effort between her kids. (Be careful!) How does kid i's maternal attention x_i vary with his talent θ_i?

(b) Alternatively, suppose that Mom knows that either kid would be miserable if he failed while his sibling succeeded, and she wants to avoid this. Her preferences are now $u(2) = u(0) > u(1)$. Once again, determine her optimal allocation of effort and comment on the relationship between θ_i and x_i.