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Homework 1

1. A die is tossed n times. Find the probability of events where:

(i) Atleast one of the outcomes is equal to 6

$= 1 -$ none of the outcomes are equal to 6.

$$= 1 - \left(\frac{5}{6}\right)^n$$

(ii) An outcome equal to 6 is observed exactly once.

$$\text{Atleast once} \begin{cases} ⓪ \text{ None of the times} \\ ① \text{ Exactly once} \\ ② \text{ Exactly twice} \\ \vdots \\ ⓝ \text{ All the time.} \end{cases}$$

$${}^nC_1 \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^{n-1}$$

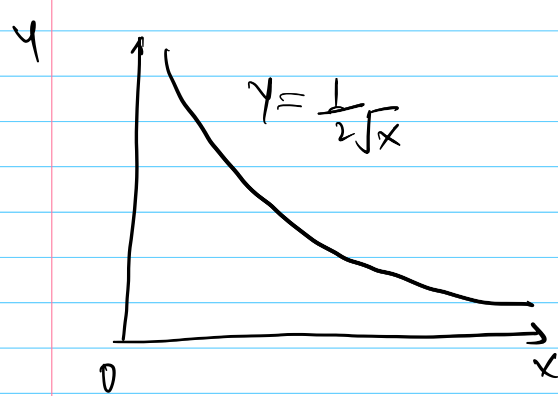
Q3 The statement is false as explained below:—

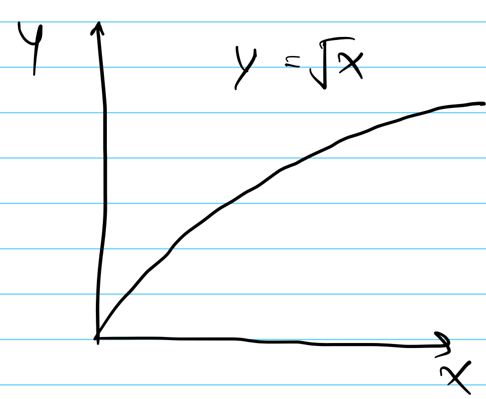
1) 'Almost everywhere' means that the random variable X has a density everywhere except a subset of measure zero.

2) A measurable function $X: F \to B$ is called a random variable. On the Borel set, a set of a singleton point has measure zero.

3) In the case where random variable X has probability density function $f(x) = 1/2\sqrt{x}$ defined on $[0, 1]$. At pt $x = 0$, consider that the density is zero.

$\Rightarrow$ f has density "almost everywhere" as it has density except on the subset $\{0\}$, a set of singleton point.





(4) The corresponding CDF, $F(x) = \sqrt{x}$ is not uniformly continuous as for a function to be uniformly continuous, all its derivates need to be bounded.
But at $x=0$, $F'(x) = 1/2\sqrt{0} = \infty$

For the statement to be satisfied, the pdf must have density everywhere on the bounded segment.

Q4 :

We assume that the marbles are distinct.

- Numerator :

No urn is empty $\Rightarrow$ There is a marble in each urn

$\Rightarrow$ $n!$ ways of placing the marbles

- Denominator $\Rightarrow$ n^n ways to place the marbles in n urns

$$P(\text{no urn is empty}) = \frac{n!}{n^n}$$

Q5 For $H(x)$ to be a distribution function, it has to fulfill the following properties:

i) Monotonicity

If $x_1 \leq x_2 \Rightarrow H(x_1) \leq H(x_2)$

Assume $x_1 \leq x_2$

$\Rightarrow G(x_1) \leq G(x_2)$ (As $G(\cdot)$ is a distn fn)

Let $y_1 = G(x_1)$ & $y_2 = G(x_2)$

$\Rightarrow y_1 \leq y_2$

$\Rightarrow f(y_1) \leq f(y_2)$ (As $f(\cdot)$ is a distn function)

$\Rightarrow F(G(x_1)) \leq F(G(x_2))$

$\Rightarrow H(x_1) \leq H(x_2)$ $\square$

No conditions required.

2) $\lim_{x \to -\infty} H(x) = 0$ and $\lim_{x \to +\infty} H(x) = 1$

$H(x) = F(G(x))$

$$\lim_{x \to -\infty} H(x) = \lim_{x \to -\infty} F(G(x))$$

$$= F\left(\lim_{x \to -\infty} G(x)\right)$$

$$= F(0)$$ (As $G(\cdot)$ is a dist^n f^n)

For $\lim_{x \to -\infty} H(x) = 0$;

$F(0)$ should be continous & $F(0) = 0$

$$\lim_{x \to +\infty} H(x) = \lim_{x \to +\infty} F(G(x))$$

$$= F\left(\lim_{x \to +\infty} G(x)\right)$$

$$= F(1)$$ (As $G(\cdot)$ is a dist^n function)

For $\lim_{x \to +\infty} H(x) = 1$:

$\circledast$ $F(1)$ should be continaeus & $F(1) = 1$.

3) Left continuity: $\lim_{x \uparrow x_0} H(x) = H(x_0)$

$\lim_{x \uparrow x_0} F(G(x))$

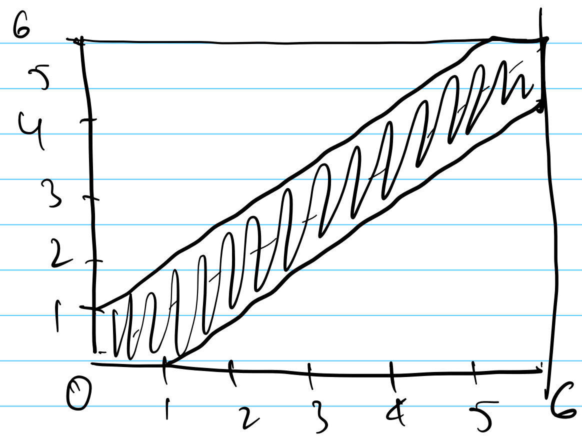
$\Rightarrow F(\lim_{x \uparrow x_0} (G(x)))$

$= F(G(x_0))$

• F should be continous at every $G(x_0)$

Q-2

(a) Date from 7-8 pm → normalize it to one hour window. The following diagram splits the one hour in 6 parts to represent the 10 minute gap in which the husband & wife can arrive



They will meet as long as they arrive in the shaded part of the diagram.

$$P(\text{date occurs}) = \frac{\underbrace{36}_{\text{area of the square}} - \underbrace{\tfrac{1}{2}(5)(5)}_{\text{area of the 2 } \Delta\text{s}}}{36}$$

$$= \frac{11}{36}$$