Oct 30, 2023.

Welfare Evaluations.

Price change : $p^0 \to p^1$

$v(p^1, m) - v(p^0, m)$

Ordinal nature of utility makes this "problematic"
Want a "standardized" welfare measure

Money metric indirect utility function
- Fix some (arbitrary) base $\bar{p}$

$\mu(\bar{p}; p^0, m) = e(\bar{p}, v(p^0, m))$

$\mu(\bar{p}; p^1, m) = e(\bar{p}, v(p^1, m))$

$\Delta\omega = \mu(\bar{p}; p^1, m) - \mu(\bar{p}, p^0, m)$

still requires reference prices $\bar{p}$

Most natural choices :—

$\bar{p} = p^0$ $\qquad\qquad$ $\bar{p} = p^1$

Equivalent variation (EV) : $\bar{p} = p^0$
Compensating variation (CV) : $\bar{p} = p^1$

$u^0 = v(p^0, m)$ $\qquad\qquad$ $u^1 = v(p^1, m)$

① EV :

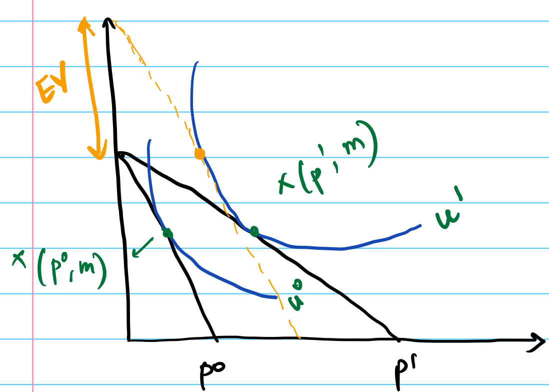
$EV = \mu(p^0; p^1, m) - \mu(p^0; p^0, m)$
$= e(p^0, u^1) - e(p^0, u^0)$
$= e(p^0, u^1) - m$

Two scenarios

1. Price changes : $p^0 \to p^1$

2. Prices fixed, cons^m gets an extra net \$ EV

EV is the no. that makes ② = ①



$p_1^1 < p_1^0$

$p_2^1 = p_2^0 = 1$

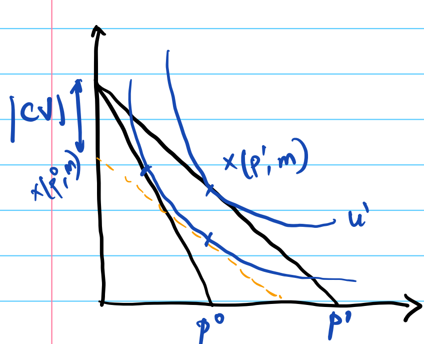
② CV

$$\bar{p} = p^1$$

$$CV = u(p^1; p^1, m) - u(p^1; p^0, m)$$

$$= e(p^1, u^1) - e(p^1, u^0)$$

$$= m - e(p^1, u^0)$$



$p_1^0 > p_1^1$

$p_2^0 = p_2^1 = 1$

COMMENTS :-

1) Many choices of utility f^n we could use for welfare
2) EV & CV are 2 most natural
3) In general, CV $\neq$ EV but will always have same sign
4) In principle, observable if we know Marshallian demand (integrability)

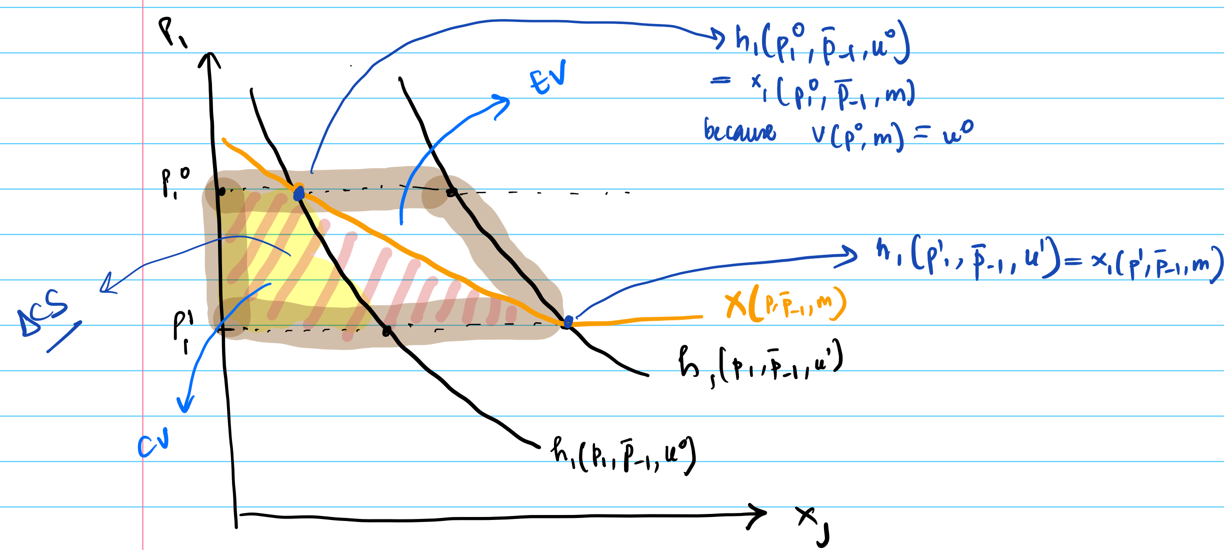
Another way to view EV, CV (& consumer surplus)

Fix $p_i^0 = p_i^1 = \bar{p}_i \quad \forall\, i \neq 1$

Recall,

$$e(p^0, u^0) = e(p^1, u^1) = m$$

$EV =$



p_1
$h_1(p_1^0, \bar{p}_{-1}, u^0)$
$= x_1(p_1^0, \bar{p}_{-1}, m)$
because $v(p^0, m) = u^0$
EV
p_1^0
p_1^1
$h_1(p_1^1, \bar{p}_{-1}, u^1) = x_1(p_1^1, \bar{p}_{-1}, m)$
$x(p_1, \bar{p}_{-1}, m)$
$h_1(p_1, \bar{p}_{-1}, u^1)$
$h_1(p_1, \bar{p}_{-1}, u^0)$
ΔCS
CV
x_1

Consumer Surplus:

$$\Delta CS = \int_{p_1'}^{p_1^0} x_1(\tilde{p}_1, \bar{p}_{-1}, m)\, d\tilde{p}_1$$

$CV \leq \Delta CS \leq EV$ in picture → holds for normal goods
$EV \leq \Delta CS \leq CV$ → for inferior goods
$CV = \Delta CS = EV$ → for quasilinear goods.

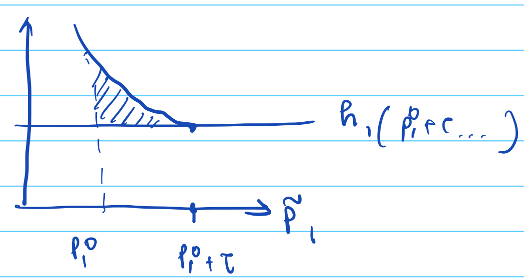
Example:

Gov't wants to raise \$T tax revenue

1. Linear tax on good 1 $p^0 = (p_1^0, p_2^0) \rightarrow (p_1^0 + \tau, p_2^0) = p'$
2. lump sum tax $T = \tau x_1(p', m)$

What are the welfare comparisons b/w 1 & 2?
Answer by calculating EV of the lump sum tax
If $EV = e(p^0, u') - m < -T$, cons is worse off under 1

$$\begin{aligned}
DWL &= -EV - T \\
&= e(p', u') - e(p^0, u') - T \\
&= \int_{p_1^0}^{p_1^0+\tau} h_1(\tilde{p}_1, p_2^0, u')\, d\tilde{p}_1 - \underbrace{\tau h_1(p_1^0 + \tau, p_2^0, u')}_{= x_1(p', m)} \\
&= \int_{p_1^0}^{p_1^0+\tau} [h_1(\tilde{p}_1, p_2^0, u') - h_1(p_1^0 + \tau, p_2^0, u')]\, d\tilde{p}_1 \geq 0
\end{aligned}$$



$DWL = -EV - T \geq 0 \Rightarrow EV \leq -T$