Final Exam - Econ 7010

December 7th, 2021

The exam contains 3 questions. **Use a separate bluebook for each question.** Be sure to write your name on each, as well as the question number.

Your answers to all questions must be fully justified to receive credit. Ensure that your final answers are presented in a clear and coherent manner. I should not have to (and will not) "search" for your answers to any question. If you cannot answer a question completely, a well-labeled diagram and/or clear explanation of the intuition behind the solution process can obtain partial credit.

You have 2 hours to complete the exam. The total number of points is 100. Budget your time wisely; you may find later questions easier than earlier ones.

Good luck!

Question 1 (40 points)

Sebastian has preferences over ice cream (x_1), donuts (x_2), and kale (x_3) described by the utility function

$$u(x) = \frac{x_1 x_2}{x_3 + 1}.$$

The price of good i is $p_i > 0$, and he has wealth m.

(a) (12 points) For each of the following properties, (i) state its definition for a general utility function (1 point each) (ii) determine whether Sebastian's preferences satisfy it. Prove your answer (2 points each).

- Local non-satiation
- Monotonicity
- Strong monotonicity
- Convexity

(b) (6 points) Find Sebastian's Marshallian demand functions and indirect utility function.

(c) (6 points) *Use the duality identities* to find Sebastian's Hicksian demand functions and expenditure function.

For the rest of the problem, let $p_1 = 1$, $p_2 = 1$, $p_3 = 2$, and $m = 10$. Concerned about its citizens eating too much junk food, the government decides to mandate that all households must buy at least one unit of kale (assume that the kale must also be consumed, i.e., it cannot be bought and just thrown away). Because of this, the price of kale has increased to $p_3' = 5$.

(d) (10 points) Sebastian is incensed by this policy, and threatens to not vote for the current government in the next election unless he is compensated for his loss in welfare due to the kale mandate. Being knowledgable in microeconomic theory, he proposes to the government two possibilities: the equivalent variation, and the compensating variation. Calculate these measures. Presuming the government wants to spend as little money as possible, which will they choose?

(e) (3 points) Concerned about mandates, the government decides to enact an alternative policy: for every unit of kale that he consumes, they will give Sebastian 1 dollar. Write down Sebastian's optimization problem, including the budget constraint.

(f) (3 points) Find Sebastian's optimal bundle. Which policy is more effective in increasing kale consumption?

Question 2 (30 points)

Paul Samuelson once defined a coward as someone who will not take a bet with 2-to-1 odds, even when you let him choose his side. You are wondering if your friend is a coward, so you decide to offer him such a bet. You will flip a loaded coin, that has a probability p_H of coming up heads and $p_T = 1 - p_H$ of coming up tails (you may assume both of you know the value of p_H, but $p_H \neq 1/2$). You allow your friend to choose heads or tails (this is what we mean by saying he can "choose his side"). After he makes his choice, you will flip the coin. If he wins, you will give him \$$2z$; if he loses, he must give you \$$z$, where $z > 0$.

Assume your friend is an expected utility maximizer with an increasing Bernoulli utility function over money $u(\cdot)$ with $u(0) = 0$. He has an initial wealth of w_0. For parts (a)-(c), you may assume $w_0 = 0$, but z can be any arbitrary positive number.

(a) (5 points) Is it possible for your friend to be a Samuelson coward? In other words, is rejecting both sides of the bet consistent with expected utility maximization for some increasing utility function over money $u(\cdot)$? Either provide a utility function that is consistent with this behavior, or prove that no such function exists. (Your function can be defined only for certain points.)

(b) (5 points) Assume now that you know your friend is in fact risk-neutral. Is it possible for him to be a Samuelson coward?

(c) (5 points) Go back to an arbitrary $u(\cdot)$. "Prove" the following statement: If u is differentiable and concave, then for all z sufficiently small, an expected utility maximizer *cannot* reject both sides of the bet. (A complete formal proof is fine, but may be difficult; a clear argument with a convincing explanation of the intuition is sufficient for full credit. What you learned from the previous parts may be helpful.)

For the remainder of the question, consider the following utility function for your friend:

$$u(x) = \frac{x^\gamma}{\gamma}.$$

(d) (5 points) Calculate the coefficient of relative risk aversion for this utility function. For which values of γ is your friend risk-averse? For which is he risk-loving?

(e) (5 points) Let $w_0 = 11$, $z = 7$, $\gamma = 1/2$, and $p_H = 2/3$. Find your friend's certainty equivalent of this lottery, assuming that he chooses heads. How does it compare to the expected value of the lottery?

(f) Let $c^*(z)$ be the certainty equivalent as a function of z. *Use the implicit function theorem* to find an equation for dc^*/dz. Is it positive or negative? Provide intuition for the result.
(You may restrict to the range $z \leq 11$, so that the utility function is well-defined for all possible lottery outcomes; all other parameters are as in the previous part.)

Question 3 (30 points)

Consider a choice set with three alternatives, $\mathcal{X} = \{x, y, z\}$, and a choice function $C(\cdot)$ defined as follows:

$$C(\{x\}) = x \qquad C(\{y\}) = y \qquad C(\{z\}) = z$$

$$C(\{x,y\}) = y \qquad C(\{y,z\}) = y \qquad C(\{x,z\}) = x$$

$$C(\{x,y,z\}) = x$$

(a) (6 points) Does this choice structure satisfy WARP?

Consider the following condition on choice structures, known as *Sen's* α:

> *Let* $Y, Z \subseteq \mathcal{X}$ *be two subsets of* $\mathcal{X}$ *such that* $Y \subseteq Z$. *If* $x \in C(Z)$ *and* $x \in Y$, *then* $x \in C(Y)$.

(b) (2 points) Translate this condition into words. (It may be helpful, though not necessary, to draw a Venn diagram.)

(c) (6 points) Does the choice structure above satisfy Sen's α?

(d) (2 points) Sen's α sometimes goes by the name of *independence of irrelevant alternatives*. Say that $x \in C(Z)$, and let $Y = Z \setminus \{w\}$ for some $w \neq x$. Using this notation, explain briefly why this alternative nomenclature makes sense.

(e) (9 points) Prove formally that WARP implies Sen's α in general; that is, given an arbitrary choice function $C(\cdot)$ show that:

   $C(\cdot)$ *satisfies WARP* $\implies$ $C(\cdot)$ *satisfies Sen's* α.

   (Hint: To show that $P \implies Q$, it is equivalent to show that "not Q" $\implies$ "not P"; this is called proof by contrapositive.)

(f) (5 points) Finally, consider the following condition, called Sen's β:

   If $x, y \in Y$, $Y \subseteq Z$, $x, y \in C(Y)$, *and* $y \in C(Z)$, *then* $x \in C(Z)$.

   Does the choice structure given above satisfy Sen's β? (Hint: Note that for any offer set $Y \subseteq \mathcal{X}$, $|C(Y)| = 1$.)