Econ7020: Macroeconomic Theory

Fall 2023

Problem Set 1. Due date: before class on September 14.

Problem 1

Consider an economy with two infinitely lived consumers. There is one good in each period. Consumer $i \in \{1, 2\}$ has the utility function $\sum_{t=0}^{\infty} \beta^t \log c_t^i$. Here $\beta \in (0, 1)$ is the common discount factor. Each of the consumers is endowed with a sequence of goods:

$$(w_0^1, w_1^1, w_2^1, w_3^1, \dots) = (2, 2, 2, 2, \dots)$$
$$(w_0^2, w_1^2, w_2^2, w_3^2, \dots) = (1, 4, 1, 4, \dots).$$

There is no production or storage technology.

(a) Describe the Sequential Markets structure for this economy, explaining when markets open, who trades with whom and so on. Define the Sequential Markets equilibrium for this economy.

(b) Describe the Arrow-Debreu Markets structure for this economy, explaining when markets are open, who trades with whom and so on. Define the Arrow-Debreu equilibrium for this economy.

(c) Carefully state a proposition that establishes the essential equivalence of the equilibrium concept in part (a) with that in part (b). Be sure to specify the relationships between the objects in the Arrow-Debreu equilibrium and those in the Sequential Markets equilibrium. [You do not have to prove this proposition].

(d) Calculate the Arrow-Debreu equilibrium for this economy. Use this answer and result from (c) to calculate the Sequential Markets equilibrium.

Problem 2

Consider a pure-exchange economy with two infinitely lived consumers, each of whom has the same utility function $U = \sum_{t=0}^{\infty} \beta^t \log c_t^i$, with $\beta \in (0,1)$. Consumption good is perishable. The streams of endowments are:

$$(w_0^1, w_1^1, w_2^1, w_3^1, \dots) = (4, 2, 4, 2, \dots)$$

and

$$(w_0^2, w_1^2, w_2^2, w_3^2, \dots) = (2, 4, 2, 4, \dots).$$

(a) Describe an Arrow-Debreu market structure for this economy, explaining when markets are open, who trades with whom, and so on. Define an Arrow-Debreu equilibrium.

(b) Define a Pareto efficient allocation for this economy. Calculate a Pareto efficient allocation by maximizing a weighted sum of utilities, $\alpha_1 U_1 + \alpha_2 U_2$.

(c) Define an Arrow-Debreu equilibrium with transfer payments. Find the transfer payments necessary to implement the Pareto efficient allocation in part (b) as equilibrium with transfers. *Note that transfer payments must be functions of model parameters* α_1, α_2 *and* β.

(d) Find the transfer payments necessary to implement the allocation $(c_t^1, c_t^2) = (3, 3)$ as an equilibrium with transfers.

Problem 3

Consider an overlapping generations economy in which there is one good in each period and each generation, except the initial one, lives for two periods. The representative consumer in generation $t, t = 1, 2, \dots$ has the utility function

$$\log c_t^t + \log c_{t+1}^t$$

and the endowment $(w_t, w_{t+1}) = (w_1, w_2)$. The representative consumer in generation 0 lives only in period 1, prefers more consumptin to less, and has the endowment $w_1^0 = w_2$. There is no fiat money.

(a) Define an Arrow-Debreu equilibrium for this economy. Calculate the unique Arrow-Debreu equilibrium. [You do not have to prove that equilibrium is unique].

(b) Define a Sequential Markets equilibrium. State a proposition (or propositions) that relate the unique SME to the unique ADE. Calculate the unique SME. [You do not have to prove that equilibrium is unique.]

(c) Define a Pareto efficient allocation for this economy.

(d) Prove that the equilibrium allocation is Pareto efficient if $w_2 > w_1$.

(e) Prove that the equilibrium allocation is not Pareto efficient if $w_2 < w_1$.