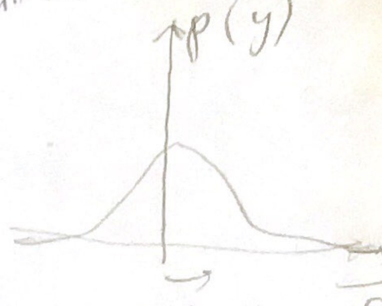
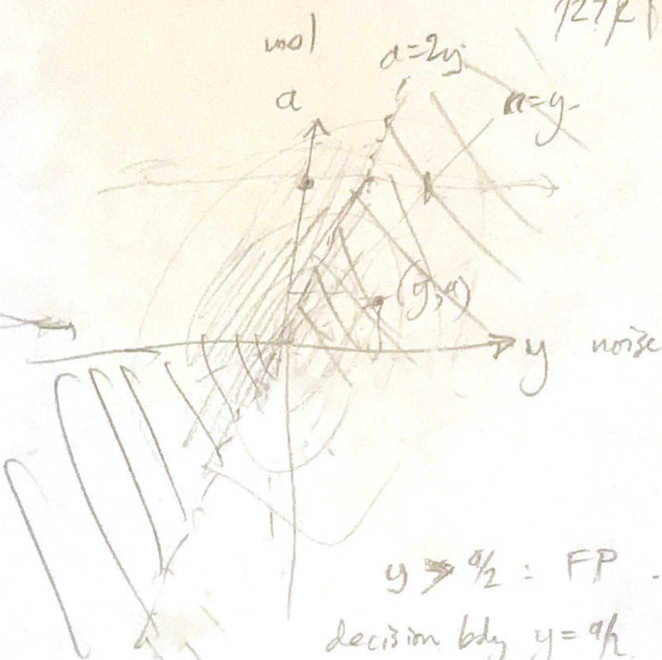
Tanya cage 1) theory results: 7/27/21

$N=1$ zero-mean

$p=q=1$ $a \sim N(0,1)$ "rand. mol"

$y = \epsilon \sim N(0, \sigma^2)$





joint $p(a,y) = p(a)\,p(y|a)$

$N(0,1)$ $N(0,\sigma^2)$ $p(\cdot y)$

meth. 1 (MaxLik)

EFPR:

$= \mathbb{E}_{\vec a} \left[\mathbb{E}_{\vec y} \, \mathbb{1}_{\|y-a\|^2 \le \|y\|^2} \right] = \frac{1}{2}\left(1 - \operatorname{erf}\left(\frac{a}{2\sqrt{2}\sigma}\right)\right)$

sig (noise)

$\|y\|^2 - \|y\|^2 - 2a\cdot y + a^2 \le 0$

$y \geq a/2$: FP

decision bdy $y = a/2$

$N>1$: $\vec a = [\cdots 0, 0, N(0,4), N(0,4), N(0,4), N(0,4), 0, 0, \cdots]$ (p pixels)

$\vec y \sim^{iid} N(0,\sigma^2)$

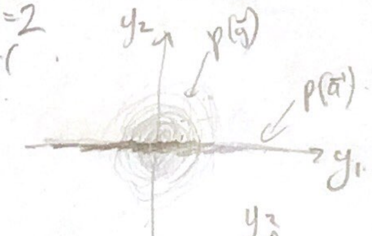
$2a\cdot y \geq a^2$

$N=2$

$p = 1\times 1$ (mol)

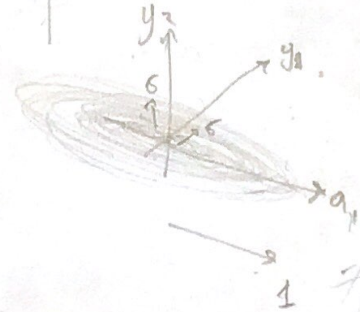
$\vec y = [y_1 \, y_2]$

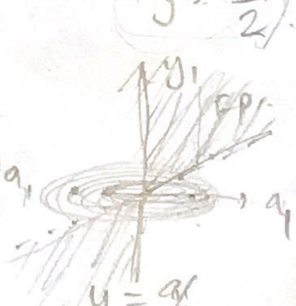
$\vec a \sim N(0,1), 0$ ($a_2 = 0$)

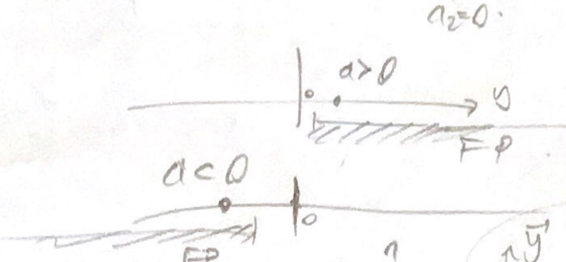


a cone!

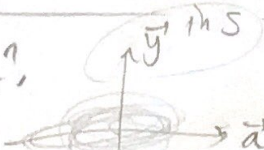
$a\cdot y \geq \frac{|a|^2}{2}$







numerical answer?



conical integral over MV Gaussian.

isotropic

$EFPR = \iint e^{-\frac{|a|^2}{2} - \frac{|y|^2}{2\sigma^2}} \, da \, dy$ domain $\left\{ \vec a, \vec y : a_1 y_1 + a_2 y_2 \geq \frac{1}{2}a_1^2 + \frac{1}{2}a_2^2 \right\}$

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