As put in the paper -

"Calculating the SVD consists of finding the eigenvalues and eigenvectors of AA' and $A'A$. The eigenvectors of $A'A$ make up the columns of V, the eigenvectors of AA' make up the columns of U. Also, the singular values in Σ are square roots of eigenvalues from AA' or $A'A$. The singular values are the diagonal entries of the σ matrix and are arranged in descending order. The singular values are always real numbers. If the matrix A is a real matrix, then U and V are also real."

Again, you can see a fully worked example of the closed form solution at the MIT Link here.

A More Common Approach

The main issue with the closed form solution (especially for us) is that it doesn't actually work when we have missing data. Instead, Simon Funk (and then many followers) came up with other solutions for finding our matrices of interest in these cases using **gradient descent**.

So all of this is to say, people don't really use the closed form solution for SVD, and therefore, we aren't going to spend a lot of time on it either. The link above is all you need to know. Now, we are going to look at the main way that the matrices in SVD are estimated, as this is what is used for estimating values in FunkSVD.

Additional Resources

Below are some additional resources in case you are looking for others that go beyond what was shown in the simplified MIT paper.

- Stanford Discussion on SVD
- Why are Singular Values Always Positive on StackExchange
- An additional resource for SVD in Python
- Using Missing Values to Improve Recommendations in SVD

NEXT