intertemporal choice

① present & future value

1) present value : the value of a future good in the present
2) future value: the value of a present good in the future

$$PV = \frac{FV}{1+r}$$

② intertemporal choice

1) intertemporal budget constraint

↳ consumer is faced w making consumption choices over his lifetime (multiple periods), with multiple incomes at these different time periods.

↳ in each of these periods, he can choose to consume or to save. saving allows for future consumption and growth of value through interest rates (price of others borrowing) in the credit market

in first period : $y_1 \geq c_1 + s_1$ ⇒ income is spent or saved

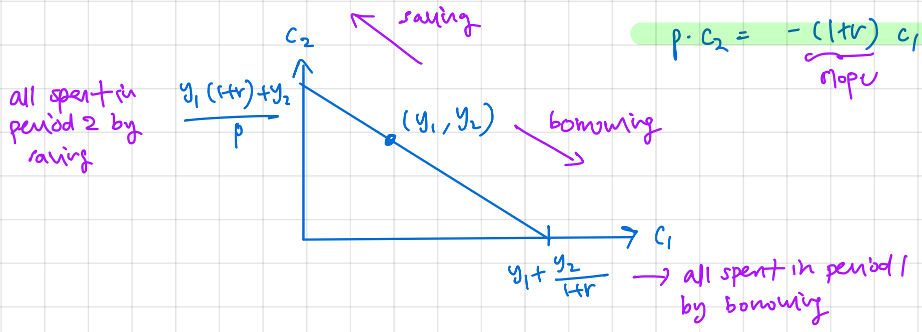
in period : $y_2 + s_1(1+r) \geq p \cdot c_2 + s_2$ ⇒ appreciated savings & income fund second period spending & saving

period 1 price as numeraire

→ if we assume all money is spent, we can connect the constraints in each period

$$c_1 + \frac{p \cdot c_2}{1+r} = y_1 + \frac{y_2}{1+r}$$

⇒ all possible combinations of c_1 and c_2 lie on or below this line



$$p \cdot c_2 = \underbrace{-(1+r)}_{\text{slope}}\, c_1 + \underbrace{y_1(1+r) + y_2}_{\text{intercept}}$$

2) intertemporal preference, utility & indifference curves

↳ an individual tends to smooth out consumption over periods rather than extremes

↳ future consumption may be valued less than current consumption

↳ other standard assumptions of well behaved utility functions hold → more is better ↳ diminishing MU

↳ no reason why utility function in each period should change, since same person

⇒ "standard" convex utility function w some weighting of future consumption can model lifetime utility & consumption choices

$$u(c_1, c_2) = \overbrace{u(c_1) + \underbrace{\beta}_{\text{weighting}} \cdot u(c_2)}^{\text{same function}}, \quad \beta \in (0,1)$$

3) accounting for inflation / price levels across periods

↳ we account for price in the model using P_1 as a numeraire, since relative, rather than absolute, price matters most.

ie. $$p = \frac{P_2}{P_1}$$

↳ the credit market provides a mechanism through which one can transfer real value from the present to future

⇒ we can incorporate inflation effect into i/r through real interest rates

$$\boxed{\begin{aligned} P_2 &= 1+\pi \\ 1+\rho &= \frac{1+r}{1+\pi} \approx 1+r-\pi \text{ when } \pi \approx 0 \end{aligned}}$$

nominal i/r

real i/r

inflation rate

4) utility maximisation

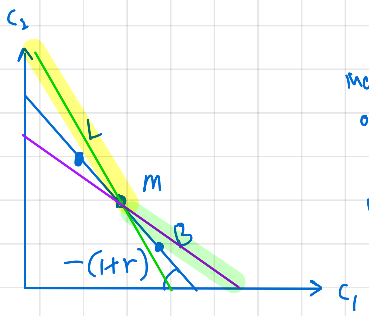
$$\max_{c_1, c_2} u(c_1, c_2) \qquad du = \frac{\partial u}{\partial c_1} dc_1 + \frac{\partial u}{\partial c_2} dc_1 = 0$$

s.t. budget constraint

③ comparative statics : changes in real i/r

i) if interest rate increases, a lender will remain a lender
if interest rate falls, a borrower will remain a borrower

⇒ proof by revealed preference



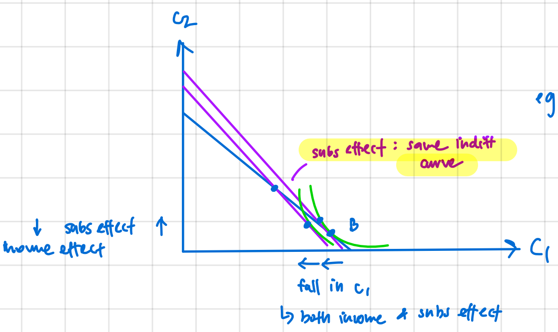
increase in interest rate causes pivot about endowment. New optimal cannot be below endowment point since it was previously affordable but not considered. Same logic for borrower when interest rate falls.

purchasing power reduced by less gains from fallen interest

2) if interest rate falls, how a lender reacts depends on income and substitution effects.
if interest rate rises, how a borrower reacts also depends on income & substitution effects.

different terms of trade (price) between present & future consumption

↳ revealed preference cannot infer anything



eg. originally a borrower. i/r increases

↳ more expensive to consume today. $c_1 \downarrow$, $c_2 \uparrow$

↳ generally able to purchase less $c_1 \downarrow$, $c_2 \downarrow$
due to loss paying interest

④ comparative statics: inflation & deflation

1) real interest rate

↳ changes in relative price levels also distort transfer of value through time via i/r

↳ we need to consider real interest rate ie. the TOT between now and future

From PV: $c_1 + \frac{c_2 \cdot p}{1+r} = m_1 + \frac{m_2}{1+r}$

suppose $p = 1 + \pi$

then TOT between present and future $= \frac{1+r}{1+\pi}$

Then $1+\rho = \frac{1+r}{1+\pi} \approx 1 + \pi - r$ when $\pi \approx 0$

2) statics analysis

1. does future income change w price level?
2. inflation affects future

} → determines pivot point

Notes

1. for scenarios w multiple consumptions in each period, just set up multiple simaltaneous equations! eg. $u = \min(a_1, 2b_1) + \min(a_2, 2b_2)$

$\Rightarrow$
$a_1 = 2b_1$ — 1
$a_2 = 2b_2$ — 2
LBC — 3
tangency MRS — 4

} solve simaltaneously

choices under uncertainty

① preferences under uncertainty

1. consumer is faced w uncertainty in that outcomes will occur with different probabilities. we use a single metric ($) to quantify state.

2. To study an individual's choices, we must model his preferences under uncertainty.
   2.1 so we assume a rational consumer uses expected utility to make choices.
   2.2. we can model the expected utility of an individual given some (c_2, c_1) scenario he is dealt and his utility function in either scenario, and variables he can control

3. suppose all states are mutually exclusive
   3.1 because they are mutually exclusive, the utilities across different states should be additive and weighted by their probabilities

⇒ so, we can model preferences under uncertainty with expected utility functions

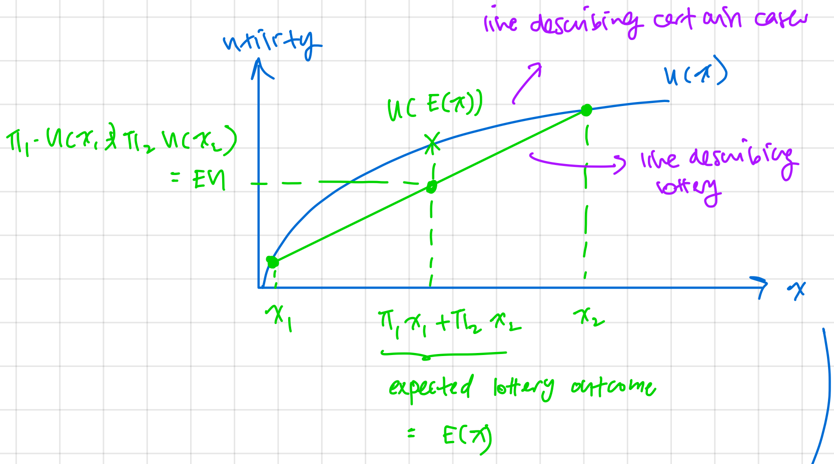
$$EU = \sum_{i=1}^{n} \pi_i \cdot U(c_i)$$

↳ same individual ⟶ same utility function across states

↳ probability weighted sum due mutual exclusivity of states

② risk aversion

1) comparing expectation of outcomes vs. expected utility



1. we can map the utility in either scenario to two points on the curve

2. Then the line connecting these points represents some weighted sum of $u(x_1)$ and $u(x_2)$. At $x = \pi_1 x_1 + \pi_2 x_2$, it is the expected utility by definition

3. If a consumer is risk averse, he will gain greater utility from a certain sum than a chance of a lot more.

↳ at corresponding x, $u(x) > EU|_{\pi_1, \pi_2}$

↳ a consumer will prefer the actual sum than the probabilistic utility he might get

(implications)

1. diminishing MU & concavity for risk averse consumers
   ↳ consumption smoothing behavior

2. convexity for risk preferring consumers
   ↳ extreme outcomes are preferred

$u(E(x)) > EU$ — certain payout preferred to lottery ⇒ risk aversion

$u(E(x)) < EU$ — lottery preferred to certain outcomes ⇒ risk loving

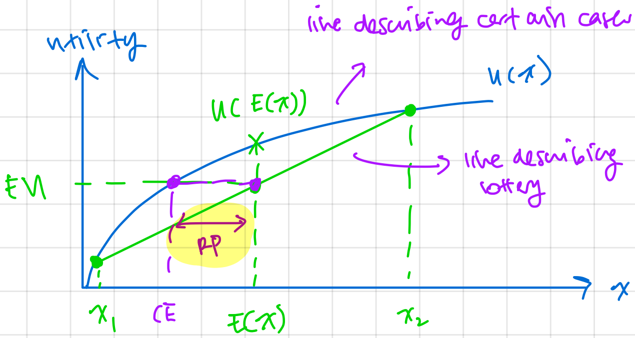
$u(E(x)) = EU$ ⇒ indifferent, risk neutral

2) certainty equivalence and risk premium

certainty equivalence

↳ to get the same expected utility, how much in certain payoffs would I need?
ie. how much do you value this uncertain lottery?

$$EU = \pi_1 x_1 + \pi_2 x_2 \ldots$$
$$CE = u^{-1}(EU)$$
$$RP = E(x) - CE$$



risk premium

↳ expected payoff of a lottery is uncertain
↳ CE measures the amount of money that would give the same utility of this uncertain set of outcomes

↳ the difference between these two measure your risk aversion because even though on average you will make more w/ E(x) (doing ∞ lotteries), you are willing to give up the difference so that you will certainly get CE
⇒ precisely because of the uncertainty

3) arrow - pratt measure
↳ intuition : we know that risk averse utility curves are concave, and that the degree of concavity measures this risk aversion

$$APM = -\frac{u''(x)}{u'(x)}$$

risk averse : $u''(x) < 0$

$u'(x) > 0$, normalises $u''(x)$

because $u'(x) > 0$ for reasonable $u(x)$ functions & we want to measure aversion

③ choice under uncertainty

↳ given different states of consumption, if we are able to choose between these bundles, we simply choose the one w/ highest expected utility, because we have no mechanism of transferring values between the states in each bundle
→ is there one? ⇒ insurance / gambling

1) states of nature : different & mutually exclusive outcomes of some random event

↳ contingent consumption plan : specification of what will be consumed in each state (the bundle of outcomes)

2) optimal contingent consumption

↳ given a random event, we are endowed w a specified contingent consumption plan

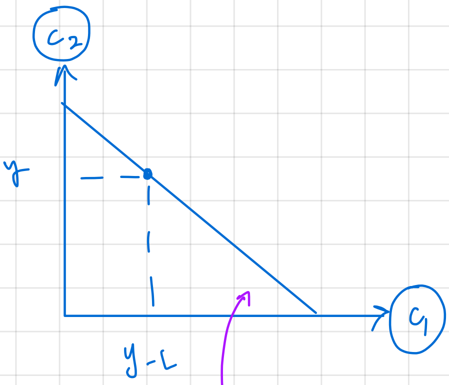
↳ suppose there is some external body w infinite amounts of money; we can use this third party to transfer consumption from one state to the other, given some ToT dictated by the third party

1. suppose a consumer has an endowment of y, with a π chance of suffering loss L.

2. suppose a third party offers to pay K in the event of loss, at the price of α. then the consumer now can transfer value from one state to another, by way of paying a fee in the non-event state.

2.1 suppose we price insurance as some linear combination of payout K.

$\alpha = \gamma K + \beta$



in both states, need to pay

$$c_2 = y - \alpha = y - \gamma K - \beta$$

$$c_1 = y - L + K - \gamma K - \beta$$

payout (choose)

$$= y - L + K(1-\gamma) - \beta$$

$$K = \frac{c_1 - y + L + \beta}{1-\gamma}$$

$$c_2 = y - \gamma\left(\frac{c_1 - y + L + \beta}{1-\gamma}\right) - \beta$$

$$c_2 = -\frac{\gamma}{1-\gamma} c_1 + \frac{y - \gamma L - \gamma\beta}{1-\gamma}$$

3. the consumer is now faced with a choice : pay some $\gamma K - \beta$ where $\gamma K - \beta < K$, so that in the other state, consumption is at a better level than endowed contingent consumption, or ignore it.

3.1 So he first finds, given pricing constraints set by third party, where his new optimal consumption will be. γ, β

3.2 He then compares the expected utility of these contingent states, and compares it to the endowment.

good state → bad state

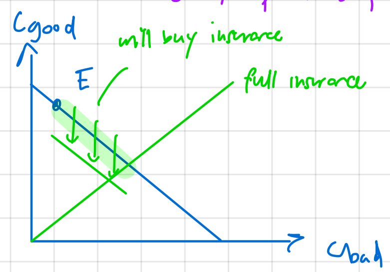
$$\max_{c_2, c_1} \; EU = \pi_1 u(c_2) + \pi_2 \cdot u(c_1)$$

$$\text{s.t.} \quad c_2 = -\frac{\gamma}{1-\gamma} c_1 + \frac{y - \gamma L - \gamma\beta}{1-\gamma}$$

sub

compare w endowment $\longleftrightarrow (c_2, c_1) \Longleftarrow \dfrac{dEU}{dc_1} = 0$

budget constraint has a discontinuity that shift the line down for all points where insurance is purchased



④ insurance

1) competitive insurance

1. insurance company expected profit

$$E(\text{profit}) = \underbrace{\gamma K + \beta}_{\text{certain revenue}} - \underbrace{\pi K}_{\text{probabilistic payout}}$$

2. in a competitive market, firms price at exactly how much society values the good or service because of the free market without distortions.

2.1 so $E(\text{profit}) = 0$

2.2 so $K(\gamma - \pi) + \beta = 0$ } "fair" insurance

↳ if $\beta = 0$, then $\gamma = \pi$.

3. Suppose $\beta = 0$, no handling fee. Then $\gamma = \pi$.

3.1 A consumer's optimal consumption given insurance w/o handling fee is governed by $\dfrac{\gamma}{1-\gamma} = \dfrac{\pi_1 \, mu_1}{\pi_2 \, mu_2} = \dfrac{\pi \, mu_1}{(1-\pi) \, mu_2}$

3.2 given $\gamma = \pi$ in a fair insurance scenario, $\dfrac{mu_1}{mu_2} = 1$, so $mu_1 = mu_2$.

3.3 Because utility functions are monotonic and strictly ordered, $mu|_x$ is strictly unique.

3.4 so $mu_1 = mu_2 \longrightarrow c_1 = c_2 \Longrightarrow$ fair insurance leaves the consumer indifferent between states

2) unfair insurance

1. suppose insurers make positive expected economic profit

1.1 $E(\text{profit}) = K(\gamma - \pi) + \beta > 0$

1.2 if $\beta = 0$, then $\gamma - \pi > 0$. insurers charge more than their economic cost.

2. If an optimal consumer were to purchase such unfair insurance, he will consume more in the good state.

2.1 $\gamma - \pi > 0$. so $\gamma > \pi$. so $\frac{\gamma}{1-\gamma} > \frac{\pi}{1-\pi}$

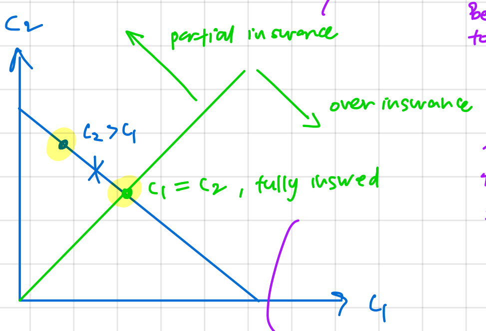
2.2 since $\frac{\gamma}{1-\gamma} = \frac{\pi}{1-\pi} \cdot \frac{MU_1}{MU_2}$, then $\frac{MU_1}{MU_2} > 1$, $MU_1 > MU_2$.

2.3 Given consumption smoothing preferences, law of diminishing MRS holds.

2.4 so $c_1 < c_2$, since MU is unique.

⇒ partial insurance

intuition: you lose some to positive profit. Because of that, your optimal is to insure less



intuition: the chance of c_2 is very low. In the case of gambling, for example. so you prefer the "bad" state where L is positive (ie you win)

intuition: full insurance when fair. Your optimal is to have no difference in states, and you pay exactly that.

Notes

1. do not count existing initial wealth into CE.
   ↳ CE is about the uncertain part of the lottery.

   eg. given ω, lottery of $\pi_1 x_1 + \pi_2 x_2$

   $$EU = u(\omega) + \pi_1 u(x_1) + \pi_2 u(x_2)$$

   $$\Rightarrow CE = u^{-1}(EU) - \omega$$

2. If any variable can be determined by the player himself, then we optimize w.r.t. to it, since he can choose!

3. Remember that given wealth is present in both scenarios → consider in both scenarios utilities!   eg. $u(y + \text{winnings})$ vs. $u(y - \text{losings})$
   included

4. "optimal insurance choice" → find payout K optimal

5. "Fully insured" → level of insurance = amount of payout desired
   ⇒ fully insured → payout K = L ⟺ indifferent between states

Monopoly and monopolistic behaviour

① profit maximisation

revenues

$TR = P(Q) \times Q$

$AR = \frac{TR}{Q}$

$MR = \frac{dTR}{dQ}$

costs

$TC = TVC + TFC$

$AC = \frac{TC}{Q}$

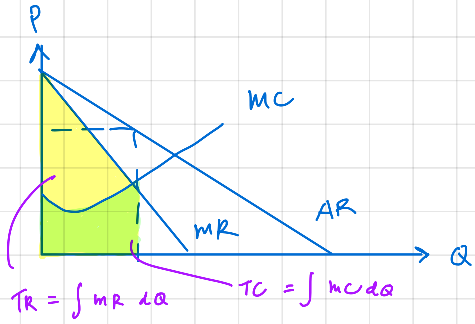
$MC = \frac{dTC}{dQ} = \frac{dTVC}{dQ}$

$\pi = TR - TC$

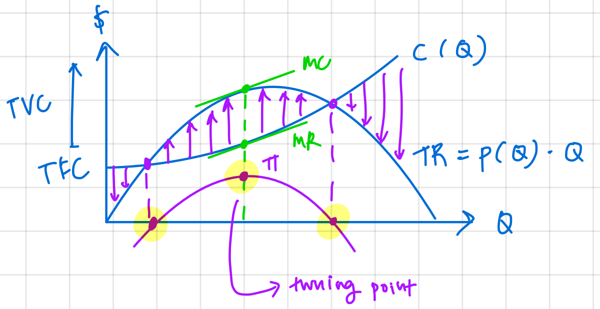
$\frac{d\pi}{dQ} = \frac{dTR}{dQ} - \frac{dTC}{dQ} = MR - MC = 0$

$\Rightarrow MC = MR$

Firm demand curve



Firm cost / revenue curves



② pricing in markets

1) markup pricing

1. price elasticity of demand : $\frac{\text{rate of change in } Q}{\text{rate of change in } P} = \frac{\frac{dQ}{Q}}{\frac{dP}{P}} = \frac{dQ}{dP} \cdot \frac{P}{Q}$

ε is always negative due to law of demand

$TR = P(Q) \cdot Q$

$MR = \frac{dP}{dQ} \cdot Q + P(Q)$

$= \frac{P(Q)}{P(Q)} \left[\frac{dP}{dQ} \cdot Q \right] + P(Q)$

$= P(Q) \left(\frac{dP}{dQ} \cdot \frac{Q}{P} + 1 \right)$

$= P(Q) \left(1 + \frac{1}{\varepsilon} \right)$

2. when firm maximises profit, $MC = MR$.

2.1 so $P \left(1 + \frac{1}{\varepsilon} \right) = MC$

2.2 $\varepsilon < 0$. so $P \left(1 - \frac{1}{|\varepsilon|} \right) = MC$

2.3 so $P = \frac{MC}{\left(1 - \frac{1}{|\varepsilon|} \right)}$

in perfect markets, PED $= -\infty$

→ markup pricing depending on elasticity of demand

2) pricing & PED

1. in perfect competion, PED = $-\infty$, so there is no markup pricing

$$\lim_{|\varepsilon| \to \infty} P = \lim_{|\varepsilon| \to \infty} \left(1 - \frac{1}{|\varepsilon|}\right) MC = MC$$

2. a profit maximising firm will never operate on the inelastic part of demand curve

2.1 when profit maximising, $MC = MR$.

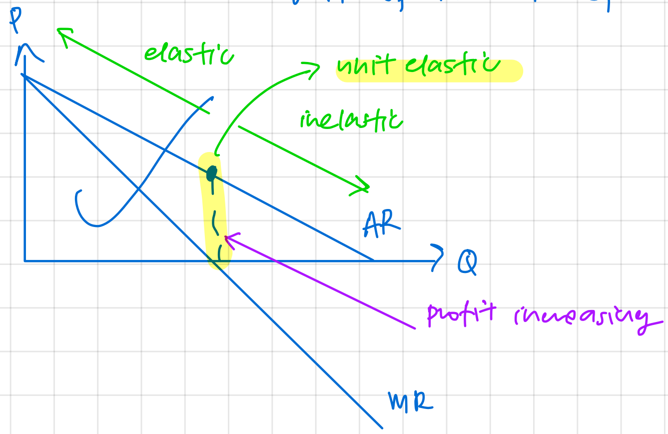
2.2 so $P \cdot \left(1 - \frac{1}{|\varepsilon|}\right) = MR = MC$

2.3 when inelastic, $|\varepsilon| < 1$, so $MR < 0$

2.4 But $MC \geq 0$. so $|\varepsilon| \geq 1$.

2.5 to profit maximise, firm must be producing at least at unit elastic or more parts of demand.

↳ elasticity it operates at depends on MC



3. when $MR = 0$, $|\varepsilon| = 1$

③ barriers to entry : imperfect pricing

↳ in perfectly competitive markets, if existing firms make profits, new firms can enter

↳ in imperfect markets, the existence of barriers to entry allow existing firms to earn positive profits and make it unprofitable for new entrants through their own pricing strategies

barriers to entry
- legal
  - patents, licensing
- structural
  - network externalities
  - nature of industry & MES
- strategic
  - limit pricing
  - predatory pricing

4) price discrimination

1° price discrimination

- firm charges maximum that each consumer is willing to pay
- outcome is pareto efficient, since there is no way to make producer and consumer both better off
- all consumer surplus captured by monopolist
- rare in practice due to difficulty of "bargaining" and isolating consumers from arbitrage

3° price discrimination

1) conditions for 3° price discrimination

   1. perfect identification of different groups ⇒ firms able to get seperate demand curves
   2. No arbitrage across groups

⇒ 3° price discrimination = charging diff. groups of consumers diff. prices for reasons not associated w/ diff. in costs of production

2) uniform pricing vs. 3° price discrimination in 2 markets

   1. suppose a market can be split into n distinct demand curves. If a firm is to charge a uniform price, then its optimisation problem is based on aggregate demand.

      1.1 Aggregation of demand curves often imply a piecewise function : based on prices where sub groups are priced out of the market

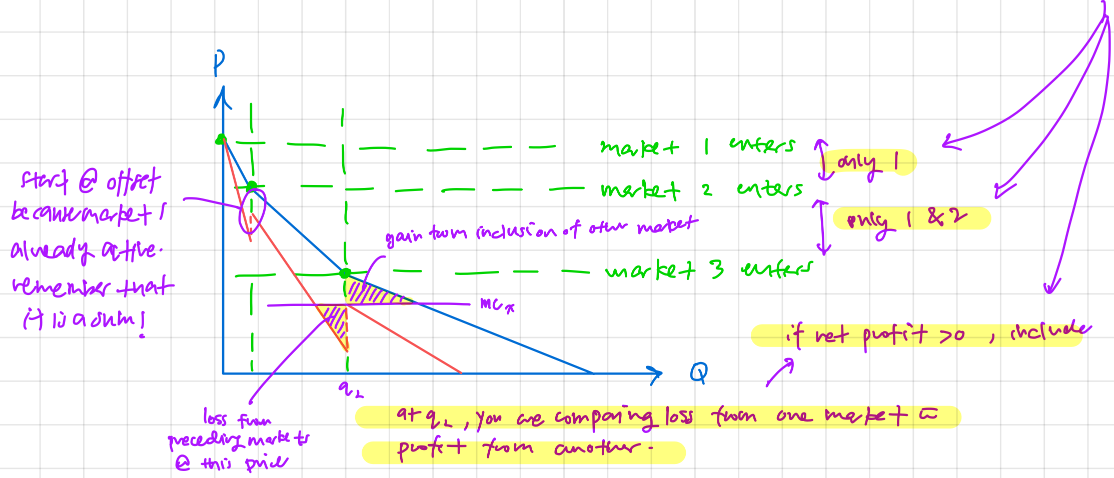
      1.2 Each sub-function has a distinct MR function. The optimisation problem for a uniform pricing firm is:

$$Q = \sum_{i=1}^{n} q_i(p) \qquad Q(p) = \begin{cases} q_a & w \leq p \leq x \\ q_b & x \leq p \leq y \\ q_c & y \leq p \leq z \end{cases}$$

$$\max_{P,Q} \quad \underbrace{\pi = p \cdot Q(p) - c(Q) \cdot Q(p)}_{\text{treatment as a single market}}$$

↳ at each range, test

      1.3 graphically, this is represented by a kinked demand curve, with kinks at boundary points where price allows inclusion of markets. A firms pricing and qt. decision is thus a balancing act of choosing a price & qt. (and by extension, markets) such that π is maximum.



at q_2, you are comparing loss from one market w profit from another.

2. However, because the markets are fundamentally distinct despite production by a single firm, the correct optimisation problem should be:

$$\boxed{\Pi = \sum_i TR_i - C\left(\sum q_i\right) \longrightarrow \text{simultaneously}, \left\{ \begin{array}{l} \dfrac{d\Pi}{dq_i} = MR_i - MC\left(\sum q_i\right) = 0 \\ \vdots \end{array} \right.}$$

2.1 $MR_i = MC$: intuition is that even spread across two markets, it is a single firm producing all units. If $MR_i > MC$, then it makes sense to produce a unit for that market since there's room for growth. If $MR_i < MC$, then the firm is making a loss in that market. Underlying mechanism is firm charging according to PED in diff. markets, subject to its total cost curve.

3) relative price elasticities in sister markets

1. $MR_i = P_i\left(1 - \frac{1}{|\varepsilon_i|}\right) = MC$

2. suppose $P_a > P_b$. But $MC_a = MC_b$. so $1 - \frac{1}{|\varepsilon_a|} < 1 - \frac{1}{|\varepsilon_b|}$

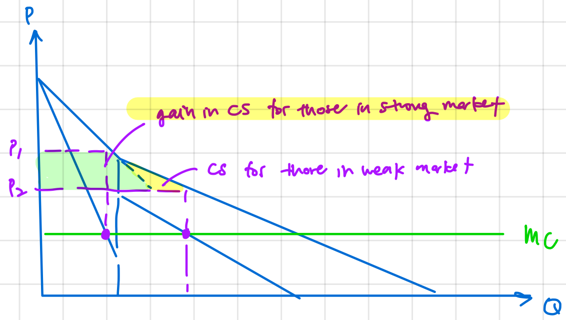
3. So $|\varepsilon_b| > |\varepsilon_a|$

intuition: a firm that price discriminates sets low prices for price sensitive groups and high prices for insensitive ones

4) consumer and producer welfare

↳ for uniform pricing, entry of secondary weaker market will mean consumers in first market have lower prices. CS ↑. But weaker market will have higher price charged. So lower CS.

⇒ note that this is dependent on MC, which determines if weak market even enters. If it does not, then as if single market.



⑤ The inefficiency of monopoly

1) surplus

1. consumer surplus : difference between WTP and P , summed

2. producer surplus : difference between WTS and P , summed

$$\boxed{\begin{aligned} CS &= \int_0^{Q^*} P(Q) - P^* \, dQ \\ PS &= \int_0^{Q^*} P^* - MC \, dQ \end{aligned}}$$

→ TFC is ignored because firm supplies at $MC = MR$, not $MC = MR + AFC$

3. Total surplus : $CS + PS$

↳ intuition : sum of total gains from consumers and producers trading

2) monopolist's deadweightloss

↳ deadweightloss : gains-to-trade not achieved by the market

i.e. $TS_{max} - TS > 0$

$$TS = CS + PS = \int_0^{Q^*} P - P^* \, dQ + \int_0^{Q^*} P^* - MC \, dQ$$

$$\frac{dTS}{dQ} = P - P^* + P^* - MC = 0 \Rightarrow P = MC$$

1. To maximise total gains to trade for all agents (i.e. pareto efficiency), $P = MC$. $TS = TS_{max} \rightarrow P = MC$

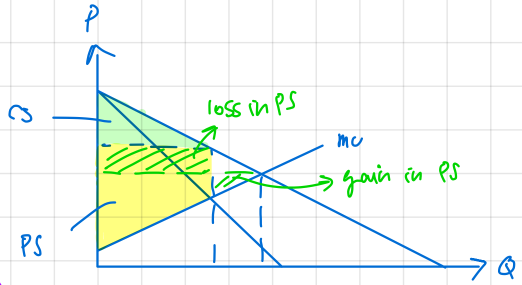
2. A monopolist pricing at $MC = MR$ leads to $TS < TS_{max}$. Hence there is some deadweight loss.

2.1 To maximise profit, $MC = MR$. An alternate interpretation is pricing such that PS is maximized.

$$PS = \int_0^{Q^*} P^* - MC \, dQ$$

price set by monopolist

$$\frac{dPS}{dQ} = P^* - MC = 0, \text{ so } MR = MC$$



2.2 $\frac{dMR}{dQ} > MR$ (MR is steeper than AR).

2.3 $\frac{dP}{dQ} < 0$ by law of demand.

2.4 so where $P^*\Big|_{MC = MR} > P\Big|_{AR = MC}$. so $P^* \neq MC$. so $TS \neq TS_{max}$.

⑥ Quantity taxation of monopolists

↳ unit tax : $mc' = mc + t$

↳ $\pi = TR - TC - Q \cdot t$, $\frac{d\pi}{dQ} = 0$

$\Rightarrow MR = mc + t$

$\Rightarrow \boxed{p' = \frac{mc + t}{1 - \frac{1}{|\varepsilon|}}}$ → tax burden $= p' - p = \frac{t}{1 - \frac{1}{|\varepsilon|}}$

tax burden depends on PED at new equilibrium

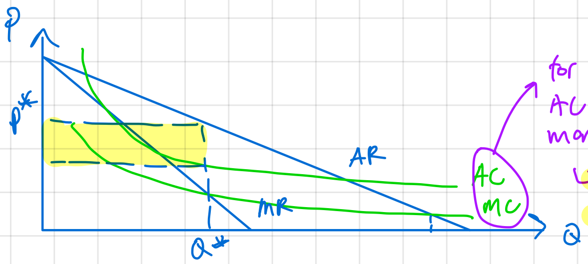
qt. tax is distortionary

⑦ natural monopoly

1) a natural monopoly

↳ a natural arises when it is economically unprofitable for ≥ 2 firms to be in a market

↳ often due to natural BTE (eg. high FC), small market demand



for natural monopolist, EOS is often so high that AC (and thus, MC) decrease over entire market demand

↳ notice that MC pricing is even worse for a natural monopolist because of this

2) regulating natural monopolies

↳ like any profit maximising monopolist, the natural monopolist incurs deadweight losses

MC pricing

↳ DWL = 0 since P = MC

↳ but firm makes an economic loss since AC > MC

↳ especially so for natural monopolist, where AC decreases throughout market demand, and MC < AC strictly

AC pricing

↳ reduces π to 0, increases TS

↳ but hard to regulate as govt. has to ascertain reasonable ATC

↳ firm has incentive to bolster ATC and no incentive to be cost efficient

Notes

1. remember than consumer/producer benefits are quantified by surplus. So whether they are better, worse off or indifferent depends on their surplus.

2. "Industry outcome"

   ↳ P, Q, π, CS, PS, TS

   ↳ eg. optimization solution has 2 valid values of Q ⇒ pick the one $\bar{w}$ higher PS

3. Firm's sustainability: based on π

4. consider that diff. taxation methods (lump-sum, qt.) and pricing regulation on monopoly can have multiple solutions. choose the one $\bar{w}$ highest TS!

5. Agg. D: aggregate Q <u>first</u>, then invert.

6. $u(0)$ is not necessarily 0!

Oligopolistic equilibrium

① oligopolistic competition

↳ in oligopoly markets, because of BTE, there are a small number of firms, each w/ market power and price setting ability

↳ note that competition can be direct (same good) or indirect (related goods)

↳ their objective is still to maximize profits. so given competition of other firms, they choose a price and quantity to sell at that they believe will maximise π.

- price → price competition: set price. let mkt. demand determine qt.
- quantity → qt. competition → set qt. let total mkt. qt. determine price.

in diff. markets, firms act differently. Choices could be made sequentially (i.e. responding to a fixed response) or simultaneously (i.e. responding at the same time, so having to predict what they might do)

→ choice of price vs. qt. competition is exogenous and dependent on industry / product / time period

best response function → more precisely, max π w.r.t. Q or P, each diff

↳ Q_A as a function of Q_B, derived from $MC = MR_{residual}$

1. suppose demand function $P(Q_A, Q_B)$ faces A and B.
2. Given some value of Q_B, A wants to choose Q_A to maximise π_A.
3. $MR_A = \frac{\partial}{\partial Q_A}(P \times Q_A) = MC \implies Q_A = f(Q_B)$ → describes the π maximising amount to produce, given some amt. produced by the competitor

   $P(Q_A + Q_B)$

   ⇒ price determined by total qt.

② simultaneous qt. competion

Cournout Competition

↳ firms produce identical product
↳ firms compete by independently but simultaneously choosing output levels, w/ knowledge that total output determines market price

1) mathematical intuition for 2 firms

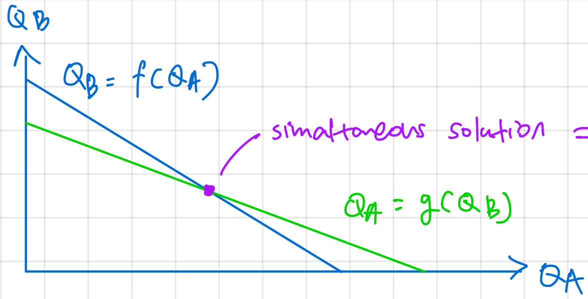
1. Both firms make choices simultaneously. So they make their decisions based on what they expect the other to do. i.e. π maximise.

   1.1 Assume: perfect info about the market & other firms' cost structure.

2. then best response function describes how A will react to B and vice versa.

   $D := P(Q_A + Q_B)$. $MC_i = MR_i$ → they must hold simultaneously

   $\Rightarrow Q_B = f(Q_A), \quad Q_A = g(Q_B)$



⟹ both firms responding optimally to the other. Cournot eqm. → no incentive to change ⟹ stable eqm.

↳ assumes "true" simultaneous-ness. In reality there is some version of price leadership due to sequential nature of time.

2) many firms in Cournot eqm: elastic demand

1. Similarly, every firm expects all other firms to behave optimally. Intersection of all best response curves in n-dimensional space (simultaneous $MC_i = MR_i(\cdots)$),

2. We can relate the firm's best response to the market share and elasticity

2.1 $$MC_i = MR_i = \frac{\partial}{\partial Q_i} \underbrace{P(Q)(Q_i)}_{TR} = \underbrace{P(Q) + \frac{\partial P}{\partial Q} \cdot \frac{\partial Q}{\partial Q_i}}_{\text{chain rule}}$$

$\frac{\partial Q}{\partial Q_i} \to Q_i$, since $Q = Q_1 + Q_2 \ldots$

$$= P(Q) + \frac{\partial P}{\partial Q} \cdot Q_i = \underbrace{P(Q)}_{\text{factorize}}\left(1 + \frac{\partial P}{\partial Q} \cdot \frac{Q_i}{P(Q)}\right)$$

mkt share, $s_i = \frac{Q_i}{Q}$

$$= P(Q)\left(1 + \underbrace{\frac{\partial P}{\partial Q} \cdot \frac{Q}{P(Q)}}_{\text{elasticity of mkt demand}} \cdot \underbrace{\frac{Q_i}{Q}}_{\text{expand}}\right) = P(Q)\left(1 - \frac{s_i}{|\varepsilon|}\right)$$

notice that as firm goes to monopoly, $s_i = \frac{Q_i}{Q} \to 1$, and you get monopolist's pricing ability based on PED. And as $s_i \to 0$, you go to $P = MC$, in PC markets. ⟹ traditional eqm. models are exactly based on Cournot eqm: firms simultaneously trying to maximize π, given their knowledge of the market's price being affected by every other firm's qt.

③ sequential quantity competition

↳ suppose firm has some first-mover advantage, and other firms have to respond to its choices. So we relax simultaneous response for 1 firm.

⟹ all other firms react based on their best response functions (simultaneously) but leader, given this expected behavior, chooses the point of maximum profit

Followers

↳ must react optimally.

$MC_i = MR_i$

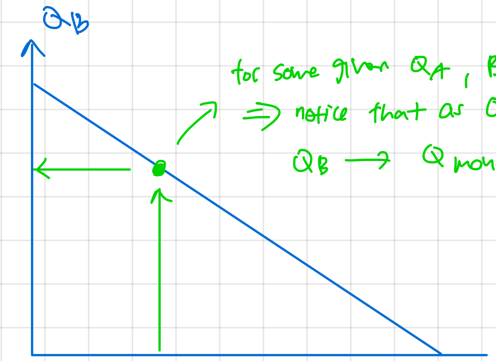
Leader

↳ will act optimally, given that followers will respond optimally to its Q choices & affect market price

$$\max.\ \pi_A = P(Q_A + Q_B \ldots) \cdot Q_A - C(Q_A)$$

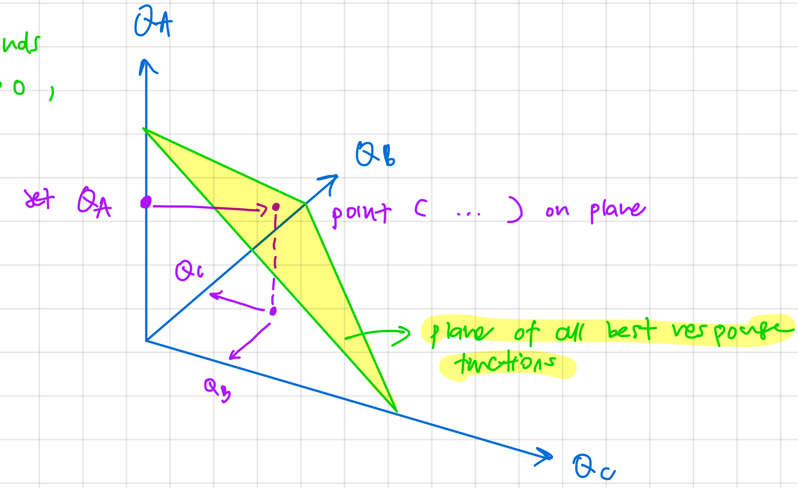
$$\text{s.t.}\quad MC = MR_i \quad \text{sub in}$$

$\vdots$



for some given Q_A, B responds
⇒ notice that as $Q_A \to 0$,
$Q_B \to Q$ monopoly

$Q_B = f(Q_A)$
best response curve of B



④ simultaneous price competition

↳ when firms simultaneously compete on price, because all rational consumers will always pick lowest price option, firms have incentive to undercut until cannot undercut anymore ⇒ $P = \min(MC_i)$, bertrand equilibrium. ⇒ $\pi = 0$

↳ assumes ≥ 2 firms w/ $MC = \min(MC_i)$, else just becomes monopoly

1) intuition

↳ we assume all consumers are rational. If a product is identical, they will always pick the cheapest.

↳ suppose A sets a price. Then B can capture all market share (assuming no limits on production, for now) by undercutting. A can do so in response.

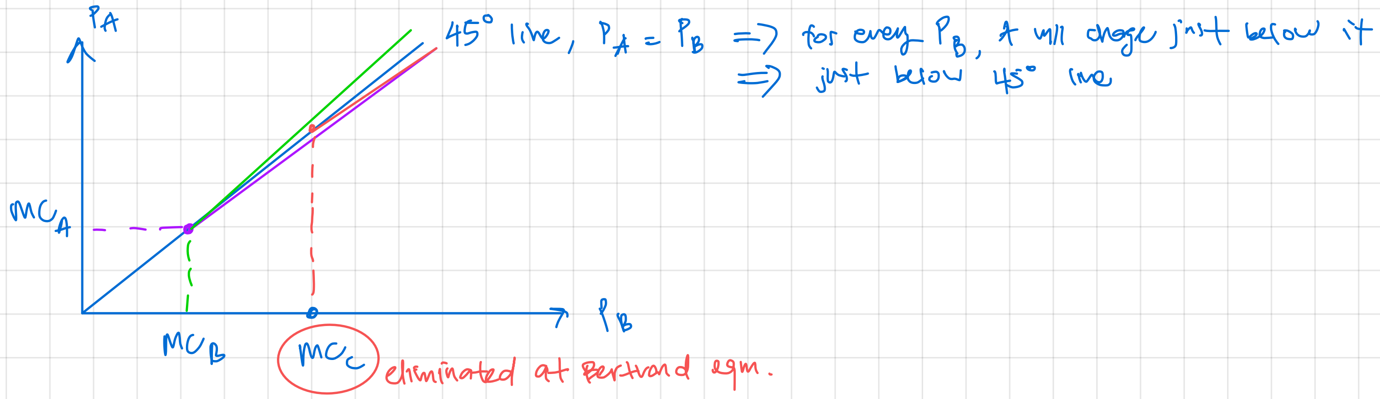
↳ all firms can keep doing so until they reach a price they cannot go below, at $P = MC$

perfect price elasticity

⇒ process that is effectively competitive bidding has created an eqm. of perfect competition where $P = MC$ ⇒ firms that cannot charge same will have 0 demand

⇒ often at $P = MC$ making a loss, so short term competition to gain mkt share

2) graphical intuition



2) Bertrand competition in <u>related</u> markets

$$Q_B = f(P_B, P_A)$$
$$Q_A = f(P_A, P_B)$$
$$\Rightarrow \pi_A = Q_A P_A, \quad \pi_B = Q_B P_B$$

$$\left.\begin{array}{l} \dfrac{d\pi_A}{dP_A} = 0 \\ \\ \dfrac{d\pi_B}{dP_B} = 0 \end{array}\right\}$$

solve simultaneously
⇒ choosing optimal P_A in <u>response</u> to P_B

⑤ <u>collusion</u>

1) the new optimisation problem

↳ when firms collude, they behave like a single firm with distributed production
↳ we assume they are happy w/ pareto efficient distribution of profits
ie. at least as well-off as without collusion

$$\max.\ \underbrace{p(Q_1 + Q_2 \dots)(Q_1 + Q_2 \dots)}_{\text{total revenue}} - \underbrace{c(Q_1) - c(Q_2) \dots}_{\text{individual costs}}$$

Ⓐ $\dfrac{d\pi}{dQ} \Rightarrow$ simultaneous satisfaction of : $MR_{total} = MC_i(Q_i)$

no room for more π from getting one firm to produce more where $MR > MC$, and none making loss

Ⓑ treat as a single firm.

$Q_1 = \frac{1}{2}Q$
ie. half MC

$TR = P \cdot Q \Rightarrow MR$
$TC = c(Q_1) + c(Q_2) \dots \Rightarrow MC =$ { for each region, always pick lowest. if equal, split production equally. ⇒ sum to get indiv Q_i over all zones

2) cheating

indiv.   shared

↳ cartel optimisation conditions assign qt. and price to each participating firm
⇒ if firm is not producing on its best response function, then given the other firms qt. and influence on market prices, it can gain from changing its qt. to optimally influence the market Q and P and exploit gains.

↳ test : see if on best response function

↳ collusion equilibrium may not be stable, especially if one-off

3) cheating vs. punishment → $\boxed{\text{Grim trigger strategy}}$: kick out of cartel after current round if cheat in current round

↳ if game is repeated, then decision to cheat should be based on lifetime profits, given cheating and being punished after

discount factor eg. $\frac{1}{1+r}$ ⇒ PV

$$\Pi_{cooperate} = \Pi + \delta(\Pi) + \delta^2(\Pi) \ldots$$

$$= \Pi(1 + \delta + \delta^2 \ldots)$$

$$= \frac{\Pi}{1-\delta}$$

Π_{cheat} vs. Π_{coop}

$\boxed{\text{tit for tat}}$ alternating Π_{cheat} & $\Pi_{compete}$

eg. $\Pi = \Pi_{cheat} + \delta \Pi_{compete} + \delta^2 \Pi_{cheat} + \delta^3 \Pi_{compete} \ldots$

↳ how Π_{cheat} is set up depends on punishment eg. one round, multiple, forever?

⑥ competition in general

↳ firms try to respond optimally by maximising profit.

competition on price

$Q_A = f(P_A, P_B)$

$TR_A = P_A \cdot Q_A \Rightarrow MR_A = \frac{dTR_A}{dP_A} = MC_A$

$\Rightarrow P_A^* = g(P_A, P_B)$

choose optimal price in response to some demand / revenue curve influenced by P_B or other factors

competition on qt.

$P_A = f(Q_A, Q_B)$

$TR_A = P_A \cdot Q_A \Rightarrow MR_A = \frac{dTR_A}{dQ_A} = MC_A$

$\Rightarrow Q_A^* = g(Q_A, Q_B)$

choose optimal Q_A given some revenue curve influenced by Q_B or other factors

simultaneous or sequential?

simultaneous

↳ firms optimizing wrt to each others' expected response

⇒ simultaneous eqn of best responses

leadership

↳ leader optimising, constrained by expectations of follower's responses

$MC_A = MR_A$ s.t. best response simultaneous of follower

collusion?

qt.

Π = compete vs. monoply ⟷ Π_{cheat} ⟷

price

$\Pi = 0$ vs. monopoly

Notes

1. cheating in a collusion can work by assuming other firms hold qt. constant
   $\Rightarrow$ then cheater moves to produce on best response.

   But if firms hold agreed price constant, then cheater can undercut and effectively be a monopoly / act as single firm

2. Bertrand eqm / competitive bidding: always start from $p = MC$. suppose $MC_1 > MC_2$, then charge at $p \approx MC_2$, since firm 2 will undercut an infinitesimally small amount.

3. Bertrand grim trigger: $\underline{0\ \pi}$ because will just compete @ $p = MC$

   $\downarrow$

   so β needed for sustained collusion is lower since punishment more harsh

4. tit for tat: alternating π_{cheat} & $\pi_{compete}$

   eg. $\pi = \pi_{cheat} + \delta \pi_{compete} + \delta^2 \pi_{cheat} + \delta^3 \pi_{compete}$.

   useful trick:

   $$\pi = \pi_{cheat} + \delta^2 \pi_{cheat} + \delta^4 \pi_{cheat} \ldots + \delta \pi_{compete} + \delta^3 \pi_{compete} \ldots$$

   $$= \frac{\pi_{cheat}}{1-\delta^2} + \delta \frac{\pi_{compete}}{1-\delta^2}$$

Fundamentals of game theory

game theory: the study of situations where agents' choices can have influence on the payoffs (and thus choices) of other agents

Assumptions: 1. rationality: all agents playing the game are rational & everyone knows everyone is rational

2. perfect information: everyone knows the game and knows that everyone else knows the game (i.e. payoffs)

best response function: function describing best response of a player given all possible choices of all other players. If a player is indifferent, then all values are a best response (by default)

① pareto efficiency and improvements

pareto optimal allocation: there is no way to make some individual better off without making someone else worse off

pareto improvement: an allocation is pareto improving if it harms no one and helps at least one agent

→ there are no pareto improvements to be made from a pareto efficient allocation

② pure strategy simultaneous games

simultaneous game: agents make decisions jointly and independently. They do not coordinate with each other.

1) pure strategy games: players can only choose between mutually exclusive choices

2) PS nash equilibrium: an allocation of pure strategies is a Nash equilibrium if for every player, their choice is optimal given all other players' choices

↳ i.e. an allocation where no agent has the incentive to deviate

↳ Nash equilibria are not pareto optimal in general

3) Dominant & dominated strategies

strictly dominant strategy: given a players set of strategies, his strictly dominant strategy is the one that regardless of all combinations of choices by other players, has the highest payoff (always worst)

strictly dominated strategy: a strictly dominated strategy is one that is inferior to at least one other strategy across all strategies of other players

↳ it is never part of the nash equilibrium

↳ always incentive to deviate

weakly dominant strategy: a strategy that is always better than or equal to all other strategies, for all opponents' plays

↳ weakly and strictly dominant strategies cannot co-exist

weakly dominated strategy: a strategy that is always worse than or equal to another strategy for all opponents' plays (at best equal to another strategy)

↳ can be part of a nash equilibrium.

1. for a given strategy, for every one of opponents' responses, find the optimal strategy

2. if same optimal strategy for all of opponent's plays, then that is the dominant strategy

3. if a strategy is never picked as optimal, that is a dominated strategy

4. if a strategy always could be picked (i.e. tie or optimal), that is a weakly dominant strategy

5. if a strategy is never the optimal but tied at least once, it is a weakly dominated strategy

eg.

B

A	e	f	g
a	3	4	5
b	0	1	2
c	1	5	4

⟹ a is dominant strategy for A

⟹ b is dominated strategy for A

4) iterative deletion of dominated strategies

↳ idea: since strictly dominated strategies are never played, we can recursively identify a dominated strategy and delete it

↳ so long as a strictly dominated strategy always exists (until there is only a single set of choices for each player left), this method will solve for the pure strategy nash equilibrium

⇒ but if not, because weakly-dominated strategies can be part of the nash equilibrium, iterative deletion will not solve for Nash eqm. in such a case in general

↓

use iterative deletion to reduce solution space, then manually solve

5) solving for pure strategy nash equilibria

(Payoff matrix: two players)

1. set up payoff matrix
2. iterative deletion while there are strictly dominated strategies
3. find optimal response for each player given other player's responses
   ⇒ intersections are nash equilibria

(best response functions: many players)

1. consider different states players might be in
2. set up equations modelling each player in each of these scenarios and/or payoff matrix (in equation form)
3. find equilibria
   ↳ inequalities / simultaneous equation
   ↳ look for incentive to deviate unilaterally

remember that players have perfect information!

(special case: symmetric games)

↳ each player faces same set of choices & payoffs ⇒ single equation modelling choice faced by by individual, solve

③ mixed strategy simultaneous games

1) mixed strategies : suppose we allow agents to choose their strategies at random, but choose the frequency (i.e. probability) with which they make these choices

2) mixed strategy nash equilibria : an equilibrium in which each agent chooses the optimal frequency with which to play his strategies given the frequency choices of the other agent

3) solving for MSNE

↳ intuition : given frequencies of possible actions of opponents, an agent will look to choose a set of frequencies of his own actions, such that on average he is indifferent between all choices he can make.

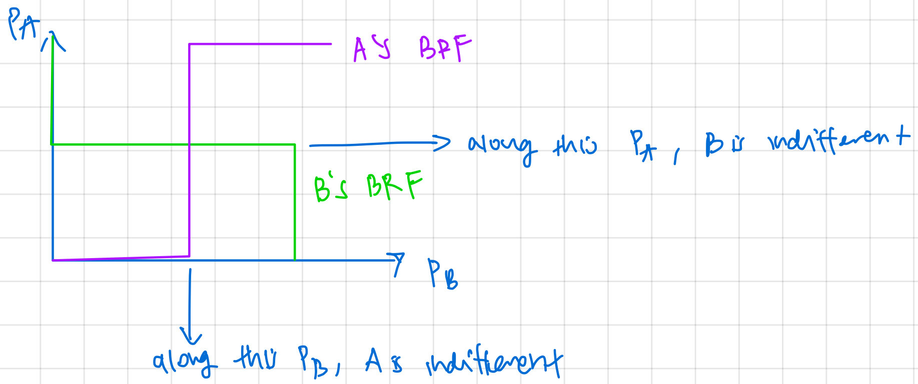
1. For each player, find expected payoff given each of his choices, according to probability distribution of opponents play

2. Form simultaneous equations by equating the expected payoffs for each of his choices

3. Solve simultaneously so that indifferent between choices

⇒ idea : because of perfect information, each player knows how frequently on average his opponents will choose a given choice. So he will adjust his own frequencies so that on average he is indifferent between his random choices

best response functions eg. suppose A only has 2 choices, B only 2 choices.



4) games, PSNE, MSNE

↳ a game have no, one or more than one PSNE or MSNE

↳ a game can have both PSNE and MSNE

④ PS repeated games

1) cooperation

↳ in repeated games with the same players, there are new strategies available to the player to encourage cooperation

↳ if players meet repeatedly, we can develop a punishment system that induces cooperation

Nash reversion strategy

↳ idea : if you cooperate, I cooperate. If you cheat once, I cheat forever (grim trigger)

Tit for tat

↳ idea: try to cooperate in the first round. For all other rounds, do what the other player did in the previous round

2) finitely repeated games

↳ in finitely repeated games, there are limited opportunities to punish another player for cheating

↳ players cooperate because they hope that cooperation will induce further cooperation in future

↳ but this requires that there will always be the possibility of future play

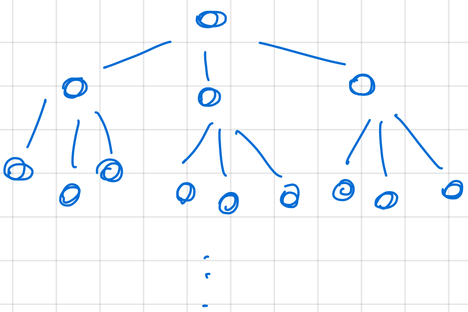
⇒ if no incentive to cooperate on the last round, then no incentive on the round before that, all the way to the start

⑤ PS sequential games

↳ in sequential games, one player gets to move first, which restricts the choices & payoffs all subsequent players have, recursively

representing sequential games

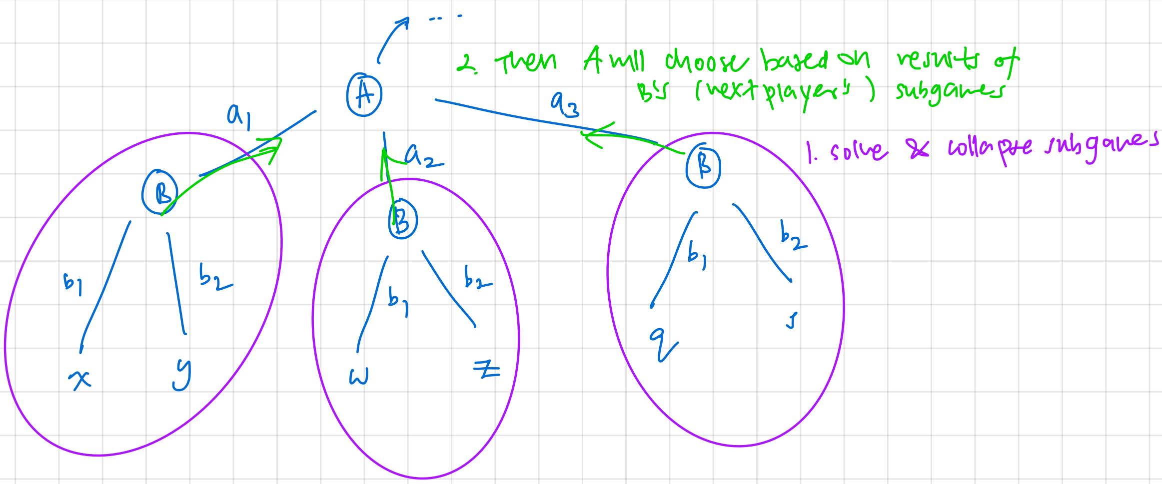
↳ we use extensive form : trees



backward induction
finding the "subgame" NE

↳ idea: every player has perfect information. The net outcome depends on how others play in respond to one's play. So by looking ahead to see what each player would rationally do at each step, one can trace the outcomes back to see what choice he should make at the first step.

1. model the game in extensive form
2. Starting from the end, let the player choose his optimal at <u>all</u> subtrees at that height. The preceding player then considers those results
3. Repeat until first player. Now trace forward to get sequence of events



↳ in a scenario where a player is indifferent between choices at a given juncture, then we don't have enough information to solve.

↳ backward induction is basically iterative deletion but applied to a sequential game

Threats in sequential games

↳ supposing that all players are rational, then a player would not consider the threat of a subsequent player choosing something that makes things worse for everyone as credible.

⑥ PS Bargaining games

↳ bargaining games are sequential games where offers are made over a set of rounds

Rubenstein bargaining

↳ Rubenstein bargaining model refers to a class of bargaining games that feature alternating offers through an infinite time horizon.

↳ let us simplify games to a finite time horizon

↳ to model for time, we consider discount factors α and β for each player

↳ we assume that if a player is indifferent between accepting or not, they will accept, since offer-er will offer very small amount rounded to 0.

backwards induction

↳ in a finite set of bargaining rounds, we know how the game might end (say, winner takes all)

↳ we run backwards induction by this line of logic: a person will offer an amount that makes the person indifferent at his turn (including discount factor) and no more (this way, optimizes himself)

eg.

	t		$t+1$
α $\boxed{A}$	5	$\longrightarrow$	2
β $\boxed{B}$	?		1

PV of gain by B in t+1 is $1 \times \beta = \beta$.
so A just needs to offer β. So A keeps $(5 - \beta)$

apply logic recursively

Intuition

↳ discount factor represents loss of payoff across time

↳ different scenarios also offer different value

↳ last player / players with good no-agreement outcomes have more bargaining power since have to offer more to match

Notes

1. sequential w BRF — sub choices / functions / variables as constraints into p

Externalities

① externalities

1) definitions

externality: the cost or benefit generated by a transaction between agents, incurred by an agent not involved in the transaction

consumption externality | production externality

2) why do externalities arise?

↳ agents are always rational. To maximise their objective, they operate at MPC = MPB. should they generate externalities, positive or negative, they do not factor it into their optimisation problem

↳ mathematically:

$u_A(x)$

$u_B(y, x)$ so B's utility / objective depends on A's choices

A consumer where $\frac{d u_A}{dx} = 0$

similar to games in that agents' actions influence others, but different in that it is only one way ⇒ impose externality

3) inefficiencies of externalities

↳ negative externality → over producing / consuming

↳ positive externality → under producing / consuming

why is it pareto inefficient?

↳ recall that a pareto efficient outcome is one where there are no pareto improvements to be made

↳ so long as there is at least one way to make at least one agent better off without harming others, then the allocation is not pareto optimal

⇒ put another way: if there is a way to make total utility higher, then suppose we redistributed them s.t. everyone was at least as well off as before, we would have excess to make one person better off → so there is a pareto improvement to be had

① Tragedy of the commons

↳ suppose we have a good that is non-excludable but rival.

non-excludable → every agent has access to it

rival ↳ usage by one agent diminishes the amt. available for another agent

↳ for these types of goods, their rival nature are often what cause externalities because every agent only considers their own calculus $MB = MC$ when they choose to consume the good. So long as entry $MB > MC$ makes sense, they will enter, and all agents already in the market will see a fall in utility due to the rival nature of the good (i.e. externality imposed)

modelling it

individual benefit diminishes as a function of total no. of users

individual cost rises as a function of total no. of users

1. set up MC and MB of entry, as a function of N, total people in market
2. So long as $MB > MR$, agent will enter, until he's indifferent

② solutions to externalities

↳ There is some optimal level of externality such that for all agents as a whole, $MC = MR$ (i.e. $MSC = MSB$)

↳ there are different strategies we can use to get agents to take into account externalities into their decisions. This depends in part on how we frame the problem.

agents in general do not consider how their choices affect others

taxation

merger

qt. limit

both

cap & trade

there is no market for agents to use prices to negotiate
⇒ property rights are poorly defined

bargaining

merger

- when agents are distinct, they only consider mb, mc w.r.t. to variables they can control, and disregard how these decision variables affect other agents
- merging affected agents with the original aligns their objectives, so the agent is now incentivised to account for how externalities from one decision variable may affect others, and select them in a way that maximises the overall objective

1. sum their objectives
2. differentiate w.r.t. to all decision variables & solve.

$$\pi_A(x, \overset{\text{externality}}{y}), \ \pi_B(z) \Rightarrow \pi(x,y,z) = \pi_A(x,y) + \pi_B(z)$$

$$\frac{\partial \pi}{\partial y} = 0, \quad \frac{\partial \pi}{\partial z} = 0, \quad \frac{\partial \pi}{\partial x} = 0 \ \Big\} \text{ optimal amt. of everything}$$

introduction of property rights

- coase theorem: if property rights of an externality are well defined and there are no transaction costs, bargaining between agents will lead to an efficient outcome regardless of the initial allocation of property

⇒ owner of externality can effectively sell it eg. polluter buys credits, advertiser sells benefit, and externality becomes a tradeable decision variable

$\pi_A(x, A) \rightarrow$ sells x for $p_x \cdot x + F$ ← suppose $\pi_A^* \big|_{x=0}$ is greater than $\pi_A(x)$ for any x. ie. A loses by even trading the right. Then a flat fee is needed to make him indifferent.

$\pi_B(x, B)$ ← buys

eg. $\pi_A^* \big|_{x=0} = 25$

$\pi_A(x, A) = 24 + F$

$\Rightarrow F \geq 1$ to sell $x \geq 0$

$$\left\{ \begin{aligned} \frac{\partial \pi_A}{\partial A} &= \frac{\partial \pi_A}{\partial x} = 0 \\ \frac{\partial \pi_B}{\partial B} &= \frac{\partial \pi_B}{\partial x} = 0 \end{aligned} \right.$$

- problem: transaction costs & ethics of defining property rights

taxation / subsidies

- in general, if we can impose a tax or subsidee, we force the agent to internalize the social cost or benefit of his actions

positive ext.

$$\boxed{MB + s = MC}$$

negative ext.

$$\boxed{MB = MC + t}$$

- problem: how much?

qt. limit

↳ impose the optimal limit onto each firm

⇒ as if merger, but distribute such that pareto improvement

↳ problem: impossible due to imperfect information

---

cap & trade

↳ a market-based solution is a more feasible implementation of qt. limit

↳ agents have different optimisation functions, and in general they do not have identical ones. So while they might all contribute to an externality, how much is optimal for them is different

↳ imposing a qt. limit entails finding this info and maintaining it. Impossible.

⇒ so instead we can impose a broad qt. limit on every agent and assign them the right to trade their excess

⇒ then total amount is optimal, and agents themselves through a market mechanism find a pareto efficient allocation

⇒ this is a levelled up formulation of bargaining, where each agent is endowed initially with some x (not just 1 agent) and they can trade it

1. set total quantity Q, max as if all merger

$$\max \sum \pi_i \Rightarrow Q^*$$

2. constrained optimisation

$$\max \left\{ \begin{array}{l} \pi_i = f(\underbrace{i}_{\text{prod.}}, \underbrace{\pi}_{\text{externality}}) \\ \vdots \end{array} \right\} \quad \frac{\partial \pi_i}{\partial i} = 0 \text{ for each}$$

$$\text{s.t. } \sum i = Q^* \quad \text{constrained}$$

Public goods

① public goods

non-excludability once produced, it is impossible to exclude someone from consuming it

non-rivalry one person's consumption of the good doesn't diminish the amount available for another person's consumption

public good a good that is non-excludable & non-rival

↳ public goods are an example of a particular kind of externality — one that everyone must consume the same amount of

② pareto efficient level of public good

1. Suppose there are i agents, each with an endowed wealth w_i. Each agent can choose between consuming c_i and contributing some funds g_i to a public good. Because the public good is non-rivalrous and non-excludable, every agent consumes the same amount of G. Suppose p is the price of good, with price of public good as the numeraire.

   1.1 budget constraint : $c_i + g_i = w_i$

   1.2 utility : $u_i(c_i, G) = \bar{u}_i$

2. To find all pareto efficient allocations of $(c_1, c_2 \dots c_n, G)$, we are trying to find all solutions that would (without loss of generality) maximize a given consumer's utility subject to all other's being held constant.

   2.1 without loss of generality, let that be consumer 1.

$$\boxed{\begin{aligned} &\max_{(c_1, c_2 \dots G)} u_1(c_1, G) \\ &\text{s.t. } p(c_1 + c_2 + \dots) + \underbrace{g_1 + g_2 \dots}_{cost(G)} = w_1 + w_2 \quad \text{— ①} \\ &\{u_i(c_i, G) = \bar{u}_i\} \quad \text{— } (n-1) \end{aligned}}$$

some arbitrary level of utility

   2.2 we can perform lagrangian optimization. The lagrangian is:

$$L = u_1(c_1, G) - \underbrace{\sum_{i=2}^{n} \lambda_i (u_i(c_i, G) - \bar{u}_i)}_{\text{all others' utilities}} - \mu\underbrace{\left(\sum_{i=1}^{n} pc_i + cost(G) - \sum_{i=1}^{n} w_i \right)}_{\text{total budget constraint}}$$

$$\left\{ \begin{aligned} &\frac{\partial L}{\partial c_i} \dots \\ &\frac{\partial L}{\partial G} \dots \end{aligned} \right\} \Rightarrow p \sum_{i=1}^{n} \frac{\frac{\partial u_i(c_i, G)}{\partial G}}{\frac{\partial u_i(c_i, G)}{\partial c_i}} = \frac{\partial cost(G)}{\partial G} \Rightarrow \boxed{p \sum_{i=1}^{n} |MRS_{c_i, G}| = MC(G)} \quad \text{— ③}$$

intuition

↳ sum of $MRS_{c_i,G}$ measures the total willingness to pay: in total, for 1 unit of G, how much c_i they would be willing to sacrifice.

↳ $MRS_{c_i, G(i)}$: how many c_i agent i is willing to chip in / trade for one more unit of G

$p \sum_{i=1}^{n} |MRS_{c_i,G}| > MC(G)$

Then collectively, there is enough total value that adding up what everyone is willing to chip in is enough to buy an additional unit of G

$p \sum_{i=1}^{n} |MRS_{c_i,G}| < MC(G)$

The collective amount everyone is willing to chip in is less than the cost of one unit of G. Everyone would be better off selling one unit of G and splitting those gains among their private c_i

$p \sum_{i=1}^{n} |MRS_{c_i,G}| = MC(G)$

No room for arbitrage, every \$ on c_i gives as much utility to the individual as how much they (& everyone else sharing the cost) is spending on G at g_i.

③ private provision & free riding

↳ if G were a private good (excludable, rival), then the optimal allocation occurs where $MRS_{c_i,G} = \frac{P_G}{P_c}$. There will be no more room for improvement, since all individuals would be maximising their utilities given their endowments.

↳ because public goods are non-excludable and non-rivalrous, individuals who do not purchase it can always enjoy it. Individuals who purchased it cannot stop them or charge them.

↳ so their payoff matrix would appear such that the dominant strategy is to not purchase it as long as cost > 0, since he is either indifferent if no one buys and better off if he free-rides someone else

⇒ left to private provision, so long as the cost of provision > 0 and pareto efficient allocation of G, $G^* > 0$, then there will exist pareto inefficiency due to free riders.

Asymmetric information

① asymmetric information

imperfect information markets are said to have imperfect information when not all agents have complete information about all agents in the market

asymmetric information markets are said to have asymmetric information when one side is better informed / has more information than the other. These markets by definition have imperfect information

② adverse selection

adverse selection adverse selection happens when agents with more or better information in the market exploit agents with less information. ie. because agents cannot determine other agents' type, price signals are distorted

pareto inefficiency & market failure

1. in a perfect market, consumers know the quality of the good - so they pay exactly how much they value it, knowing how much utility they gain out of it
   1.1 price mechanism serves as an efficient mechanism for distributing resources, no under- or over- production

2. suppose we have asymmetric information such that one party only has knowledge of the distribution of goods quality in the market.

3. then rational agents, depending on their risk profile, are only willing to pay their certainty equivalence (so risk neutral will pay $E(x)$)

4. sellers know this, so sellers who are only willing to sell above CE will not enter the market

5. Buyers also know how sellers react, leaving only low-quality sellers in the market. In some scenarios, this could cyclically lead to the market collapsing

adverse selection given quality choice

↳ suppose now sellers can choose whether to sell high/low quality goods. How will they choose, and what is the stable equilibrium?

1. case 1: only high quality
2. case 2: only low quality
3. case 3: mix
4. case 4: no market

1. how much are consumers willing to pay
2. Do sellers have incentive to deviate?

no

yes, stable

③ solutions to adverse selection

(regulation) banning low-quality goods ⟶ then side with less information can have a priori information to decide price to offer

(getting signals)

↳ suppose there are different quality of sellers, but buyers cannot distinguish them, and so offer the same price. This is known as a pooling equilibrium

eg. employee wages

$$w_{pool} = (1-q)\, w_{low} + q\, w_{high} \quad , \quad w_{high} > w_{low}$$

↳ sellers have the option to spend some money and acquire some signal of quality. assuming that the cost of such a signal is different for different quality sellers, a separating equilibrium occurs when it is rational for a high quality seller to acquire the signal but not for a low quality seller:

cost of signal: $c_{high} < c_{low}$

1. $w_{high} - c_{high} > w_{low}$ ⟹ (1) utility gain after obtaining signal than without for high quality
2. $w_{high} - c_{low} < w_{low}$ ⟹ (2) low ability sellers worse off acquiring signals and so choose not to

↳ but, assuming signals do nothing to quality, they reduce total surplus since resources have to be expended acquiring those signals

④ moral hazard

(moral hazard) when agents are able to transfer risk without the other agent knowing and so take unnecessary risks

(inefficiency of moral hazard)

↳ suppose agents pay to transfer risk to another agent (eg. insurance)

↳ rationally, the merchant will charge a price that is representative of the risk of payout:

$$E(\pi) = (\underset{\text{rate}}{\gamma} - \underset{\text{true probability}}{\pi})\, \underset{\text{payout}}{K}$$

↳ in an instance of asymmetric information, the insured agent may engage in riskier behavior without the insurer knowing. Then the insurer will not know π, and will not be able to act optimally

⑤ solutions to moral hazard

regulation enforce the reduction of risky behavior (e.g. annual car checks)

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incentive contracting under certainty

↳ abstractly, a worker is hired by a principal to do a task. Because of asymmetric information, only the worker knows how much effort, x, he puts in

the principal's problem: design an incentive contract that induce the rational worker to optimize the principal's payoff. The contract must fulfil two constraints to work:

1. participation: the contract must minimally provide the worker a level of utility that is equal to his opportunity cost of doing this: ie. leaves him indifferent ie. will want to work

2. incentive compatibility: The contract must be such that the optimal solution to the worker's utility maximisation problem is the π maximising level of effort. ie. will work optimally

$$\forall x \geq 0, \quad s(f(x^*)) - c(x^*) \geq s(f(x)) - c(x)$$

rental contracts

↳ principal keeps a lumpsum R & worker keeps remaining

$$s(f(x)) = f(x) - R$$

wage contracts

↳ wage per unit effort + lump sum

$$s(f(x)) = wx + K$$

take it or leave it

↳ worker gets all or nothing

$$s(f(x)) = \begin{cases} L & \text{if } x = x^* \\ 0 & \text{otherwise} \end{cases}$$

The worker: optimize payoffs, u

$$\max_x u(x) = \max(\underbrace{\bar{u}}_{\substack{\text{reserve}\\ \text{opp. cost}}}, \underbrace{s((f(x))}_{\substack{\text{reward}\\ \text{for output}}} - \underbrace{c(x)}_{\substack{\text{cost of}\\ \text{effort}}})$$

$$\Rightarrow \frac{du}{dx} = 0 \Rightarrow x$$

The principal: optimize π

↳ let A be a set of decision variables for incentive scheme $s(y) = s(f(x))$

$$\pi = \underbrace{f(x)}_{\text{earnings}} - \underbrace{s(f(x))}_{\text{payout to worker}}$$

1. find x^*. $\frac{d\pi}{dx} = 0 \Rightarrow x^*$

2. Solve for decision variables A such that incentive & participation compatible

incentive enforced

$$\max_A \pi = f(x^*) - s(f(x^*), A)$$

participation

$$\text{s.t. } s(f(x^*), A) - c(x^*) \geq \bar{u}$$

Incentive contracting under uncertainty

↳ suppose there is some uncertainty in the production process, so effort does not deterministically translate into output

⇒ then agents maximize <u>expected</u> payoff, and will make choices according to their risk preference

revenue-sharing

↳ worker receives a share of the output

$s(f(x)) = \alpha f(x), \quad 0 < \alpha < 1$

<u>The worker</u>: optimize expected payoffs

$$\max_x E[u(x)] = E[\max(\underbrace{\bar{u}}_{\text{reserve opp. cost}}, \underbrace{s((f(x))}_{\text{reward for output}} - \underbrace{c(x)}_{\text{cost of effort}})]$$

$\Rightarrow \frac{du}{dx} = 0 \Rightarrow x$

<u>The principal</u>: optimize $E(\pi)$

↳ let A be a set of decision variables for incentive scheme $s(y) = s(f(x))$

$$E(\pi) = E[\underbrace{f(x)}_{\text{earnings}} - \underbrace{s(f(x))}_{\text{payout to worker}}]$$

1. find x^*. $\frac{dE\pi}{dx} = 0 \Rightarrow x^*$

2. Solve for decision variables A such that incentive & participation compatible

$$\max_A E(\pi) = E[f(x^*) - s(f(x^*), A)]$$

incentive enforced

$$\text{s.t. } \underbrace{E[s(f(x^*), A) - c(x^*)]}_{E[u(x^*)]} \geq \bar{u}$$

participation