

## 1. Number systems

### ① conversion between bases

1) whole numbers: subtraction of  $\text{Base}^n$

2) fraction: repeated multiplication until fractional product is 0 to sufficient dp

$$\text{eg. } 0.3125_{10} = 0.0101_2$$

$$0.3125 \times 2 = 0.625$$

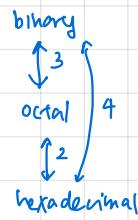
$$0.625 \times 2 = 1.25$$

$$0.25 \times 2 = 0.50$$

$$0.50 \times 2 = 1.00$$

### 3) combined bases

| Hexadecimal | Binary | Decimal |
|-------------|--------|---------|
| 0           | 0000   | 0       |
| 1           | 0001   | 1       |
| 2           | 0010   | 2       |
| 3           | 0011   | 3       |
| 4           | 0100   | 4       |
| 5           | 0101   | 5       |
| 6           | 0110   | 6       |
| 7           | 0111   | 7       |
| 8           | 1000   | 8       |
| 9           | 1001   | 9       |
| A           | 1010   | 10      |
| B           | 1011   | 11      |
| C           | 1100   | 12      |
| D           | 1101   | 13      |
| E           | 1110   | 14      |
| F           | 1111   | 15      |



### 4) range and overflow

$$\begin{array}{c} | \\ N \text{ bits} - 2^N \text{ values} \end{array} \quad \begin{array}{c} | \\ \text{max} \xrightarrow{-1} \text{min} \end{array}$$

### 5) binary addition and subtraction

$$\cdot A - B = A + (-B) - \text{negate}$$

- determining overflows: positive + positive  $\rightarrow$  negative  
(i) (i) (i)
- negative + negative  $\rightarrow$  positive  
(i) (i) (i)

## 4- ISAs

1) address space: N bit address  $\rightarrow 2^N$  space

2) endianness

big endian

↳ msbyte stored in lowest address (going up)

little endian

↳ msbyte stored in highest address (going down)

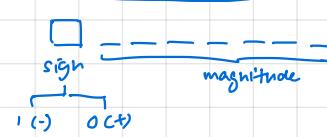


### 3) instruction set size

- move bits = move instructions
- use single unique opcodes to distinguish
- "stage-wide" splitting

## ② representing negative numbers

### Sign & magnitude



• range:  $\pm (2^{N-1})$   
 $-2^{N-1} \dots -1, 0, 1 \dots 2^{N-1}$

• to negative: flip sign

### 1's complement

$$-(2^{N-1})$$

$$\begin{array}{c} | \\ 2^6 2^5 2^4 2^3 2^2 2^1 1 \end{array}$$

• to negative: flip all bits

• how to obtain:

$$\begin{aligned} -X &= 2^N - X \quad (\text{binary}) \\ -X &= B^N - B^m - X \quad (\text{general}) \end{aligned}$$

convert to binary

$/$  no. of integer digits       $/$  no. of fractional digits

• binary addition: if end carry, add back to result

• range:  $\pm (2^{N-1})$

0111 1111 127 largest  
1000 0000 -127 smallest  
0000 0000 +0  
1111 1111 -0

### Excess representation

• range =  $-2^{N-1}$  to  $2^{N-1} - 1$

• convert binary to decimal - excess ( $\text{excess} = 2^{N-1}$ )

### 2's complement

$$-2^7$$

$$\begin{array}{c} | \\ 2^6 2^5 2^4 2^3 2^2 2^1 1 \end{array}$$

range:  $-2^{N-1}$  to  $2^{N-1} - 1$

0111 1111 127 max  
1000 0000 -128 min  
0000 0000 0

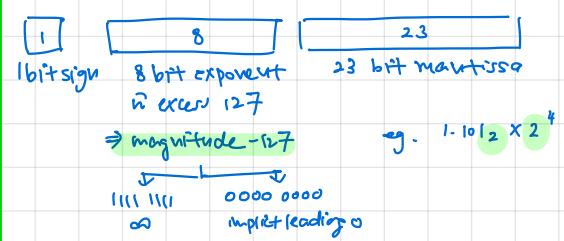
• to negative: invert all, binary addition +1

• how to obtain:

$$\begin{aligned} -X &= 2^N - X \quad (\text{binary}) \\ -X &= B^N - X \end{aligned}$$

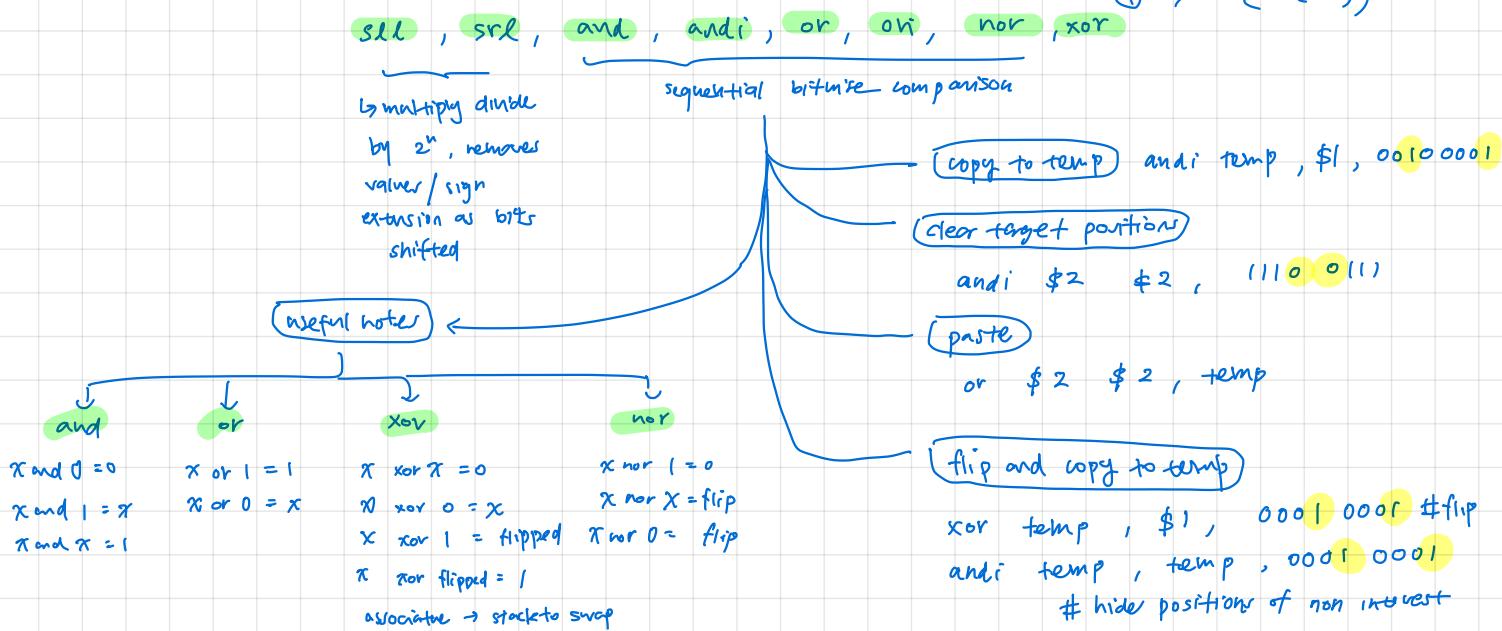
• addition: binary addition, discard end carry

## ③ IEEE 754 floating point representation



## 2. Assembly & MIPS

### ① useful logical operations & masking



## 3. C programming

### ① computation

i) variable assignment : assignment returns to LHS, cascading

| Operator Type                | Operator  | Associativity | value update                  |
|------------------------------|---|---------------|-------------------------------|
| Primary expression operators | <code>() expr++ expr--</code>                     | Left to right | $\xleftarrow{-\text{op}-}$    |
| Unary operators              | <code>* &amp; + - ++expr --expr (typecast)</code> | Right to left | $\xrightarrow{\text{+ op +}}$ |
| Binary operators             | <code>*</code> / %                                | Left to right |                               |
|                              | <code>+ -</code>                                  |               |                               |
| Assignment operators         | <code>= += -= *= /= %=</code>                     | Right to left |                               |

### 2) mixed type operations & truncating

|                                |       |            |   |
|--------------------------------|-------|------------|---|
| <code>int m = 10/4;</code>     | means | $m = 2;$   | $m = \frac{\text{int}}{\text{int}}$               |
| <code>float p = 10/4;</code>   | means | $p = 2.0;$ | $p = \frac{\text{int}}{\text{int}}$               |
| <code>int n = 10/4.0;</code>   | means | $n = 2;$   | $n = \frac{\text{int}}{\text{float}}$             |
| <code>float q = 10/4.0;</code> | means | $q = 2.5;$ | $q = \frac{\text{int}}{\text{float}}$             |
| <code>int r = -10/4.0;</code>  | means | $r = -2;$  | Caution!<br>$r = \frac{\text{int}}{\text{float}}$ |

### 3) types

- ↳ int : 4 bytes
- ↳ no boolean types (0 or 1)
- ↳ float : 4 bytes
- ↳ double : 8 bytes
- ↳ char : single quotes, 1 byte

### ② pointers

i) declaration and initialisation

`type * pointer_name;`  $\rightarrow$  points to random address if not initialised

`type * pointer_name = &a;`

ii) changing addr.: `pointer_name = &a;`

iii) de-referencing: `* pointer_name = value;`

### ③ functions and variable scope

i) scope: local functions are local

### ④ arrays

- 1) continuous memory location
- 2) cannot initialise after declaration  $\rightarrow$  random if not at all initialised
- 3) a is fixed pointer to `a[0]`
- 4) `sizeof()`  $\rightarrow$  expression  $\rightarrow$  size in bytes of object representation of return value.
- 5) array sizes depend on type  $\rightarrow$  int, double etc. = 4  $\rightarrow$  char[] = String = 1

### ⑤ strings

- 1) input & output, functions  $\rightarrow$  see notes
- 2) importance of '\0'  $\Rightarrow$  distinguishes String from char[]

### ⑥ structures

- 1) reading and modifying: `struct-variable.field`
- 2) structure variable pointer points to first field in struct
- 3) pointer is mutable
- 4) pass by value by default
- 5) arrow operator

$(\ast \text{struct\_pointer}).\text{field} \Leftrightarrow \text{struct\_pointer} \rightarrow \text{field}$

$\Leftrightarrow \ast(\text{struct\_pointer}. \text{field})$   
equivalent, - precedence over  $\ast$

# MIPS Reference Data

①



## ARITHMETIC CORE INSTRUCTION SET

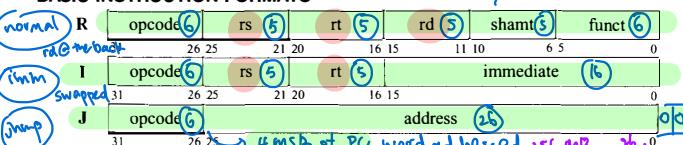
② OPCODE / FMT / FT / FUNCT / FUNCT (Hex)

### CORE INSTRUCTION SET

| NAME, MNEMONIC              | FOR-MAT | OPERATION (in Verilog)                                  |
|-----------------------------|---------|---|
| Add                         | add     | R [rd] = R[rs] + R[rt]                                  |
| Add Immediate               | addi    | I R[rt] = R[rs] + SignExtImm                            |
| Add Imm. Unsigned           | addiu   | I R[rt] = R[rs] + SignExtImm                            |
| Add Unsigned                | addu    | R R[rd] = R[rs] + R[rt]                                 |
| And                         | and     | R R[rd] = R[rs] & R[rt]                                 |
| And Immediate               | andi    | I R[rt] = R[rs] & ZeroExtImm                            |
| Branch On Equal             | beq     | I if(R[rs]==R[rt]) PC=PC+4+BranchAddr                   |
| Branch On Not Equal         | bne     | I if(R[rs]!=R[rt]) PC=PC+4+BranchAddr                   |
| Jump                        | j       | J PC=JumpAddr   |
| Jump And Link               | jal     | J R[31]=PC+8;PC=JumpAddr                                |
| Jump Register               | jr      | R PC=R[rs]  |
| Load Byte Unsigned          | lbu     | I R[rt]={24'b0,M[R[rs]+SignExtImm](7:0)}                |
| Load Halfword Unsigned      | lhu     | I R[rt]={16'b0,M[R[rs]+SignExtImm](15:0)}               |
| Load Linked                 | ll      | I R[rt]=M[R[rs]+SignExtImm]                             |
| Load Upper Imm.             | lui     | I R[rt]={imm, 16'b0} <i>clears lower 16 bits</i>        |
| Load Word                   | lw      | I R[rt]=M[R[rs]+SignExtImm]                             |
| Nor                         | nor     | R R[rd] = ~ (R[rs]   R[rt])                             |
| Or                          | or      | R R[rd] = R[rs]   R[rt]                                 |
| Or Immediate                | ori     | I R[rt] = R[rs]   ZeroExtImm                            |
| Set Less Than               | slt     | R R[rd] = (R[rs] < R[rt]) ? 1 : 0                       |
| Set Less Than Imm.          | slti    | I R[rt] = (R[rs] < SignExtImm) ? 1 : 0                  |
| Set Less Than Imm. Unsigned | sltiu   | I R[rt] = (R[rs] < SignExtImm) ? 1 : 0                  |
| Set Less Than Unsigned      | situ    | R R[rd] = (R[rs] < R[rt]) ? 1 : 0                       |
| Shift Left Logical          | sll     | R R[rd] = R[rt] << shamt                                |
| Shift Right Logical         | srl     | R R[rd] = R[rt] >> shamt                                |
| Store Byte                  | sb      | I M[R[rs]+SignExtImm](7:0)= R[rt](7:0)                  |
| Store Conditional           | sc      | I M[R[rs]+SignExtImm] = R[rt]; R[rt] = (atomic) ? 1 : 0 |
| Store Halfword              | sh      | I M[R[rs]+SignExtImm](15:0)= R[rt](15:0)                |
| Store Word                  | sw      | I M[R[rs]+SignExtImm] = R[rt]                           |
| Subtract                    | sub     | R R[rd] = R[rs] - R[rt]                                 |
| Subtract Unsigned           | subu    | R R[rd] = R[rs] - R[rt]                                 |

- (1) May cause overflow exception
- (2) SignExtImm = { 16{immediate[15]}, immediate }
- (3) ZeroExtImm = { 16{1'b0}, immediate }
- (4) BranchAddr = { 14{immediate[15]}, immediate, 2'b0 }
- (5) JumpAddr = { PC+4[31:28], address, 2'b0 }
- (6) Operands considered unsigned numbers (vs. 2's comp.)
- (7) Atomic test&set pair; R[rt] = 1 if pair atomic, 0 if not atomic

### BASIC INSTRUCTION FORMATS



OPCODE / FUNCT (Hex)

NAME, MNEMONIC FOR-MAT OPERATION

Branch On FP True bc1t FI if(FPcond)PC=PC+4+BranchAddr (4) 11/8/1/-

Branch On FP False bc1f FI if(!FPcond)PC=PC+4+BranchAddr(4) 11/8/0/-

|      | Hexadecimal                   | Binary | Decimal |
|------|-------------------------------|--------|---------|
| Div  | 0 / 20 <sub>hex</sub> / 10000 | 00000  | 0       |
| Divi | 8 <sub>hex</sub> 001100       | 100000 | 1       |
| FPA  | 9 <sub>hex</sub>              | 1001   | -y      |
| Dou  | 0 / 21 <sub>hex</sub> / 00000 | 100000 | 2       |
| FPC  | FP C                          | 10010  | -y      |
| FPC  | 0 / 24 <sub>hex</sub> / 10000 | 100000 | Dou     |
| Dou  | C hex 001100                  | 100000 | FP E    |
| Dou  | 4 <sub>hex</sub> 000000       | 100000 | FP E    |
| Dou  | 5 <sub>hex</sub> 000001       | 100001 | FP N    |
| Dou  | 2 <sub>hex</sub> 000000       | 100000 | FP N    |
| Dou  | 3 <sub>hex</sub>              | 10011  | FP S    |
| Dou  | 8 <sub>hex</sub>              | 10000  | FP S    |
| Dou  | 9 <sub>hex</sub>              | 10001  | FP S    |
| Load | A                             | 1010   | 10      |
| Load | B                             | 1011   | 11      |
| Load | C                             | 1100   | 12      |
| Load | D                             | 1101   | 13      |
| Mul  | E                             | 1110   | 14      |
| Mul  | F                             | 1111   | 15      |

sdc1 I M[R[rs]+SignExtImm] = F[rt]; (2) 3d/-/-/-

M[R[rs]+SignExtImm+4] = F[rt+1]

### FLOATING-POINT INSTRUCTION FORMATS

| FR | opcode | fmt   | ft    | fs        | fd  | funct |
|----|--------|-------|-------|-----------|-----|-------|
| 31 | 26 25  | 21 20 | 16 15 | 11 10     | 6 5 | 0     |
| FI | opcode | fmt   | ft    | immediate |     |       |
| 31 | 26 25  | 21 20 | 16 15 |           |     |       |

### PSEUDOINSTRUCTION SET

| NAME                         | MNEMONIC | OPERATION                   |
|------------------------------|----------|-----------------------------|
| Branch Less Than             | blt      | if(R[rs]<R[rt]) PC = Label  |
| Branch Greater Than          | bgt      | if(R[rs]>R[rt]) PC = Label  |
| Branch Less Than or Equal    | bie      | if(R[rs]<=R[rt]) PC = Label |
| Branch Greater Than or Equal | bge      | if(R[rs]>=R[rt]) PC = Label |
| Load Immediate               | li       | R[rd] = immediate           |
| Move                         | move     | R[rd] = R[rs]               |

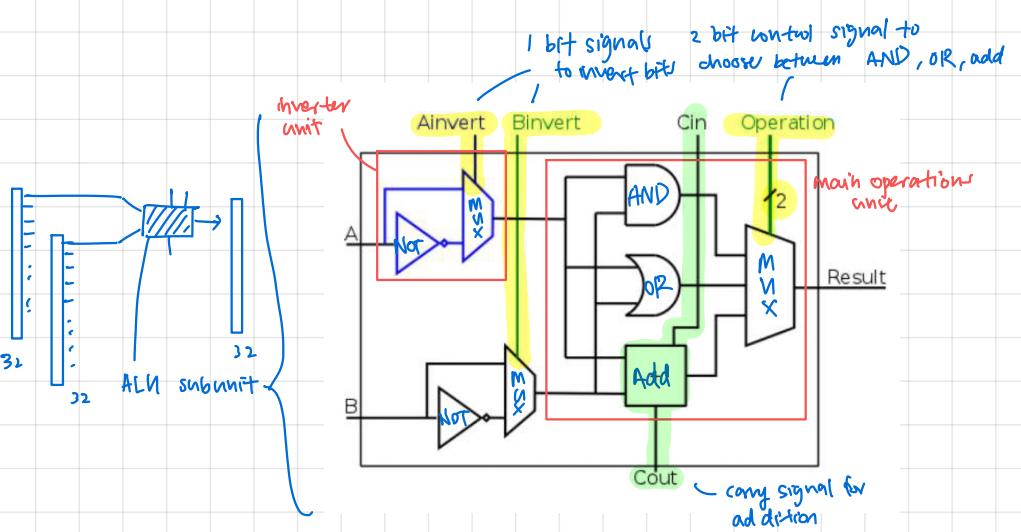
### REGISTER NAME, NUMBER, USE, CALL CONVENTION

| NAME      | NUMBER | USE   | PREERVED ACROSS A CALL? |
|-----------|--------|---|-------------------------|
| \$zero    | 0      | The Constant Value 0                                  | N.A.                    |
| \$at      | 1      | Assembler Temporary                                   | No                      |
| \$v0-\$v1 | 2-3    | Values for Function Results and Expression Evaluation | No                      |
| \$a0-\$a3 | 4-7    | Arguments   | No                      |
| \$t0-\$t7 | 8-15   | Temporaries   | No                      |
| \$s0-\$s7 | 16-23  | Saved Temporaries                                     | Yes                     |
| \$t8-\$t9 | 24-25  | Temporaries   | No                      |
| \$k0-\$k1 | 26-27  | Reserved for OS Kernel                                | No                      |
| \$gp      | 28     | Global Pointer  | Yes                     |
| \$sp      | 29     | Stack Pointer   | Yes                     |
| \$fp      | 30     | Frame Pointer   | Yes                     |
| \$ra      | 31     | Return Address  | Yes                     |

## 6. Control

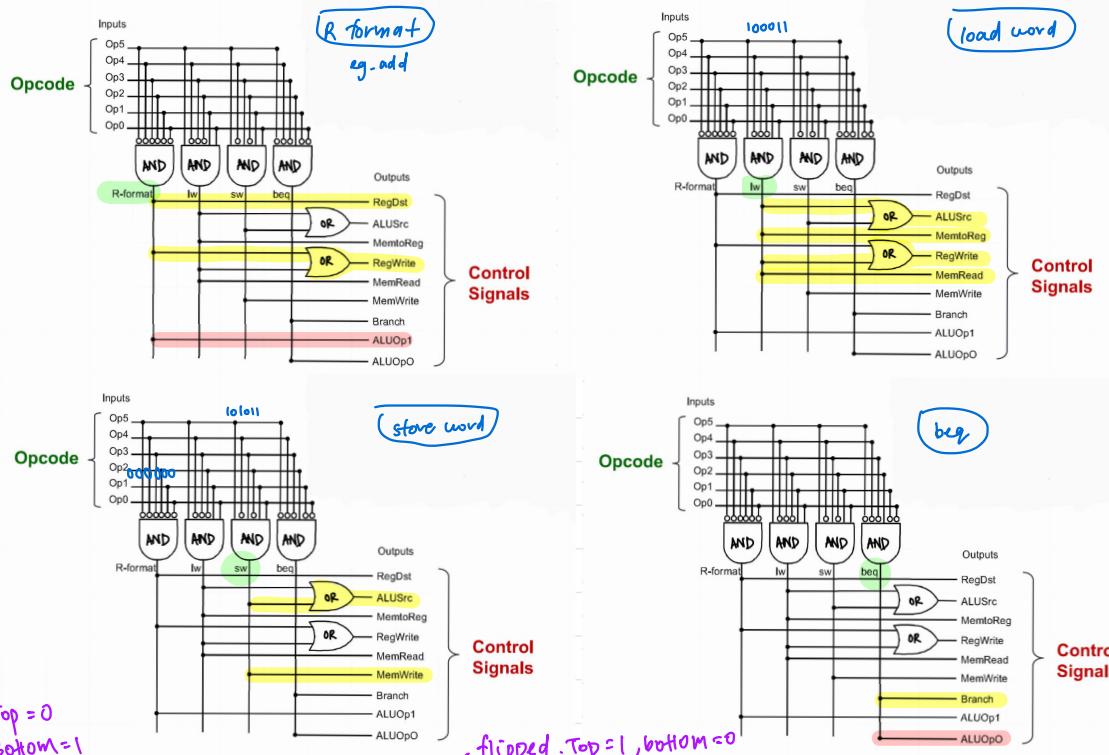
### ① ALU implementation

| ALU control lines | Function         |
|-------------------|------------------|
| 0000              | AND              |
| 0001              | OR               |
| 0010              | add              |
| 0110              | subtract         |
| 0111              | set on less than |
| 1100              | NOR              |



### ② combinational circuits for signals

#### D) generating non-ALU signals



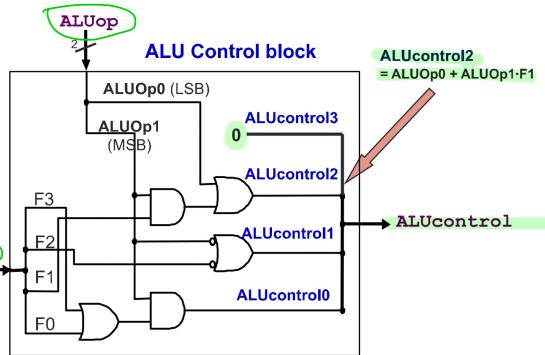
|        | RegDst | ALUSrc | MemToReg | RegWrite | MemRead | MemWrite | Branch | ALUop        |
|--------|--------|--------|----------|----------|---------|----------|--------|--------------|
|        |        |        |          |          |         |          |        | op1      op0 |
| R-type | 1      | 0      | 0        | 1        | 0       | 0        | 0      | 1 0          |
| lw     | 0      | 1      | 1        | 1        | 1       | 0        | 0      | 0 0          |
| sw     | X      | 1      | X        | 0        | 0       | 1        | 0      | 0 0          |
| beq    | X      | 0      | X        | 0        | 0       | 0        | 1      | 0 1          |

I-type: 0 1 0 1 0 0 0 1 0

|        | Opcode<br>(Op[5:0] == Inst[31:26]) |     |     |     |     |     | Value in Hexadecimal |
|--------|------------------------------------|-----|-----|-----|-----|-----|----------------------|
|        | Op5                                | Op4 | Op3 | Op2 | Op1 | Op0 |                      |
| R-type | 0                                  | 0   | 0   | 0   | 0   | 0   | 0                    |
| lw     | 1                                  | 0   | 0   | 0   | 1   | 1   | 23                   |
| sw     | 1                                  | 0   | 1   | 0   | 1   | 1   | 2B                   |
| beq    | 0                                  | 0   | 0   | 1   | 0   | 0   | 4                    |

I-type: 0 0 0 0 0 0 0

## 2) generating ALUctrl signals



|     | ALUop |     | Funct Field<br>( F[5:0] == Inst[5:0] ) |    |    |    |    |    |      | ALU control |
|-----|-------|-----|--|----|----|----|----|----|------|-------------|
|     | MSB   | LSB | F5                                     | F4 | F3 | F2 | F1 | F0 |      |             |
| lw  | 0     | 0   | X                                      | X  | X  | X  | X  | X  | 0010 | add         |
| sw  | 0     | 0   | X                                      | X  | X  | X  | X  | X  | 0010 | sub         |
| beq | 0X    | 1   | X                                      | X  | X  | X  | X  | X  | 0110 | add         |
| add | 1     | 0X  | XX                                     | XX | 0  | 0  | 0  | 0  | 0010 | sub         |
| sub | 1     | 0X  | XX                                     | XX | 0  | 0  | 1  | 0  | 0110 | add         |
| and | 1     | 0X  | XX                                     | XX | 0  | 1  | 0  | 0  | 0000 | sub         |
| or  | 1     | 0X  | XX                                     | XX | 0  | 1  | 0  | 1  | 0001 | add         |
| slt | 1     | 0X  | XX                                     | XX | 1  | 0  | 1  | 0  | 0111 | sub         |

| Instruction opcode | ALUOp | Instruction operation | Funct field | Desired ALU action | ALU control input |
|--------------------|-------|-----------------------|-------------|--------------------|-------------------|
| LW                 | 00    | load word             | XXXXXX      | add                | 0010              |
| SW                 | 00    | store word            | XXXXXX      | add                | 0010              |
| Branch equal       | 01    | branch equal          | XXXXXX      | subtract           | 0110              |
| R-type             | 10    | add                   | 100000      | add                | 0010              |
| R-type             | 10    | subtract              | 100010      | subtract           | 0110              |
| R-type             | 10    | AND                   | 100100      | AND                | 0000              |
| R-type             | 10    | OR                    | 100101      | OR                 | 0001              |
| R-type             | 10    | set on less than      | 101010      | set on less than   | 0111              |

## ③ cycle time

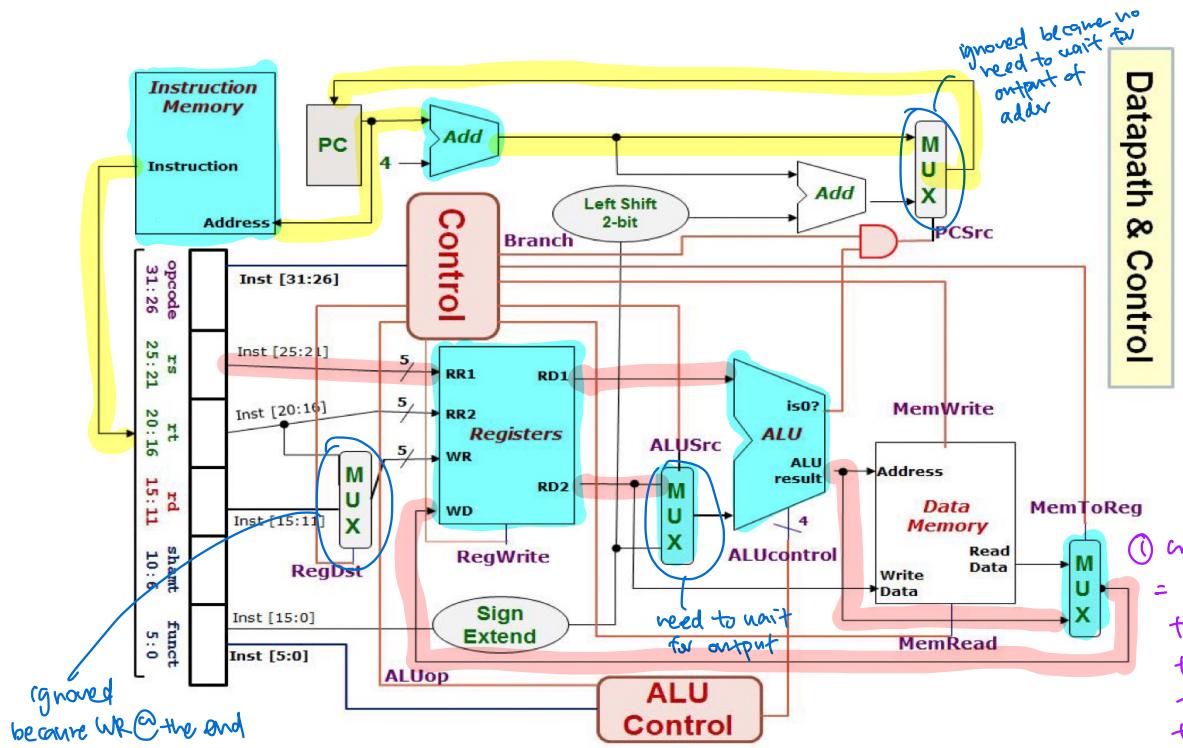
- single cycle implementation :  $T = \text{time of longest possible process}$
- multi cycle implementation :  $T = \text{time of longest single stage}$

| Dec | Hx     | Oct | Char                     | Dec | Hx     | Oct   | Html  | Chr | Dec | Hx     | Oct   | Html | Chr | Dec    | Hx     | Oct | Html | Chr    |        |   |
|-----|--------|-----|--------------------------|-----|--------|-------|-------|-----|-----|--------|-------|------|-----|--------|--------|-----|------|--------|--------|---|
| 0   | 0 000  | NUL | (null)                   | 32  | 20 040 | &#32; | Space |     | 64  | 40 100 | &#64; | Ø    | 96  | 60 140 | &#96;  | ~   | 128  | 80 100 | &#128; | ÿ |
| 1   | 1 001  | SOH | (start of heading)       | 33  | 21 041 | &#33; | !     | !   | 65  | 41 101 | &#65; | A    | 97  | 61 141 | &#97;  | a   | 129  | 81 041 | &#129; | à |
| 2   | 2 002  | STX | (start of text)          | 34  | 22 042 | &#34; | "     | "   | 66  | 42 102 | &#66; | B    | 98  | 62 142 | &#98;  | b   | 130  | 82 042 | &#130; | â |
| 3   | 3 003  | ETX | (end of text)            | 35  | 23 043 | &#35; | #     | #   | 67  | 43 103 | &#67; | C    | 99  | 63 143 | &#99;  | c   | 131  | 83 043 | &#131; | ç |
| 4   | 4 004  | EOT | (end of transmission)    | 36  | 24 044 | &#36; | \$    | \$  | 68  | 44 104 | &#68; | D    | 100 | 64 144 | &#100; | d   | 132  | 84 044 | &#132; | đ |
| 5   | 5 005  | ENQ | (enquiry)                | 37  | 25 045 | &#37; | %     | %   | 69  | 45 105 | &#69; | E    | 101 | 65 145 | &#101; | e   | 133  | 85 045 | &#133; | é |
| 6   | 6 006  | ACK | (acknowledge)            | 38  | 26 046 | &#38; | &     | &   | 70  | 46 106 | &#70; | F    | 102 | 66 146 | &#102; | f   | 134  | 86 046 | &#134; | ƒ |
| 7   | 7 007  | BEL | (bell)                   | 39  | 27 047 | &#39; | '     | '   | 71  | 47 107 | &#71; | G    | 103 | 67 147 | &#103; | g   | 135  | 87 047 | &#135; | ߱ |
| 8   | 8 010  | BS  | (backspace)              | 40  | 28 050 | &#40; | (     | (   | 72  | 48 110 | &#72; | H    | 104 | 68 150 | &#104; | h   | 136  | 88 050 | &#136; | ߲ |
| 9   | 9 011  | TAB | (horizontal tab)         | 41  | 29 051 | &#41; | )     | )   | 73  | 49 111 | &#73; | I    | 105 | 69 151 | &#105; | i   | 137  | 89 051 | &#137; | ߳ |
| 10  | A 012  | LF  | (NL line feed, new line) | 42  | 2A 052 | &#42; | *     | *   | 74  | 4A 112 | &#74; | J    | 106 | 6A 152 | &#106; | j   | 138  | 8A 052 | &#138; | ߴ |
| 11  | B 013  | VT  | (vertical tab)           | 43  | 2B 053 | &#43; | +     | +   | 75  | 4B 113 | &#75; | K    | 107 | 6B 153 | &#107; | k   | 139  | 8B 053 | &#139; | ߵ |
| 12  | C 014  | FF  | (NP form feed, new page) | 44  | 2C 054 | &#44; | ,     | ,   | 76  | 4C 114 | &#76; | L    | 108 | 6C 154 | &#108; | l   | 140  | 8C 054 | &#140; | ߷ |
| 13  | D 015  | CR  | (carriage return)        | 45  | 2D 055 | &#45; | -     | -   | 77  | 4D 115 | &#77; | M    | 109 | 6D 155 | &#109; | m   | 141  | 8D 055 | &#141; | ߸ |
| 14  | E 016  | SO  | (shift out)              | 46  | 2E 056 | &#46; | ,     | ,   | 78  | 4E 116 | &#78; | N    | 110 | 6E 156 | &#110; | n   | 142  | 8E 056 | &#142; | ߹ |
| 15  | F 017  | SI  | (shift in)               | 47  | 2F 057 | &#47; | /     | /   | 79  | 4F 117 | &#79; | O    | 111 | 6F 157 | &#111; | o   | 143  | 8F 057 | &#143; | ߻ |
| 16  | 10 020 | DLE | (data link escape)       | 48  | 30 060 | &#48; | 0     | 0   | 80  | 50 120 | &#80; | P    | 112 | 70 160 | &#112; | p   | 144  | 80 060 | &#144; | ߴ |
| 17  | 11 021 | DC1 | (device control 1)       | 49  | 31 061 | &#49; | 1     | 1   | 81  | 51 121 | &#81; | Q    | 113 | 71 161 | &#113; | q   | 145  | 81 061 | &#145; | ߵ |
| 18  | 12 022 | DC2 | (device control 2)       | 50  | 32 062 | &#50; | 2     | 2   | 82  | 52 122 | &#82; | R    | 114 | 72 162 | &#114; | r   | 146  | 82 062 | &#146; | ߶ |
| 19  | 13 023 | DC3 | (device control 3)       | 51  | 33 063 | &#51; | 3     | 3   | 83  | 53 123 | &#83; | S    | 115 | 73 163 | &#115; | s   | 147  | 83 063 | &#147; | ߷ |
| 20  | 14 024 | DC4 | (device control 4)       | 52  | 34 064 | &#52; | 4     | 4   | 84  | 54 124 | &#84; | T    | 116 | 74 164 | &#116; | t   | 148  | 84 064 | &#148; | ߸ |
| 21  | 15 025 | NAK | (negative acknowledge)   | 53  | 35 065 | &#53; | 5     | 5   | 85  | 55 125 | &#85; | U    | 117 | 75 165 | &#117; | u   | 149  | 85 065 | &#149; | ߷ |
| 22  | 16 026 | SYN | (synchronous idle)       | 54  | 36 066 | &#54; | 6     | 6   | 86  | 56 126 | &#86; | V    | 118 | 76 166 | &#118; | v   | 150  | 86 066 | &#150; | ߷ |
| 23  | 17 027 | ETB | (end of trans. block)    | 55  | 37 067 | &#55; | 7     | 7   | 87  | 57 127 | &#87; | W    | 119 | 77 167 | &#119; | w   | 151  | 87 067 | &#151; | ߷ |
| 24  | 18 030 | CAN | (cancel)                 | 56  | 38 070 | &#56; | 8     | 8   | 88  | 58 130 | &#88; | X    | 120 | 78 170 | &#120; | x   | 152  | 88 070 | &#152; | ߷ |
| 25  | 19 031 | EM  | (end of medium)          | 57  | 39 071 | &#57; | 9     | 9   | 89  | 59 131 | &#89; | Y    | 121 | 79 171 | &#121; | y   | 153  | 89 071 | &#153; | ߷ |
| 26  | 1A 032 | SUB | (substitute)             | 58  | 3A 072 | &#58; | :     | :   | 90  | 5A 132 | &#90; | Z    | 122 | 7A 172 | &#122; | z   | 154  | 8A 072 | &#154; | ߷ |
| 27  | 1B 033 | ESC | (escape)                 | 59  | 3B 073 | &#59; | :     | :   | 91  | 5B 133 | &#91; | [    | 123 | 7B 173 | &#123; | [   | 155  | 8B 073 | &#155; | ߷ |
| 28  | 1C 034 | FS  | (file separator)         | 60  | 3C 074 | &#60; | <     | <   | 92  | 5C 134 | &#92; | \    | 124 | 7C 174 | &#124; | \   | 156  | 8C 074 | &#156; | ߷ |
| 29  | 1D 035 | GS  | (group separator)        | 61  | 3D 075 | &#61; | =     | =   | 93  | 5D 135 | &#93; | ]    | 125 | 7D 175 | &#125; | ]   | 157  | 8D 075 | &#157; | ߷ |
| 30  | 1E 036 | RS  | (record separator)       | 62  | 3E 076 | &#62; | >     | >   | 94  | 5E 136 | &#94; | ^    | 126 | 7E 176 | &#126; | ~   | 158  | 8E 076 | &#158; | ߷ |
| 31  | 1F 037 | US  | (unit separator)         | 63  | 3F 077 | &#63; | ?     | ?   | 95  | 5F 137 | &#95; | _    | 127 | 7F 177 | &#127; | DEL | 159  | 8F 077 | &#159; | ߷ |

(arithmetic)

op rd rs rt

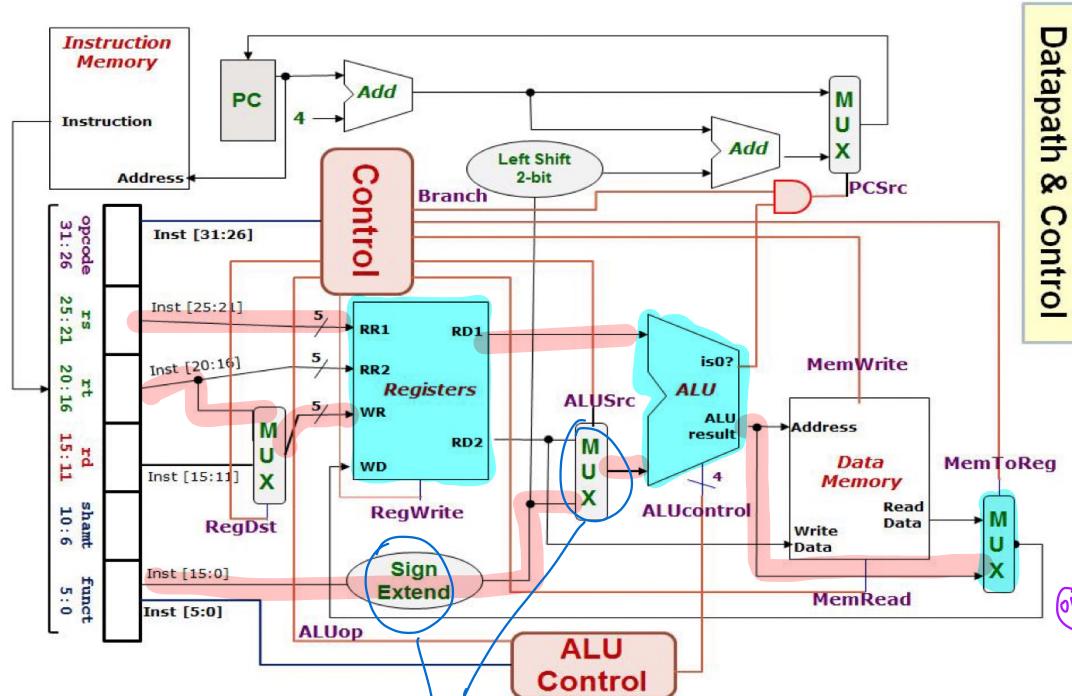
② others: instruction + register + mux (reg dst)  
instruction + control ... only when control > reg  
instruction + ADD + mux (PC src)



|        | RegDst | ALUSrc | MemTo Reg | Reg Write | Mem Read | Mem Write | Branch | ALUop |     |
|--------|--------|--------|-----------|-----------|----------|-----------|--------|-------|-----|
|        |        |        |           |           |          |           |        | op1   | op0 |
| R-type | 1      | 0      | 0         | 1         | 0        | 0         | 0      | 1     | 0   |

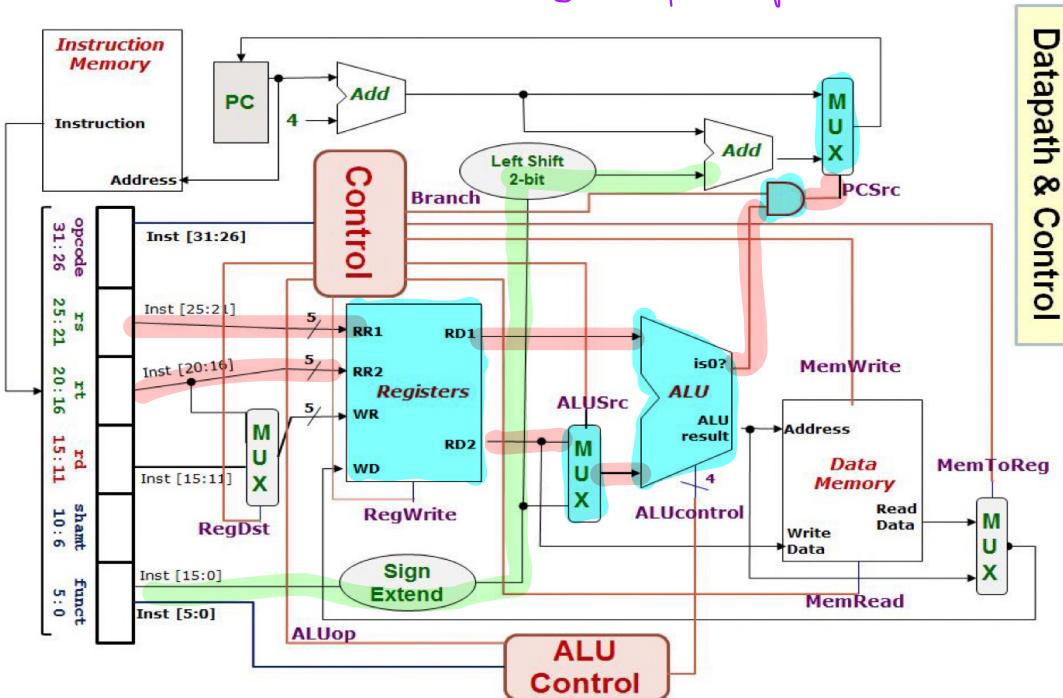
(immediate arithmetic)

opi nt rs mm



branching) beg, bne rs rt imm

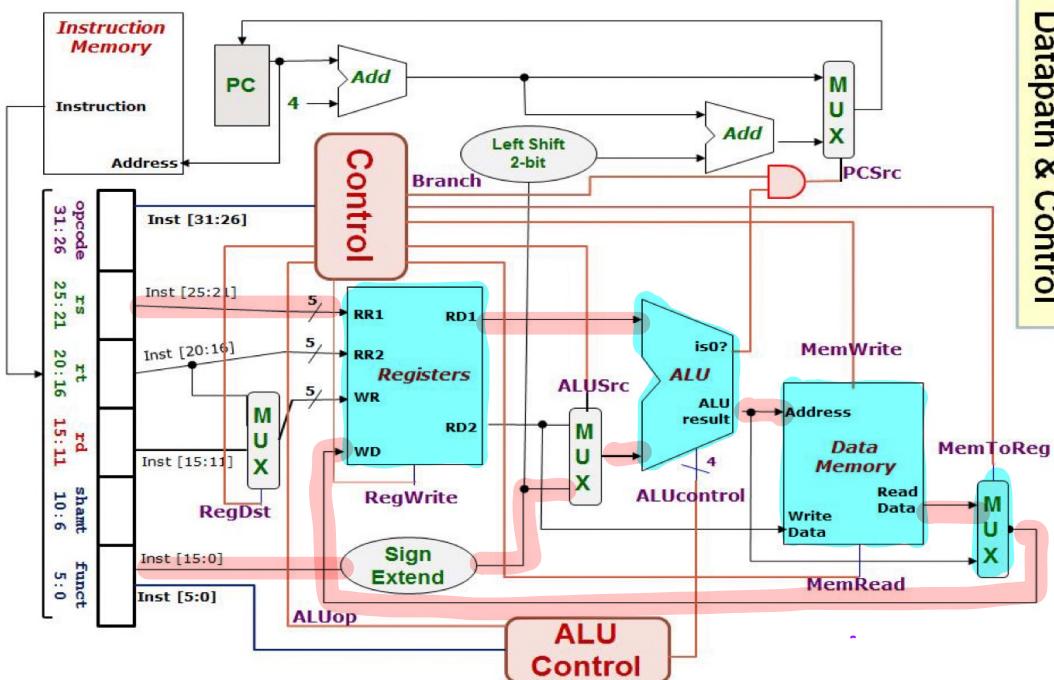
$$\textcircled{1} \text{ critical path} = \text{register} + \text{instruction } f + \text{mux} + \text{ALU} + \text{AND} + \text{mux } (\text{PC succ})$$



or  
signature  
+ shift  
+ add  
+ mix (PCsrc)

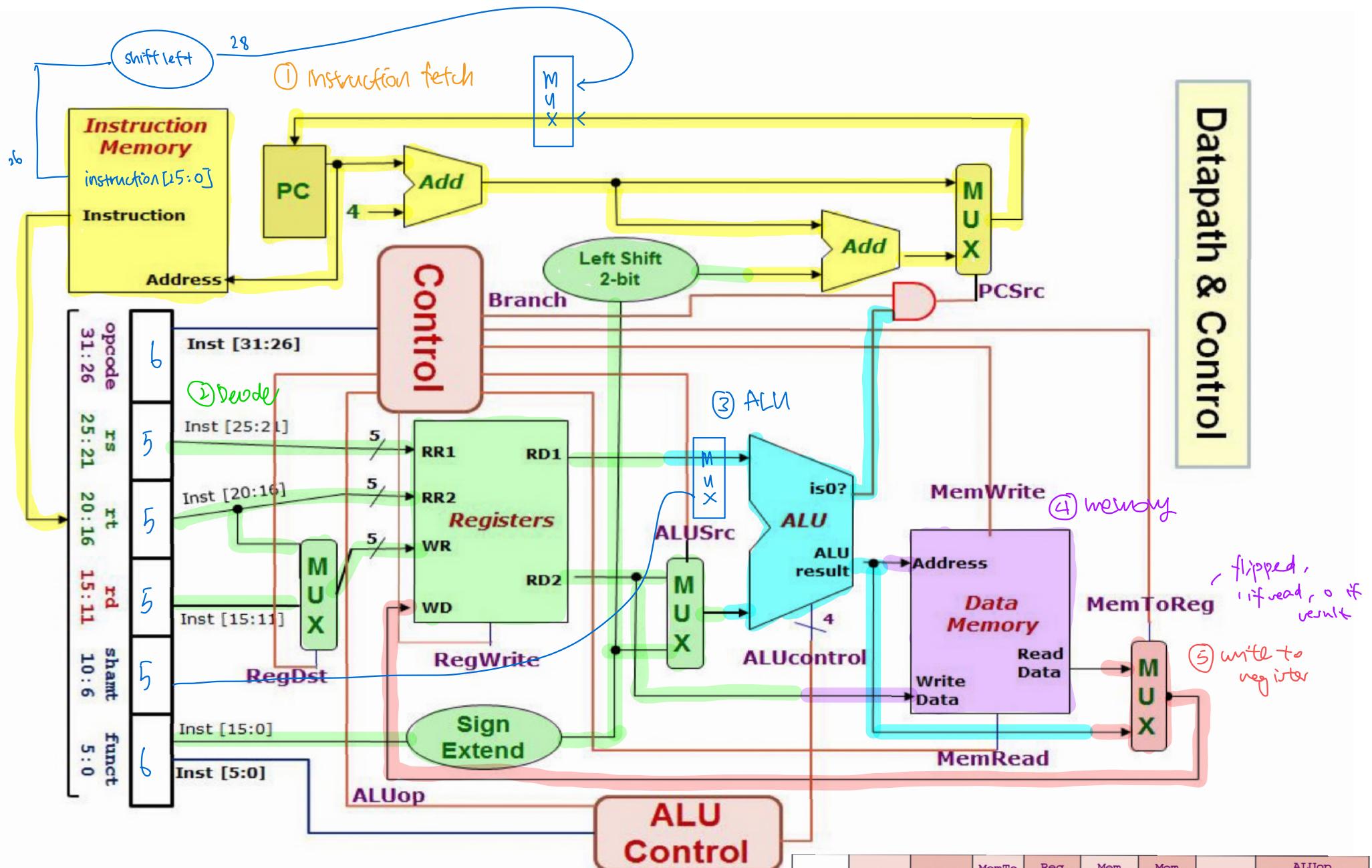
|        | RegDst | ALUSrc | MemTo Reg | Reg Write | Mem Read | Mem Write | Branch | ALUop |     |
|--------|--------|--------|-----------|-----------|----------|-----------|--------|-------|-----|
|        |        |        |           |           |          |           |        | op1   | op0 |
| R-type | 1      | 0      | 0         | 1         | 0        | 0         | 0      | 1     | 0   |
| lw     | 0      | 1      | 1         | 1         | 1        | 0         | 0      | 0     | 0   |
| sw     | X      | 1      | X         | 0         | 0        | 1         | 0      | 0     | 0   |
| beq    | X      | 0      | X         | 0         | 0        | 0         | 1      | 0     | 1   |

(load/store) lw / sw rt imm(rs)



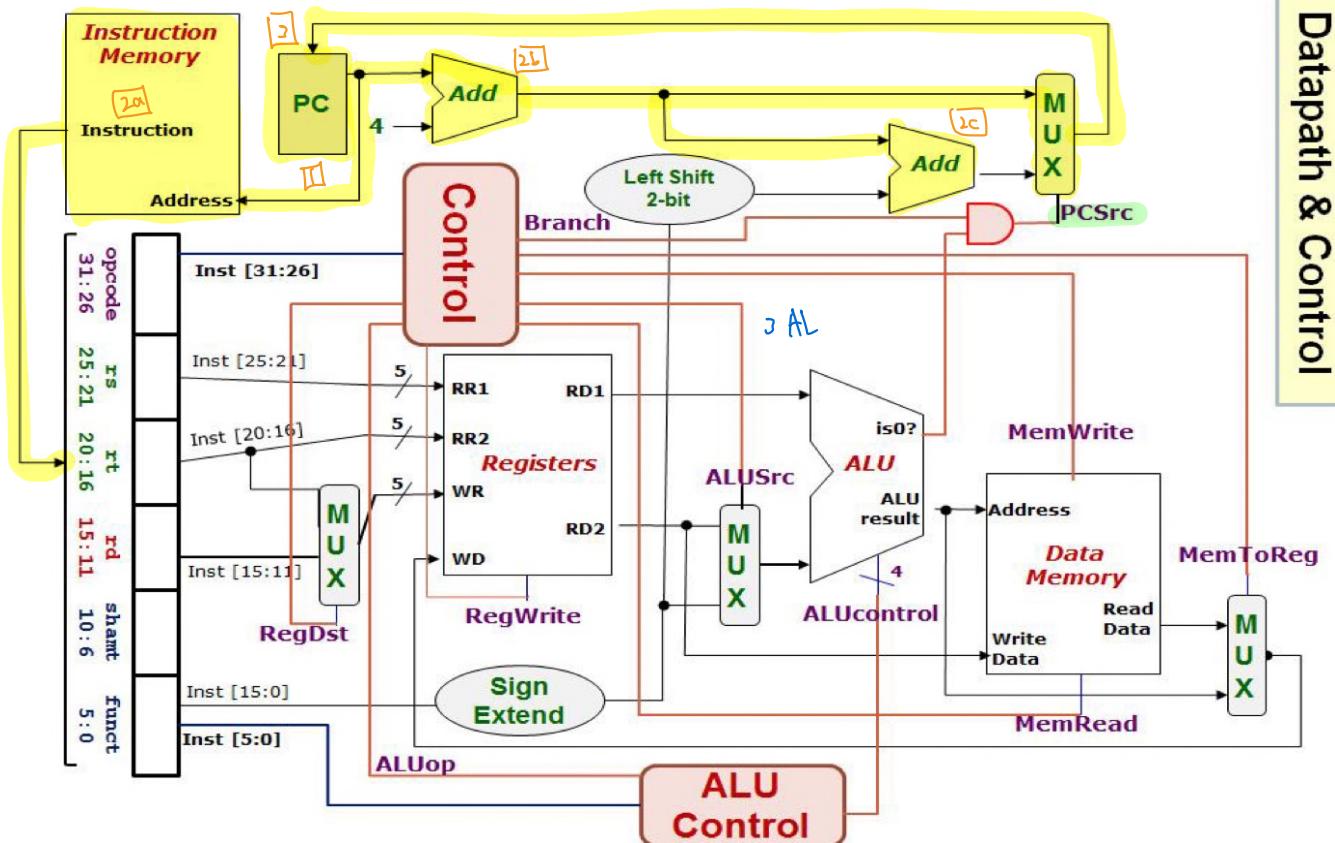
paths = instruction  
+ register (or) signed word  
+ ALU  
+ memory

#### 4. Datapath



|        | RegDst | ALUSrc | MemToReg | RegWrite | MemRead | MemWrite | Branch | ALUop |
|--------|--------|--------|----------|----------|---------|----------|--------|-------|
|        | op1    | op0    |          |          |         |          |        |       |
| R-type | 1      | 0      | 0        | 1        | 0       | 0        | 0      | 1 0   |
| lw     | 0      | 1      | 1        | 1        | 1       | 0        | 0      | 0 0   |
| sw     | X      | 1      | X        | 0        | 0       | 1        | 0      | 0 0   |
| beq    | X      | 0      | X        | 0        | 0       | 0        | 1      | 0 1   |

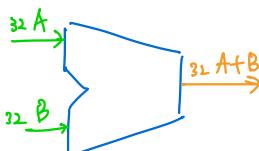
## ① Instruction Fetch Stage



## Components

### 1) Adder

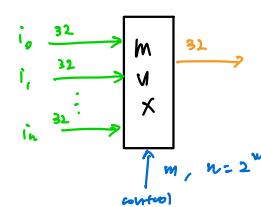
↳ combinational logic circuit to implement addition of two numbers



### 2) Multiplexer

↳ circuit that selects one input from multiple based on m bit control signal

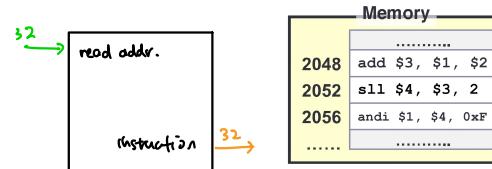
↳ select *i*th line if control signal = *i*



### 3) Instruction memory

↳ sequential circuit that has internal states to store information

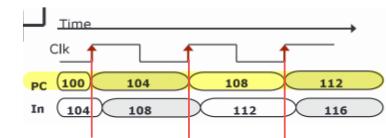
↳ given instruction address as input, outputs content(instruction) at address



## ① Instruction Fetch

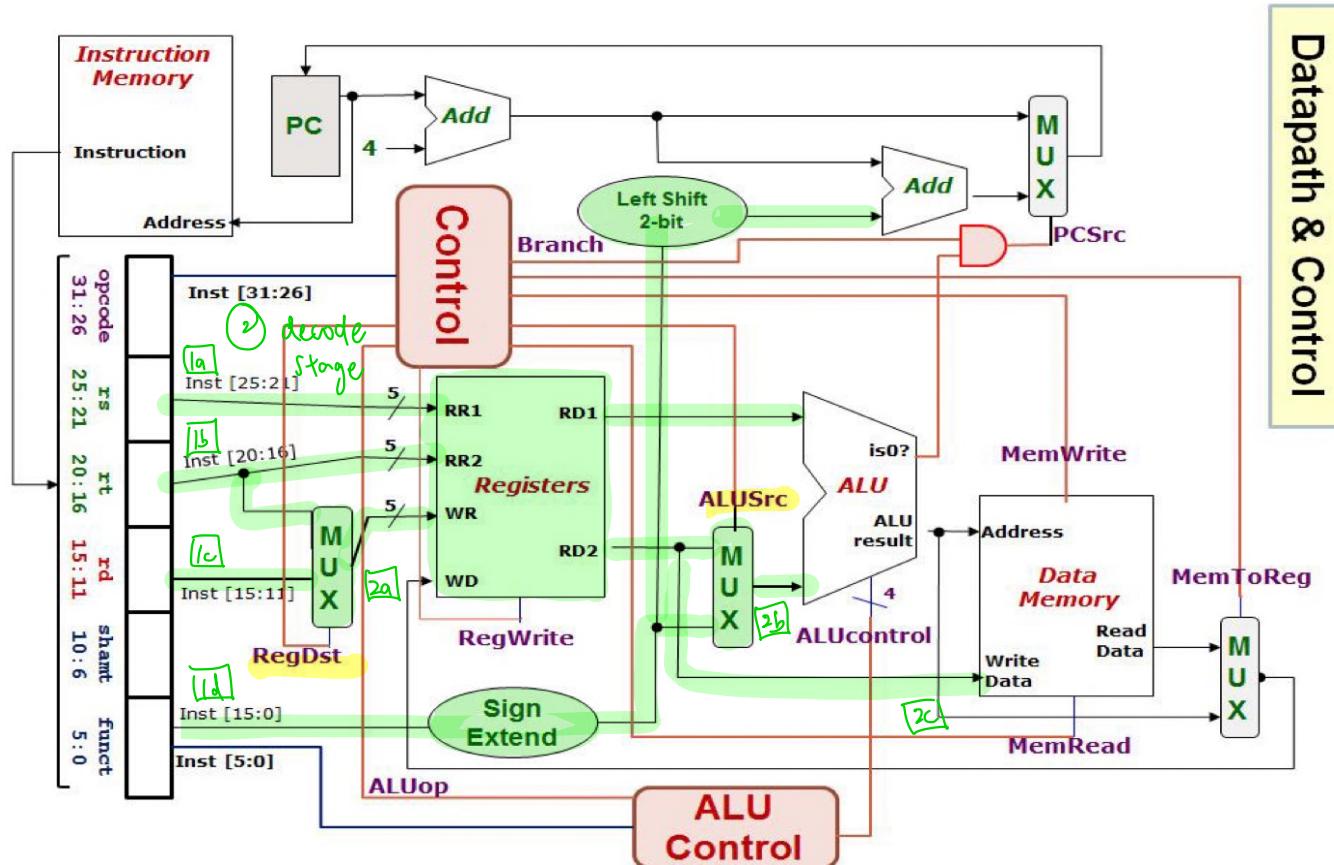
### process

- 1) During first half of clock period **PC** is read, sent to memory
- 2) Instruction is loaded from instruction memory and sent to decode stage
- 2b) adder adds **PC** value, **PC+4** is also added to sign extended 16 bit immediate
- 2c) multiplexer chooses which (**PC+4** or **PC+4 + immediate**) to return depending on **PC/Src**
- 3) **PC** is updated with new value at the next rising edge



### 4) PC (program counter)

↳ special register that stores address of current instruction being executed



## ② Decoder stage

### (process)

- value in rs is loaded to RR1 to RD1 and output
- value in rt is loaded to RR2, MUX and RD2 and output
- value in rd is loaded to MUX1
- 16 bit immediate is sign extended and loaded to MUX2, as well as left shifted 2 bits and loaded to adder at fetch stage

2a. MUX1 decides between rd and rt to load into WR using RegDst signal.  $0 \Rightarrow rt, 1 \Rightarrow rd$

2b. MUX2 decides between RD2 and immediate value to send as output using ALUsrc signal.  
 ↳  $0 \Rightarrow RD2, 1 \Rightarrow \text{immediate}$

|        |                             |
|--------|-----------------------------|
| R mode | $[op   rs   rt   rd   shf]$ |
| I mode | $[op   rs   rt   imm]$      |

op  
read

2c. RD2 signal is posted to Data memory, for writing to memory if needed.

### (Components)

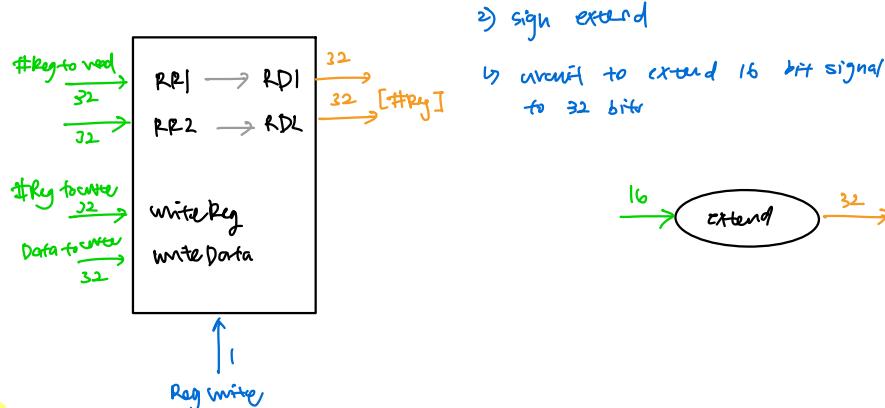
#### 1) register file

↳ a collection of 32 registers

↳ each register is 32 bits wide, can be read / written by specifying reg. number

↳ read at most two registers per instruction, write at most one register per instruction

↳ RegWrite is a control signal to indicate writing of register : (True, 0 False)



#### 2) sign extend

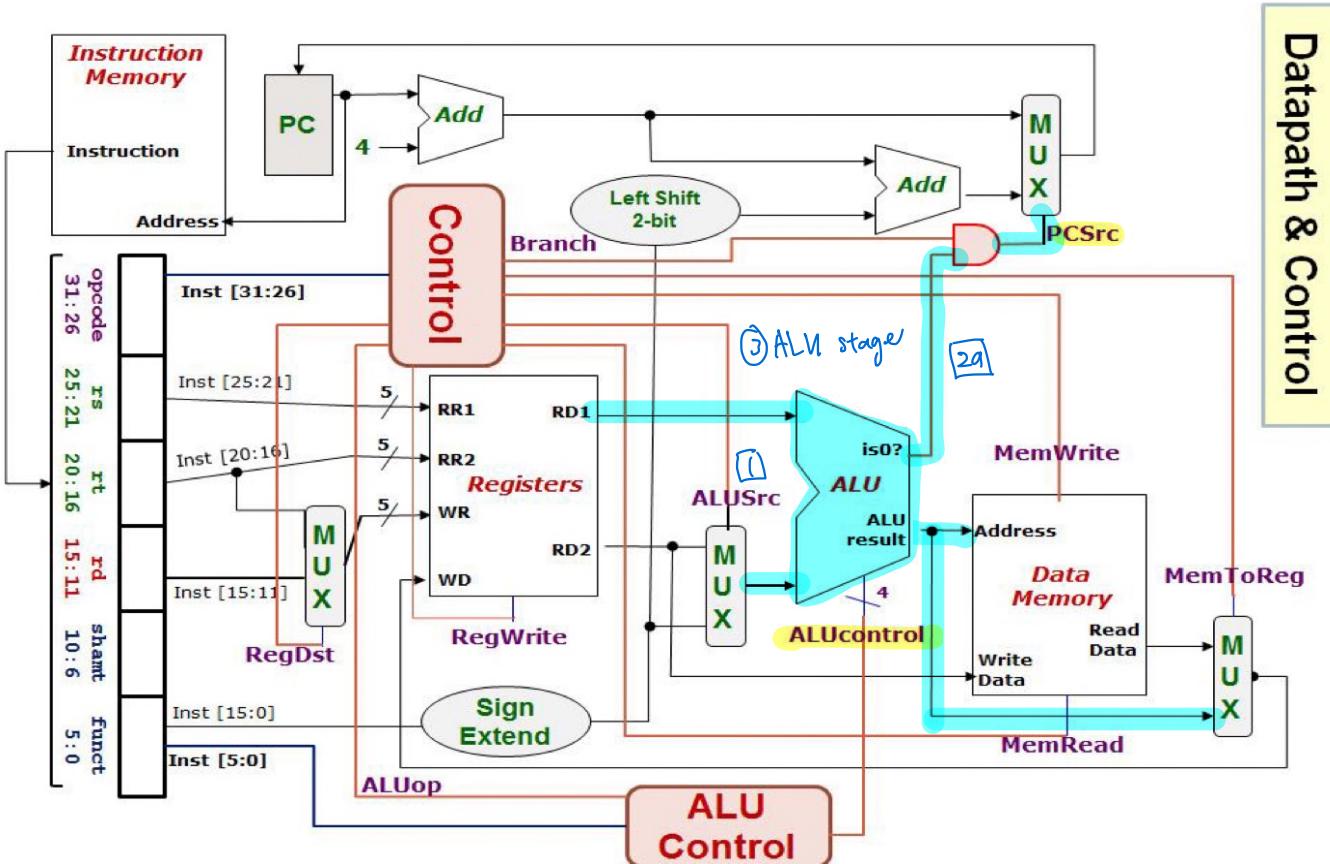
↳ circuit to extend 16 bit signal to 32 bits



#### 3) 2-bit left shift

↳ circuit to shift 32 bit signal





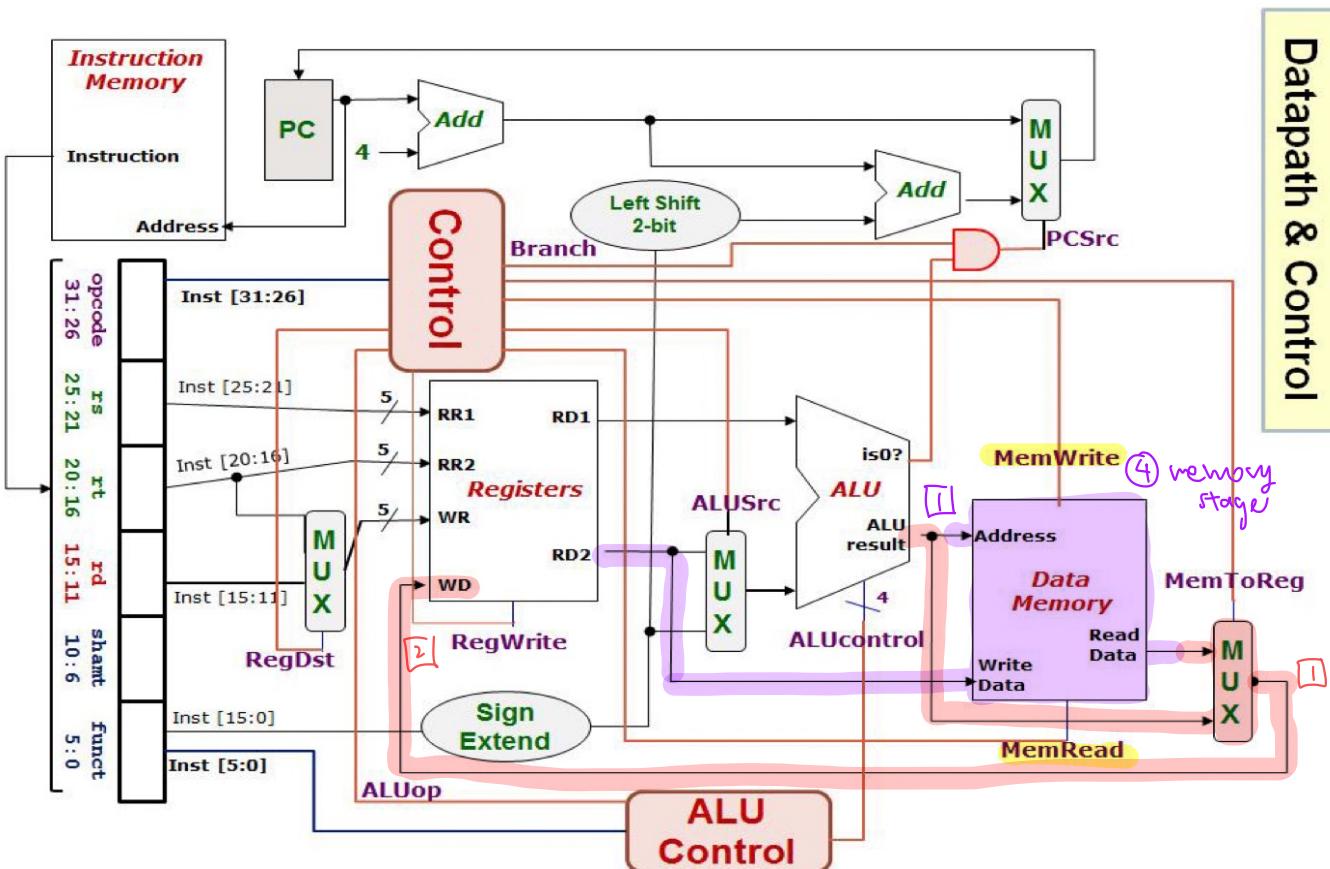
### ③ ALU computation

(process)

1. operands are read into ALU
- 2a. ALU does  $op1 - op2$  to test if  $op1 == op2$ . signal is sent to AND gate to form control signal **PCSrc** in control output, to decide if next instruction is  $PC + 4$  or  $PC + 4 + \text{immediate } \times 4$
- 2b. ALU does  $op1 + op2$ , sends it to memory as well as MUX for later decision of whether to send read data from memory or result directly from ALU.

(components)

- ↳ Arithmetic/Logic Unit
- ↳ combinational circuit to implement arithmetic and logical operations
- ↳ uses 4 bit **ALUcontrol** control signal to indicate operation
- ↳ outputs 32 bit result and 1 bit  $op1 == op2$  result



(4) memory stage

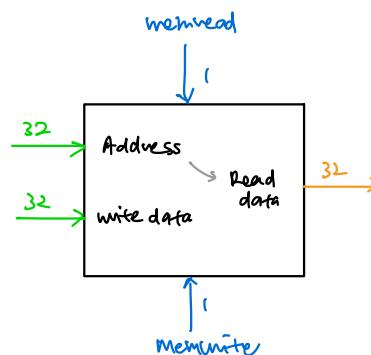
1. output from ALU is read as an address and output to RD to mux.
2. if memwrite = 1, then output from RD2 is written to memory at address from ALU.

(5) write stage

1. mux routes correct result to register file depending on MemToReg signal
2. written in register WR determined at decode stage

(Components)

- a) Data memory
- b) storage element for data
- c) takes in memory address from ALU and data to be written from RD2
- d) depending on memwrite / memread signals, will write to memory or output to read data. only 1 can be asserted at a time. so signal is default / 00000...)



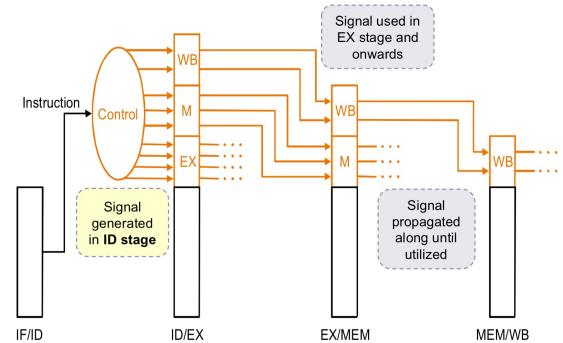
## 13 - Pipelining

### ① Stages and datapath (write implementation)

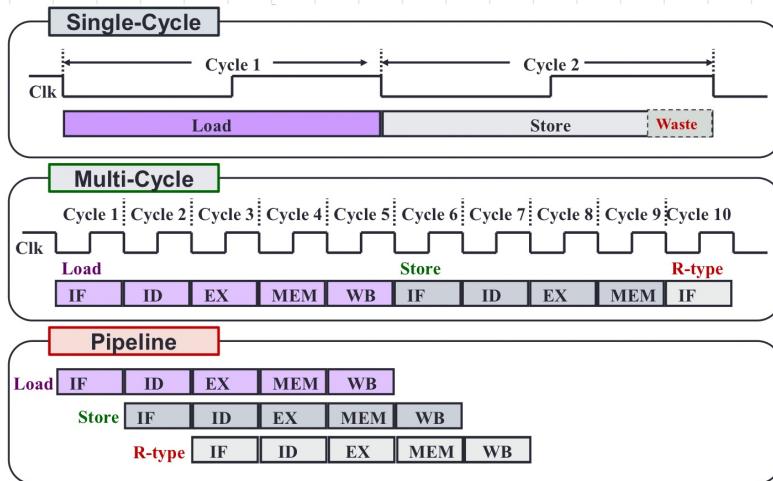
i) (intuition): to keep instructions separate, we use special 'pipeline registers' to store and pass forward relevant information to the next execution stage

ii) control: pass on relevant signals through pipeline registers

|        | EX Stage |        |       |     | MEM Stage |           |        | WB Stage  |           |
|--------|----------|--------|-------|-----|-----------|-----------|--------|-----------|-----------|
|        | RegDst   | ALUSrc | ALUop |     | Mem Read  | Mem Write | Branch | MemTo Reg | Reg Write |
|        |          |        | op1   | op0 |           |           |        |           |           |
| R-type | 1        | 0      | 1     | 0   | 0         | 0         | 0      | 0         | 1         |
| lw     | 0        | 1      | 0     | 0   | 1         | 0         | 0      | 1         | 1         |
| sw     | X        | 1      | 0     | 0   | 0         | 1         | 0      | X         | 0         |
| beq    | X        | 0      | 0     | 1   | 0         | 0         | 1      | X         | 0         |



### ③ Performance of processors



(single cycle)

$$CT = \sum T_i$$

$$T = n \text{ instructions} - CT$$

$$\text{cycles} = n \text{ instructions}$$

(multicycle)

$$CT = \max(T_i) \quad T = n \text{ instructions} \cdot \text{average CPI} \cdot CT$$

$$\text{cycles} = \text{variable} \Rightarrow \text{average}$$

(pipelined)

$$CT = \max(T_k) + \text{Overhead} \quad T = \text{cycles} \cdot CT$$

$$\text{cycles} = n \text{ instructions} + \underbrace{N-1}_{\text{stages wasted}} \text{ to fill up pipeline}$$

$$\text{Speedup}_{\text{pipeline}} = \frac{\text{Time}_{\text{seq}}}{\text{Time}_{\text{pipeline}}}$$

$$= \frac{I \times \sum_{k=1}^N T_k}{(I+N-1) \times (\max(T_k) + T_d)}$$

$$= \frac{I \times N \times T_k}{(I+N-1) \times T_k}$$

$$\approx \frac{I \times N \times T_k}{I \times T_k}$$

$$\approx N$$

Conclusion:

Pipeline processor can gain N times speedup, where N is the number of pipeline stages

### Delayed branch

#### Observation:

- Branch outcome takes X number of cycles to be known
- X cycles stall  $\Rightarrow$  no. of 'slots' to put after branch

#### Idea:

- Move non-control dependent instructions into the X slots following a branch
  - Known as the **branch-delay slot**
- These instructions are executed **regardless of the branch outcome**

↳ either better off or no difference

eg.

#### Non-delayed branch

```
or $8, $9, $10
add $1, $2, $3
sub $4, $5, $6
beq $1, $4, Exit
xor $10, $1, $11
```

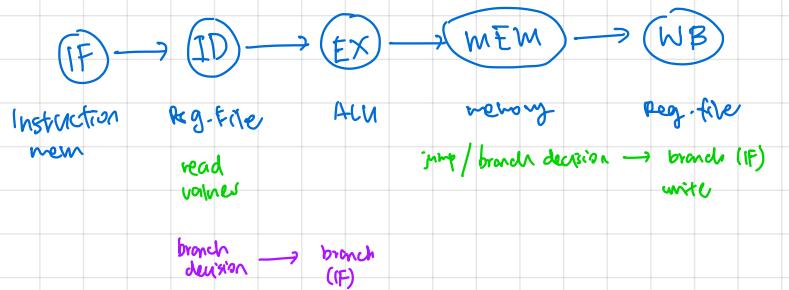
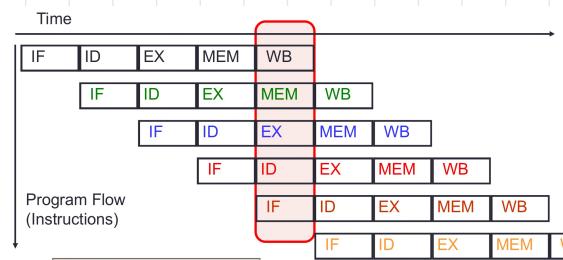
- The "or" instruction is moved into the delayed slot:

- Get executed regardless of the branch outcome
- Same behavior as the original code!

#### Delayed branch

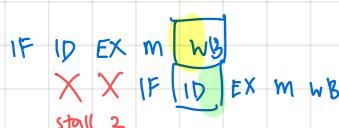
```
add $1, $2, $3
sub $4, $5, $6
beq $1, $4, Exit
or $8, $9, $10
xor $10, $1, $11
```

#### ④ Hazards



1. spot RAW dependencies (affects next two instructions)

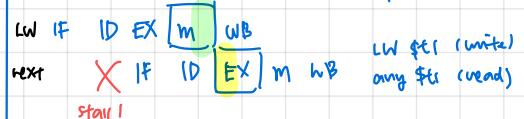
(no forwarding) writer in first half, read in second



(normal forwarding) ALU out → ALU in for next cycle



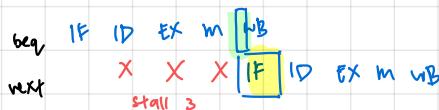
(LW forwarding) mEm out ⇒ ALU in for next cycle



2. spot jumps and branches - Based on conditions and RAW dependencies, stall accordingly

#### ③ Branching

Decision in stage 3, next PCaddr in stage 4, so branch in stage 5. Addr supplied by pipeline register.



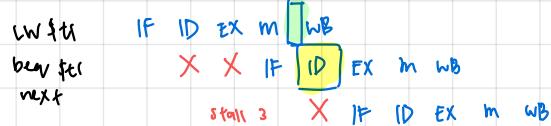
RAW on pre-beq same as non-branching.

#### early branching w/o forwarding

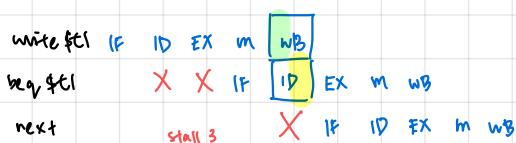
① (normal) Decision & next PCaddr made in stage 2 so IF in stage 3, supplied by pipeline reg.



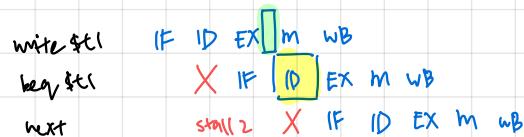
③ (LW RAW) Dependent value retrieved in stage 4. Then forward from pipeline register to ID of next cycle. Stall 2. Branch decision and next PC calculated in stage 2. stall 1. Branch in stage 3.



③ (RAW/LW RAW) result written in stage 5. read in second half. next PC calculated in stage 2. branch in 3.



② (RAW) Dependent value calculated in stage 3. Forward from pipeline reg. to next ID. stall 1. then stall 1 for ID stage to make branch decision & calculate new PC. branch in stage 3.



#### Branch prediction

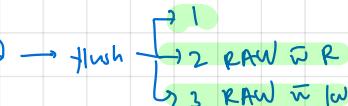
Simple prediction:

- All branches are assumed to be **not taken**
- Fetch the successor instruction and start pumping it through the pipeline stages

When the actual branch outcome is known:

- Not taken:** Guessed correctly → No pipeline stall
- Taken:** Guessed wrongly → Wrong instructions in the pipeline → **Flush** successor instruction from the pipeline

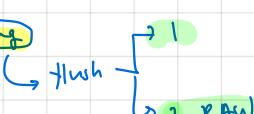
if (early branching) → (with forwarding)



(Info forwarding)

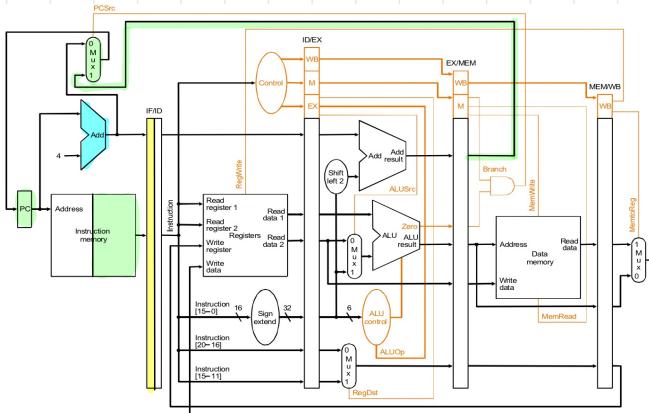
(Normal)

flush 3.



## (Naive implementation)

### 1. Instruction fetch



#### (First half)

MUX chooses next instruction, passes to IM.

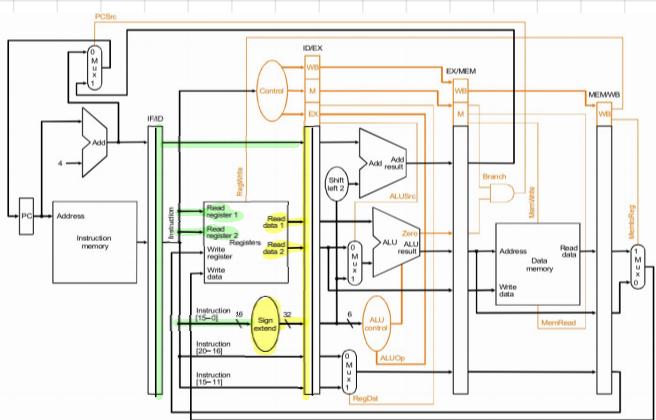
Adder calculates PC+4

#### (Second half)

IF/ID register receives : instruction read

PC+4

### 2. Instruction decode



#### (First half)

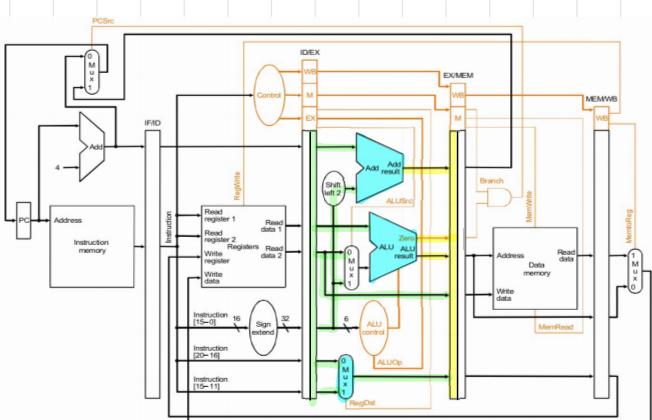
IF/ID supplies register numbers / 16 bit immediate

#### (Second half)

data values read from register file }  
sign extended immediate }  
PC + 4 pass found }  
WR pass forward }

ID/EX receiver

### 3. Execution



#### (First half)

ID/EX supplies : data values read from register file → ALU  
sign extended immediate }  
PC + 4 pass found }  
WR pass forward }

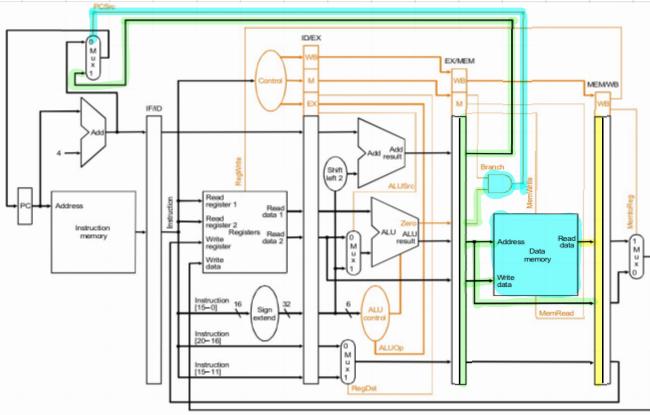
PC+4 + imm  
computed

WR or RT (for SW) chosen by MUX

#### (Second half)

EX/ME M receives : PC + 4 + 4 x imm  
ALU result  
is Zero? signal  
Data read 2 from RF

#### 4. Mem stage



(first half)

EX/MEM supplies : Put 4 + 4ximm for next inst.

ALU result for adder.

RD2 writing into mem

ALU result is zero?

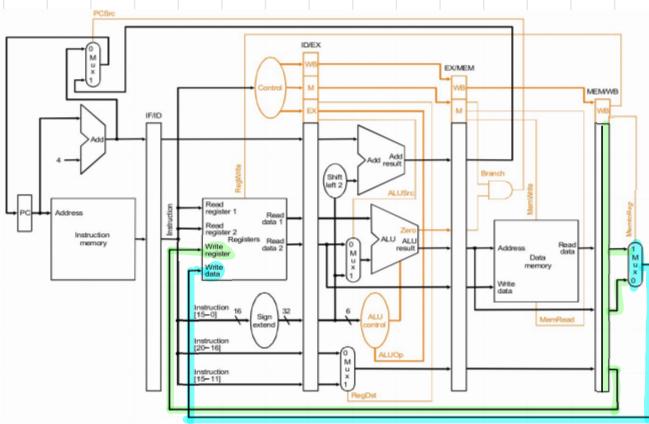
memory reads or writes data - rather than in stage 3, to reduce is zero signal used by AND to decide branch

(second half)

MEM/WB receives : data from memory

ALU result

#### 5. WB stage



(first half)

MEM/WB Supplies : ALU result & data from mem

WR register

MUX chooses and writes to RF

(second half)

nothing (RF used by stage 2 to read)

## 14- Caching

### ① Hits & misses

**Hit:** Data is in cache (e.g., X)

- **Hit rate:** Fraction of memory accesses that hit
- **Hit time:** Time to access cache

**Miss:** Data is not in cache (e.g., Y)

- **Miss rate** =  $1 - \text{Hit rate}$
- **Miss penalty:** Time to replace cache block + hit time

#### Average Access Time

$$= \text{Hit rate} \times \text{Hit Time} + (1 - \text{Hit rate}) \times \text{Miss penalty}$$

### Compulsory misses

- On the first access to a block; the block must be brought into the cache *e.g. first call*
- Also called **cold start misses** or **first reference misses**

ALL

### Conflict misses

- Occur in the case of **direct mapped cache** or **set associative cache**, when several blocks are mapped to the same block/set *e.g. something like this*
- Also called **collision misses** or **interference misses**

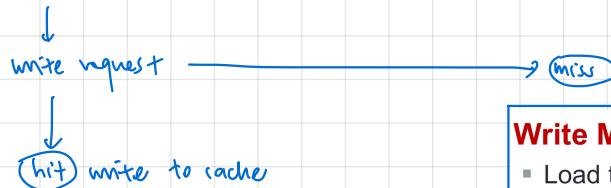
SA direct

### Capacity misses

- Occur when blocks are discarded from cache as cache cannot contain all blocks needed  
*i.e. was once there, but removed due to other call*  $\Rightarrow$  **cache size dependent**  
size ↑, miss ↓

ALL

### ② Write policy (sw instructions)



Cache and main memory are inconsistent

- Modified data only in cache, not in memory!

#### Solution 1: Write-through cache

- Write data both to cache and to main memory

#### Solution 2: Write-back cache

- Only write to cache
- Write to main memory only when cache block is replaced (evicted)

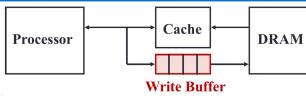
### Write Miss option 1: Write allocate

- Load the complete block into cache
- Change only the required word in cache
- Write to main memory depends on write policy

### Write Miss option 2: Write around

- Do not load the block to cache
- Write directly to **main memory only**

#### write through cache (both)



#### Problem:

- Write will operate at the speed of main memory!

#### Solution:

- Put a write buffer between cache and main memory
  - Processor: writes data to cache + write buffer
  - Memory controller: write contents of the buffer to memory (kinda like async)

#### write back cache (cache, memory @ evictions)

#### Problem:

- Quite wasteful if we write back every evicted cache blocks

#### Solution:

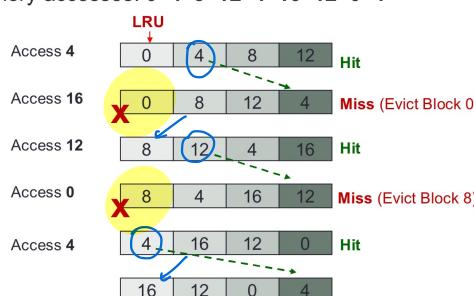
- Add an additional bit (**Dirty bit**) to each cache block
- Write operation will change dirty bit to 1
  - Only cache block is updated, no write to memory
- When a cache block is replaced:
  - Only write back to memory if dirty bit is 1  $\Rightarrow$  bit reset

*↳ so writing from cache to memory only when necessary*

### ③ Block replacement policy (SA or FA caches)

Least Recently Used policy in action:

- 4-way SA cache
- Memory accesses: 0 4 8 12 4 16 12 0 4



1. write down order of memory accesses
2. write out cache access horizontally

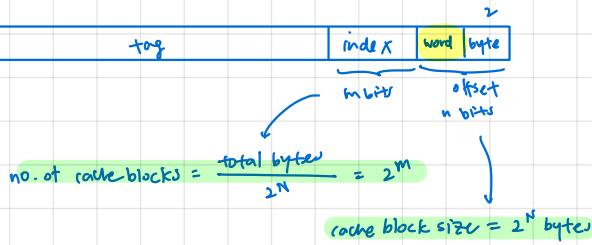
words 0-3

|         |       |         |     |
|---------|-------|---------|-----|
| index 0 | I 0-3 | I 8-11  | ... |
| Index 1 | I 4-7 | I 12-15 | ... |

## ④ Mapping policy & Cache types

### Direct mapped cache

(how it works) one row, one set.

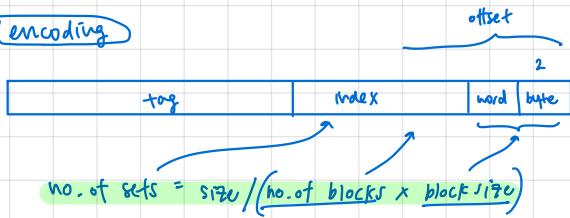


### Set Associative Cache

(how it works)

n-way : cache made up of distinct rows of sets , each set able to hold n blocks  
 $\Rightarrow$  reduce conflict misses

### encoding



1. calculate encoding: tag, index , block , word , byte .

2. create table

|         | word 0 | word 1 | word 2 | word 3 |
|---------|--------|--------|--------|--------|
| index 0 |        |        |        |        |
| index 1 |        |        |        |        |
| :       |        |        |        |        |

3 . look at word/byte bit to see where it will go

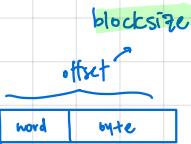
4. Determine hit/miss & collisions

### Fully associative cache

(how it works)

$\hookrightarrow$  no mapping policy. Just added.

### encoding



## 7. Boolean algebra

### ① Boolean values and operators

i) operator precedence NOT(') > AND () > OR (+)

ii) laws & theorems

| Identity laws                                 | Dual form                                   |
|---|---|
| $A + 0 = 0 + A = A$                           | $A \cdot 1 = 1 \cdot A = A$                 |
| Inverse/complement laws                       |   |
| $A + A' = 1$                                  | $A \cdot A' = 0$                            |
| Commutative laws                              |   |
| $A + B = B + A$                               | $A \cdot B = B \cdot A$                     |
| Associative laws *                            |   |
| $A + (B + C) = (A + B) + C$                   | $A \cdot (B \cdot C) = (A \cdot B) \cdot C$ |
| Distributive laws                             |   |
| $A \cdot (B + C) = (A \cdot B) + (A \cdot C)$ | $A + (B \cdot C) = (A + B) \cdot (A + C)$   |

| Idempotency   | Dual form   |
|---|---|
| $X + X = X$   | $X \cdot X = X$                                       |
| One element / Zero element                                  | ,   |
| $X + 1 = 1$   | $X \cdot 0 = 0$                                       |
| Involution  |   |
| $(X')' = X$   |   |
| Absorption 1  |   |
| $X + X \cdot Y = X$   | $X \cdot (X + Y) = X$                                 |
| Absorption 2  |   |
| $X + X' \cdot Y = X + Y$                                    | $X \cdot (X' + Y) = X \cdot Y$                        |
| DeMorgans' (can be generalised to more than 2 variables)    |   |
| $(X + Y)' = X' \cdot Y'$                                    | $(X \cdot Y)' = X' + Y'$                              |
| Consensus   |   |
| $X \cdot Y + X \cdot Z + Y \cdot Z = X \cdot Y + X \cdot Z$ | $(X+Y) \cdot (X'+Z) \cdot (Y+Z) = (X+Y) \cdot (X'+Z)$ |

3) duality: if the AND/OR operators & 1/0 in a boolean equation are swapped, the equation remains valid.  
 ↳ logically equivalent. ⇒ useful in proofs or simplification

### ② Standard forms

i) building blocks

**literal** single boolean variable  
 $\rightarrow x'$

**product term** logical product of literals  
 or a single literal

$x \cdot y \cdot z \cdot \dots$

**sum term** logical sum of literals  
 or a single literal

$\pi, x + y + z$

ii) combinations

**sum of products** a single product term or a logical sum of several

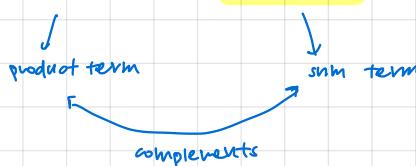
$$x \quad x + y \quad x + y + z \quad y \cdot z + x \cdot y^{'}$$

**product of sum** a single sum term or logical product of several sum terms

$$\pi \quad x \cdot (y + z) \quad (x + y) \cdot (\pi + z)$$

### ③ Canonical standard forms

i) minterms & maxterms of n variables → contain n literals from all the variables



| x | y | Minterms      |          | Maxterms  |          |
|---|---|---------------|----------|-----------|----------|
|   |   | Term          | Notation | Term      | Notation |
| 0 | 0 | $x \cdot y'$  | $m_0$    | $x + y$   | $M_0$    |
| 0 | 1 | $x \cdot y$   | $m_1$    | $x + y'$  | $M_1$    |
| 1 | 0 | $x' \cdot y'$ | $m_2$    | $x' + y$  | $M_2$    |
| 1 | 1 | $x' \cdot y$  | $m_3$    | $x' + y'$ | $M_3$    |

notice its inversion when 0,0

ii) canonical SOP and POS

↳ SOP of minterms / POS of max terms



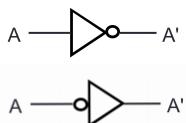
iii) conversion between SOP and POS ⇒ complement

$$\Sigma(1, 4, 5, 6, 7) = \Pi M(0, 2, 3)$$

## 8. Logic circuits

### ① Logic gates and truth tables

Not



|   |    |
|---|----|
| A | A' |
| 0 | 1  |
| 1 | 0  |

AND



| A | B | $A \cdot B$ |
|---|---|-------------|
| 0 | 0 | 0           |
| 0 | 1 | 0           |
| 1 | 0 | 0           |
| 1 | 1 | 1           |

OR



| A | B | $A + B$ |
|---|---|---------|
| 0 | 0 | 0       |
| 0 | 1 | 1       |
| 1 | 0 | 1       |
| 1 | 1 | 1       |

XOR



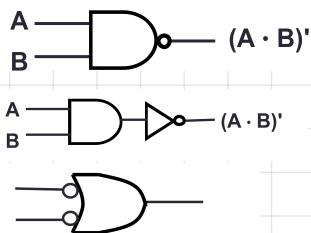
| A | B | $A \oplus B$ |
|---|---|--------------|
| 0 | 0 | 0            |
| 0 | 1 | 1            |
| 1 | 0 | 1            |
| 1 | 1 | 0            |

XNOR



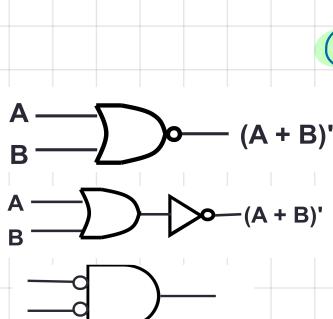
| A | B | $(A \oplus B)'$ |
|---|---|-----------------|
| 0 | 0 | 1               |
| 0 | 1 | 0               |
| 1 | 0 | 0               |
| 1 | 1 | 1               |

NAND



| A | B | $(A \cdot B)'$ |
|---|---|----------------|
| 0 | 0 | 1              |
| 0 | 1 | 1              |
| 1 | 0 | 1              |
| 1 | 1 | 0              |

Negative-OR



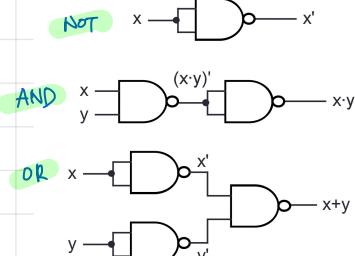
| A | B | $(A + B)'$ |
|---|---|------------|
| 0 | 0 | 1          |
| 0 | 1 | 0          |
| 1 | 0 | 0          |
| 1 | 1 | 0          |

Negative-AND

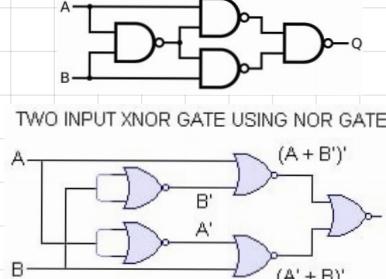
### ② Complete logic sets and universal gates

1) complete sets of logic (AND/OR/NOT) with NAND & NOR, also (AND, NOT)

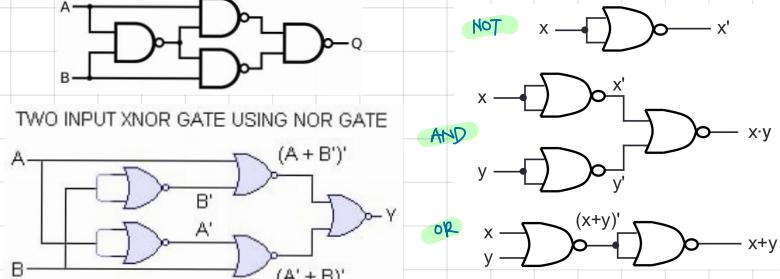
NAND



XOR

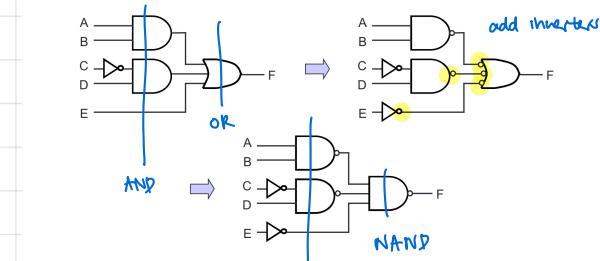


NOR

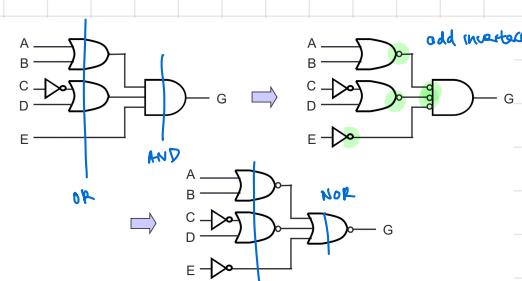


### 2) 2-level circuitry (NOT discounted)

SOP AND/OR | NAND



POS OR/AND | NOR

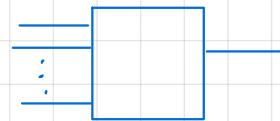


## 10. Combinational circuit

### ① Circuit delays

↳ logic gate  $\bar{t}$  delay  $t$ ; inputs stable at  $t_1, t_2 \dots$

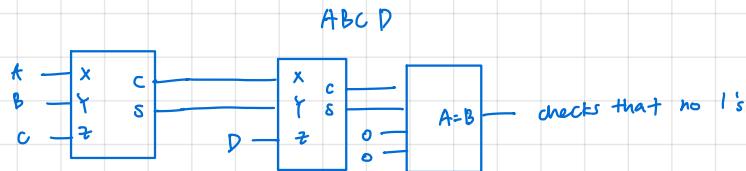
$\Rightarrow$  time at which output is stable is  $\max(t_1, t_2 \dots) + t$



$$\max(t_1 \dots t_i) + t$$

### ② Problem solving: common cases

1. To count the number of 1's: add all positions eg. using full adders + magnitude comp.



2. To multiply / divide  $\Rightarrow$  do right or left shift

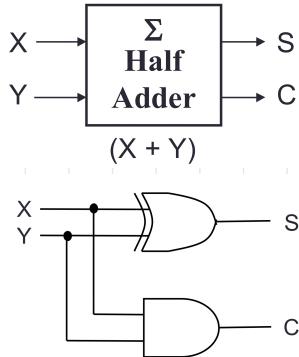
3. When examining truth table, pay attention to adjacent output = 1 terms to identify what matters & what doesn't

4. systematically look at subgroups  $\square \square \square \square$ , commonly first / last n

5. When a lot eg.  $\{m, r, i, b, f \dots\} =$  consider using SOP for  $F'$  instead, then we  $\text{Do}$  or  $\text{NOR}$  gate

$$\begin{array}{r} 000100 \\ 000110 \end{array}$$

(half adder) implements  $X+Y \rightarrow \text{sum} + \text{carry}$

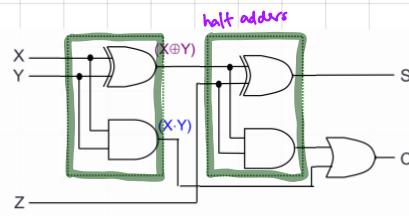


| X | Y | C | S |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

$$C = X \cdot Y$$

$$S = X \oplus Y$$

(full adder) implements  $X+Y+Z \rightarrow \text{sum} + \text{carry}$

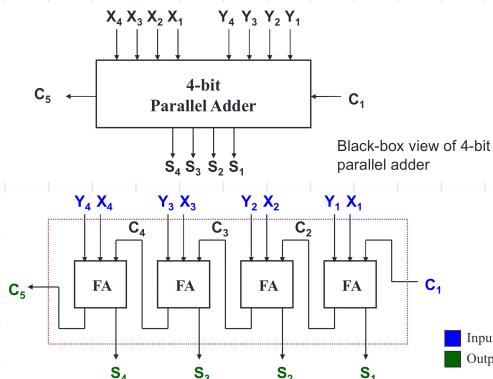


$$C = X \cdot Y + (X \oplus Y) \cdot Z$$

$$S = X \oplus Y \oplus Z$$

| X | Y | Z | C | S |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 |

(parallel adder) 9 bit in, 5 bits out. normally implemented by cascading



(BCD to excess 3 converter)

converts 0 to 9 to excess 3 representation ( $BCD + 3$ )

|    | BCD |   |   |   | Excess-3 |   |   |   |  |
|----|-----|---|---|---|----------|---|---|---|--|
|    | A   | B | C | D | W        | X | Y | Z |  |
| 0  | 0   | 0 | 0 | 0 | 0        | 0 | 1 | 1 |  |
| 1  | 0   | 0 | 0 | 1 | 0        | 1 | 0 | 0 |  |
| 2  | 0   | 0 | 1 | 0 | 0        | 1 | 0 | 1 |  |
| 3  | 0   | 0 | 1 | 1 | 0        | 1 | 1 | 0 |  |
| 4  | 0   | 1 | 0 | 0 | 0        | 1 | 1 | 1 |  |
| 5  | 0   | 1 | 0 | 1 | 1        | 0 | 0 | 0 |  |
| 6  | 0   | 1 | 1 | 0 | 1        | 0 | 0 | 1 |  |
| 7  | 0   | 1 | 1 | 1 | 1        | 0 | 1 | 0 |  |
| 8  | 1   | 0 | 0 | 0 | 0        | 1 | 1 | 1 |  |
| 9  | 1   | 0 | 0 | 1 | 1        | 1 | 0 | 0 |  |
| 10 | 1   | 0 | 1 | 0 | X        | X | X | X |  |
| 11 | 1   | 0 | 1 | 1 | X        | X | X | X |  |
| 12 | 1   | 1 | 0 | 0 | X        | X | X | X |  |
| 13 | 1   | 1 | 0 | 1 | X        | X | X | X |  |
| 14 | 1   | 1 | 1 | 0 | X        | X | X | X |  |
| 15 | 1   | 1 | 1 | 1 | X        | X | X | X |  |

$$W = A + B \cdot C + B \cdot D$$

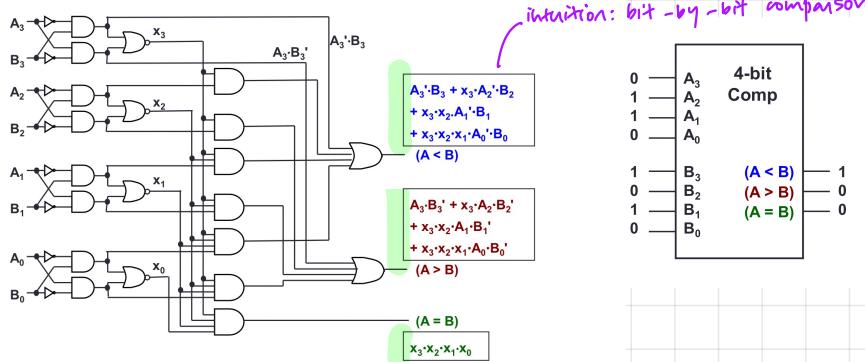
$$X = B' \cdot C + B' \cdot D + B \cdot C \cdot D'$$

$$Y = C \cdot D + C' \cdot D'$$

$$Z = D'$$

(magnitude comparators) compare unsigned values to check if  $A > B$ ,  $A = B$ ,  $A < B$

↳ to consider usage, see if one column always one (ie  $\geq$ ), or equal, etc.

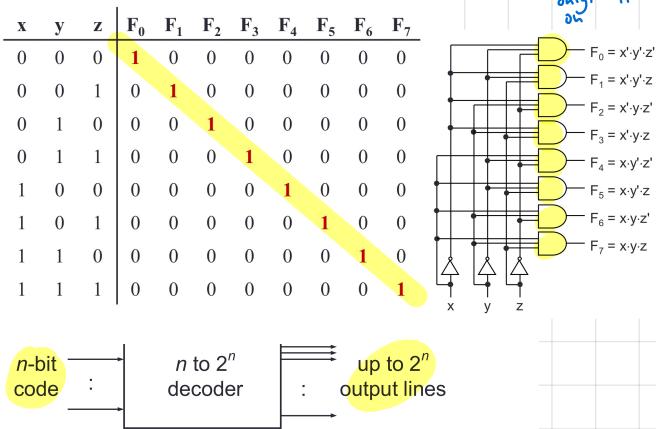


|   |       |            |
|---|-------|------------|
| 0 | $A_3$ | 4-bit Comp |
| 1 | $A_2$ |            |
| 1 | $A_1$ |            |
| 0 | $A_0$ |            |
| 1 | $B_3$ | $(A < B)$  |
| 0 | $B_2$ | $(A > B)$  |
| 1 | $B_1$ | $(A = B)$  |
| 0 | $B_0$ |            |

## ② Decoders

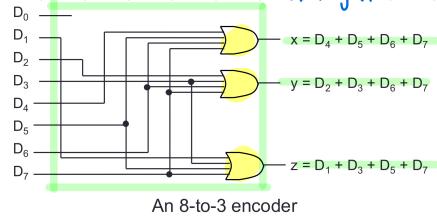
### 11. MSI circuits

(how it works) : selects an output line from  $2^n$  of them using a  $n$ -bit input,  $A \cdot B \cdot C' \Rightarrow$  AND only off



## ③ Encoders

(how it works) compresses  $2^n$  input to  $n$  bits w/ OR gates

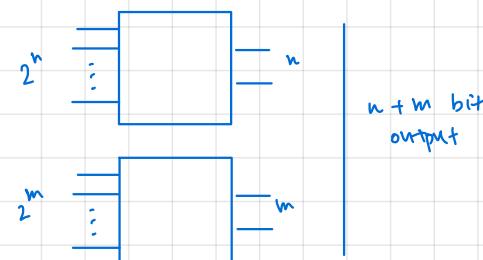


| Inputs         |                |                |                |                |                |                |                | Outputs |   |   |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|---------|---|---|
| D <sub>0</sub> | D <sub>1</sub> | D <sub>2</sub> | D <sub>3</sub> | D <sub>4</sub> | D <sub>5</sub> | D <sub>6</sub> | D <sub>7</sub> | x       | y | z |
| 1              | 0              | 0              | 0              | 0              | 0              | 0              | 0              | 0       | 0 | 0 |
| 0              | 1              | 0              | 0              | 0              | 0              | 0              | 0              | 0       | 0 | 1 |
| 0              | 0              | 1              | 0              | 0              | 0              | 0              | 0              | 0       | 1 | 0 |
| 0              | 0              | 0              | 1              | 0              | 0              | 0              | 0              | 0       | 1 | 1 |
| 0              | 0              | 0              | 0              | 1              | 0              | 0              | 0              | 1       | 0 | 0 |
| 0              | 0              | 0              | 0              | 0              | 1              | 0              | 0              | 1       | 0 | 1 |
| 0              | 0              | 0              | 0              | 0              | 0              | 1              | 0              | 1       | 1 | 0 |
| 0              | 0              | 0              | 0              | 0              | 0              | 0              | 1              | 1       | 1 | 1 |

(composite decoders) if two or more inputs = 1, input is the highest priority takes precedence.

| Inputs         |                |                |                | Outputs |   |   |
|----------------|----------------|----------------|----------------|---------|---|---|
| D <sub>0</sub> | D <sub>1</sub> | D <sub>2</sub> | D <sub>3</sub> | f       | g | v |
| 0              | 0              | 0              | 0              | X       | X | 0 |
| 1              | 0              | 0              | 0              | 0       | 0 | 1 |
| X              | 1              | 0              | 0              | 0       | 1 | 1 |
| X              | X              | 1              | 0              | 1       | 0 | 1 |
| X              | X              | X              | 1              | 1       | 1 | 1 |

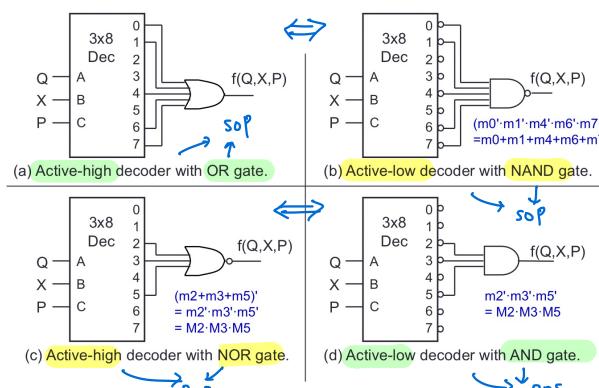
Constructing larger encoders



(implementing functions) boolean function in SOP, we get

(2/2)

$$f(Q, X, P) = \sum m(0, 1, 4, 6, 7) = \prod M(2, 3, 5)$$



#### ④ multiplexers

(how it works)  $2^n$  input lines,  $n$  selection lines,  $m$  (identical) output lines. " $2^n : m$  mux."

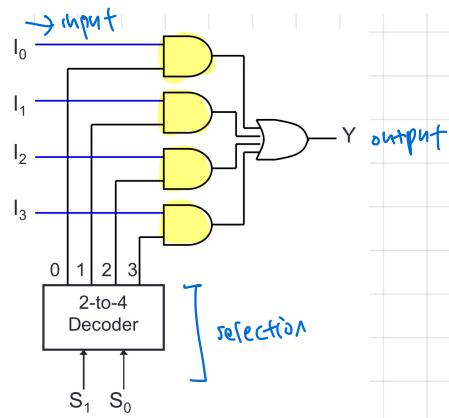
↳ implemented by AND of input & decoder selection, then OR.

Output of multiplexer is

"sum of the (product of data lines and selection lines)"

Example: Output of a 4-to-1 multiplexer is:

$$Y = I_0 \cdot (S_1 \cdot S_0) + I_1 \cdot (S_1 \cdot S_0') + I_2 \cdot (S_1' \cdot S_0) + I_3 \cdot (S_1' \cdot S_0')$$

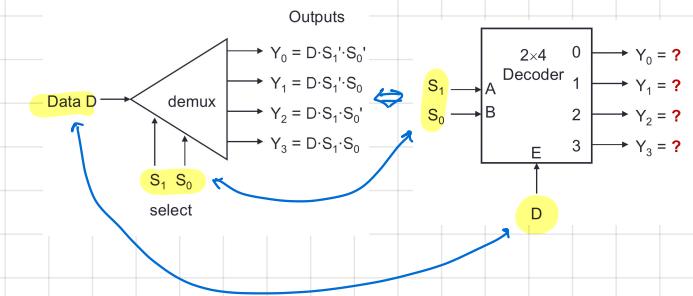


|   |   |
|---|---|
| 0 | 1 |
| 1 | 0 |
| 1 | 1 |

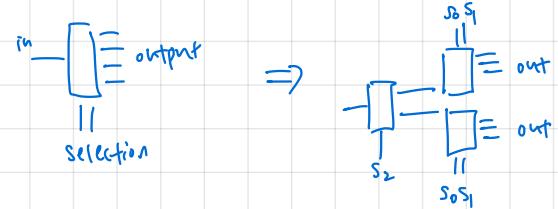
#### ⑤ De multiplexers

(how it works) given data input and  $n$  bit selection line, directs data ( $1 \text{ bit}$ ) to one of  $2^n$  lines.

↳ identical to encoder, where data  $\approx$  enables

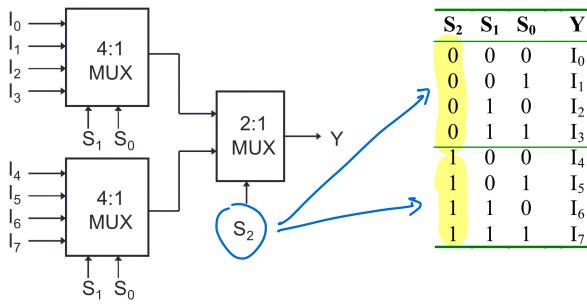


#### constructing comp - demultiplexers



#### constructing larger multiplexers

breaking up bits to choose  
⇒ use msb



#### implementing functions

boolean SOP functions

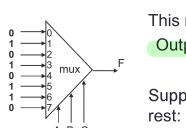
1. Express in sum-of-minterms form.

Example:

$$F(A, B, C) = A' \cdot B' \cdot C + A' \cdot B \cdot C + A \cdot B' \cdot C + A \cdot B \cdot C' \\ = \Sigma m(1, 3, 5, 6)$$

2. Connect  $n$  variables to the  $n$  selection lines.

3. Put a '1' on a data line if it is a minterm of the function, or '0' otherwise.



This method works because:

$$\text{Output} = l_0 \cdot m_0 + l_1 \cdot m_1 + l_2 \cdot m_2 + l_3 \cdot m_3 \\ + l_4 \cdot m_4 + l_5 \cdot m_5 + l_6 \cdot m_6 + l_7 \cdot m_7$$

Supplying '1' to  $l_1, l_3, l_5, l_6$ , and '0' to the rest:

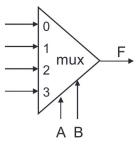
$$\text{Output} = m_1 + m_3 + m_5 + m_6$$

#### using smaller multiplexers

3. Draw the truth table for function, by grouping inputs by selection line values, then determine multiplexer inputs by comparing input line (C) and function (F) for corresponding selection line values.

| A | B | C | F | MUX input |
|---|---|---|---|-----------|
| 0 | 0 | 0 | 1 | 1         |
| 0 | 0 | 1 | 1 | 1         |
| 0 | 1 | 0 | 0 | C         |
| 0 | 1 | 1 | 1 | 0         |
| 1 | 0 | 0 | 0 | 0         |
| 1 | 0 | 1 | 0 | C'        |
| 1 | 1 | 0 | 1 | 0         |
| 1 | 1 | 1 | 0 | C'        |

regardless of C,  
F = 1  
F = 0  
regardless of C,  
F = 0  
F = c'



## 12. sequential circuits

### ① analysis of sequential circuits

↳ sequential circuits : built from logic gates and flip flops

↳ intuition behind sequential circuit: for outputs that are previous-state dependent e.g. reading machine

1. identify and derive boolean expressions for flip flop inputs and system outputs

2. derive state equations for flip flop states

3. derive state table  $\Rightarrow m$  flip flops (states) and  $n$  inputs  $\Rightarrow 2^{mn}$  rows

↳ fill in inputs & states combinations

↳ use flip flop input functions to get inputs e.g.  $KA$ ,  $JA$  ...

↳ use characteristic tables & output functions to get new states & outputs

4. draw state diagrams

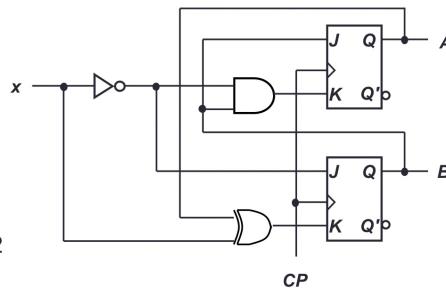


Figure 2

Obtain the **flip-flop input functions** from the circuit:

$$JA = B$$

$$KA = B \cdot x'$$

$$JB = x'$$

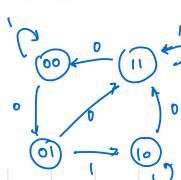
$$KB = A' \cdot x + A \cdot x' = A \oplus x$$

$$\begin{aligned} JA &= B \\ KA &= B \cdot x' \end{aligned}$$

$$\begin{aligned} JB &= x' \\ KB &= A' \cdot x + A \cdot x' = A \oplus x \end{aligned}$$

Fill the **state table** using the above functions, knowing the characteristics of the flip-flops used.

| Present state | Input | Next state |   | Flip-flop inputs |                |    |    |    |    |
|---------------|-------|------------|---|------------------|----------------|----|----|----|----|
|               |       | A          | B | A <sup>+</sup>   | B <sup>+</sup> | JA | KA | JB | KB |
| 0 0           | 0     | 0          | 0 | 0                | 0              | 0  | 0  | 1  | 0  |
| 0 1           | 0     | 0          | 1 | 0                | 1              | 0  | 0  | 0  | 1  |
| 1 0           | 0     | 1          | 0 | 1                | 1              | 1  | 1  | 1  | 0  |
| 1 1           | 0     | 1          | 1 | 0                | 0              | 0  | 0  | 0  | 0  |
| 0 0           | 1     | 1          | 0 | 1                | 0              | 1  | 0  | 0  | 1  |
| 0 1           | 1     | 1          | 1 | 0                | 0              | 1  | 0  | 1  | 1  |
| 1 0           | 1     | 0          | 0 | 1                | 0              | 0  | 0  | 1  | 1  |
| 1 1           | 1     | 0          | 1 | 0                | 1              | 1  | 1  | 0  | 0  |
| 1 1           | 1     | 1          | 1 | 1                | 1              | 1  | 1  | 0  | 0  |



↳ sinks & self correction

if circuit is able to (without inputs) move from an invalid to valid state in a finite no. of steps

state that once a circuit enters, never leaves out

### ② Design of sequential circuits

1. From state diagram identify no. of states (flip-flops), inputs and outputs

2. Fill in state table

↳ fill in combinations of states & inputs

↳ from state diagram fill in next states

↳ using excitation table, fill in flip flop inputs

↳ use k-maps to derive flip flop input expressions, implement

From state table, get flip-flop input functions.

| Present state | Input | Next state |   | Flip-flop inputs |                |    |    |    |    |
|---------------|-------|------------|---|------------------|----------------|----|----|----|----|
|               |       | A          | B | A <sup>+</sup>   | B <sup>+</sup> | JA | KA | JB | KB |
| 0 0           | 0     | 0          | 0 | 0                | 0              | 0  | x  | x  | x  |
| 0 0           | 1     | 0          | 1 | 0                | 1              | 0  | x  | x  | x  |
| 0 1           | 0     | 1          | 0 | 1                | 1              | 1  | x  | x  | x  |
| 0 1           | 1     | 0          | 1 | 0                | 1              | 0  | x  | x  | x  |
| 1 0           | 0     | 1          | 0 | 0                | 0              | 0  | x  | x  | x  |
| 1 0           | 1     | 1          | 1 | 1                | 0              | 1  | x  | x  | x  |
| 1 1           | 0     | 1          | 1 | 1                | 0              | 0  | x  | x  | x  |
| 1 1           | 1     | 0          | 0 | 0                | 1              | 1  | x  | x  | x  |

| Present state | Input | Next state |   | Flip-flop inputs |                |    |    |    |    |
|---------------|-------|------------|---|------------------|----------------|----|----|----|----|
|               |       | A          | B | A <sup>+</sup>   | B <sup>+</sup> | JA | KA | JB | KB |
| 0 0           | 0     | 0          | 0 | 0                | 0              | 0  | x  | x  | x  |
| 0 0           | 1     | 0          | 1 | 0                | 1              | 0  | x  | x  | x  |

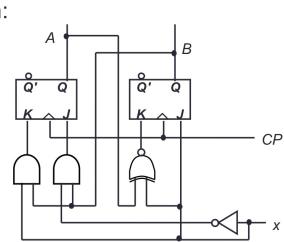
| Present state | Input | Next state |   | Flip-flop inputs |                |    |    |    |    |
|---------------|-------|------------|---|------------------|----------------|----|----|----|----|
|               |       | A          | B | A <sup>+</sup>   | B <sup>+</sup> | JA | KA | JB | KB |
| 0 0           | 0     | 0          | 0 | 0                | 0              | 0  | x  | x  | x  |
| 0 0           | 1     | 0          | 1 | 0                | 1              | 0  | x  | x  | x  |

Flip-flop input functions:

$$\begin{aligned} JA &= B \cdot x' \\ KA &= B \cdot x \end{aligned}$$

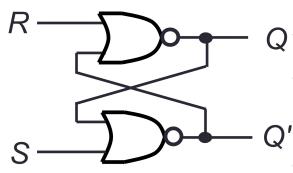
$$\begin{aligned} JB &= x \\ KB &= (A \oplus x)' \end{aligned}$$

Logic diagram:



S-R

ungated SR latch



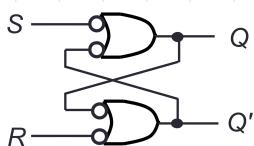
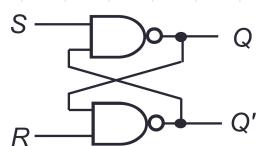
| S | R | $Q(t+1)$ | Comments      |
|---|---|----------|---------------|
| 0 | 0 | $Q(t)$   | No change     |
| 0 | 1 | 0        | Reset         |
| 1 | 0 | 1        | Set           |
| 1 | 1 | ?        | Unpredictable |

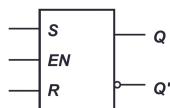
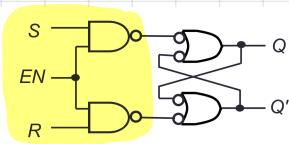
| Q | $Q^+$ | S | R |
|---|-------|---|---|
| 0 | 0     | 0 | X |
| 0 | 1     | 1 | 0 |
| 1 | 0     | 0 | 1 |
| 1 | 1     | X | 0 |

$$Q^t = ?$$

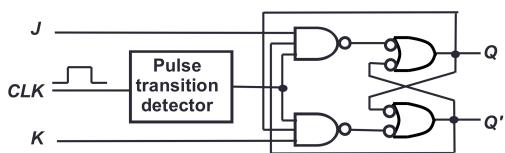
ungated active-low SR latch (inputs are flipped)



gated SR latch / flip flop



(T flip flop) valid version of SR



| J | K | $Q(t+1)$ | Comments  |
|---|---|----------|-----------|
| 0 | 0 | $Q(t)$   | No change |
| 0 | 1 | 0        | Reset     |
| 1 | 0 | 1        | Set       |
| 1 | 1 | $Q(t)'$  | Toggle    |

$$Q^t = J \cdot Q + K' \cdot Q'$$

↳ has no invalid output. Same as SR, but invalid replaced by toggle.

↳ Q and  $Q^t$  are fed back to pulse-triggered NAND gates

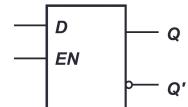
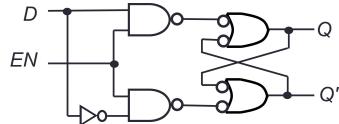
(D) Direct

gated D latch / flip flop

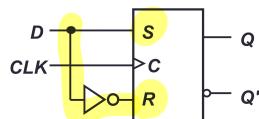
$$Q^+ = D$$

| Q | $Q^+$ | D |
|---|-------|---|
| 0 | 0     | 0 |
| 0 | 1     | 1 |
| 1 | 0     | 0 |
| 1 | 1     | 1 |

| D | $Q(t+1)$ |
|---|----------|
| 0 | 0        |
| 1 | 1        |

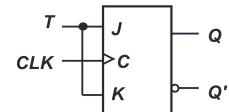
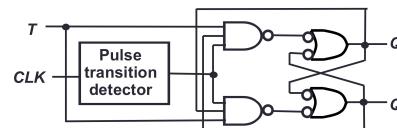


Convert S-R flip-flop into a D flip-flop: add an inverter.



↳ gets rid of undesirable invalid SR states, but loses ability to 'no change'

(T flip flop) Toggle



| Q | $Q^+$ | T |
|---|-------|---|
| 0 | 0     | 0 |
| 0 | 1     | 1 |
| 1 | 0     | 1 |
| 1 | 1     | 0 |

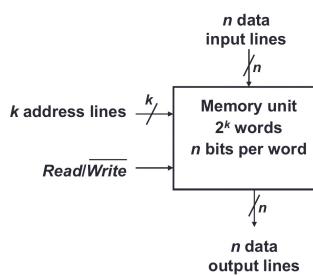
| T | $Q(t+1)$ |
|---|----------|
| 0 | $Q(t)$   |
| 1 | $Q(t)'$  |

↳ toggles. Formed from JK by tying inputs together.

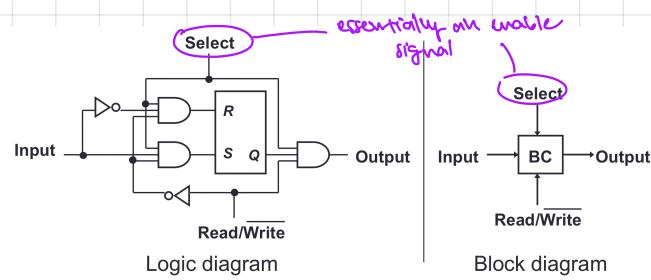
### ① memory

#### 1) the memory unit

| Memory Enable | Read/Write | Memory Operation        |
|---------------|------------|-------------------------|
| 0             | X          | None                    |
| 1             | 0          | Write to selected word  |
| 1             | 1          | Read from selected word |



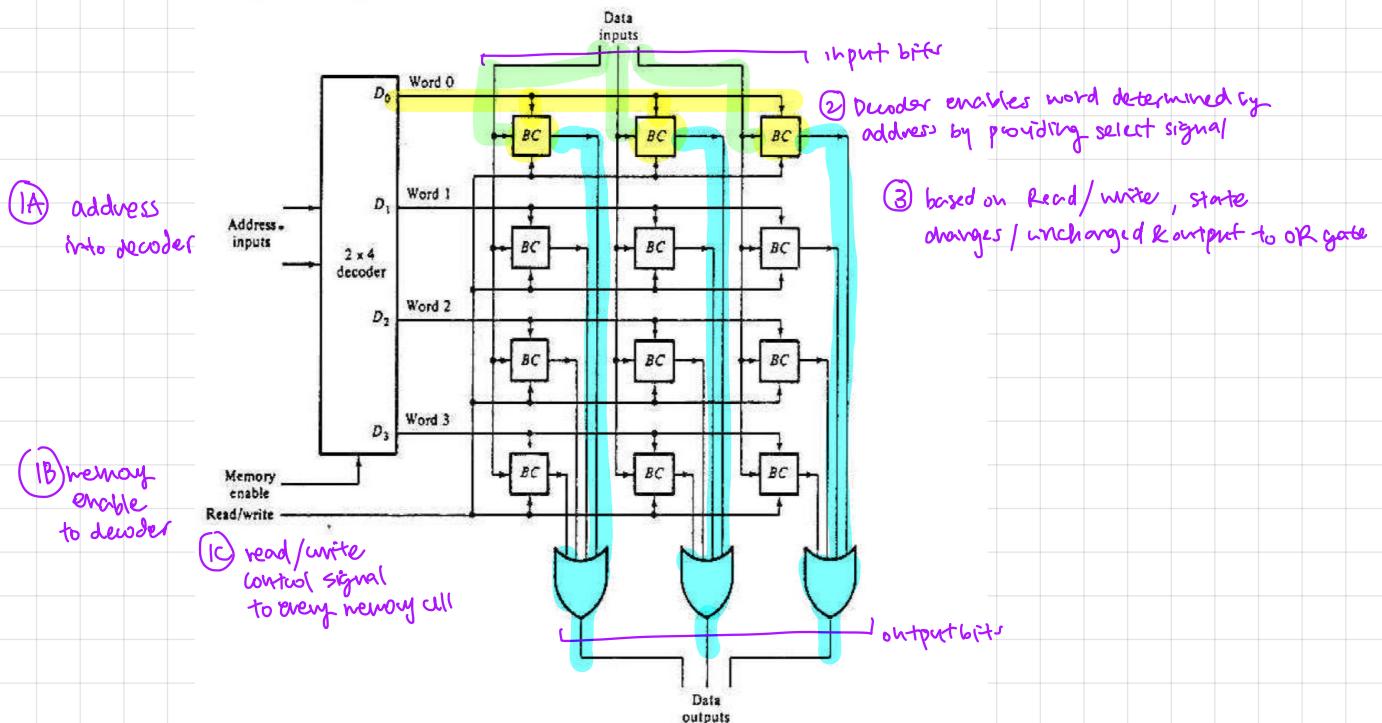
#### 2) memory cells of static RAM: flip-flops are used



added wiring around SR flip-flop is so that behaviour follows memory control signals

read = 1 → unchanged  
write = 0 → Q = input  
select → enable

#### 3) arrays of memory cells to build memory



#### 4) arrays of memory chips to build larger memory

