One period consumer : work leisure tradeoff

① the consumer's role in a single period economy

The economy is made up of consumers, firms and the government.
We start by modeling everything in some unit of real consumption.

Consumers suppose their main goal is to consume and enjoy leisure time.
↳ endowed w/ h hours in the period. they choose between working for a wage or leisure, and spend all earnings by consuming goods

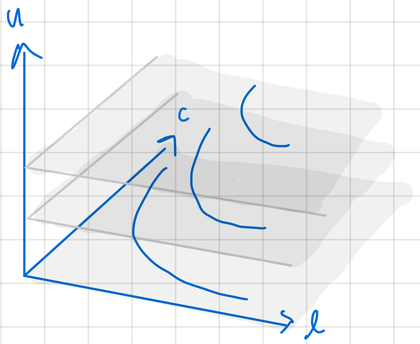
↳ they pay taxes to the government
↳ they can also own firms, and so earn dividends / π of firms they own.

② choosing how much to work given constraints

1) consumption – leisure preference

↳ for a single period, we can model his work-leisure preference with a utility function that is well behaved, as a function of c and ℓ, because they give the consumer "satisfaction" (work to consume)

more is better → downward sloping
diversity is good → diminishing MRS



2) the budget constraint

outflows : taxes, consumption spending
inflows : dividends from capital investment, wage given hours worked

$\pi + w\cdot(h-\ell) = c + t \iff c = -w\cdot\ell + wh + \pi - t$

↳ notice that t is mandatory. If $\pi > t$, then consumer need not work.
If $\pi = 0$, then consumer must work enough to cover taxes.

3) optimal choice

$$\max_{c,\ell} u(c,\ell) \quad s.t. \quad \pi + w(h-\ell) = c + t$$

$$\Rightarrow \quad MRS_{\ell,c} = \frac{MU_\ell}{MU_c} = w \text{ at optimal.}$$

(intuition)

if $\frac{MU_\ell}{MU_c} > w$

↳ consumer values 1 hr of leisure more than what 1 hour of work can give in consumption

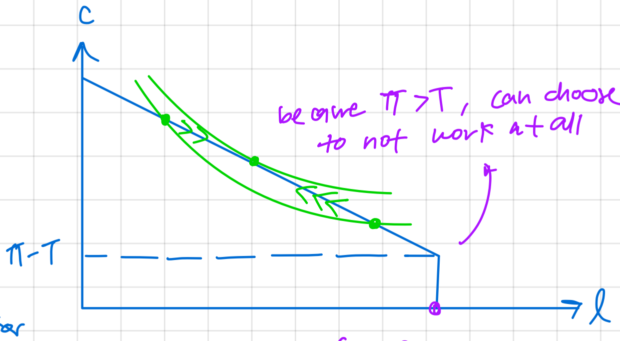
⇒ trade more consumption for leisure

if $\frac{MU_\ell}{MU_c} < w$

↳ consumer values 1 hour of leisure less than what 1 hour of work can give in consumption

⇒ trade more leisure for consumption

(graphically)



③ Labour supply

↳ consumer is a price taker in the labour market.

↳ given a certain wage rate, disposable income and his own work-leisure preference, he is willing to work a certain number of hours. We call those hours his labour supply function.

$$n^s(w) = h - \ell^*(w)$$

optimal leisure given wage rate

(how does w influence $n^s(w)$?)

1) substitution effect:

↳ w is the price of ℓ relative to c

↳ change in w, change in ℓ due to ToT between the two

} $w\uparrow, \ell\downarrow \Rightarrow N_s\uparrow$, vice versa

$\frac{dN_s}{dw} > 0$

2) income effect:

↳ leisure is a normal good

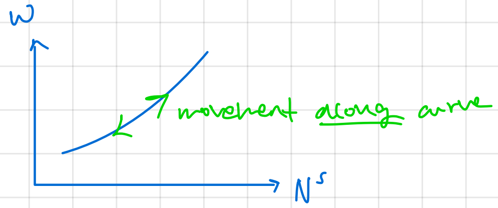
↳ change in w, change in income, want to consume more leisure

} $w\uparrow, \ell\uparrow \Rightarrow N_s\downarrow$, vice versa

$\frac{dN_s}{dw} < 0$

⇒ net effect is ambiguous. But data suggests substitution dominates

↳ $\frac{dN_s}{dw} > 0$, upward sloping supply curve

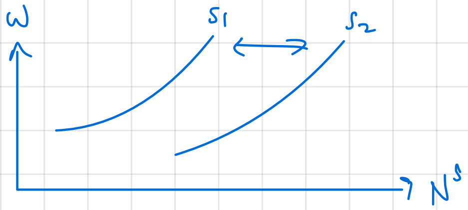


④ changes in disposable income

↳ suppose change in π / change in t

↳ leisure is a normal good. ↑ disposable income, ↑l ⇒ N_s ↓

exogeneous variable, shift in curve



one-period firm

① Role of the firm in an economy

- the firm is owned by consumers
- it uses both capital and hired labor (supplied by consumers) to produce goods
- profits are paid to consumers who are shareholders (owners) of the firm

② The production function

- we can model a firm's production by a production function that relates capital, labor input and technology sophistication to output. models production capability.
- in a single period model (the short-term) we assume that capital is a fixed factor of production, and labor is variable.

$$Y = \underbrace{z}_{\text{total factor of productivity: captures tech sophistication}} F(\underbrace{K}_{\text{capital employed}}, \underbrace{N}_{\text{hours of labor worked}})$$

properties of the production function

1. the production function exhibits constant RTS

$$zF(x \cdot K, x \cdot N) = x \cdot zF(K, N)$$

- assumption means scale doesn't matter, and it is convenient to use representative firms

2. output increases when K or N increases.
   - as K increases

$$\frac{\partial Y}{\partial K} = mpK > 0 \qquad \frac{\partial Y}{\partial N} = mpN > 0$$

3. Diminishing returns

mpN crowding out causes inefficiencies, so each unit of labor wastes some resources that could be otherwise used on productive work

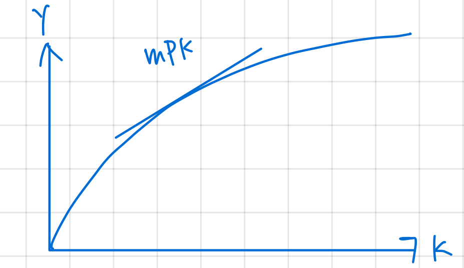
⟹ reflected in concavity of mpN

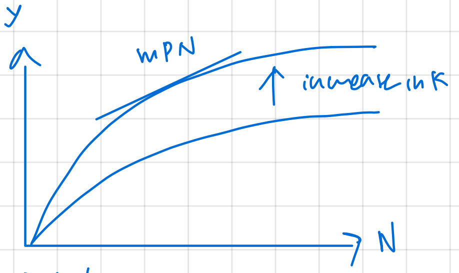
mpK dilution of impact due to underutilization of capital

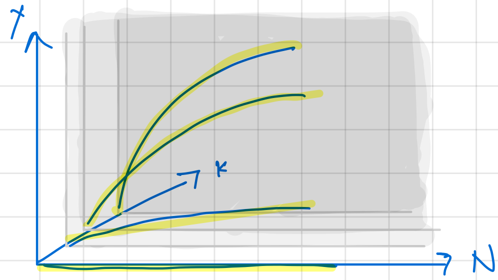
4. Both labor and capital are needed. $F(K, 0) = F(0, N) = 0$

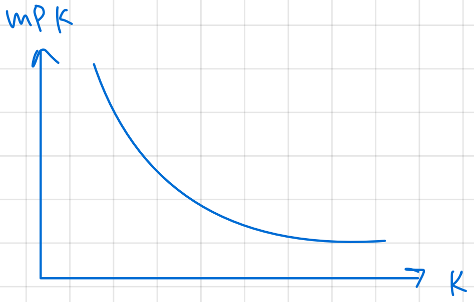
5. mpN increases w/ K i.e. $\frac{d\, mpN}{dK} > 0$
   - each worker has more capital to work with → more productive

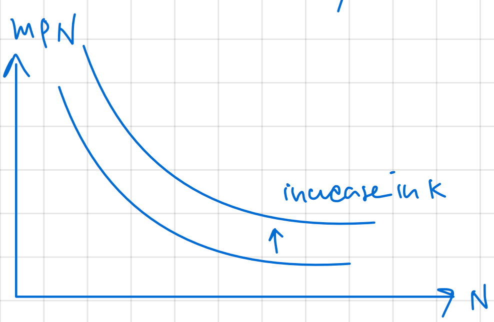
graphically











③ π maximisation

↳ given some wage rate and production capability, the firm can really only decide how much labour is the optimal to hire such that it maximises π.

↳ we assume K is endowed.

$$\pi = \underbrace{z F(K, N^d)}_{\text{revenue in units of consumption}} - \underbrace{w N^d}_{\text{labor (variable) cost in units of consumption}}$$

$$\frac{d\pi}{dN_d} = mpN - w = 0 \implies \text{firm maximises } \pi \text{ by setting } mpN = w.$$

intuition

$mpN > w$

Each unit of labor costs w but produces mpN. Total output increases more than input. Should hire more.

$mpN < w$

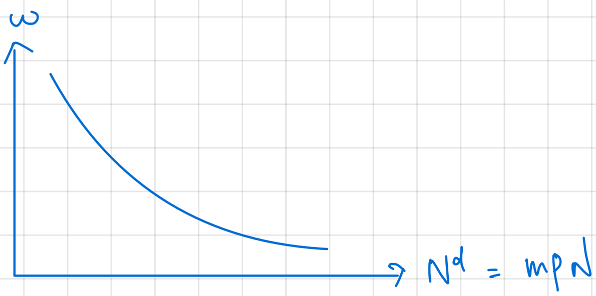
Each unit of labor produces less than they produce. making a loss on each hour employed.

④ Labour demand

↳ to maximize profit, for a given wage rate, firms will hire the qt. of labour where $mPN = w$.

↳ mpN is a function of N. so $N^d(w) = mpN$

↳ since $mpN > 0$ and diminishing mpN, $N^d(w)$ is downward sloping and diminishing.



⑤ <u>output supply</u>

↳ we can model each firm's willingness and ability to produce goods with the production function, since it will produce at a given optimal N^d.

Government and the full one-period model

① Government

↳ simplistically, in a simple closed economy model, we can think of the government using taxation to fund exogeneous expenditure

↳ This expenditure involves taking goods from the private sector

$$G = T$$

② goods demand

↳ in this model, consumers and govt. consume goods produced by firms.

↳ govt. exogeneously decides how much, while consumers choose by optimizing lifetime utility, subject to exogeneous variables like endowment, wage, preferences.

$\Rightarrow$ aggregate demand, $Y^d = \sum_{i=1}^{n} c_i + G = C + G$

③ Representative agents

1) Representative consumer & firm

↳ on average, we can aggregate all consumers and represent them as a single representative consumer, or at least distinct consumer(s)

↳ these representative consumers help us to simplify the model by abstraction

↳ we can aggregate by summation of their consumption / leisure choices since the constraints / optimisation function (preferences) are scale invariant

↳ we can apply the same logic to firms, since their production functions exhibit constant returns to scale

③ competitive equilibrium

1) agents and markets

↳ in the one-period economy, consumers, given their endowments, decide how much to work and consume, supplying labor

↳ firms, given their production capabilities, are hiring this labor for wages and producing goods that are consumed. So they are coupled with consumers through the goods and labor markets.

↳ the government exogeneously purchases goods and funds them by taxation.

competitive equilibrium in the labor market

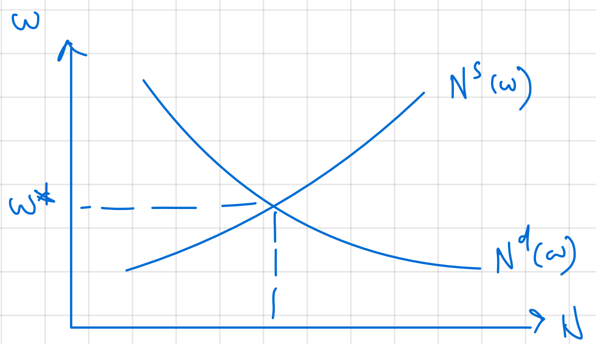
A competitive equilibrium is a set of endogeneous quantities, c^*, N^*, T and Y^* and real wage w^* such that given the exogeneous variables G, z and K, the following conditions are satisfied:

1. The representative consumer chooses C and l (and so, N^s) to maximize lifetime utility subject to his budget constraint, given T and π and w as a price taker

2. the representative firm chooses N^d to maximize profits, producing $Y = zF(K, N^d)$ and earning profit $\pi = Y - N^d \cdot w$, treating wage w as given.

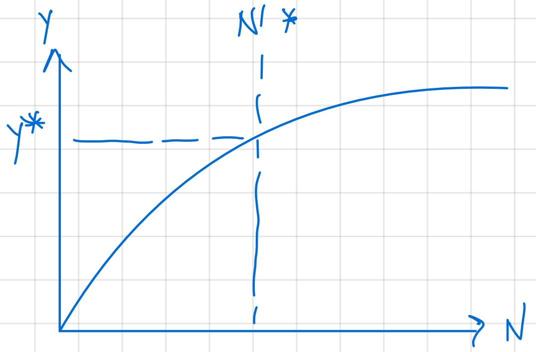
3. The government budget constraint is satisfied, so $G = T$. Taxes paid by consumers are equal to the exogeneous qt. of government spending.

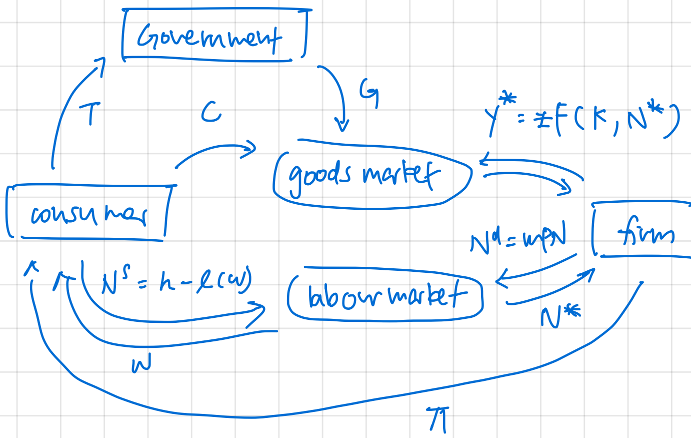
4. The labor market clears — that is, $N^d = N^s$. By walras law, the goods market also clears.



$$Y^* = C + G$$

$$C = wN^s + \pi - T$$
$$= wN^s + (Y - wN^d) - G$$
$$\Rightarrow \text{if } N^s = N^d, \text{ then } Y^* = C + G$$





The concept of intertemporal consumption and a credit market

① The credit market

↳ we want to introduce the notion of time. Given that an individual is endowed w different amounts in different periods, there is a new degree of freedom in which individuals can borrow from others in one period and repay these loans in the next period.

↳ individuals who loan charge a price — that is, interest.

1) present value: the value of a future good in the present
2) future value: the value of a present good in the future

$$PV = \frac{FV}{1+r}$$

let's simplify the concept by considering an individual endowed w y_j in $t=j$, and he has to decide how much to consume in each period, c_j.

② intertemporal budget constraint

↳ consumer is faced w making consumption choices over his life time (multiple periods), with multiple endowments at these different time periods. We abstract complex variables to "income" or "wealth"

↳ in each of these periods, he can choose to consume or to save. saving allows for future consumption and growth of value through interest rates (price of others borrowing) in the credit market

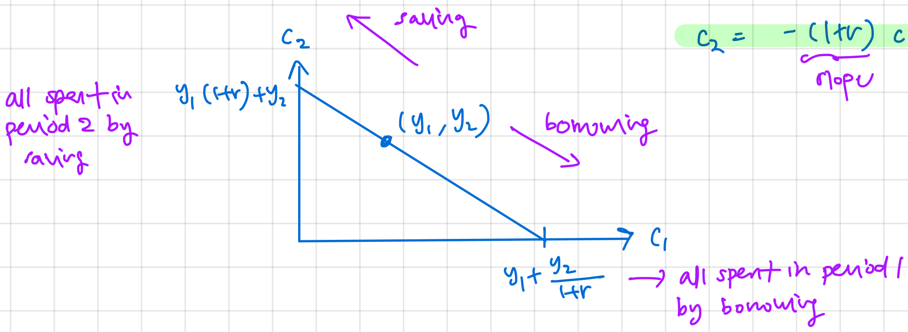
in first period: $y_1 \geq c_1 + s_1$ ⇒ income is spent or saved

in period: $y_2 + s_1(1+r) \geq c_2 + s_2$ ⇒ appreciated savings & income fund second period spending & saving

→ if we assume all money is spent, we can connect the constraints in each period $s_2 = 0$

$$c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r}$$ lifetime wealth

⇒ all possible combinations of c_1 and c_2 lie on or below this line



$$c_2 = \underbrace{-(1+r)}_{\text{slope}}\, c_1 + \underbrace{y_1(1+r) + y_2}_{\text{intercept}}$$

③ intertemporal preference, utility & indifference curves

- an individual tends to smooth out consumption over periods rather than extremes
- future consumption may be valued less than current consumption
- other standard assumptions of well behaved utility functions hold
  - more is better
  - diminishing mU
- no reason why utility function in each period should change, since same person

⇒ "standard" convex utility function w/ some weighting of future consumption can model lifetime utility & consumption choices

$$u(c_1, c_2) = \underbrace{u(c_1)}_{\text{same function}} + \underbrace{\beta}_{\text{weighting (discount factor)}} \cdot u(c_2), \quad \beta \in (0,1)$$

time separable utility function

④ optimal choice

- given a preference for consumption and budget constraint across periods, the consumer chooses the optimal level of consumption.

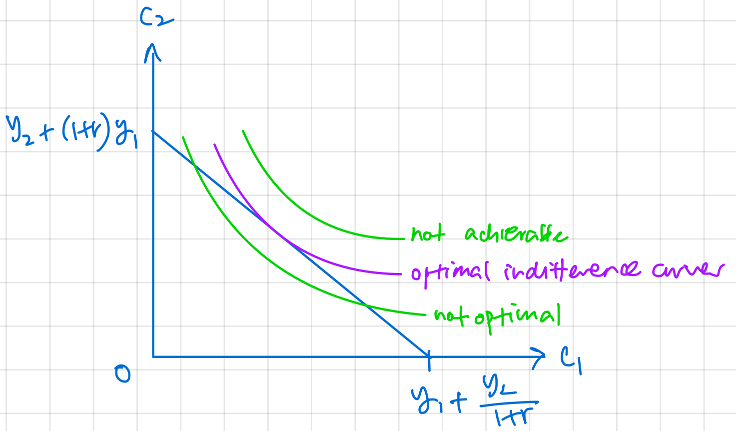
use FOC method : substitute, then diff. ⇒ solve

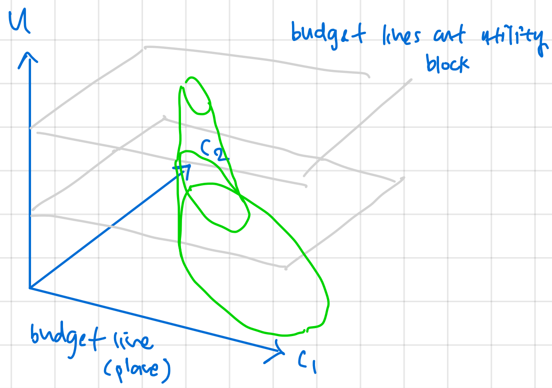
$$\max_{c_1, c_2} u(c_1, c_2) \quad \text{s.t. budget constraint}$$

$$du = \frac{\partial u}{\partial c_1} dc_1 + \frac{\partial u}{\partial c_2} dc_1 = 0$$

$$\frac{u'(c_1^*)}{\beta u'(c_2^*)} = 1+r$$

⇒ consumer makes choice in earliest period. He just follows through in second period





⑤ optimal consumption path

↳ given incomes across two periods, a consumer chooses how much to consume in either period depending on his preferences (β, how much he values future consumption relative to the present) and the cost of transferring consumption between periods ($1+r$)

⇒ based on his consumption path across periods, we can infer r or β

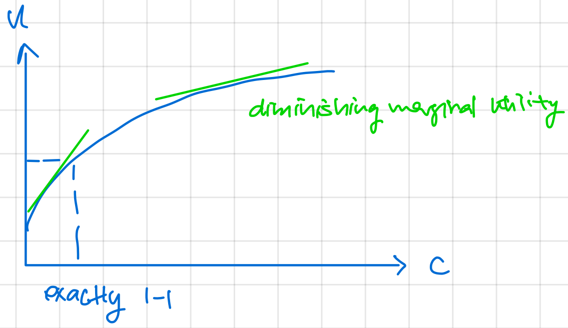
0. At optimal consumption basket, tangency condition holds. $\dfrac{u'(c_1^*)}{\beta u'(c_2^*)} = 1+r$

1. utility functions are ordinal, more is better $u' > 0$ and DMU $u'' < 0$

2. $u'(c_1^*) > u'(c_1^*) \longleftrightarrow c_1^* < c_2^*$

$u'(c_1^*) < u'(c_2^*) \longleftrightarrow c_1^* > c_2^*$

$u'(c_1^*) = u'(c_2^*) \longleftrightarrow c_1^* = c_2^*$



3. $\boxed{\dfrac{u'(c_1^*)}{\beta u(c_2^*)} = 1+r \longleftrightarrow \dfrac{u(c_1^*)}{u(c_2^*)} = \dfrac{1+r}{\beta}}$

intuition: no room to increase lifetime utility because cost of shifting one unit from present to future is precisely the cost of doing so ⇒ no room for "arbitrage"

4. From consumption path we can infer utility and thus relative magnitudes of $1+r$ and discount factor β.

$1+r = \beta \longleftrightarrow u(c_1^*) = u(c_2^*) \longleftrightarrow c_1^* = c_2^*$

↳ interest rate compensates exactly for impatience

$1+r < \beta \longleftrightarrow u(c_1^*) > u(c_2^*) \longleftrightarrow c_1^* > c_2^*$

↳ too impatient; interest rate too low s.t. I cannot save enough to consume more in future to make up for the fact that it's in future

$1+r > \beta \longleftrightarrow u(c_1^*) < u(c_2^*) \longleftrightarrow c_1^* < c_2^*$

↳ the interest gain to finance future consumption is more than sufficient to allow me to consume more in future, s.t. I can make up for the fact that I value future consumption less

$\dfrac{dr^*}{d\beta}$?

↳ describes how r^* compensates for patience as patience changes

⑥ consumer demand, C

↳ given consumption across two periods, we can model the price of consumption in either period as a function of interest rate, since that's the price to shift consumption between periods.

how does C_i change w/ i/r ?

1) substitution effect: diff gradient, same ID curve
2) income effect: same gradient, diff ID curve

⇒ do income first then substitution
⇒ ask: is endowment still affordable?

i/r ↑ → relatively more expensive to consume in period 1, and need to pay more interest in second period / gain more interest in second period

↳ lenders stay lenders
↳ borrowers unknown

→ borrower
substitution : c_1^* ↓, c_2^* ↑
income : c_1^* ↓, c_2^* ↓
loss from interest smoothed over both periods
c_1^* ↓, c_2^* ?

→ lender
substitution : c_1^* ↓, c_2^* ↑
income : c_1^* ↑, c_2^* ↑
gains from interest smoothed over both periods
c_1^* ?, c_2^* ↑

i/r ↓ → relatively less expensive to consume in period 1, need to pay less interest over lifetime, gain less from interest over lifetime

↳ borrowers stay borrowers
↳ lenders unknown

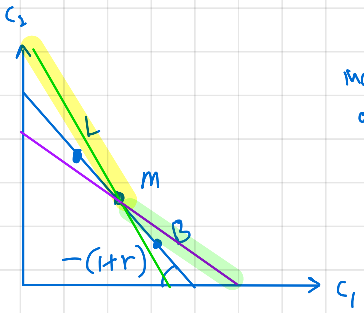
→ borrower
substitution : c_1^* ↑, c_2^* ↓
income : c_1^* ↑, c_2^* ↑
c_1^* ↑, c_2^* ?

→ lender
substitution : c_1^* ↑, c_2^* ↓
income : c_1^* ↓, c_2^* ↓
c_2^* ↓, c_1^* ?

insights from LBC & revealed preference

↳ if interest rate increases, a lender will remain a lender
↳ if interest rate falls, a borrower will remain a borrower

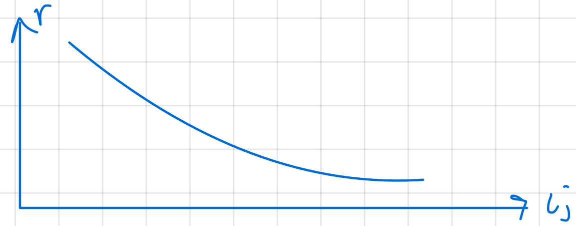
⇒ proof by revealed preference



increase in interest rate causes pivot about endowment. New optimal cannot be below endowment point since it was previously affordable but not considered. Same logic for borrower when interest rate falls.

which effect dominates?

↳ data generally suggests that substitution effect dominates, so $\frac{d c_j}{dr} < 0$

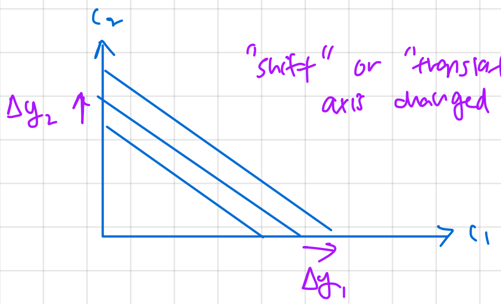


changes in income

1) changes to LBC

↳ change in income in either or both period(s) will change lifetime income

↳ we account for "temporary" changes in income through single period increases, while "permanent" through multi period changes.



"shift" or "translation" depends on which axis changed

2) changes in demand

↳ changes in LBC → income effect. in general $\frac{d c_i}{d y_j} > 0$

3) magnitude of change: consumption smoothing

↳ we assume consumption goods are normal goods.

↳ because of consumption smoothing behaviour, consumers tend to spread changes in income over consumption in multiple periods

→ $mps_{i,j}$, $mpc_{i,j}$ give us information about how c_i changes to y_j.

$$mpc_{i,j} = \frac{d c_i^*}{d y_j}$$

$$mps_{i,j} = \frac{d s_i^*}{d y_j}$$

1. solve for c_i using FOC
2. differentiate
3. magnitude indicates sensitivity

$mpc_{i,j} + mps_{i,j} = 1$

savings & consumption decisions in each period are dependent

7 competitive equilibrium w/ the credit market

1. suppose there are N individuals in the economy. Each of them maximise lifetime utility by saving / borrowing. Assume they are all interest rate takers because they all borrow / lend a relatively insignificant amount.

2. given a market interest rate, individuals trying to maximise lifetime utility will decide to save / borrow / spend. If total borrowing > total saving, then r rises. If the converse is true, then r falls. r adjusts until total borrowing = total saving for a given time period, at r^*.

2.1 At this point, demand for present funds exactly matches supply, and the credit market clears. Intuitively, because saving is coupled with spending and all income in each period goes to either, we know that total income must equal total consumption, since income that was not spent directly was lent and spent by borrowers in the same period.

2.2 More mathematically, by walras' law, for n markets, if n-1 markets have cleared, then the last (goods) market must also clear at $r = r^*$

3. competitive equilibrium is achieved.

competitive: individual maximisers lend & borrow thru in the credit markets price mechanism, without caring about others

equilibrium: all resources are allocated for each time period, with no shortage or excess

3.1 The credit market clears at r^*, where total borrowing = total saving. and total income = total spending

competitive equilibrium

⇒ a competitive equilibrium is an allocation $\{c_1^j, c_2^j, s_1^j, s_2^j\}_{j=1}^{n_j}$, giving the amount consumed & saved in each period by each individual of type j at an eqm. interest rate r^*, such that:

↳ for each individual taking r^* as given, $(c_1^{j*}, c_2^{j*}, s_1^{j*}, s_2^{j*})$ is the solution to his lifetime utility maximisation problem

↳ the credit market clears at interest rate r^*, where total borrowing = total saving

$$\textstyle\sum_i s_t^i(r^*) = 0 = \sum_i y_t^i - \sum_i c_t^i$$

(i^{th} consumer's saving in the t^{th} period

suppose there is a single consumer

⇒ then he spends exactly his income in each period, since there is no one to borrow from

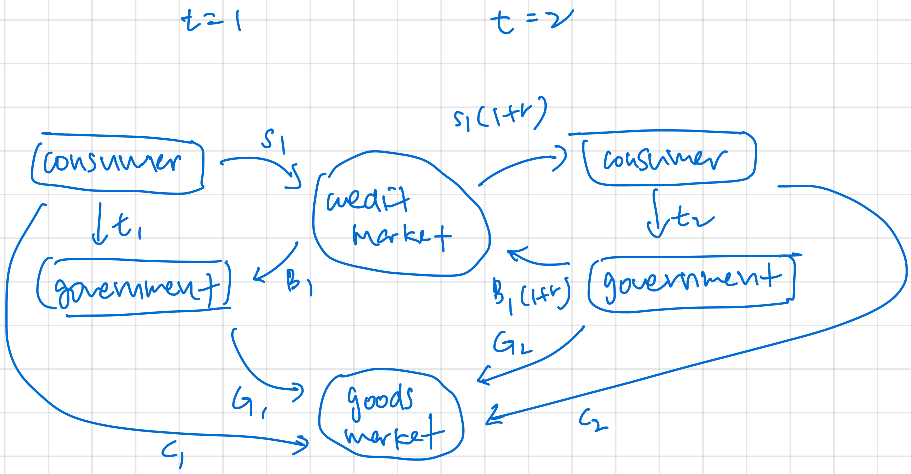
Government and the credit market

① Government in a two period model

1) role of government

↳ in a 2 period model, a government can now exogeneously decide its spending across two periods, funded by taxation or borrowing/lending in the credit market.

↳ to explore government - credit market interactions, we consider the simplest case of a 2-period consumer; there are only the goods & credit markets.



2) budget constraint

expenditures & revenues

- government spending, G_i
- taxation, T_i
- borrowing from the credit market by issuing bonds, B_i (can be negative, lending surplus

like saving (but borrowing) in the case of consumers

$$G_i = T_i + B_i$$

$$\begin{cases} G_1 = T_1 + B_1 \\ G_2 = T_2 - (1+r)B_1 \end{cases}$$

repaying borrowed

The government's LBC

$$G_1 + \frac{G_2}{1+r} = T_1 + \frac{T_2}{1+r}$$

exogeneous

② competitive equilibrium w/ consumers in the credit market

1. suppose we have an economy of N people, each paying t_t^i taxes and given income y_t^i in each period, trying to maximise lifetime utility.

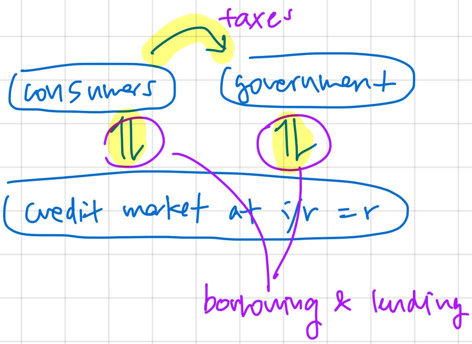
   1.1 the consumer's LBC:

$$c_1 + \frac{c_2}{1+r} = \underbrace{\overbrace{y_1 + \frac{y_2}{1+r}}^{\text{PV of income}} - \overbrace{\left(t_1 + \frac{t_2}{1+r}\right)}^{\text{PV of taxes}}}_{\text{PV of lifetime disposable income}}$$

   1.2 the government's LBC:

$$G_1 + \frac{G_2}{1+r} = N \cdot t_1 + \frac{N \cdot t_2}{1+r}$$

2. consumers and the government participate in the credit market, with consumers behaving competitively to maximise lifetime utility, and the government imposing exogenous taxes & spending.



   2.1 Given some tax rate and some spending, a government may need to borrow from the credit market, or lend in the case of tax surplus. This influences the market i/r since it affects the availability of funds.

   2.2 consumers with their disposable income, depending on their preferences & the prevailing i/r, decide to borrow (spend now) or save (lend to others to spend now).

   2.3 r adjusts through the price mechanism until there is no excess or shortage.

   2.4 At this point, the market clears.

$$\sum s_t^i = B_t \qquad \sum y_t^i = \sum c_t^i + G_t$$

directly through taxes, or indirectly via savings and bonds

directly, or through consumer/govt. lending

Definition

3 Ricardian equivalence

Theorem: for a given level of govt. expenditure for the first & second period, the exact timing of taxes has no impact on the real economy.

i.e. the consumption choices and real i/r do not change when you change taxes in any period, so long as you fix G ⇒ because it will be financed through private savings from Δ disposable income

1) intuition

↳ individuals consume out of lifetime wealth

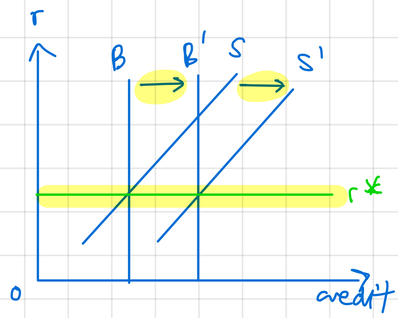
↳ so long as he knows govt. will spend G_1 now and G_2 later, he knows that a tax cut now means a tax raise later, and vice versa. So the change in disposable income from changes in taxes enters the credit market (eg. consumers save for future taxes), which are used to compensate for a govt.'s deficit / surplus based on the tax changes.

⇒ new govt. borrowing matches new consumer lending → competitive eqm., no change in c^* or r^*, only S^* and B^*

2) mathematically

1. suppose a government changes T_1, for a fixed G.

$$G_1 = T_1 + B_1 = T_1' + B_1'$$
$$G_2 = T_2 - (1+r) B_1 = T_2' - (1+r') B_1'$$
$$G_1 + \frac{G_2}{1+r} = T_1 + \frac{T_2}{1+r} = T_1' + \frac{T_2'}{1+r'}$$



2. We can abstract the consumers in the market to a single representative consumer, so that private lending and borrowing are encapsulated away.

2.1 i.e. if $B_i = 0$, then $S_i = 0$ by competitive equilibrium.

2.2 It follows that the consumer's LBC is:

$$y_1 = C_1 + S_1 + T_1 = C_1' + S_1' + T_1'$$
$$y_2 = C_2 + (1+r) S_1 + T_2 = (1+r') S_1' + T_2' + C_2'$$

3. At competitive equilibrium, $S_i = B_i$, $Y_i = G_i + C_i$ for markets to clear.

3.1 G_i, Y_i are fixed, so c_i is fixed.

⇒ we can see that if y_1, y_2, G_1, G_2 are fixed, then c_i^* is unchanged. Only S_1 changes in response to changes in T_1 and T_2.

3) implications

↳ we can think of government expenditure & borrowing, like a consumer, as a "forced" manner of resource allocation across time for the whole of society

↳ The Ricardian equivalence tells us that there is no free lunch: government spending must be funded by taxation, and for a given degree of "forced" expenditure, consumers will be no different over their lifetimes because they will eventually have the same lifetime disposable income. We cannot create value out of thin air.

→ neither better nor worse

4) assumptions

1. changes in taxes are identical for every consumer.
↳ if change in tax is different for each consumer, then they face different impacts on their calculus to maximise lifetime utility. consumption baskets will not be unchanged in general.

2. Debt is paid off eventually by the same consumers
↳ if across two periods, it is the same set of consumers, then the burden they accrue must be paid off by them. So they would not be better off.

↳ But if they are different groups, then you cannot conclude they will be the same — since their representative consumers are different!

3. Taxes are non-distortionary
↳ lump-sum taxes are non-distortionary since they do not influence per-unit consumption and choices

↳ unit taxes affect price signals and so distort consumption choices. If a distortionary tax is changed, then the decision landscape facing the consumer is different, and we may or may not be better off.

4. credit markets are perfect
↳ The equivalence assumes that govt. surplus can be borrowed by consumers and vice versa and price signals are not distorted. And because credit flows smoothly as an intermediate, the final outcome is the same.

↳ in reality, diff. interest rates due to uncertainties and credit constraints restrict the flow of credit: so changes in taxes are not exactly offset by shifts in the credit market.

5 Credit market failure & social security

Pay-as-you-go systems

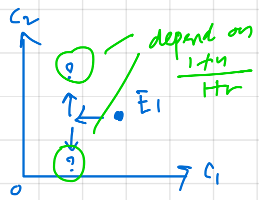
↳ there is no way for past generations to trade with future generations (think as either generations' representative consumers)

↳ if value is created by nature (eg. larger population, more resources) in future, a govt. might want to infinitely spread these resources to previous generations

1. suppose each generation pays t in the first period, then receives b benefit in some future period.

1.1 t collected in present generation used to pay for previous generation's benefits.

1.2 Balancing of budget; benefit to previous generation:



$$N_t\, b = N_{t+1} \cdot t$$

$$b = (1+n)\, t$$

system means that endowment shifts inwards in period 1, ? in period 2

2. consumer only receives benefit in next period. So his LBC is:

present value of future benefit

$$c_1 + \frac{c_2}{1+r} = y_1 - t + \frac{y_2}{1+r} + \frac{b}{1+r}$$

2.1 whether he is better off depends on $\frac{b}{1+r}$ vs. t. If $\frac{b}{1+r} > t$, then better off. So:

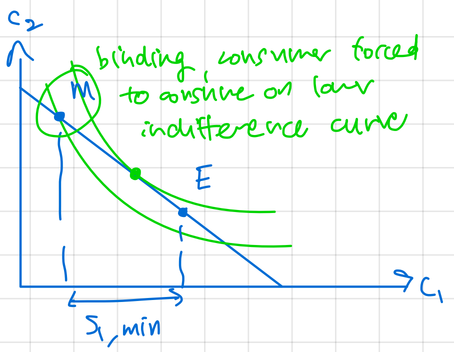
$$\frac{b}{1+r} = \frac{1+n}{1+r} \cdot t \implies \text{depends on whether } \frac{1+n}{1+r} > 1.$$

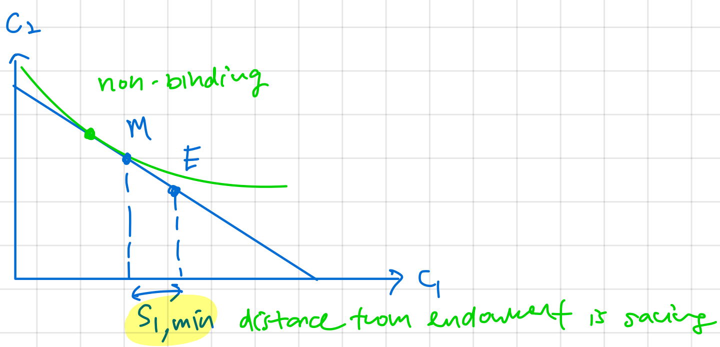
⟹ If natural growth of resources exceeds growth of resources transferred through credit market, consumer will be better off. First generation is <u>always</u> better off, since they paid nothing but got b in next period.

Fully-funded social security

1) imposition of minimum savings

↳ either it binds, and consumers are not consuming at optimal basket, or consumers are no better off (assuming forced savings through perfect credit market)





2) mechanism of binding

non-binding

binding

excess is saved and put in credit market

save $S_{1,min}$ under system

notice distinction where money goes

the two-period representative consumer

① the consumer in a 2 period economy

The economy is made up of consumers, firms and the government.
We start by modeling everything in some unit of real consumption.

$\boxed{\text{consumers}}$ suppose their main goal is to consume and enjoy leisure time throughout their lifetime.

↳ endowed w/ h hours in a period. they choose between working for a wage or leisure, and spend earnings by consuming goods or saving it for future consumption.
↳ they pay taxes to the government in either period.
↳ they can also own firms, and so earn dividends / π of firms they own.

② consumer's full utility maximization problem

↳ suppose we have a consumer trying to maximize lifetime utility. In each period, he has to pay taxes, receive dividends, decide how much to work, how much to consume (and save)

$$\max_{c_1, c_2, \ell_1, \ell_2} u(c_1, \ell_1) + \beta\, u(c_2, \ell_2)$$

$$\text{s.t.} \quad c_1 + \frac{c_2}{1+r} = w_1(h-\ell_1) + \frac{w_2(h-\ell_2)}{1+r} \quad \text{wage income}$$

$$+ \pi_1 + \frac{\pi_2}{1+r} \quad \text{dividend income}$$

$$- \left(t_1 + \frac{t_2}{1+r}\right) \quad \text{tax expenditure}$$

1. with his LBC, the consumer is faced w/ some trades.

distinct sets

$\frac{dc_1}{d\ell_1} = -w_1$	$\frac{dc_2}{d\ell_1} = -(1+r)w_1$
$\frac{dc_1}{d\ell_2} = -\frac{w_2}{1+r}$	$\frac{dc_2}{d\ell_2} = -w_2$
$\frac{dc_1}{dc_2} = -(1+r)$	$\frac{dc_2}{dc_1} = -\frac{1}{1+r}$

as price takers, these are trade prices of leisure vs. work, present vs. future consumption

2. Differentiate to get simultaneous equations using chain rule. Here, we write eqns based on c_1.

$$\boxed{\begin{aligned}
\frac{\partial u}{\partial c_2} &= \frac{\partial u(c_1,l_1)}{\partial c_1}\frac{dc_1}{dc_2} + \beta\frac{\partial u(c_2,l_2)}{\partial c_2} = 0\\
\frac{\partial u}{\partial l_1} &= \frac{\partial u(c_1,l_1)}{\partial l_1} + \frac{\partial u(c_1,l_1)}{\partial c_1}\cdot\frac{dc_1}{dl_1} = 0\\
\frac{\partial u}{\partial l_2} &= \frac{\partial u(c_1,l_1)}{\partial c_1}\cdot\frac{dc_1}{dl_2} + \beta\frac{\partial u(c_2,l_2)}{\partial l_2} = 0
\end{aligned}}$$

③ labour supply

↳ consumer is a price taker in the labour market.
↳ consumer is also a price taker in the credit market.
↳ given a certain interest rate, his disposable income in either period, the wage rate and his own work-leisure preference, he is willing to work a certain number of hours in either period to maximise lifetime utility.

$$\Rightarrow \quad n_t^s(w_t, r) = h - l_t^*(w_t, r)$$

l_t^*: optimal leisure in period t, given interest rate & wage rate w_t

period 1 change → immediate
period 2 change → expected change

how does n_t^s change $\bar{c}$ w_t?

1) substitution effect:

↳ w_t is the price of l_t relative to c
↳ change in w_t, change in l_t due to ToT between the two } $w\uparrow, l\downarrow \Rightarrow N_s\uparrow$, vice versa

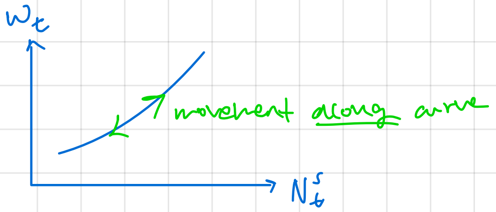
$\frac{dN_t^s}{dw_t} > 0$

2) income effect:

↳ leisure is a normal good
↳ change in w_t, change in income, want to consume more leisure } $w\uparrow, l\uparrow \Rightarrow N_s\downarrow$, vice versa

$\frac{dN_t^s}{dw_t} < 0$

⇒ net effect is ambiguous. But data suggests substitution dominates
↳ $\frac{dN_t^s}{dw} > 0$, upward sloping supply curve



how does n_1^s change w r?

1) substitution effect

↳ ToT between present & next period work = $\frac{w_1(1+r)}{w_2}$ → if you choose to work 1h in future, you lose $w_1(1+r)$, gain w_2

↳ ↑ r, ↑ price of current to future leisure, work more now $\frac{dN_1^s}{dr} > 0$, $\frac{dN_2^s}{dr} < 0$

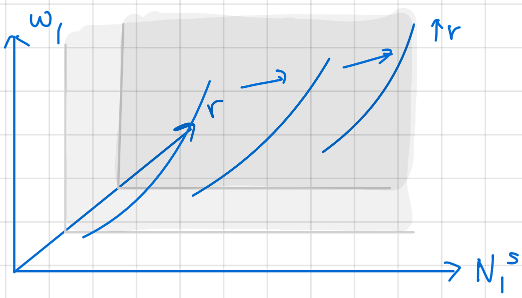
↳ work more now to gain more from interest

2) income effect

↳ ↑ r, same savings give more over lifetime. l is normal good.

↳ ↑ r, work less now $\frac{dN_1^s}{dr} < 0$

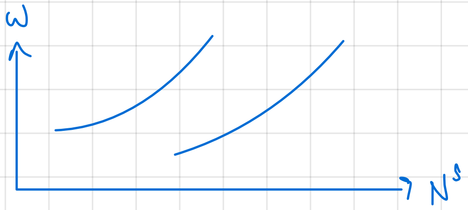
⇒ data suggests substitution effect dominates ⇒ $\frac{dN_1^s}{dr} > 0$, r↑, rightward shift



how does n_t^s change w disposable income?

↳ suppose disposable income changes by t or π — exogeneous variable, shift in curve

↳ leisure is a normal good. ↑ disposable income, ↑l ⇒ N_s ↓



$\frac{dN_i^s}{dy_j^D} > 0$

4 consumer demand, C

↳ given consumption across two periods, we can model the price of consumption in either period as a function of interest rate, since that's the price to shift consumption between periods.

how does C_i change w/ i/r ?

1) substitution effect: diff gradient, same ID curve
2) income effect: same gradient, diff ID curve

⇒ do income first then substitution
⇒ ask: is endowment still affordable?

i/r ↑ → relatively more expensive to consume in period 1, and need to pay more interest in second period / gain more interest in second period

↳ lenders stay lenders
↳ borrowers unknown

borrower
- substitution : $c_1^* \downarrow$, $c_2^* \uparrow$
- income : $c_1^* \downarrow$, $c_2^* \downarrow$

loss from interest smoothed over both periods

⇒ $c_1^* \downarrow$, c_2^* ?

lender
- substitution : $c_1^* \downarrow$, $c_2^* \uparrow$
- income : $c_1^* \uparrow$, $c_2^* \uparrow$

gains from interest smoothed over both periods

⇒ c_1^* ?, $c_2^* \uparrow$

i/r ↓ → relatively less expensive to consume in period 1, need to pay less interest over lifetime, gain less from interest over lifetime

↳ borrowers stay borrowers
↳ lenders unknown

borrower
- substitution : $c_1^* \uparrow$, $c_2^* \downarrow$
- income : $c_1^* \uparrow$, $c_2^* \uparrow$

⇒ $c_1^* \uparrow$, c_2^* ?

lender
- substitution : $c_1^* \uparrow$, $c_2^* \downarrow$
- income : $c_1^* \downarrow$, $c_2^* \downarrow$

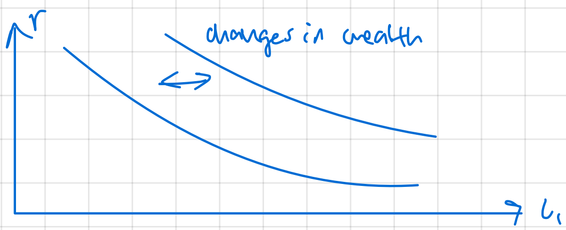
⇒ $c_2^* \downarrow$, c_1^* ?

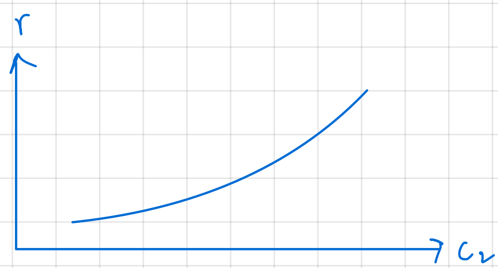
current demand

↳ observe that $\frac{dc_1}{dr} < 0$, $\frac{dc_2}{dr} > 0$ since substitution effect dominates

⇒ as r rises, current consumption falls

⇒ as r rises, <u>expected</u> future consumption rises





changes in income

↳ suppose change in t_i or π_i ⇒ change in lifetime earnings

↳ changes in LBC → income effect. in general $\frac{d c_i}{d y_j} > 0$

↳ we assume consumption goods are normal goods.

↳ because of consumption smoothing behaviour, consumers tend to spread changes in income over consumption in multiple periods

→ $mps_{i,j}$, $mpc_{i,j}$ give us information about how c_i changes to y_j.

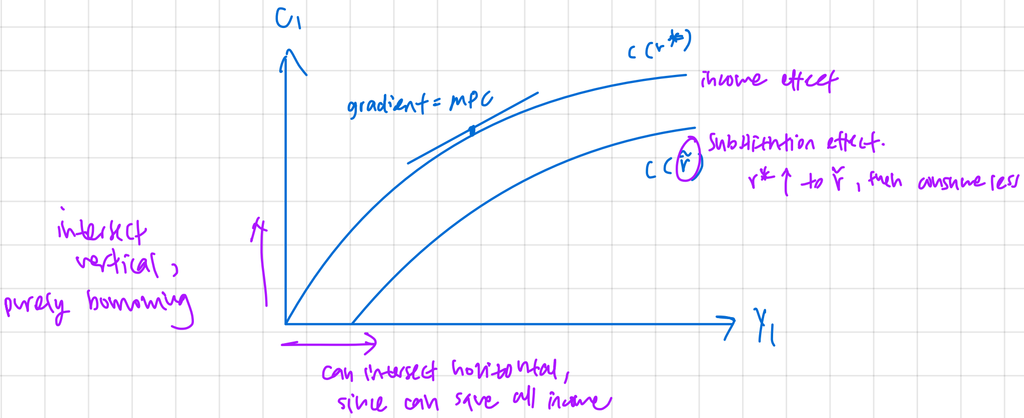
$$mpc_{i,j} = \frac{d c_i^*}{d y_j}$$

$$mps_{i,j} = \frac{d s_i^*}{d y_j}$$

1. solve for c_i using FOC
2. differentiate
3. magnitude indicates sensitivity

$mpc_{i,j} + mps_{i,j} = 1$

savings & consumption decisions in each period are dependent



The two-period representative firm

① production function and law of motion of capital

1) production across two periods

↳ as with the representative firm in the one period model, the firm produces output modelled by a production function

total factor of productivity: captures tech sophistication

$$Y_t = z_t F(K_t, N_t)$$

hours of labour worked

capital employed

properties of the production function

1. the production function exhibits constant RTs

$$zF(x \cdot K, x \cdot N) = x \cdot zF(K, N)$$

↳ assumption means scale doesn't matter, and it is convenient to use representative firms

2. output increases when K or N increases.

↳ as K increases

$$\frac{\partial Y}{\partial K} = MPK > 0 \qquad \frac{\partial Y}{\partial N} = MPN > 0$$

3. Diminishing returns

MPN crowding out causes inefficiencies, so each unit of labour wastes some resources that would be otherwise used on productive work $\frac{\partial^2 Y}{\partial N_t^2} < 0$

⟹ reflected in concavity of MPN

MPK dilution of impact due to underutilization of capital $\frac{\partial^2 Y}{\partial K_t^2} < 0$

4. Both labour and capital are needed. $F(K, 0) = F(0, N) = 0$

5. MPN increases w K ie. $\frac{d\,MPN_t}{dK_t} > 0$

↳ each worker has more capital to work with → more productive

2) investment & law of motion of capital

↳ in the 2-period model, we assume a firm is endowed with a fixed amount of capital K_1 in the first period (short term).

↳ in the second period, the firm has the flexibility to invest in capital for the next period

↳ also, capital depreciates

$\boxed{\text{law of motion of capital}}$

$$\overbrace{K_{t+1}}^{\text{next period capital}} = \underbrace{K_t(1-\delta)}_{\text{leftover capital after depreciation}} + \overbrace{I_t}^{\text{investment spending}}$$

② lifetime π maximisation

↳ the firm's goal is to maximize lifetime π

↳ it does so by choosing optimal N_1, N_2 and $I_1 \leftrightarrow K_2$, taking r, w_1 and w_2 as given (price takers)

$\boxed{\text{formalising the objective}}$

$$\pi_1 = z_1 F_1(K_1, N_1) - w_1 N_1 - I_1$$

$$\pi_2 = z_2 F_2(K_2, N_2) - w_2 N_2 + \underbrace{(1-\delta) K_2}_{\text{sale of leftover assets, since finite time horizon}}$$

$$I_1 = K_2 - K_1(1-\delta)$$

$$\max_{N_1, N_2, K_2 \leftrightarrow I_1} \pi_1 + \frac{\pi_2}{1+r}$$

$$\Rightarrow \max_{N_1, N_2, K_2} z_1 F_1(K_1, N_1) - w_1 N_1 - K_2 + K_1(1-\delta) + \frac{z_2 F_2(K_2, N_2)}{1+r} - \frac{w_2 N_2}{1+r} + \frac{(1-\delta) K_2}{1+r}$$

$\boxed{\text{FOCs governing the optimum}}$

$$\frac{\partial \pi_T}{\partial N_1} = mpN_1 - w_1 = 0 \Rightarrow mpN_1 = w_1$$

$$\frac{\partial \pi_T}{\partial N_2} = \frac{mpN_2 - w_2}{1+r} = 0 \Rightarrow mpN_2 = w_2$$

} firm will hire labor up till 1 unit of labor contributes to output exactly he's worth & no more, given diminishing returns

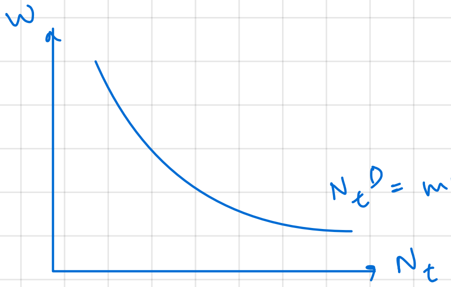
$$\frac{\partial \pi_T}{\partial K_2} = -1 + \frac{mpK_2(1-\delta)}{1+r} = 0 \Rightarrow mpK_2 = r + \delta$$

} firm will invest in capital up till 1 unit of capital in second period is equal to opportunity cost of purchase: implicit (i/r forgone) & explicit (depreciation loss)

→ marginal benefit = marginal cost

③ firm labour demand

↳ As in the one period model, a firm will hire at exactly where $mpN_t = w_t$ ie. $w = mpN_t = N_t^D$



anything that changes mpN_t will change N_t^D

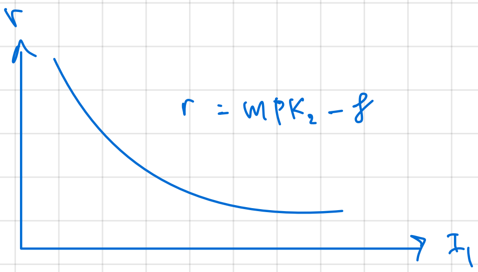
ΔK_t $\quad$ ΔZ_t

$\Delta mpN_t \rightarrow \Delta N_t^D$

④ investment demand

↳ firm will invest in capital for future period in so far as it increases total π. ie. it will invest exactly where $mpK_2 = r + \delta$

$\Rightarrow$ $r = mpK_2 - \delta$ describes the optimal investment for a given interest rate



anything that changes mpK_2 will change I_1

↳ remember that $K_2 = K_1(1-\delta) + I_1$

higher starting K_1

↓

higher K_2, lower mpK_2, since diminishing mpK

↓

$\downarrow mpK_2$, $\downarrow I_1$

ΔZ_2

↓

ΔmpK_2, ΔI_1

The full two-period model

① an overview of how markets come together

↳ so far, we have three agents in the economy: consumers, firms and government, and all trades are done in real value (units of consumption)

↳ these agents interact with one another in the current period through the labour and goods market, and across time through the credit market.

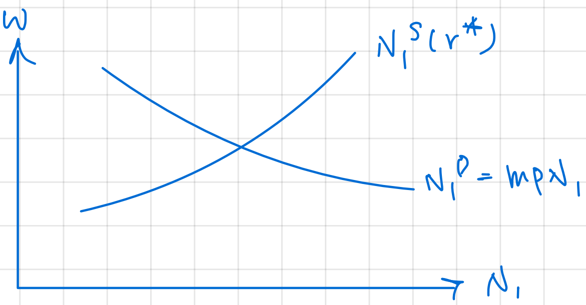
⇒ but how do these markets interact with one another?

② current labour market

↳ at a given interest rate, wage rate and level of wealth, consumers are willing to work at N_1^S.

↳ given a certain production function and labour productivity, at a given wage rate, firms want workers to work N_1^D.

⇒ market clears when $N_1^S = N_1^D$

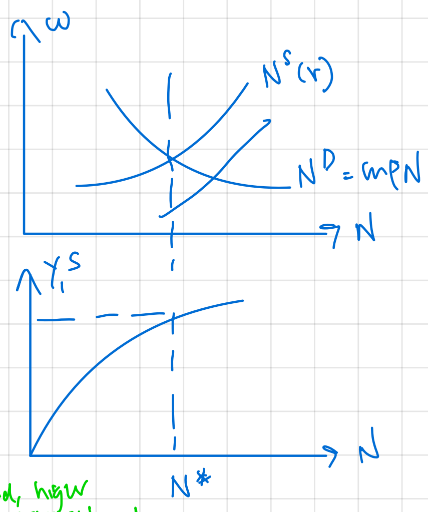


② goods market

1) output supply

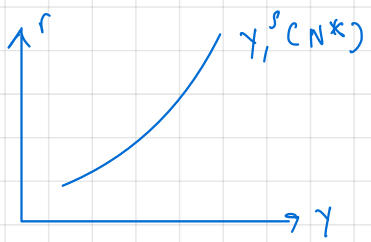
↳ output supply models qt. of goods produced at a given interest rate, for a representative firm using K capital and hiring N* labour.

↳ for current output supply, K is fixed, so $Y_1^S = z_1 F(K_1, N_1^*)$ is dependent on equilibrium in the labour market.

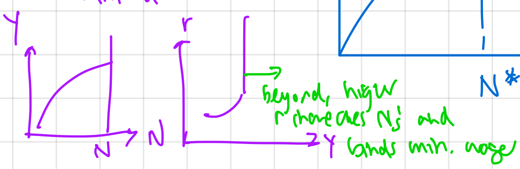


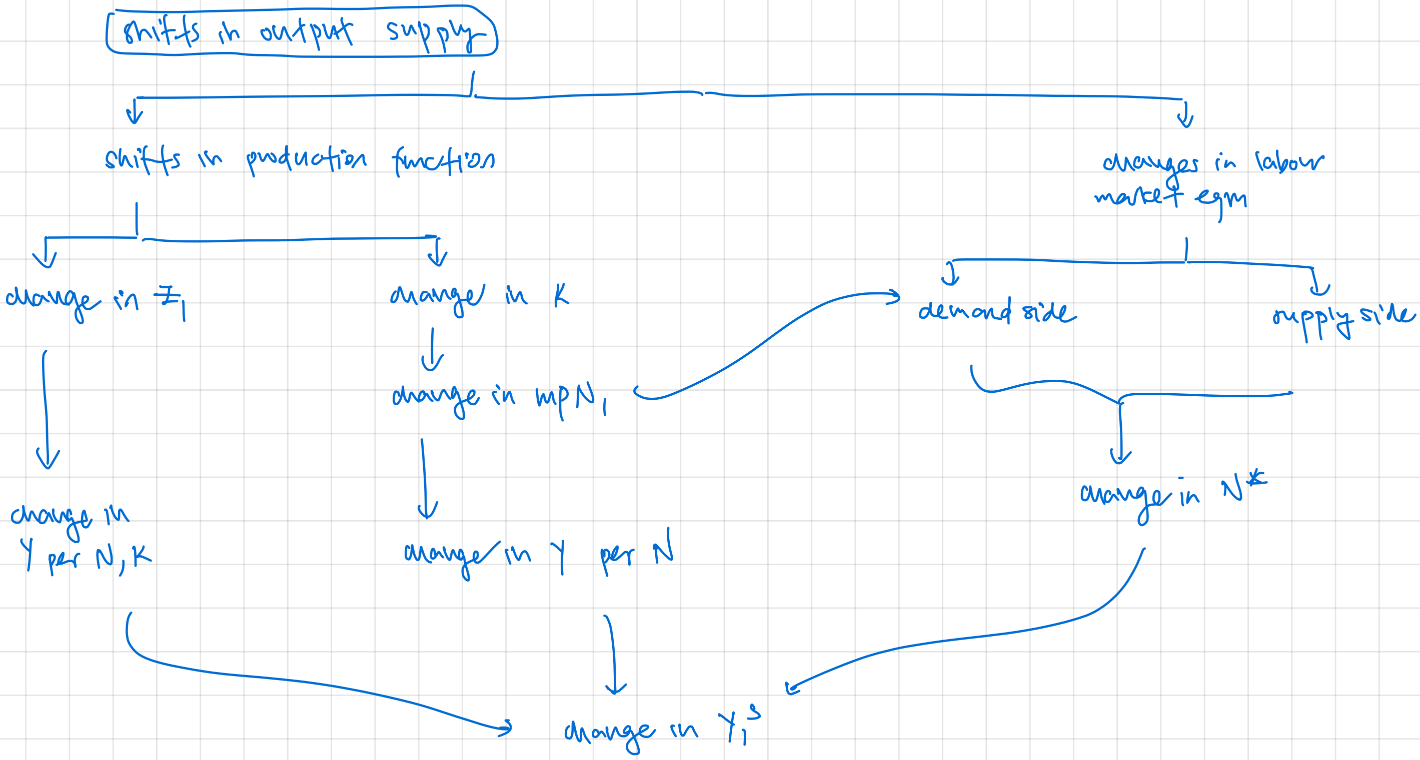
(how is output supply shaped?)

1. as r ↑, $N^S(r)$ ↑ since substitution effect dominates
2. $N_1^* \to \tilde{N}_1$, Y_1^S ↑ ⇒ upward sloping



if min. wage is binding, N is stuck, y_1^S is kinked





2) output demand

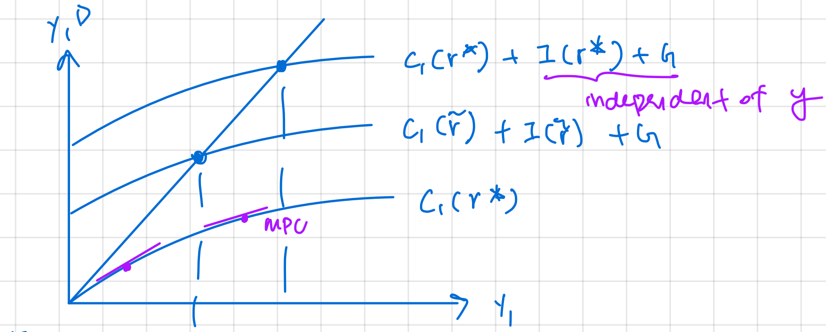
↳ goods are demanded by every agent in the economy : government G_t, consumers C_t and firms I_t.

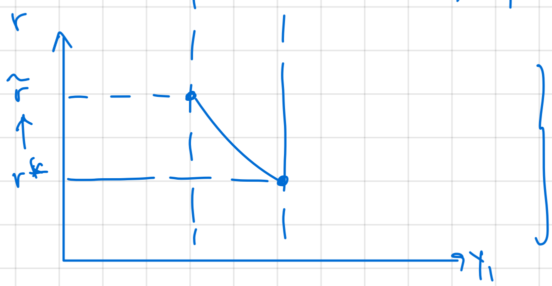
↳ I_t and C_t are i/r sensitive. Aggregating ⟹ $Y_t^D = C_t(r_{t-1}) + I_t(r_{t-1}) + G$

how is output demand shaped ?

1. As $r\uparrow$, $C_1\downarrow$ since substitution effect dominates
2. Also, as income rises, C_1 is affected due to income effects.
3. As $r\uparrow$, $I_1\downarrow$ since gains from investing in alternative assets rises and opportunity cost of investement rises
4. G stays constant.

⟹ output demand lies at $Y_1^D = Y$, ie. where demand for goods induced by the income from those goods (consumers) is equal.





⟹ so Y_1^D is downward sloping

shifts in output demand: any factor that affects C, I or G, except r

C: change in wealth π, t, in either period

I_1: $\Delta TFP_2 \rightarrow \Delta MPK_2$
$\Rightarrow I_1 = MPK_2 - f$

G: change in spending

③ The complete intertemporal model

competitive equilibrium

Given that the government's spending and taxing $\{G_1, G_2, T_1, T_2\}$ in such a way that its LBC holds, a competitive equilibrium is an allocation $\{C_1^*, C_2^*, S_1^*, S_2^*\}$ and $\{N_1^{S*}, N_2^{S*}, N_1^{D*}, N_2^{D*}, I_1^*\}$ and prices $\{r_1^*, w_1^*, w_2^*\}$ such that:

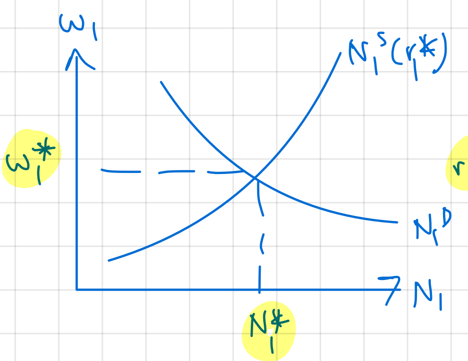
1. the representative consumer chooses $\{C_1^*, C_2^*, S_1^*, S_2^*, N_1^{S*}, N_2^{S*}\}$ in order to maximise his lifetime utility, taking $\{r^*, w_1^*, w_2^*\}$ as given

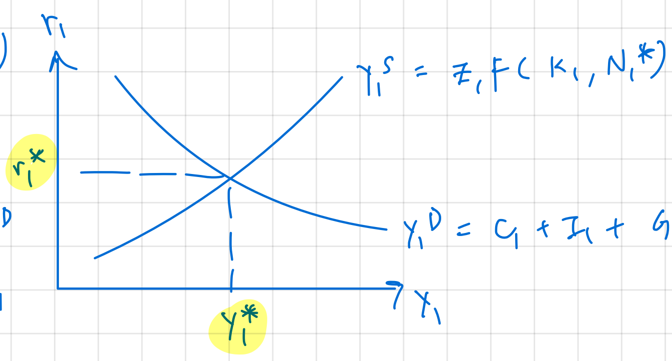
2. the representative firm maximises its present value of profits by choosing $\{N_1^{D*}, N_2^{D*}, I_1^*\}$, taking $\{r^*, w_1^*, w_2^*\}$ as given

3. the goods market clears, goods supplied equals goods demanded for each period. $Y_t^* = Y_t^S = Y_t^D = C_t^*(r^*) + I_t^*(r^*) + G$ for $t = 1, 2$, at $r = r^*$

4. the labor market clears in each period at w_t^*, when labor supply equals labor demand. $N_t^* = N_t^S = N_t^D$ for $t = 1, 2$

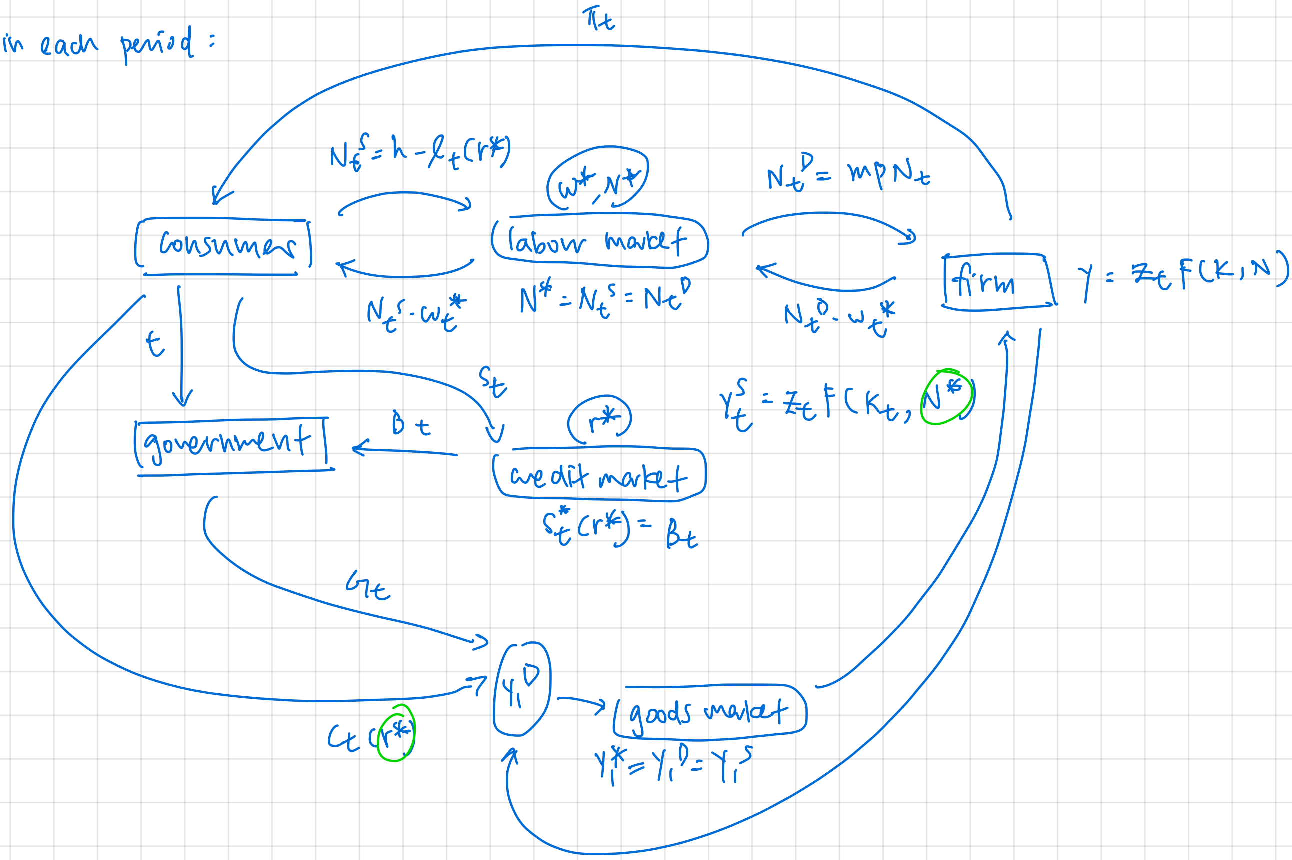
By walras law, if goods and labor markets clear in each period, then the credit market also clears in every period. $S_t^* = B_t^*$





a high level view

in each period:



$I_1(r^*)$ $\begin{cases} r = mpK_2 - \delta \Rightarrow K_2 \text{ subject} \\ K_2 = I_1 + (K_1 - \delta) \\ \Rightarrow I_1 \end{cases}$

④ analysis of shocks to general equilibrium

↳ exogeneous variables: $G_t = T_t$, K_1, z_t ⟹ changes in any one can lead to a change in today's equilibrium

change in K_1 shock that damages capital

1. suppose currently in equilibrium.
2. effects on goods market

   2.1 K_1 falls, so $Y_1^S = z_1 F(K_1, N_1)$ falls.

   2.2 also, profit maximising I_1 where $mpK_2 = r + \delta$. $K_2 = I_1 + (1-\delta)K_1$

   2.3 since diminishing mpK_2, K_1 falls, K_2 falls, mpK_2 rises. so more investment.

   2.4 Y_1^D rises.

3. price mechanism. clearing of goods, labour and credit markets by walras' law.

4. Net effects on r_1, Y_1, w_1, N_1

$\boxed{\text{change in } z_1}$ shock that affects current period productivity

1. suppose currently in equilibrium.

2. effects on labour market & output supply

   2.1 change in circumstances leads to change in z_1

   2.2 To profit maximize, hire where $MPN_1 = w_1$

   2.3 change in MPN_1, change in qt. of labour demanded at every wage.

3. no effects on goods demand

4. price mechanism. clearing of goods, labour and credit markets by walras' law.

5. Net effects on r_1, Y_1, w_1, N_1

$\boxed{\text{change in } z_2}$ expected change in future productivity

1. suppose currently in equilibrium.

2. no effects on labour market & output supply

3. effects on goods market

   3.1 change in circumstances, change in z_2, change in MPK_2

   3.2 lifetime profit maximizing level of I_1 where $MPK_2 = r + \delta$, $K_2 = I_1 + (K_1 - \delta)$

   3.3 change in MPK_2, more/less productive per unit, change in I_1, shift in I_1 and Y_1^d.

   3.4 consumer expects higher income in period 2. So $C_1^d \uparrow$.

4. price mechanism. clearing of goods, labour and credit markets by walras' law.

5. Net effects on r_1, Y_1, w_1, N_1. Resolution of ambiguities.

$\boxed{\text{change in } G_1}$

1. suppose currently in equilibrium.

2. explanation of changes

   2.1 ΔG, must balance LBC, ΔT (lifetime taxes). By Ricardian Equivalence, timing does not matter

   2.2 consumer lifetime wealth changes by $-\Delta T$

3. effect on labour market and goods supply

   3.1 change in ΔT, income effect, change in N_1^s at r^*

   3.2 change in labour market to $\tilde{N}_1$, shift in Y_1^s

4. effect on goods demand

4.1 C falls due to ΔT and income effect by mpc

4.2 C rises due to overall change δ by mpc

4.3 Y_1^D rises by ΔG

4.4 if mpc is constant, $\delta = \Delta Y_1^D = \Delta G$.

5. price mechanism. Clearing of goods, labour and credit markets by walras' law.

6. Net effects on r_1, Y_1, w_1, N_1. Resolution of ambiguities.

$\boxed{\text{price mechanism}}$

1. at r^*, not in equilibrium
2. demand $\leftrightarrow$ supply, Δr

2.1 as Δr, $\Delta I(r)$ and $\Delta C(r)$ due to opp. cost of investment and dominant substitution effect where relative price of consumption today to tomorrow is $(1+r)$

2.2 as Δr, $\Delta N_1^S(r)$ due to dominant substitution effect, where relative price of leisure-time today to tomorrow is $\frac{w_1(1+r)}{w_2}$

3. equilibrium, net effects on w_1, N_1, Y_1, r_1. Resolution of ambiguities
+ change in cond I

ΔG

↓

LBC : $G_1 + \frac{G_2}{1+r} = T_1 + \frac{T_2}{1+r} = T$

$\Delta G \Rightarrow \Delta T$

labour mkt

ΔT, income effect, $\boxed{\Delta N_1^S}$

↓

$\boxed{\Delta Y_1^S}$

goods market

ΔG, $\Delta C = 0$

↓

$\boxed{\Delta Y_1^D}$

$\boxed{\Delta K_1}$

↓

goods market

$I_1 \big|_{\pi max}$: $MPK_2 = r_1 + \delta$

$K_2 = I_1 + K_1(1-\delta)$

MPK_2 diminishing

$\Rightarrow \Delta K_1 \Rightarrow \Delta MPK_2 \Rightarrow \Delta I_1$

↓

$\boxed{\Delta Y_1^D}$

$Y_1^S = z_1 F(K_1, N_1)$

↓

$\boxed{\Delta Y_1^S}$

Z_1

$\downarrow$

ΔMPN_1

$\downarrow$

labour market

π max N_1^D @ $MPN_1 = w$

$\Delta MPN_1 \Rightarrow \boxed{\Delta N_1^D}$

Z_2

$\downarrow$

ΔMPK_2

$\downarrow$

goods market

π max I_1 @ $MPK_2 = r + \delta$

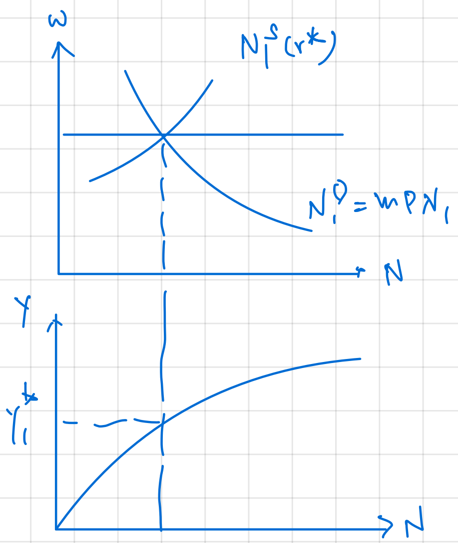
$K_2 = I_1 + K_1(1-\delta) \Rightarrow K_2 \propto Z_1$

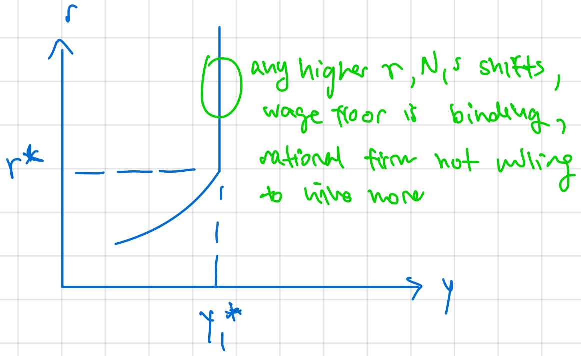
$\downarrow$

$\boxed{\Delta Y_1^D}$

Notes :

1. wage floor in labour market





change in G_1 eg. increase

1. lets suppose the economy was originally in equilibrium

2. G_1 increases to $\tilde{G}_1$, and G_2 is unchanged

2.1 As this increases the present value of life time expenditure of the government, the present value of taxes must also rise. By the Ricardian equivalence, it does not matter when these taxes are imposed.

$$G_1 + \frac{G_2}{1+r} = T_1 + \frac{T_2}{1+r} = \tau \quad \Delta G_1 = \Delta(\tau)$$

2.2 The increase in taxation leads to a fall in lifetime value of wealth

Description of effects

3. In the labor market, this has the effect of increasing labor supply at the current interest rate.

3.1 The fall in present value of lifetime wealth implies a fall in consumption of leisure in the current period since it is a normal good. So consumers work more, since $N_1^S = h - l_1$

3.2 Labor supply shifts right from N_1^S to $\tilde{N}_1^S$

3.3 This has the effect of shifting output supply right from Y_1^S to $\tilde{Y}_1^S$

4. The increase in G_1 also increases output demand in the goods market

4.1 $Y_1^D = C_1 + I_1 + G_1$. so $G_1 + \Delta G$ increases output demand.

4.2 Also, because the consumer's lifetime wealth has fallen due to increased taxation, assuming mpc is constant, consumption spending falls by $|mpc\,(\Delta \text{wealth})|$.

4.3 Simultaneously, the net change in output demand (and thus, income) will affect consumption demand by $mpc \cdot f$, since f is total change in goods demand.

4.4 mathematically: $f = \Delta G - mpc\,\Delta G + mpc\, f \Rightarrow f = \Delta G = \Delta Y_1^D$

4.5 So goods demand increases by ΔG, shifting from Y_1^D to $\tilde{Y}_1^D$

5. At the original equilibrium interest rate r^*, output demand exceeds output supply. So the goods market cannot be in equilibrium.

5.1 interest rate rises due to excess demand, which has the effect of also dampening consumer demand (since substitution effect dominates, and individuals want to save more) and investment spending, since returns from alternative assets rise and opportunity cost of investment rises. These are represented by a movement along the demand curve.

5.2 As interest rate rises, consumers work more since the price of leisure today is more expensive than the price of leisure tomorrow, given by $\frac{w_1(1+r)}{w_2}$ and the substitution effect dominates. This has the effect of increasing labor supply, shifting from $\tilde{N}_1^S$ to $\bar{N}_1^S$ and moving along $\tilde{Y}_1^S$.

6. Equilibrium in the goods market is achieved when equilibrium r_1 has risen sufficiently so that $Y_1^d = Y_1^S$ at $\hat{r}$ and $\tilde{Y}_1$ in the goods market, and in the labor market, $\bar{N}_1^S = \tilde{N}_1^D$ at the new wage rate $\tilde{w}^*$ and labor market equilibrium $\tilde{N}_1^*$.

7. Net effects

7.1 In the labour market there is an unambiguous increase in N_1 to $\tilde{N}_1^*$ and fall in w_1 to $\tilde{w}_1^*$

7.2 In the goods market, if we assume that demand rise exceeds supply fall, a reasonable assumption since supply shifts occur indirectly through the labour market, then r_1 rises to $\tilde{r}_1^*$ and Y_1 rises to $\tilde{r}_1^*$.

8. Present government expenditure has the effect of crowding out private spending

8.1 $\Delta Y_1^D = \Delta G_1$, but $\Delta Y < \Delta G$. $\frac{\Delta Y}{\Delta G} < 1$

8.2 ΔG_1 has led to disequilibrium in the goods market, since demand exceeded supply. To restore equilibrium, interest had to rise, which led to a fall in c_1 due to dominating substitution effect.

8.3 It also led to a fall in I_1, which has the effect of decreasing K_2 and future productive capacity.

money

① what is money?

i) definition

1. a medium of exchange — accepted in exchange for goods because it can be used in trade for other goods
2. a store of value — relatively stable value, can use it to trade current for future goods
3. unit of account

fiat money

commodity-backed money

ii) measuring the amount of money

M0: (or **monetary base** or **narrow money** or **outside money**) refers to currency in circulation and deposits of depository institutions with the Fed

M1: M0 + currency plus liquid assets including demand deposits, travelers' checks, etc (transaction deposits); intended to measure assets most widely used by private sector in transactions

M2: M1 + savings deposits, small-denomination savings deposits, retail money market mutual funds (savings deposits); not used directly in transactions but easily exchanged for currency

② money in the intertemporal model

i) what does money model?

people use money to transact because we don't have ideal "real units"

↳ in real life, a barter economy cannot work because of a double coincidence of wants

↳ money is the solution: a traded placeholder of value ie. we need money for all transactions

2) how much money do we pay for a unit of consumption?

↳ money as a unit is a placeholder for some unit(s) of consumption.

↳ But how many units? Suppose there are more dollars in the economy than units of real consumption at equilibrium - what then?

the terms of trade between a dollar and a unit of consumption changes to reflect the dollars & units of consumption available

concept of price level: The ToT between a unit of money and a unit of consumption

3) nominal & real variables

↳ money as a placeholder of real value (how much money = real unit is determined by the money market) introduces the notion of price.

↳ nominal value : value of x in units of money

↳ real value : value of x in some ideal unit of value / consumption

inflation rate: change in price level across periods from now to then $i_t = \frac{P_t}{P_{t+1}}$

Fisher relation

↳ from one period to the next: $1 \Rightarrow 1+r$

↳ but in nominal terms, price levels may have changed across periods

↳ we need to relate nominal to real variables.

normalizing for price levels

$$1+r_t = (1+R_t)\frac{P_t}{P_{t+1}} = \frac{1+R_t}{1+i_t} \Rightarrow r_t = R_t - i_t - r_t i_t$$

$$R_t \approx r_t - i_t \text{ if } (r_t i_t) \text{ is small}$$

rewriting the optimization problem: multiply real variables w/ price in period t

consumers

$$\max_{c_1, c_2, l_1, l_2, s_1} u(c_1, c_2, l_1, l_2)$$

$$\text{s.t. } P_1(c_1+s_1) = P_1[w_1(h-l_1) + \pi_1 - T_1]$$

$$P_2(c_2+T_2) = P_2[(1+r)s_1 + \pi_2 + w_2(h-l_2)]$$

government

$$P_1 G_1 = P_1(B_1 + T_1)$$

$$P_2[G_2 + \underbrace{(1+r)(B_1)}_{\text{real variable}}] = P_2 T_2$$

firms

$$\max \; P_1\pi_1 + \frac{P_2\pi_2}{1+r}$$

4) The money market

↳ how is this ToT determined? ⇒ the money market — balancing demand for units of representation & supply

money supply

↳ suppose the central bank is able to control the amount of money circulating in the economy. Possibly by law or buyback operations.

⇒ so M^S_t is an exogeneous variable like G, decided by the central bank. Note that like G, we haven't strictly modelled why they might want to do so

money demand

↳ in an ideal barter economy, we would be able to measure our wealth in units of consumption and trade those. Generalizing to money, we would ideally want money to do just that : $1 = 1 "unit of consumption"

⇒ that is real money demand : breaking down all real units of consumption / value produced in the economy into tradeable units

units of tradeable currency desired (real demand) ⇒ $L(Y_t, r)$

Y_t: more transactions, more money, $\frac{\partial L}{\partial Y} > 0$

r: real interest rate. higher i/r, higher opp cost of consuming now, need less units for consumption, $\frac{\partial L}{\partial r} < 0$

↳ but because there is some fixed units of money circulating in the economy, some ToT between unit of real consumption & $ is determined by the market.

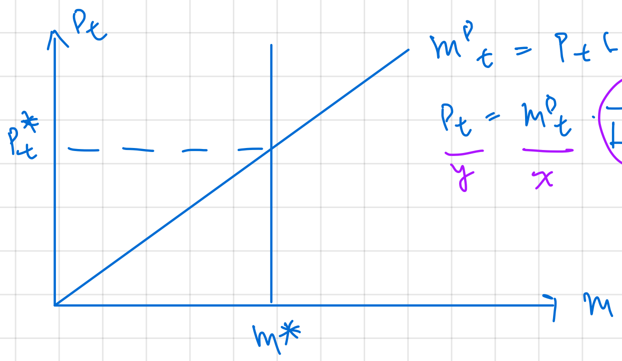
⇒ to convert to nominal money demand, we multiply by P_t

$$m^D_t = P_t L(Y_t, R_t) = P_t L(Y_t, r_t - i_t - i_t r_t)$$

(R_t) ↳ in nominal terms

equilibrium in the money market

↳ ToT (price level) is determined by infinitely inelastic supply (controlled) and nominal demand for money.



$m^D_t = P_t L(Y_t, R_t)$

$\underset{y}{P_t} = \underset{x}{m^D_t} \cdot \frac{1}{L(Y_t R_t)}$ $\frac{1}{\text{real demand}}$ ⇒ gradient

⇒ money market clears when money supplied = money demanded in each period

$$m_t^* = m^{S*}_t = m^{D*}_t$$

④ The money intertemporal model

In a cash-in-advance economy, given $\{G_t, T_t, M^S_t\}$ exogeneously, a competitive equilibrium is the allocation $\{C_t^*, S_t^*, m_t^*, N_t^{S*}, N_t^{D*}, I_t^*\}$ and prices $\{r_t^*, w_t^*, P_t^*\}$ for all $t \in \{1,2\}$ such that:

1. The representative consumer chooses $\{C_t^*, S_t^*, M_t^*, N_t^{S*}\}$ such that he is maximising lifetime utility, taking $\{r_t^*, w_t^*, P_t^*\}$ as given

2. The representative firm chooses $\{N_t^{D*}, I_t^*\}$ taking $\{P_t^*, w_t^*, r_t^*\}$ as given to maximise life-time profit

3. The labour market clears at $\{w_t^*, N_t^*\}$ in each time period when labour supply N_t^{S*} = labour demand N_t^{D*}

4. The goods market clears at Y_t^* at each time period when goods supplied $Y_t^S = Y_t^D = C_t^* + I_t^* + G_t = Y_t^*$

5. The credit market clears at r_t^* when total savings S_t^* = total borrowing B_t^* by the government

6. The money market clears at m_t^*, P_t^* when money supplied M_s = money demanded $m_t^{D*} = m^*$

⑤ money neutrality

Definition: A model displays money neutrality when a change in the quantity of money leaves the real variables unchanged, and therefore only nominal variables are affected.

⇒ money intertemporal model exhibits money neutrality

Intuition

↳ qt. of money is determined exogeneously by central bank in our model

↳ it is also not a decision variable in our model, because real variables are solved for in the labour, goods & credit markets, and the price level is then determined given real output and r^* in the money market.

eg. suppose ΔM_t^S and markets initially in eqm

⇒ $\Delta M^S_t \rightarrow$ no change in any real variables, no change in $L(Y^*, R^*)$

$\rightarrow$ but $P^* \rightarrow \tilde{P}$

$$\Rightarrow \frac{\Delta M_t}{M_t} = \frac{\Delta P_t}{P_t}$$

⑥ The liquidity - money (LM) curve

the LM curve A curve that shows all (r_t, Y_t) pairs where the money market is in equilibrium. It is effectively a representation of the money market clearing condition in the goods market at a given m^* and p^*

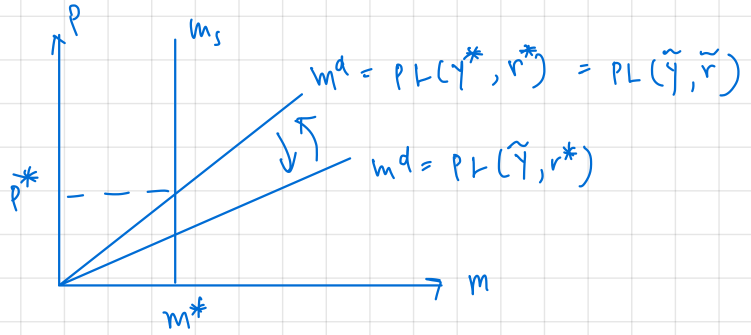
how is the LM curve shaped?

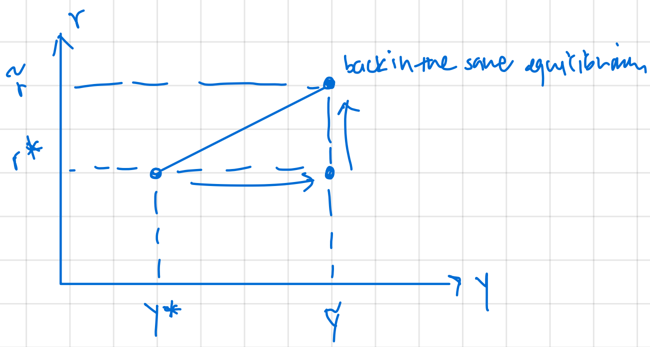
↳ a given money market equilibrium is described by p^* and m^*.

↳ at a given $r = r^*$, if we increase y to $\tilde{y}$, money demand increases since more money is needed for transactions. Real demand rises and so prices fall. But this is no longer the same equilibrium.

↳ to restore the original equilibrium, we will need to lower money demand. A rise in interest rate will do so, since individuals will want to hold less money.

⟹ the LM curve is upward sloping

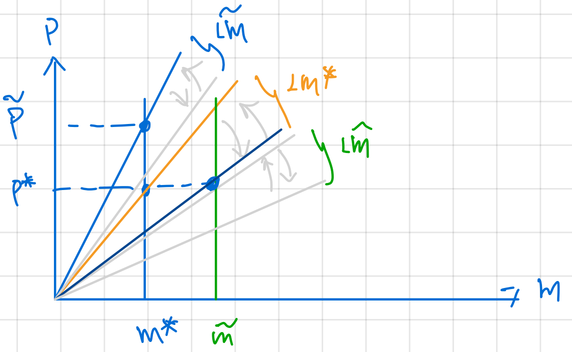


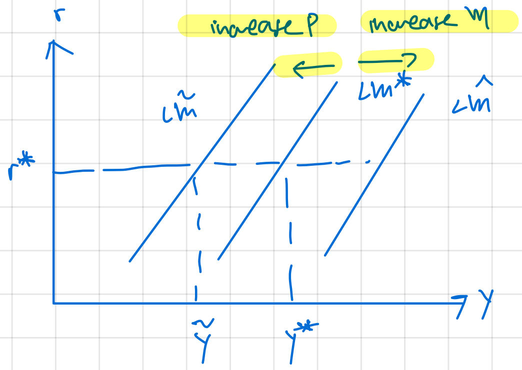


how do we interpret the LM curve?

↳ the LM curve describes the relationship between y and r that must hold to maintain the money market at a given equilibrium m^*, p^*

↳ suppose we want to describe a eqm at m^*, $\tilde{p}$, $\tilde{p} > p^*$. That would be describing a scenario with smaller <u>real</u> money demand, or equivalently, at every r, a smaller y. ⟹ leftward shift in LM





↳ suppose we want to describe $\tilde{m}, p^*$, $\tilde{m} > m^*$. Then that would be equivalent to real money supply $\frac{m}{p}$ increasing, as if at a given interest rate, y has increased.

↳ movement <u>along</u> LM curve : along an LM curve, $\frac{M}{P}$ is constant. As Y increases (number of transactions increase), r must intuitively also increase since more people borrow to support spending

Business cycles

① what are business cycles?

business cycles: fluctuations about trend in real GDP

peaks and troughs: Turning points, where peaks are positive deviations & troughs negative.

amplitude: the maximum deviation from trend

frequency: number of peaks in real GDP that occur per year

boom: a series of positive deviations from trend culminating in a peak

recession: a series of negative deviations from trend culminating in a trough

② features of business cycles

1. deviations from trends are persistent — they do so for a while at a time
   ↳ note that we threshold to consider something as deviating
2. no real regularity in amplitude or frequency of fluctuations about trend. Size and length of time between peaks / troughs vary

③ comovement of economic variables

↳ while real GDP fluctuates in irregular patterns, other macroeconomic variables tend to fluctuate together with each other and real GDP in patterns that exhibit strong regularities

⇒ we can find these patterns in variability using statistics. To study patterns in deviation, we detrend the data to get their deviation

cyclical: A variable is pro/counter-cyclical if its deviation from trend are positively/negatively correlated with deviations from trend of real GDP. If not correlated, a variable is acyclical.

timing: if a variable x tends to aid in the prediction of the future path of another variable y, we say that x is leading variable and y is the lagging variable. If they move at the same time, then they are coincident.

④ observations of some macroeconomic variables

variable	cyclicality	timing
consumption	pro-	coincident
investment	pro-	coincident
price level	counter-	coincident
employment	pro-	lagging
real wages	pro-	?
avg. labour productivity	pro-	coincident
nominal money supply	pro-	leading

⇒ how do macro models
model & explain these observations?

Business cycle models

① Neoclassical vs. new keynesians

↳ modern economists agree that macroeconomic models should have microfoundations : describe preferences, endowments & optimizing behavior of agents, market equilibria modelled

↳ but neo-classical economists believe that prices adjust quickly, and markets function best without unnecessary interference

↳ neo-keynsians believe that prices are sticky — in the short run, price may fail to adjust and markets do not always clear — so there is a place for active government intervention

② Neoclassical model : Real business cycle model

↳ key idea : can a money intertemporal model where prices adjust quickly match & explain business cycles & key economic data ?

1) TFP & Solow residual

↳ in the money intertemporal model, we model production output $Y_t^s = z_t F(K_t, N_t) = Y^D$ at equilibrium.

Solow residual : a statistical measure of TFP z_t

↳ observation : the detrended solow residual tracks closely w/ business cycle data of real GDP (measure of Y_t) detrended

⇒ question : can random fluctuations in z_t in the intertemporal model replicate business cycles ?

2) modelling business cycle fluctuations as TFP shocks

↳ if predictions by model match the data, then we have an understanding of business cycles

variable	cyclicality	timing
consumption	pro-	coincident
investment	pro-	coincident
price level	counter-	coincident
employment	pro-	lagging
real wages	pro-	?
avg. labor productivity	pro-	coincident
nominal money supply	pro-	leading
TFP	pro-	coincident

persistent productivity shocks

↳ the solow residual is a persistent variable — when it is above (below) trend, it tends to stay there

↳ so when we model it, we can consider it as a change in <u>both</u> z_1 & z_2

(Δz_1 & Δz_2 in money intertemporal model)

↳ suppose we model $\uparrow z_1$ and $\uparrow z_2$. we expect - $C_1\uparrow$, $I_1\uparrow$, $N_1\uparrow$, $w_1\uparrow$, $Y/N_1\uparrow$, $ms_1\uparrow$, $P_1\downarrow$

1. An increase in z_1 has the effect of increasing mpN_1. So, at the current wage w_1, firm will hire more, increasing qt. of labour demanded at every wage level.

   1.1 this is represented by a shift from N_1^D to $\tilde{N}_1^D$ in the labour market.

   1.2 So the labour market adjusts to a new equilibrium at $\tilde{w}_1$ and $\tilde{N}_1$ where $\tilde{N}_1^D = N_1^S$

   1.3 this has the effect of increasing output supply from Y_1^S to $\tilde{Y}_1^S$ in the goods market.

2. An increase in z_2 has the effect of increasing mpK_2. Taking interest rate as given r^*_1 the profit maximising firm will choose to invest more in capital since the returns from capital will be greater than interest returns.

   2.1 Also, consumers expect higher income in period 2, and so consume more in period 1 due to income effect.

   2.3 these are represented by a shift from Y_1^D to $\tilde{Y}_1^D$.

3. There are simultaneous shifts in Y_1^D and Y_1^S. But it is likely that $\Delta Y_1^S > \Delta Y_1^D$, since the changes in output supply arise from actual changes in z_1, while changes in demand arise from <u>anticipated</u> changes in z_2.

4. At the current interest rate r^*, $Y_1^S > Y_1^D$. So the goods market cannot be in equilibrium.

   4.1 with a shortage of demand for goods, consumers will borrow less, causing the interest rate to fall. As the interest rate falls, consumption rises since the price of consumption today to tomorrow at $(1+r)$ falls and the substitution effect dominates. The opportunity cost of investment in capital also falls as the returns from alternative assets fall, so I_1^D increases.

   4.2 These are represented by a movement along $\tilde{Y}_1^D$.

   4.3 As interest rate falls, the relative price of leisure today to tomorrow at $\frac{w_1(1+r)}{w_2}$ falls, and since the substitution effect dominates, labour supply falls.

   4.4 So output supply also falls, as a movement along $\tilde{Y}_1^S$

   4.5 These effects continue until the goods market clears at $\tilde{r}_1$ and $Y_1^S = Y_1^D = \tilde{Y}_1$.

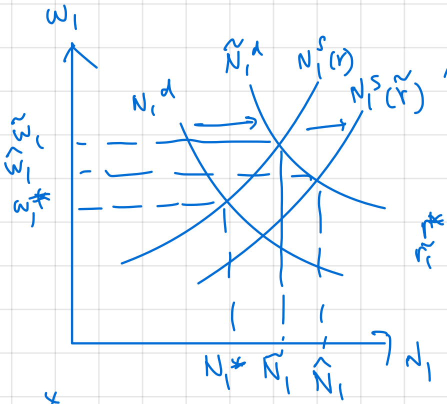
4.6 And in the labor market $N_1^s(r_1^*)$ has shifted to the right to $N_1^s(\tilde{r}_1)$ and $N_1^s = N_1^d = \tilde{N}_1$ and $\tilde{w}_1$ has fallen to $\hat{w}_1$
is higher, so $\hat{w}_1$ remains higher than w_1^*.

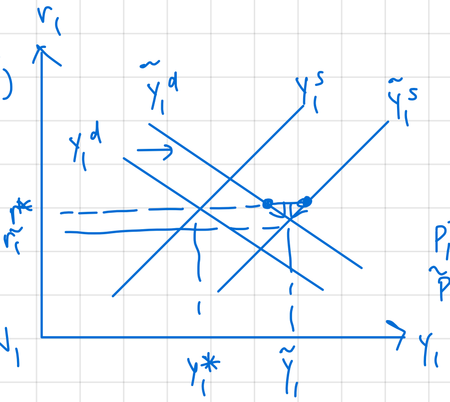
5. In the money market, as real output has risen and real interest rate has fallen, there is a rise in money demand. So the price level falls from P^* to $\tilde{P}$.

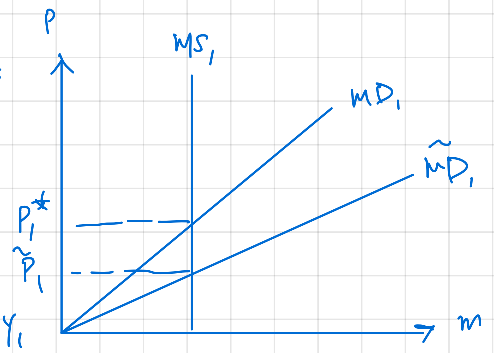
6. In all, we can observe that as Z_1 and Z_2 have increased:

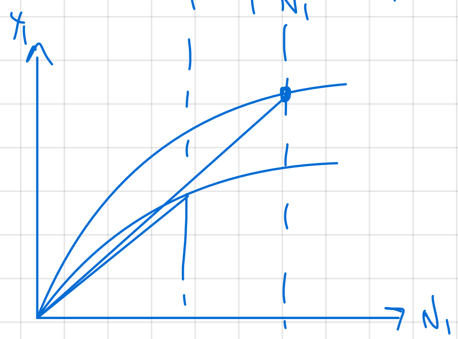
6.1 C_1, I_1, N_1, w_1, Y_1 and Y_1/N_1 have risen.

6.2 r_1 and P_1 have fallen.









eg. contracts — price is fixed, m is fixed

shock hits, price signal cannot adjust

not @ mkt clearing price

endogenous money

↳ data suggests that money supply is procyclical and leading in more recent years, and the comovement of price level & real output has fallen

⇒ we can explain by way of endogenous money supply response — central bank expects changes to price level and so preemptively raises money supply in advance — so MS leads real GDP and price & output are decoupled

③ implications & critiques of RBC

1) implications

neutrality of money — because markets clear as expected, money is neutral, consistent w/ our analysis of the money intertemporal model

no role for government in stabilization

1. money is neutral, so changes in MS have no real effects
2. business cycles are optimal responses to random shocks to TFP, so there are no inefficiencies to be corrected

↳ note that more elaborate models do account for distortions that require intervention

eg. distortionary effect of taxes

2) critiques

1. no real intuitive explanation for where TFP shocks come from
2. measurement errors — solow residual that tracks GDP closely may not be capturing true TFP due to labour hoarding during recessions

④ keynsian business cycle theory

1) fundamentals of the new keynsian model

↳ like neoclassical models, new keynsian models are also rooted in microeconomic foundations

↳ however, a fundamental difference is an assumption of sticky prices in the money intertemporal model

↳ these sticky prices are justified by contracting or menu costs — so firms are unable to charge their optimal price

⇒ prices in a given period are fixed before the period, and firms must commit to them. so we take P_t as fixed.

2) modelling sticky prices in the money intertemporal model

money market

↳ because prices are fixed, we take both M^S and P as exogeneous and fixed

Goods market

↳ we can model the fact that money market must always clear in the goods market by using the LM curve to represent the fixed state of the money market

⇒ the goods market equilibrium is where Y^d and LM intersect

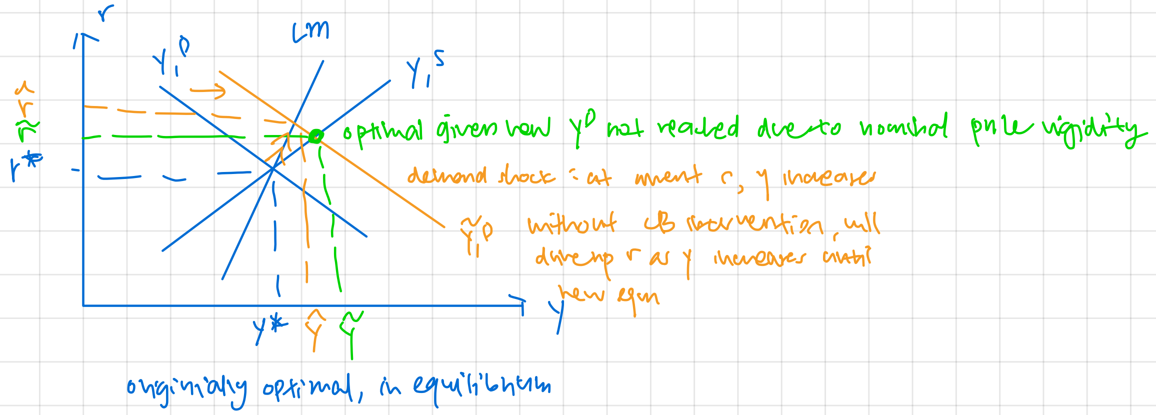
↳ Intuition:

money is needed for all transactions. In a model with flexible prices, prices adjust simultaneously with the underlying real goods demand & supply, so all agents are acting optimal

However, in the short run, prices are fixed due to menu costs or contracting. So, prices no longer serve as an effective transmission mechanism to signal real changes in Y_1^d and Y_1^s.

Prices also serve as the price at which firms are willing to transact. As a result, we now have "demand creating its own supply".

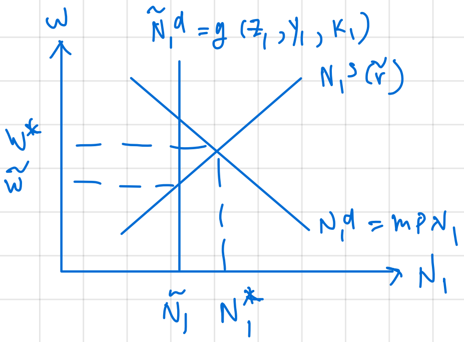
The LM curve describes combinations of Y and r at which the money market clears. Without central bank intervention, because prices are nominally rigid, y and r will adjust along the LM curve through i/r changes, as will C(r) and I(r) along the LM curve



Labour market

↳ because goods market equilibrium is not where $Y_1^d = Y_1^s$, but rather where $Y_1^d =$ LM, firms are not producing at their optimal level of output — simply wherever the goods market clears. So they hire labour accordingly.

⇒ labour demand, $N_1^d \neq MPN_1$, but infinitely elastic. $\tilde{N}_1^d = g(z_1, Y_1, K_1)$

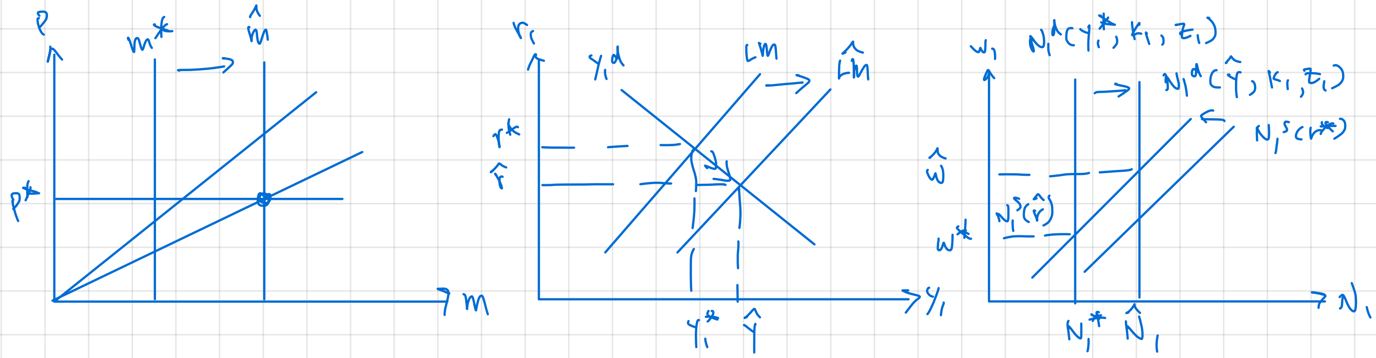


3) comparative statics in the NK model

exogenous changes in m_1

1. change in m_1. P is fixed. so Y and r must shift to reach new equilibrium in the money market. illustrated by LM curve shift. ⇒ $\hat{Y}$, $\hat{r}$, $\hat{m}$, P^*

2. change in Y_1. so, change in N_1^d since change in labour input needed. ⇒ $\hat{w}$, $\hat{N}$

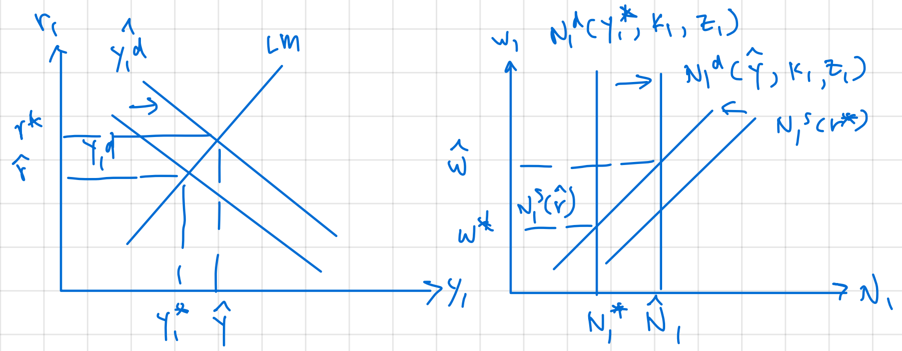


⇒ monetary transmission mechanism, non-neutrality of money

changes in Y_1^d

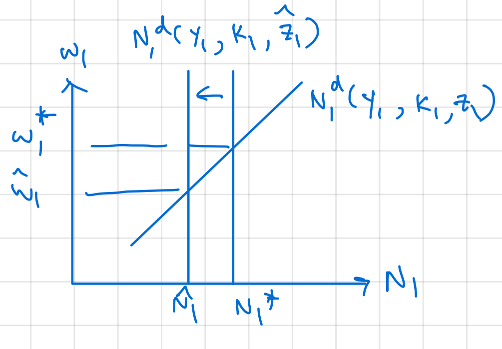
1. exogenous change in G_1, G_2, z_2. Then Y_1^d will change. Y_1^s also might, but since eqm. condition not involving Y_1^s, ignore

2. shift in Y_1^d, change in Y to $\hat{Y}$ in goods market. Firms need to hire more. So w shift to $\hat{w}$, N_1 to $\hat{N}_1$.



changes in z_1

1. no change in the money market
2. no change in the goods market
3. change in z_1, so firms can produce more w/ less labour. lower labour demand. w_1 falls to $\hat{w}_1$ and N_1 to $\hat{N}_1$ along N_1^s.



4) including endogeneous money market intervention

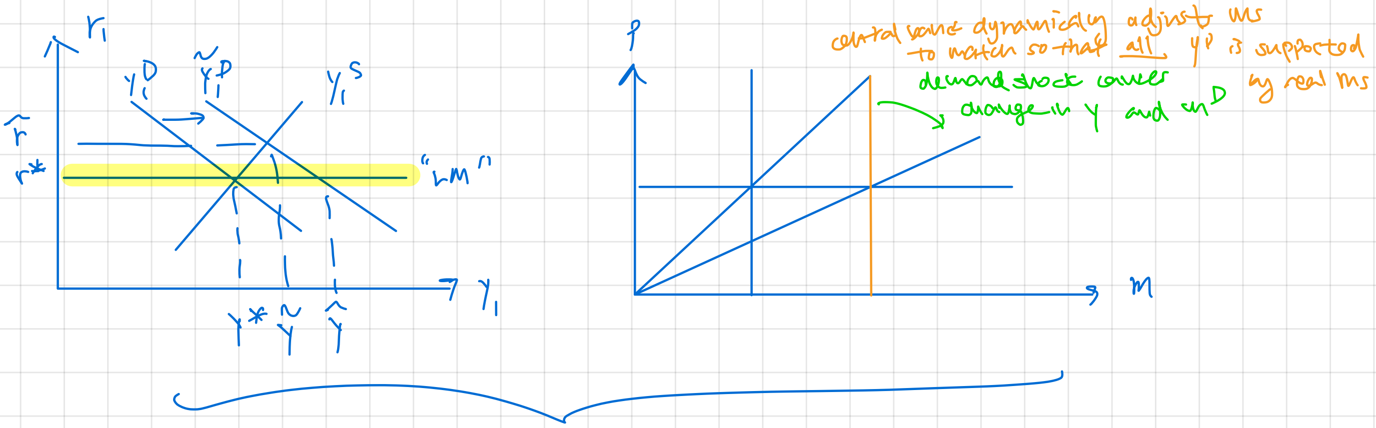
↳ suppose we model that central bank intervenes to provide laxity in money supply so that despite price rigidity, no Y–r tradeoff

i.e. ms changes to keep r constant

$\boxed{\text{goods and money markets}}$

↳ without CB intervention, because real money supply is rigid, Y and r adjust to demand shocks along LM curve

↳ if CB intervenes to maintain r via ms to support changes in Y w/o changing cost of borrowing, then "LM" is flat.



as borrow more to purchase @ current prices, no change to r because CB provides laxity via ms

5) business cycles as expected changes Z_{t+1}

↳ Z_{t+1} models expectations about the future, and are in some ways random

$\uparrow Z_{t+1}$

expected $\uparrow Y_2$, income effect

expected $\uparrow MPK_2$

$N_1^S \downarrow$ → $Y_1^S \downarrow$

$C_1 \uparrow$ → $Y_1^d \uparrow$

$\uparrow I_1$ → $Y_1^d \uparrow$

procyclical

$Y_1^d \uparrow$ → $N_1 \uparrow$, $w_1 \uparrow$

optimal : $Y_1^d = Y_1^S$

sticky price : $Y_1^d = LM$

} output gap

5 implications of the New Keynsian model

1) non-neutrality of money

↳ in a model ē flexible prices, prices serve as effective signals of underlying real variables. ie. they adjust to real variables. So, they do not in themselves change anything real.

↳ in the New Keynsian model, prices are not free to adjust, and because they are normally the intermediaries between demand and supply, they now have real effects

eg. ΔMS (nominal variable) but P is fixed

$\Rightarrow$ so r changes

$\Rightarrow$ optimising consumers change their purchasing decisions in current period

$\Rightarrow$ firms change their I_1 decisions

$\Rightarrow$ firms change their outputs Y and thus $\underbrace{\text{labour demand}}_{w,\ N}$

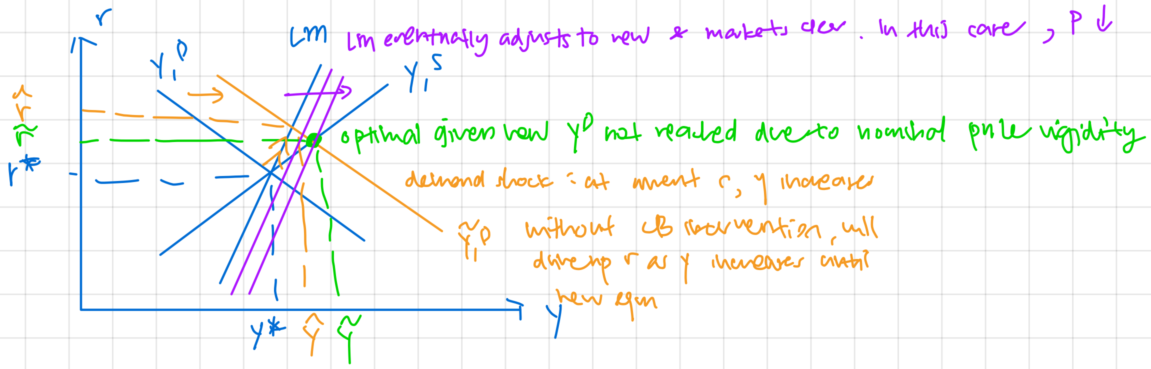
} Y, r

2) optimal policy

Stabilisation policy

↳ NK models are based on the idea that P does not adjust in the short run, causing output gaps (positive or negative). inefficiencies since $Y_1^d \neq Y_1^s$, etc.

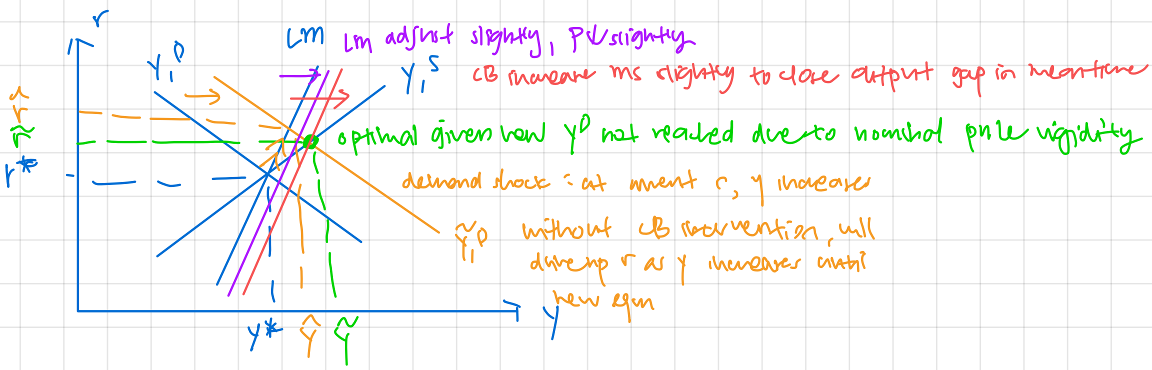
↳ left on their own, these prices will eventually adjust and markets will clear



originally optimal, in equilibrium

↳ intervention can also smooth these by shifting LM via MS until it catches up on its own

↳ LM curve describes the state of the money market — suggests that ΔMS could lead to changes in LM and eqm r, y. By figuring out where Y_1^d and Y_1^s would intersect given some shock, a central bank could adjust ΔMS to tune real money supply such that $y_1^d = LM = y_1^s$ and achieve efficiency while waiting for P to adjust

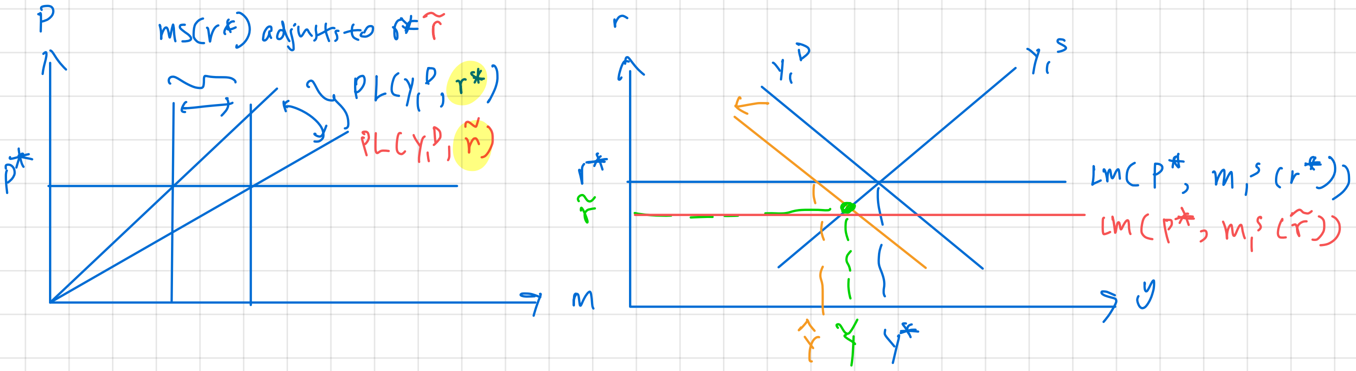


originally optimal, in equilibrium

$\boxed{\text{monetary policy } \bar{c} \text{ interest rate targets}}$

↳ endogeneously, we have it that CB adjusts to changes in Y_1^D so that real money supply is lax and r is stagnant

↳ as above, in the short run because P is rigid, we want to adjust ms so that the degree of laxity in the credit market is such that as y increases, r also increases, to coincide where $\tilde{y}_1^D = \tilde{y}_1^S$.



initially in equilibrium

demand shock. CB's ms laxity @ interest rate target r^* allows y to fall.

CB adjusts ms laxity to always match y such that r fixed at $\tilde{r}$ in the credit market

r causes changes in rational consumer & firms I(r), C(r) choices ⇒ y adjusts to $\tilde{y}$

liquidity trap

↳ suppose $r = 0$

↳ $r < 0$ is not offered, because if it were, you can arbitrage by simply holding cash and then depositing it

⇒ when r cannot go lower, notice that m_D is virtually flat and cannot lower any more → LM cannot move expansionary-ly anymore

fiscal policy

↳ as our analysis $\bar{c}$ Δy_1^d, ΔG can restore equilibrium by matching the output gap

↳ notice that because money is not neutral: P, w are fixed, then there is no change in r, and equivalently there is no crowding out due to G

government intervention
helps to smooth over inefficiencies in the short run

3) critiques of NK models

1. questionable theory underlying price stickiness
   ↳ maybe prices seem unoptimally sticky, but are actually optimally sticky
   eg. contracts

   ↳ menu costs are very small compared to changing output

⑥ Neoclassical vs. neo keynsian models

neoclassical	neo keynsian
all markets clear, efficient	not all markets clear, no competitive eqm, output gap
⇒ no role for government	⇒ role for government

fluctuations due to $\Delta z_1, z_2$

both fit data on comovement relatively well