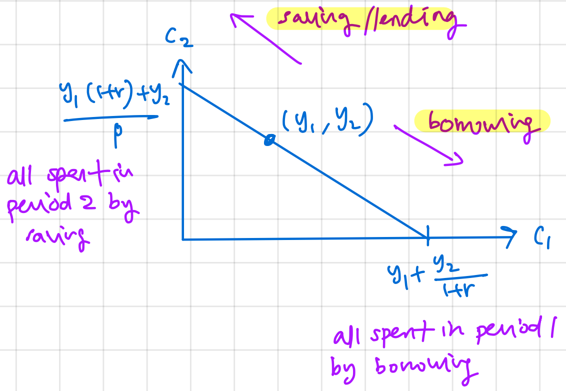
Intertemporal choice

LBC

$$PV = \frac{FV}{1+r}$$

$$c_1 + \frac{p \cdot c_2}{1+r} = y_1 + \frac{y_2}{1+r}$$

$$p \cdot c_2 = -(1+r)c_1 + y_1(1+r) + y_2$$



inflation

$$\frac{P_2}{P_1} = 1+\pi$$

$$1+\rho = \frac{1+r}{1+\pi} \approx 1+r-\pi \text{ when } \pi \approx 0$$

ρ: real i/r; r: nominal i/r; π: inflation rate

utility maximisation

$$U(c_1, c_2) = U(c_1) + \beta \cdot U(c_2)$$

same function

$\beta \in (0,1)$ — weighting

$$\max_{c_1, c_2} U(c_1, c_2)$$

s.t. budget constraint

sub & FOC

or

$$\frac{MU_{c_1}}{MU_{c_2}} = 1+r, \text{ sub LBC}$$

comparative statics

change in i/r

1) substitution effect: diff gradient, same ID curve
2) income effect: same gradient, diff ID curve

⇒ do income first then substitution

⇒ ask: is endowment still affordable?

i/r ↑ → relatively more expensive to consume in period 1, and need to pay more interest in second period / gain more interest in second period

↳ lenders stay lenders

↳ borrowers unknown

- borrower
  - substitution: c_1^* ↓, c_2^* ↑
  - income: c_1^* ↓, c_2^* ↓
  - loss from interest smoothed over both periods
  - c_1^* ↓, c_2^* ?
- lender
  - substitution: c_1^* ↓, c_2^* ↑
  - income: c_1^* ↑, c_2^* ↑
  - gains from interest smoothed over both periods
  - c_1^* ?, c_2^* ↑

i/r ↓ → relatively less expensive to consume in period 1, need to pay less interest over lifetime, gain less from interest over lifetime

↳ borrowers stay borrowers

↳ lenders unknown

- borrower
  - substitution: c_1^* ↑, c_2^* ↓
  - income: c_1^* ↑, c_2^* ↑
  - c_1^* ↑, c_2^* ?
- lender
  - substitution: c_1^* ↑, c_2^* ↓
  - income: c_1^* ↓, c_2^* ↓
  - c_2^* ↓, c_1^* ?

choices under uncertainty

① preferences under uncertainty

Expected utility

$$EU = \sum_{i=1}^{n} \pi_i \, u(c_i)$$

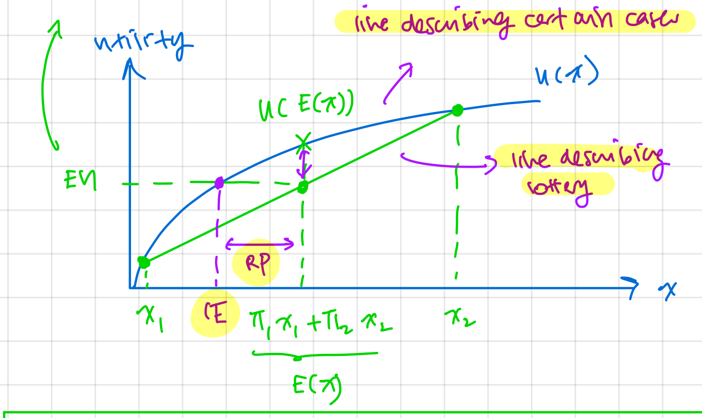
→ same individual, same u function across states

→ probability weighted sum due to mutual exclusivity of states

② risk aversion

EU vs. $u(E(x))$

$\pi_1 \cdot u(x_1) + \pi_2 u(x_2)$



$u(E(x)) > EU$ certain payout preferred to lottery ⇒ risk aversion

$u(E(x)) < EU$ lottery preferred to certain outcomes ⇒ risk loving

$u(E(x)) = EU$ ⇒ indifferent, risk neutral

1. diminishing MU & concavity for risk averse consumers
   ↳ more wealth, less risk averse
   ↳ consumption smoothing behavior

2. convexity for risk preferring consumers
   ↳ extreme outcomes are preferred

certainty equivalence & risk premium

$EU = \pi_1 x_1 + \pi_2 x_2 \ldots$

$CE = u^{-1}(EU)$ → certain money value to get same utility as expected returns

$RP = E(x) - CE$

the difference between these two measure your risk aversion because even though on average you will make more w/ $E(x)$ (doing ∞ lotteries), you are willing to give up the difference so that you will certainly get CE

arrow-pratt measure

↳ risk averse curves are concave & degree of concavity measures risk preference

$$APM = -\frac{u''(x)}{u(x)}$$

risk averse: $u''(x) < 0$

$u(x) > 0$, normalises $u''(x)$

because $u'(x) > 0$ for reasonable $u(x)$ functions & we want to measure aversion

③ optimal contigent consumption

1. set up contingent consumption ↳ compare w/ endowment
2. get EU
3. optimize simultaneously wrt to unknowns

⇒ making a choice to change EU

↳ eg. how much to bet, how much to buy

↳ maximise EU wrt. to variable of choice

⇒ closed form eqn of optimal value of variable

⇒ solve simultaneously w/ max EU given ∞ unknowns

④ insurance

↳ given some payout K and fee $\gamma K + \beta$, consumer has to choose whether he wants to buy.

⇒ see if given these constraints, optimal consumption is better than endowment

$c_2 = y - (\gamma K + \beta)$

$c_1 = y - L + K - (\gamma K + \beta)$

$\max_{c_1, c_2} EU$

$\therefore c_2 = -\frac{\gamma}{1-\gamma} c_1 + \frac{y - \gamma L - \gamma \beta}{1-\gamma}$

$\frac{\gamma}{1-\gamma} = \frac{\pi_1 MU_1}{\pi_2 MU_2}$

$$E(\pi) = \gamma K + \beta - \pi K$$

fair $E(\pi) = 0$

$MU_1 = MU_2$,

$c_1 = c_2$

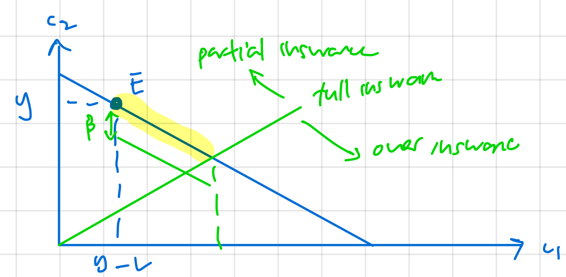
$u(c_1) = u(c_2)$

unfair $E(\pi > 0)$

$MU_1 > MU_2$

$c_1 < c_2$

$u(c_1) < u(c_2)$



Monopoly

① π maximisation

revenues

$TR = P(Q) \times Q$

$AR = \frac{TR}{Q}$

$MR = \frac{dTR}{dQ}$

costs

$TC = TVC + TFC$

$AC = \frac{TC}{Q}$

$MC = \frac{dTC}{dQ} = \frac{dTVC}{dQ}$

$\pi = TR - TC$

$\frac{d\pi}{dQ} = \frac{dTR}{dQ} - \frac{dTC}{dQ} = MR - MC = 0$

$\Rightarrow MC = MR$

② markup pricing

$PED = \frac{\text{rate of change in } Q}{\text{rate of change in } p}$

$= \frac{dQ}{dp} \cdot \frac{P}{Q}$

ε is always negative due to law of demand

$P = \frac{MC}{\left(1 - \frac{1}{|\varepsilon|}\right)}$ $\Big\}$ $MR = P \cdot \left(1 - \frac{1}{|\varepsilon|}\right)$

1. in PC, $P = MC$ because $PED = -\infty$
2. π maximising firm will never operate on inelastic part, since $MR < 0$ there.

③ inefficiency of monopoly

$CS = \int_0^{Q^*} P(Q) - P^* \, dQ$

$PS = \int_0^{Q^*} P^* - MC \, dQ$

④ natural monopoly & regulation

qt. taxation

$MC' = MC + t$

$P' = \frac{MC + t}{1 - \frac{1}{|\varepsilon|}}$

$\Big\}$ tax burden $= P' - P = \frac{t}{1 - \frac{1}{|\varepsilon|}}$

lump-sum taxation

$\pi' = \pi - T$

AC pricing $P = AC, \pi = 0$

MC pricing $P = MC, \pi < 0$

⑤ price discrimination

uniform pricing

$Q = \sum_{i=1}^{n} q_i(p)$

$$Q(p) = \begin{cases} q_a & w \leq P \leq x \\ q_b & x \leq P \leq y \\ q_c & y \leq P \leq z \end{cases}$$

rest @ each range

$\max_{P,Q} \underbrace{\pi = P \cdot Q(P) - C(Q) \cdot Q(P)}_{\text{treatment as a single market}}$

3° price discrimination

$\pi = \sum_i TR_i - C\left(\sum_i q_i\right)$

simultaneously,

$\frac{d\pi}{dq_i} = MR_i - MC\left(\sum q_i\right) = 0$

$\vdots$

relative PED in sister markets

$MR_i = P_i\left(1 - \frac{1}{|\varepsilon_i|}\right) = MC$

$P_a > P_b$. $\quad |\varepsilon_b| > |\varepsilon_a|$

$\Rightarrow$ a firm that price discriminates sets low prices for price sensitive groups and high prices for insensitive ones

oligopoly

↳ firms try to respond optimally by maximising profit.

competition on price

$Q_A = f(P_A, P_B)$

$TR_A = P_A \cdot Q_A$

$MR_A = \frac{d\,TR_A}{dP_A} = MC_A$

best response price

$P_A^* = g(P_A, P_B)$

choose optimal price in response to some demand / revenue curve influenced by P_B or other factors

competition on qt.

$P_A = f(Q_A, Q_B)$

$TR_A = P_A \cdot Q_A$

$MR_A = \frac{d\,TR_A}{dQ_A} = MC_A$

best response qt.

$Q_A^* = g(Q_A, Q_B)$

choose optimal Q_A given some revenue curve influenced by Q_B or other factors

---

simultaneous or sequential?

simultaneous

firms optimizing wrt to each others' expected response

⇒ simultaneous eqn of best responses

simultaneous qt. ⇒ cournot eqm.

simultaneous price ⇒ Bertrand

leadership

↳ leader optimising, constrained by expectations of follower's responses

$\max_{Q_A, P_A} \pi_A$ ← sub into, then optimize leader

s.t. ... best response functions
... of followers, simultaneously
...

sequential qt. ⇒ Stackelberg

---

collusion?

↳ when firms collude, they behave like a single firm with distributed production

↳ we assume they are happy $\bar{c}$ pareto efficient distribution of profits

$$\max.\ \underbrace{p(Q_1 + Q_2 \ldots)(Q_1 + Q_2 \ldots)}_{\text{total revenue}}$$

$$- \underbrace{c(Q_1) - c(Q_2) \ldots}_{\text{individual costs}}$$

$\frac{d\pi}{dQ} \Rightarrow MR = MC_i(Q_i)$

no room for more π from getting one firm to produce more where $MR > MC$, and none making loss

$TR = P \cdot Q \Rightarrow MR$

$TC = c(Q_1) + c(Q_2) \ldots$

$MC =$ for each region, always pick lowest.
if equal, split production equally.
⇒ sum to get indiv Q_i over all zones

cheating vs. punishment

π_{cheat} → best response to cartel members's fixed variable (P or Q)

⇒ grim trigger : kick out of cartel after current round if cheat

$\pi_{coop} = \pi + \delta(\pi) + \delta^2(\pi) \ldots$

$= \frac{\pi}{1-\delta}$

⇒ compare π_{coop} vs. π_{cheat} vs π_{comp}

Game theory

nash equilibrium an allocation where for every player, their choice is optimal given all other players' choices

i.e. no incentive to deviate for any player

pure strategy games players choose strategies

mixed strategy games players choose frequency of employing each strategy

---

① PS games

dominant & dominated

1. for every opponent's play, find the optimal strategy
2. same optimal for all opponents' plays & unique (never tied) ⇒ **strictly dominant**
3. never picked or tied ⇒ **strictly dominated**
4. always optimal and tied at least once ⇒ **weakly dominant**
5. never solely optimal but tied at least once ⇒ weakly dominated

---

solving w/ iterative deletion

1. find strictly dominated strategy and delete it
2. recurse. If no more

solving w/ best response

two players

many players

Fundamentals of game theory

① pure strategy simultaneous games

Dominant & dominated strategies

strongly dominant strategy with highest payoff no matter what opponents choose — always best response

weakly dominant strategy that is at least as good, if not better than all other strategies, for all opponents choices — always at least tied for best response

strongly dominated strategy that is always the worst choice

weakly dominated strategy that is always worst but tied for worst at least once

never best response, but never best response $\not\Rightarrow$ dominated

neither sometimes worst, sometimes best

---

iterative deletion of dominated strategies

1. find strictly dominated strategy, delete it
2. recurse

solving for pure strategy nash equilibria

Payoff matrix : two players

1. set up payoff matrix
2. iterative deletion while there are strictly dominated strategies
3. find optimal response for each player given other player's responses
   ⇒ intersections are nash equilibria

best response functions : many players

1. consider different states players might be in
2. set up equations modelling each player in each of these scenarios and/or payoff matrix (in equation form)

remember that players have perfect information!

3. find equilibria
   ↳ inequalities / simultaneous equations of BRF
   ↳ look for incentive to deviate unilaterally

$\max u_A$
s.t. $u_i,\ i \in \{1, 2, \ldots\}$

special case : symmetric games

↳ each player faces same set of choices & payoffs ⇒ single equation modelling choice faced by by individual, solve

② mixed strategy simultaneous games

⇒ idea: because of perfect information, each player knows how frequently on average his opponents will choose a given choice. So he will adjust his own frequencies so that on average he is indifferent between his random choices

solving for MSNE

↳ intuition: given frequencies of possible actions of opponents, an agent will look to choose a set of frequencies of his own actions, such that on average he is indifferent between all choices he can make.

1. For each player, find expected payoff given each of his choices, according to probability distribution of opponents play

$$E(\text{payoff to } A \mid A \text{ plays } x)$$

2. Form simultaneous equations by equating the expected payoffs for each of his choices

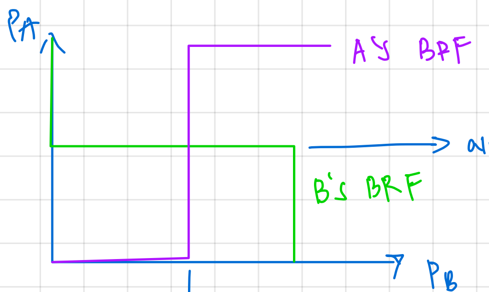
$$E(\text{payoff to } A \mid A \text{ plays } x) = E(\text{payoff to } A \mid A \text{ plays } y) = \dots$$

3. Solve simultaneously so that indifferent between choices ⇒ P_B

4. Repeat for every player

best response functions

eg. suppose A only has 2 choices, B only 2 choices.



→ along this P_A, B is indifferent

along this P_B, A is indifferent

③ PS repeated games

$$\boxed{\pi_{\text{don't cheat}} = \frac{\pi_{\text{don't cheat}}}{1-\delta}}$$

Nash reversion strategy

↳ idea : if you cooperate, I cooperate. If you cheat once, I cheat forever (grim trigger)

$$\boxed{\pi_{\text{cheat}} = \pi_{\text{cheat}} + \frac{\delta}{1-\delta}(\pi_{\text{don't coop}})}$$

Tit for tat

↳ idea: try to cooperate in the first round. For all other rounds, do what the other player did in the previous round

$$\boxed{\pi_{\text{cheat}} = \frac{\pi_{\text{cheat}}}{1-\delta^2} + \frac{\delta}{1-\delta^2}(\pi_{\text{don't coop}})}$$

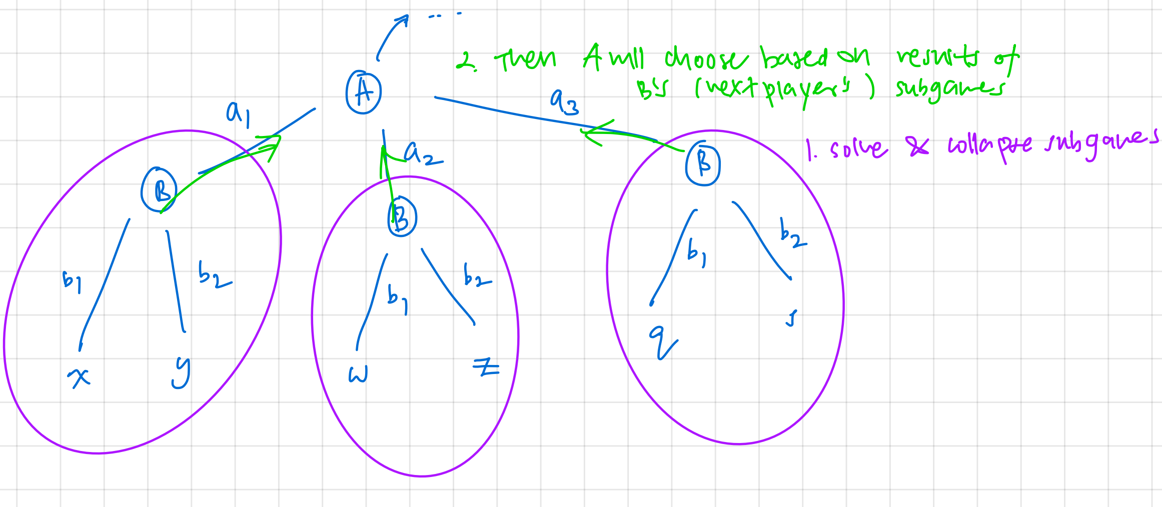
---

④ PS sequential games

backward induction finding the "subgame" NE

↳ idea: every player has perfect information. The net outcome depends on how others play in respond to one's play. So by looking ahead to see what each player would rationally do at each step, one can trace the outcomes back to see what choice we should make at the first step.

1. model the game in extensive form
2. Starting from the end, let the player choose his optimal at <u>all</u> subtrees of that height. The preceding player then considers those results
3. Repeat until first player. Now trace forward to get sequence of events



↳ in a scenario where a player is indifferent between choices at a given juncture, then we don't have enough information to solve.

⑤ PS Bargaining games

↳ bargaining games are sequential games where offers are made over a set of rounds

backwards induction

↳ in a finite set of bargaining rounds, we know how the game might end (say, winner takes all)

↳ we run backwards induction by this line of logic: a person will offer an amount that makes the person indifferent at his turn (including discount factor) and no more (this way, optimizes himself)

1. consider final state of final player. what is his guaranteed surplus?
2. Previous player knows this, will offer present value of that surplus + cost.
   2.1 Now, what is his surplus?

   $\boxed{\text{surplus} = \text{earnings} - \text{offer}}$

3. repeat until beginning

$A \xrightarrow[\delta_B \times \text{surplus} + \text{cost}]{\text{offer}} B$ surplus?

Externalities

① externalities

positive externality: $\dfrac{\partial u_B}{\partial x_A} > 0$

negative externality: $\dfrac{\partial u_B}{\partial x_A} < 0$

② social optimal

$$\frac{d\,TU}{dx} = 0 \iff MSB = MSC$$

merger

↳ merging affected agents with the original aligns their objectives, so the agent is now incentivised to account for how externalities from one decision variable may affect others, and select them in a way that maximises the overall objective

1. sum their objectives
2. differentiate w.r.t. to all decision variables & solve.

$$\pi_A(\overset{\text{externality}}{x}, y), \ \pi_B(z) \Rightarrow \pi(x,y,z) = \pi_A(x,y) + \pi_B(z)$$

$$\frac{\partial \pi}{\partial y} = 0, \quad \frac{\partial \pi}{\partial z} = 0, \quad \frac{\partial \pi}{\partial x} = 0 \ \Big\} \text{ optimal amt. of everything}$$

---

introduction of property rights

↳ coase theorem: if property rights of an externality are well defined and there are no transaction costs, bargaining between agents will lead to an efficient outcome regardless of the initial allocation of property

⇒ owner of externality can effectively sell it eg. polluter buys credits, advertiser sells benefit, and externality becomes a tradeable decision variable

$\pi_A(x, A) \rightarrow$ sells x for $\underbrace{p_x \cdot x}_{\text{revenue}} + \underbrace{F}_{\text{reservation cost}}$

$\pi_B(x, B) \leftarrow$ buys

← suppose $\pi_A^* \big|_{x=0}$ is greater than $\pi_A(x)$ for any x. ie. A loses by even trading the right. Then a flat fee is needed to make him indifferent.

$$\frac{\partial \pi_A}{\partial A} = \frac{\partial \pi_A}{\partial x} = 0$$

$$\frac{\partial \pi_B}{\partial B} = \frac{\partial \pi_B}{\partial x} = 0$$

↳ problem: transaction costs & ethics of defining property rights

---

taxation / subsidies

↳ in general, if we can impose a tax or subsidee, we force the agent to internalize the social cost or benefit of his actions

positive ext.

$$MB + s = MC$$

negative ext.

$$MB = MC + t$$

↳ problem: how much?

qt. limit

↳ impose the optimal limit onto each firm

⇒ as if merger, but distribute such that pareto improvement

↳ problem: impossible due to imperfect information

---

cap & trade

⇒ this is a levelled up formulation of bargaining, where each agent is endowed initially with some x (not just 1 agent) and they can trade it

1. set total quantity Q, max as if all merger

   $\max \sum \pi_i \Rightarrow x^* \longrightarrow$ split among each to buy & sell

2. constrained optimization

   $\pi_i =$ revenue $-$ cost $+$ revenue/cost of trade of credits

   s.t. sum of externalities $=$ quota

   initial $x_{i,0} = \frac{x^*}{n}$ split equally, can sell no more than $x_{i,0}$

⇒ optimize simultaneously

Public goods

① private provision & free riding

↳ if G were a private good (excludable, rival), then the optimal allocation occurs where $MRS_{ci,G} = \frac{P_G}{P_c}$. There will be no more room for improvement, since all individuals would be maximising their utilities given their endowments.

↳ because public goods are non-excludable and non-rivalrous, individuals who do not purchase it can always enjoy it. individuals who purchased it cannot stop them or charge them.

↳ so their payoff matrix would appear such that the dominant strategy is to not purchase it as long as cost > 0, since he is either indifferent if no one buys and better off if he free-rides someone else

⇒ left to private provision, so long as the cost of provision > 0 and pareto efficient allocation of G, $G^* > 0$, then there will exist pareto inefficiency due to free riders.

② pareto efficient level of public good

1. Suppose there are i agents, each with an endowed wealth w_i. Each agent can choose between consuming c_i and contributing some funds g_i to a public good. Because the public good is non-rivalrous and non-excludable, every agent consumes the same amount at G. Suppose p is the price of good, with price of public good as the numeraire.

$$\boxed{\begin{aligned} &\text{budget constraint}: \quad c_i + g_i = w_i \\ &\text{utility}: \quad u_i(c_i, G) = \bar{u}_i \end{aligned}}$$

2. To find all pareto efficient allocations of $(c_1, c_2 \dots c_n, G)$, we are trying to find all solutions that would (without loss of generality) maximise a given consumer's utility subject to all other's being held constant. let that be consumer 1.

$$\max_{(c_1, c_2 \dots G)} u_1(c_1, G)$$

$$\text{s.t. } p(c_1 + c_2 + \dots) + \underbrace{g_1 + g_2 \dots}_{cost(G)} = w_1 + w_2 \quad \text{— ①}$$

some arbitrary level of utility

$$\{u_i(c_i, G) = \bar{u}_i\}_i, \quad i \in [2, n] \quad \text{— } \textcircled{n-1}$$

3. we can perform lagrangian optimisation. The lagrangian is:

$$L = u_1(c_1, G) - \underbrace{\sum_{i=2}^{n} \lambda_i (u_i(c_i, G) - \bar{u}_i)}_{\text{all other's utilities}} - \mu \underbrace{\left(\sum_{i=1}^{n} p c_i + cost(G) - \sum_{i=1}^{n} w_i \right)}_{\text{total budget constraint}}$$

$$\left\{ \begin{array}{l} \frac{\partial L}{\partial c_i} \dots \\ \frac{\partial L}{\partial G} \dots \end{array} \right\} \Rightarrow p \sum_{i=1}^{n} \frac{\frac{\partial u_i(c_i, G)}{\partial G}}{\frac{\partial u_i(c_i, G)}{\partial c_i}} = \frac{\partial\, cost(G)}{\partial G} = \boxed{p \sum_{i=1}^{n} |MRS_{c_i, G}| = MC(G)} \quad \text{— ②}$$

Intuition

↳ sum of $MRS_{c_i,G}$ measures the total willingness to pay : in total, for 1 unit of G, how much c_i they would be willing to sacrifice.

↳ $MRS_{c_i, G(i)}$: how many c_i agent i is willing to chip in / trade for one more unit of G

$$p\sum_{i=1}^{n}|MRS_{c_i,G}| > MC(G)$$

Then collectively, there is enough total value that adding up what everyone is willing to chip in is enough to buy an additional unit of G

$$p\sum_{i=1}^{n}|MRS_{c_i,G}| < MC(G)$$

The collective amount everyone is willing to chip in is less than the cost of one unit of G. Everyone would be better off selling one unit of G and splitting those gains among their private c_i

$$p\sum_{i=1}^{n}|MRS_{c_i,G}| = MC(G)$$

No room for arbitrage, every \$ on c_i gives as much utility to the individual as how much they (& everyone else sharing the cost) is spending on G at g_i.

Asymmetric information

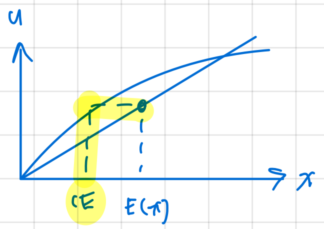
① adverse selection

adverse selection adverse selection happens when agents with more or better information in the market exploit agents with less information. ie. because agents cannot determine other agents' type, price signals are distorted

consumer willingness to pay

payment = certainty equivalence

↳ for risk neutral, $CE = E(x)$



pareto inefficiency & market failure

1. in a perfect market, consumers know the quality of the good - so they pay exactly how much they value it, knowing how much utility they gain out of it
   1.1 price mechanism serves as an efficient mechanism for distributing resources, no under- or over- production

2. suppose we have asymmetric information such that one party only has knowledge of the distribution of goods quality in the market.

3. then rational agents, depending on their risk profile, are only willing to pay their certainty equivalence (so risk neutral will pay $E(x)$)

4. sellers know this, so sellers who are only willing to sell above CE will not enter the market

5. Buyers also know how sellers react, leaving only low-quality sellers in the market. in some scenarios, this could cyclically lead to the market collapsing

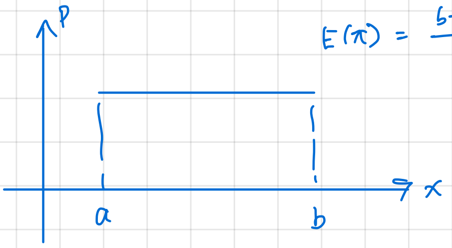
willingness to pay $= E(x) + k$
sellers will sell if $E(x) + k \geq x$

or some other value

$E(x) + k = x$ — those above x will leave the market

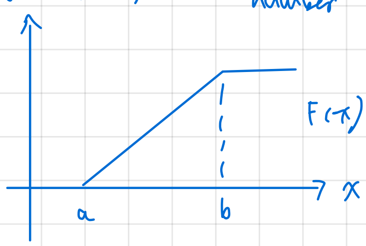
$$E(x) = \int_a^b x \cdot p(x)\, dx$$

uniform distribution



$$E(x) = \frac{b-a}{2}$$

cumulative $p \xrightarrow{x_n}$ cumulative number



$$F(x) = \begin{cases} 0 & x < a \\ \dfrac{x-a}{b-a} & a \leq x \leq b \\ 1 & x > b \end{cases}$$

---

adverse selection given quality choice

↳ suppose now sellers can choose whether to sell high/low quality goods. How will they choose, and what is the stable equilibrium?

1. case 1: only high quality
2. case 2: only low quality
3. case 3: mix
4. case 4: no market

1. how much are consumers willing to pay
2. Do sellers have incentive to deviate?

no

yes, stable

③ solutions to adverse selection

(regulation) banning low-quality goods → then side with less information can have a priori information to decide price to offer

---

(getting signals)

↳ suppose there are different quality of sellers, but buyers cannot distinguish them, and so offer the same price. This is known as a pooling equilibrium

eg. employee wages

$$\boxed{w_{pool} = (1-q)\,w_{low} + q\,w_{high}\ , \quad w_{high} > w_{low}}$$

↳ sellers have the option to spend some money and acquire some signal of quality. assuming that the cost of such a signal is different for different quality sellers, a separating equilibrium occurs when it is rational for a high quality seller to acquire the signal but not for a low quality seller:

$$\boxed{\text{cost of signal: } c_{high} < c_{low}}$$

1. $w_{high} - c_{high} > w_{low}$ ⟹ (1) utility gain after obtaining signal than without for high quality
2. $w_{high} - c_{low} < w_{low}$ ⟹ (2) low ability sellers worse off acquiring signals and so choose not to

↳ but, assuming signals do nothing to quality, they reduce total surplus since resources have to be expended acquiring those signals

---

④ moral hazard

(moral hazard) when agents are able to transfer risk without the other agent knowing and so take unnecessary risks

(inefficiency of moral hazard)

↳ suppose agents pay to transfer risk to another agent (eg. insurance)
↳ rationally, the merchant will charge a price that is representative of the risk of payout.

$$E(\pi) = (\gamma - \pi)\,K$$

γ = rate, π = true probability, K → payout

↳ in an instance of asymmetric information, the insured agent may engage in riskier behavior without the insurer knowing. Then the insurer will not know π, and will not be able to act optimally

⑤ solutions to moral hazard

regulation enforce the reduction of risky behavior (eg. annual car checks)

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incentive contracting under certainty

↳ abstractly, a worker is hired by a principal to do a task. Because of asymmetric information, only the worker knows how much effort, x, he puts in

the principal's problem: design an incentive contract that induce the rational worker to optimize the principal's payoff. The contract must fulfil two constraints to work:

1. participation: the contract must minimally provide the worker a level of utility that is equal to his opportunity cost of doing this: ie. leaves him indifferent ie. will want to work

2. incentive compatibility: The contract much be such that the optimal solution to the worker's utility maximisation problem is the π maximising level of effort. ie. will work optimally

$$\forall x \geq 0, \quad s(f(x^*)) - c(x^*) \geq s(f(x)) - c(x)$$

rental contracts

↳ principal keeps a lump sum R & worker keeps remaining

$$s(f(x)) = f(x) - R$$

wage contracts

↳ wage per unit effort + lump sum

$$s(f(x)) = wx + K$$

take it or leave it

↳ worker gets all or nothing

$$s(f(x)) = \begin{cases} L & \text{if } x = x^* \\ 0 & \text{otherwise} \end{cases}$$

The worker: optimise payoffs, u

$$\max_x u(x) = \max(\underbrace{\bar{u}}_{\text{reserve opp. cost}}, \underbrace{s((f(x))}_{\text{reward thru output}} - \underbrace{c(x)}_{\text{cost of effort}})$$

$$\Rightarrow \frac{du}{dx} = 0 \Rightarrow x$$

The principal: optimise π

↳ set A be a set of decision variables for incentive scheme $s(y) = s(f(x))$

$$\pi = \underbrace{f(x)}_{\text{earnings}} - \underbrace{s(f(x))}_{\text{payout to worker}}$$

1. find x^*. $\frac{d\pi}{dx} = 0 \Rightarrow x^*$

2. Solve for decision variables A such that incentive & participation compatible

incentive enforced

$$\max_A \pi = f(x^*) - s(f(x^*), A)$$

participation

$$\text{s.t. } s(f(x^*), A) - c(x^*) \geq \bar{u}$$

Incentive contracting under uncertainty

↳ suppose there is some uncertainty in the production process, so effort does not deterministically translate into output

⇒ then agents maximize <u>expected</u> payoff, and will make choices according to their risk preference

revenue-sharing

↳ worker receives a share of the output

$s(f(x)) = \alpha f(x), \quad 0 < \alpha < 1$

The worker: optimize expected payoffs

$$\max_x E[u(x)] = E[\max(\underbrace{\bar{u}}_{\text{reserve opp. cost}}, \underbrace{s(f(x))}_{\text{reward for output}} - \underbrace{c(x)}_{\text{cost of effort}})]$$

$$\Rightarrow \frac{du}{dx} = 0 \Rightarrow x$$

The principal: optimize $E(\pi)$

↳ let A be a set of decision variables for incentive scheme $s(y) = s(f(x))$

$$E(\pi) = E[\underbrace{f(x)}_{\text{earnings}} - \underbrace{s(f(x))}_{\text{payout to worker}}]$$

1. find x^*: $\frac{dE\pi}{dx} = 0 \Rightarrow x^*$

2. Solve for decision variables A such that incentive & participation compatible

incentive enforced

$$\max_A E(\pi) = E[f(x^*) - s(f(x^*), A)]$$

participation

$$\text{s.t. } \underbrace{E[s(f(x^*), A) - c(x^*)]}_{E[u(x^*)]} \geq \bar{u}$$