CAP 379
Artificial Intelligence Lab

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Lab 02
Predicate Calculus

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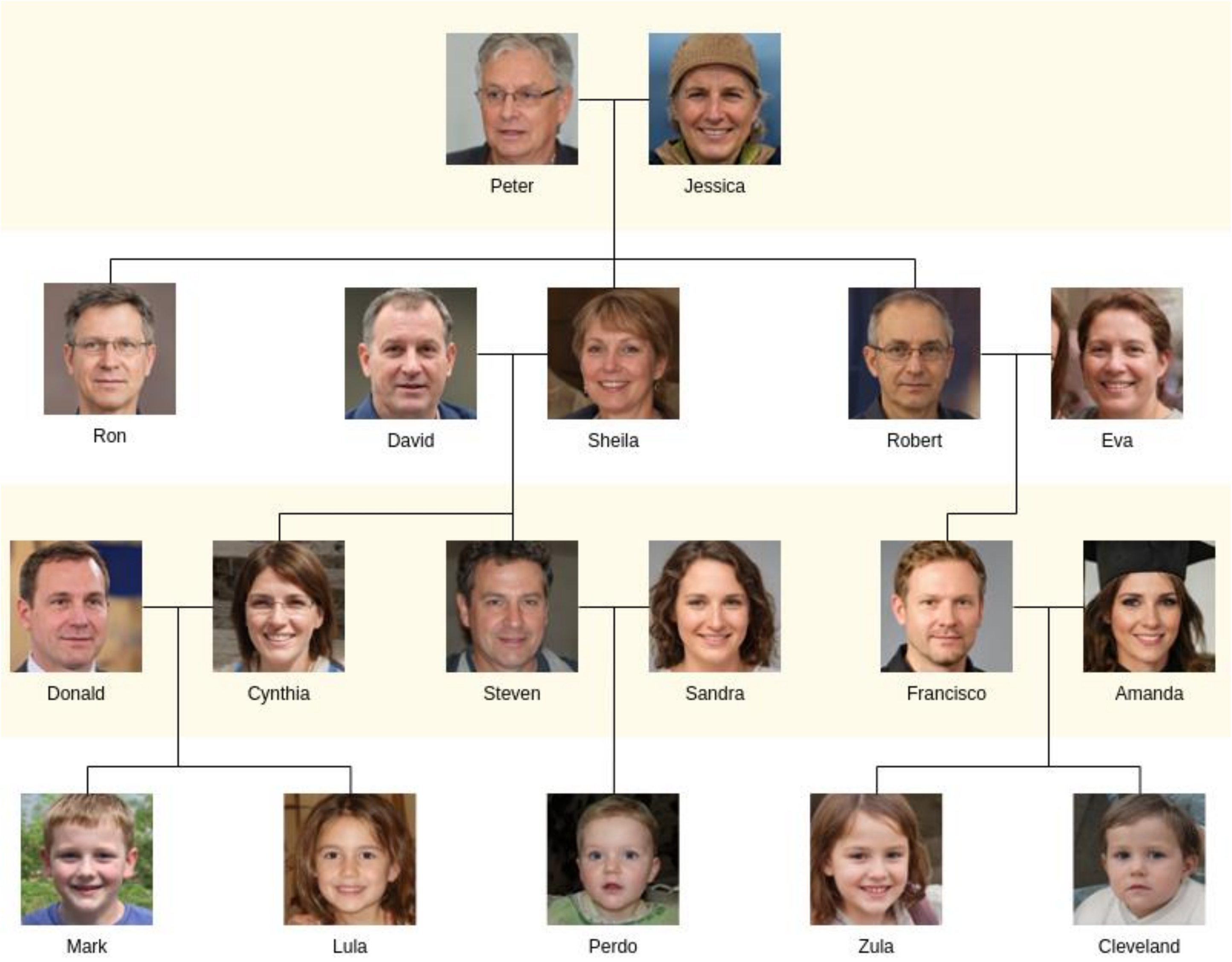
Predicate Calculus: Family Tree Example in Prolog



Limitations of Propositional Logic

- **Static Nature**:
  - Propositional logic requires explicit enumeration of all relationships.
  - Dynamic relationships (e.g., "Find the nth ancestor") are cumbersome.
- **No Variables for Quantifiers**:
  - Statements like "All descendants of Peter" require predicate logic, not propositional logic.
- **Limited Expressiveness**:
  - Cannot generalize rules for complex reasoning (e.g., inheritance of traits or roles).

Predicate Logic

- To overcome these limitations, predicate logic introduces:
  - **Variables** for dynamic queries.
  - **Quantifiers** (`forall`, `exists`) for general rules.
  - **Advanced Reasoning** for more intelligent systems.

Predicate Calculus

- **Propositional Calculus:** Uses atomic symbols (e.g., P, Q) to represent entire propositions. Lacks the ability to describe components of propositions.
- **Predicate Calculus:** Allows representation of relationships between objects and properties, using predicates and variables.

Features

- **Predicates:**
  - Represents a relationship between objects.
  - Example: `weather(tuesday, rain)`.
- **Variables:**
  - Generalize assertions about classes of objects.
  - Example: `∀X (weather (X, rain))` → "It rains every day."
- **Inference Rules:**
  - Access and manipulate predicate calculus expressions.

Syntax of Predicate Calculus

- **Symbols:**
  - Alphabet: Letters (`A-Z, a-z`), Digits (`0-9`), Underscore (`_`).
  - Symbols start with a letter and may contain legal characters.
- **Types of Symbols:**
  - **Constants:** Specific objects, properties (lowercase). Example: george, blue.
  - **Variables:** Represent classes of objects (uppercase). Example: X, Day.
  - **Functions:** Map elements from a domain to a range (lowercase). Example: father(david).
  - **Predicates:** Define relationships. Example: likes(george, kate).
- **Reserved Truth Symbols:**
  - true, false.

Well-Formed Expressions

- **Constants:** Start with a lowercase letter.
  - Example: `blue, rain.`
- **Variables:** Start with an uppercase letter.
  - Example: `X, Day.`
- **Function Expressions:** Function symbol followed by arguments.
  - Example: `father(david), price(bananas).`
- **Atomic Sentences:** Predicate followed by arguments (arity matters).
  - Example: `likes(george, kate), friends(bill, george).`

Evaluation

- Replacing a function with its value is called **evaluation**.
  - **Example**: If `father(david) = george`, then `friends(father(david), allen)` evaluates to `friends(george, allen)`.

Definitions

- **Terms:** Constants, variables, or function expressions.
  - Example: `X, mother(sarah), cat.`
- **Predicates:** Define relations of arity n.
  - Example: `likes(X, Y), friends(bill, george).`

Examples

- **Predicate with constants:** `likes(george, kate).`
- **Predicate with variables:** `friends(X, Y).`
- **Functions as arguments:** `friends(father(david), father(andrew)).`

Core Concepts

- **Predicate Symbols and Atomic Sentences**:
  - Predicate symbols begin with lowercase letters.
  - Predicates have an **arity**, which defines the number of arguments (e.g., `mother(eve, abel)` has arity 2).
  - An **atomic sentence** is formed by applying a predicate to the correct number of terms enclosed in parentheses and separated by commas.
  - **Truth values** `(true, false)` are also considered atomic sentences.
- **Logical Connectives**:
  - The connectives `∧ (and), ∨ (or), ¬ (not), → (implies), and ≡ (equivalent)` are used to combine atomic sentences into more complex expressions.

Core Concepts

- **Quantifiers**:
  - **Universal Quantifier** (∀): Indicates that a sentence applies to all elements in the domain.
    - Example: `∀ X likes(X, ice_cream)` means "everyone likes ice cream."
  - **Existential Quantifier** (∃): Indicates that a sentence applies to at least one element in the domain.
    - Example: `∃ Y friends(Y, peter)` means "someone is a friend of Peter."
- **Defining Complex Sentences**: Sentences in predicate calculus can be built recursively:
  - Base: Atomic sentences.
  - Negation: If s is a sentence, so is ¬s.
  - Conjunction/Disjunction: If `s1` and `s2` are sentences, so are `s1 ∧ s2` and `s1 ∨ s2`.
  - Implication/Equivalence: If `s1` and `s2` are sentences, so are `s1 → s2` and `s1 ≡ s2`.
  - Quantification: If X is a variable and s a sentence, ∀X s and ∃X s are also sentences.

Example: Biblical Genealogy

Using predicates to describe relationships:

- **Atomic predicates:**
  - `mother(eve, abel)`
  - `mother(eve, cain)`
  - `father(adam, abel)`
  - `father(adam, cain)`

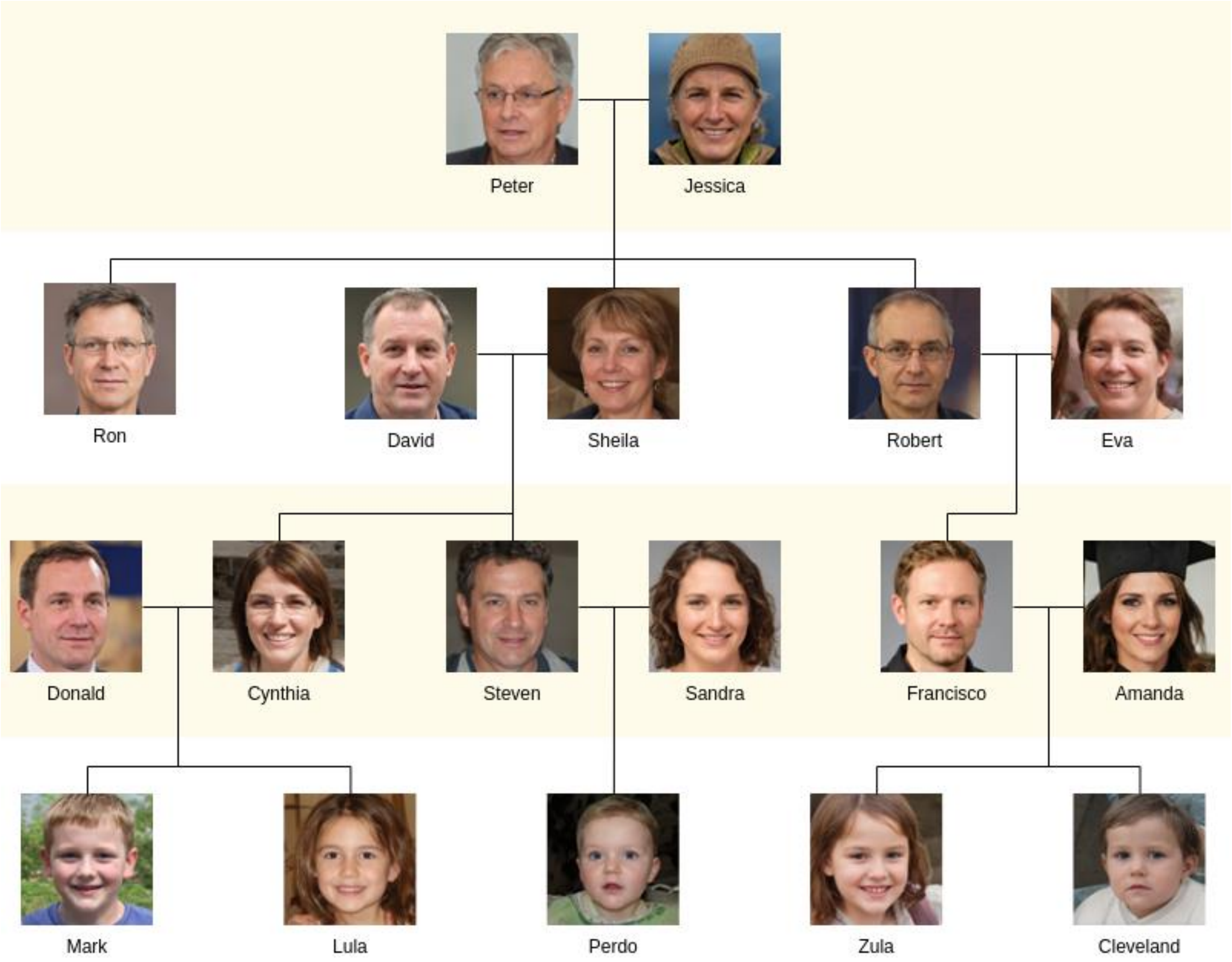
- **Derived relationships:**
  - Parent definition: $\forall X \forall Y (father(X, Y) \lor mother(X, Y) \rightarrow parent(X, Y))$
  - Sibling definition: $\forall X \forall Y \forall Z (parent(X, Y) \land parent(X, Z) \rightarrow sibling(Y, Z))$
  - Inference: From these rules, we can deduce `sibling(cain, abel)`.

Well-formedness and Examples

- **Valid Atomic Sentences**:
  - `equal(plus(two, three), five)`
  - `equal(plus(2, 3), seven)` (though this is false under standard arithmetic, it is still syntactically valid).
- **Complex Sentences**:
  - `∃ X foo(X, two, plus(two, three)) ∧ equal(plus(two, three), five)`
  - `(foo(two, two, plus(two, three))) → (equal(plus(three, two), five) ≡ true)`

Back to Example

Filename: family-tree_predicate_logic.pl



Step 1: Facts

- Define individuals
  - individual(peter).
- Define parent-child relationships
  - parent(peter, ron).
- Define gender
  - male(peter).

Step 2: Define Rules

- Grandparent Relationships
- Sibling Relationships
- Ancestor Relationships
- Descendant Relationships
- Cousin Relationships
- Intelligent Inference

Step 2: Define Rules

- Grandparent Relationships

  grandparent(X, Z) :- parent(X, Y), parent(Y, Z).
- Sibling Relationships

  sibling(X, Y) :- parent(P, X), parent(P, Y), X \= Y.
- Ancestor Relationships

  ancestor(X, Y) :- parent(X, Y).

  ancestor(X, Y) :- parent(X, Z), ancestor(Z, Y).
- Descendant Relationships

  descendant(X, Y) :- ancestor(Y, X)
- Cousin Relationships

  cousin(X,Y) :- parent(P, X), parent(P, Y), X \= Y.

  cousin(X, Y) :- grandparent(Z, X), grandparent(Z, Y), X \= Y.

Step 2: Define Rules

- Intelligent Inference
  - Check if X and Y are related
    - related(X, Y) :- ancestor(Z, X), ancestor(Z, Y).
- Universal Quantifier (∀)
- Existential Quantifier (∃)

Step 3: Queries

- Who are the children of Peter?
  - parent(peter, X).
- Who are the siblings of Ron?
  - sibling(ron, X).
- Who are Peter's grandchildren?
  - grandparent(peter, X).