Histogram, Filtering in Image Processing

DR. SUKANTA GHOSH

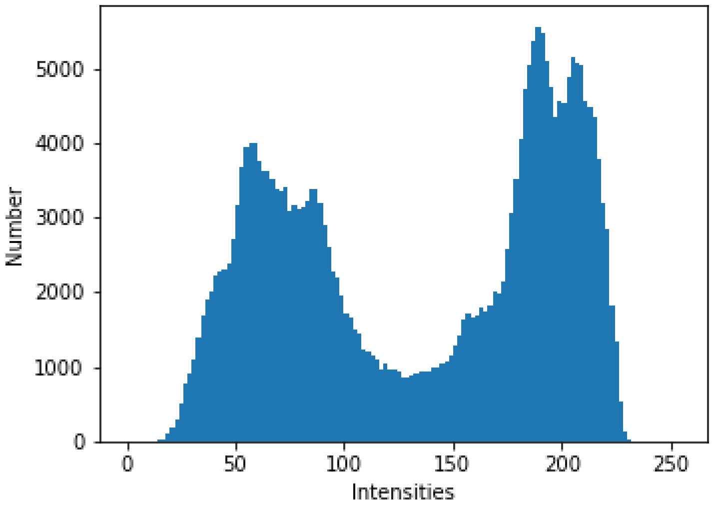
Histogram

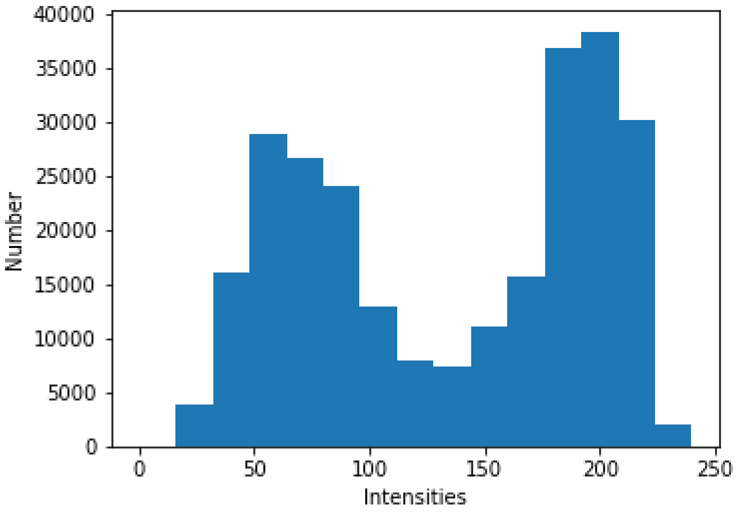
- The histogram of a digital image depicts how the intensities of its pixels are distributed.
- It is the discrete function *h* such that: $h(i) = n_i$
- where n_i is the number of pixels with intensity i.

- The histograms are displayed as a bar plot, constituted as a set of bins.
- The number (hence the width) of the bins is chosen by the user.
- For example, we choose 128 bins and 16 bins. Both histograms lie on [0,255] which is the intensity range of the image.

Example


0
100
200
300
400
500
0
100
200
300
400
500

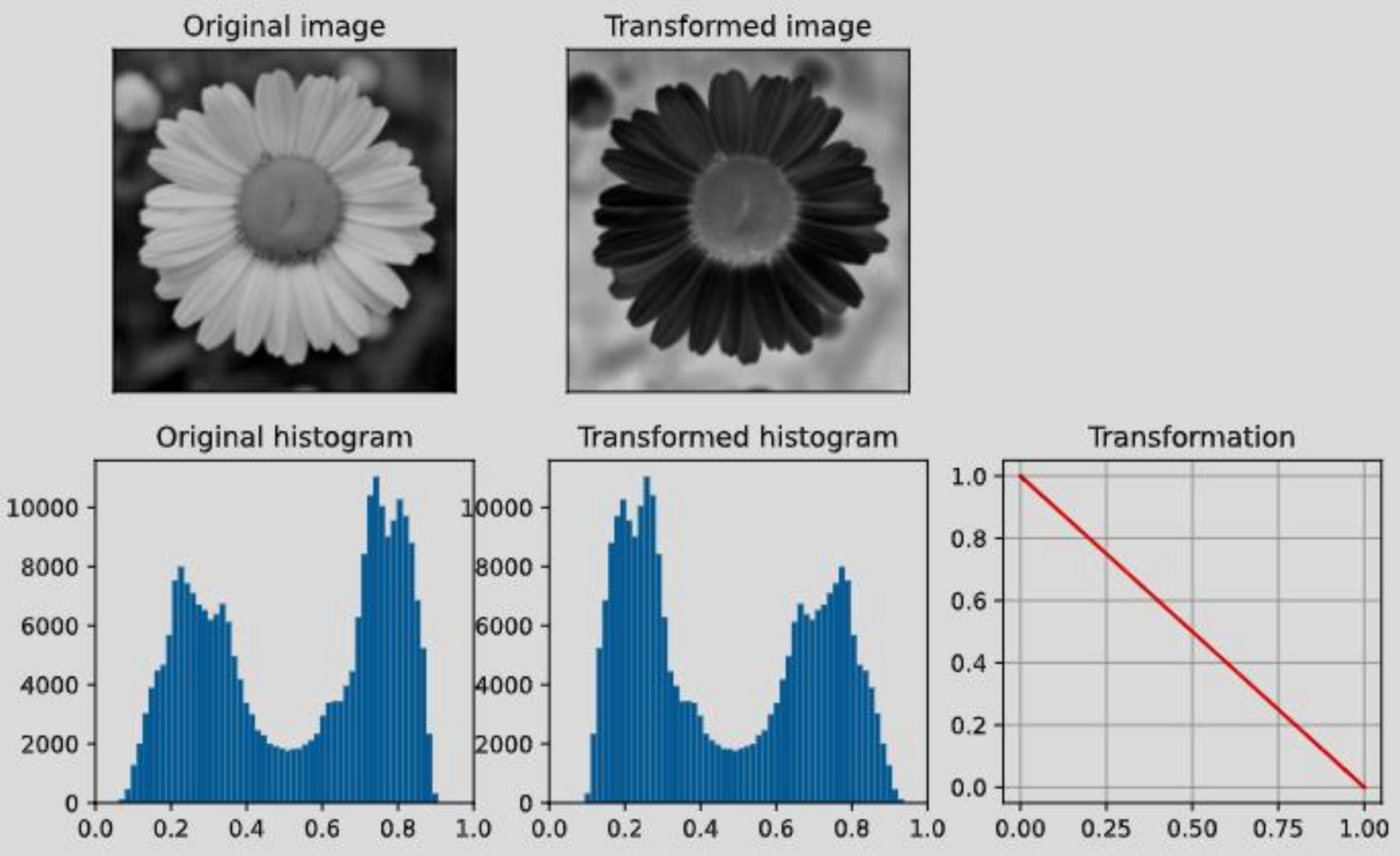

Number
Intensities
0
1000
2000
3000
4000
5000
0
50
100
150
200
250


Number
Intensities
0
5000
10000
15000
20000
25000
30000
35000
40000
0
50
100
150
200
250

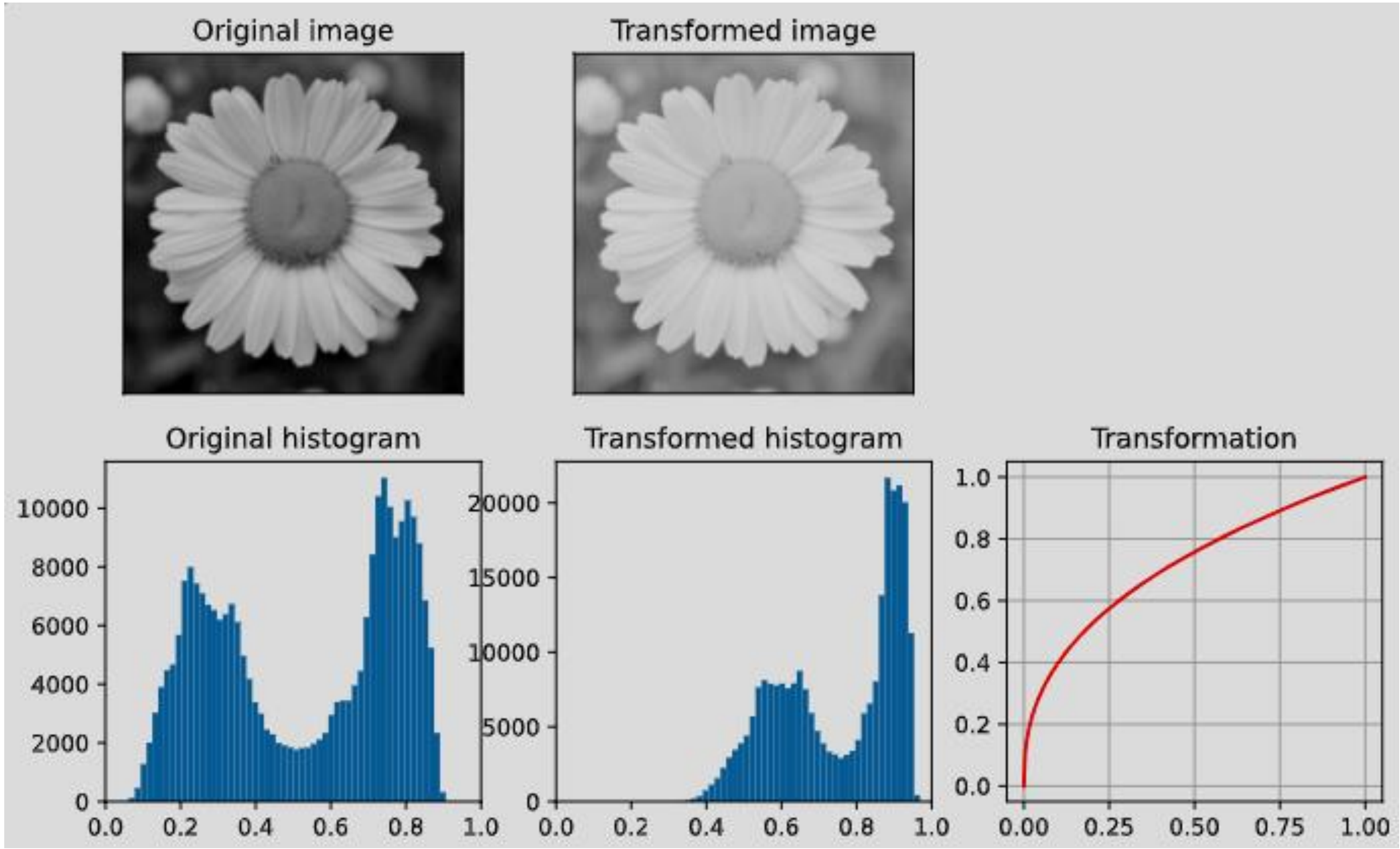
Histogram transformations

- A histogram transformation consists in applying a mathematical function to the intensity distribution.
- Generally, the transformations are useful to improve the visual quality of an image but are rarely needed inside an automatic processing.
- The transform, denoted T, is applied on the pixel intensities to change their values: $j = T(i)$

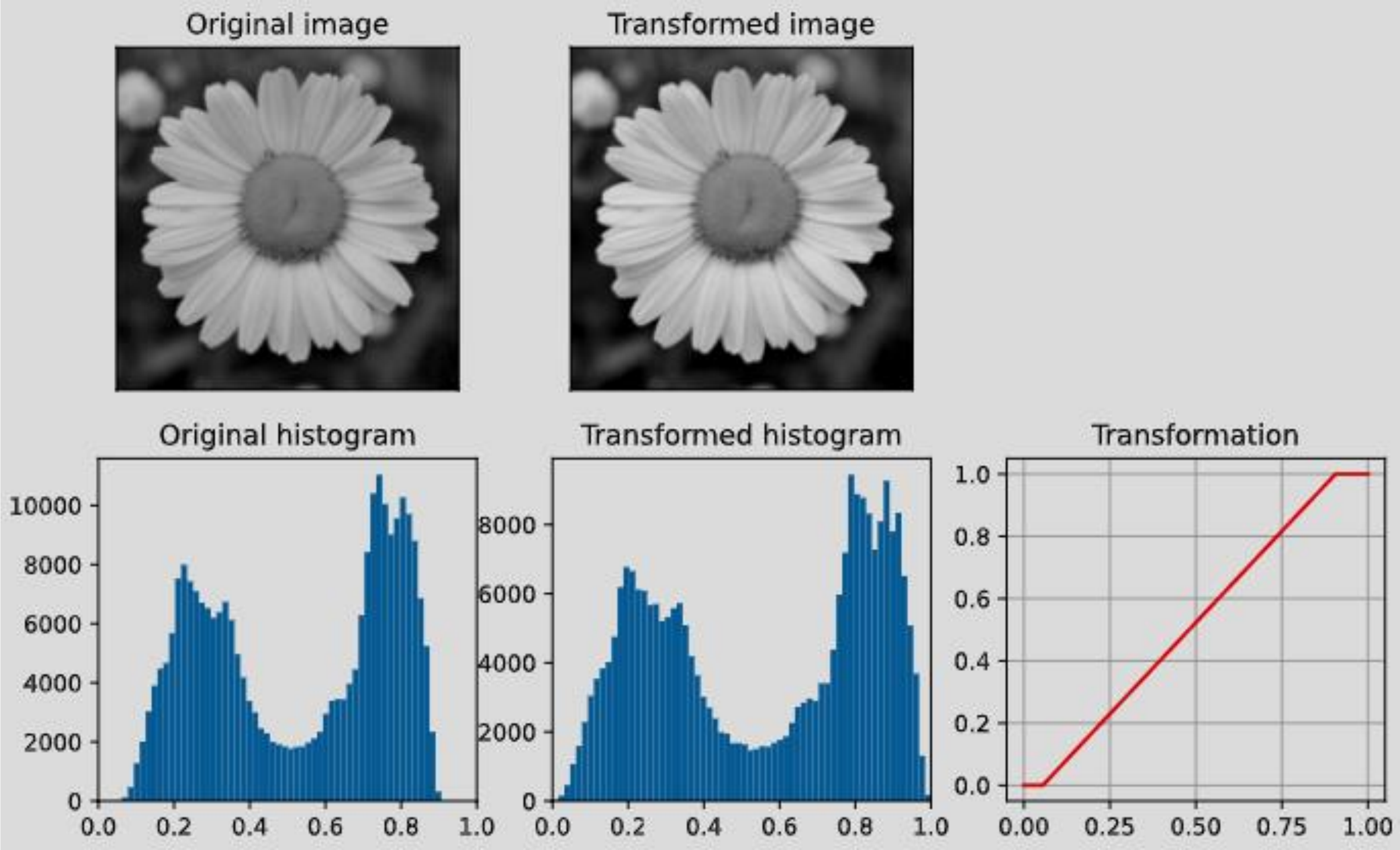
Negative image: $T(i) = 1 - i$



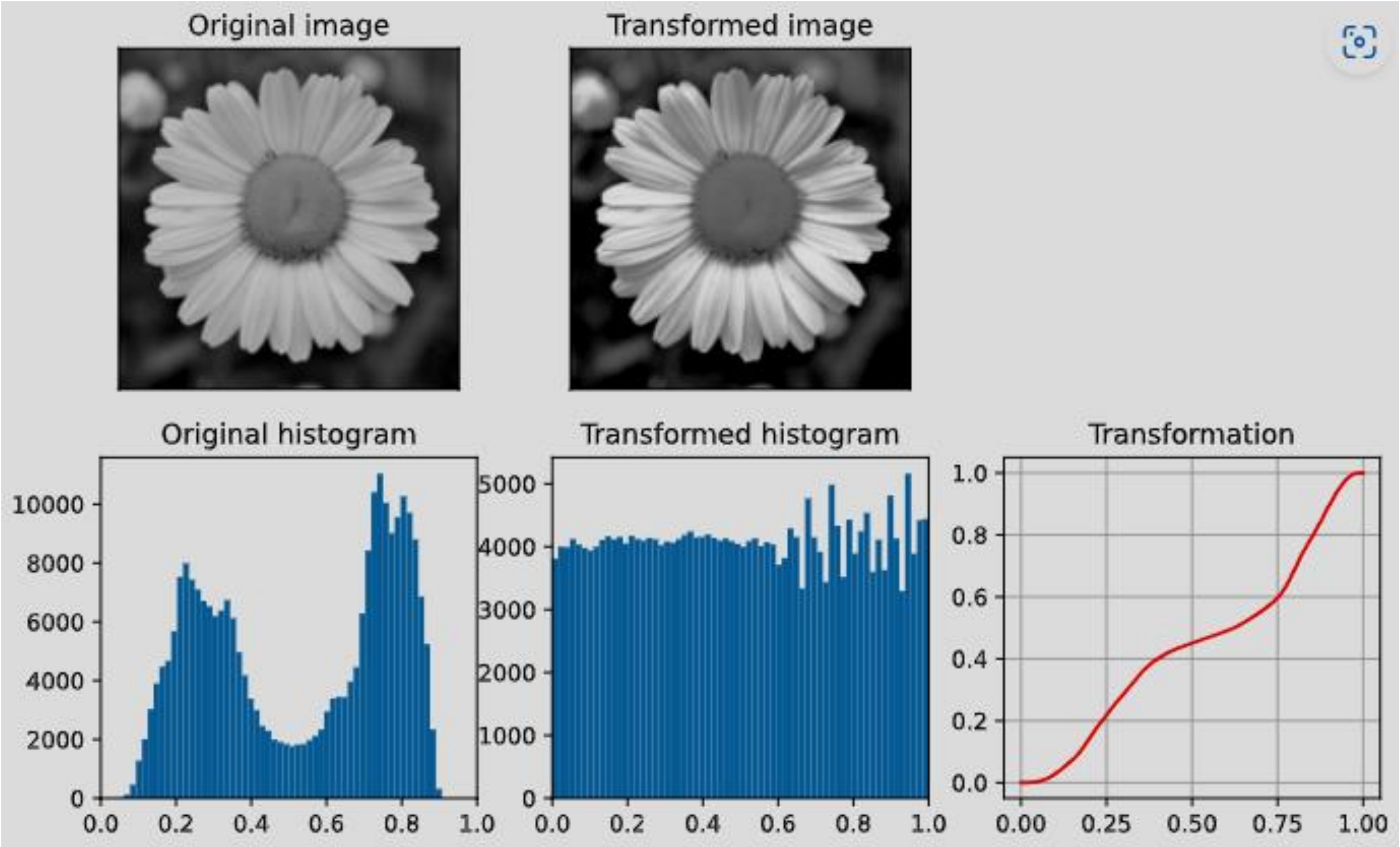
Gamma correction: $T(i) = i^{\gamma}$



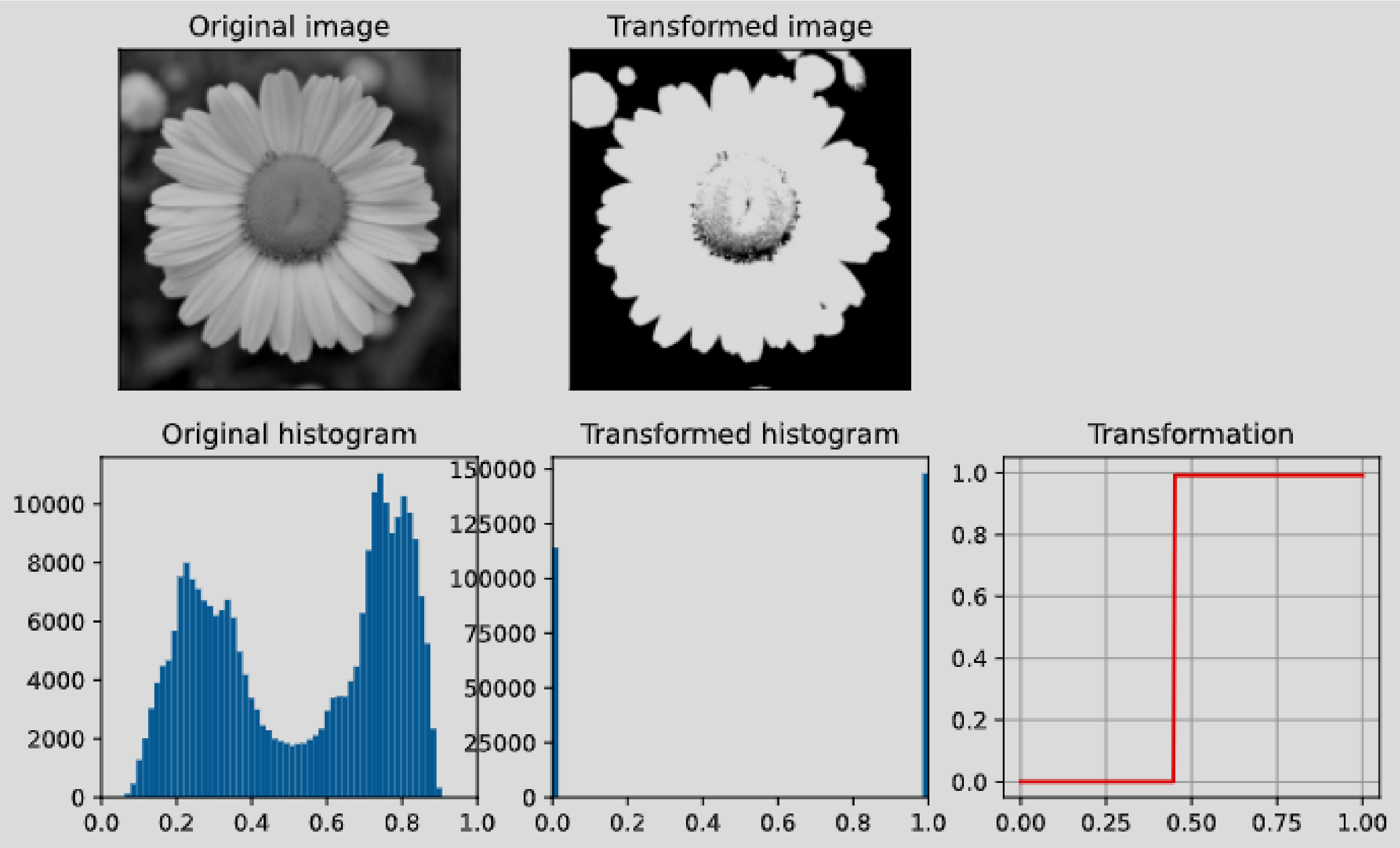
Histogram spreading: $T(i) = \frac{i - i_{min}}{i_{max} - i_{min}}$



Histogram equalization: $T(i) = \frac{i}{MN}\sum_{k=0}^{i} n_k$



Thresholding



Filtering

- Filtering consists of amplifying or attenuating some frequencies in the image.
- This is achieved with two mathematical objects, namely the Convolution and the Fourier transform.

Convolution

- Many image processing results come from a modification of one pixel with respect to its neighbors.
- When this modification is similar in the entire image g, it can be mathematically defined using a second image h which defines the neighbor relationships.
- This results in a third image f. This is called convolution.

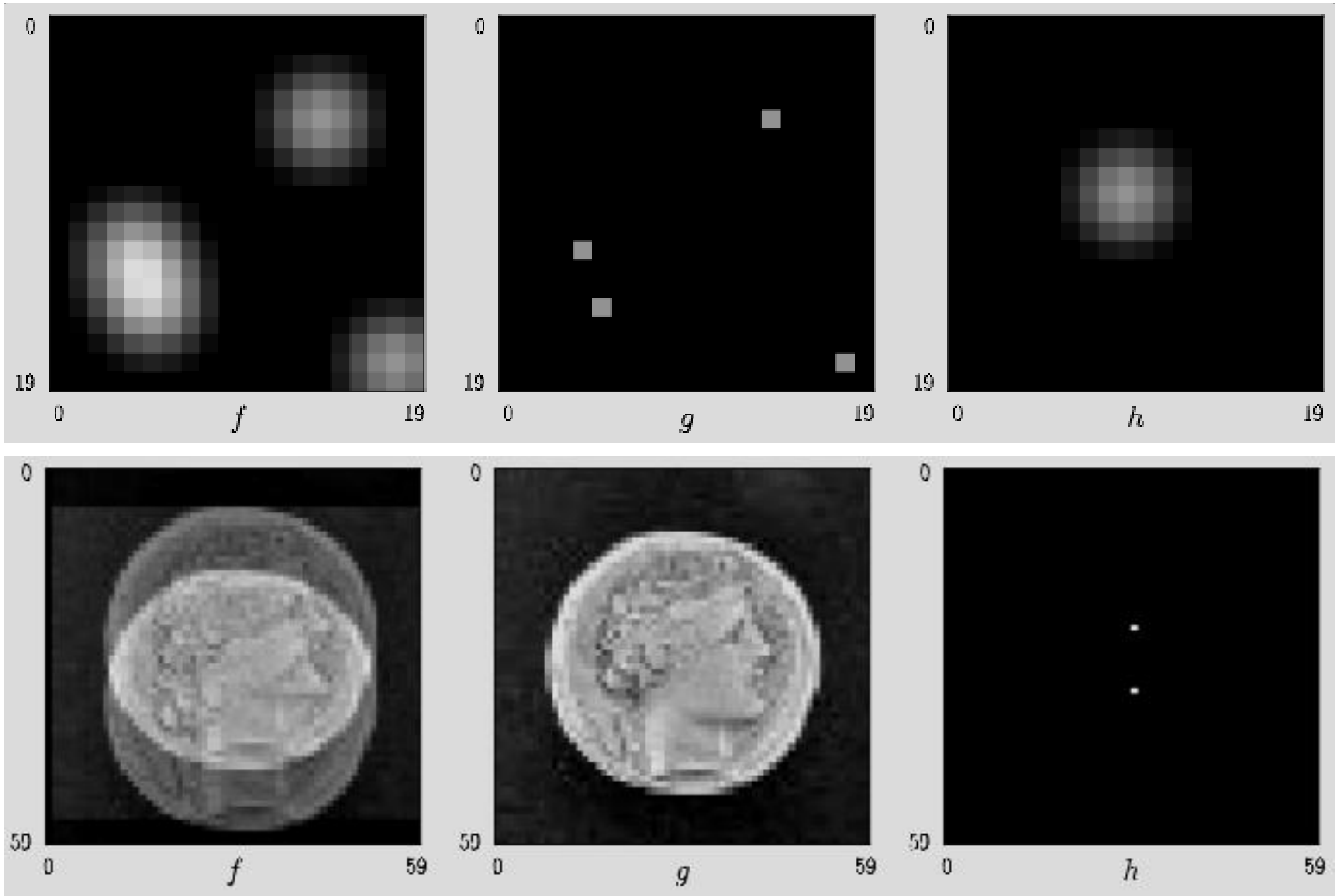
$$f(x,y) = (g * h)(x,y) = \sum_m \sum_n g(x-m, y-n)\, h(m,n)$$

Convolution

$$
\begin{array}{|c|c|c|c|c|}
\hline
f_{1,1} & f_{1,2} & f_{1,3} & f_{1,4} & f_{1,5} \\ \hline
f_{2,1} & f_{2,2} & f_{2,3} & f_{2,4} & f_{2,5} \\ \hline
f_{3,1} & f_{3,2} & f_{3,3} & f_{3,4} & f_{3,5} \\ \hline
f_{4,1} & f_{4,2} & f_{4,3} & f_{4,4} & f_{4,5} \\ \hline
f_{5,1} & f_{5,2} & f_{5,3} & f_{5,4} & f_{5,5} \\ \hline
\end{array}
=
\begin{array}{|c|c|c|c|c|}
\hline
g_{1,1} & g_{1,2} & g_{1,3} & g_{1,4} & g_{1,5} \\ \hline
g_{2,1} & g_{2,2} & g_{2,3} & g_{2,4} & g_{2,5} \\ \hline
g_{3,1} & g_{3,2} & g_{3,3} & g_{3,4} & g_{3,5} \\ \hline
g_{4,1} & g_{4,2} & g_{4,3} & g_{4,4} & g_{4,5} \\ \hline
g_{5,1} & g_{5,2} & g_{5,3} & g_{5,4} & g_{5,5} \\ \hline
\end{array}
*
\begin{array}{ccc c}
-1 & 0 & +1 & \\
\begin{array}{|c|c|c|}
\hline
h_{-,-} & h_{-,0} & h_{-,+} \\ \hline
h_{0,-} & h_{0,0} & h_{0,+} \\ \hline
h_{+,-} & h_{+,0} & h_{+,+} \\ \hline
\end{array}
& & &
\begin{array}{c}
-1 \\ 0 \\ +1
\end{array}
\end{array}
$$

$$
f_{2,2} = g_{3,3}h_{-,-} + g_{3,2}h_{-,0} + g_{3,1}h_{-,+} + g_{2,3}h_{0,-} + g_{2,2}h_{0,0} + g_{2,1}h_{0,+} + g_{1,3}h_{+,-} + g_{1,2}h_{+,0} + g_{1,1}h_{+,+}
$$

Convolution



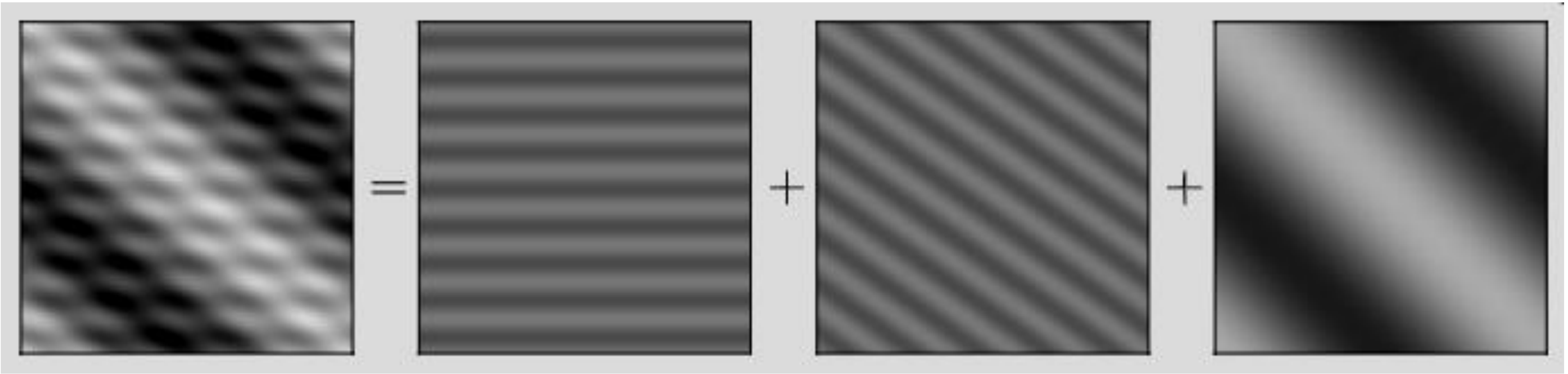
Convolution

- As a mathematical operation, the convolution has several properties.
  - The neutral element of convolution is an image filled with zeros but the pixel at the center equals 1.
  - The convolution is commutative.
  - The convolution is distributive with respect to the addition.
  - The convolution is bilinear.
  - The convolution is associative.

Fourier transform

- The (2D) Fourier transform is a very classical tool in image processing.
- It is the extension of the well known Fourier transform for signals which decomposes a signal into a sum of complex oscillations (actually, complex exponential).
- In image processing, the complex oscillations always come by pair because the pixels have real intensities.

Fourier transform



Filtering

- The operation of filtering consists of selecting some frequencies in the images.
- Filtering an image is performed through convolution of the image with a point spread function (PSF) that represents the filter.
- Similarly, it can be achieved by multiplying the DFT of the image and the PSF.
- In the example below, the first row in the images is in the spatial domain: the filtered image is the convolution of the original image and the kernel (PSF).
- The second row shows the same information in the Fourier domain, thus the DFT of the filtered image is the multiplication (point to point) of the DFT of the original image and the kernel.

Filtering

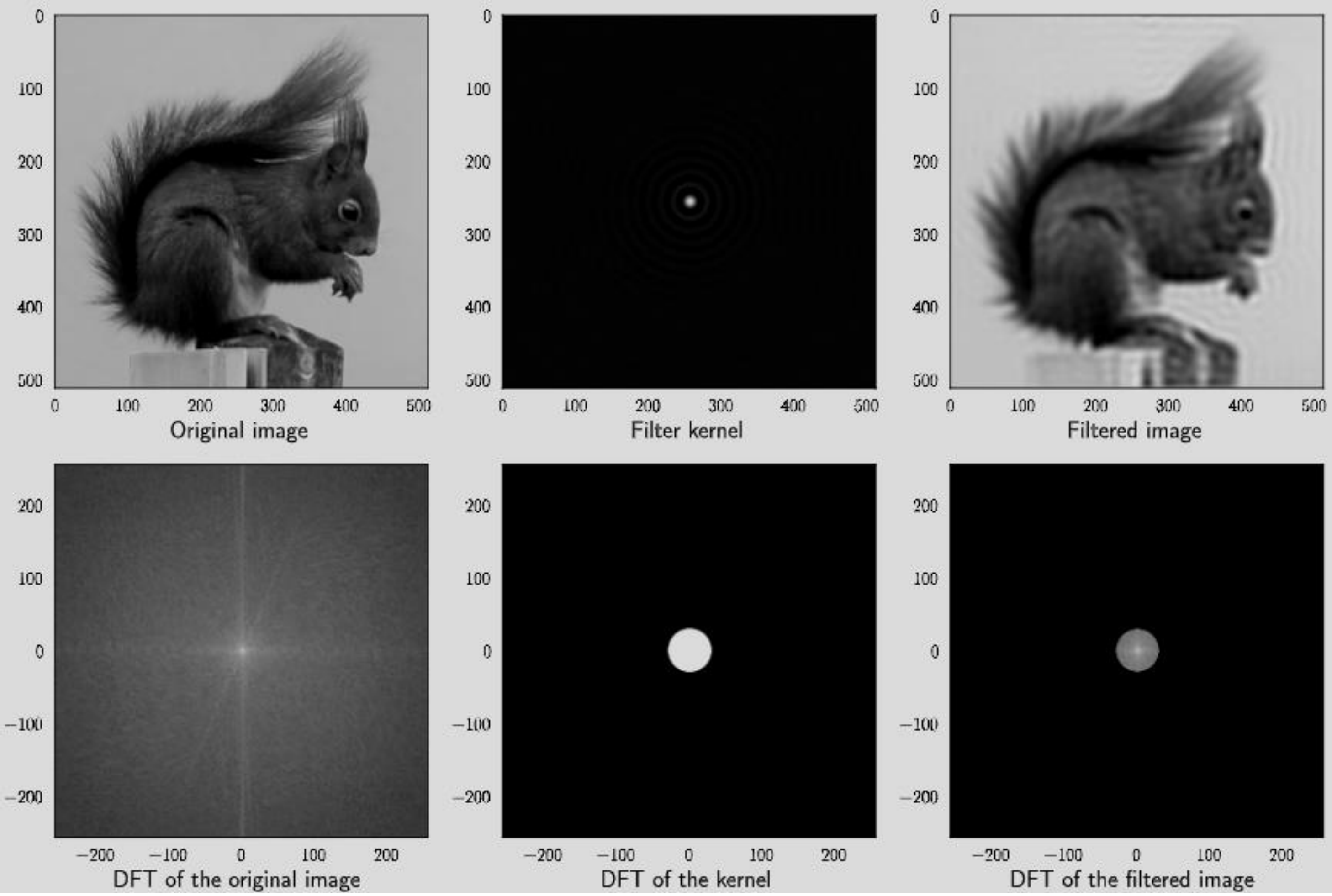

Original image
Filter kernel
Filtered image
DFT of the original image
DFT of the kernel
DFT of the filtered image

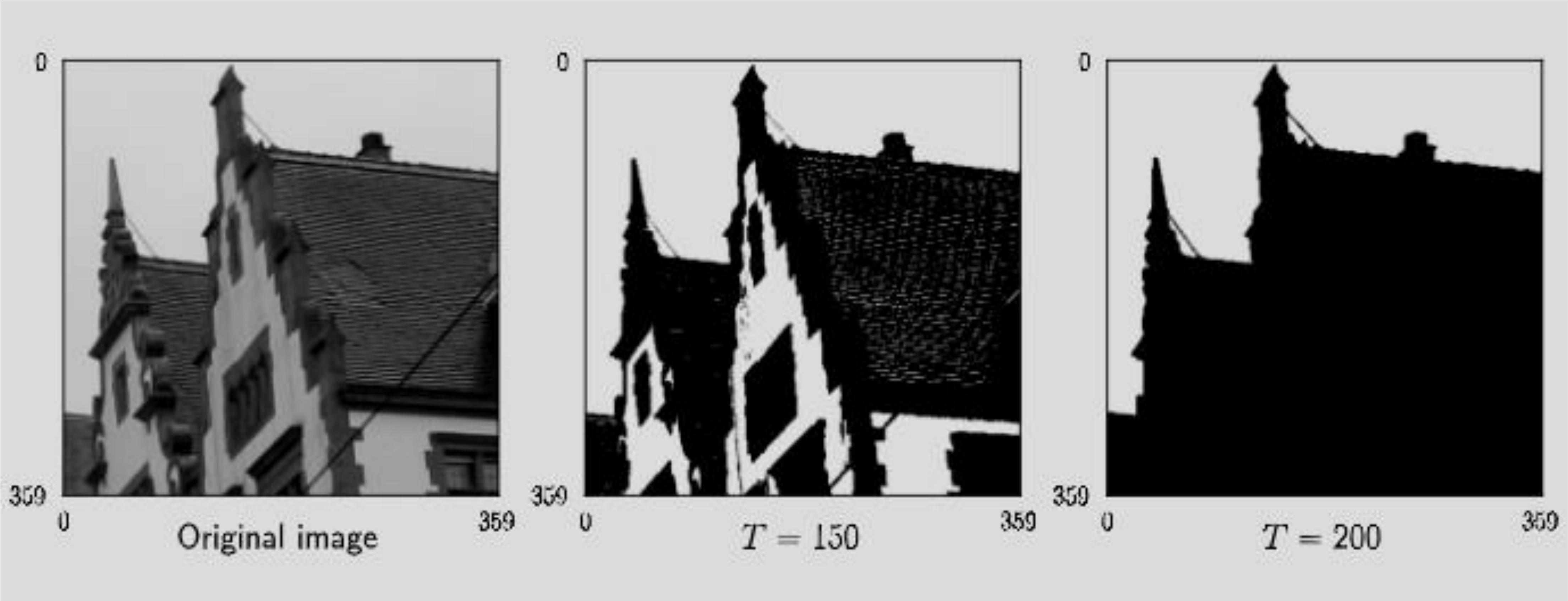
Image Segmentation

- Image segmentation consists of partitioning an image f according to a certain criterion.
- This means that the image is divided into regions R_i that are both mutually disjoint and collectively cover the entire image.
- Two pixels in the same region satisfy the criterion, but two pixels in two adjacent regions do not.

Histogram thresholding
Binary thresholding

- A very simple method of segmentation consists of associating with each pixel of the image f a binary number that depends on the intensity of the pixels and on a threshold T.
- $g(m,n) = \begin{cases} 0 & f(m,n) < T \\ 1, & f(m,n) \geq T \end{cases}$
- This method is called "binarization".
- It gives a segmentation into two classes, depending on the intensity of the pixels of a grayscale image.
- Segmentation depends on the value of T. Therefore, the histogram is very useful to choose the threshold.
- Threshold value is chosen with help of Otsu method.

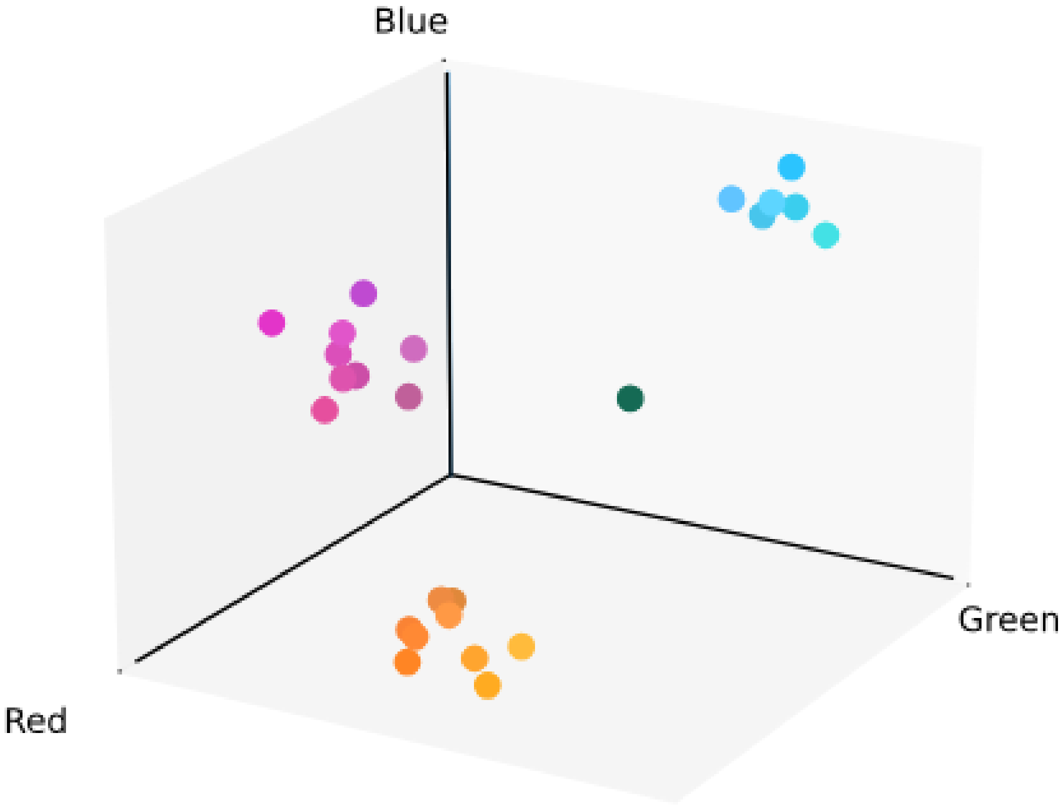
Histogram thresholding
Binary thresholding



Clustering

- Thresholding applies well on a grayscale image, for which it is easy to define a threshold from the modes of the histogram.
- However, this approach cannot be applied to a color or multiband image because there is no histogram.
- Each pixel in a B-band image can be represented by a point in a B-dimensional space.
- By doing so, pixels with similar colors form groups in the space.
- Clustering methods consist of defining groups of pixels. Therefore, all the pixels in the same group define a class in the segmented image.
- A classical clustering method for image segmentation is the k-means method .

Clustering



Algorithm: K-means

1. Initialize (randomly) the K centroids
2. While the centroids vary:

    STEP A For each point:

    1. Calculate the distances from the point to all centroids
    2. Assign the point to the nearest group

    STEP B Calculate the centroid of each group

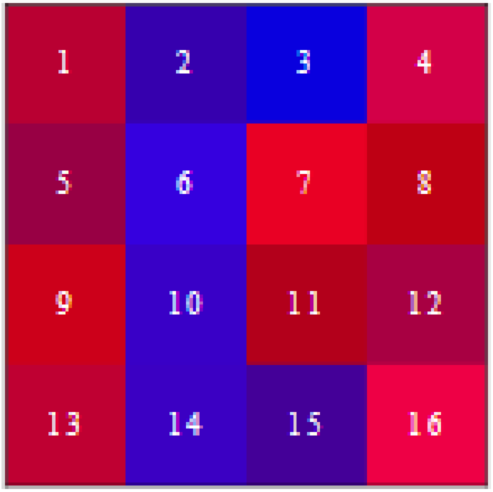
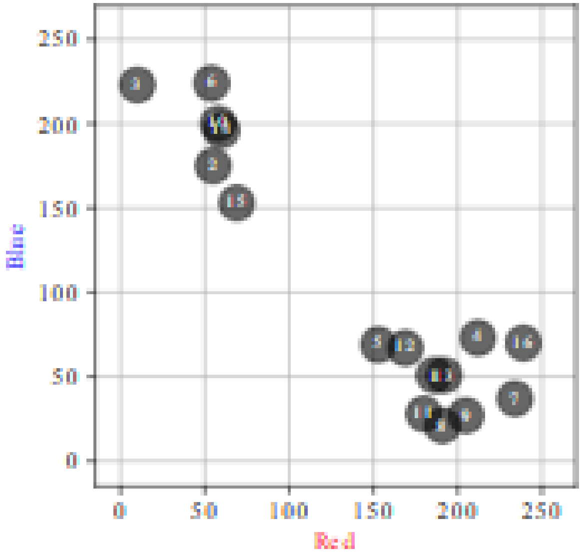


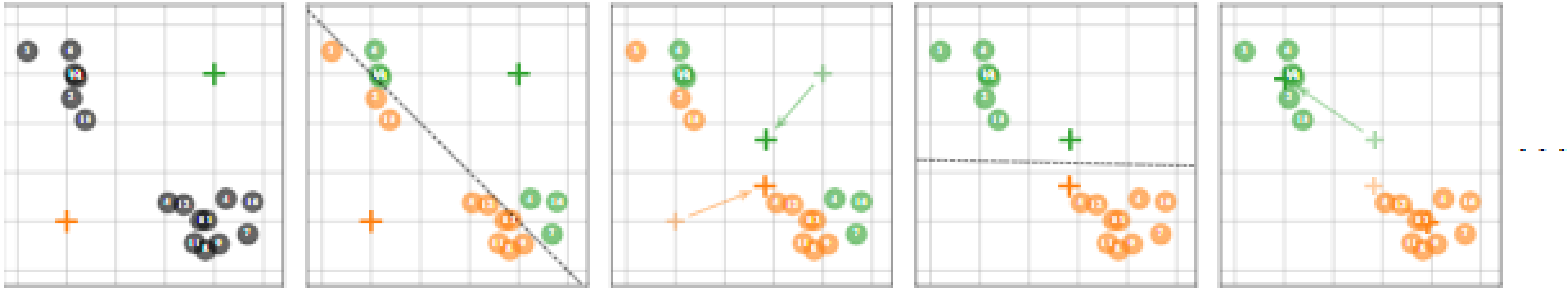
Image with bands red and blue.
Numbers are the pixel indices.

186 51	55 175	10 223	212 73
153 69	54 224	234 37	191 21
205 27	58 200	180 28	169 67
192 51	60 196	69 153	239 70

Intensities (red and blue)
of the pixels.



Pixels plotted in
the (red,blue) space.



Initialization

Iteration 1
Step A

Iteration 1
Step B

Iteration 2
Step A

Iteration 2
Step B

Progression of the K-means algorithm (only the initialization and the frst two iterations are illustrated).
Centroids are represented by +, the dotted line indicates the boundary between the two classes.

How to evaluate a segmentation?

Sensibility (*sensibilité*)

$$\frac{\mathrm{TP}}{\mathrm{TP}+\mathrm{FN}}$$

Specificity (*spécificité*)

$$\frac{\mathrm{TN}}{\mathrm{TN}+\mathrm{FP}}$$

Dice coefficient (*coefficient de Dice*)

$$\frac{2\,\mathrm{TP}}{2\,\mathrm{TP}+\mathrm{FP}+\mathrm{FN}} = \frac{2\,|f \cap f^*|}{|f|+|f^*|}$$

Jaccard coefficient (*coefficient de Jaccard*)

$$\frac{\mathrm{TP}}{\mathrm{TP}+\mathrm{FP}+\mathrm{FN}} = \frac{|f \cap f^*|}{|f \cup f^*|}$$

Feature Detection

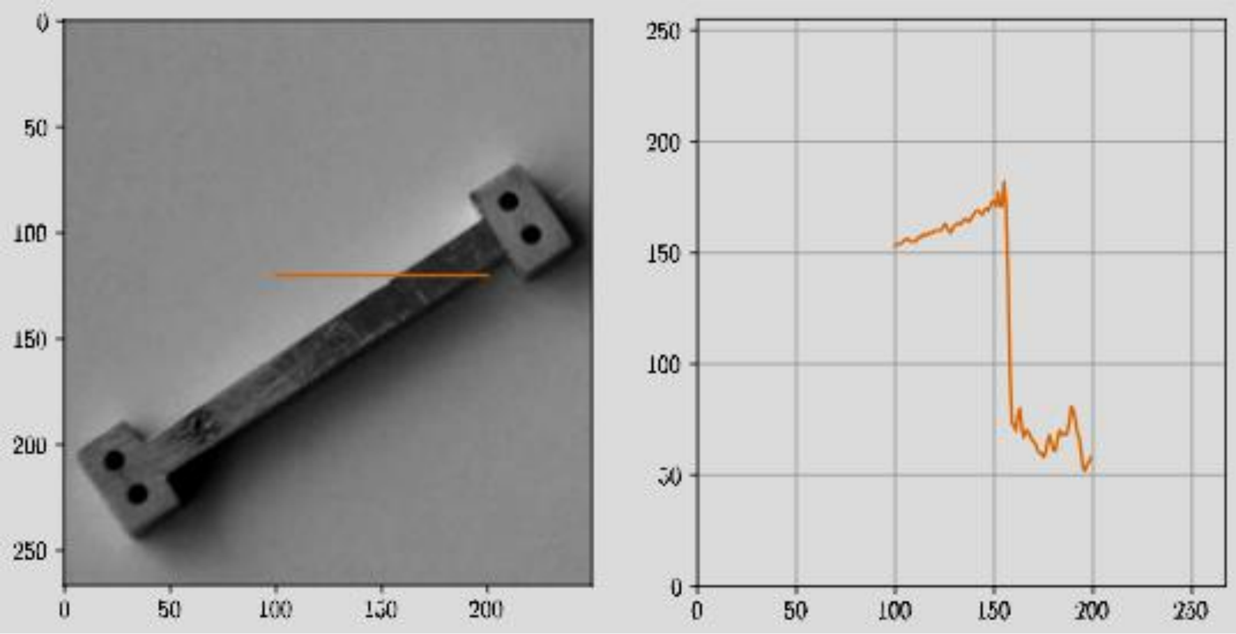
- Feature detection deals with the searching for specific marks in an image, such as edges or corners of the objects, geometric shapes, and particular patterns.
- Feature detection extracts high-level information from the image.
- Many applications need feature detection, for example: measuring the detected objects, recognizing a specific object, or classifying images according to their characteristics.

Edge detection

- An edge in an image is the frontier that delimits two objects.
- Therefore, edge detection is useful for identifying or measuring objects, or segmenting the image.

Edge detection

- Edges are characterized by a rapid variation in the intensity of the pixels.
- The given figure represents the brightness profile along a horizontal line in the image.
- One sees that the outline of the industrial piece shows a sudden decrease in the brightness of the pixels.



Edge detection

- From this example, it appears that derivation is an efficient tool for highlighting the edges.
- An edge can be detected by analyzing the first derivative of the intensity profile.
- Gradient operators are very simple methods for detecting edges. They use the first derivative and can be calculated using a convolution.