CAP 781
MACHINE LEARNING

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UNIT – IV
Image Processing_02

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Introduction to Mathematical Operations in DIP

- **Array vs. Matrix Operation**

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \qquad B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

Array product operator

$$A \;.*\; B = \begin{bmatrix} a_{11}b_{11} & a_{12}b_{12} \\ a_{21}b_{21} & a_{22}b_{22} \end{bmatrix}$$

Array product

Matrix product operator

$$A * B = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

Matrix product

Introduction to Mathematical Operations in DIP

- **Linear vs. Nonlinear Operation**

$$H\left[f(x,y)\right]=g(x,y)$$

$$H\left[a_i f_i(x,y)+a_j f_j(x,y)\right]$$

$$=H\left[a_i f_i(x,y)\right]+H\left[a_j f_j(x,y)\right]$$

Additivity

$$=a_i H\left[f_i(x,y)\right]+a_j H\left[f_j(x,y)\right]$$

Homogeneity

$$=a_i g_i(x,y)+a_j g_j(x,y)$$

H is said to be a **linear operator;**

H is said to be a **nonlinear operator** if it does not meet the above qualification.

Arithmetic Operations

- Arithmetic operations between images are array operations. The four arithmetic operations are denoted as

$$s(x,y) = f(x,y) + g(x,y)$$
$$d(x,y) = f(x,y) - g(x,y)$$
$$p(x,y) = f(x,y) \times g(x,y)$$
$$v(x,y) = f(x,y) \div g(x,y)$$

Example: Addition of Noisy Images for Noise Reduction

Noiseless image: $f(x,y)$

Noise: $n(x,y)$ (at every pair of coordinates (x,y), the noise is uncorrelated and has zero average value)

Corrupted image: $g(x,y)$

$$g(x,y) = f(x,y) + n(x,y)$$

Reducing the noise by adding a set of noisy images, $\{g_i(x,y)\}$

$$\overline{g}(x,y) = \frac{1}{K}\sum_{i=1}^{K} g_i(x,y)$$

Example: Addition of Noisy Images for Noise Reduction

$$\bar{g}(x,y) = \frac{1}{K}\sum_{i=1}^{K} g_i(x,y)$$

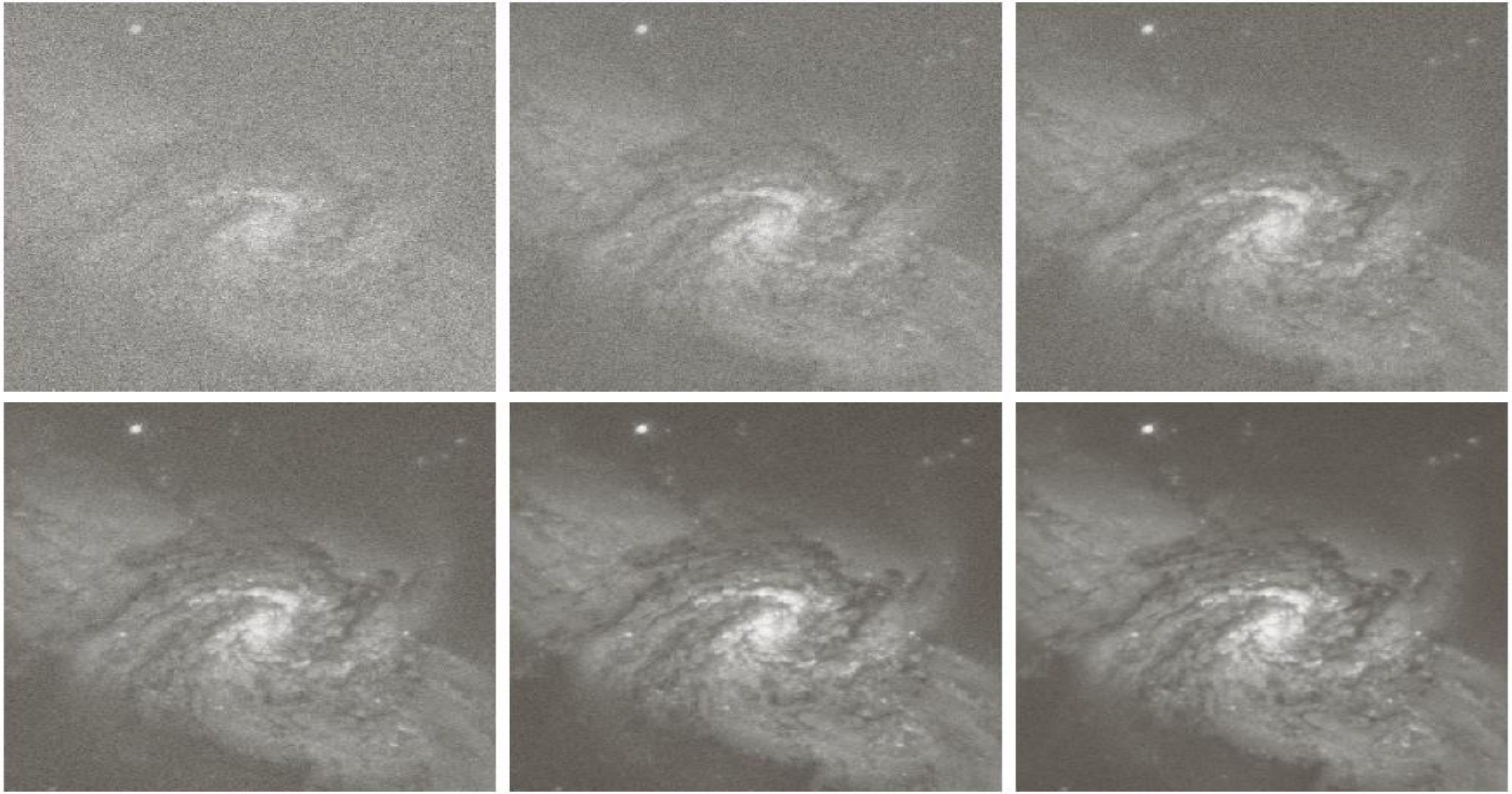
$$\begin{aligned}
E\{\bar{g}(x,y)\} &= E\left\{\frac{1}{K}\sum_{i=1}^{K} g_i(x,y)\right\} \\
&= E\left\{\frac{1}{K}\sum_{i=1}^{K}\left[f(x,y) + n_i(x,y)\right]\right\} \\
&= f(x,y) + E\left\{\frac{1}{K}\sum_{i=1}^{K} n_i(x,y)\right\} \\
&= f(x,y)
\end{aligned}$$

$$\begin{aligned}
\sigma^2_{\bar{g}(x,y)} &= \sigma^2_{\frac{1}{K}\sum_{i=1}^{K} g_i(x,y)} \\
&= \sigma^2_{\frac{1}{K}\sum_{i=1}^{K} n_i(x,y)} = \frac{1}{K}\sigma^2_{n(x,y)}
\end{aligned}$$

Example: Addition of Noisy Images for Noise Reduction

- In astronomy, imaging under very low light levels frequently causes sensor noise to render single images virtually useless for analysis.

- In astronomical observations, similar sensors for noise reduction by observing the same scene over long periods of time. Image averaging is then used to reduce the noise.



a	b	c
d	e	f

FIGURE 2.26 (a) Image of Galaxy Pair NGC 3314 corrupted by additive Gaussian noise. (b)–(f) Results of averaging 5, 10, 20, 50, and 100 noisy images, respectively. (Original image courtesy of NASA.)

An Example of Image Subtraction: Mask Mode Radiography

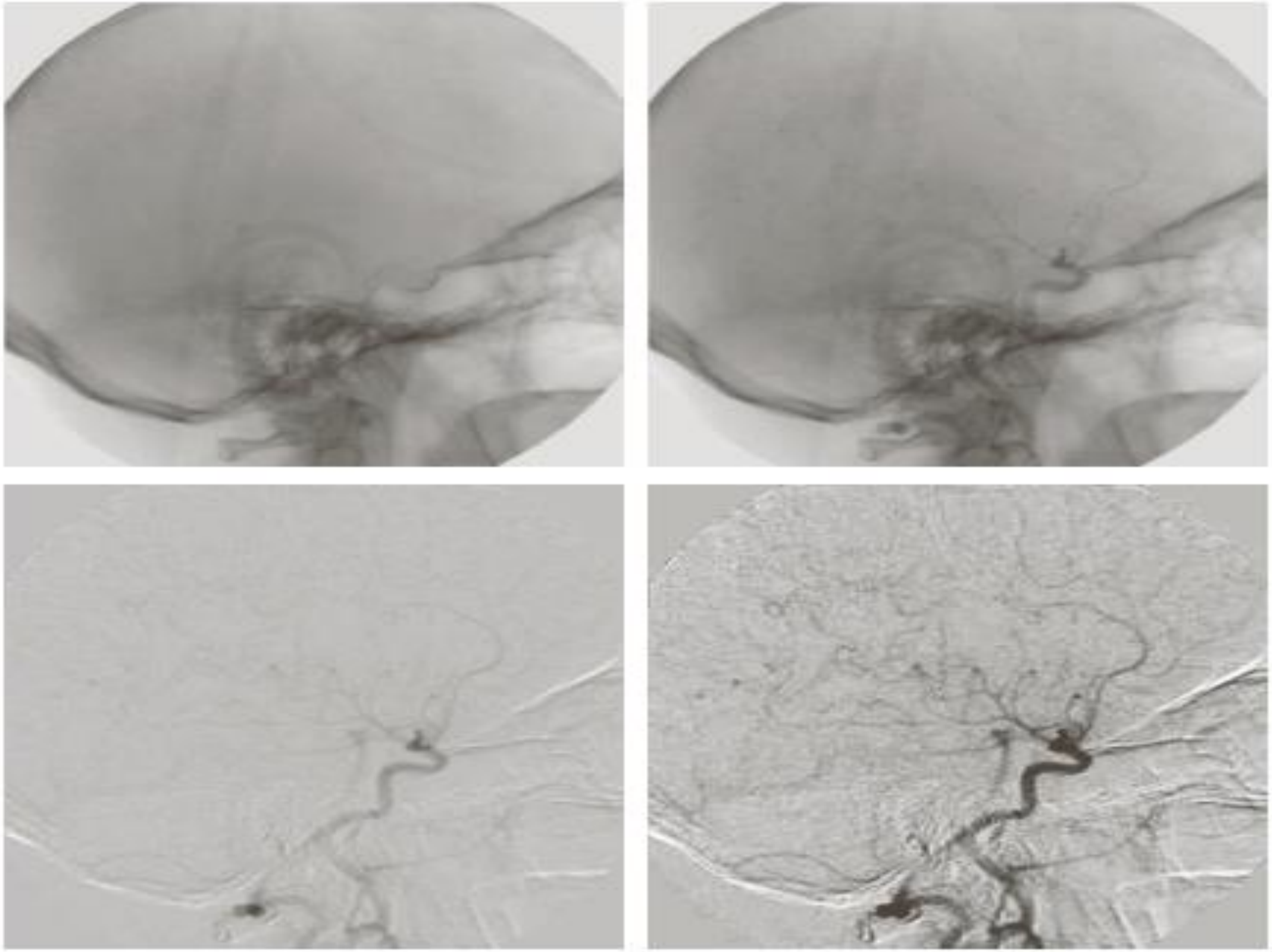
Mask $h(x,y)$: an X-ray image of a region of a patient's body

Live images $f(x,y)$: X-ray images captured at TV rates after injection of the contrast medium

Enhanced detail $g(x,y)$

$$g(x,y) = f(x,y) - h(x,y)$$

The procedure gives a movie showing how the contrast medium propagates through the various arteries in the area being observed.



a b
c d

FIGURE 2.28 Digital subtraction angiography. (a) Mask image. (b) A live image. (c) Difference between (a) and (b). (d) Enhanced difference image. (Figures (a) and (b) courtesy of The Image Sciences Institute, University Medical Center, Utrecht, The Netherlands.)

An Example of Image Multiplication

a b c

FIGURE 2.29 Shading correction. (a) Shaded SEM image of a tungsten filament and support, magnified approximately 130 times. (b) The shading pattern. (c) Product of (a) by the reciprocal of (b). (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)

Spatial Operations

- Single-pixel operations

  Alter the values of an image's pixels based on the intensity.

$$s = T(z)$$

e.g.,

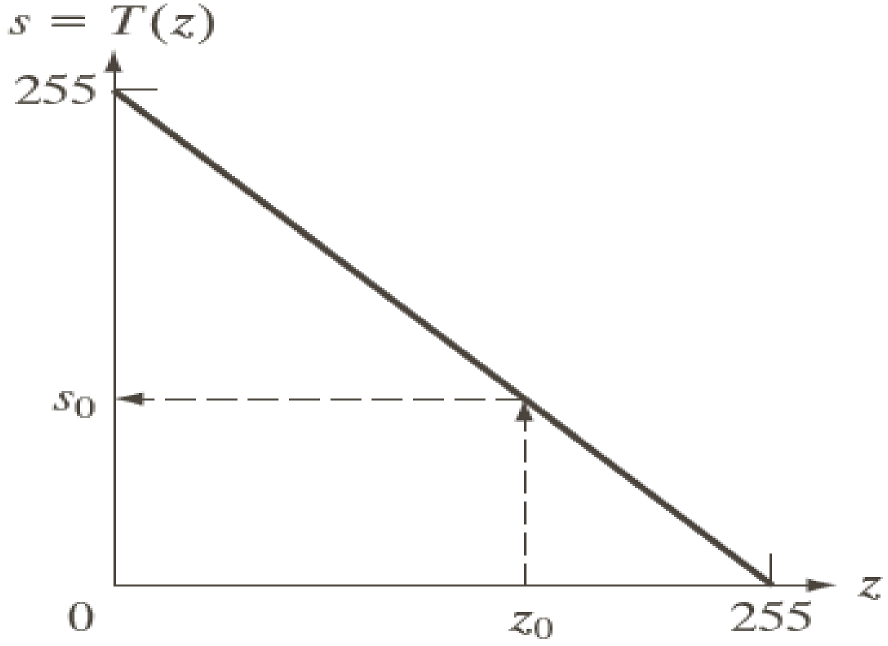
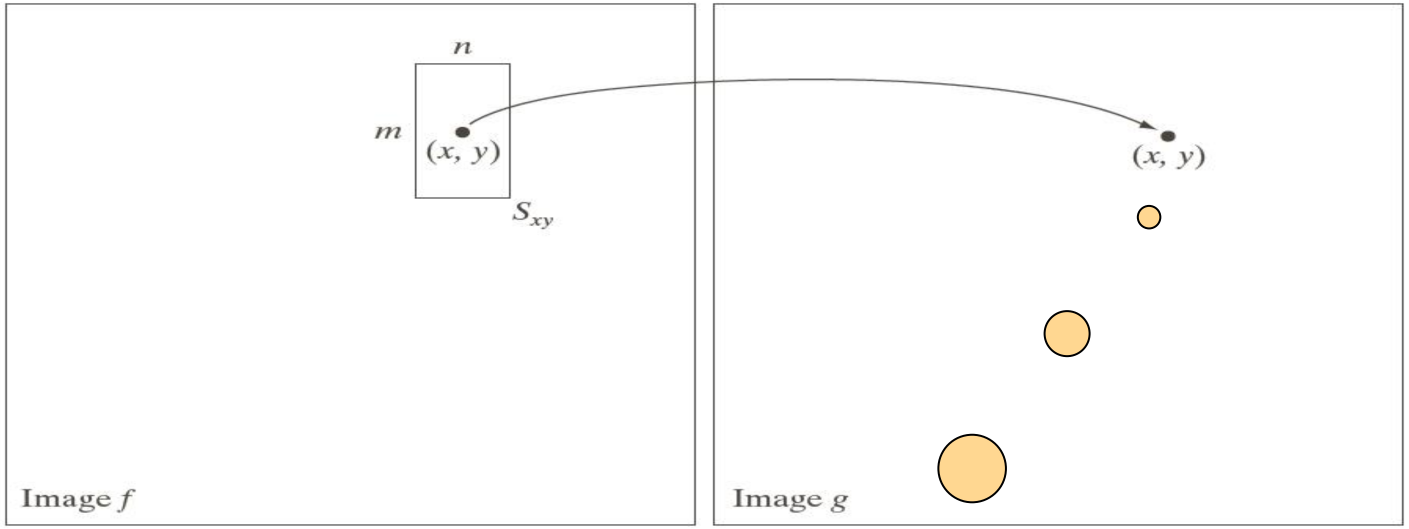


FIGURE 2.34 Intensity transformation function used to obtain the negative of an 8-bit image. The dashed arrows show transformation of an arbitrary input intensity value z_0 into its corresponding output value s_0.

Spatial Operations

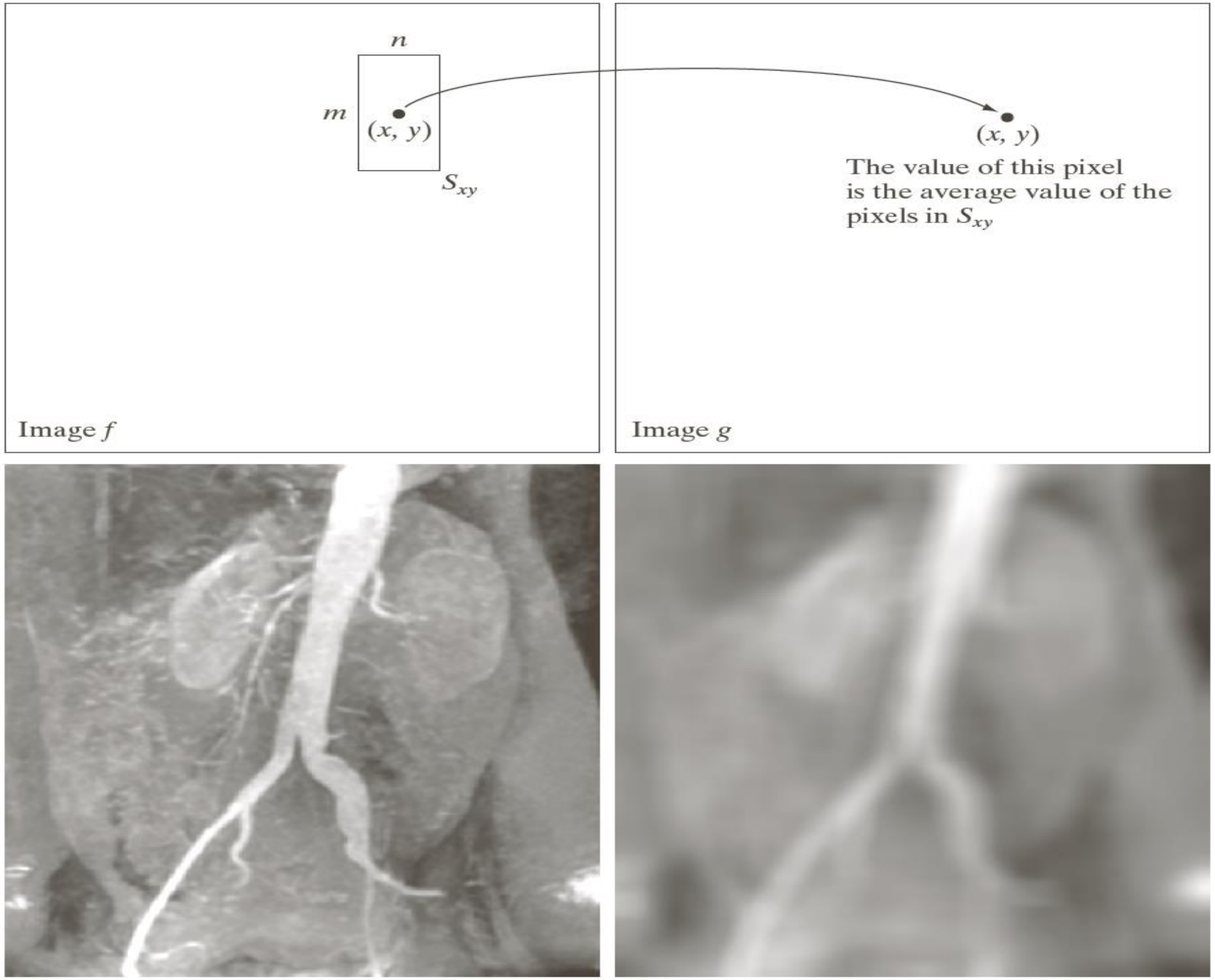
- Neighborhood operations



The value of this pixel is determined by a specified operation involving the pixels in the input image with coordinates in S_{xy}

Spatial Operations

- Neighborhood operations



Intensity Transformation
and
Spatial Filtering

Spatial Domain vs. Transform Domain

- **Spatial domain**

  image plane itself, directly process the intensity values of the image plane

- **Transform domain**

  process the transform coefficients, not directly process the intensity values of the image plane

Spatial Domain Process

$$g(x, y) = T[f(x, y)])$$

$$f(x, y): \text{input image}$$

$$g(x, y): \text{output image}$$

$$T: \text{an operator on } f \text{ defined over}$$

$$\text{a neighborhood of point } (x, y)$$

Spatial Domain Process

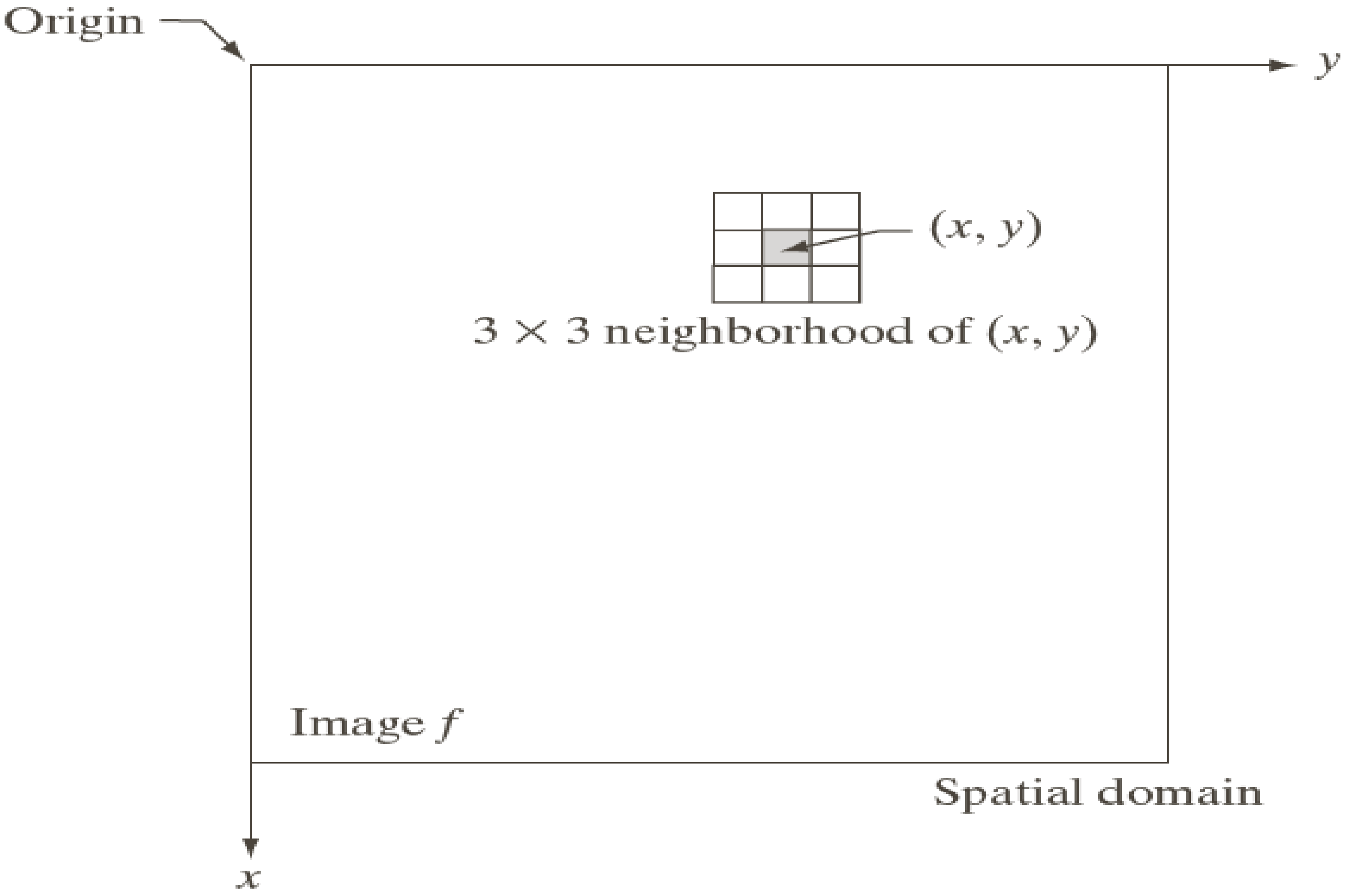
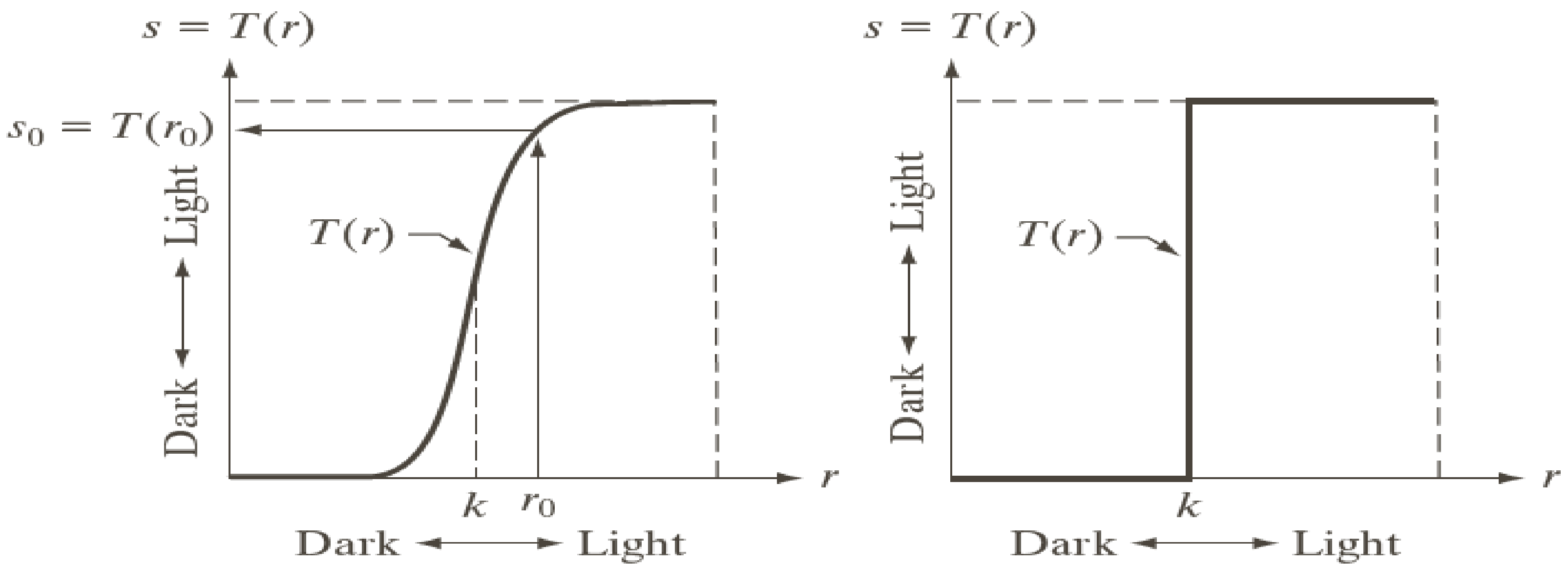


FIGURE 3.1
A 3 × 3 neighborhood about a point (x, y) in an image in the spatial domain. The neighborhood is moved from pixel to pixel in the image to generate an output image.

Spatial Domain Process

Intensity transformation function

$$s = T(r)$$



a b

FIGURE 3.2 Intensity transformation functions. (a) Contrast-stretching function. (b) Thresholding function.

Some Basic Intensity Transformation Functions

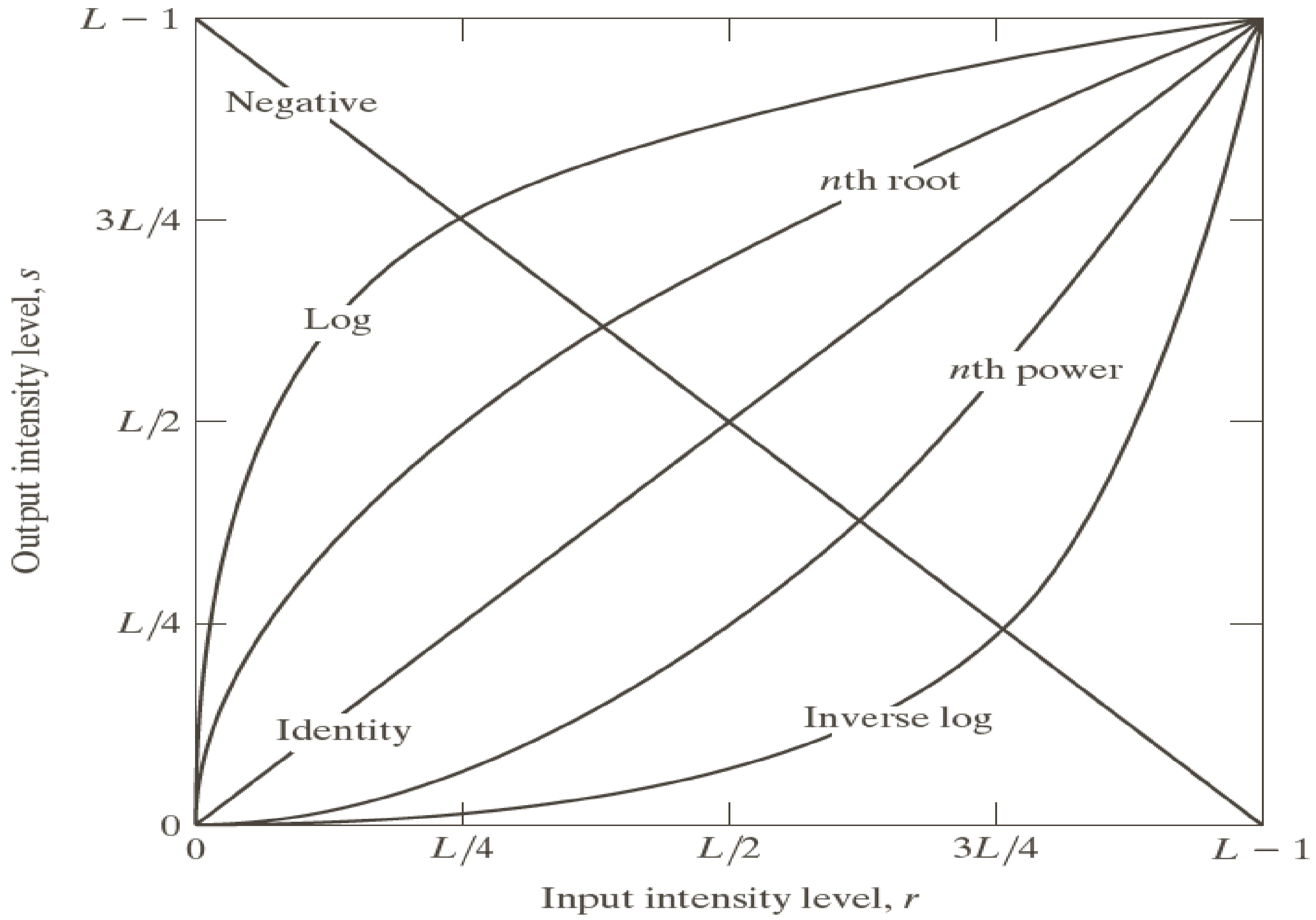


FIGURE 3.3 Some basic intensity transformation functions. All curves were scaled to fit in the range shown.

Image Negatives

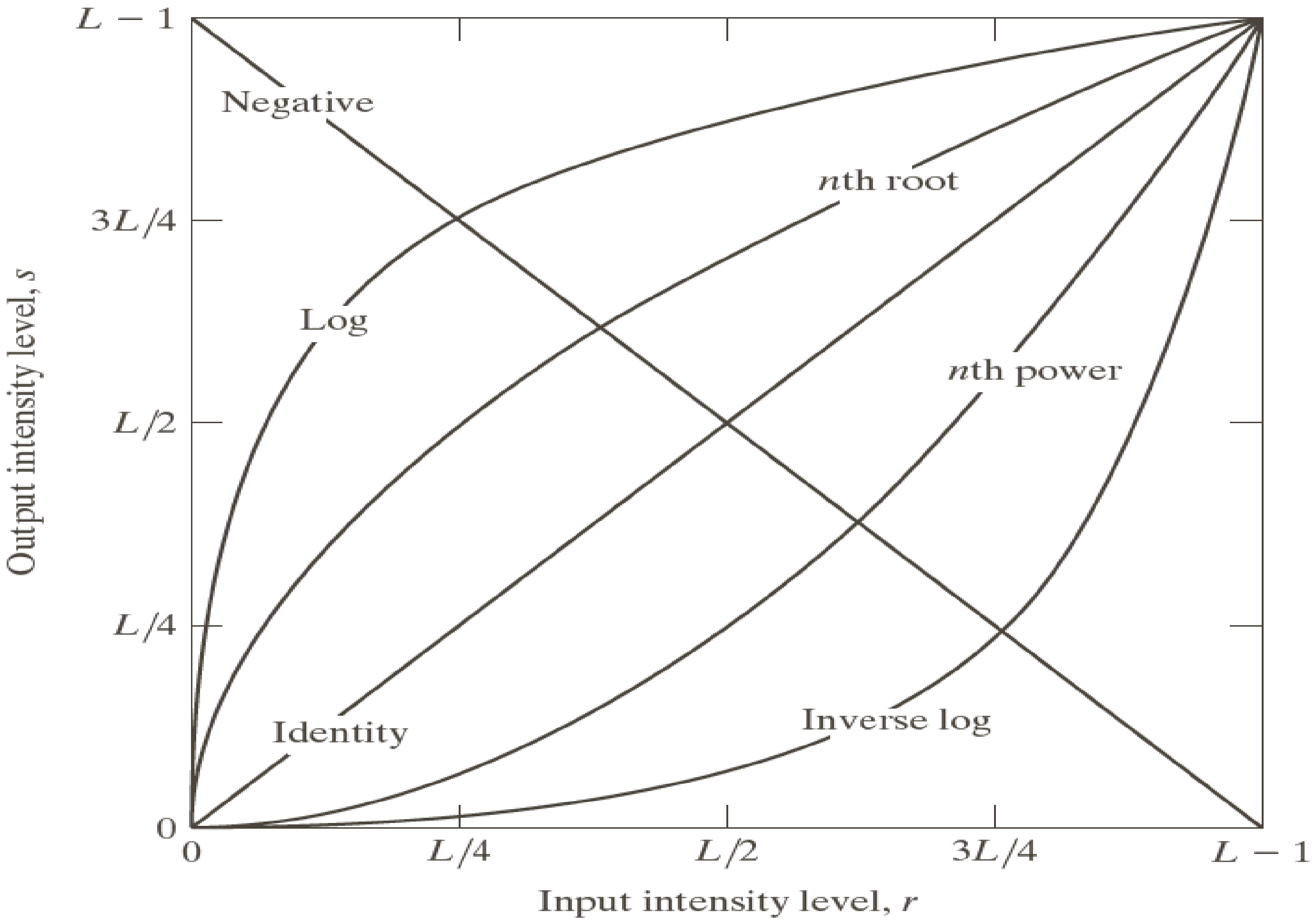
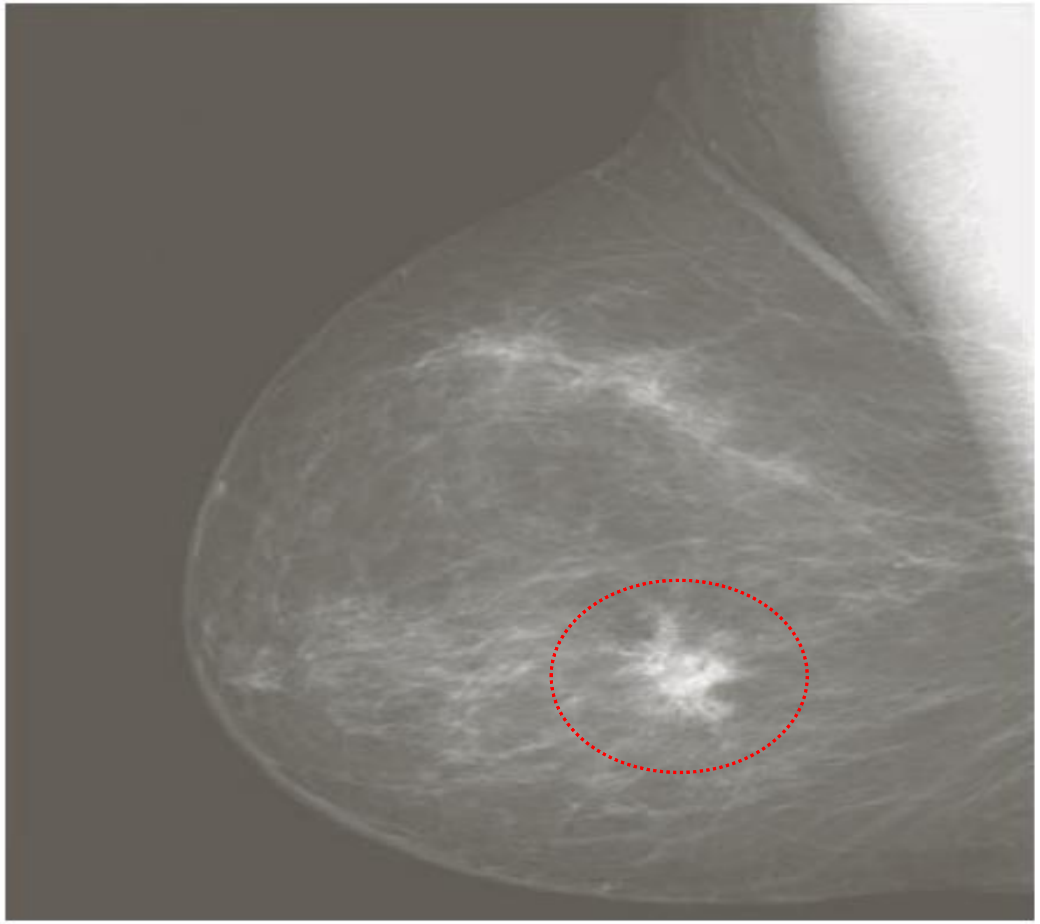


Image negatives

$$s = L - 1 - r$$

Example: Image Negatives

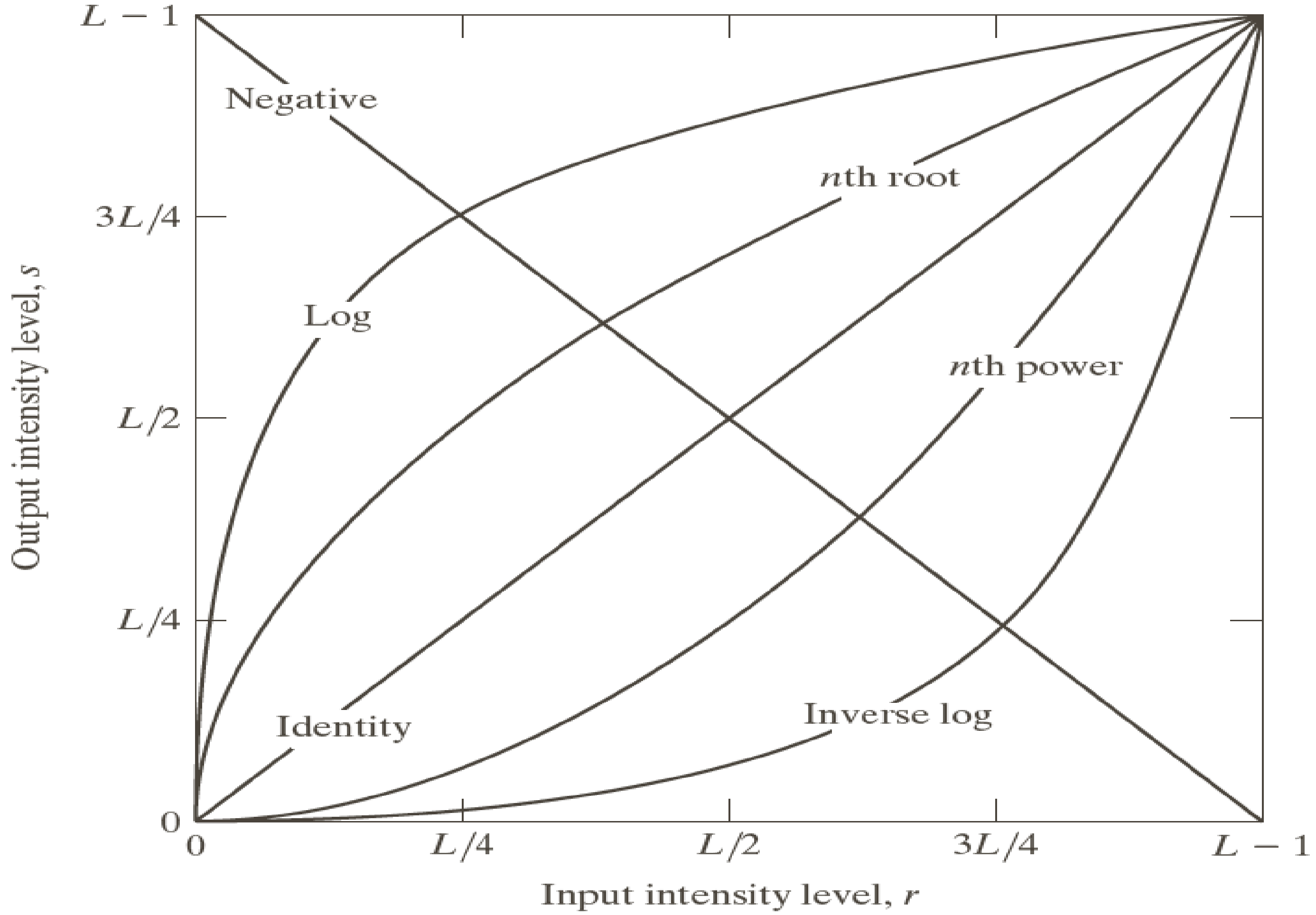


a b

FIGURE 3.4 (a) Original digital mammogram. (b) Negative image obtained using the negative transformation in Eq. (3.2-1). (Courtesy of G.E. Medical Systems.)

Small lesion

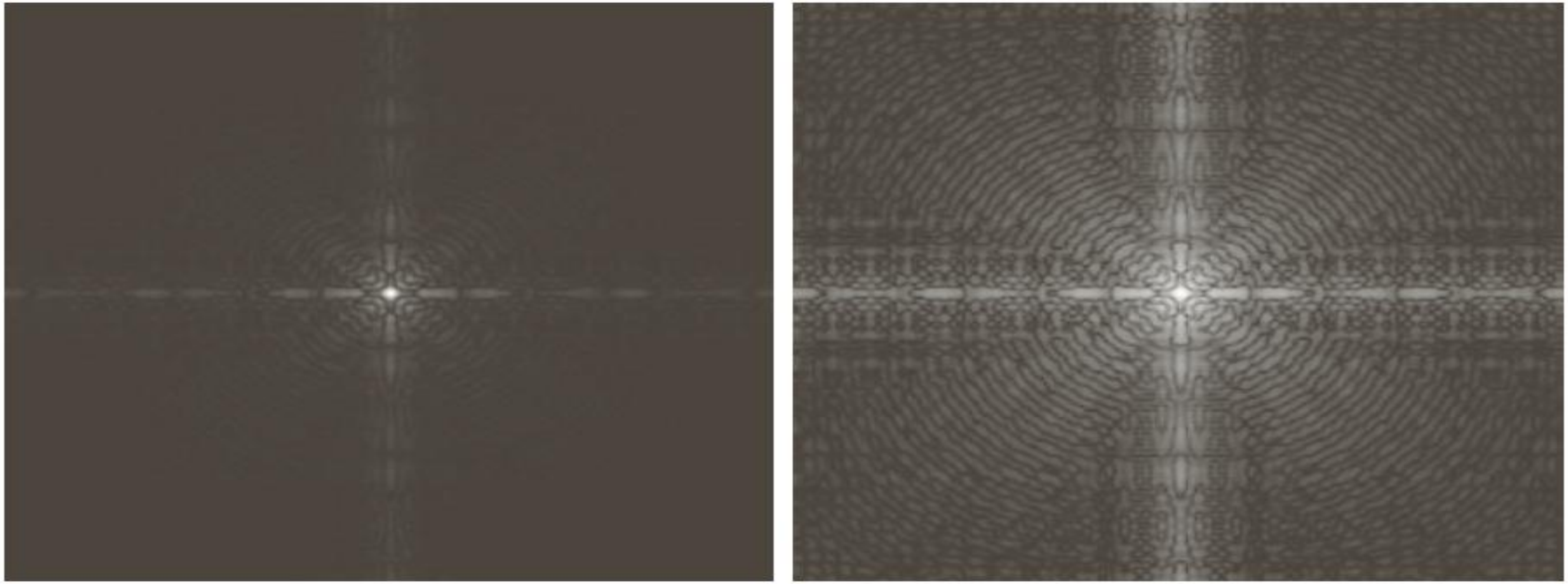
Log Transformations



Log Transformations

$$s = c\log(1+r)$$

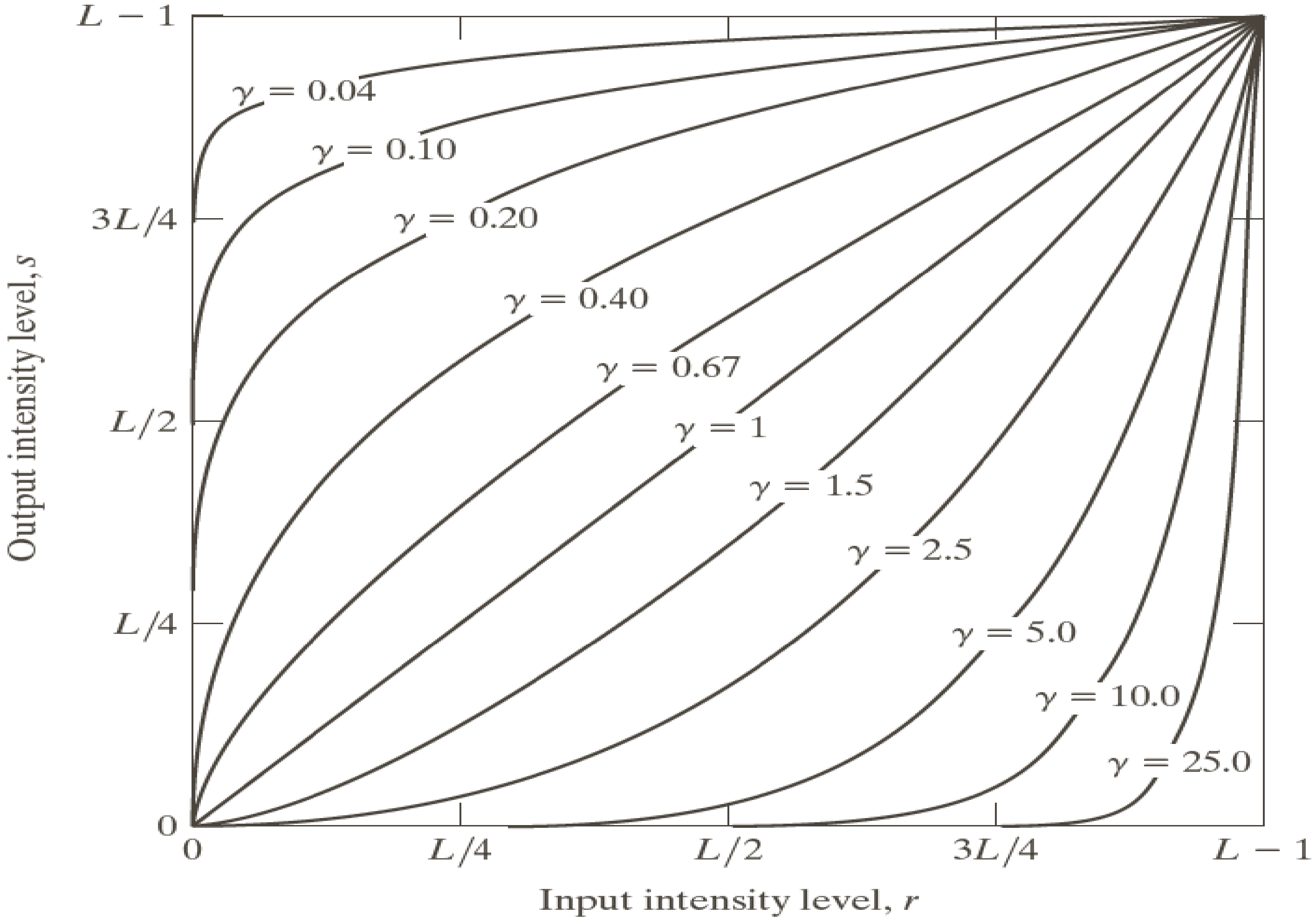
Example: Log Transformations



a b

FIGURE 3.5 (a) Fourier spectrum. (b) Result of applying the log transformation in Eq. (3.2-2) with $c = 1$.

Power-Law (Gamma) Transformations



$$s = cr^{\gamma}$$

FIGURE 3.6 Plots of the equation $s = cr^{\gamma}$ for various values of γ ($c = 1$ in all cases). All curves were scaled to fit in the range shown.

Example: Gamma Transformations

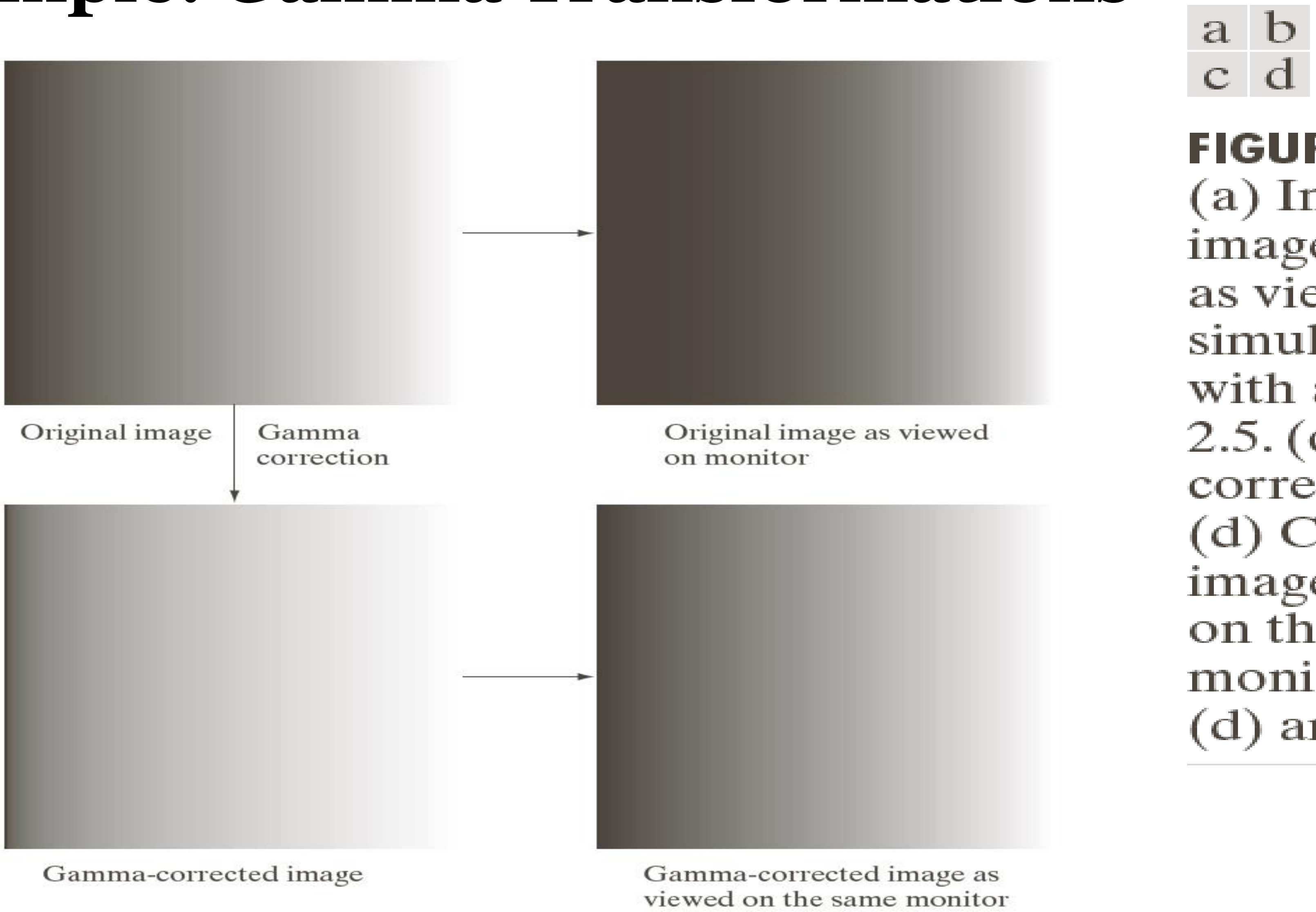
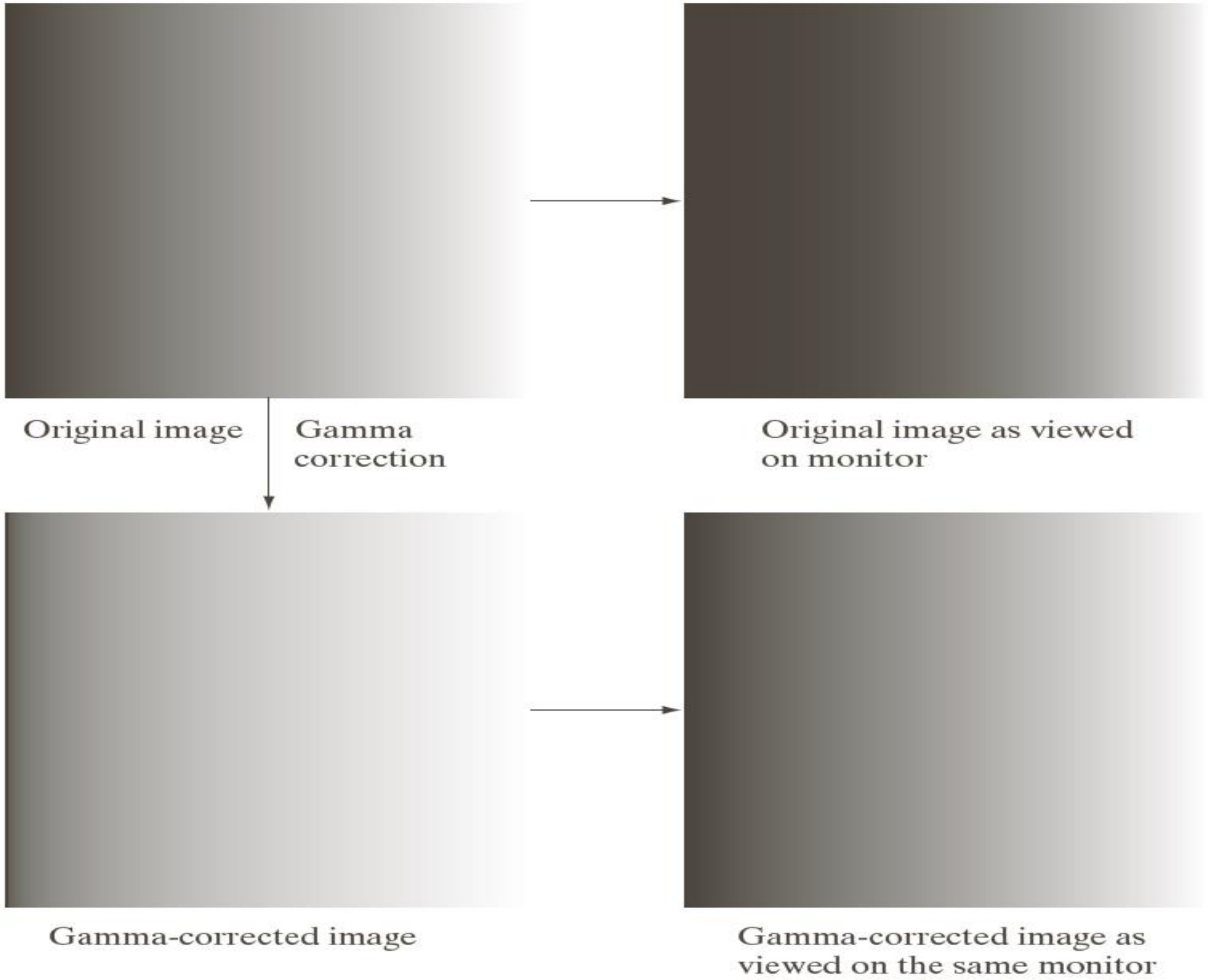


FIGURE 3.7 (a) Intensity ramp image. (b) Image as viewed on a simulated monitor with a gamma of 2.5. (c) Gamma-corrected image. (d) Corrected image as viewed on the same monitor. Compare (d) and (a).

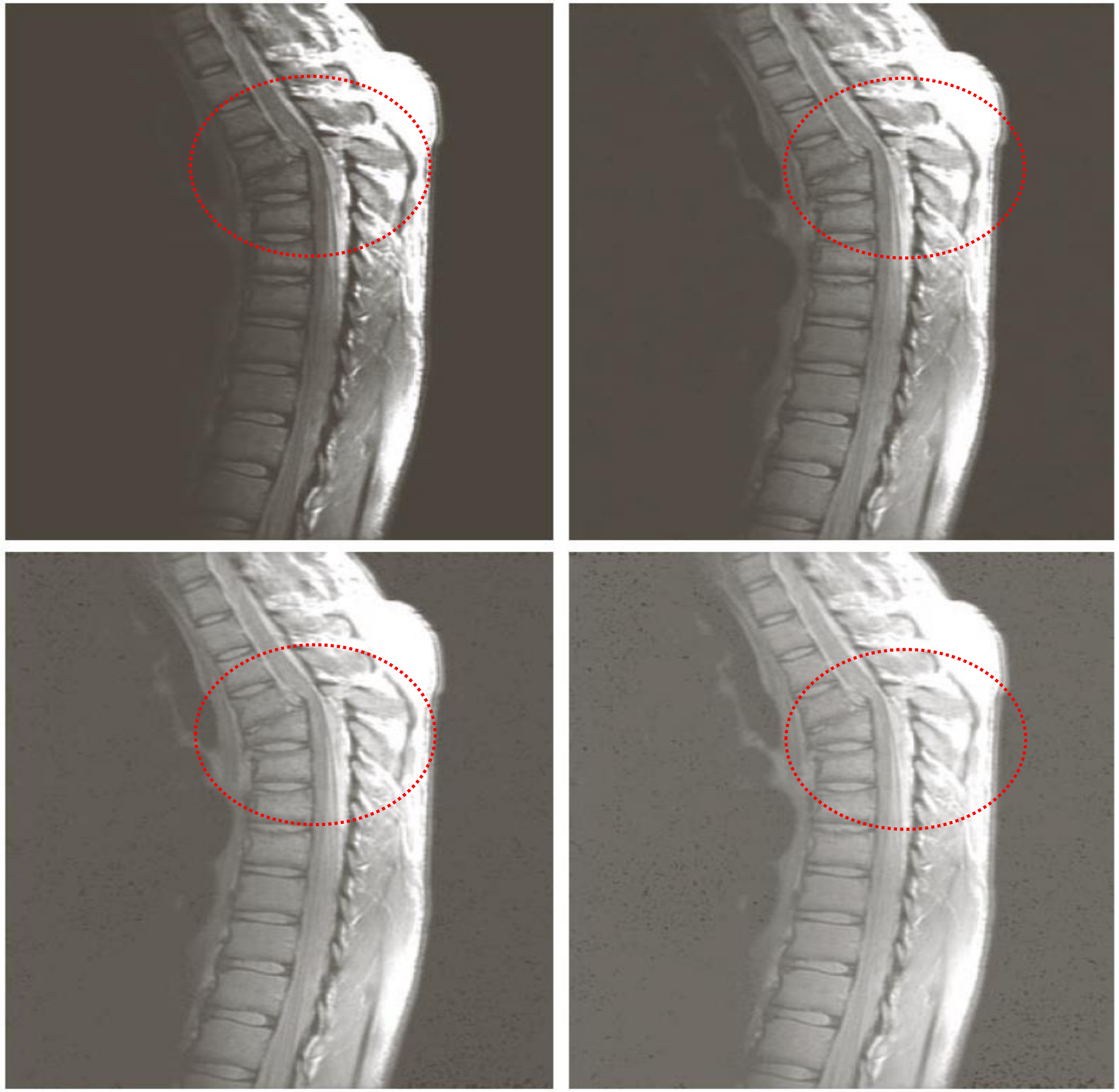
Example: Gamma Transformations



Cathode ray tube (CRT) devices have an intensity-to-voltage response that is a power function, with exponents varying from approximately 1.8 to 2.5

$$s = r^{1/2.5}$$

Example: Gamma Transformations



a	b
c	d

FIGURE 3.8 (a) Magnetic resonance image (MRI) of a fractured human spine. (b)–(d) Results of applying the transformation in Eq. (3.2-3) with $c = 1$ and $\gamma = 0.6, 0.4,$ and 0.3, respectively. (Original image courtesy of Dr. David R. Pickens, Department of Radiology and Radiological Sciences, Vanderbilt University Medical Center.)

Example: Gamma Transformations

a	b
c	d

FIGURE 3.9 (a) Aerial image. (b)–(d) Results of applying the transformation in Eq. (3.2-3) with $c = 1$ and $\gamma = 3.0, 4.0,$ and 5.0, respectively. (Original image for this example courtesy of NASA.)

Piecewise-Linear Transformations

- **Contrast Stretching**

  — Expands the range of intensity levels in an image so that it spans the full intensity range of the recording medium or display device.

- **Intensity-level Slicing**

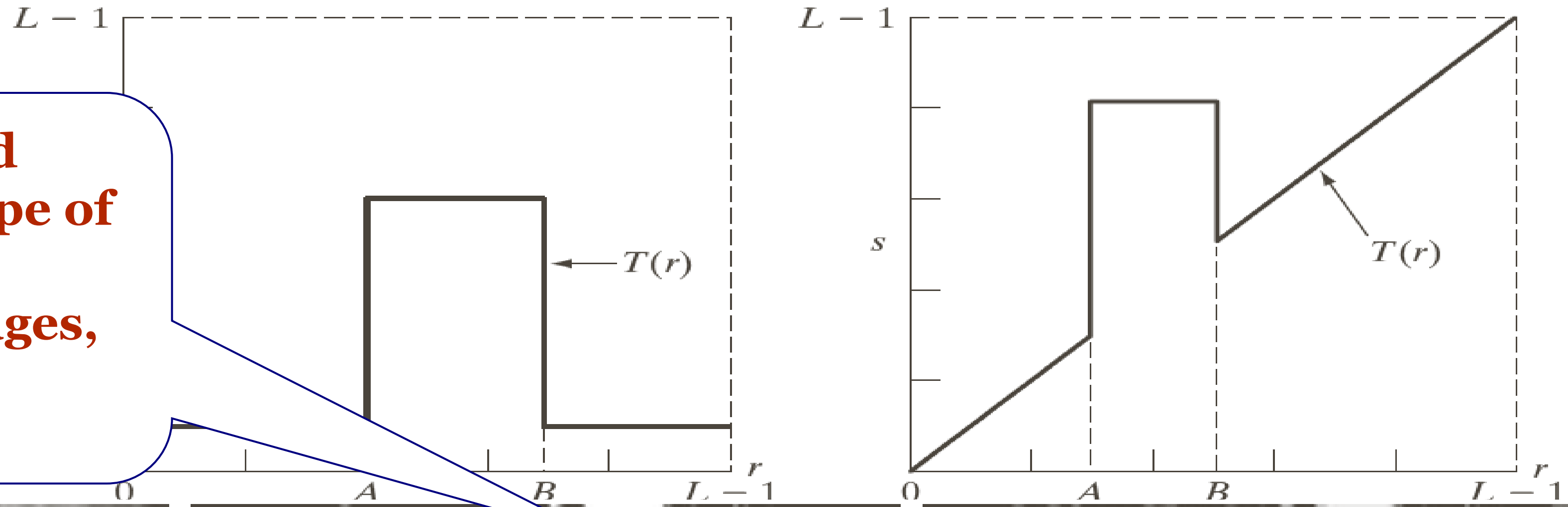
  — Highlighting a specific range of intensities in an image often is of interest.

a	b
c	d

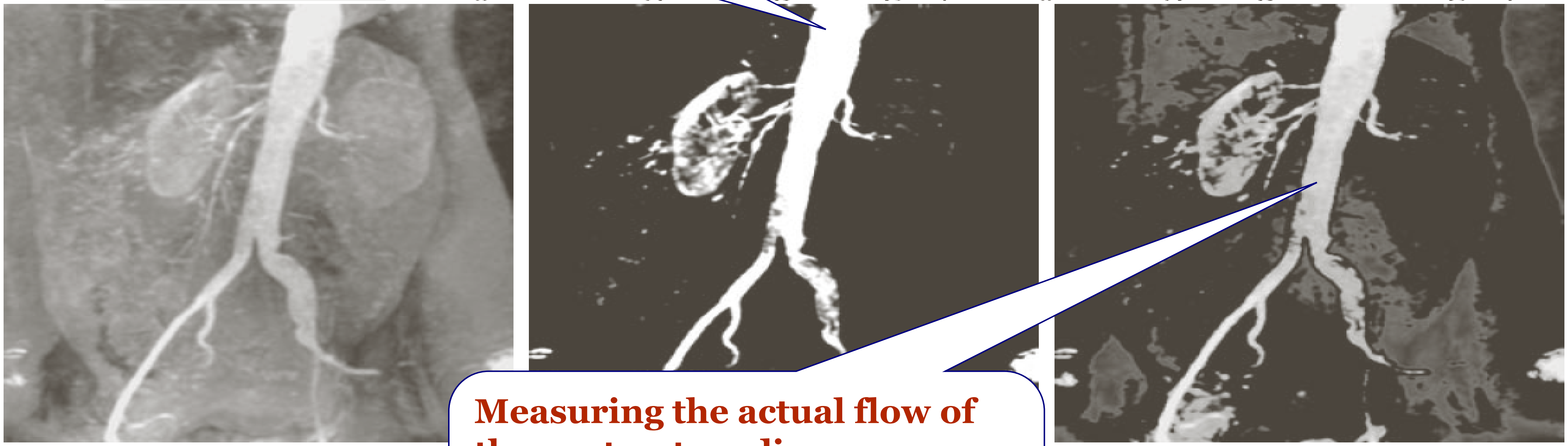
FIGURE 3.10 Contrast stretching. (a) Form of transformation function. (b) A low-contrast image. (c) Result of contrast stretching. (d) Result of thresholding. (Original image courtesy of Dr. Roger Heady, Research School of Biological Sciences, Australian National University, Canberra, Australia.)

a b

FIGURE 3.11 (a) This



Highlight the major blood vessels and study the shape of the flow of the contrast medium (to detect blockages, etc.)



Measuring the actual flow of the contrast medium as a function of time in a series of images

a b c

FIGURE 3.12 (a) Aortic angiog… …mation of the type illustrated in Fig. 3.11(a), with the range of inte… …end of the gray scale. (c) Result of using the transformation in Fig… …ack, so that grays in the area of the blood vessels and kidneys were preserved. (Original image courtesy of Dr. Thomas R. Gest, University of Michigan Medical School.)

Bit-plane Slicing

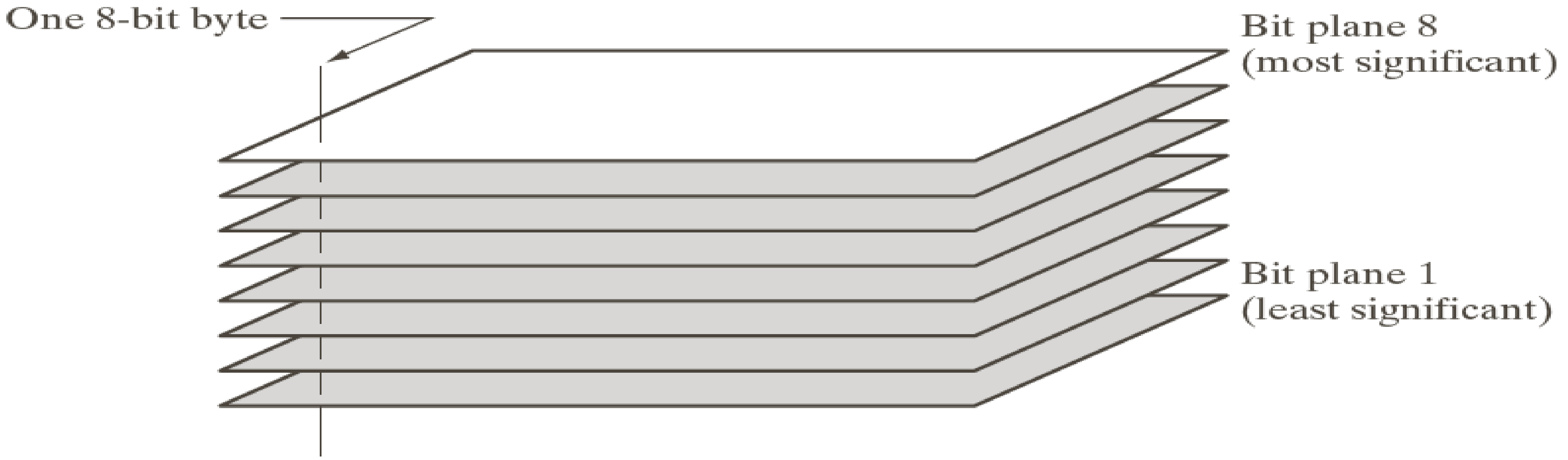
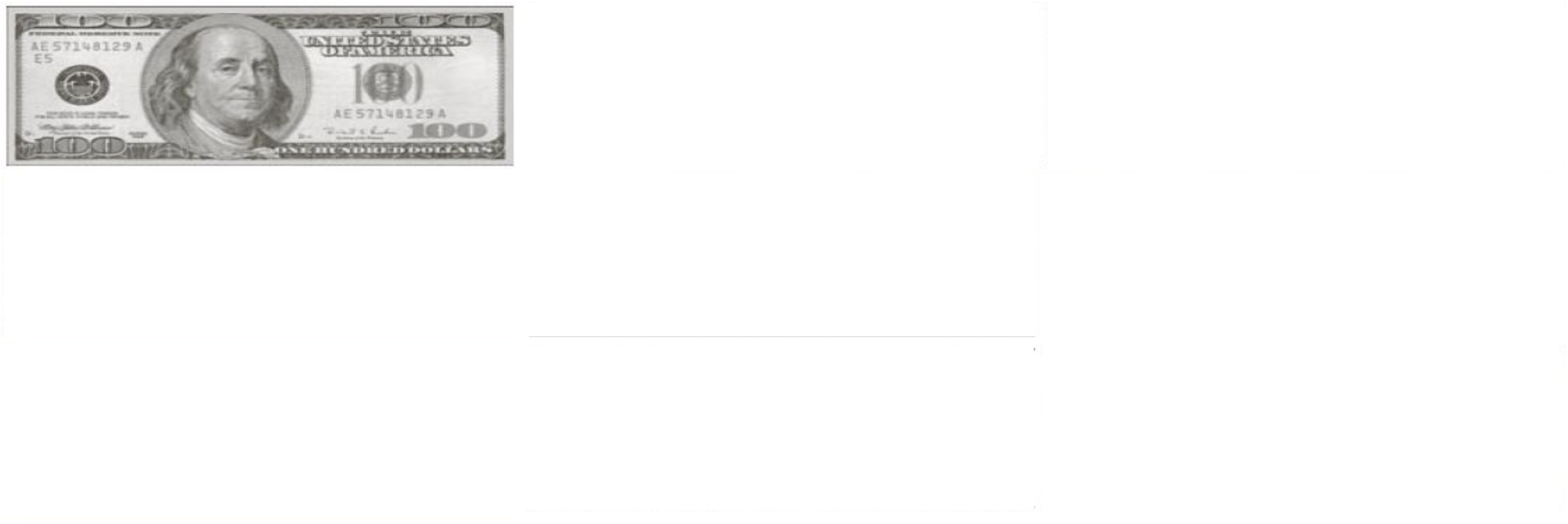


FIGURE 3.13
Bit-plane representation of an 8-bit image.

Bit-plane Slicing



a	b	c
d	e	f
g	h	i

FIGURE 3.14 (a) An 8-bit gray-scale image of size 500 × 1192 pixels. (b) through (i) Bit planes 1 through 8, with bit plane 1 corresponding to the least significant bit. Each bit plane is a binary image.

Bit-plane Slicing

FIGURE 3.15 Images reconstructed using (a) bit planes 8 and 7; (b) bit planes 8, 7, and 6; and (c) bit planes 8, 7, 6, and 5. Compare (c) with Fig. 3.14(a).