CSC418 A2 : PART A

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1) a) $P_1 = (0, 0.2828)$

$P_2 = (0.1818, 0.5143)$

$P_3 = (0.0952, 0.6734$

$P_4 = (-0.1053, 0.4466)$

$$L_{P_1P_2} = (0, 0.2828, 1) \times (0.1818, 0.5143, 1)$$

$$= \begin{bmatrix} j & j & k \\ 0 & 0.2828 & 1 \\ 0.1818 & 0.5143 & 1 \end{bmatrix}$$

$$= -0.2315i + 0.1818j - 0.051413k$$

$$= \begin{bmatrix} -0.2315 \\ 0.1818 \\ -0.0514 \end{bmatrix}$$

$$L_{P_3P_4} = (0.0952, 0.6734, 1) \times (-0.1053, 0.4466, 1)$$

$$= \begin{bmatrix} i & j & k \\ 0.0952 & 0.6734 & 1 \\ -0.1053 & 0.4466 & 1 \end{bmatrix}$$

$$= 0.2268i - 0.2005j + 0.113425k$$

$$= \begin{bmatrix} 0.2268 \\ -0.2005 \\ 0.1134 \end{bmatrix}$$

$$L_{P_2P_3} = (0.1818,\ 0.5143,\ 1) \times (0.0952,\ 0.6734,\ 1)$$

$$= \begin{bmatrix} i & j & k \\ 0.1818 & 0.5143 & 1 \\ 0.0952 & 0.6734 & 1 \end{bmatrix}$$

$$= -0.1591i - 0.0866j + 0.0734k$$

$$= \begin{bmatrix} -0.1591 \\ -0.0866 \\ 0.0735 \end{bmatrix}$$

$$L_{P_4P_1} = (-0.1053,\ 0.4466,\ 1) \times (0,\ 0.2828,\ 1)$$

$$= \begin{bmatrix} i & j & k \\ -0.1053 & 0.4466 & 1 \\ 0 & 0.2828 & 1 \end{bmatrix}$$

$$= 0.1638i + 0.1053j - 0.0297788k$$

$$= \begin{bmatrix} 0.1638 \\ 0.1053 \\ -0.0298 \end{bmatrix}$$

Vanishing points are where P_{L1L2} and P_{L3L4} intersect, and where P_{L2L3} and P_{L4L1} intersect.

$$V_1 = L_{P_1P_2} \times L_{P_3P_4}$$

$$= \begin{bmatrix} i & j & k \\ -0.2315 & 0.1818 & -0.0514 \\ 0.2268 & -0.2005 & 0.1134 \end{bmatrix}$$

$$= 0.0103i + 0.0146j + 0.0052k = \begin{bmatrix} 0.0103 \\ 0.0146 \\ 0.0052 \end{bmatrix}$$

$$\approx \begin{bmatrix} 0.0103/0.0052 \\ 0.0146/0.0052 \\ 0.0052/0.0052 \end{bmatrix} \approx \begin{bmatrix} 1.9895 \\ 2.8161 \\ 1 \end{bmatrix}$$

$$V_2 = L_{P_2P_3} \times L_{P_4P_1}$$

$$= \begin{bmatrix} i & j & k \\ -0.1591 & -0.0866 & 0.0735 \\ 0.1638 & 0.1053 & -0.0298 \end{bmatrix}$$

$$= -0.0052i + 0.0073j - 0.0027k$$

$$= \begin{bmatrix} -0.0052 \\ 0.0073 \\ -0.0027 \end{bmatrix} \approx \begin{bmatrix} -0.0052/-0.0027 \\ 0.0073/-0.0027 \\ -0.0027/-0.0027 \end{bmatrix}$$

$$\approx \begin{bmatrix} 2.0080 \\ -2.8407 \\ 1 \end{bmatrix}$$

The two vanishing points are

$(1.9895, 2.8161)$ and $(2.0080, -2.8407)$

b) Let a line be:

$$(x_0 + y_0 + z_0) + t(D_x, D_y, D_z)$$

where (X_0, Y_0, Z_0) is a point
and (D_x, D_y, D_z) is a direction

$\longrightarrow$

If camera at (0,0,0) does not lie on the line, then camera and line define a plane

Image Projection of line is the intersection of the plane and image project plane.

This can be represented parametrically as:

$$(x(t), y(t)) = \left[\frac{X_0 + T_x t}{Z_0 + T_z t}, \frac{Y_0 + T_y t}{Z_0 + T_z t}\right] f$$

If $T_z \neq 0$, let $t \to \infty$, so the vanishing point can be shown as

$$(x_v, y_v) = f\left(\frac{T_x}{T_z}, \frac{T_y}{T_z}\right)$$

If we were to follow this process with another line with the same direction, it would result in the same (x_v, y_v). Thus, two parallel lines will have the same vanishing point.

c) Get the dot product of the two vanishing points:

$$V_1 \cdot V_2 = (1.9895, 2.8161) \cdot (2.008, -2.8407)$$

$$= (1.9895)(2.008) + (2.8161)(-2.8407)$$
$$= -4.005$$

If they were orthogonal, the dot product would be 0, not -4.005. Thus, r_1 and r_2 are not orthogonal.

d) Distance from origin to $(1.9895, 2.8161)$

$= \sqrt{(1.9895)^2 + (2.8161)^2}$

$= 3.4480$

Distance from origin to $(2.008, -2.8407)$

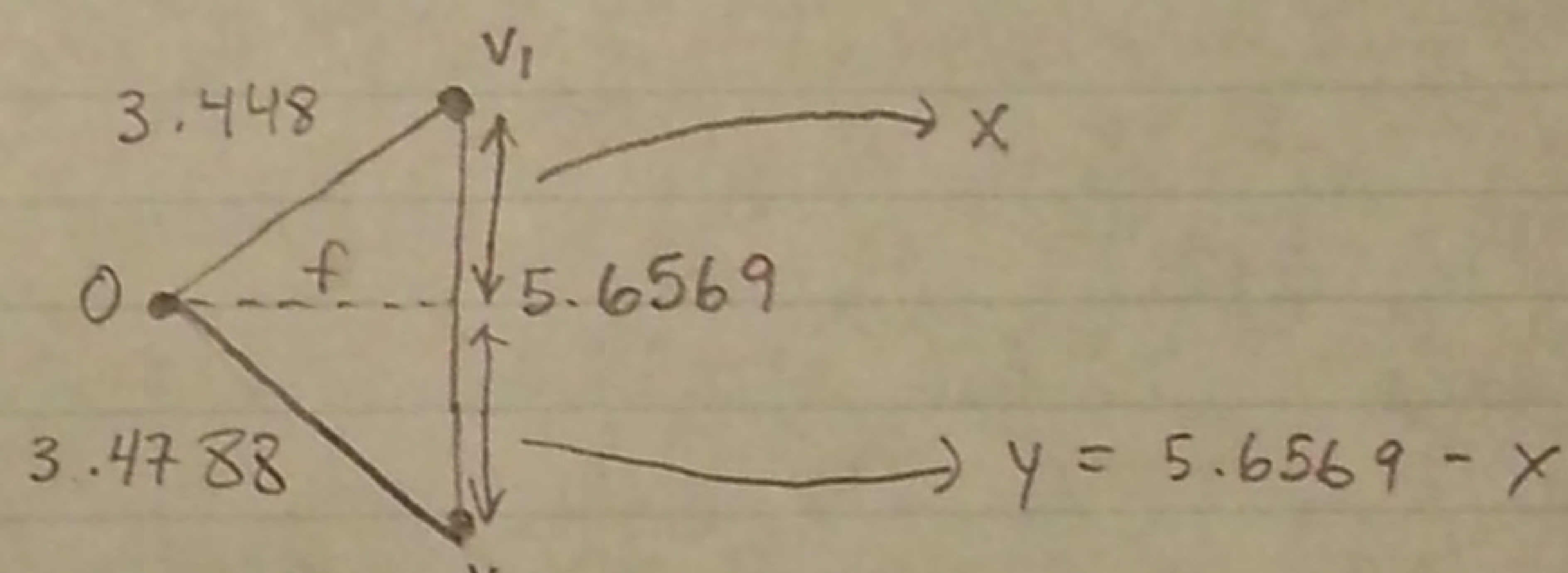
$= \sqrt{(2.008)^2 + (-28407)^2}$

$= 3.4788$

Distance between vanishing points:

$= \sqrt{(2.008 - 1.9895)^2 + (-28407 - 2.8161)^2}$

$= 5.6569$



$x^2 + f^2 = (3.448)^2 = 11.8884 \Rightarrow f^2 = 11.8884 - x^2$

$f^2 + y^2 = (3.4788)^2$

$\Rightarrow f^2 + (5.6569 - x)^2 = 12.1017$

$\Rightarrow 11.8884 - \cancel{x^2} + 32 + \cancel{x^2} - 11.3137x$

$\Rightarrow x = 2.8096$

$f^2 = 11.8884 - (2.8096)^2$

$\Rightarrow f = 1.999 \Rightarrow f = 2$

The focal length is 2.

c) We can find normal by crossing vanishing points

$$n = v_1 \times v_2$$

$$= \begin{bmatrix} i & j & k \\ 1.9895 & 2.8161 & 1 \\ 2.0080 & -2.8407 & 1 \end{bmatrix}$$

$$= 5.6568i + 0.0185j - 11.3062k$$

$$= \begin{bmatrix} 5.6568 \\ 0.0185 \\ 11.3062 \end{bmatrix}$$

2) $x(t) = \sin(2\pi t) + 2t$
$y(t) = t^2$ $\quad 0 \le t \le 5$

multiply $x(t)$ by $\cos\theta$
z would by $x(t)$ times $\sin\theta$

so:

$$x(t) = (\sin(2\pi t) + 2t)\cos\theta$$
$$y(t) = t^2$$
$$z(t) = (\sin(2\pi t + 2t)\sin\theta$$

3) a) $x_1(t) = x_2(u) \Rightarrow 3\sin(2\pi t) = u$
$y_1(t) = y_2(u) \Rightarrow 3\cos(2\pi t) = 0$
$\Rightarrow \cos(2\pi t) = 0$
$\Rightarrow t = n/2 - 1/4, \; n \in \mathbb{Z}$

In lecture, it was said to keep $0 \le t \le 1$,
thus, $t = \frac{1}{4}, \frac{3}{4}$

$z_1(t) = z_2(u)$

$\Rightarrow 2\sqrt{2} = \sqrt{u^2-1}$

$\Rightarrow (2\sqrt{2})^2 = (\sqrt{u^2-1})^2$

$\Rightarrow 8 = u^2-1$

$\Rightarrow 9 = u^2$

$\Rightarrow \pm 3 = u$

In lecture, it was said to keep u positive, thus $u = 3$

So, using $t = \frac{1}{4}, \frac{3}{4}$ and $u=3$, we find the point of intersection is $(3, 0, \sqrt{8}) = (3, 0, 2\sqrt{2})$

b) $f_1'(t) = (x_1'(t), y_1'(t), z_1'(t))$

$f_1'(t) = (6\pi\cos(2\pi t), -6\pi\sin(2\pi t), 0)$

$f_1'(\frac{1}{4}) = (6\pi\cos(2\pi(3)),$
$-6\pi\sin(2\pi(3)),$
$0\quad)$

$= (0, -6\pi, 0)$

$f_2'(u) = (x_2'(u), y_2'(u), z_2'(u))$

$= (1, 0, \frac{1}{2\sqrt{u^2-1}} \cdot 2u)$

$= (1, 0, \frac{u}{\sqrt{u^2-1}})$

$f_2'(3) = (1, 0, \frac{3}{\sqrt{8}})$

c) $(0, -6\pi, 0) \times (1, 0, \frac{3}{\sqrt{8}})$

$$= \begin{bmatrix} i & j & k \\ 0 & -6\pi & 0 \\ 1 & 0 & 3/\sqrt{8} \end{bmatrix}$$

$$= -\frac{18\pi}{\sqrt{8}} i - 0j + 6\pi k$$

$$= \begin{bmatrix} -18\pi/\sqrt{8} \\ 0 \\ 6\pi \end{bmatrix}$$

$(1, 0, 3/\sqrt{8}) \times (0, -6\pi, 0)$

$$= \begin{bmatrix} i & j & k \\ 1 & 0 & 3/\sqrt{8} \\ 0 & -6\pi & 0 \end{bmatrix}$$

$$= \frac{18\pi}{\sqrt{8}} i + 0j - 6\pi k$$

$$= \begin{bmatrix} 18\pi/\sqrt{8} \\ 0 \\ -6\pi \end{bmatrix}$$

Since $\begin{bmatrix} -18\pi/\sqrt{8} \\ 0 \\ 6\pi \end{bmatrix}$ has positive z, we choose it as it faces out from the surface

$$= \begin{bmatrix} -9\pi/\sqrt{2} \\ 0 \\ 6\pi \end{bmatrix}$$

Normalize:

$$\frac{(-9\pi/\sqrt{2},\ 0,\ 6\pi)}{\sqrt{\left(-\frac{9\pi}{\sqrt{2}}\right)^2 + 0^2 + (6\pi)^2}}$$

$$= \frac{(-9\pi/\sqrt{2},\ 0,\ 6\pi)}{\sqrt{\frac{81\pi^2}{2} + 36\pi^2}}$$

$$= \frac{(-9\pi/\sqrt{2},\ 0,\ 6\pi)}{\sqrt{\frac{153\pi^2}{2}}}$$

$$= \frac{1}{\sqrt{\frac{153\pi^2}{2}}}\left(-\frac{9\pi}{\sqrt{2}},\ 0,\ 6\pi\right)$$

d) $\nabla f(x,y,z) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right)$

$$= (2x,\ 2y,\ -2z)$$

e) $\nabla f(3, 0, \sqrt{8}) = (2(3),\ 2(0),\ -2(\sqrt{8}))$

$$= (6, 0, -2\sqrt{8})$$
$$= (6, 0, -4\sqrt{2})$$

Normal is in opposite direction from the on computed in c.

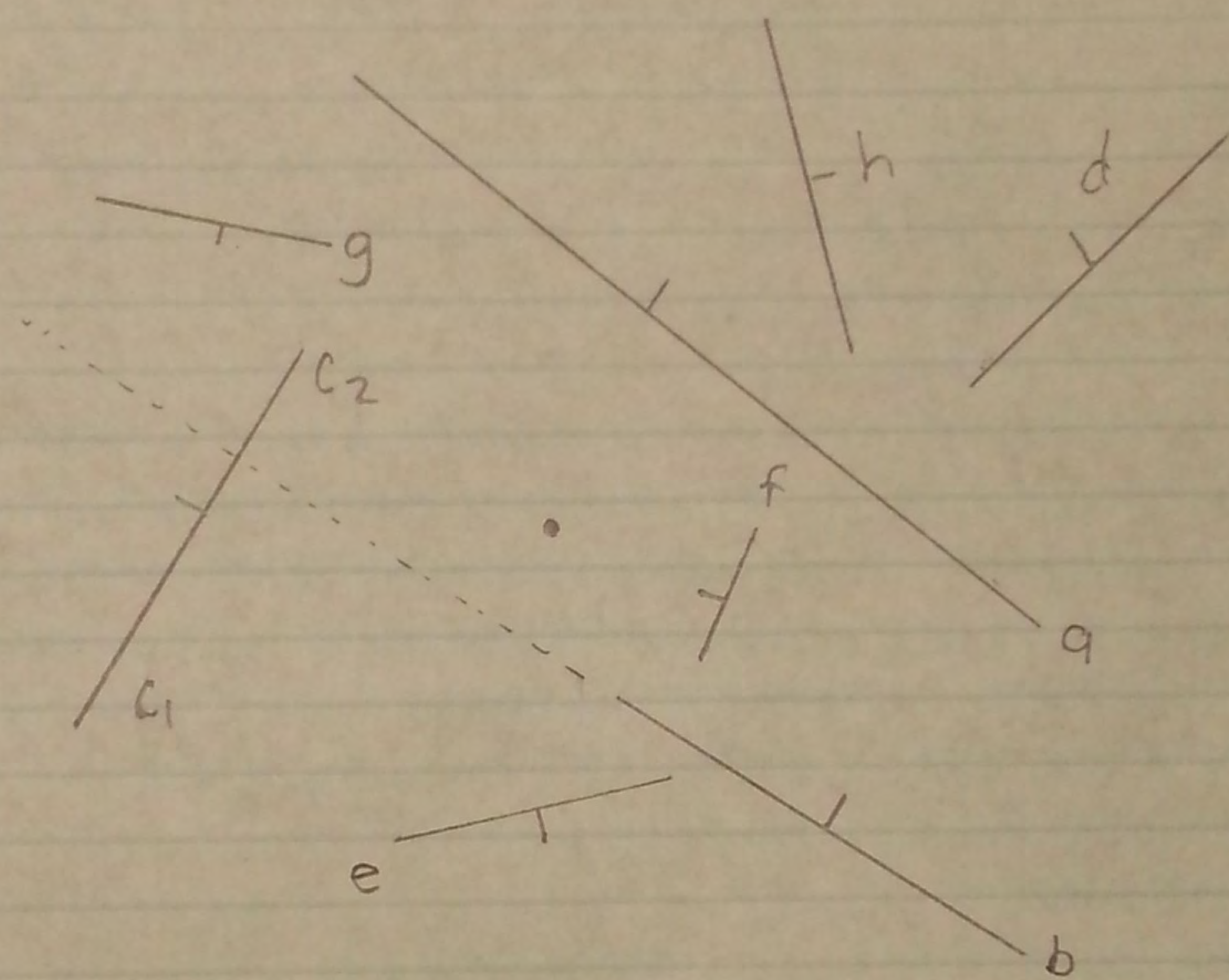
$$n = \frac{(6, 0, -4\sqrt{2})}{\sqrt{6^2 + 0^2 (-4\sqrt{2})^2}} = \frac{(6, 0, -4\sqrt{2})}{\sqrt{36 + 32}}$$

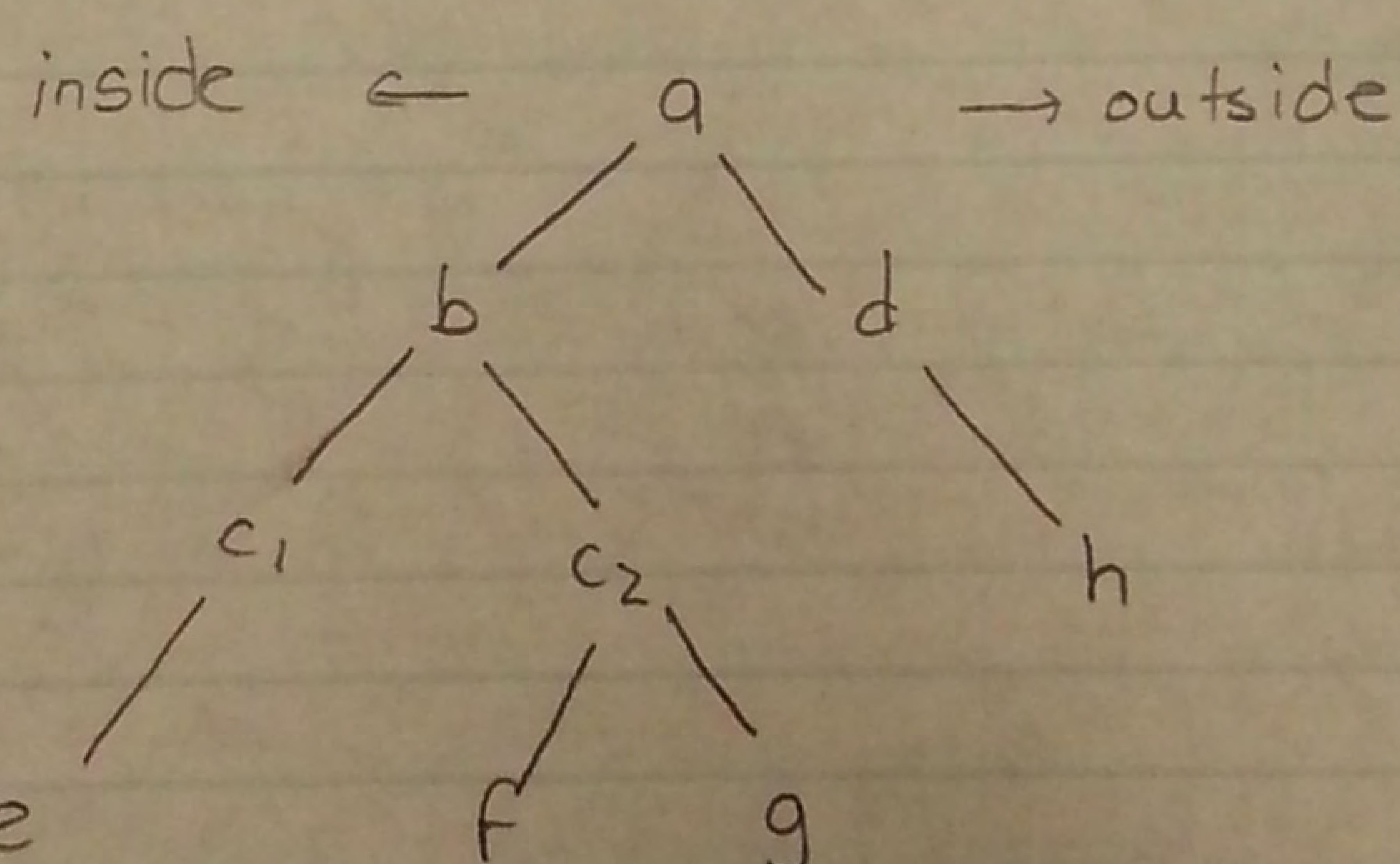
$$= \frac{1}{\sqrt{68}}(6, 0, -4\sqrt{2}) = \frac{1}{2\sqrt{17}}(6, 0, -4\sqrt{2})$$

4) a) You can exclude h

The view of h is completely obstructed, and it's normal is pointing away from the general direction of the camera

b)





c) h d a g c_2 f b c_1 e