

* Notes About Code:

In addition to **SudokuSolver.py**, two additional files have been submitted:

- **SudokuHelpers.py** – contains helper methods for SudokuSolver.py
- **SudokuIO.py** - contains methods to read/write to files, and outputting to the screen

SudokuSolver contains the main algorithms and main. **Without the two additional files, the program won't compile/run, so please download them aswell.**

Brute Force Algorithm

This implementation attempts to solve the sudoku puzzle using an exhaustive search method. Initially, scan the puzzle for a coordinates list of all empty cells (by row and column). Since we are brute forcing, we need every single permutation of possible values we can insert into empty cell values. Retrieving all possible blanks was retrieved using the itertools module's product method, which provided permutative lists of size of the number of blank cells found. Starting with the first permutation (e.g. (1, 1, 1, ...)), iterate through all possibilities, and check if it applying the permutation to the puzzle results in a goal state, until a permutation is found that solves the puzzle.

Back Tracking Algorithm

This implementation attempts to solve the sudoku puzzle using the back-tracking constraint satisfaction problem method, using recursion to back-track if a path leads to a situation where a legal placement of a value is not possible. The algorithm was adapted from pseudo code in slide 24 of CSP lecture. As a base case to the recursion, check if puzzle is in goal state and return true if so. Else, get the first empty cell. For each value that can be put in the empty cell (1-9), assign the value to the empty cell. Check if this creates conflicts, and choose the next value if it does and so on. If constraints are ok, modify puzzle with the value and then recurse this function with the rest of the empty cells, returning true with each successful placement. If this value doesn't lead to the solution, the recursive call will eventually return false, then try the next value if one exists. Check constraints before assigning the value, or else it'll accidentally find itself as duplicate. If all values are tried in the cell, and a legal placement is not possible, reset the cell to an empty cell as we have tried all values without success, and return false. The recursive function will then backtrack to an earlier placed space to try a different state.

Forward Checking + MRV Algorithm

This implementation extends back tracking, and attempts to solve the puzzle using a minimum remaining values heuristics method, using recursion to back-track if path leads to inability to place values legally. As a base case to the recursion, check if puzzle is in goal state and return true if so. Else, get list of all empty cells, and the legal values they can currently take, sorted by minimum restricted values, i.e. the cells which have the fewest legal options. For each value that can legally be put in the chosen empty cell, modify the puzzle by placing value in empty cell. Then recurse this function with the rest of the empty cells, re-evaluating the mrv cells. If this value doesn't lead to the solution, the recursive call will eventually return false, then try the next legal value if one exists. If all legal values have been tried in the cell, and a legal placement is not possible, reset the cell to an empty cell as we have tried all values without success, and return false. The recursive function will then backtrack to an earlier placed space to try a different state.

Comparing performance of search algorithms for test cases listed in the appendix			
Algorithms:	Brute Force	Back-Tracking	Forward Checking with MRV
puzzle1.txt (Test case 1)			
Total clock time (ms)	N/A Too Long	5.69796562195	11.7399692535
Total search time (ms)	N/A Too Long	5.65981864929	11.6810798645
Nodes generated	?	687	41
puzzle2.txt (Test case 2)			
Total clock time (ms)	N/A Too Long	64.5699501038	20.1671123505
Total search time (ms)	N/A Too Long	64.5248889923	20.1210975647
Nodes generated	?	8008	58
puzzle3.txt (Test case 3)			
Total clock time (ms)	N/A Too Long	117.506980896	62.9210472107
Total search time (ms)	N/A Too Long	117.410182953	62.8731250763
Nodes generated	?	14340	167
puzzle4.txt (Test case 4)			
Total clock time (ms)	N/A Too Long	37.0609760284	33.0669879913
Total search time (ms)	N/A Too Long	37.0230674744	33.0278873444
Nodes generated	?	4621	86
puzzle5.txt (Test case 5)			
Total clock time (ms)	N/A Too Long	20.4617977142	28.9080142975
Total search time (ms)	N/A Too Long	20.41888237	28.8689136505
Nodes generated	?	2359	66

Results Analysis:

For all five puzzles, running brute force ran too long, unable to yield the clock/search time and number of nodes generated for any of them. This is expected, as trying every permutation possible has a worst case complexity of 9^n , where n is the number of blank cells. Taking puzzle 1 as an example, which has 41 empty cells, there is 9^{41} permutations of puzzle state, and surfing through these is an exhaustive method.

When comparing the number of nodes generated between the back tracking and FC-MRV methods, we get expected results for all puzzles where the number of nodes generated with FC-MRV are significantly smaller than BT (eg. For puzzle 3, FC-MRV only expanded 1.2% of the nodes expanded by BT. While theoretically, the worst case complexity for both BT and FC-MRV remains 9^n (n = # of empty cells), best and average case significantly increase, with BT able to find solutions faster due to the fact it checks constraints after placing a possible value and back tracking when stuck, instead of starting from scratch, and FC-MRV faster due to the fact it checks constraints prior to placement, with the order of said placement determined dynamically based on the MRV heuristic, instead of a predefined static order.

While FC-MRV generated fewer nodes, unexpected results were observed where in some cases, the BT method performed faster time-wise compared to FC-MRV, even when generating substantially lesser nodes (e.g. puzzle 1 took double time with FC-MRV, puzzle 4 ran roughly the same time as BT, and puzzle 5 was a little longer than BT). This can be attributed to the fact that in some scenarios, a significantly larger amount of computation is required to determine the MRV heuristic, compared to simply backtracking. Another interesting unexpected result was that most puzzles gave different solutions when using BT and FC-MRV, however both were correct valid solutions. With BT searching linearly and back tracking, it makes sense it was able to find an early solution out of several in its search, compared to FC-MRV searching optimally based on the heuristic, which could attribute to why in some cases, BT performed faster.