# Magnetic fields of solenoids

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## Introduction and Theoretical Review

Magnetism has been known to humans since ancient times. The Greeks knew minerals that have magnetic properties. First compasses were used probably around the year 1000. But many of the properties of magnets used nowadays in every day life were discovered much later. As electricity was studied in the 1800's scientists began to realize the connection between electricity and magnetism. They realized that electric current is directly responsible for the creation of magnetic fields [4].

Charged particles can pass through an electrical conductor and we call this electricity. These particles can be electrons or ions. The net rate of passage of electric charge through a volume or a surface is the definition of a current. Charge carriers are particles that are traveling and in electric circuits, they are frequently electrons crossing inside a wire. The symbol $I$ is used to describe current most commonly [1].

Ohm's law is a basic equation that relates the current flowing through a conductor to the voltage $V$ and resistance R, i.e., $V = IR$. The current is measured in amperes, the potential difference $V$ across the conductor is measured in volts, and the resistance $R$ is measured in ohms. Ohm's law shows the proportionality between $V$ and $I$ suggesting that $R$ is constant and unrelated to the current [1].

Alternating or direct current can exist. The transport of electric charge in alternating current (AC) systems periodically flips direction. A sine wave is the most common waveform of an alternating current circuit. Electromagnetic waves can be produced with time varying currents and their importance in the telecommunication sector to transmit data cannot be overstated. [1].

Direct current (DC) refers to a system in which electric charge moves only in one direction and with constant velocity.

It can be generated by batteries, solar cells, and electric machines.

Direct current can flow through a conductor like a wire, but it can also flow through semiconductors, insulators, or even in vacuum as in electron or ion beams. The ampere is defined as the passage of electric charge across a surface at one coulomb per second. An ammeter is the device usually used to measure electric current [1].

Magnetic fields are created by electric currents and are used in motors, generators, inductors and transformers. They can cause however heating in ordinary conductors and light bulbs rely on this process to emit light. When an electric current travels through a coil of wires in an electromagnet, it behaves like a magnet. When the current is turned off, the coil immediately loses its magnetism. A magnetic field is created by an electric current. The magnetic field can be represented as a pattern of circular field lines surrounding the wire that continues as long as current flows [1].

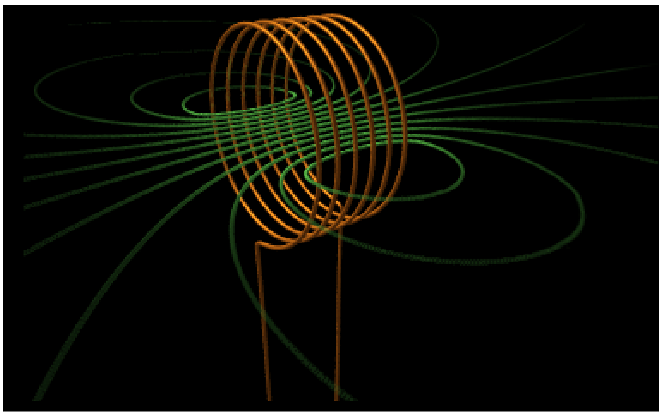

**Figure 1:** *Magnetic field lines (green) through a coil when current is flowing (image taken from [1])*

It is necessary to have a solenoid to create an uniform magnetic field. At their center the field has nearly constant density. The are multiple applications for this. A solenoid is a helical coil of twisted wire with many circular loops. [4]

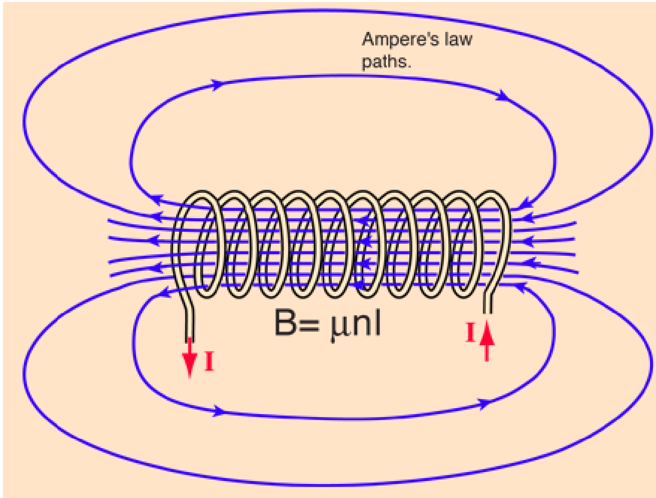


**Figure 2:** *Magnetic field lines and the equation for a magnetic field strength at the center of a solenoid (image taken from [2])*

As is visible in Figure 2, the magnetic field is equal to $B = \mu n I$, where n=N/L is the number of turns per unit length called the turn density. The magnetic field $B$ is proportional to the current $I$ in the coil. The permeability $\mu$ is equal to the permeability of free space multiplied by the relative permeability ($\mu = \mu_0 k$)[2]. The equation for magnetic field strength is directly derived from Ampere's Law which states that the total current passing through an area bounded by a closed curve is the line integral of the magnetic field around that curve [4].

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I \tag{1}$$

## Experimental Design

The number of turns per unit length is always the same therefore the equation $B = \mu n I$ can be used to find the current if the magnetic field strength at the center of the solenoid is known. The opposite also holds true. If we look at equation (2) we see that L (the length of a solenoid) is inversely proportional to the magnetic field. It is evident that making the solenoid longer does not increase the magnetic field. What does have an effect is increasing the number of turns over a given length. This raises the magnetic field strength. Looking at the equation below proves this [3]:

$$B = \frac{\mu_0 N I}{L} \tag{2}$$

If there is the same current within two solenoids, $\mu_0$ is a constant then the only non-constants are the number of turns $N$ and the length $L$:

$$\frac{N}{L} \tag{3}$$

As evident, doubling the turns to $2N$ and the length to $2L$ does not affect this proportion. The values that have been doubled cancel each other out:

$$\frac{2N}{2L} = \frac{N}{L} \tag{4}$$

This ratio can be written shortly as a lowercase $n$ to simplify:

$$\frac{N}{L} = n \tag{5}$$

These units of $n$ are in turns per unit of length. For example, looking at Figure 3 below we see that $n = 2$ turns per centimeter [3]

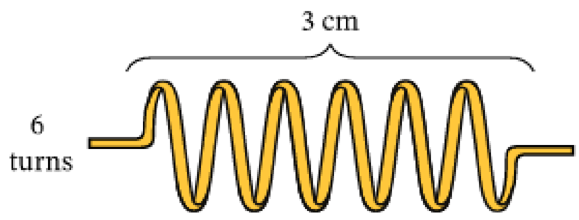


**Figure 3**

But if we had 9 turns in the same coil with a length of 3 centimeters we would have $n$ equal to 3 which would have given us a higher $B$ in the equation. Therefore increasing $n$ is what will increase the magnetic field strength and a closely bound coil will have stronger magnetic field at its center than one with distant loops.

### Manufacturing process

The process used in the manufacturing of the coils is shown in the pictures below (Figures 5,6,7). First cylinders with different diameters were chosen. Then a thin layer of white duct tape-like material was folded around the cylinder according to the length the solenoid would have. Afterward, the coil was woven by hand around the cylinder on top of the duct tape and when the desired shape was achieved glue was deposited on all regions of the wire plus tape.

Then the process required some waiting time for the glue to harden and finally, the coils were removed. The coils are supposed to be shaped like in Figure 4. Table 1 gives the parameters of the coils.

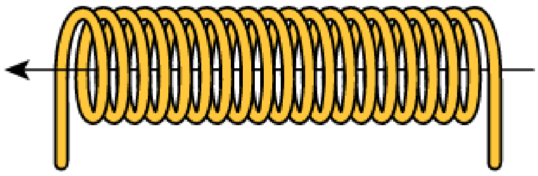

**Figure 4**



**Figure 5:** *Cylinder with a coil woven on top*

**Figure 6:** *Thinner cylinder with another coil*

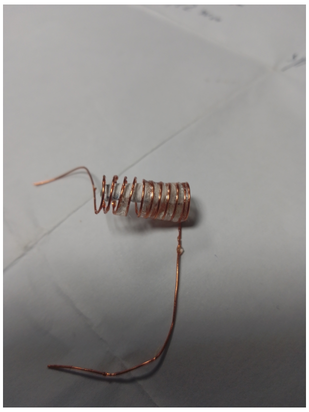

**Figure 7:** *Coil 6*

**Figure 8:** *All the prepared coils*

**Table 1:** *Coil parameters*

| Coil | OD | ID | N |
|---|---|---|---|
| $C_1$ | 4 $mm$ | 0.35 $mm$ | 8 |
| $C_2$ | 5 $mm$ | 0.35 $mm$ | 7 |
| $C_3$ | 4 $mm$ | 0.5 $mm$ | 7 |
| $C_5$ | 6 $mm$ | 0.75 $mm$ | 9 |
| $C_6$ | 5.7 $mm$ | 0.35 $mm$ | 12 |
| $C_7$ | 2.5 $mm$ | 0.75 $mm$ | 13 |
| $C_8$ | 5.7 $mm$ | 0.75 $mm$ | 19 |
| $C_9$ | 6 $mm$ | 0.22 $mm$ | 19 |

A probe head was chosen so that soldering can be done to coil number 6 (Figure 9). First, the insulation on the wire was removed with a sharp knife - the ends were grazed. Then using tin the coil was soldered to the probe head (Figure 10). Finally, the frequency was tuned and matched (Figure 12).

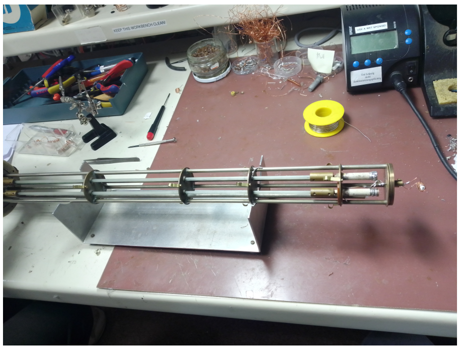

**Figure 9:** *Probe head*

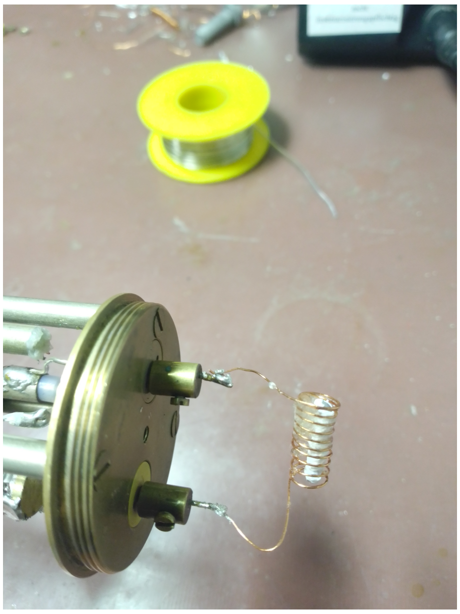

**Figure 10:** *Coil 6 soldered on the probehead*

The coils are supposed to behave like an RLC circuit with frequency $\omega$.

$$\omega = \frac{1}{\sqrt{L(C_T + C_M)}} \tag{6}$$

Where $C_M$ is the matching capacitor and $C_T$ is the tuning capacitor. $L$ is the inductance of the coil, also the entire coil has resistance $R$.

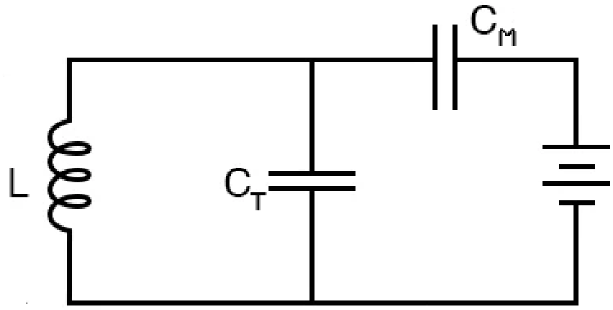


**Figure 11:** *The circuit overview*

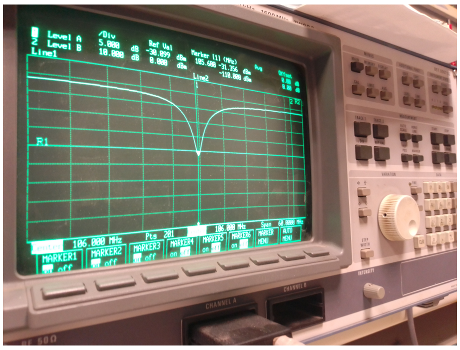

**Figure 12:** *Tuning*

Then the probe head with the coil was put inside the superconducting magnet (Figure 13).

**Figure 13:** *The superconducting magnet*

We sent an electrical pulse into the circuit with a frequency close to the natural frequency and this created an alternating magnetic field inside the coil. The atoms resonated close to this frequency and they emitted a signal back which we then detected. The results are shown in the experimental results section.

## Simulations

Comsol software was used to simulate the magnetic field in the solenoids. A helix with the coil parameters was built and a point inside the helix for the point evaluations of the magnetic field. The physics added into the simulation include Ampere's law and magnetic insulation of the edges in the environment of the simulation, which is a sphere. The helix was modeled as a coil with 1 Ampere of current running through it and the number of turns and wire cross-section were added accordingly.

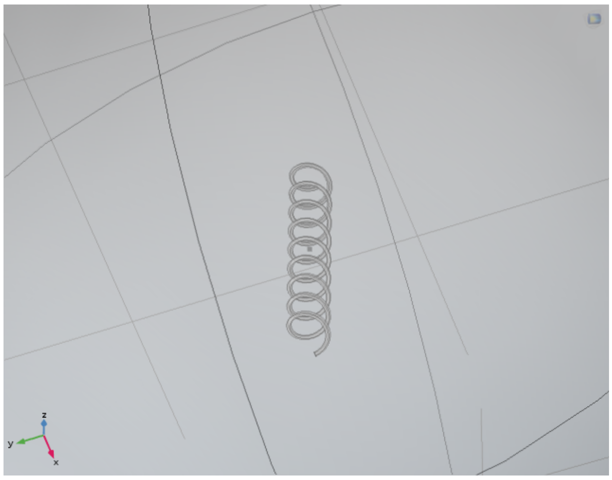

**Figure 14:** *Helical shape of the coils with the point inside*

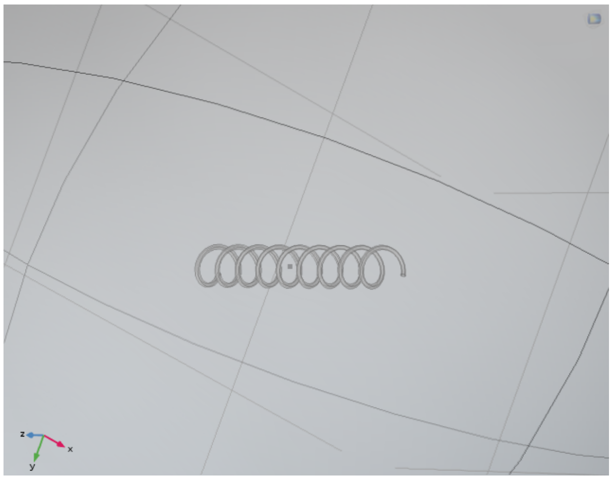

**Figure 15:** *Same as Figure 9 but from a sideways perspective*

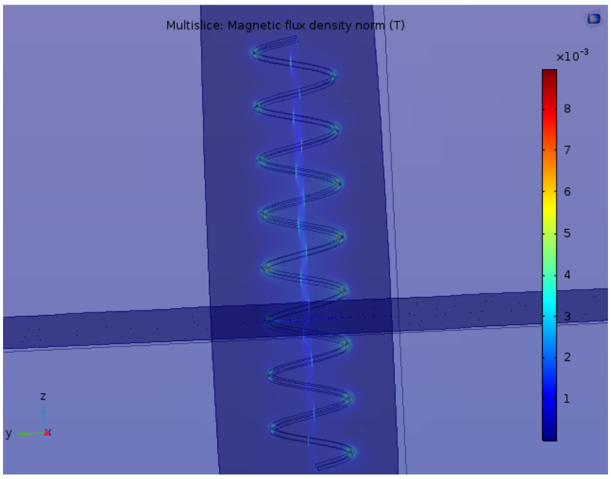


**Figure 16:** *Coil 1*

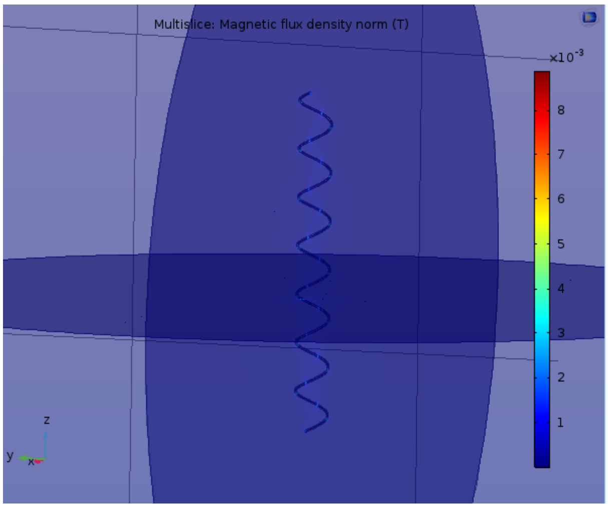


**Figure 17:** *Coil 2*

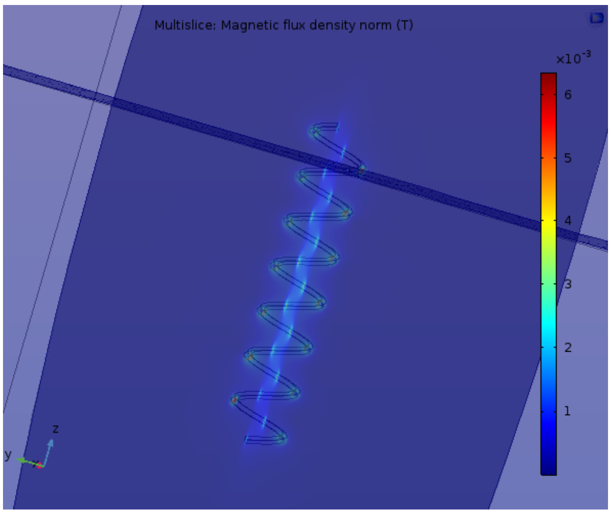


**Figure 18:** *Coil 3*

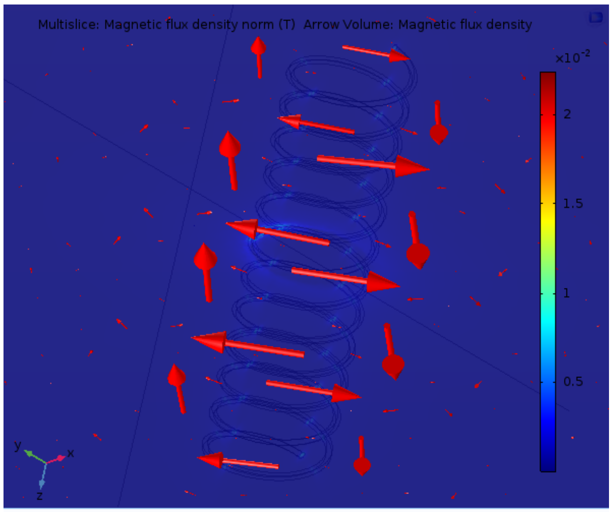


**Figure 19:** *Coil 6*

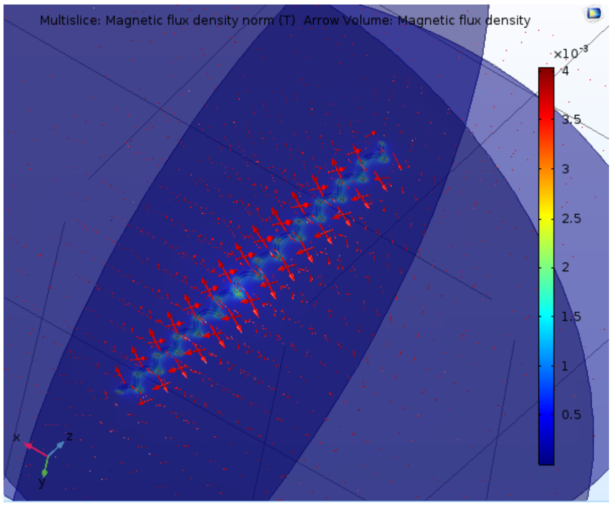


**Figure 20:** *Coil 7*

Point evaluations were done at the center of each solenoid and magnetic field strength was analyzed in $x$,$y$, and $z$ directions. The magnitudes of the vectors were calculated according to $\sqrt{x^2+y^2+z^2}$ and are given in table 3 below. Table 2 also gives the field in the z-direction.

**Table 2:** *Magnitude magnetic field z-direction*

| Coil | Magnetic field strength (Tesla) |
|---|---|
| $C_1$ | 1.7415 $mT$ |
| $C_2$ | 0.2029 $mT$ |
| $C_3$ | 1.2771 $mT$ |
| $C_5$ | 0.2265 $mT$ |
| $C_6$ | 0.1996 $mT$ |
| $C_7$ | 0.4995 $mT$ |
| $C_8$ | 0.5302 $mT$ |
| $C_9$ | 0.7090 $mT$ |

**Table 3:** *Magnitude magnetic field*

| Coil | Magnetic field strength (Tesla) |
|---|---|
| $C_1$ | 1.7414 $mT$ |
| $C_2$ | 0.2303 $mT$ |
| $C_3$ | 40.4122 $mT$ |
| $C_5$ | 0.2689 $mT$ |
| $C_6$ | **8.5673** $mT$ |
| $C_7$ | 0.8508 $mT$ |
| $C_8$ | 0.5521 $mT$ |
| $C_9$ | 0.7093 $mT$ |

## Experimental Observations

The frequency of copper was measured

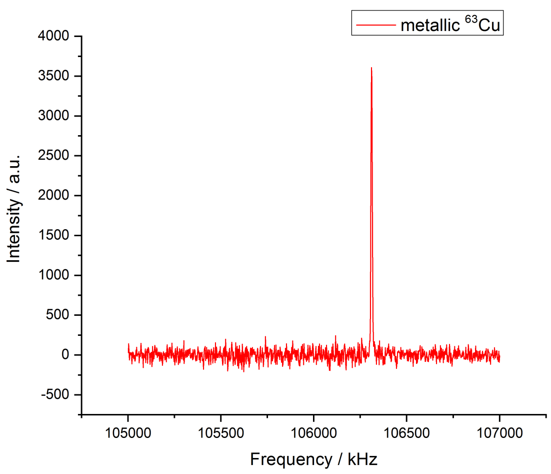


**Figure 21**

The frequency at the highest peak was analyzed using Origin software and is given in table 4 below

| | |
|---|---|
| Kilo Hertz | 106310.5 |
| Intensity (a.u) | 3607.3 |

**Table 4**

The Q factor was calculated using the baseline -7 dBM.

$$Q = \frac{\omega_0}{\omega_1 - \omega_2} \tag{7}$$

$$Q = \frac{105.6}{102.6 - 108.3} = \frac{105.6}{5.7} \tag{8}$$

$$Q = 18.52 \tag{9}$$

The following formula for the magnetic field in the coil can be used for more precise calculations:

$$B_1 = \sqrt{\frac{\mu_0 Q P}{2\omega V_{coil}}} \tag{10}$$

Where Q is the Q factor equal to 18.52, P is the power of the pulse equal to 40 Watts, $\omega$ is the frequency of the pulse we send and V is the volume of the coil equal to $\pi r^2 h$. We calculate V to be 274.75 $mm^3$ or $2.7475 * 10^-7$ $m^3$

We calculate the B field using equation (10):

$$\sqrt{\frac{4*\pi*10^{-7}*18.52*40}{2*106000000*2.7475*10^{-7}}} \tag{11}$$

$$B_1 = 3.99*10^{-3} \tag{12}$$

Therefore the experiment gives us a value of $4mT$

## Conclusion

We produced different coils with different geometry and we did simulations with the same parameters. The magnetic field was calculated experimentally for one of the coils. There seems to be however a certain difference between the simulated value for coil 6 and the observed value. The experimental one is 4 mT while the one from the simulations is 8.56 mT. This could be due to systematic errors in the experimental part like random errors or uncertainties. The electricity flow through the coil might not be perfect. However, it is also possible that the simulations are not precise, since Comsol is a tricky software and not easy to use. There are many parameters to keep in mind while designing a simulation with it and a number of things could have gone wrong while trying to fashion the coils. For example, the software detects the coils as linear only sometimes and gives errors if one tries to make it a circular one. In the future, more robust methods should be used in the manufacturing of coils and in simulations as well.

## References

[1] https://en.wikipedia.org/wiki/Electric_current

[2] http://hyperphysics.phy-astr.gsu.edu/hbase/magnetic/solenoid.html

[3] https://www.nagwa.com/en/explainers/186157825721/

[4] Physics for Scientists and Engineers , Second Edition , Randal D. Knight , pages 998, 1000, 1048, https://www.academia.edu/30142465/_Paul_A_Tipler_Gene_Mosca_Physics_for_Scientist