①

$$\Delta S_j^{i,k} = \Delta SL_j^{i-1,k} \cdot C_j^{k-1} \beta_j \cancel{+ \frac{\Delta\Phi_j^{i-1,k}}{g} C_j^{k-1} \beta_j} - T_0^{k-1} [C_j^{k-1} \beta_j - C_0^{k-1} \beta_0]$$

$$\Delta SL = \Delta G - \Delta R + \frac{\Delta\Phi}{g}.$$ (R, i.e. U, vertical disp)

j: Time step

i: inner loop ($\Delta S \rightleftharpoons \Delta SL$ ~~and~~ $\cancel{\Delta\Phi}$)

k: outer loop (initial topo, ocean functions)

$$T_0^k = T(t_p) + \Delta SL^k(t_p)$$

ΔS is the ocean load change relative to initial stage.

ΔSL is the sea level change (in ocean and ocean), $\Delta SL(t_p) = -T(t_p) - (-T_0)$

"$-T$" is bathymetry.

For stage j. total surface load is $(\Delta I_j + \Delta S_j) \doteq \Delta L_j$

$$\Delta I_j + \Delta S_j = \rho_i \Delta I_j - \rho_w T_0 [O_j - O_0] + \rho_w \Delta SL_j \cdot O_j \qquad (O_j = C_j \beta_j)$$

If we define $\Delta IT_j = \left[\Delta I_j - T_0 [O_j - O_0] \frac{\rho_w}{\rho_i}\right] \cdot \rho_i$, or use ice height for load.

Then we will find

$$C \equiv \Delta\Phi_j / g = -\frac{1}{A_j} \cdot \frac{\rho_I}{\rho_w} \underline{\iint \Delta IT_j \, d\Omega} - \frac{1}{A_j} \underline{\iint (\Delta G - \Delta R)_j O_j \, d\Omega}$$

$$\Delta SL = \Delta G - \Delta R + C$$ (this is water height)

which is same as our old code assume ocean area does not change with time.

(e.g. Paulson Eq 8)

②

So, we define ~~surface load as~~

~~ice~~ input surface load as $\Delta I - T_0[O_j - O_0]$,

which has two parts, ice load change and water load change related to ocean/land transition (topo correction)

Note the ~~topo corr~~ ~~water load change~~

topo correction should be expressed as "ice equivilant height", not just water height.

§ Assume ~~the~~ we have done one outer loop with initial topo T_0^k, O_j^k,

How to perform new outer iteration $k+1$ based on the ~~per~~ previous iteration's solution ΔG^k, ΔR^k?

① What we need is $\Delta I^{k+1} - T_0^{k+1}[O_j^{k+1} - O_0^{k+1}]$,

So ΔI^{k+1}, T_0^{k+1}, O_j^{k+1}

② To get O_j^{k+1}, we just need know T_j^{k+1}, and $O_j^{k+1} = 1$ when $T_j^{k+1} < 0$.

~~T_j^{k+1}~~ Since $\Delta SL(t_p) = -T(t_p) - (-T_0)$

$\Delta SL(t_j) = -T(t_j) - (-T_0)$

$\therefore T(t_j) = T(t_p) + \Delta SL(t_p) - \Delta SL(t_j)$

or, $T^{k+1}(t_j) = T(t_p) + \Delta SL^k(t_p) - \Delta SL^k(t_j)$

When $j=0$, we also got T_0^{k+1}, and $\Delta SL^k(t_0) = 0$.

③ ΔI^{k+1}, we want to do floating ice check, based on ice height and T_j^{k+1}. (topography)

④ With that, we only need to know how to get $\Delta SL^k(t_j)$

③

§ calculate $\Delta SL^k(t_j)$, with ΔG_j^k, ΔR_j^k, $\underbrace{\{\Delta I_j - T_0[O_j - O_0]\}^k}_{\Delta IT_j^k}$.

$$\Delta SL^k(t_j) = \Delta G_j^k - \Delta R_j^k + C_j^k$$

$$C_j^k = -\frac{1}{A_j^k}\left[\iint \frac{\rho_i}{\rho_w} \Delta IT_j^k \, d\Omega + \iint (\Delta G - \Delta R)_j^k O_j^k \, d\Omega\right]$$

(here ΔI is grounding ice)

Done.

~~with~~ P.S.1 with $\Delta SL^k(t_j)$, we already know T_0^{k+1}, O_j^{k+1}

so, why not directly use T_0^{k+1} for $\Delta I_j^{k+1} - T_0^{k+1}[O_j^{k+1} - O_0^{k+1}]$.

I think this is better, but Kendall use $\Delta I_j^{k+1} - T_0^k[O_j^k - O_0^k]$.

PS.2 In the term $\Delta I_j - T_0[O_j - O_0]$, note it is actually

$$\rho_i \Delta I_j - \rho_w T_0[O_j - O_0]$$