From Monopoly to Competition: Optimal Contests Prevail

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Abstract

We study competition among contests in a general model that allows for an arbitrary and heterogeneous space of contest design, where the goal of the contest designers is to maximize the contestants' sum of efforts. Our main result shows that optimal contests in the monopolistic setting (i.e., those that maximize the sum of efforts in a model with a single contest) form an equilibrium in the model with competition among contests. Under a very natural assumption these contests are in fact dominant, and the equilibria that they form are unique. Moreover, equilibria with the optimal contests are Pareto-optimal even in cases where other equilibria emerge. In many natural cases, they also maximize the social welfare.

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1 Introduction

Many important economic and social interactions may be viewed as contests. The designer aims to maximize her abstract utility (e.g. workers' productivity, sales competitions, innovative ideas for new projects, useful information from contestants) by forming a contest, and contestants exert effort in hopes of winning a prize. The design of optimal contests is by now well understood in the monopolistic (single-contest) setting. In particular, in many cases, a winner-takes-all contest is optimal in terms of maximizing either the sum of contestants' efforts or the single maximal effort (e.g. Barut and Kovenock, 1998; Kalra and Shi, 2001; Moldovanu and Sela, 2001; Terwiesch and Xu, 2008; Chawla et al., 2019).

While most of the existing literature on contest design focuses on a monopolistic contest with an exogenously given set of participants, in reality, many times, there are multiple contests on a market and these contests must compete to attract participants, which induces a participation vs. effort trade-off. Although the optimal contest in the single-contest setting induces maximal effort exertion *after* contestants choose to participate in their contests, contestants might at the same time be discouraged from choosing the more demanding contests. To attract participants, it seems helpful to design lucrative and easy contests that leave a large fraction of the total surplus to contestants. Thus, these two aspects appear to be contradicting.

Previous literature has already started to acknowledge this issue with few models that formally studied it (e.g., Azmat and Möller (2009); Stouras et al. (2020)). In particular, Azmat and Möller (2009) conclude that, despite the competitive environment, contest designers should still choose effort-maximizing contests since the effort aspect dominates the participation aspect. However, it is not clear how robust this conclusion really is, since these previous papers analyze models that are restricted in two main aspects. First, they assume that all contests have the same total prize to offer.[1] Second, and perhaps even more important, they restrict the choice of a contest and assume that designers choose a multiple-prize contest where contestants' winning probabilities for each prize are determined by a Tullock success function that is exgonenous and identical for all contest designers. A main appeal of a Tullock contest as a model to winner-determination in real-life contests is that it captures

[1] Previous analysis seems to significantly rely on this assumption. It also formally assumes exactly two competing contests, although this aspect may be more easily generalized.

the fact that the efforts $e_1, ..., e_k$ of the k contestants in a certain contest cannot be fully observed by the contest designer. The winning probability may be following a Tullock contest success function $e_i^\tau/(\sum_{j=1}^k e_j^\tau)$ using some parameter τ to capture partial-observability by letting contestants with higher efforts win with higher probabilities (as τ increases, effort observability is better). With such a motivation, it seems that a contest designer always has the strategic option to artificially reduce her ability to observe effort, e.g., using a Tullock contest success function with some parameter $\tau' < \tau$. Combining various prize structures with a limited choice of the parameter τ is a natural way to expand the set of possible contests to consider. This is not captured in previous models. Alternatively, effort-observability can be endogenously determined (e.g., online contests could use better technology to increase effort observability). In this case, contest designers have the freedom to increase τ where in the limit as $\tau \to \infty$ we have an all-pay auction where the contestant with the highest effort wins with certainty. Contest success functions may depend on the number of contestants in other complex ways and many additional examples of natural classes of contests exist (Corchón, 2007). Naturally, as the strategic flexibility of contest designers increases, existing outcomes may no longer be in equilibrium, and even if they do remain in equilibrium, additional more attractive equilibria might emerge.

This paper provides a more general framework and analysis of competition among multiple contests. To be consistent with previous literature our starting point is the model of Azmat and Möller (2009) which we generalize in order to capture the two main aspects discussed in the previous paragraph: a general contest design space and asymmetric contest designers. In a high-level, the model is composed of three phases. In the first phase, contest designers choose their contests (and commit to them) from a class of contests available to them which could be any arbitrary class of contests. In the second step, after seeing the contests chosen by designers, each contestant chooses (possibly in a random way) one contest to participate in. Finally, in each contest, contestants invest effort by playing a symmetric Nash equilibrium (which previous literature has shown to exist, see details in Section 2). Designers aim to maximize the sum of efforts exerted in their own contests and contestants aim to maximize the reward they receive minus their effort.

The bottom line of our analysis is that optimal (effort-maximizing) contests in the monopolistic setting are still in equilibrium even when significantly increasing the strategic flexibility of contest designers. In other words, effort indeed dominates participation in the aforementioned trade-off for the competing designers. In fact, these contests remain the unique equilibrium in many interesting and natural cases (although, as we show, not always). Moreover, even when additional contests emerge as equilibria, choosing effort-maximizing contests is a Pareto-optimal outcome for contest designers maintaining the attractiveness (for the contest designers) of these types of contests. These conclusions hold regardless of the number of designers, the total rewards they have, and the classes of contests they can choose from.

Technically, we identify two properties that characterize the class of effort-maximizing contests and show that if every contest designer chooses a contest that satisfies these two properties then we are at an equilibrium which is Pareto optimal for the designers. The first property, which we term Monotonically Decreasing Utility (MDU) simply says that a contestant's symmetric-equilibrium utility in the single contest game decreases as the number of contestants increases. The second property, which we term Maximal Rent Dissipation (MRD), is defined with respect to the space of possible contests $\mathcal{S}_i$ that contest designer i has. A specific contest $C_i \in \mathcal{S}_i$ has maximal rent dissipation if, for any other contest $C_i' \in \mathcal{S}_i$ that could be a possible choice for designer i, and for any number of contestants, k, the contestant's symmetric-equilibrium utility in the single contest C_i when there are k contestants is not larger than the contestant's equilibrium utility in the single contest C_i' when there are k contestants. Thus, C_i minimizes the contestants utilities and therefore maximizes the utility of the contest designer, among all contests available to designer i. In this sense, C_i is "optimal".

Going back to Tullock contests and prize structures, the generality of our framework yields, as corollaries to our main result, that: (1) Choosing all-pay auctions is an equilibrium for designers who can only adjust their observability of effort (i.e. the Tullock parameter τ) but must give the entire reward to the winner (i.e. winner-takes-all); in fact we show that choosing any $\tau \geq 2$ is an equilibrium. (2) Choosing winner-takes-all contests is an equilibrium for designers who can only adjust prize structures while the parameter τ is exogenous. (3) Choosing the winner-takes-all all-pay auction is an equilibrium for the designers when they can adjust both prize structures and effort observability.

1.1 Additional related literature

In this subsection we review two strands of literature on contest design. One strand of literature considers optimality of contests in a monopolistic (single contest) setting, in terms of revenue for the designer, agent participation, etc. A second, more recent strand, considers equilibrium outcomes when multiple contest designers compete over agents' participation and effort. Our result is interesting in the way it ties together these domains: We show that effort-maximizing contests (those that were identified in the first literature strand) are in an equilibrium in our general model, that belongs to and follows the second strand. The takeaway message to a contest designer is that, in case she is interested to maximize the sum of contestants efforts, introducing competition does not change her basic goal of maximizing over effort extraction, given a fixed set of contestants.

1.1.1 Optimal contests in the monopolistic setting

In the monopolistic setting, several works study optimal multiple-prize contest design with the objective of maximizing sum of efforts. Under different assumptions, most of these papers arrive at the same conclusion that the optimal contest is the winner-takes-all contest where the full prize is offered to the single contestant exerting the highest effort. For example, in all-pay auctions where contestants' efforts are fully observable, Barut and Kovenock (1998); Moldovanu and Sela (2001) show that a winner-takes-all all-pay auction is optimal assuming contestants having either linear or concave cost functions (interestingly, for convex cost functions their results vary). An exception is Glazer and Hassin (1988) who show that the optimal contest should offer equal prizes to all players except for the player with the lowest effort if players value the prize money by a strictly concave utility function. We consider linear utilities and linear cost functions as in Barut and Kovenock (1998) so the winner-takes-all is optimal. When contestants' efforts are not fully observable and their winning probabilities for different prizes are assumed to follow a Tullock success function, the optimal prize structure is once again a winner-takes-all (Clark and Riis, 1998). The winner-takes-all is also optimal in a stochastic-quality model (e.g., Kalra and Shi, 2001; Ales et al., 2017) where a contestant who exerts effort e_i produces a submission with random quality $Q_i = e_i + Z_i$ where Z_i follows some noise distribution. In this case, prizes are allocated based on the submission qualities. If the designer must offer a single prize, then the all-pay auction is optimal among all possible contests, as it induces full rent dissipation – contestants' utilities are reduced to zero and their sum of efforts is maximized (Baye et al., 1996).

A line of the literature including Schweinzer and Segev (2012); Baye et al. (1996); Alcalde and Dahm (2010); Ewerhart (2017) studies symmetric equilibria and rent dissipation in optimal contests. Our formal results, which analyze contests that admit symmetric equilibria and certain dissipation properties, have concrete applications thanks to existence and characterization results provided in these papers.

1.1.2 Competition among contests

Previous works (e.g., Stouras et al., 2020; Azmat and Möller, 2009) on contest competition suggest that either participation or effort may dominate in the aforementioned trade-off under different assumptions. For example, Stouras et al. (2020) show that, for designers who wish to maximize the highest quality, participation outweighs effort and hence the designers will set multiple prizes when the quality of submission is sensitive to their effort. When the quality of submission is not sensitive to effort, they show that the effort aspect is dominating and hence the designers will offer a single prize which induces the maximum effort exertion in the single-contest setting. On the other hand, Azmat and Möller (2009) suggest that, for designers maximizing the sum of efforts, the effort aspect is always dominating, and hence a single prize should be offered. As mentioned before, we generalize their work in several aspects.

DiPalantino and Vojnovic (2009) consider an incomplete information competition among contests. They focus on participation issues rather than on the strategic choices of contest designers, by assuming that all contests are all-pay. They explicitly characterize the relationship between contestants' participation behavior and contests' rewards, and find that rewards yield logarithmically diminishing returns with respect to participation levels. Körpeoğlu et al. (2017) consider an incomplete information contest model where contestants can participate in multiple contests, and contest designers use winner-takes-all contests while strategically choosing rewards to maximize the maximal submission quality minus reward. They show that, in several cases, contest designers benefit from contestants' participation in multiple contests.

2 Competition Among Contests: Model and Preliminaries

2.1 A single-contest game

A contest designer designs a contest among several contestants in order to maximize the sum of efforts exerted by the contestants in return for some reward to be divided among them according to some winning rule determined by the designer.

Formally, a contest C is composed of a reward R and a family of contest success functions $\boldsymbol{f}^k : \mathbb{R}^k_{\geq 0} \to [0,1]^k$ for each number of contestants $k > 0$. Contestants exert efforts $(e_1, \ldots, e_k) \in \mathbb{R}^k_{\geq 0}$ to compete for the reward. Each contestant i receives a fraction $f^k_i(e_1, \ldots, e_k)$ of the reward, where $f^k_i(e_1, \ldots, e_k)$ is the i-th coordinate of the vector $\boldsymbol{f}^k(e_1, \ldots, e_k)$. In a stochastic quality model (e.g., the additive-noise model[2]), $f^k_i(e_1, \ldots, e_k)$ is the expected fraction of reward received by contestant i. We allow general functions $f^k_i(\cdot)$ and only require that $\sum_{i=1}^k f^k_i(e_1, \ldots, e_k) \leq 1$. The utility of a contestant is the reward she gets minus the effort she exerts: $f^k_i(e_1, \ldots, e_k)R - e_i$. The utility of the contest designer is the sum of efforts $\sum\limits_{i=1}^k e_i$. When $k = 0$ the designer's utility is 0.

Definition 2.1. *A contest is* anonymous *if its contest success functions* $\boldsymbol{f}^k : \mathbb{R}^k_{\geq 0} \to [0,1]^k$ *satisfy, for any* $k > 0$*, for any* $(e_1, \ldots, e_k) \in \mathbb{R}_{\geq 0}$ *and any permutation* π *of* $(1, \ldots, k)$*,*

$$\boldsymbol{f}^k\left(e_{\pi(1)}, \ldots, e_{\pi(k)}\right) = \left(f^k_{\pi(1)}(e_1, \ldots, e_k), \ldots, f^k_{\pi(k)}(e_1, \ldots, e_k)\right).$$ [3]

Definition 2.2. *A contest* fully allocates the reward *if its contest success functions* $\boldsymbol{f}^k : \mathbb{R}^k_{\geq 0} \to [0,1]^k$ *satisfy, for any* $k > 0$ *and any* $(e_1, \ldots, e_k) \in \mathbb{R}^k_{\geq 0}$*,* $\sum_{i=1}^k f^k_i(e_1, \ldots, e_k) = 1$.

Example 2.3. *A Tullock contest (or, more accurately, a single-prize Tullock contest)*

[2]In an additive-noise contest model, a contestant who exerts effort e_i produces a submission with random quality $Q_i = e_i + Z_i$ where Z_i follows some noise distribution. The contest designer observes Q_i but not e_i and allocate rewards to contestants based on their Q_i's.

[3]This is the same definition as in Alcalde and Dahm (2010); it is equivalent to requiring that $f^k_i(e_1, \ldots, e_k) = f^k_{\pi(i)}(\tilde{e}_1, \ldots, \tilde{e}_k)$ where $\tilde{e}_{\pi(j)} = e_j$ for all j.

parameterized by $\tau \in [0, +\infty]$ has the following contest success function:

$$f_i^k(e_1, \ldots, e_k) = \begin{cases} \frac{e_i^\tau}{\sum\limits_{j=1}^{k} e_j^\tau} & \textit{if } e_j > 0 \textit{ for some } j \in \{1, \ldots, k\} \\ \frac{1}{k} & \textit{otherwise} \end{cases}$$

When $\tau = +\infty$, the contest becomes an "All Pay Auction (APA)" where the contestant with the highest effort wins with certainty (to maintain anonymity, if several contestants exert the highest effort, they all win with equal probability). A Tullock contest is anonymous and it fully allocates the reward.

Definition 2.4. *Denote by $\mathcal{C}_R$ the set of all contests with reward R that are anonymous, fully allocate the reward and have a symmetric Nash equilibrium among k contestants for all $k > 0$.*

For example, Alcalde and Dahm (2010) and Baye et al. (1996) show that Tullock contests with parameters $\tau \in [0, \infty)$ and $\tau = \infty$ admit a symmetric Nash equilibrium; thus, $\mathcal{C}_R$ contains all Tullock contests with reward R. Other examples of contests that admit symmetric Nash equilibria are given in, e.g., the seminal works of Hirshleifer (1989); Nti (1997), a survey by Corchón (2007), as well as later works such as Amegashie (2012).

We assume throughout the paper that all contestants in the same contest will play a symmetric Nash equilibrium of that contest. Formally, for every contest $C \in \mathcal{C}_R$ we fix a (mixed strategy) symmetric Nash equilibrium, i.e., $e_1, \ldots, e_i, \ldots, e_k$ are i.i.d. random variables that follow a distribution F defined by a mixed strategy Nash equilibrium. Since $C \in \mathcal{C}_R$ is anonymous, in the symmetric Nash equilibrium all contestants get an equal expected fraction of the reward and hence their expected utilities are identical. We denote their identical expected utility by $\gamma_C(k) = \mathbb{E}_{e_1,\ldots,e_k \sim F}\left[f_i^k(e_1, \ldots, e_k)R - e_i\right]$. Moreover, since $C \in \mathcal{C}_R$ fully allocates the reward, we must have $\mathbb{E}_{e_1,\ldots,e_k \sim F}\left[f_i^k(e_1, \ldots, e_k)\right] = \frac{1}{k}$ and hence

$$\gamma_C(k) = \frac{R}{k} - \mathbb{E}_{e_i \sim F}\left[e_i\right]. \tag{1}$$

We note that when $k = 1$, $\gamma_C(1) = R$, because the single contestant will not exert any effort. We also have $\gamma_C(k) \geq 0$ for any $k > 0$ since a contestant can always choose to exert zero effort and guarantee non-negative utility. Moreover, since $e_i \geq 0$, we always have $\gamma_C(k) \le \frac{R}{k}$. We can use $\gamma_C(k)$ to express the utility of a contest designer in a contest $C \in \mathcal{C}_R$ with $k \ge 1$ contestants by rearranging (1):

$$\mathbb{E}_{e_1,\dots,e_k \sim F}\left[\sum_{i=1}^{k} e_i\right] = k\,\mathbb{E}_{e_i \sim F}\left[e_i\right] = R - k\gamma_C(k). \tag{2}$$

Note that we assume that the utility of a contest designer is the expected sum of efforts, even if this is non-observable. This fits settings like workplace contests that aim to improve workers' productivity. More generally, in the additive noise model, expected sum of qualities is equal to expected sum of efforts since the expected noise is usually assumed to be zero.

2.2 A contest competition game

In this paper we study a game where multiple contest designers compete by choosing their contest success functions. Contestants observe the different contests and choose in which one to participate.

Definition 2.5. *A complete-information* contest competition game *is denoted by* $CCG(m, n, (R_i)_{i=1}^m, (\mathcal{S}_i)_{i=1}^m)$, *where* $m \ge 2$ *is the number of contest designers,* $n \ge 1$ *is the number of contestants,* $R_i > 0$ *is the reward of contest* i, *and* $\mathcal{S}_i \subseteq \mathcal{C}_{R_i}$. *The game has two phases:*

1. ***Designers choose contests**. Each designer* i *chooses a contest* $C_i \in \mathcal{S}_i$ *simultaneously. Contestants observe the chosen contests* $(C_1, \dots, C_m)$.

2. ***Contestants play a normal-form game of choosing in which contest to participate**. A pure strategy of each contestant in this game is to choose one contest. Importantly, contestants may play a mixed strategy, meaning that each contestant* ℓ *participates in each contest* C_i $(i = 1, ..., m)$ *with some probability* $p_{\ell i}$, $\sum_{i=1}^{m} p_{\ell i} = 1$. *We denote the vector of probabilities chosen by contestant* ℓ *by* $\boldsymbol{p}_\ell = (p_{\ell 1}, ..., p_{\ell m})$.

After Nature assigns contestants to contests, utilities are as follows. If there are $k \ge 1$ *contestants participating in contest* C_i, *then each of these contestants gains utility* $\gamma_{C_i}(k)$ *and contest designer* i *gains utility* $R_i - k\gamma_{C_i}(k)$. *If* $k = 0$ *then the utility of the contest designer is* 0.

The first important element of our model is the space $\mathcal{S}_i$ of all possible contests a designer can (strategically) choose. For example, this could be the space of all Tullock contests, i.e., the parameter τ becomes a strategic choice. Some of the previous literature views τ as an exogenous parameter representing how accurately the designer is able to observe the ranking of efforts performed by contestants. Even so, it seems plausible that the designer chooses an "ignorance is bliss" approach where she lowers the τ value (thus, observes efforts' ranking less accurately) in order to encourage participation. Alternatively, another example for $\mathcal{S}_i$ could be the space of all prize structures where the ranking is determined according to a specific Tullock contest with a fixed exogenous τ, as in Azmat and Möller (2009).[4]

We remark that this model implicitly assumes that when a contestant decides on the level of effort to exert in the contest she participates in, she knows the total number of contestants k in the same contest. In practice, contestants can observe the number of participants when they are in physical contests (like sport contests) or when the contest designer chooses to reveal this information. Moreover, Myerson and Wärneryd (2006) show that contest designers have an incentive to do so because the expected aggregate effort in a contest with a commonly known number of participants is in general higher than that in a contest where the contestants do not see the number of participants.

In the second phase each contestant has a finite number m of possible actions and the game is symmetric, hence there must exist at least one symmetric (mixed strategy) Nash equilibrium (Nash, 1951). We will assume in all our results that the contestants play this symmetric equilibrium, i.e., we will only consider equilibria in which the probability vector of every contestant is the same ($\boldsymbol{p}_1 = \boldsymbol{p}_2 = \cdots = \boldsymbol{p}_n$). Example D.3 discusses the case where the contestants choose an asymmetric equilibrium. Formally, we denote by $\boldsymbol{p}(C_1, \ldots, C_m) \in \mathbb{R}^m$ the probability vector chosen by the contestants at their symmetric equilibrium when the designers choose contests $(C_1, \ldots, C_m)$ in the first phase.[5]

Given that designers choose contests $\boldsymbol{C} = (C_1, \ldots, C_m)$ in the first phase of the

[4] Since we assume contests that fully allocate the reward, each f^k can be derived from prize structures of at most k prizes. Azmat and Möller (2009) allow for any number of prizes, although having more prizes than contestants is not beneficial for the contest designer.

[5] If there are multiple symmetric equilibria, we allow $\boldsymbol{p}(C_1, \ldots, C_m)$ to be any one of those. All our conclusions hold regardless of which symmetric equilibrium the contestants play. In addition, we show in Lemma 2.13 that the symmetric equilibrium is unique if a certain condition (that is satisfied, e.g., by all Tullock contests) holds.

game and contestants participate in contests with probabilities $(p_1, \ldots, p_m) = \boldsymbol{p}(\boldsymbol{C})$ in equilibrium, the contestants' utility is as follows. For a contestant who participates in C_i, the number of contestants among the other $n-1$ contestants who also participate in C_i follows the binomial distribution $\mathrm{Bin}(n-1, p_i)$. Therefore, the expected utility of a contestant participating in C_i, denoted by $\beta(C_i, p_i)$, equals

$$\beta(C_i, p_i) = \mathbb{E}_{k \sim \mathrm{Bin}(n-1, p_i)} \left[\gamma_{C_i}(k+1) \right] = \sum_{k=0}^{n-1} \binom{n-1}{k} p_i^k (1-p_i)^{n-1-k} \gamma_{C_i}(k+1). \quad (3)$$

Denote the set of indices of contests in which contestants participate with positive probability (i.e., the support of $\boldsymbol{p}(\boldsymbol{C})$) by

$$\mathrm{Supp}(\boldsymbol{C}) = \{ i : p_i(\boldsymbol{C}) > 0 \}. \quad (4)$$

Claim 2.6 (Equilibrium condition)**.** *Suppose that designers choose contests $\boldsymbol{C} = (C_1, \ldots, C_m)$ in the first phase of the game and contestants participate in contests with probabilities $(p_1, \ldots, p_m) = \boldsymbol{p}(\boldsymbol{C})$ in equilibrium. Then,*

- *If $i \in \mathrm{Supp}(\boldsymbol{C})$, then $\beta(C_i, p_i) \geq \beta(C_j, p_j)$ for any $j = 1, \ldots, m$.*
- *Thus, if $i, j \in \mathrm{Supp}(\boldsymbol{C})$, then $\beta(C_i, p_i) = \beta(C_j, p_j)$.*

Proof. $(p_1, \ldots, p_m)$ is a symmetric equilibrium, i.e., given that all other participants play $(p_1, \ldots, p_m)$, a player's best response is to play $(p_1, \ldots, p_m)$ herself. Therefore, the expected utility of choosing to participate in contest i is at least as high as choosing to participate in contest j, as contest i is assigned a positive probability $p_i > 0$. □

2.3 Equilibrium among contest designers

We use $\boldsymbol{C} = (C_i, \boldsymbol{C}_{-i}) = (C_1, \ldots, C_m)$ to denote the contests (strategies) chosen by all designers, where $\boldsymbol{C}_{-i}$ denotes the contests chosen by designers other than i. Let $u_i(C_i, \boldsymbol{C}_{-i})$ be the expected utility of contest designer i given that contestants use $\boldsymbol{p}(C_i, \boldsymbol{C}_{-i})$. Formally, by (2) the utility of the designer of contest C_i equals $R_i - k\gamma_{C_i}(k)$ when there are $k \geq 1$ participants. Since each contestant participates in C_i independently with probability $p_i = p_i(C_i, \boldsymbol{C}_{-i})$, the total number k of participants in C_i follows the binomial distribution $\mathrm{Bin}(n, p_i)$, and hence the designer's expected utility equals

$$u_i(C_i, \boldsymbol{C}_{-i}) = \mathbb{E}_{k\sim\mathrm{Bin}(n,p_i)}\left[(R_i - k\gamma_{C_i}(k)) \cdot \mathbb{1}[k \geq 1]\right]. \tag{5}$$

Since a contest between the designer and the multiple contestants is a constant-sum game where the overall utility of all players (i.e., the welfare) equals the total reward R_i whenever there is at least one contestant, designer i's expected utility $u_i(C_i, \boldsymbol{C}_{-i})$ can be written as the expected welfare $R_i[1-(1-p_i)^n]$ minus the sum of contestants' expected utilities obtained from contest i, $np_i\beta(C_i, p_i)$. Formally,

Claim 2.7.

$$u_i(C_i, \boldsymbol{C}_{-i}) = R_i\left[1-(1-p_i)^n\right] - np_i\beta(C_i, p_i). \tag{6}$$

Proof.

$$\begin{aligned}
u(C_i, \boldsymbol{C}_{-i}) = \mathbb{E}_{k\sim\mathrm{Bin}(n,p_i)}\left[(R_i - k\gamma_{C_i}(k)) \cdot \mathbb{1}[k \geq 1]\right] &= \sum_{k=1}^{n}\binom{n}{k}p_i^k(1-p_i)^{n-k}(R_i - k\gamma_{C_i}(k)) \\
&= \sum_{k=1}^{n}\binom{n}{k}p_i^k(1-p_i)^{n-k}R_i - \sum_{k=1}^{n}\binom{n}{k}p_i^k(1-p_i)^{n-k}k\gamma_{C_i}(k) \\
&= R_i\left[1-(1-p_i)^n\right] - \sum_{k=1}^{n}\binom{n}{k}p_i^k(1-p_i)^{n-k}k\gamma_{C_i}(k) \\
&= R_i\left[1-(1-p_i)^n\right] - np_i\sum_{k=1}^{n}\binom{n-1}{k-1}p_i^{k-1}(1-p_i)^{n-k}\gamma_{C_i}(k) \\
&= R_i\left[1-(1-p_i)^n\right] - np_i\,\mathbb{E}_{k'\sim\mathrm{Bin}(n-1,p_i)}\left[\gamma_{C_i}(k'+1)\right],
\end{aligned}$$

which equals $R_i\left[1-(1-p_i)^n\right] - np_i\beta(C_i, p_i)$ by (3). $\square$

In this paper we analyze the following solution concepts for the contest competition game:

Definition 2.8. *Given some $CCG(m, n, (R_i)_{i=1}^m, (\mathcal{S}_i)_{i=1}^m)$,*

- *A contest $C_i \in \mathcal{S}_i$ is* dominant *if $\forall C_1' \in \mathcal{S}_1, ..., C_m' \in \mathcal{S}_m$, $u_i(C_i, \boldsymbol{C'}_{-i}) \geq u_i(C_i', \boldsymbol{C'}_{-i})$.*
- *A tuple of contests $(C_1, \ldots, C_m)$, where $C_i \in \mathcal{S}_i$ for all i, is a* contestant-symmetric subgame-perfect equilibrium *if $u_i(C_i, \boldsymbol{C}_{-i}) \geq u_i(C_i', \boldsymbol{C}_{-i}), \forall C_i' \in \mathcal{S}_i, \forall i = 1, \ldots, m$.*

For simplicity and also for practical purposes, we do not consider the case where designers play mixed strategies (i.e., distributions over multiple contests).

2.4 Additional important properties of contests

Our results will use the following three properties of contests:

Definition 2.9.

- *A contest $C_i \in \mathcal{C}_{R_i}$ has* monotonically decreasing utility (MDU) *if $\gamma_{C_i}(1) \geq \gamma_{C_i}(2) \geq \cdots \geq \gamma_{C_i}(n)$. In words, the symmetric Nash equilibrium expected utility of a contestant is decreasing as the number of contestants increases.*

- *A contest $C_i \in \mathcal{S}_i \subseteq \mathcal{C}_{R_i}$ has* maximal rent dissipation (MRD) *in $\mathcal{S}_i$ if for any $C_i' \in \mathcal{S}_i$ and any $k = 1, \ldots, n$, $\gamma_{C_i}(k) \leq \gamma_{C_i'}(k)$. Let $\mathrm{MRD}(\mathcal{S}_i) \subseteq \mathcal{S}_i$ denote the set of all contests with maximal rent dissipation in $\mathcal{S}_i$. In words, an MRD contest maximizes the designer's utility regardless of the number of contestants which is equivalent to minimizing the symmetric Nash equilibrium expected utility of contestants.*

- *A contest $C_i \in \mathcal{C}_{R_i}$ has* full rent dissipation *if $\gamma_{C_i}(1) = R_i$ and $\gamma_{C_i}(k) = 0$ for $k = 2, \ldots, n$.*

Claim 2.10. *If C_i has monotonically decreasing utility, then for $p < p'$, $\beta(C_i, p) > \beta(C_i, p')$.*

The proof of the claim is in Appendix A.

Claim 2.11. *Let $T_i \in \mathrm{MRD}(\mathcal{S}_i)$, and let $C_i \in \mathcal{S}_i$ be any other contest in $\mathcal{S}_i$. Then for any $p \in [0, 1]$, $\beta(T_i, p) \leq \beta(C_i, p)$.*

Proof. By the definition of maximal rent dissipation contest, $\gamma_{C_i}(k+1) \geq \gamma_{T_i}(k+1)$ for all $k = 0, \ldots, n-1$, thus

$$\beta(C_i, p) = \mathbb{E}_{k \sim \mathrm{Bin}(n-1,p)} \left[\gamma_{C_i}(k+1)\right] \geq \mathbb{E}_{k \sim \mathrm{Bin}(n-1,p)} \left[\gamma_{T_i}(k+1)\right] = \beta(T_i, p). \qquad \square$$

Note that a full rent dissipation contest C_i has monotonically decreasing utility and has maximal rent dissipation in any set $\mathcal{S}_i \subseteq \mathcal{C}_{R_i}$ that contains it. It is known that APA has full rent dissipation (Baye et al., 1996) and in fact, as a corollary of Ewerhart (2017) we observe that every Tullock contest with parameter $\tau \geq 2$ has full rent dissipation. Thus, the class of Tullock contests contains maximal rent dissipation contests (namely, those with $\tau \geq 2$). Also, it is a class of contests that have monotonically decreasing utility:

Lemma 2.12 (Corollary of Baye et al., 1996; Schweinzer and Segev, 2012; Ewerhart, 2017)**.** *Let C_τ be a Tullock contest with reward R and with parameter $\tau \in [0, +\infty]$. Then, $\gamma_{C_\tau}(k) = R(\frac{1}{k} - \frac{k-1}{k^2}\tau)$ if $\frac{k}{k-1} > \tau$ and $\gamma_{C_\tau}(k) = 0$ if $\frac{k}{k-1} \leq \tau$. For $\tau = +\infty$, $\gamma_{C_\tau}(1) = R$ and $\gamma_{C_\tau}(k) = 0$ for $k \geq 2$. As corollaries,*

- *Every Tullock contest has monotonically decreasing utility.*
- *Every Tullock contest with parameter $\tau \geq 2$ has full rent dissipation.*
- *If $\mathcal{S}$ is the set of all Tullock contests with parameter τ in some range whose maximum $\tau^{\max}$ is well defined and at most 2, then the Tullock contest with $\tau^{\max}$ is the only contest in* $\mathrm{MRD}(\mathcal{S})$.

A proof of this lemma is given in Appendix A. Proposition 2 of Nti (1997) shows a large class of contests that generalize Tullock contests with $\tau \leq 1$ and have the MDU property. MDU contests have another useful property:

Lemma 2.13. *If $C_1, \ldots, C_m$ are MDU contests then the contestants' symmetric equilibrium $\boldsymbol{p}(C_1, \ldots, C_m)$ is unique.*

Proof. Let $\boldsymbol{p} = (p_1, \ldots, p_m)$ and $\boldsymbol{p}' = (p'_1, \ldots, p'_m)$ be two symmetric equilibria for contestants. If they are different, then there exist i, j such that $p_i > p'_i$ and $p_j < p'_j$. Then we get the following contradiction

$$\begin{aligned}
&\beta(C_i, p'_i) > && (p'_i < p_i, C_i \text{ has MDU, Claim 2.10}) \\
&\beta(C_i, p_i) \geq && (p_i > 0, \text{Claim 2.6}) \\
&\beta(C_j, p_j) > && (p_j < p'_j, C_j \text{ has MDU, Claim 2.10}) \\
&\beta(C_j, p'_j) \geq && (p'_j > 0, \text{Claim 2.6}) \\
&\beta(C_i, p'_i). &&
\end{aligned}$$

□

3 Main Results: Equilibria in Contest Competition Games

Our first main result shows that choosing a maximal rent dissipation contest with a monotonically decreasing utility is a subgame-perfect equilibrium of the CCG game, and, moreover, that such a maximal rent dissipation contest is a dominant contest when the set of all possible contests contains only contests with monotonically decreasing utilities:

Theorem 3.1.

1. *Fix any* $CCG(m, n, (R_i)_{i=1}^m, (\mathcal{S}_i)_{i=1}^m)$ *where each* $\mathcal{S}_i \subseteq \mathcal{C}_{R_i}$ *contains a maximal rent dissipation contest that has monotonically decreasing utility, denoted by* $T_i \in \mathrm{MRD}(\mathcal{S}_i)$. *Then,* $(T_1, \ldots, T_m)$ *is a contestant-symmetric subgame-perfect equilibrium.*

2. *Moreover, if each* $\mathcal{S}_i$ *only contains contests with monotonically decreasing utility, then* T_i *is a dominant contest for each designer* i.

The full proof, as well as most other proofs in this section are deferred to Appendix B. In a very high-level, the argument for why MRD contests constitute an equilibrium for the contest competition game is the following. Consider any contest designer i. Suppose each of the n contestants participates in designer i's contest with some probability p_i (assuming a symmetric participation equilibrium). According to Claim 2.7, designer i's expected utility equals

$$u_i(C_i, \boldsymbol{C}_{-i}) = R_i \left[1 - (1 - p_i)^n\right] - np_i\beta(C_i, p_i),$$

where we recall that $\beta(C_i, p_i)$ is each contestant's expected utility conditioning on her already participating in C_i. Now, suppose that contest designer i switches to a contest C'_i that requires less effort from the contestants (namely, leaving more utility to the contestants) and hence increases the participation probability to $p'_i = p_i + \Delta p$. The welfare term is increased by $\Delta p \frac{\partial R_i[1-(1-p_i)^n]}{\partial p_i} = n\Delta p R_i(1 - p_i)^{n-1}$. A contestant's utility in contest i will be increased (here we will use the condition that other contests C_j, $j \neq i$, are MDU contests, as explained in the formal proof) and suppose it is increased to $\beta(C_i', p_i') = \beta(C_i, p_i) + \Delta\beta$. The utility of each contestant is increased by

$$\begin{aligned} p_i'\beta(C_i', p_i') - p_i\beta(C_i, p_i) &= (p_i + \Delta p)(\beta(C_i, p_i) + \Delta\beta) - p_i\beta(C_i, p_i) \\ &= \Delta p\beta(C_i, p_i) + p_i\Delta\beta + \Delta p\Delta\beta \\ &> \Delta p\beta(C_i, p_i) \\ &\geq \Delta p R_i(1 - p_i)^{n-1}, \end{aligned}$$

where the last inequality is because a contestant obtains utility R_i when no other contestants participate in C_i, which happens with probability $(1 - p_i)^{n-1}$. Thus, the overall increase of contestants' utility is greater than $n\Delta p R_i(1 - p_i)^{n-1}$, outweighing the increase of the welfare term, so the designer's utility is decreased.

Theorem 3.1 has the following implication regarding APA (or any other full rent dissipation contest):

Corollary 3.2.

1. *Fix any $CCG(m, n, (R_i)_{i=1}^m, (\mathcal{S}_i)_{i=1}^m)$ where each $\mathcal{S}_i \subseteq \mathcal{C}_{R_i}$ contains a full rent dissipation contest (e.g.,* APA*), denoted by F_i. Then, $(F_1, \ldots, F_m)$ is a contestant-symmetric subgame-perfect equilibrium.*

2. *Let $\mathcal{T}_{R_i}$ be the set of all Tullock contests with reward R_i. Then,* APA *and any other Tullock contest with $\tau \geq 2$ is a dominant contest for every designer in $CCG(m, n, (R_i)_{i=1}^m, (\mathcal{T}_{R_i})_{i=1}^m)$.*

3. *If $\mathcal{S}_i$ is the set of all Tullock contests with parameter τ_i in some range whose maximum $\tau_i^{\max}$ is well defined and at most 2. Then, the Tullock contest with $\tau_i^{\max}$ is the only dominant contest for every designer in $CCG(m, n, (R_i)_{i=1}^m, (\mathcal{S}_i)_{i=1}^m)$.*

Theorem 3.1 shows a specific type of contestant-symmetric subgame-perfect equilibria. The following example shows that if the sets $\mathcal{S}_i$ also contain contests that do not satisfy the condition of monotonically decreasing utility, other types of equilibria may exist, and the equilibria that the theorem shows is not dominant:

Example 3.3. *Let $m = 2, n = 6$, $R_1 = R_2 = 1$, both $\mathcal{S}_1$ and $\mathcal{S}_2$ consist of two contests: the* APA *contest and a contest C that gives the reward for free when the number of participants $k = 5, 6$ and runs* APA *otherwise. We thus have $\gamma_C = (1, 0, 0, 0, 1/5, 1/6)$, which is not monotonically decreasing. We claim that (C, C) is a contestant-symmetric* *subgame-perfect equilibrium and that* APA *is not a best-response to* C *(and therefore not a dominant contest): When designers choose* (C, C)*, by symmetry, contestants participate in either contest with equal probability* $(0.5, 0.5)$*. By direct computation (e.g., using* (6)*), the expected utility of each designers is* $(0.7812, 0.7812)$*. Now suppose designer* 1 *switches to* APA*. The probabilities* $(p_1, p_2) = \boldsymbol{p}(\text{APA}, C)$ *in the contestants' symmetric mixed strategy Nash equilibrium must satisfy, according to Claim 2.6,* $\beta(\text{APA}, p_1) = \beta(C, p_2)$ *(assuming* $p_1, p_2 > 0$*). By numerical methods, we find that* $(p_1, p_2) = (0.4061, 0.5939)$*. The expected utility of designers is then* $(0.7761, 0.7323)$*. Since* $0.7761 < 0.7812$*, designer 1 will not switch to* APA*. By symmetry, designer 2 will not switch to* APA*. Hence,* (C, C) *is an equilibrium, and* APA *is not a dominant contest.*

In this example, for every k, $\boldsymbol{f}^k$ uses a Tullock contest with a parameter τ_k that depends on k (namely, $\tau_k = 0$ for $k = 5, 6$ and $+\infty$ otherwise). An example where we use $\tau_k = 1$ instead of $\tau_k = 0$ for some k could be constructed in a similar way.[6]

When the sets $\mathcal{S}_i$ contain only contests with monotonically decreasing utility, the contestant-symmetric subgame-perfect equilibria that Theorem 3.1 describes are the only possible equilibria:

Theorem 3.4. *Fix any* $CCG(m, n, (R_i)_{i=1}^m, (\mathcal{S}_i)_{i=1}^m)$ *where each* $\mathcal{S}_i \subseteq \mathcal{C}_{R_i}$ *only contains contests with monotonically decreasing utility. Assume* $\text{MRD}(\mathcal{S}_i) \neq \emptyset$ *for each* i*. Pick* $T_i \in \text{MRD}(\mathcal{S}_i)$*, and let* $\tilde{p}_i = p_i(T_1, \ldots, T_m)$ *be the probability a contestant participates in contest* T_i *in the equilibrium of contestants, and let* $P = \text{Supp}(\boldsymbol{T}) = \{i : \tilde{p}_i > 0\}$ *be the set of indices of contests in which contestants participate with positive probability when the contests are* $(T_1, \ldots, T_m)$*. Then*

1. *for any contestant-symmetric subgame-perfect equilibrium* $(C_1, \ldots, C_m)$*,*

$$p_i(C_1, \ldots, C_m) = \tilde{p}_i.$$

2. *if* $|P| \geq 2$*, then* $(C_1, \ldots, C_m) \in \mathcal{S}_1 \times \cdots \times \mathcal{S}_m$ *is a contestant-symmetric subgame-*

[6]In particular, $m = 2, n = 10$, $R_1 = R_2 = 1$, both $\mathcal{S}_1$ and $\mathcal{S}_2$ consist of two contests: the APA contest and a contest C with $\gamma_C = (1, 0, 0, 0, 0, 0, 1/49, 1/64, 1/81, 1/100)$, that is, choosing Tullock contest with $\tau_k = 1$ when $7 \leq k \leq 10$ and $\tau_k = +\infty$ otherwise. Then (C, C) is a contestant-symmetric subgame-perfect equilibrium, and APA is not a dominant contest for either designer. See Example D.1 for details. Moreover, Example D.2 shows that even if S_1 contains only contests with monotonically decreasing utility, APA may not be a dominant contest for designer 1.

perfect equilibrium if and only if $C_i \in \mathrm{MRD}(\mathcal{S}_i), \forall i \in P$.[7]

3. *if* $|P| = 1$*, let* $P = \{i_0\}$*, then* $(C_1, \ldots, C_m) \in \mathcal{S}_1 \times \cdots \times \mathcal{S}_m$ *is a contestant-symmetric subgame-perfect equilibrium if and only if* $\gamma_{C_{i_0}}(n) = \gamma_{T_{i_0}}(n)$.[8]

In the symmetric-reward case we can show that $|P| = m$ which makes the statement shorter:

Corollary 3.5. *In the symmetric-reward case, i.e.,* $R_1 = \cdots = R_m$*,* $(C_1, \ldots, C_m) \in \mathcal{S}_1 \times \cdots \times \mathcal{S}_m$ *is a contestant-symmetric subgame-perfect equilibrium if and only if* $C_i \in \mathrm{MRD}(\mathcal{S}_i)$ *for all* $i \in \{1, \ldots, m\}$*.*

Proof. We need to show that $|P| = m$ in this symmetric-reward case. Assume by contradiction that there exists i such that $\tilde{p}_i = 0$. Since $\sum_{\ell=1}^{m} \tilde{p}_\ell = 1$, there exists $j \neq i$ such that $\tilde{p}_j > 0$. Then by Claim 2.10, $\beta(T_j, \tilde{p}_j) < \beta(T_j, 0) = R_j = R_i = \beta(T_i, \tilde{p}_i)$. However, this contradicts the equilibrium condition (Claim 2.6) which states that $\tilde{p}_j > 0$ implies $\beta(T_j, \tilde{p}_j) \geq \beta(T_i, \tilde{p}_i)$. Therefore, we conclude that $\tilde{p}_i > 0$ for all $i \in \{1, \ldots, m\}$, i.e., $|P| = m$ as required. □

Thus, the case of symmetric rewards is a "clear cut" while the general case is more involved. The following example demonstrates the need for this distinction using a setting with highly asymmetric rewards (see Appendix D.2 for a proof).

Example 3.6. *Consider* $m \geq 3$ *contests and* n *contestants. Contest 1 has reward* $R_1 = 1$*, and each of others has reward* $R_j = \left(\frac{m-1}{m-2}\right)^{n-1} + 1$*. Each set* $\mathcal{S}_i$ *contains all monotonically decreasing utility contests (hence contains* APA*). Then for any contest* $C_1 \in \mathcal{S}_1$*,* $(C_1, T_2, \ldots, T_m)$ *where* $T_j = \mathrm{APA} \in \mathrm{MRD}(\mathcal{S}_j)$ *for* $j = 2, \ldots, m$ *is a contestant-symmetric subgame-perfect equilibrium. In this equilibrium,* $p_1(C_1, T_2, \ldots, T_m) = 0$*, and* $p_j(C_1, T_2, \ldots, T_m) = \frac{1}{m-1} > 0$ *for any* $j = 2, \ldots, m$*.*

Finally, the equilibria in Theorem 3.1 are Pareto optimal for the contest designers:

Definition 3.7.

[7] If $p_i(C_1, ..., C_m) = 0$ then contest i could be anything: If $p_i(C_1, \ldots, C_m) = 0$, then the utility for agent i is 0, which cannot be improved by choosing any other contest C'_i as $(C_i, \boldsymbol{C}_{-i})$ is an equilibrium. Moreover, by Claim B.2, $p_i(C'_i, \boldsymbol{C}_{-i})$ must be 0 as well, so the choice of C'_i does not affect the choices of contests of other designers.

[8] As $p_{i_0}(C_1, \ldots, C_m) = 1$, with probability 1 there are n contestants in contest i_0, thus the contest success functions of contest i_0 for $k \neq n$ have no effect on the utility calculation for the contestants' best response and could be anything.

- *For two strategy profiles $\hat{\boldsymbol{C}} = (\hat{C}_1, \ldots, \hat{C}_m), \boldsymbol{C} = (C_1, \ldots, C_m) \in \mathcal{S}_1 \times \cdots \times \mathcal{S}_m$ of the contest competition game $CCG(m, n, (R_i)_{i=1}^m, (\mathcal{S}_i)_{i=1}^m)$, we say $\boldsymbol{C}$ is a* Pareto improvement *of $\hat{\boldsymbol{C}}$, if $u_i(\boldsymbol{C}) \geq u_i(\hat{\boldsymbol{C}})$ for all $i \in \{1, \ldots, m\}$ and $u_i(\boldsymbol{C}) > u_i(\hat{\boldsymbol{C}})$ for at least one $i \in \{1, \ldots, m\}$.*

- *We say a strategy profile $\hat{\boldsymbol{C}} = (\hat{C}_1, \ldots, \hat{C}_m)$ is* Pareto optimal, *if there is no Pareto improvement of it.*

Theorem 3.8. *The equilibria in Theorem 3.1 are Pareto optimal.*

4 Welfare Optimality

Throughout this section, let $\boldsymbol{C} = (C_1, \ldots, C_m)$ be a tuple of contests. Denote the sum of designers' expected utilities, the sum of contestants' expected utilities, and their sum by

$$W_D(\boldsymbol{C}) = \sum_{i=1}^m u_i(\boldsymbol{C}), \quad W_C(\boldsymbol{C}) = n \sum_{i=1}^m p_i \beta(C_i, p_i), \quad W_S(\boldsymbol{C}) = W_D(\boldsymbol{C}) + W_C(\boldsymbol{C})$$

where $p_i = p_i(\boldsymbol{C})$. By equilibrium condition (Claim 2.6) we have $\beta(C_i, p_i) = \beta(C_j, p_j)$ for all $i, j \in \mathrm{Supp}(\boldsymbol{C})$. Denote this constant by $u_c(\boldsymbol{C})$. This is a contestant's expected utility in any contest in which she participates with positive probability. Note that by definition $p_i = 0$ for any $i \notin \mathrm{Supp}(\boldsymbol{C})$, so $\sum_{i \in \mathrm{Supp}(\boldsymbol{C})} p_i = \sum_{i=1}^m p_i = 1$. As a result,

$$W_C(\boldsymbol{C}) = n \sum_{i=1}^m p_i \beta(C_i, p_i) = n \sum_{i \in \mathrm{Supp}(\boldsymbol{C})} p_i \beta(C_i, p_i) = n \sum_{i \in \mathrm{Supp}(\boldsymbol{C})} p_i u_c(\boldsymbol{C}) = n u_c(\boldsymbol{C}).$$

For $W_S(\boldsymbol{C})$, note that whenever at least one contestant participates in contest i, the sum of expected utilities of designer i and the participants in that contest equals R_i. Define the random variables $k_1, ..., k_m$ as the number of contestants in contests $C_1, \ldots, C_m$. We can represent $W_S(\boldsymbol{C})$ as

$$W_S(\boldsymbol{C}) = \mathbb{E}_{k_1, \ldots, k_m} \left[\sum_{i=1}^m R_i \mathbb{1}[k_i \geq 1] \right] = \sum_{i=1}^m R_i \, \mathbb{E}_{k_i \sim \mathrm{Bin}(n, p_i)} \left[\mathbb{1}[k_i \geq 1] \right] \tag{7}$$

$$= \sum_{i=1}^m R_i \left[1 - (1 - p_i(\boldsymbol{C}))^n \right] = \sum_{i=1}^m R_i - \sum_{i=1}^m R_i (1 - p_i(\boldsymbol{C}))^n. \tag{8}$$

Our main conclusion in this section is that the equilibria $\boldsymbol{T}$ of Theorem 3.1 obtain welfare optimality in several quite natural cases:

Theorem 4.1. *Consider a contest competition game* $CCG(m, n, (R_i)_{i=1}^m, (\mathcal{S}_i)_{i=1}^m)$ *and fix some* $T_i \in \mathrm{MRD}(\mathcal{S}_i)$ *that also have monotonically decreasing utility. Then the equilibrium* $(T_1, \ldots, T_m)$ *maximizes* W_S *in each one of the following cases:*

1. *Unrestricted contest design: for all* i, $\mathcal{S}_i = \mathcal{C}_{R_i}$.
2. APA *is a possible contest:* $\mathrm{APA} \in \mathcal{S}_i$ *for every* i. *(We note that* APA *can be replaced with any other full rent dissipation contest.)*
3. *A symmetric CCG:* $R_1 = \cdots = R_m = R$ *and* $\mathcal{S}_1 = \cdots = \mathcal{S}_m = \mathcal{S} \subset \mathcal{C}_R$.
4. *An MRD-symmetric CCG:* $R_1 = \cdots = R_m = R$ *and* $\mathrm{MRD}(\mathcal{S}_1) = \cdots = \mathrm{MRD}(\mathcal{S}_m)$.

Note that the second case generalizes the first case and the fourth case generalizes the third case (since every symmetric CCG is also MRD-symmetric). The proof of this theorem is given in Appendix C. The following example shows that the conclusion of Theorem 4.1 does not hold in general.

Example 4.2. *Take* $m = 2, n = 2, R_1 = R_2 = 1$. *Let* C, T *be Tullock contests with* $\tau_C = 1, \tau_T = 1.2$. *It can be verified that* $\gamma_C = (1, \frac{1}{4})$ *and* $\gamma_T = (1, \frac{1}{5})$. *Suppose* $\mathcal{S}_1 = \{C\}$ *and* $\mathcal{S}_2 = \{C, T\}$. *Then* $\mathrm{MRD}(\mathcal{S}_1) = \{C\}$ *and* $\mathrm{MRD}(\mathcal{S}_2) = \{T\}$. *Due to symmetry,* $\boldsymbol{p}(C, C) = (\frac{1}{2}, \frac{1}{2})$, *so*

$$W_S(C, C) = 2 - 2\left(1 - \frac{1}{2}\right)^2 = \frac{3}{2}.$$

We can determine $\boldsymbol{p}(C, T) = (\tilde{p}_1, 1 - \tilde{p}_1)$ *by solving* $\beta(C, \tilde{p}_1) = 1 - \tilde{p}_1 + \frac{1}{4}\tilde{p}_1 = \beta(T, 1 - \tilde{p}_1) = \tilde{p}_1 + \frac{1}{5}(1 - \tilde{p}_1)$ *hence* $\tilde{p}_1 = \frac{16}{31}$ *and* $\tilde{p}_2 = 1 - \tilde{p}_1 = \frac{15}{31}$. *So*

$$W_S(C, T) = 2 - (1 - \tilde{p}_1)^2 - (1 - \tilde{p}_2)^2 = \frac{1441}{961} < \frac{3}{2}.$$

Remark 4.3. *One can construct a similar example for every* m, n, *and* $R_1, \ldots, R_m$. *Specifically, construct* $\mathcal{S}_i$ *as follows: consider equation set*

$$R_i \sum_{k=1}^{n} \binom{n-1}{k-1} \tilde{p}_i^{k-1} (1 - \tilde{p}_i)^{n-k} \gamma_{C_i}(k) = R, \quad \forall i = 1, \ldots, m.$$

Find a solution $(R, \{\gamma_{C_i}(k)\}_{i=1,\dots,m,k=2,\dots,n})$ of it, and choose a monotonically decreasing utility contest T_i that has higher rent dissipation than C_i for every i. If $T_1, \dots, T_m$ satisfy $p_i(T_1, \dots, T_m) \neq \tilde{p}_i$ for some i, then $W_S(T_1, \dots, T_m) < W_S(C_1, \dots, C_m)$, which means $(T_1, \dots, T_m)$ does not maximize W_S any more, and $\mathcal{S}_i = \{C_i, T_i\}$ is then a counterexample.

Although the total welfare is not always maximized at the equilibria of Theorem 3.1, it turns out that the contestants' welfare is *always* minimized at these equilibria outcomes, as the next theorem shows.[9]

Theorem 4.4. *Fix some $CCG(m, n, (R_i)_{i=1}^m, (\mathcal{S}_i)_{i=1}^m)$. Let $\boldsymbol{T} = (T_1, \dots, T_m)$ be one of the equilibria in Theorem 3.1, i.e., $T_i \in \mathrm{MRD}(\mathcal{S}_i)$ and T_i has monotonically decreasing utility. Then for any $\boldsymbol{C} \in \mathcal{S}_1 \times \dots \times \mathcal{S}_m$, $W_C(\boldsymbol{T}) \leq W_C(\boldsymbol{C})$.*

Proof. Let $\tilde{p}_i = p_i(\boldsymbol{T})$ and $p_i = p_i(\boldsymbol{C})$. We assume without loss of generality $p_1 > 0$ (i.e. $1 \in \mathrm{Supp}(\boldsymbol{C})$). Then $u_c(\boldsymbol{C}) = \beta(C_1, p_1)$.

If $p_1 \leq \tilde{p}_1$, then $\tilde{p}_1 > 0$, which implies $u_c(\boldsymbol{T}) = \beta(T_1, \tilde{p}_1)$. Since T_1 has monotonically decreasing utility, we have $\beta(T_1, \tilde{p}_1) \leq \beta(T_1, p_1)$ by Claim 2.10. Since T_1 is also the maximal rent dissipation contest of $\mathcal{S}_1$, we get $\beta(T_1, p_1) \leq \beta(C_1, p_1)$ by Claim 2.11. These inequalities together yield

$$u_c(\boldsymbol{T}) = \beta(T_1, \tilde{p}_1) \leq \beta(T_1, p_1) \leq \beta(C_1, p_1) = u_c(\boldsymbol{C}).$$

Otherwise $p_1 > \tilde{p}_1$, then there exists $i \in \{2, \dots, m\}$ such that $\tilde{p}_i > p_i$. Note that this implies $\tilde{p}_i > 0$, so $u_c(\boldsymbol{T}) = \beta(T_i, \tilde{p}_i)$, and,

$$\begin{aligned} u_c(\boldsymbol{T}) =& \beta(T_i, \tilde{p}_i) \overset{T_i \text{ has MDU, Claim 2.10}}{\leq} \beta(T_i, p_i) \overset{T_i \in \mathrm{MRD}(\mathcal{S}_i)\text{, Claim 2.11}}{\leq} \\ & \beta(C_i, p_i) \overset{p_1 > 0\text{, Claim 2.6}}{\leq} \beta(C_1, p_1) = u_c(\boldsymbol{C}). \end{aligned}$$

In either case we would get $u_c(\boldsymbol{T}) \leq u_c(\boldsymbol{C})$, hence $W_C(\boldsymbol{T}) = n u_c(\boldsymbol{T}) \leq n u_c(\boldsymbol{C}) = W_C(\boldsymbol{C})$. □

Theorems 4.1 and 4.4 together immediately imply:

[9] We already know that each such equilibrium outcome is Pareto-optimal for the designers. However, this does not immediately imply that these equilibrium outcomes minimize the contestants welfare since (1) the game is not constant-sum as $W_S(\boldsymbol{C})$ depends on the p_i's and (2) Pareto-optimal outcomes need not necessarily maximize the aggregate designers' utility.

Corollary 4.5. *Consider a contest competition game* $CCG(m, n, (R_i)_{i=1}^m, (\mathcal{S}_i)_{i=1}^m)$ *and fix some* $T_i \in \mathrm{MRD}(\mathcal{S}_i)$ *that have monotonically decreasing utility. Then the equilibrium* $(T_1, \ldots, T_m)$ *maximizes* W_D *in the four cases of Theorem 4.1.*

Thus, for example, (APA, ..., APA) maximizes the designers welfare and minimizes the contestants welfare in the case of unrestricted contest design.

5 Summary and Discussion

This paper studies a complete-information competition game among contest designers. First, each designer chooses a contest. Second, each contestant chooses (possibly in a random way) in which contest to participate. Third, a symmetric equilibrium outcome is realized in each contest (contestants choose effort levels; the reward is allocated to them based on their realized efforts). The resulting utility of each contest designer is the sum of efforts invested in their respective contests. The resulting utility of each contestant is the reward she receives minus the effort she invests.

Our main results characterize a certain type of contests which form an equilibrium (and may even be dominant) in this game. These equilibria are Pareto-optimal for the contest designers. Under natural conditions, these are the only possible equilibria. In addition, these equilibria maximize the social welfare (while minimizing the contestants' aggregate welfare) when designers are unrestricted in their choice of a contest, or when they are all restricted in the same way. Our results yield several conclusions regarding Tullock contests. For example, if contest designers are restricted to choose a Tullock contest with some parameter τ then any Tullock contest with $\tau \geq 2$ (e.g., APA where $\tau = \infty$) is a dominant contest for every designer.

It may be interesting for future work to further examine and relax several of our assumptions:

The Participation Model: We make several assumptions that (although natural in many cases) could be further relaxed. First, one may assume that contestants cannot observe the total number of contestants in the contest they chose which may be the reality in large electronic/online contests (but is less realistic in small physical contests). Second, a contestant may be able to participate in more than one contest simultaneously, e.g., in up to some fixed maximal number of contests. Another option is to study a budgeted participation model where the total effort of each contestant could be split among several contests (see e.g. Lavi and Shiran-Shvarzbard (2020)). Third, if we allow contestants to choose an asymmetric equilibrium in response to the designers' contest success functions, our results no longer hold. Example D.3 in the Appendix shows that (APA, APA) is not an equilibrium when we have two Tullock contests and three contestants that may choose an asymmetric equilibrium for their participation probabilities. It can be interesting to understand the asymmetric case as well.

Stochastic Ability: When a contestant has a cost of effort that depends on a stochastic (and private) ability, the single contest game between a designer and $k \geq 1$ contestants is not constant-sum. Importantly, our argument in this paper uses its constant-sum property when contestants' abilities are a fixed constant; this property allows us to relate the designer's utility with contestants' utilities which can then be characterized by the equilibrium condition for the contestants. When the game is not constant-sum, our argument cannot be applied directly. Another obstacle to analyzing stochastic ability is the lack of explicit characterization of contestants' utilities in a contest (even in a single Tullock contest) with stochastic ability.

Non-MDU contests: When the prize structure can depend on the number of participants, e.g., choosing a different Tullock contest depending on the number of participants, the MDU property will typically be violated, see e.g., Example 3.3. This opens the possibility of new "exotic" contests with new types of equilibria that do not fall within Theorem 3.1.

Risk-averse contestants: In Appendix D.4 we show an example with two contest designers that can choose some Tullock contest with $\tau \in [0, 2]$. With risk-neutral contestants we showed that $(\tau = 2, \tau = 2)$ is the unique contestant-symmetric subgame-perfect equilibrium (Corollary 3.2 and Corollary 3.5). The example shows that with risk-averse contestants $(\tau = 2, \tau = 2)$ is no longer a contestant-symmetric subgame-perfect equilibrium. It can be interesting to investigate how contestants' risk perceptions affect the qualitative conclusions of this work. In particular, we conjecture that with risk-loving participants, our results continue to hold.

A Missing Proofs from Section 2

A.1 Proof of Claim 2.10

Claim 2.10. *If C_i has monotonically decreasing utility, then for $p < p'$, $\beta(C_i, p) > \beta(C_i, p')$.*

Proof. By definition,

$$\begin{aligned}\beta(C_i, p) &= (1-p)^{n-1} R_i + \sum_{k=1}^{n-1} \binom{n-1}{k} p^k (1-p)^{n-1-k} \gamma_{C_i}(k+1) \\ &= (1-p)^{n-1} \frac{R_i}{2} + (1-p)^{n-1} \frac{R_i}{2} + \sum_{k=1}^{n-1} \binom{n-1}{k} p^k (1-p)^{n-1-k} \gamma_{C_i}(k+1).\end{aligned}$$

Let

$$\tilde{\gamma}(k+1) = \begin{cases} \frac{R_i}{2} & k = 0; \\ \gamma_{C_i}(k+1) & \text{otherwise.} \end{cases}$$

We can write $\beta(C_i, p) = (1-p)^{n-1} \frac{R_i}{2} + \mathbb{E}_{k \sim \text{Bin}(n-1,p)} [\tilde{\gamma}(k+1)]$. We recall that $\gamma_{C_i}(k)$, the utility of a contestant in a contest with k contestants, is at most $\frac{R_i}{k}$; in particular, $\gamma_{C_i}(2) \le \frac{R_i}{2}$. Therefore, the sequence

$$\tilde{\gamma}(1) = \frac{R_i}{2} \ge \tilde{\gamma}(2) \ge \cdots \ge \tilde{\gamma}(n)$$

is decreasing under the assumption that C_i has monotonically decreasing utility. Due to the first order stochastic dominance of a binomial distribution with a higher p parameter over another binomial distribution with a lower p parameter (see e.g., Wolfstetter, 1999), we have

$$\mathbb{E}_{k \sim \text{Bin}(n-1,p)} [\tilde{\gamma}(k+1)] \ge \mathbb{E}_{k \sim \text{Bin}(n-1,p')} [\tilde{\gamma}(k+1)]$$

which implies

$$\begin{aligned}\beta(C_i, p) &= (1-p)^{n-1} \frac{R_i}{2} + \mathbb{E}_{k \sim \text{Bin}(n-1,p')} [\tilde{\gamma}(k+1)] \\ &> (1-p')^{n-1} \frac{R_i}{2} + \mathbb{E}_{k \sim \text{Bin}(n-1,p')} [\tilde{\gamma}(k+1)] = \beta(C_i, p').\end{aligned}$$

□

A.2 Proof of Lemma 2.12

Lemma 2.12 (Corollary of Baye et al., 1996; Schweinzer and Segev, 2012; Ewerhart, 2017)**.** *Let C_τ be a Tullock contest with reward R and with parameter $\tau \in [0, +\infty]$. Then, $\gamma_{C_\tau}(k) = R(\frac{1}{k} - \frac{k-1}{k^2}\tau)$ if $\frac{k}{k-1} > \tau$ and $\gamma_{C_\tau}(k) = 0$ if $\frac{k}{k-1} \leq \tau$. For $\tau = +\infty$, $\gamma_{C_\tau}(1) = R$ and $\gamma_{C_\tau}(k) = 0$ for $k \geq 2$. As corollaries,*

- *Every Tullock contest has monotonically decreasing utility.*
- *Every Tullock contest with parameter $\tau \geq 2$ has full rent dissipation.*
- *If $\mathcal{S}$ is the set of all Tullock contests with parameter τ in some range whose maximum $\tau^{\max}$ is well defined and at most 2, then the Tullock contest with $\tau^{\max}$ is the only contest in* $\mathrm{MRD}(\mathcal{S})$*.*

Proof. Proposition 2 of Schweinzer and Segev (2012) shows that for any $k \geq 2$ such that $\frac{k}{k-1} \geq \tau$, $\gamma_{C_\tau}(k) = R(\frac{1}{k} - \frac{k-1}{k^2}\tau)$; in particular, $0 \leq \gamma_{C_\tau}(k) \leq \frac{1}{k}R < R = \gamma_{C_\tau}(1)$. Corollary 5 of Ewerhart (2017) shows that if $\frac{k}{k-1} < \tau$, then $\gamma_{C_\tau}(k) = 0$. Baye et al. (1996) show that for $\tau = +\infty$, $\gamma_{C_\tau}(k) = 0$ for $k \geq 2$.

To prove the third corollary of the lemma, we note that for any fixed k, $\gamma_{C_\tau}(k) = R(\frac{1}{k} - \frac{k-1}{k^2}\tau)$ is a non-negative decreasing function of τ for $0 \leq \tau \leq \frac{k}{k-1}$, and $\gamma_{C_\tau}(k) = 0$ for $\tau > \frac{k}{k-1}$, so $\gamma_{C_{\tau^{\max}}}(k) \leq \gamma_{C_{\tau'}}(k)$ for any $\tau' \leq \tau^{\max}$.

We now turn our attention to the first two corollaries. If $\tau > 2$, then $\frac{k}{k-1} < \tau$ for any $k \geq 2$, so C_τ has full rent dissipation. If $\tau = 2$, then for $k = 2$ we $\gamma_{C_\tau}(k) = R(\frac{1}{k} - \frac{k-1}{k^2}\tau) = 0$, and for any $k \geq 3$ we have $\gamma_{C_\tau}(k) = 0$ since $\frac{k}{k-1} < \tau$, so C_τ has full rent dissipation. For any $\tau < 2$, we let K_τ be the first $k \in \{2, 3, ...\}$ such that $\frac{K_\tau}{K_\tau - 1} < \tau$ (let $K_\tau = \infty$ for $\tau \leq 1$). In order to show that C_τ has monotonically decreasing utility, we only need to show that $\gamma_{C_\tau}(k)$ is monotonically decreasing in k in the range $2 \leq k < K_\tau$. Consider the function $f(x) = \frac{1}{x} - \frac{x-1}{x^2}\tau$ for $2 \leq x < K_\tau$. We take its derivative:

$$f'(x) = -\frac{1}{x^2} - \frac{x^2 - 2(x-1)x}{x^4}\tau = -\frac{1}{x^2} + \frac{x-2}{x^3}\tau.$$

Because $\tau \leq \frac{x}{x-1}$ for $x < K_\tau$, we have

$$f'(x) \leq -\frac{1}{x^2} + \frac{x-2}{x^3}\frac{x}{x-1} = \frac{1}{x^2}\left(-1 + \frac{x-2}{x-1}\right) = -\frac{1}{x^2} \cdot \frac{1}{x-1} \leq 0.$$

Thus, $f(x)$ is monotonically decreasing, and so is $\gamma_{C_\tau}(k)$. $\square$

B Missing Proofs from Section 3

Throughout this section we assume that $\mathcal{S}_i \subseteq \mathcal{C}_{R_i}$ contains a maximal rent dissipation contest with monotonically decreasing utility, which we denote by $T_i \in \mathrm{MRD}(\mathcal{S}_i)$.

B.1 Analysis of the contest competition game: proof of Theorem 3.1

Theorem 3.1 immediately follows from the following lemma, which is our main technical lemma. It shows that for any designer i, choosing T_i is always a best response if other designers choose contests with monotonically decreasing utility.

Lemma B.1. *Fix any $CCG(m, n, (R_i)_{i=1}^m, (\mathcal{S}_i)_{i=1}^m)$ where for any i, $\mathcal{S}_i \subseteq \mathcal{C}_{R_i}$ contains a maximal rent dissipation contest T_i that has monotonically decreasing utility. Fix some designer i and for all $j \neq i$ fix some $\hat{C}_j \in \mathcal{S}_j$ with monotonically decreasing utility. Then $u_i(T_i, \hat{\boldsymbol{C}}_{-i}) \geq u_i(C_i, \hat{\boldsymbol{C}}_{-i})$ for all $C_i \in \mathcal{S}_i$.*

We prove a useful claim before proving the lemma. The claim is about the change of contestants' participation equilibrium when a designer switches her contest. Intuitively, we expect that, with all other contests left unchanged, a designer (say, designer 1) setting her contest to have less rent dissipation results in higher participation (properties (a) and (b) in the next claim). A more surprising property is that if participants are unwilling to participate in contest 1 under maximal rent dissipation, then no other contest will be lucrative enough to attract them (property (c)).

Claim B.2. *Following the notations in Lemma B.1, we let $(\hat{p}_1, \hat{p}_2, \ldots, \hat{p}_m) = \boldsymbol{p}(T_1, \hat{C}_2, \ldots, \hat{C}_m)$, and for any $C_1 \in \mathcal{S}_1$, let $(p_1, p_2, \ldots, p_m) = \boldsymbol{p}(C_1, \hat{C}_2, \ldots, \hat{C}_m)$. Then,*

(a) $p_1 \geq \hat{p}_1$*;*

(b) $p_j \leq \hat{p}_j$ *for all* $j \in \{2, \ldots, m\}$*;*

(c) $\hat{p}_i = 0 \implies p_i = 0$ *for all* $i \in \{1, 2, \ldots, m\}$*.*

Proof. **Proof of (a).** Assume by contradiction $\hat{p}_1 > p_1 \geq 0$, then there is some designer $j \neq 1$ with $\hat{p}_j < p_j$, because the probabilities sum to one. We now have the contradiction:

$$\begin{aligned}
&\beta(T_1, \hat{p}_1) \geq && (\hat{p}_1 > 0,\ \text{Claim 2.6})\\
&\beta(\hat{C}_j, \hat{p}_j) > && (\hat{p}_j < p_j, \hat{C}_j \text{ has MDU, Claim 2.10})\\
&\beta(\hat{C}_j, p_j) \geq && (p_j > 0,\ \text{Claim 2.6})\\
&\beta(C_1, p_1) \geq && (T_1 \in \mathrm{MRD}(\mathcal{S}_1), \text{ Claim 2.11})\\
&\beta(T_1, p_1) > && (p_1 < \hat{p}_1, T_1 \text{ has MDU, Claim 2.10})\\
&\beta(T_1, \hat{p}_1).
\end{aligned}$$

Proof of (b). From (a) we know

$$\sum_{j=2}^{m} \hat{p}_j = 1 - \hat{p}_1 \geq 1 - p_1 = \sum_{j=2}^{m} p_j. \tag{9}$$

Assume by contradiction that for some $j_0 \in \{2, 3, \ldots, m\}$, $p_{j_0} > \hat{p}_{j_0} \geq 0$, then by (9) there must exist $j \in \{2, 3, \ldots, m\}$ with $j \neq j_0$ such that $\hat{p}_j > p_j \geq 0$. We therefore have the contradiction

$$\begin{aligned}
&\beta(\hat{C}_j, p_j) > && (p_j < \hat{p}_j, \hat{C}_j \text{ has MDU, Claim 2.10})\\
&\beta(\hat{C}_j, \hat{p}_j) \geq && (\hat{p}_j > 0, \text{Claim 2.6})\\
&\beta(\hat{C}_{j_0}, \hat{p}_{j_0}) > && (\hat{p}_{j_0} < p_{j_0}, \hat{C}_{j_0} \text{ has MDU, Claim 2.10})\\
&\beta(\hat{C}_{j_0}, p_{j_0}) \geq && (p_{j_0} > 0, \text{Claim 2.6})\\
&\beta(\hat{C}_j, p_j).
\end{aligned}$$

Proof of (c). For any $i \in \{2, \ldots, m\}$, (c) is an immediate corollary of (b). Consider $i = 1$, and assume by contradiction that $p_1 > \hat{p}_1 = 0$. Then there is some designer $j \neq 1$ with $p_j < \hat{p}_j$ because the probabilities sum to one. We then have the contradiction

$$\begin{aligned}
&\beta(T_1, \hat{p}_1) = && (\hat{p}_1 = 0)\\
&\gamma_{T_1}(1) = && (\text{By definition})
\end{aligned}$$

$$
\begin{aligned}
& R_1 \geq && \left(\gamma_{C_1}(k+1) \leq \frac{R_1}{k+1} \leq R_1 \text{ by Eq. (1)}\right) \\
& \mathbb{E}_{k\sim\mathrm{Bin}(n-1,p_1)}\left[\gamma_{C_1}(k+1)\right] = \beta(C_1, p_1) \geq && (p_1 > 0, \text{Claim 2.6}) \\
& \beta(\hat{C}_j, p_j) > && (p_j < \hat{p}_j, \hat{C}_j \text{ has MDU, Claim 2.10}) \\
& \beta(\hat{C}_j, \hat{p}_j) \geq && (\hat{p}_j > 0, \text{Claim 2.6}) \\
& \beta(T_1, \hat{p}_1). && \square
\end{aligned}
$$

Proof of Lemma B.1. Without loss of generality, we only prove it for designer $i = 1$. Suppose that when designer 1 chooses T_1 and all other designers j choose some contests $\hat{C}_j$ with monotonically decreasing utility, contestants choose participation probabilities $(\hat{p}_1, \hat{p}_2, \dots, \hat{p}_m) = \boldsymbol{p}(T_1, \hat{C}_2, \dots, \hat{C}_m)$. According to Claim 2.7, the expected utility of designer 1, denoted by $\hat{u}_1$, equals

$$\hat{u}_1 = u_1(T_1, \hat{C}_2, \dots, \hat{C}_m) = R_1\left[1 - (1-\hat{p}_1)^n\right] - n\hat{p}_1\beta(T_1, \hat{p}_1). \tag{10}$$

When designer 1 switches to any other contest $C_1 \in \mathcal{S}_1$, letting $(p_1, p_2, \dots, p_m) = \boldsymbol{p}(C_1, \hat{C}_2, \dots, \hat{C}_m)$, the expected utility of designer 1 becomes

$$u_1 = u_1(C_1, \hat{C}_2, \dots, \hat{C}_m) = R_1\left[1 - (1-p_1)^n\right] - np_1\beta(C_1, p_1). \tag{11}$$

Our goal is to show that $\hat{u}_1 \geq u_1$.

If $\hat{p}_1 = 0$, then by (c) of Claim B.2 we have $p_1 = 0$ and hence $\hat{u}_1 = \hat{u}_1 = 0$. The conclusion holds.

Now assume $\hat{p}_1 > 0$. If $\hat{p}_j = 0$ for all $j \in \{2, \dots, m\}$, then by (c) of Claim B.2 we have $p_j = 0$ for all j. This implies $\hat{p}_1 = p_1 = 1$ and hence $\hat{u}_1 = R_1 - n\gamma_{T_1}(n)$ and $u_1 = R_1 - n\gamma_{C_1}(n)$. Since $\gamma_{T_1}(n) \leq \gamma_{C_1}(n)$ by the assumption that T_1 has maximal rent dissipation, we have $\hat{u}_1 \geq u_1$.

Now we consider the case where $\hat{p}_j > 0$ for some $j \in \{2, \dots, m\}$. Because each contestant participates in both T_1 and $\hat{C}_j$ with positive probability, by equilibrium condition (Claim 2.6), we must have

$$\beta(T_1, \hat{p}_1) = \beta(\hat{C}_j, \hat{p}_j).$$

By (a) of Claim B.2, $p_1 \geq \hat{p}_1 > 0$, so each contestant participates in C_1 with positive probability, and by equilibrium condition (Claim 2.6),

$$\beta(C_1, p_1) \geq \beta(\hat{C}_j, p_j).$$

According to Claim 2.10, $\beta(\hat{C}_j, p)$ is a monotonically decreasing function of p, and by (b) of Claim B.2, $p_j \leq \hat{p}_j$. Therefore, we have $\beta(\hat{C}_j, p_j) \geq \beta(\hat{C}_j, \hat{p}_j)$ and hence

$$\beta(C_1, p_1) \geq \beta(\hat{C}_j, p_j) \geq \beta(\hat{C}_j, \hat{p}_j) = \beta(T_1, \hat{p}_1). \tag{12}$$

Plugging (12) into (11), we get

$$u_1 \leq R_1[1 - (1 - p_1)^n] - np_1\beta(T_1, \hat{p}_1).$$

Now we define function

$$f(p) = R_1[1 - (1 - p)^n] - np\beta(T_1, \hat{p}_1). \tag{13}$$

We take its derivative:

$$\begin{aligned}
f'(p) &= nR_1(1-p)^{n-1} - n\beta(T_1, \hat{p}_1) \\
&= nR_1(1-p)^{n-1} - n\sum_{k=0}^{n-1}\binom{n-1}{k}\hat{p}_1^k(1-\hat{p}_1)^{n-1-k}\gamma_{T_1}(k+1) \\
&= nR_1(1-p)^{n-1} - n(1-\hat{p}_1)^{n-1}R_1 - n\sum_{k=1}^{n-1}\binom{n-1}{k}\hat{p}_1^k(1-\hat{p}_1)^{n-1-k}\gamma_{T_1}(k+1) \\
&\leq nR_1(1-p)^{n-1} - nR_1(1-\hat{p}_1)^{n-1}.
\end{aligned}$$

For $p > \hat{p}_1$, $(1-p)^{n-1} < (1-\hat{p}_1)^{n-1}$, so $f'(p) < 0$. Thus, $f(p)$ is monotonically decreasing in the range $[\hat{p}_1, 1]$, which implies

$$\hat{u}_1 = f(\hat{p}_1) \geq f(p_1) \geq u_1, \tag{14}$$

concluding the proof.

B.2 Full characterization of equilibria for MDU contests: proof of Theorem 3.4

Additional properties of contestants' participation game. The following two claims use the notation and assumptions of Lemma B.1, specifically, we fix a contest competition game $CCG(m, n, (R_i)_{i=1}^m, (\mathcal{S}_i)_{i=1}^m)$, where for every i, $\mathrm{MRD}(\mathcal{S}_i)$ contains at least one contest T_i, and T_i has monotonically decreasing utility. The first claim and its proof are similar to item (c) of Claim B.2.[10]

Claim B.3. *Let $\tilde{p}_i = p_i(T_1, \dots, T_m)$. For any strategy profile $(C_1, \dots, C_m) \in \mathcal{S}_1 \times \dots \times \mathcal{S}_m$, let $p_i = p_i(C_1, \dots, C_m)$. Then for any $i \in \{1, \dots, m\}$, $\tilde{p}_i = 0$ implies $p_i = 0$.*

Proof. Assume by contradiction there exists i such that $\tilde{p}_i = 0$ and $p_i > 0$. Then there exists $j \neq i$ such that $\tilde{p}_j > p_j$. By Claim 2.6 we have $\beta(T_j, \tilde{p}_j) \geq \beta(T_i, \tilde{p}_i) = R_i$ and $\beta(C_i, p_i) \geq \beta(C_j, p_j)$. By Claim 2.11 (since $T_j \in \mathrm{MRD}(\mathcal{S}_j)$), we have $\beta(C_j, p_j) \geq \beta(T_j, p_j)$. By Claim 2.10, since T_j is a MDU contest and $\tilde{p}_j > p_j$, we have $\beta(T_j, p_j) > \beta(T_j, \tilde{p}_j)$. Finally, it is obvious that $\beta(C_i, p_i) \leq R_i$. Combining these inequalities, we get

$$\beta(T_j, \tilde{p}_j) \geq \beta(T_i, \tilde{p}_i) = R_i \geq \beta(C_i, p_i) \geq \beta(C_j, p_j) \geq \beta(T_j, p_j) > \beta(T_j, \tilde{p}_j),$$

which is a contradiction. □

When $\hat{\boldsymbol{C}}_{-i}$ are all MDU contests, Lemma B.1 states that T_i is a dominant contest for designer i. Thus, if we have $u_i(T_i, \hat{\boldsymbol{C}}_{-i}) = u_i(C_i, \hat{\boldsymbol{C}}_{-i})$ for some $C_i \in \mathcal{S}_i$, then C_i is a best response to $\hat{\boldsymbol{C}}_{-i}$. The next claim shows that, in this case, the equilibrium outcome in the two contestants' participation games $(T_i, \hat{\boldsymbol{C}}_{-i})$ and $(C_i, \hat{\boldsymbol{C}}_{-i})$ is identical.

Claim B.4. *If $u_i(T_i, \hat{\boldsymbol{C}}_{-i}) = u_i(C_i, \hat{\boldsymbol{C}}_{-i})$ for some $C_i \in \mathcal{S}_i$, then $\hat{p}_i = p_i$ and $\beta(T_i, \hat{p}_i) = \beta(C_i, p_i)$, where $\hat{p}_i = p_i(T_i, \hat{\boldsymbol{C}}_{-i})$ and $p_i = p_i(C_i, \hat{\boldsymbol{C}}_{-i})$.*

Proof. Without loss of generality, we only prove it for contest designer $i = 1$. Following the notation in Section B.1 (Eq. (10), Eq. (11) and Eq. (13)), define

$$\hat{u}_1 = u_1(T_1, \hat{\boldsymbol{C}}_{-1}) = R_1 \left[1 - (1 - \hat{p}_1)^n\right] - n\hat{p}_1\beta(T_1, \hat{p}_1),$$

[10] We note the differences: (1) here every contest changes, while in Claim B.2 only one contest changes, and (2) here we require all contests to be maximal rent dissipation (and MDU), while in Claim B.2 we only require them to be MDU.

$$
\begin{aligned}
u_1 &= u_1(C_1, \hat{\boldsymbol{C}}_{-1}) = R_1\left[1-(1-p_1)^n\right] - np_1\beta(C_1, p_1),\\
f(p) &= R_1[1-(1-p)^n] - np\beta(T_1, \hat{p}_1).
\end{aligned}
$$

Recall that by the assumption in the statement of the claim we have $\hat{u}_1 = u_1$. Consider the following three cases:

- If $\hat{p}_1 = 0$, then by (c) of Claim B.2, $p_1 = 0 = \hat{p}_1$. And $\beta(T_1, \hat{p}_1) = \beta(C_1, p_1) = R_1$.
- If $\hat{p}_1 = 1$, then by (a) of Claim B.2, $1 \geq p_1 \geq \hat{p}_1 = 1$ hence $p_1 = \hat{p}_1 = 1$. Therefore,

  $$R_1 - n\beta(T_1, \hat{p}_1) = \hat{u}_1 = u_1 = R_1 - n\beta(C_1, p_1).$$

  This immediately implies $\beta(T_1, \hat{p}_1) = \beta(C_1, p_1)$.
- Otherwise, $0 < \hat{p}_1 < 1$, so there exists some $j \in \{2, \ldots, m\}$ such that $\hat{p}_j > 0$ and therefore Eq. (14) in the proof of Lemma B.1 holds. Furthermore, since $\hat{u}_1 = u_1$, all the inequalities in (14) become equalities. Thus,

  $$f(\hat{p}_1) = f(p_1) = u_1. \tag{15}$$

  By (a) of Claim B.2, $p_1 \geq \hat{p}_1$, so by strict monotonicity of $f(p)$ in the range $p \in [\hat{p}_1, 1]$, Eq. (15) implies $p_1 = \hat{p}_1$ and in addition

  $$R_1[1-(1-p_1)^n] - np_1\beta(T_1, \hat{p}_1) = f(p_1) = u_1 = R_1[1-(1-p_1)^n] - np_1\beta(C_1, p_1),$$

  which directly implies $\beta(T_1, \hat{p}_1) = \beta(C_1, p_1)$ since $p_1 = \hat{p}_1 > 0$. $\square$

Proof of the "$\Longrightarrow$" direction of items 2 and 3 of Theorem 3.4. Under the notation of Theorem 3.4, for every contestant-symmetric subgame-perfect equilibrium $(C_1, ..., C_m)$, recall that $\mathrm{Supp}(C_1, \ldots, C_m) = \{i : p_i(C_1, \ldots, C_m) > 0\}$. We defer the proof that $\mathrm{Supp}(C_1, \ldots, C_m) = P$ and first prove a useful lemma:

Lemma B.5.

- *If* $|\mathrm{Supp}(C_1, \ldots, C_m)| > 1$ *then for any* $i \in \mathrm{Supp}(C_1, \ldots, C_m)$, $C_i \in \mathrm{MRD}(\mathcal{S}_i)$.
- *If* $\mathrm{Supp}(C_1, \ldots, C_m) = \{i_0\}$ *then* $p_{i_0}(C_1, \ldots, C_m) = 1$ *and* $\gamma_{C_{i_0}}(n) = \gamma_{T_{i_0}}(n)$.

Proof. Recall that T_i is a dominant contest and hence a best response to $\boldsymbol{C}_{-i}$. Since $(C_1, \ldots, C_m)$ is a contestant-symmetric subgame-perfect equilibrium, C_i is also a best response. Applying Claim B.4, we get $p_i := p_i(C_i, \boldsymbol{C}_{-i}) = p_i(T_i, \boldsymbol{C}_{-i})$ and

$$\beta(T_i, p_i) = \beta(T_i, p_i(T_i, \boldsymbol{C}_{-i})) = \beta(C_i, p_i(C_i, \boldsymbol{C}_{-i})) = \beta(C_i, p_i). \tag{16}$$

By definition

$$\begin{aligned} \beta(T_i, p_i) &= \sum_{k=0}^{n-1} \binom{n-1}{k} p_i^k (1-p_i)^{n-1-k} \gamma_{T_i}(k+1) \\ = \beta(C_i, p_i) &= \sum_{k=0}^{n-1} \binom{n-1}{k} p_i^k (1-p_i)^{n-1-k} \gamma_{C_i}(k+1). \end{aligned}$$

If $p_i = 1$, then $\beta(T_i, p_i) = \gamma_{T_i}(n) = \beta(C_i, p_i) = \gamma_{C_i}(n)$, this corresponds to the second case of the lemma.

Otherwise, for any $i \in \{1, \ldots, m\}$, $p_i < 1$. We prove the first case of the lemma, i.e. for any $i \in \mathrm{Supp}(C_1, \ldots, C_m)$, $C_i \in \mathrm{MRD}(\mathcal{S}_i)$. Actually, as $0 < p_i < 1$, $\binom{n-1}{k} p_i^k (1 - p_i)^{n-1-k} > 0$ for every $k = 0, \ldots, n-1$. Moreover, $T_i \in \mathrm{MRD}(\mathcal{S}_i)$ implies $\gamma_{T_i}(k+1) \leq \gamma_{C_i}(k+1)$ for every $k = 0, \ldots, n-1$. As a result, for (16) to hold, we must have $\gamma_{T_i}(k+1) = \gamma_{C_i}(k+1)$ for every $k = 0, \ldots, n-1$, which indicates $C_i \in \mathrm{MRD}(\mathcal{S}_i)$. This completes the proof of the lemma. $\square$

Comparing Lemma B.5 with the conclusion of the " $\Longrightarrow$ " direction, we are left to prove $\mathrm{Supp}(C_1, \ldots, C_m) = P$. The $\mathrm{Supp}(C_1, \ldots, C_m) \subseteq P$ result is just a direct implication of Claim B.3. We then prove $P \subseteq \mathrm{Supp}(C_1, \ldots, C_m)$. Denote $p_i(C_1, \ldots, C_m)$ by p_i for simplicity. Assume towards a contradiction that there exists $i \in \{1, \ldots, m\}$ such that $\tilde{p}_i > p_i = 0$. Then there exists $j \neq i$ such that $p_j > \tilde{p}_j$. Note that this implies $p_j > 0$. Therefore, by Lemma B.5, either $C_j \in \mathrm{MRD}(\mathcal{S}_j)$, or $p_j = 1$ and $\gamma_{C_j}(n) = \gamma_{T_j}(n)$. In either case we have $\beta(T_j, p_j) = \beta(C_j, p_j)$. By equilibrium condition (Claim 2.6), $\beta(C_j, p_j) \overset{(p_j>0)}{\geq} \beta(C_i, p_i) \overset{(p_i=0)}{=} R_i$ and $\beta(T_i, \tilde{p}_i) \overset{(\tilde{p}_i>0)}{\geq} \beta(T_j, \tilde{p}_j)$. By Claim 2.10, $\beta(T_j, \tilde{p}_j) > \beta(T_j, p_j)$ and $R_i = \beta(T_i, p_i) > \beta(T_i, \tilde{p}_i)$. These inequalities together yield

$$\beta(T_j, \tilde{p}_j) > \beta(T_j, p_j) = \beta(C_j, p_j) \geq \beta(C_i, p_i) = R_i = \beta(T_i, p_i) > \beta(T_i, \tilde{p}_i) \geq \beta(T_j, \tilde{p}_j),$$

which is a contradiction. Therefore, we conclude that $p_i = 0$ implies $\tilde{p}_i = 0$ for any $i \in \{1, \dots, m\}$. This completes the proof.

Proof of the "$\Longleftarrow$" direction of items 2 and 3 of Theorem 3.4. To prove this direction we first assume $|P| > 1$. Assume $(C_1, \dots, C_m)$ is any strategy profile satisfying $C_i \in \mathrm{MRD}(\mathcal{S}_i)$ for any $i \in P$. To prove that it is a contestant-symmetric subgame-perfect equilibrium, we only need to show that for any i, C_i is designer i's best response when the other designers choose $\boldsymbol{C}_{-i}$. Note that Lemma B.1 already guarantees that for $i \in P$, C_i is designer i's best response, so we are left to show that this also holds for those $i \notin P$. Assume $\tilde{p}_i = 0$, then for any $C_i' \in \mathcal{S}_i$, by Claim B.3, $p_i(C_i', \boldsymbol{C}_{-i}) = 0$, which implies that designer i gets zero utility no matter which C_i' she chooses. So C_i is indeed one of her best responses. To conclude, C_i is designer i's best response for any i, which implies that $(C_1, \dots, C_m)$ is a contestant-symmetric subgame-perfect equilibrium.

We then assume $|P| = 1$. Suppose $P = \{i_0\}$, and assume $(C_1, \dots, C_m)$ is any strategy profile satisfying $\gamma_{C_{i_0}}(n) = \gamma_{T_{i_0}}(n)$. We need to show that for any i, C_i is designer i's best response when the other designers choose $\boldsymbol{C}_{-i}$. This time Claim B.3 promises that for any $i \neq i_0$ and any $C_i' \in \mathcal{S}_i$, $p_i(C_i', \boldsymbol{C}_{-i}) = 0$, so C_i is designer i's best response. And by the same claim, $p_i(C_{i_0}, \boldsymbol{C}_{-i_0}) = p_i(T_{i_0}, \boldsymbol{C}_{-i_0}) = 0$ for any $i \neq i_0$, so $p_{i_0}(C_{i_0}, \boldsymbol{C}_{-i_0}) = p_{i_0}(T_{i_0}, \boldsymbol{C}_{-i_0}) = 1$. As a result,

$$\beta(C_{i_0}, p_{i_0}(C_{i_0}, \boldsymbol{C}_{-i_0})) = \gamma_{C_{i_0}}(n) = \gamma_{T_{i_0}}(n) = \beta(T_{i_0}, p_{i_0}(T_{i_0}, \boldsymbol{C}_{-i_0})),$$

and

$$u_{i_0}(C_{i_0}, \boldsymbol{C}_{-i_0}) = R_{i_0} - n\beta(C_{i_0}, p_{i_0}(C_{i_0}, \boldsymbol{C}_{-i_0})) = R_{i_0} - n\beta(T_{i_0}, p_{i_0}(T_{i_0}, \boldsymbol{C}_{-i_0})) = u_{i_0}(T_{i_0}, \boldsymbol{C}_{-i_0}).$$

In other words, C_{i_0} has equal utility for designer i_0 as her best response T_{i_0}, which implies that C_{i_0} is also designer i_0's best response. To conclude, C_i is designer i's best response for any i, so $(C_1, \dots, C_m)$ is a contestant-symmetric subgame-perfect equilibrium. This completes the proof.

Proof of item 1 of Theorem 3.4. For any contestant-symmetric subgame-perfect equilibrium $(C_1, \dots, C_m)$, we claim that $(\tilde{p}_1, \dots, \tilde{p}_m)$ is a symmetric equilibrium for the contestants under $(C_1, \dots, C_m)$; then, since the contestants' symmetric equilibrium is unique according to Lemma 2.13, we must have $p_i(C_1, \dots, C_m) = \tilde{p}_i$, which completes the proof. Consider $\beta(C_i, \tilde{p}_i)$ for all $i \in \{1, \ldots, m\}$. If $i \notin P$, then $\tilde{p}_i = 0$ and $\beta(C_i, \tilde{p}_i) = R_i = \beta(T_i, \tilde{p}_i)$. If $i \in P$, then by item 2 and 3, either $C_i \in \mathrm{MRD}(\mathcal{S}_i)$ or $\tilde{p}_i = 1$ and $\gamma_{C_i}(n) = \gamma_{T_i}(n)$, and we have $\beta(C_i, \tilde{p}_i) = \beta(T_i, \tilde{p}_i)$ in either case. So $\beta(C_i, \tilde{p}_i) = \beta(T_i, \tilde{p}_i)$ for all $i \in \{1, \ldots, m\}$. Then applying equilibrium condition (Claim 2.6) for the case where designers choose $(T_1, \ldots, T_m)$, we get $\beta(C_i, \tilde{p}_i) = \beta(T_i, \tilde{p}_i) = \beta(T_j, \tilde{p}_j) = \beta(C_j, \tilde{p}_j) \geq \beta(T_\ell, \tilde{p}_\ell) = \beta(C_\ell, \tilde{p}_\ell)$ for any $i, j \in P$ and $\ell \notin P$. We therefore know that when designers choose $(C_1, \ldots, C_m)$, $(\tilde{p}_1, \ldots, \tilde{p}_m)$ is still a best response for any contestant when all the other contestants use $(\tilde{p}_1, \ldots, \tilde{p}_m)$, which means that $(\tilde{p}_1, \ldots, \tilde{p}_m)$ is a contestants' symmetric equilibrium.

B.3 Pareto efficiency of the equilibria: proof of Theorem 3.8

Assume $\boldsymbol{T} = (T_1, \ldots, T_m)$ is the contestant-symmetric subgame-perfect equilibrium in Theorem 3.1 for $CCG(m, n, (R_i)_{i=1}^n, (\mathcal{S}_i)_{i=1}^n)$, and $\boldsymbol{C} = (C_1, \ldots, C_m) \in \mathcal{S}_1 \times \cdots \times \mathcal{S}_m$ is any other strategy profile of the designers. We will show that $\boldsymbol{C}$ is not a Pareto improvement of $\boldsymbol{T}$ which proves the theorem. Denote by $(\tilde{p}_1, \ldots, \tilde{p}_m) = \boldsymbol{p}(\boldsymbol{T})$ and $(p_1, \ldots, p_m) = \boldsymbol{p}(\boldsymbol{C})$ the symmetric equilibria contestants play under $\boldsymbol{T}$ and $\boldsymbol{C}$, respectively. If $p_i = \tilde{p}_i$ for any i, then as T_i is the maximal rent dissipation contest of $\mathcal{S}_i$, by Claim 2.11 we have

$$\beta(T_i, p_i) \leq \beta(C_i, p_i)$$

As a result, for any designer i,

$$\begin{aligned} u_i(\boldsymbol{T}) &= R_i[1 - (1 - \tilde{p}_i)^n] - n\tilde{p}_i\beta(T_i, \tilde{p}_i) \\ &= R_i[1 - (1 - p_i)^n] - np_i\beta(T_i, p_i) \\ &\geq R_i[1 - (1 - p_i)^n] - np_i\beta(C_i, p_i) = u_i(\boldsymbol{C}), \end{aligned}$$

showing that $\boldsymbol{C}$ is not a Pareto improvement of $\boldsymbol{T}$.

Otherwise, there exist i, j such that $p_i > \tilde{p}_i$ and $p_j < \tilde{p}_j$. Note that this implies $p_i, \tilde{p}_j > 0$, so by equilibrium condition (Claim 2.6), we get

$$\beta(C_i, p_i) \geq \beta(C_j, p_j), \tag{17}$$

and

$$\beta(T_j, \tilde{p}_j) \geq \beta(T_i, \tilde{p}_i). \tag{18}$$

Since $p_j < \tilde{p}_j$ and T_j is a monotonically decreasing utility contest, by Claim 2.10 we have

$$\beta(T_j, \tilde{p}_j) < \beta(T_j, p_j), \tag{19}$$

Moreover, as T_j is a maximal rent dissipation contest in $\mathcal{S}_j$, we have

$$\beta(T_j, p_j) \leq \beta(C_j, p_j) \tag{20}$$

by Claim 2.11. Combining these inequalities together, we get

$$\beta(T_i, \tilde{p}_i) \overset{(18)}{\leq} \beta(T_j, \tilde{p}_j) \overset{(19)}{<} \beta(T_j, p_j) \overset{(20)}{\leq} \beta(C_j, p_j) \overset{(17)}{\leq} \beta(C_i, p_i).$$

Now we consider the utilities of designer i in $\boldsymbol{T}$ and $\boldsymbol{C}$. We have

$$\begin{aligned} u_i(\boldsymbol{T}) &= R_i[1 - (1 - \tilde{p}_i)^n] - n\tilde{p}_i\beta(T_i, \tilde{p}_i), \\ u_i(\boldsymbol{C}) &= R_i[1 - (1 - p_i)^n] - np_i\beta(C_i, p_i). \end{aligned}$$

Similarly to (13), we define $f(p) = R_i[1 - (1 - p)^n] - np\beta(T_i, \tilde{p}_i)$ and have

$$f'(p) \leq nR_i(1 - p)^{n-1} - nR_i(1 - \tilde{p}_i)^{n-1} < 0$$

for $p > \tilde{p}_i$, which implies that $f(p)$ is a strictly decreasing function of p when $p \geq \tilde{p}_i$. Therefore, as $p_i > \tilde{p}_i$, we have $f(p_i) < f(\tilde{p}_i)$. As a result,

$$\begin{aligned} u_i(\boldsymbol{T}) = f(\tilde{p}_i) > f(p_i) &= R_i[1 - (1 - p_i)^n] - np_i\beta(T_i, \tilde{p}_i) \\ &\geq R_i[1 - (1 - p_i)^n] - np_i\beta(C_i, p_i) = u_i(\boldsymbol{C}), \end{aligned}$$

which indicates that $\boldsymbol{C}$ cannot be a Pareto improvement of $\boldsymbol{T}$, concluding the proof.

C Missing Proofs from Section 4

C.1 Proof of Theorem 4.1

For any $\boldsymbol{C} = (C_1, \ldots, C_m)$, let $\boldsymbol{p}(\boldsymbol{C}) = (p_1, \ldots, p_m)$. By Eq. (7),

$$W_S(C_1, \ldots, C_m) = \sum_{i=1}^{m} R_i - \sum_{i=1}^{m} R_i(1 - p_i)^n.$$

Then by Hölder's inequality,

$$\left(\sum_{i=1}^{m} R_i(1-p_i)^n\right)^{\frac{1}{n}} \left(\sum_{i=1}^{m} R_i^{-\frac{1}{n-1}}\right)^{\frac{n-1}{n}} = \left(\sum_{i=1}^{m} \left(R_i^{\frac{1}{n}}(1-p_i)\right)^n\right)^{\frac{1}{n}} \left(\sum_{i=1}^{m} \left(R_i^{-\frac{1}{n}}\right)^{\frac{n}{n-1}}\right)^{\frac{n-1}{n}}$$
$$\geq \sum_{i=1}^{m} \left(R_i^{\frac{1}{n}}(1-p_i)\right)\left(R_i^{-\frac{1}{n}}\right) = \sum_{i=1}^{m}(1-p_i) = m-1.$$

So

$$\sum_{i=1}^{m} R_i(1-p_i)^n \geq \left(\frac{m-1}{\left(\sum_{i=1}^{m} R_i^{-\frac{1}{n-1}}\right)^{\frac{n-1}{n}}}\right)^n = \frac{(m-1)^n}{\left(\sum_{i=1}^{m} R_i^{-\frac{1}{n-1}}\right)^{n-1}},$$

and

$$W_S(C_1, \ldots, C_m) = \sum_{i=1}^{m} R_i - \sum_{i=1}^{m} R_i(1-p_i)^n \leq \sum_{i=1}^{m} R_i - \frac{(m-1)^n}{\left(\sum_{i=1}^{m} R_i^{-\frac{1}{n-1}}\right)^{n-1}}. \tag{21}$$

We will show that $W_S(T_1, \ldots, T_m)$ is equal to the right-hand side in the following cases.

The case when $\mathcal{S}_i$ contains a full rent dissipation contest for every i. In this case, since $T_i \in \mathrm{MRD}(\mathcal{S}_i)$, we have that T_i is also a full rent dissipation contest. Let $\tilde{p}_i = p_i(T_1, \ldots, T_m)$. If $\tilde{p}_i = 0$ for some i, then by Claim B.3, $p_i(C_1, \ldots, C_m) = 0$. Thus $W_S(\boldsymbol{T}) = W_S(\boldsymbol{T}_{-i})$ and $W_S(\boldsymbol{C}) = W_S(\boldsymbol{C}_{-i})$. We can thus assume that $\tilde{p}_i > 0$ for every i. Then by Claim 2.6,

$$\beta(T_1, \tilde{p}_1) = \beta(T_2, \tilde{p}_2) = \cdots = \beta(T_m, \tilde{p}_m). \tag{22}$$

As T_i has full rent dissipation, we have $\beta(T_i, \tilde{p}_i) = R_i(1-\tilde{p}_i)^{n-1}$. Substitute this into (22), we get

$$R_1^{\frac{1}{n-1}}(1-\tilde{p}_1) = R_2^{\frac{1}{n-1}}(1-\tilde{p}_2) = \cdots = R_m^{\frac{1}{n-1}}(1-\tilde{p}_m).$$

So $\forall i = 1, \ldots, m$,

$$\frac{1-\tilde{p}_j}{1-\tilde{p}_i} = \frac{R_j^{-\frac{1}{n-1}}}{R_i^{-\frac{1}{n-1}}}, \quad \forall j = 1, \ldots, m. \tag{23}$$

Note that $\sum_{j=1}^{m}(1-\tilde{p}_j) = m-1$. Fixing any i and summing (23) for $j = 1$ to m, we obtain

$$\frac{m-1}{1-\tilde{p}_i} = \sum_{j=1}^{m} \frac{1-\tilde{p}_j}{1-\tilde{p}_i} = \frac{\sum_{j=1}^{m} R_j^{-\frac{1}{n-1}}}{R_i^{-\frac{1}{n-1}}}.$$

So

$$1-\tilde{p}_i = \frac{m-1}{\sum_{j=1}^{m} R_j^{-\frac{1}{n-1}}} \cdot R_i^{-\frac{1}{n-1}},$$

and therefore,

$$W_S(T_1, \ldots, T_m) = \sum_{i=1}^{m} R_i - \sum_{i=1}^{m} R_i(1-\tilde{p}_i)^n = \sum_{i=1}^{m} R_i - \frac{(m-1)^n}{\left(\sum_{j=1}^{m} R_j^{-\frac{1}{n-1}}\right)^{n-1}},$$

which meets the bound given by (21). This completes the proof for this case.

The case of MRD-symmetric strategy space. Note that by definition any two contests $T, T' \in \mathrm{MRD}(\mathcal{S}_i)$ must have the same $\boldsymbol{\gamma}$ vector (i.e., for any $k = 1, \ldots, n$, $\gamma_T(k) = \gamma_{T'}(k)$). The following lemma therefore proves this case:

Lemma C.1. *In a $CCG(m, n, (R_i)_{i=1}^m, (\mathcal{S}_i)_{i=1}^m)$ with $R_1 = \cdots = R_m = R$ (and $\mathcal{S}_i$'s can be different), for any strategy profile $\boldsymbol{C}' = (C_1', \ldots, C_m')$ where for every i, $C_i' \in \mathcal{S}_i$ is a MDU contest, and has the same $\boldsymbol{\gamma}$ vector (i.e., for any $k = 1, \ldots, n$, and any $i, j \in \{1, \ldots, m\}$, $\gamma_{C_i'}(k) = \gamma_{C_j'}(k)$), $\boldsymbol{C}'$ maximizes W_S.*

Proof. When the rewards are the same, (21) becomes

$$W_S(C_1, \ldots, C_m) = mR - R\sum_{i=1}^{m}(1-p_i)^n \le mR - R\frac{(m-1)^n}{m^{n-1}}$$

for any $(C_1, \ldots, C_m) \in \mathcal{S}_1 \times \cdots \times \mathcal{S}_m$. On the other hand, since any two C_i' and C_j'

have the same $\boldsymbol{\gamma}$ vector it follows that

$$\beta\left(C_1', \frac{1}{m}\right) = \cdots = \beta\left(C_m', \frac{1}{m}\right).$$

So $(p_1, \ldots, p_m) = \left(\frac{1}{m}, \ldots, \frac{1}{m}\right)$ is a symmetric equilibrium for the contestants under $(C_1', \ldots, C_m')$. Moreover, by Lemma 2.13, it is the unique symmetric equilibrium, so $p_i(\boldsymbol{C}') = \frac{1}{m}$. Therefore

$$W_S(C_1', \ldots, C_m') = \sum_{i=1}^{m} R_i - \sum_{i=1}^{m} R_i(1-p_i(\boldsymbol{C}'))^n = mR - R\frac{(m-1)^n}{m^{n-1}} \geq W_S(C_1, \ldots, C_m),$$

which completes our proof. □

D Additional Examples

D.1 Non-MDU contests

Theorem 3.1 (or Corollary 3.2) show that a sufficient condition for a set of contests to be in equilibrium in a contest competition game is that the contests have both MRD and MDU (or simply having full rent dissipation). Example 3.3 then shows that this condition is not necessary by giving an example of equilibria consisting of non-MDU contests; also, it shows that the MRD and MDU contests are not dominant when the sets $\mathcal{S}_i$ contain non-MDU contests. But that example makes use of an unnatural Tullock contest with parameter $\tau_k = 0$, where the reward is given to contestants for free. Here we give another example using a more natural Tullock contest with $\tau_k = 1$.

Example D.1. *Let $m = 2, n = 10$, $R_1 = R_2 = 1$, both $\mathcal{S}_1$ and $\mathcal{S}_2$ consist of two contests: the* APA *contest and a contest C with $\gamma_C = (1, 0, 0, 0, 0, 0, 1/49, 1/64, 1/81, 1/100)$ (that is, choosing Tullock contest with $\tau_k = 1$ when there are $7 \leq k \leq 10$ contestants and $\tau_k = +\infty$ otherwise). We claim that (C, C) is a contestant-symmetric subgame-perfect equilibrium: When designers choose (C, C), by symmetry, contestants participate in either contest with equal probability $(0.5, 0.5)$. By direct computation (e.g. using (6)), the expected utility of each designers is*

$$(0.9658, 0.9658).$$

Now suppose designer 1 switches to APA*. The probabilities* $(p_1, p_2) = \boldsymbol{p}(\text{APA}, C)$ *in the contestants' symmetric mixed strategy Nash equilibrium must satisfy, according to Claim 2.6,* $\beta(\text{APA}, p_1) = \beta(C, p_2)$ *(assuming* $p_1, p_2 > 0$*). By numerical methods, we find that* $(p_1, p_2) = (0.4125, 0.5875)$*. Then the expected utility of designers becomes*

$$(0.9607, 0.9509).$$

Since $0.9607 < 0.9658$*, designer 1 will not switch to* APA*. By symmetry, designer 2 will not switch to* APA*, either. Hence,* (C, C) *is an equilibrium. Also,* APA *is not a dominant contest because it is not a best-response to* C*.*

Concerning dominant contests in contest competition games, the second part of Theorem 3.1 show that the MRD and MDU contests (e.g., the full rent dissipation contest APA) are dominant if all sets $\mathcal{S}_i$ only contain MDU contests. Example 3.3 and Example D.1 show that APA is not dominant for designer 1 when both sets $\mathcal{S}_1$ and $\mathcal{S}_2$ contain non-MDU contests in a two-designer game. Here we give another example where APA is not dominant even if $\mathcal{S}_1$ only contains MDU contests (while S_2 still contains non-MDU contests).

Example D.2. *Consider two contest designers* ($m = 2$) *with* $R_1 = R_2 = 1$ *and ten contestants* ($n = 10$)*. Designer 2 chooses contest* C *which is a Tullock contest whose parameter* τ_k *depends on the number of contestants* k*: if* $k \leq 5$*, she plays* APA*; if* $k \geq 6$*, she chooses* $\tau_k = 1$*. Consider designer 1 that chooses from the set of all Tullock contests. We show that* APA *is not a dominant contest for designer 1 by showing that the Tullock contest with* $\tau = 1.2$ *is a better response for designer 1 than* APA*.*

According to Lemma 2.12, the contest utility functions for contestants satisfy:

$$\gamma_C(k) = \begin{cases} 1 & k = 1 \\ 0 & 2 \leq k \leq 5 \\ \frac{1}{k^2} & k \geq 6 \end{cases} \quad \gamma_{\text{APA}}(k) = \begin{cases} 1 & k = 1 \\ 0 & k \geq 2 \end{cases} \quad \gamma_{\tau=1.2}(k) = \begin{cases} 1 & k = 1 \\ \frac{1}{k} - \frac{1.2(k-1)}{k^2} & 2 \leq k \leq 5 \\ 0 & k \geq 6 \end{cases}$$

Using the contestants' equilibria equations (Claim 2.6), this induces participation equilibria probabilities of:

$$\hat{p}_1 \approx 0.366965, \quad p_1 \approx 0.519786,$$

where $(\hat{p}_1, \hat{p}_2) = \boldsymbol{p}(\mathrm{APA}, C)$ *and* $(p_1, p_2) = \boldsymbol{p}(\tau = 1.2, C)$.

This translates to designer 1 utility of ≈ 0.929759 *choosing* APA, *and* ≈ 0.930121 *choosing* $\tau = 1.2$.

D.2 Proof of Example 3.6

Example 3.6. *Consider* $m \geq 3$ *contests and* n *contestants. Contest 1 has reward* $R_1 = 1$*, and each of others has reward* $R_j = \left(\frac{m-1}{m-2}\right)^{n-1} + 1$*. Each set* $\mathcal{S}_i$ *contains all monotonically decreasing utility contests (hence contains* APA*). Then for any contest* $C_1 \in \mathcal{S}_1$*,* $(C_1, T_2, \ldots, T_m)$ *where* $T_j = \mathrm{APA} \in \mathrm{MRD}(\mathcal{S}_j)$ *for* $j = 2, \ldots, m$ *is a contestant-symmetric subgame-perfect equilibrium. In this equilibrium,* $p_1(C_1, T_2, \ldots, T_m) = 0$*, and* $p_j(C_1, T_2, \ldots, T_m) = \frac{1}{m-1} > 0$ *for any* $j = 2, \ldots, m$.

Proof. When for all $i = 1, \ldots, m$, contest designer i chooses $T_i = \mathrm{APA}$, we have that for any $j = 2, \ldots, m$,

$$\beta\left(T_j, \frac{1}{m-1}\right) = R_j\left(1 - \frac{1}{m-1}\right)^{n-1} = \left[\left(\frac{m-1}{m-2}\right)^{n-1} + 1\right]\left(\frac{m-2}{m-1}\right)^{n-1} > 1 = R_1 = \beta(T_1, 0),$$

which implies that $\left(0, \frac{1}{m-1}, \ldots, \frac{1}{m-1}\right)$ is a symmetric equilibrium for contestants in $(T_1, \ldots, T_m)$, i.e. $p_i(T_1, \ldots, T_m) = \begin{cases} \frac{1}{m-1}, & i = 2, \ldots, m \\ 0, & i = 1 \end{cases}$. Then the claim that any $(C_1, T_2, \ldots, T_m)$, with $T_j = \mathrm{APA}$ for $j = 2, \ldots, m$ and C_1 be an arbitrary contest in $\mathcal{S}_1$, is a contestant-symmetric subgame-perfect equilibrium with $p_i(C_1, T_2, \ldots, T_m) = p_i(T_1, \ldots, T_m) = \begin{cases} \frac{1}{m-1}, & i = 2, \ldots, m \\ 0, & i = 1 \end{cases}$ follows from Theorem 3.4. $\square$

D.3 Asymmetric participation equilibrium

To demonstrate the importance of the symmetry aspect of the contestant-symmetric subgame-perfect equilibrium to our results, we consider an example where we allow contestants to play an asymmetric equilibrium in response to the designers' contest success functions; the conclusion that APA contests form an equilibrium now no longer holds.

Example D.3. *Consider two contest designers ($m = 2$) with reward $R_1 = R_2 = 1$ and three contestants ($n = 3$). The designers choose Tullock contests with parameter $\tau \in [0, \infty]$. We show that if the contestants may play an asymmetric equilibrium for their participation,* $(\mathrm{APA}, \mathrm{APA})$ *may not be an equilibrium for the designers. We compare designer's 1 utility of choosing either $\tau = \mathrm{APA}$ or $\tau = 1.5$ in response to designer 2 choosing* APA.

If designer 1 chooses APA *and the contestants play the symmetric equilibria of $p_1 = p_2 = \frac{1}{2}$, the designers' expected utilities are the same. Moreover, notice that under any realization of the participation probabilities, there is some contest with at least 2 contestants and some contest with at most 1 contestant. By the full rent dissipation property of* APA*, we conclude that under any realization the sum of utilities for the designers is 1, and so each has an expected utility of $\frac{1}{2}$.*

If designer 1 chooses $\tau = 1.5$, the contestants may play an asymmetric equilibrium where contestants 1,2 choose contest 1 and contestant 3 chooses contest 2. This is an equilibrium because contestant 3 gets the full reward with no effort, and for contestant 1 (w.l.o.g., the same argument applies to contestant 2), given the other contestants' choices and that changing her choice to contest 2 leads to an expected utility of zero, the minimal possible, it is a best response to stay in contest 1.

Under this asymmetric participation equilibrium, the utility for contest 1's designer is 0.75 *(according to Lemma 2.12, $u_1 = R_1 - k\gamma_{\tau=1.5}(k) = 1 - 2 \cdot 0.125 = 0.75$), higher than its utility by setting* APA*. We thus conclude that there is a better response than* APA *for contest designer 1 to the* APA *set by contest designer 2, and therefore* $(\mathrm{APA}, \mathrm{APA})$ *is not an equilibrium. This stands in contrast with the results of Theorem 3.1 which assumes symmetric equilibrium.*

D.4 Risk averse contestants

Consider an example CCG with $m = 2, n = 2, R_1 = R_2 = 1$ and $\mathcal{S}_i$ for both contest designers is the set of Tullock contests with $\tau \in [0, 2]$. With risk-neutral contestants we showed that $(\tau = 2, \tau = 2)$ is the unique contestant-symmetric subgame-perfect equilibrium (Corollary 3.2 and Corollary 3.5). We show via an example that with risk-averse contestants $(\tau = 2, \tau = 2)$ is no longer a contestant-symmetric subgame-perfect equilibrium.

Definition D.4. *A risk-averse contestant in a contest has a twice differentiable, strictly increasing in* $[0,1]$*, concave utility function* $a : \mathcal{R} \to \mathcal{R}$*, with* $a(0) = 0, a(1) = 1$.

In our example we use the utility function $a(x) = 1 - (1-x)^4$. Schweinzer and Segev (2012) show that, in a single Tullock contest with parameter $\tau \in [0,2]$, there exists a pure strategy symmetric Nash equilibrium (see also the proof of Lemma 2.12). Since it is pure, the efforts exerted in this equilibrium are the same whether contestants are risk-neutral or risk-averse. A contestant's utility in the competition game is thus (following Eq. (3)):

$$\beta_{\text{averse}}(C_i, p_i) = \mathbb{E}_{k \sim \text{Bin}(n-1,p_i)}\left[a(\gamma_{C_i}(k+1))\right] = \sum_{k=0}^{n-1} \binom{n-1}{k} p_i^k (1-p_i)^{n-1-k} a(\gamma_{C_i}(k+1)). \tag{24}$$

In particular, for $\tau = 2$,

$$\beta_{\text{averse}}(\tau = 2, p_i) = \mathbb{E}_{k \sim \text{Bin}(n-1,p_i)}\left[a(\gamma_{\tau=2}(k+1))\right] = (1-p_i)^{n-1} a(\gamma_{\tau=2}(1)) = (1-p_i)^{n-1}.$$

Claim 2.6 (equilibrium condition) continues to hold, and the definition of designer utilities remains the same as they remain risk-neutral (Eq. (5)). We show that with risk-averse contestants $\tau = 1$ is a better response than $\tau = 2$ to $\tau = 2$. By symmetry, if both designers set $\tau = 2$, we have $p_1 = p_2 = \frac{1}{2}$ and designer 1's utility is $p_1^2 \cdot (1 - 2\gamma_{\tau=2}(2)) = \frac{1}{4}$ (the formula for $\gamma_\tau(k)$ is given in Lemma 2.12). If designer 1 sets $\tau = 1$, by the equilibrium condition for risk-averse participants, we have

$$(1-p_1)a(1) + p_1 a(\gamma_{\tau=1}(2)) = \beta_{\text{averse}}(\tau = 1, p_1) = \beta_{\text{averse}}(\tau = 2, p_2) = 1 - p_2 = p_1,$$

where $\gamma_{\tau=1}(2) = \frac{1}{4}$, which yields $p_1 = \frac{256}{337}$, and the utility for designer 1 is $p_1^2(1 - 2\gamma_{\tau=1}(2)) \approx 0.288529$. This establishes that $(\tau = 2, \tau = 2)$ is not an equilibrium. It can be verified that a symmetric equilibrium exists for this setting with $\tau = \frac{2}{3}$.

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