① Find the derivation of the following formula:

$f(z) = \log_e(1+z)$, where $z = x^T x$, $x \in \mathbb{R}^d$

Soln:

Given, $f(z) = \log_e(1+z)$

By chain rule:

$$\frac{d}{dx} f(z) = \frac{d f(z)}{dz} \cdot \frac{dz}{dx}$$

$$\Rightarrow \frac{d f(z)}{dx} = \frac{d \log_e(1+z)}{dz} \cdot \frac{d}{dx}(x^T x)$$

$$= \frac{1}{1+z} \cdot 2 \cdot x^T$$

$$\frac{dz}{dx} = \frac{d\, x^T x}{dx}$$

$$= 2x^T$$

by Metrics differative rule

$$\therefore \frac{d f(z)}{dx} = \frac{2x^T}{1 + x^T x}$$