Applications of Information Theory (5LSF0) 2023

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May 16, 2023

Assignment #3
Module 4: Convolutional & Turbo Codes

1. We transmit an information-word $(x(1), x(2), x(3), x(4), x(5))$ over an inter-symbol-interference (ISI) channel. This information-word is preceded and followed by zeroes, hence

$$\begin{aligned} x(t) &= 0 \text{ for integer } t \notin \{1,2,3,4,5\} \\ x(t) &\in \{-1,1\} \text{ for } t \in \{1,2,3,4,5\}. \end{aligned}$$

All 32 information-words are equally likely. For the ISI channel for integer times t we have that

$$y(t) = x(t) + x(t-1) + n(t)$$

where the probability density function of the noise $n(t)$ is given by

$$p(n) = \frac{1}{\sqrt{2\pi}} \exp(-\frac{n^2}{2}),$$

thus $n(t)$ is Gaussian. Received sequence: $y(1) = +0.3$, $y(2) = +0.2$, $y(3) = +0.1$, $y(4) = -1.1$, $y(5) = +2.5$ and $y(6) = +0.5$. Decode the information-word with a decoder that minimizes the word-error probability. Show first that the decoder minimizes (squared) Euclidean distance.

2. Provide a MATLAB implementation of the log-domain BCJR algorithm that computes the a-posteriori bit probabilities for the non-systematic convolutional encoder in slide 17 (left-hand side) of lecture 3.2, and codeword-transmission over a binary symmetric channel (BSC) with cross-over probability 0.1. Assuming transmission is performed in this channel, and the received channel-output sequence is $\underline{y} = \big(y_1, y_2, \cdots, y_{2(T+2)}\big) = (10, 11, 00, 11, 10, 11, 11, 00)$, compute the a-posteriori bit probabilities

$$\Pr\{U_t = 1 | \underline{y} = (10, 11, \cdots, 00)\}, \text{ for } t = 1, 2, \cdots, 6. \tag{1}$$

Compute the information bit estimates that minimize the bit error probability. Make your code flexible, i.e., based on a finite-state description of the convolutional code.

Note: Both question 1 and 2 involve Matlab. The document should contain 1) a report no longer than 4 pages consisting of explanation and simulation results, and 2) the corresponding executable Matlab `.m` file.

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