1. Part 1. Do any two questions. In each question, include all definitions other that "graph" necessary to make the statement meaningful.

    (a) State and prove Menger's Theorem.

    (b) State and prove Turan's Theorem.

    (c) State and prove Vizing's Theorem.

2. Part 2. Do any two questions.

    (a) Prove that a connected graph is Eulerian if and only if every minimal edge cut contains an even number of edges.

    (b) Describe Kruskal's Algorithm and prove that it constructs a minimum weight spanning tree of a connected, weighted graph (G, w).

    (c) Prove that every k-connected graph with independence number at most $k - 1$ has a Hamilton path.

3. Part 3. Do any three questions.

    (a) Prove that every cubic graph without cut edges has a 2-factor. Give an example to show that the statement is not true for graphs with cut edges.

    (b) Prove that every tree has at most one perfect matching and, further, that a tree T has a perfect matching if and only if $T - v$ has exactly one odd component for every $v \in V$.

    (c) Without using the four colour rheorem, prove that every planar graph is five-colourable.

    (d) A *King* in a tournament is a vertex x so that, for every $y \in V - \{x\}$ there is a directed path of length two from x to y. Prove that every tournament has a King.