Combinatorics Ph.D. Candidacy Exam Syllabus

1 Enumeration

- Basic counting: permutations and combinations, arrangements and selections with repetition

- Identities involving binomial coefficients

- The Pigeonhole Principle

- Advanced counting numbers, including multinomial coefficients, Stirling numbers of the first and second kind, Bell numbers, and Catalan numbers

- The Principle of Inclusion and Exclusion

- Arrangements with forbidden positions and Rook polynomials

- Polya's Enumeration Theorem

2 Generating Functions

- Combinatorics of the ordinary power series generating function and the exponential generating function

- Derivation of generating functions for counting numbers

- Partitions of integers

3 Recurrence Relations

- Recurrence relations as models

- Fibonacci numbers

- Divide and conquer relations

- Solutions to linear homogeneous recurrence relations with constant coefficients

- Solutions to non-homogeneous recurrence relations

- Solutions to recurrence relations via generating functions

4 Finite Ramsey Theory

- Ramsey's theorem

- Calculation of Bounds for Ramsey numbers

5 Combinatorial Designs (Balanced Incomplete Block Designs)

- Kirkman's problem, Fisher's inequality, Steiner Triple systems, symmetric design, the Bruck-Ryser-Chowla Theorem, resolvable designs, derived and residual designs, construction methods, Wilson's theorem, t-designs, Teirlinck's theorem

6 Partially Ordered Sets

- Chains, antichains, Dilworth's theorem

- Mobius inversion

- Linear extensions

- Lattices

- The structure of Boolean Algebras (see Grimaldi)

7 Generation of Combinatorial Objects

- Permutations

- Subsets of a set

- Gray Codes

8 References

1. D.I.A. Cohen, Basic Techniques of Combinatorial Theory, Wiley, New York, 1978.

2. R.P. Grimaldi, Discrete and Combinatorial Mathematics, An Applied Introduction, Addison-Wesely, New York, 1989.

3. M. Hall, Combinatorial Theory, Blaisdell, London, 1967.

4. E.M. Reingold, J. Neivergelt and N. Deo, Combinatorial Algorithms: Theory and Practice, Prentice-Hall, Englewood Cliffs, N.J., 1977.