A6

2.1 Basecase: Since empty tree doesn't have node. the base case is when m-array has one node.

~~P~~ We need to prove that $I(1)=0$ $L(1)=1$ Since node is 1, ~~it's~~ it does not have children, it's a leaf. Therefore it's not a internal node,

$I(1)=(n-1)/m = (1-1)/m = 0$

$L(1) = [(m-1)n+1]/m = [m-1+1]/m = 1$

Induction case: the tree is using ~~two~~ m subnode to present N, $N = n_1+n_2 \cdots + n_m + 1$

By Induction. $I(n_1) = n_1 - 1/m$. $L(n_1) = [(m-1)n_1+1]/m$.

$I(n_m) = n_m - 1/m$ $L(n_m) = [(m-1)n_m+1]/m$.

$I(n)$ will be the combination of subinternal nodes

$L(n)$ will be the combination of leaves

We need to prove $L(n) = [(m-1)n+1]/m$.

$I(n) = (n-1)/m$.

<1> prove $I(n) = \frac{n-1}{m}$

$$I(n) = I(n_1) \cdots + I(n_m) + 1$$

$$= \frac{n_1-1}{m} + \frac{n_2-1}{m} \cdots \frac{n_m-1}{m} + 1$$

$$= \frac{n_1+n_2 \cdots + n_m - m}{m} + 1$$

$$= \frac{n_1+n_2 \cdots + n_m}{m} = \frac{n-1}{m}$$

∴ It's proven.

<2> prove $L(n) = n - I(n)$

$$= n - \frac{n-1}{m} = \frac{mn-n+1}{m} = \frac{(m-1)n+1}{m}$$

$\square$

2.2. Base case: when internal node is 0. In that case, it only when the first node appears therefore, node = 1, since a node will be either node or leaf, so it's leaf. Leaf = 1.

Therefore $L(0) = (m-1) \times 0 + 1 = 1$

$N(0) = m \times 0 + 1 = 1$

It's proven.

Induction case: When internal node is i, In that case, assume subtree's internal node is $i_1, \cdots i_m$. We have $i = i_1 + \cdots i_m + 1$

By induction. $L(i_1) = (m-1)i_1 + 1$ $\quad N(i_1) = m i_1 + 1$

$\vdots$ $\quad\quad \vdots$

$L(i_m) = (m-1)i_m + 1$ $\quad N(i_m) = m i_m + 1$

We need to prove $L(i) = (m-1)i + 1$

$N(i) = mi + 1$

Since $L(i) = L(i_1) \cdots + L(i_m)$

$= (m-1)i_1 + 1 \cdots + (m-1)i_m + 1$

$= (m-1)(i_1 + \cdots i_m) + 1 \times m$

$= (m-1)(i-1) + m$ since $(i_1 + \cdots i_m = i-1)$

$= (m-1)(i-1) + m - 1 + 1$

$\therefore$ $= (m-1)i + 1$

Since $N(i) = L(i) + i$

$N(i) = (m-1)i + 1 + i = mi + 1$

$\square$

2.3

Base Case: Since it's impossible to have 0 leaves, $l=1$ will be the base case.

When $l=1$, it will be also a node, however, since $i=n-l$, $i=0$.

So we suppose to have $i=0$ $n=1$

Therefore $N(1)=(m\times 1-1)/(m-1)=1$

$I(1)=(1-1)/m-1=0$.

Induction case: Since the leaves number is the combination of all subtree's leave, we have

$l=l_1+\cdots l_m$.

By induction $N(l_1)=(ml_1-1)/(m-1)$ $\quad I(l_1)=(l_1-1)/(m-1)$

$\vdots$ $\quad\vdots$

$N(l_m)=(ml_m-1)/(m-1)$ $\quad I(l_m)=(l_m-1)/(m-1)$

<1> prove $N(l)=(ml-1)/(m-1)$

$$N(l)=N(l_1)+\cdots N(l_m)+1$$

$$(ml_1-1)\cdots \quad ml_m-1)/(m-1)+1.$$

$$=[m(l_1+\cdots l_m)-m]/(m-1)+1$$

$$=m(l-1)/(m-1)+1$$

$$=\frac{ml-m+m-1}{m-1}$$

$$=\frac{ml-1}{m-1}$$

<2> Prove $I(l)=(l-1)/(m-1)$

$$I(l)=N(l)-l$$

$$=\frac{ml-1}{m-1}-l=\frac{ml-1-ml+l}{m-1}=\frac{l-1}{m-1}$$

$\square$.

2.4 Since the Number of leaves are 100, and $m=4$, the number of Nodes is.

$(4\times 100-1)/(4-1)=133$, the number of internal nodes will be $133-100=33$

Therefore there are 133 people see the letter, and 33 send the letter.