Deep RL Foundations in 6 Lectures

Lecture 3: Policy Gradient and Advantage Estimation

Pieter Abbeel

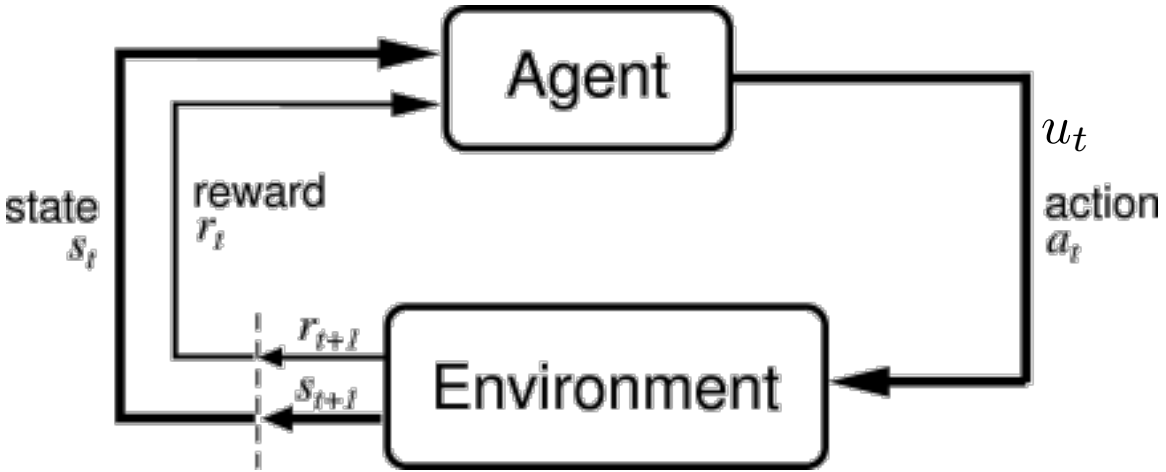
Lecture Series

- Lecture 1: MDPs Foundations and Exact Solution Methods
- Lecture 2: Deep Q-Learning
- ***Lecture 3: Policy Gradients, Advantage Estimation***
- Lecture 4: TRPO, PPO
- Lecture 5: DDPG, SAC
- Lecture 6: Model-based RL

Outline for This Lecture

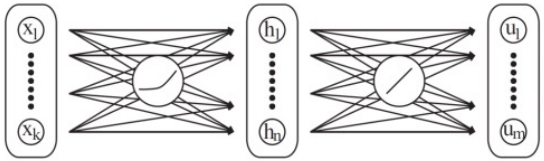
- Policy Gradient derivation
- Temporal decomposition
- Baseline subtraction
- Value function estimation
- Advantage Estimation (A2C/A3C/GAE)

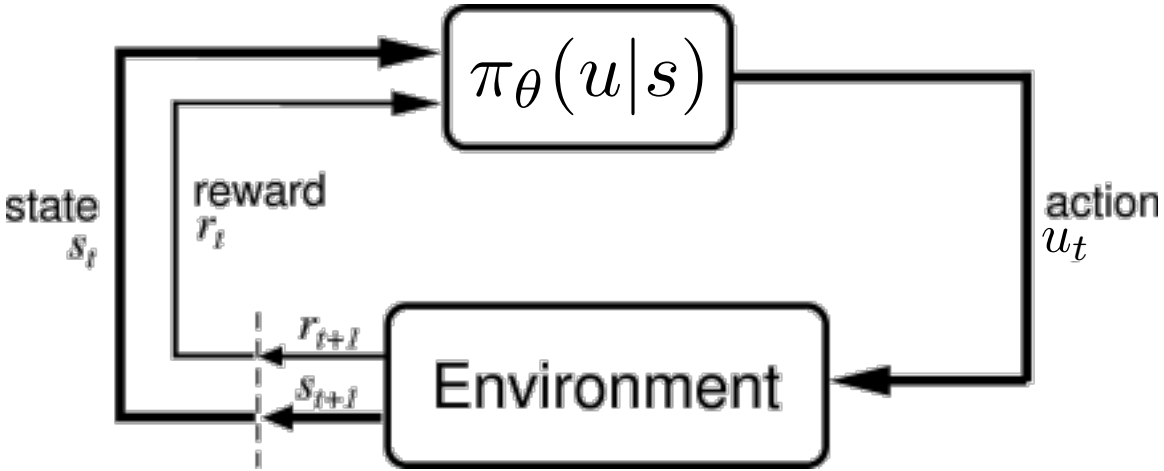
Reinforcement Learning



[Figure source: Sutton & Barto, 1998]

Policy Optimization





[Figure source: Sutton & Barto, 1998]

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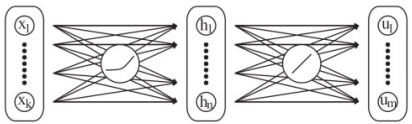

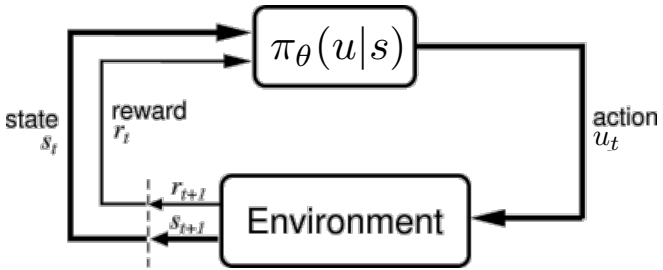

- Consider control policy parameterized by parameter vector θ

$$\max_\theta \quad \mathrm{E}[\sum_{t=0}^{H} R(s_t)|\pi_\theta]$$

- Stochastic policy class (smooths out the problem):

$\pi_\theta(u|s)$: probability of action u in state s

[Figure source: Sutton & Barto, 1998]

Why Policy Optimization

- Often π can be simpler than Q or V
  - E.g., robotic grasp
- V: doesn't prescribe actions
  - Would need dynamics model (+ compute 1 Bellman back-up)
- Q: need to be able to efficiently solve $\arg\max_u Q_\theta(s, u)$
  - Challenge for continuous / high-dimensional action spaces*

*some recent work (partially) addressing this:

NAF: Gu, Lillicrap, Sutskever, Levine ICML 2016

Input Convex NNs: Amos, Xu, Kolter arXiv 2016

Deep Energy Q: Haarnoja, Tang, Abbeel, Levine, ICML 2017

Likelihood Ratio Policy Gradient

We let τ denote a state-action sequence $s_0, u_0, \ldots, s_H, u_H$. We overload notation: $R(\tau) = \sum_{t=0}^{H} R(s_t, u_t)$.

$$U(\theta) = \mathrm{E}[\sum_{t=0}^{H} R(s_t, u_t); \pi_\theta] = \sum_{\tau} P(\tau; \theta) R(\tau)$$

In our new notation, our goal is to find θ:

$$\max_\theta U(\theta) = \max_\theta \sum_{\tau} P(\tau; \theta) R(\tau)$$

Likelihood Ratio Policy Gradient

$$U(\theta) = \sum_{\tau} P(\tau;\theta) R(\tau)$$

Taking the gradient w.r.t. θ gives

$$\nabla_\theta U(\theta) = \nabla_\theta \sum_{\tau} P(\tau;\theta) R(\tau)$$

[Aleksandrov, Sysoyev, & Shemeneva, 1968]
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$$\begin{aligned} \nabla_\theta U(\theta) &= \nabla_\theta \sum_{\tau} P(\tau;\theta) R(\tau) \\ &= \sum_{\tau} \nabla_\theta P(\tau;\theta) R(\tau) \end{aligned}$$

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\end{aligned}$$

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&= \sum_{\tau} P(\tau;\theta) \nabla_\theta \log P(\tau;\theta) R(\tau)
\end{aligned}$$

Approximate with the empirical estimate for m sample paths under policy π_θ:

$$\nabla_\theta U(\theta) \approx \hat{g} = \frac{1}{m} \sum_{i=1}^{m} \nabla_\theta \log P(\tau^{(i)};\theta) R(\tau^{(i)})$$

[Aleksandrov, Sysoyev, & Shemeneva, 1968]
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Likelihood Ratio Gradient: Validity

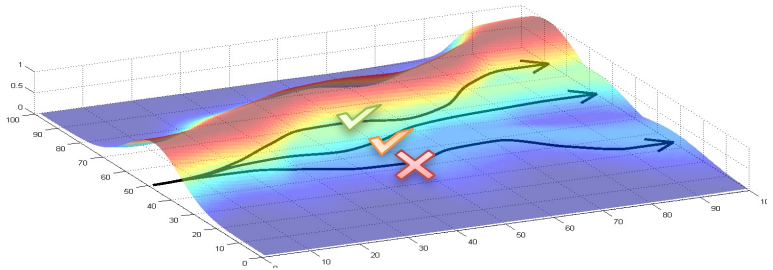
$$\nabla U(\theta) \approx \hat{g} = \frac{1}{m} \sum_{i=1}^{m} \nabla_\theta \log P(\tau^{(i)}; \theta) R(\tau^{(i)})$$

- Valid even when
  - R is discontinuous and/or unknown
  - Sample space (of paths) is a discrete set

Likelihood Ratio Gradient: Intuition

$$\nabla U(\theta) \approx \hat{g} = \frac{1}{m} \sum_{i=1}^{m} \nabla_\theta \log P(\tau^{(i)}; \theta) R(\tau^{(i)})$$

- Gradient tries to:
  - Increase probability of paths with positive R
  - Decrease probability of paths with negative R



! Likelihood ratio changes probabilities of experienced paths, does not try to change the paths (<-> Path Derivative)

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- ***Temporal decomposition***
- Baseline subtraction
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- Advantage Estimation (A2C/A3C/GAE)

Let's Decompose Path into States and Actions

$$\nabla_\theta \log P(\tau^{(i)};\theta) = \nabla_\theta \log \left[\prod_{t=0}^{H} \underbrace{P(s_{t+1}^{(i)}|s_t^{(i)},u_t^{(i)})}_{\text{dynamics model}} \cdot \underbrace{\pi_\theta(u_t^{(i)}|s_t^{(i)})}_{\text{policy}} \right]$$

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$$= \nabla_\theta \left[\sum_{t=0}^{H} \log P(s_{t+1}^{(i)}|s_t^{(i)},u_t^{(i)}) + \sum_{t=0}^{H} \log \pi_\theta(u_t^{(i)}|s_t^{(i)}) \right]$$

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$$= \nabla_\theta \sum_{t=0}^{H} \log \pi_\theta(u_t^{(i)}|s_t^{(i)})$$

$$= \sum_{t=0}^{H} \underbrace{\nabla_\theta \log \pi_\theta(u_t^{(i)}|s_t^{(i)})}_{\text{no dynamics model required!!}}$$

Likelihood Ratio Gradient Estimate

The following expression provides us with an unbiased estimate of the gradient, and we can compute it without access to a dynamics model:

$$\hat{g} = \frac{1}{m} \sum_{i=1}^{m} \nabla_\theta \log P(\tau^{(i)}; \theta) R(\tau^{(i)})$$

Here:

$$\nabla_\theta \log P(\tau^{(i)}; \theta) = \sum_{t=0}^{H} \underbrace{\nabla_\theta \log \pi_\theta(u_t^{(i)} | s_t^{(i)})}_{\text{no dynamics model required!!}}$$

Unbiased means:

$$\mathrm{E}[\hat{g}] = \nabla_\theta U(\theta)$$

Likelihood Ratio Gradient Estimate

- As formulated thus far: unbiased but very noisy
- Fixes that lead to real-world practicality
  - Baseline
  - Temporal structure
  - And next lecture: Trust region / natural gradient

Likelihood Ratio Gradient: Intuition

$$\nabla U(\theta) \approx \hat{g} = \frac{1}{m} \sum_{i=1}^{m} \nabla_\theta \log P(\tau^{(i)}; \theta) R(\tau^{(i)})$$

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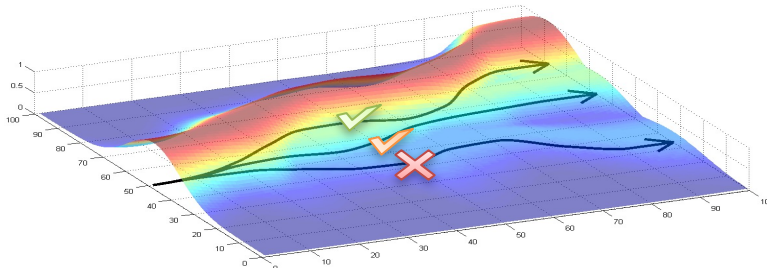

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Likelihood Ratio Gradient: Baseline

$$\nabla U(\theta) \approx \hat{g} = \frac{1}{m} \sum_{i=1}^{m} \nabla_\theta \log P(\tau^{(i)}; \theta) R(\tau^{(i)})$$

→ Consider baseline b: $\nabla U(\theta) \approx \hat{g} = \frac{1}{m} \sum_{i=1}^{m} \nabla_\theta \log P(\tau^{(i)}; \theta)(R(\tau^{(i)}) - b)$

$$\mathbb{E}\left[\nabla_\theta \log P(\tau;\theta) b\right]$$
$$= \sum_\tau P(\tau;\theta) \nabla_\theta \log P(\tau;\theta) b$$
$$= \sum_\tau P(\tau;\theta) \frac{\nabla_\theta P(\tau;\theta)}{P(\tau;\theta)} b$$
$$= \sum_\tau \nabla_\theta P(\tau;\theta) b$$
$$= \nabla_\theta \left(\sum_\tau P(\tau) b \right) \quad = b \nabla_\theta (\sum_\tau P(\tau)) = b \times 0$$

OK as long as baseline doesn't depend on action in logprob(action)

still unbiased!
[Williams 1992]

More Temporal Structure and Baseline

- Current estimate:

$$\hat{g} = \frac{1}{m}\sum_{i=1}^{m} \nabla_\theta \log P(\tau^{(i)};\theta)(R(\tau^{(i)}) - b)$$

$$= \frac{1}{m}\sum_{i=1}^{m} \left(\sum_{t=0}^{H-1} \nabla_\theta \log \pi_\theta(u_t^{(i)}|s_t^{(i)})\right)\left(\sum_{t=0}^{H-1} R(s_t^{(i)}, u_t^{(i)}) - b\right)$$

$$= \frac{1}{m}\sum_{i=1}^{m} \left(\sum_{t=0}^{H-1} \nabla_\theta \log \pi_\theta(u_t^{(i)}|s_t^{(i)})\left[\left(\sum_{k=0}^{t-1} R(s_k^{(i)}, u_k^{(i)})\right) + \left(\sum_{k=t}^{H-1} R(s_k^{(i)}, u_k^{(i)})\right) - b\right]\right)$$

Doesn't depend on $u_t^{(i)}$

Ok to depend on $s_t^{(i)}$

- Removing terms that don't depend on current action can lower variance:

$$\frac{1}{m}\sum_{i=1}^{m}\sum_{t=0}^{H-1} \nabla_\theta \log \pi_\theta(u_t^{(i)}|s_t^{(i)})\left(\sum_{k=t}^{H-1} R(s_k^{(i)}, u_k^{(i)}) - b(s_t^{(i)})\right)$$

[Policy Gradient Theorem: Sutton et al, NIPS 1999; GPOMDP: Bartlett & Baxter, JAIR 2001; Survey: Peters & Schaal, IROS 2006]

Baseline Choices

- Good choice for b?
  - Constant baseline: $b = \mathbb{E}\left[R(\tau)\right] \approx \frac{1}{m}\sum_{i=1}^{m} R(\tau^{(i)})$
  - Optimal Constant baseline: $b = \frac{\sum_i \left(\nabla_\theta \log P(\tau^{(i)};\theta)\right)^2 R(\tau^{(i)})}{\sum_i \left(\nabla_\theta \log P(\tau^{(i)};\theta)\right)^2}$
  - Time-dependent baseline: $b_t = \frac{1}{m}\sum_{i=1}^{m}\sum_{k=t}^{H-1} R(s_k^{(i)}, u_k^{(i)})$
  - State-dependent expected return:

$$b(s_t) = \mathbb{E}\left[r_t + r_{t+1} + r_{t+2} + \ldots + r_{H-1}\right] = V^\pi(s_t)$$

→ Increase logprob of action proportionally to how much its returns are better than the expected return under the current policy

[See: Greensmith, Bartlett, Baxter, JMLR 2004 for variance reduction techniques.]

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Monte Carlo Estimation of V^π

$$\frac{1}{m}\sum_{i=1}^{m}\sum_{t=0}^{H-1}\nabla_\theta \log \pi_\theta(u_t^{(i)}|s_t^{(i)})\left(\sum_{k=t}^{H-1} R(s_k^{(i)},u_k^{(i)}) - V^\pi(s_k^{(i)})\right)$$

How to estimate?

- Init $V^\pi_{\phi_0}$
  - Collect trajectories $\tau_1,\ldots,\tau_m$
  - Regress against empirical return:

$$\phi_{i+1} \leftarrow \arg\min_\phi \frac{1}{m}\sum_{i=1}^{m}\sum_{t=0}^{H-1}\left(V_\phi^\pi(s_t^{(i)}) - \left(\sum_{k=t}^{H-1} R(s_k^{(i)},u_k^{(i)})\right)\right)^2$$

Bootstrap Estimation of V^π

- Bellman Equation for V^π

$$V^\pi(s) = \sum_u \pi(u|s) \sum_{s'} P(s'|s,u)[R(s,u,s') + \gamma V^\pi(s')]$$

- Init $V^\pi_{\phi_0}$
  - Collect data {s, u, s', r}
  - Fitted V iteration:

$$\phi_{i+1} \leftarrow \min_\phi \sum_{(s,u,s',r)} \|r + V^\pi_{\phi_i}(s') - V_\phi(s)\|_2^2 + \lambda\|\phi - \phi_i\|_2^2$$

Vanilla Policy Gradient

Algorithm 1 "Vanilla" policy gradient algorithm

Initialize policy parameter θ, baseline b

for iteration=$1, 2, \ldots$ **do**

Collect a set of trajectories by executing the current policy

At each timestep in each trajectory, compute

the *return* $R_t = \sum_{t'=t}^{T-1} \gamma^{t'-t} r_{t'}$, and

the *advantage estimate* $\hat{A}_t = R_t - b(s_t)$.

Re-fit the baseline, by minimizing $\|b(s_t) - R_t\|^2$,

summed over all trajectories and timesteps.

Update the policy, using a policy gradient estimate $\hat{g}$,

which is a sum of terms $\nabla_\theta \log \pi(a_t \mid s_t, \theta) \hat{A}_t$

end for

~ [Williams, 1992]

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- ***Advantage Estimation (A2C/A3C/GAE)***

Recall Our Likelihood Ratio PG Estimator

$$\frac{1}{m}\sum_{i=1}^{m}\sum_{t=0}^{H-1}\nabla_\theta \log \pi_\theta(u_t^{(i)}|s_t^{(i)})\left(\sum_{k=t}^{H-1} R(s_k^{(i)},u_k^{(i)}) - V^\pi(s_k^{(i)})\right)$$

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- Estimation of Q from *single* roll-out

$$Q^\pi(s,u) = \mathbb{E}[r_0 + r_1 + r_2 + \cdots | s_0 = s, a_0 = a]$$

Recall Our Likelihood Ratio PG Estimator

$$\frac{1}{m}\sum_{i=1}^{m}\sum_{t=0}^{H-1}\nabla_\theta \log \pi_\theta(u_t^{(i)}|s_t^{(i)})\left(\sum_{k=t}^{H-1} R(s_k^{(i)},u_k^{(i)}) - V^\pi(s_k^{(i)})\right)$$

- Estimation of Q from *single* roll-out

$$Q^\pi(s,u) = \mathbb{E}[r_0 + r_1 + r_2 + \cdots | s_0 = s, a_0 = a]$$

- = high variance per sample based / no generalization

Further Refinements

$$\frac{1}{m}\sum_{i=1}^{m}\sum_{t=0}^{H-1}\nabla_\theta \log \pi_\theta(u_t^{(i)}|s_t^{(i)})\left(\sum_{k=t}^{H-1} R(s_k^{(i)},u_k^{(i)}) - V^\pi(s_k^{(i)})\right)$$

- Estimation of Q from *single* roll-out

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= high variance per sample based / no generalization

  - Reduce variance by discounting

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- Estimation of Q from *single* roll-out

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= high variance per sample based / no generalization

- Reduce variance by discounting
- Reduce variance by function approximation (=critic)

Variance Reduction by Discounting

$$Q^{\pi}(s,u) = \mathbb{E}[r_0 + r_1 + r_2 + \cdots | s_0 = s, a_0 = a]$$

→ introduce discount factor as a hyperparameter to improve estimate of Q:

$$Q^{\pi,\gamma}(s,u) = \mathbb{E}[r_0 + \gamma r_1 + \gamma^2 r_2 + \cdots | s_0 = s, a_0 = a]$$

Variance Reduction by Function Approximation

$$Q^{\pi,\gamma}(s,u) = \mathbb{E}[r_0 + \gamma r_1 + \gamma^2 r_2 + \cdots \mid s_0 = s, u_0 = u]$$

Variance Reduction by Function Approximation

$$
\begin{aligned}
Q^{\pi,\gamma}(s,u) &= \mathbb{E}[r_0 + \gamma r_1 + \gamma^2 r_2 + \cdots \mid s_0 = s, u_0 = u] \\
&= \mathbb{E}[r_0 + \gamma V^{\pi}(s_1) \mid s_0 = s, u_0 = u]
\end{aligned}
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Variance Reduction by Function Approximation

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Variance Reduction by Function Approximation

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&= \mathbb{E}[r_0 + \gamma r_1 + \gamma^2 V^{\pi}(s_2) \mid s_0 = s, u_0 = u] \\
&= \mathbb{E}[r_0 + \gamma r_1 + +\gamma^2 r_2 + \gamma^3 V^{\pi}(s_3) \mid s_0 = s, u_0 = u] \\
&= \cdots
\end{aligned}
$$

- ***Async Advantage Actor Critic (A3C)*** *[Mnih et al, 2016]*
  - $\hat{Q}$ one of the above choices (e.g. k=5 step lookahead)

Variance Reduction by Function Approximation

$$
\begin{aligned}
Q^{\pi,\gamma}(s,u) &= \mathbb{E}[r_0 + \gamma r_1 + \gamma^2 r_2 + \cdots \mid s_0 = s, u_0 = u] && (1-\lambda) \\
&= \mathbb{E}[r_0 + \gamma V^{\pi}(s_1) \mid s_0 = s, u_0 = u] && (1-\lambda)\lambda \\
&= \mathbb{E}[r_0 + \gamma r_1 + \gamma^2 V^{\pi}(s_2) \mid s_0 = s, u_0 = u] && (1-\lambda)\lambda^2 \\
&= \mathbb{E}[r_0 + \gamma r_1 + +\gamma^2 r_2 + \gamma^3 V^{\pi}(s_3) \mid s_0 = s, u_0 = u] \\
&= \cdots && (1-\lambda)\lambda^3
\end{aligned}
$$

- ***Generalized Advantage Estimation (GAE)*** *[Schulman et al, ICLR 2016]*
  - $\hat{Q}$ = lambda exponentially weighted average of all the above

Variance Reduction by Function Approximation

$$
\begin{aligned}
Q^{\pi,\gamma}(s,u) &= \mathbb{E}[r_0 + \gamma r_1 + \gamma^2 r_2 + \cdots \mid s_0 = s, u_0 = u] && (1-\lambda) \\
&= \mathbb{E}[r_0 + \gamma V^{\pi}(s_1) \mid s_0 = s, u_0 = u] && (1-\lambda)\lambda \\
&= \mathbb{E}[r_0 + \gamma r_1 + \gamma^2 V^{\pi}(s_2) \mid s_0 = s, u_0 = u] && (1-\lambda)\lambda^2 \\
&= \mathbb{E}[r_0 + \gamma r_1 + +\gamma^2 r_2 + \gamma^3 V^{\pi}(s_3) \mid s_0 = s, u_0 = u] \\
&= \cdots && (1-\lambda)\lambda^3
\end{aligned}
$$

- ***Generalized Advantage Estimation (GAE)*** *[Schulman et al, ICLR 2016]*
  - $\hat{Q}$ = lambda exponentially weighted average of all the above
- ~ TD(lambda) / eligibility traces [Sutton and Barto, 1990]

Policy Gradient with A3C or GAE

- Policy Gradient + Generalized Advantage Estimation:
  - Init π_{θ_0} $V^{\pi}_{\phi_0}$
  - Collect roll-outs {s, u, s', r} and $\hat{Q}_i(s,u)$
  - Update:

$$\phi_{i+1} \leftarrow \min_{\phi} \sum_{(s,u,s',r)} \|\hat{Q}_i(s,u) - V^{\pi}_{\phi}(s)\|_2^2 + \kappa\|\phi - \phi_i\|_2^2$$

$$\theta_{i+1} \leftarrow \theta_i + \alpha \frac{1}{m} \sum_{k=1}^{m} \sum_{t=0}^{H-1} \nabla_\theta \log \pi_{\theta_i}(u_t^{(k)}|s_t^{(k)}) \left(\hat{Q}_i(s_t^{(k)}, u_t^{(k)}) - V^{\pi}_{\phi_i}(s_t^{(k)})\right)$$

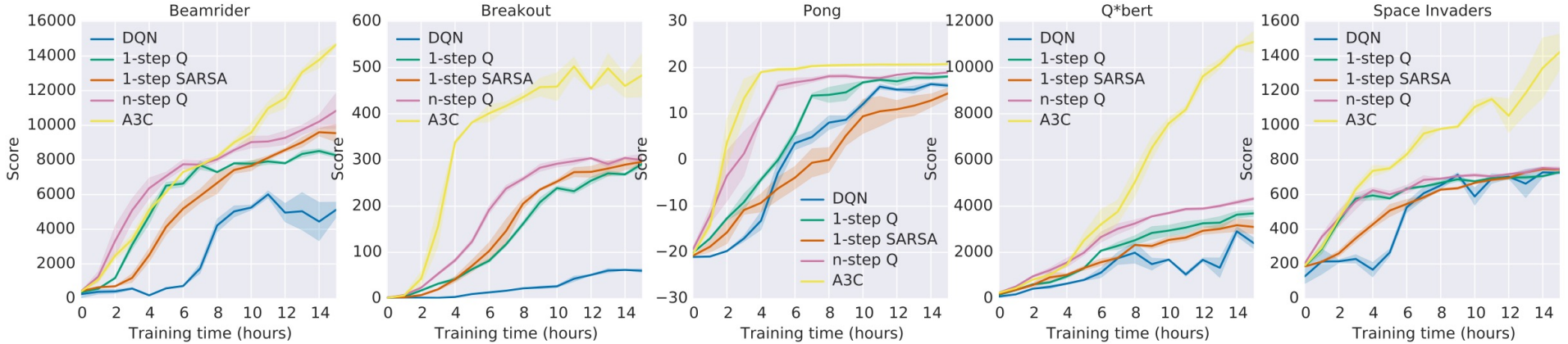
Note: many variations, e.g. could instead use 1-step for V, full roll-out for pi:

$$\phi_{i+1} \leftarrow \min_{\phi} \sum_{(s,u,s',r)} \|r + V^{\pi}_{\phi_i}(s') - V_{\phi}(s)\|_2^2 + \lambda\|\phi - \phi_i\|_2^2$$

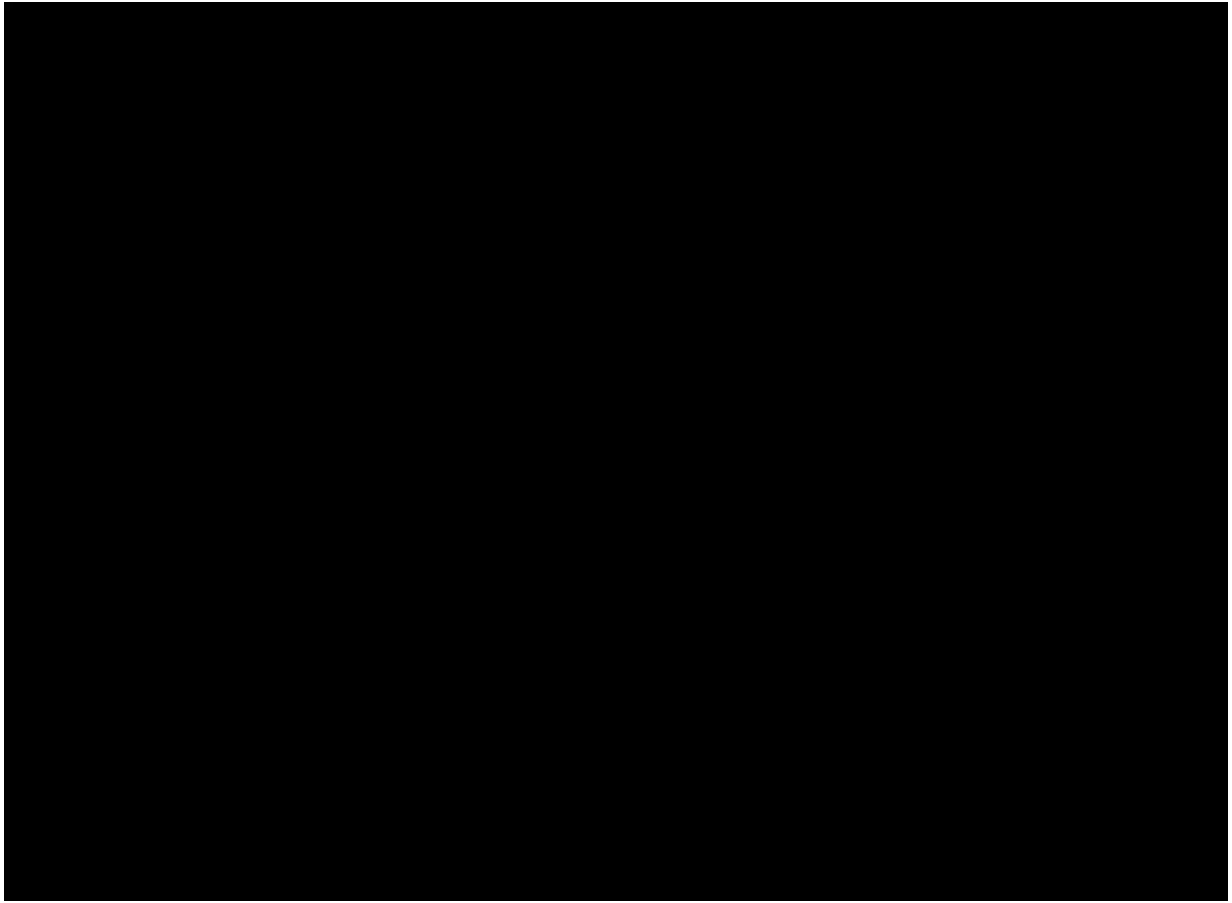
$$\theta_{i+1} \leftarrow \theta_i + \alpha \frac{1}{m} \sum_{k=1}^{m} \sum_{t=0}^{H-1} \nabla_\theta \log \pi_{\theta_i}(u_t^{(k)}|s_t^{(k)}) \left(\sum_{t'=t}^{H-1} r_{t'}^{(k)} - V^{\pi}_{\phi_i}(s_{t'}^{(k)})\right)$$

Async Advantage Actor Critic (A3C)

- [Mnih et al, ICML 2016]
  - Likelihood Ratio Policy Gradient
  - n-step Advantage Estimation



A3C -- labyrinth

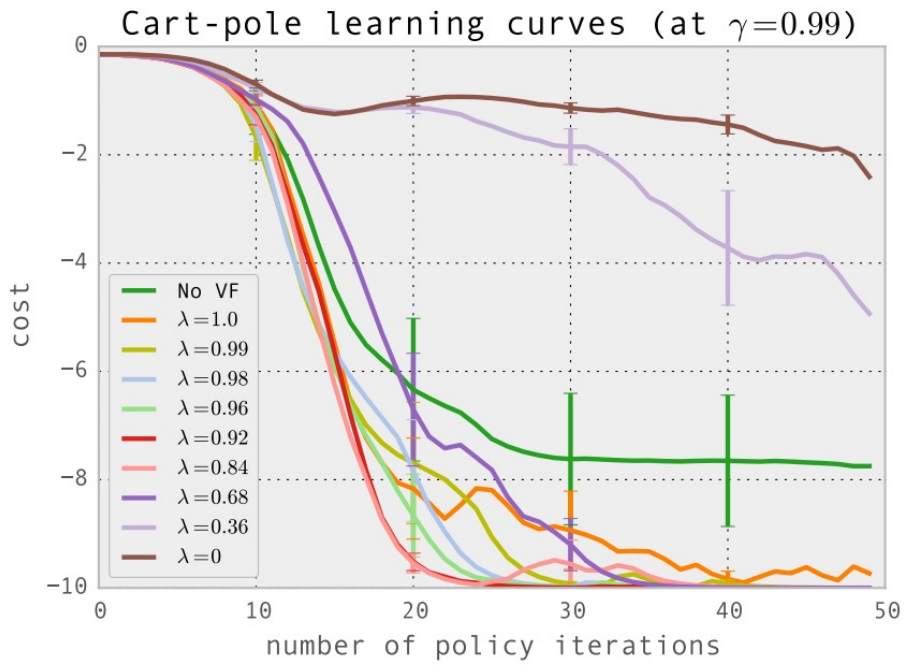


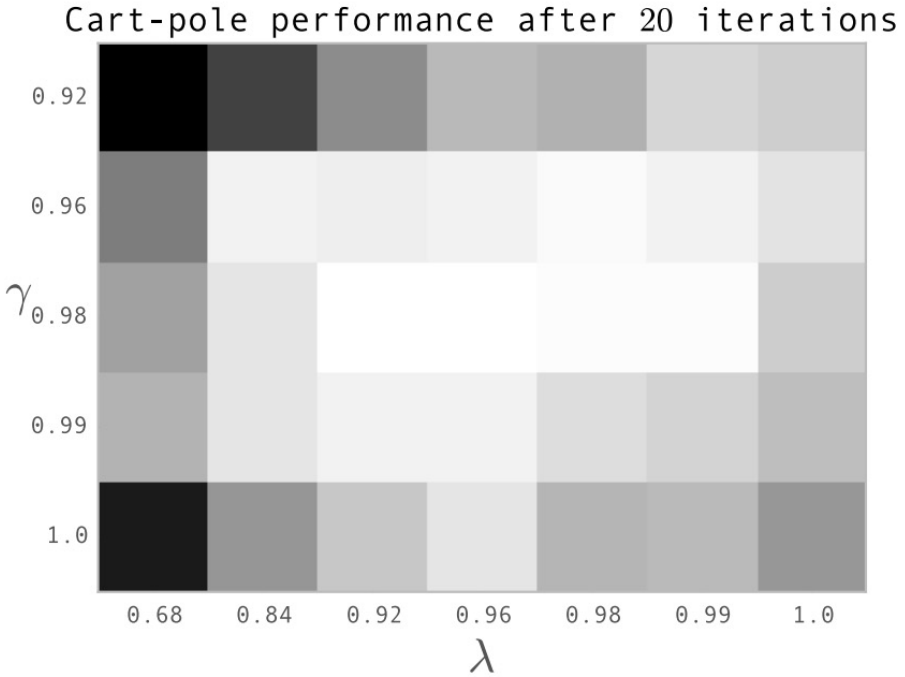
Example: Toddler Robot

[Tedrake, Zhang and Seung, 2005]

[Video: TODDLER – 40s]

GAE: Effect of gamma and lambda





[Schulman et al, 2016 -- GAE]

Summary of This Lecture

- Policy Gradient derivation
- Temporal decomposition
- Baseline subtraction
- Value function estimation
- Advantage Estimation (A2C/A3C/GAE)