Statistics 100B   **Instructor: Nicolas Christou**

Probability Distributions - Summary

Discrete Distributions				
Distribution	Probability Mass Function	Mean	Variance	Moment-generating Function
Binomial	$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$ $x = 0, 1, \cdots, n$	np	$np(1-p)$	$[pe^t + (1-p)]^n$
Geometric	$P(X = x) = (1-p)^{x-1} p$ $x = 1, 2, \cdots$	$\frac{1}{p}$	$\frac{1-p}{p^2}$	$\frac{pe^t}{1-(1-p)e^t}$
Negative Binomial	$P(X = x) = \binom{x-1}{r-1} p^r (1-p)^{x-r}$ $x = r, r+1, \cdots$	$\frac{r}{p}$	$\frac{r(1-p)}{p^2}$	$\left[\frac{pe^t}{1-(1-p)e^t}\right]^r$
Hypergeometric	$P(X = x) = \frac{\binom{r}{x}\binom{N-r}{n-x}}{\binom{N}{n}}$ $x = 0, 1, \cdots, n$ if $n \leq r$, $x = 0, 1, \cdots, r$ if $n > r$	$\frac{nr}{N}$	$n\frac{r}{N}\frac{N-r}{N}\frac{N-n}{N-1}$	Fairly complicated!
Poisson	$P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!}$ $x = 0, 1, \cdots$	λ	λ	$exp[\lambda(e^t - 1)]$
Continuous Distributions				
Distribution	Probability Density Function	Mean	Variance	Moment-generating Function
Uniform	$f(x) = \frac{1}{b-a}$ $a \leq x \leq b$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$	$\frac{e^{tb}-e^{ta}}{t(b-a)}$
Gamma	$f(x) = \frac{x^{\alpha-1}e^{-\frac{x}{\beta}}}{\beta^\alpha \Gamma(\alpha)}, \ \alpha, \beta > 0, \ x \geq 0$	$\alpha\beta$	$\alpha\beta^2$	$(1 - \beta t)^{-\alpha}$
Exponential	$f(x) = \lambda e^{-\lambda x}, \ \lambda > 0, \ x \geq 0$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$	$(1 - \frac{1}{\lambda}t)^{-1}$
Beta	$f(x) = \frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha,\beta)}$ $\alpha > 0, \ \beta > 0, \ 0 \leq x \leq 1$	$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$	
Normal	$f(x) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2}$ $-\infty < x < +\infty$	μ	σ^2	$e^{\mu t + \frac{t^2\sigma^2}{2}}$

Remarks:

Binomial:	X represents the number of successes among n trials.
Geometric:	X represents the number of trials needed until the first success.
Negative Binomial:	X represents the number of trials needed until r successes occur.
Hypergeometric:	X represents the number of items among the n selected that comes from the r group.
Poisson:	X represents the number of events that occur in time, area, etc.