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2014 Mathematical Contest in Modeling (MCM) Summary Sheet

Summary

To determine the role or influence of academic research, we need to build a network depending on citation or co-author relationship, analyze the properties of this network, and develop influence measure to determine the relative influence of authors or papers within the network. Similarly, we can use the similar way to analyze other social networks to help us make some decisions or explain some phenomena.

In Task I, we analyze the most famous co-author network, the Erdos 1 co-author network. First, we use a adjacency matrix to represent the co-author relationship between these 511 coauthors of Erdos and build the co-author network. Then, we choose ***density***, ***centrality*** *(degree, betweenness, eigenvector, closeness)*, ***cliques and sub-groups analysis***, and ***core-periphery analysis*** to analyze the properties of this network by using UCINET.

In Task II, to develop the influence measure and determine who has significant influence in the Erdos 1 network, we choose the four centralities (*degree, betweenness, eigenvector, closeness*) as evaluation indexes. In the basic analysis, we analyze them respectively and get a rough result. And in the further analysis, we use ***Analytic Hierarchy Process*** and ***Varience Analysis*** to combine these four factors and get the final influence measure. Last, we have found the authors who have significant influence in this network.

In Task III, we need to analyze another type of influence measure by analyzing some foundational papers. First, we build two networks of papers depending on citation and co-author relationship. Then, we analyze these two networks respectively. In this part, we build our own way to calculate *"**closeness centrality in a non-complete graph**"* and *"**degree centrality with weight**"*. Next, we also use ***AHP*** to combine these factors and finally found the most influential paper. Last, we discuss the similar way to analyze the influence of researchers, universities, and departments.

In Task IV, we implement our algorithm to a complete different area. We choose the TOP.35 valuable movie actors in Hollywood and build their network depending on co-star relationship.
Then, we use the similar method to develop the influence measure and compare their relative influence.

In Task V, we summarize the general method of analyzing a social network and its practical application in assessing influence and making decision.

What's more, we've analyzed the strengths and weaknesses of our algorithm.

Keywords:
Network influence, Centrality, Co-author and Citation network, Social Network

Are You Influential? Networks Will Tell You!

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1.Introduction

One of the techniques to determine influence of academic research is to build and measure properties of citation and co-author network. And the most famous co-author network is the one of Erdos.

In our paper, we mainly try to analyze the properties of Erods co-author network, and develop the influence measure. Based on this basic analysis, we expand our measure to other fields and conclude the general steps to assess a social network.

2.Solution of task I

2.1 Description and Analysis

In this task, we need to use the data in Erdos1.htm to build the co-author network of Erdos1 authors (except Erdos himself), and analyze the properties of the network.

Thus, we will accomplish two tasks:

1. Build the 511 notes network of Erdos1 co-authors. Each note represents a co-author with Erdos, and the co-authorship between them makes up their links.

2. According to Social Network Analysis Theory[1], we choose *density, centrality, Cliques and sub-groups analysis*, and *Core—Periphery analysis* to evaluate the properties of the network.

2.2 Build the co-author network of the Erdos1 authors

In this part, we use Adjacency Matrix[2] to represent the co-author relationship between the Erdos1 authors. Specific steps are as follows:

First, we number the researchers from the file Erdos1, the whole table shown in Appendix I.

Second, we build a 511*511 adjacency matrix, then fill it with “1” indicating that the two authors have co-authorship, and “0” indicating not. Part of the matrix is illustrated in Figure 2. It is obvious that this is a symmetric matrix.

	1	2	3	4	5	6	7	8	9	10
1	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	1
7	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	1	0	0	0	0

Figure 1

For instance, in the Figure 1 the values of (6, 10) and (10, 6) are “1”, which means the author 6 and the author 10 have co-author relationship. And all the others do not have.

Third, through some specific searching algorithms, we find out and import the 511*511 adjacency matrix into UCINET, a software specialized in analyzing social networks, to draw the social network graph of the co-author network.(illustrate in Figure 2)

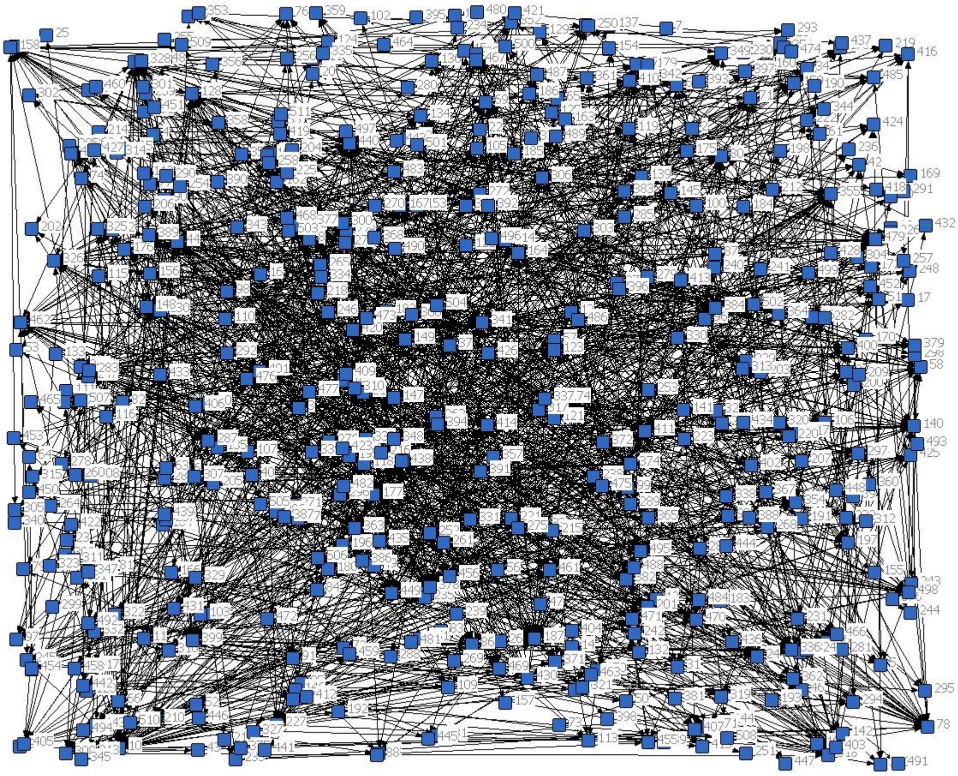

Figure 2

2.3 Analyze the properties of this network

According to the Social Network Analysis[1], there are many properties of a social network. For a given graph $G \coloneqq (V, E)$ with v vertices and e edges, define p_i, p_s, p_t are vertices in G. And we choose some of the properties as follows :

1.Density[3]:

For undirected simple graphs, the graph density is defined as:

$$D = \frac{2|E|}{|V|\,(|V|-1)}$$

The maximum number of edges is ½ |V| (|V|−1), so the maximal density is 1 (for complete graphs) and the minimal density is 0. It represents how tight the respective notes contact with each other.

2.Centrality:

Centrality refers to a group of metrics that aim to quantify the "importance" or "influence" (in a variety of senses) of a particular node (or group) within a network.

We use four common methods of measuring "centrality" :

(1) Degree centrality:

It is defined as the number of ties that a node has. If a note has higher degree centrality than other notes, it means the note is the center of the network and has greater “power” than others. The function to calculate it is:

$$deg(p_i) = \sum_{j=1}^{v} e_{ij} = \sum_{j=1}^{v} e_{ji}$$

where e_{ij}=1 when p_i directly connects with p_j, otherwise e_{ij}=0. (undirected graph)

The "degree centralization" represents the centrality of the whole network, and the closer to "1" it is, the more central tendency the network will have.

(2) Betweenness centrality:

It quantifies the number of times a node acts as a bridge along the shortest path between two other nodes. It is a measure for quantifying the control of a note on the communication between other notes in a social network, and the higher it is, the more powerful the note will be. The function to calculate it is:

$$bet(p_i) = \sum_{s \neq t \neq i} n_i$$

where n_i=1 when p_i is on one of the shortest paths between p_s and p_t, otherwise n_i=0.

Similarly, there is a property named "betweenness centralization" for the whole network. And the higher it is, the more dependent on some notes the whole network is.

(3) Eigenvector centrality:

It is a measure of the influence of a node in a network. If the eigenvector centrality of a note is higher, it means the note is more important.

The function to calculate it is:

$\mathbf{Ax} = \boldsymbol{\lambda}\mathbf{x}$. **A** is adjacency matrix of G, $\boldsymbol{\lambda}$is its eigenvalue.

$$eig(p_i) = x_i$$

where x_i is the i^{th} dimension of eigenvector correspondent to the greatest eigenvalue.

(4) Closeness(or farness) centrality:

It can be regarded as a measure of how long it will take to spread information from a note to all other nodes sequentially. The farness of a node s is defined as the sum of its distances to all other nodes, and its closeness is defined as the inverse of the farness. Thus, the more central a node is, the lower its total distance to all other nodes will be.

The function to calculate it is:

$$far(p_i) = \sum_{j=1, j \neq i}^{v} l_{ji}$$

where l_{ji} is the length between p_i and p_j.

Similarly, there is a metric named "closeness centralization" for the whole network. If the closeness centralization of a network is high, it demonstrates the notes in the network are quite different from each other.

3.Cliques and sub-groups analysis

If some notes in the network are very close to each other, they can be considered as a sub-group, and the analysis of these sub-groups is called "*Cliques and sub-groups analysis*".

There are many methods for cliques and sub-groups analysis. We use k-plex analysis. A k-plex is a maximal subgraph with the following property: each vertex of the induced subgraph is connected to ***n-k*** other vertices at least, where ***n*** is the number of vertices in the induced subgraph.

By using cliques and sub-groups analysis, we can know if the network has cliques and sub-groups, and the most important factor in cliques and sub-groups analysis is E-I Index, which range from -1 to 1, the closer to “1” it is, the more cliques and sub-groups there are in the network, and the closer to “-1” it is, the less cliques and sub-groups there are in the network

4.Core—Periphery analysis

The Core—Periphery analysis is aim to determine which notes are play a core role in a social network, and which notes are considered as periphery.

And it is always combined with “Price law”[4] in Bibliometrics. According to the Price law, if the number of papers of a author is more than $0.749\sqrt{N_{\max}}$ ($N_{\max}$ is the number of papers of an author who produces the most papers), it means the author is a core author.

2.4 Analyzing and results

In this part, we use UCINET to analyze the properties mentioned in 2.2. The analyzing process and the results are as follows:

2.4.1 Density analysis:

The outcome is illustrated in Figure 3:

Density	No. of Ties
0.0124	3234.0000

Figure 3

The density of the network is 0.0124, which is lower than general social networks. Although these authors all have co-authorship with Erdos, this sparse network demonstrates authors in the network are not related to each other closely. At the same time we can tell that Erdos has cooperated with other researchers extensively, which to some extent, contributes to his great success in many different areas.

2.4.2 Centrality analysis:

(1)Degree centrality:

The result is illustrated in Figure 4 and Figure 5:

Figure 4 shows the TOP.20 of degree centrality in the co-author network.

Figure 5 shows the descriptive statistics.

	1 Degree	2 NrmDegree	3 Share
10	52.000	10.196	0.016
165	43.000	8.431	0.013
187	43.000	8.431	0.013
44	43.000	8.431	0.013
378	42.000	8.235	0.013
479	40.000	7.843	0.012
148	39.000	7.647	0.012
438	38.000	7.451	0.012
440	35.000	6.863	0.011
341	32.000	6.275	0.010
177	32.000	6.275	0.010
78	31.000	6.078	0.010
287	30.000	5.882	0.009
128	30.000	5.882	0.009
355	29.000	5.686	0.009
180	29.000	5.686	0.009
462	29.000	5.686	0.009
249	29.000	5.686	0.009
326	27.000	5.294	0.008
449	26.000	5.098	0.008

Figure 4

		1 Degree	2 NrmDegree	3 Share
1	Mean	6.329	1.241	0.002
2	Std Dev	7.763	1.522	0.002
3	Sum	3234.000	634.118	1.000
4	Variance	60.260	2.317	0.000
5	SSQ	51260.000	1970.781	0.005
6	MCSSQ	30792.768	1183.882	0.003
7	Euc Norm	226.407	44.393	0.070
8	Minimum	0.000	0.000	0.000
9	Maximum	52.000	10.196	0.016

Network Centralization = 8.99%
Heterogeneity = 0.49%. Normalized = 0.29%

Figure 5

From the results, we can see that the degree centralization of the whole network is 8.99%. It is very low, which means this network almost do not have the tendency to gather core vertices.

And, form Figure 4, we can get the TOP.20 authors that have higher degree centrality, which means they are core authors and act more important role in the network.

(2)Betweenness centrality:

The result is illustrated in Figure 6 and Figure 7.

Figure 6 shows the TOP.20 of betweenness centrality in the co-author network.

Figure 7 shows the descriptive statistics.

	1 Betweenness	2 nBetweenness
187	9334.665	7.192
438	8885.603	6.846
449	8025.152	6.183
10	7355.721	5.667
355	7089.246	5.462
479	7026.445	5.413
44	6838.232	5.268
165	6427.991	4.952
148	6152.162	4.740
385	6045.155	4.657
341	5667.675	4.367
180	5296.368	4.081
336	4503.635	3.470
389	4429.884	3.413
249	3914.382	3.016
399	3787.207	2.918
440	3699.084	2.850
405	3640.531	2.805
378	3451.660	2.659
307	3134.779	2.415

Figure 6

DESCRIPTIVE STATISTICS FOR EACH MEASURE

		1 Betweenness	2 nBetweenness
1	Mean	600.826	0.463
2	Std Dev	1262.978	0.973
3	Sum	307022.000	236.544
4	Variance	1595112.250	0.947
5	SSQ	999569088.000	593.331
6	MCSSQ	815102336.000	483.834
7	Euc Norm	31615.963	24.358
8	Minimum	0.000	0.000
9	Maximum	9334.665	7.192

Network Centralization Index = 6.74%

Figure 7

The betweenness centralization of the network is 6.74%. It is also very low, which means the network do not depend on some notes to serve as necessary bridges for communication. To some extent, it shows this network is not much possible to divide to sub-groups.

And in Figure 6, there are 20 authors that play more important role as “bridge” in the network. They control quantity of resource and connect important researchers.

(3)Eigenvector centrality:

The result is illustrated in Figure 8 and Figure 9.

Figure 8 shows the TOP.20 of eigenvector centrality in the co-author network.

Figure 9 shows the descriptive statistics.

	1	2
	Eigenvec	nEigenvec
10	0.263	37.171
378	0.233	33.004
44	0.211	29.84
165	0.202	28.62
148	0.201	28.391
479	0.187	26.485
440	0.181	25.637
177	0.178	25.136
462	0.176	24.905
128	0.163	23.101
287	0.161	22.745
341	0.154	21.799
78	0.152	21.567
261	0.145	20.567
430	0.145	20.498
326	0.14	19.746
290	0.139	19.681
438	0.136	19.163
21	0.13	18.381
403	0.129	18.237

Figure 8

Descriptive Statistics

		1	2
		Eigenvec	nEigenvec
1	Mean	0.021	2.994
2	Std Dev	0.039	5.493
3	Sum	10.819	1529.968
4	Variance	0.002	30.175
5	SSQ	1.000	20000.000
6	MCSSQ	0.771	15419.176
7	Euc Norm	1.000	141.421
8	Minimum	-0.000	-0.000
9	Maximum	0.263	37.171
10	N of Obs	511.000	511.000

Network centralization index = 35.83%

Figure 9

From the results, we can see the 20 authors with heavier weight in the network, have much significant influence and play a more important role.

But we can also see that the difference between eigenvector centrality is small, as the variance is only 0.002. So it means many authors in the network contribute similar weight in this network, and there is no author in the absolute center.

(4)Closeness(or farness) centrality:

The result is illustrated in Figure 10 and Figure 11.

Figure 10 shows the TOP.20 of closeness centrality in the co-author network.

Figure 11 shows the descriptive statistics.

	1	2
	Farness	nCloseness
10	1192.000	39.094
44	1201.000	38.801
165	1207.000	38.608
148	1208.000	38.576
438	1226.000	38.010
479	1235.000	37.733
440	1256.000	37.102
378	1258.000	37.043
287	1264.000	36.867
449	1273.000	36.606
462	1278.000	36.463
128	1284.000	36.293
341	1288.000	36.180
187	1290.000	36.124
249	1296.000	35.957
504	1299.000	35.874
78	1303.000	35.764
389	1305.000	35.709
430	1306.000	35.681
180	1309.000	35.600

Figure 10

Statistics

		1	2
		Farness	nCloseness
1	Mean	1780.869	26.988
2	Std Dev	328.480	4.583
3	Sum	831666.000	12603.299
4	Variance	107899.383	21.006
5	SSQ	1531477504.000	349944.750
6	MCSSQ	50389012.000	9809.574
7	Euc Norm	39134.098	591.561
8	Minimum	1192.000	14.045
9	Maximum	3318.000	39.094

Network Centralization = 24.29%

Figure 11

Form the result, we can see that the centralization of the whole network is 24.29%, it is relatively high and represents that there is great difference between the notes in the network.

And Figure 10 shows the TOP.20 authors with higher closeness centrality. They have lower total distance to all other nodes and higher influence in the network, and have greater ability of spreading information.

2.4.3 Cliques and sub-groups analysis

The result is illustrated in Figure 12 and Figure 13

Figure 12 shows 349 sub-groups in the network divided by UCINET

Figure 13 shows E-I index.

```
349 k-plexes found.

   1:  1 308 318 321
   2:  6 10 21 378 440 462
   3:  6 21 258 378 440 462
   4:  6 80 258 378 462
   5:  8 31 69 282
   6:  8 31 282 284
   7:  8 41 69 337
   8:  8 69 78 337
   9:  8 69 282 454
  10:  8 69 337 411 454
  11:  8 78 165 337
  12:  8 282 284 495
  13:  8 297 337 411 454
  14:  8 297 411 470
  15:  10 17 249 341
  16:  10 21 78 140 165
  17:  10 21 78 165 440 462
  18:  10 21 140 165 287
  19:  10 21 140 165 378
  20:  10 21 165 287 326
```

Figure 12

Whole Network Results

	1 Freq	2 Pct	3 Possible	4 Density
1 Internal	3122.000	0.965	254550.000	0.012
2 External	112.000	0.035	6060.000	0.018
3 E-I	-3010.000	-0.931	-248490.000	-0.953

Figure 13

From the results, we can see that the number of sub-groups is close to the number of authors, which means the tendency to gather sub-groups in this network almost do not exist.

Indeed, in the Figure 13, the E-I index of this network is -0.953. It is very close to -1, which also represents there is almost no sub-group within this co-author network.

2.4.4 Core—Periphery analysis

The result is illustrated in Figure 14:

```
Core/Periphery Class Memberships:

  1:  6 10 17 21 44 58 69 71 78 80 128 134 140 148 158 164 165 177 178 180
  2:  1 2 3 4 5 7 8 9 11 12 13 14 15 16 18 19 20 22 23 24 25 26 27 28 29 30
```

Figure 14

Figure 14 shows the core authors and periphery authors. The authors following “1:”are core authors, and the authors following “2:” are periphery authors.

2.5 Strengths and Weaknesses

We have analyzed the properties in many ways, and assessed the network in each aspect, so analysis above is complete and detailed.

But we have not considered the number of their coauthor experiences, the influence of their works, and the time of their cooperation.

3.Solution of task II

3.1 Description and Analysis

In this task, we need to determine who in this Erdos1 network have significant influence within the network. We will use the results we have got in Task I.

Since Task I is analyzing the properties of the whole network, we need to choose some of them relating to individual influence analysis. Apparently, centrality is aim to quantify the "importance" or "influence" (in a variety of senses) of a particular node (or group) within a network, so we choose the four centralities(degree centrality, betweenness centrality, eigenvector centrality, closeness centrality) to determine the influence of different notes in the network.

3.2 Basic analysis

In this part, we will respectively analyze the four centralities to determine the relative influence of the authors within this network.

1.Degree centrality:

Part of result in Task I is:

	1 Degree	2 NrmDegree	3 Share
10	52.000	10.196	0.016
165	43.000	8.431	0.013
187	43.000	8.431	0.013

Degree centrality is defined as the number of ties that a node has.

From the results above, author 10 has highest degree centrality, which means he has co-author relationship with many other authors in the network. He has more communication with other researchers, so can be considered as a core author.

2.Betweeness centrality:

Part of result in Task I is:

	Betweenness	nBetweenness
187	9334.665	7.192
438	8885.603	6.846
449	8025.152	6.183

Betweenness centrality represents the importance of the author to serve as a “bridge” in the network.

Author 187 has the highest betweenness centrality, so he serves as an important bridge in this network, controlling many resources and connecting many researchers.

3.Eigenvector centrality:

Part of result in Task I is:

	Eigenvec	nEigenvec
10	0.263	37.171
378	0.233	33.004
44	0.211	29.84

Eigenvector centrality represents the weight of notes in the network.

Author 10 has highest eigenvector centrality, which means the author occupied the highest weight in the network, so that he has the most significant influence in this respect.

4.Closeness(or farness) centrality:

Part of result in Task I is:

	Farness	nCloseness
10	1192.000	39.094
44	1201.000	38.801
165	1207.000	38.608

Closeness centrality is a measure of how long it will take to spread information from a note to all other nodes sequentially.

Still, author 10 takes less distance to contact with other notes, which means he locates in the center of this network.

Take all the four factors analyzed above into consideration, the degree centrality, eigenvector centrality and closeness centrality of author 10 are all the highest. Besides, his betweenness centrality is in the forth place. Therefore, it can be roughly concluded that author 10 is the most influential in this co-author network.

3.3 Further analysis

In the basic model we only analyze the four centralities respectively, but did not combine them together. So in this part, we will try to synthesize these four factors together to build a final model of influence measure.

The final influence measure is required to contain all notes, but closeness centrality can only cope with complete graph. So we abandon closeness centrality and use the other three to build the influence measure.

3.3.1 Data normalization

First, we need to normalize the data, because the three centralities are in different magnitude. We use the function below to normalize the data:

$$V_{adjusted} = \frac{V_{original} - V_{\min}}{V_{\max} - V_{\min}}$$

Part of the normalized results shows in Figure 15:

1		Degree		Betweenness		Eigenvec
2	10	1	187	1	1	0.030418
3	165	0.82692308	438	0.951893078	2	0.015209
4	187	0.82692308	449	0.859715051	3	0.003802
5	44	0.82692308	10	0.788000533	4	0
6	378	0.80769231	355	0.759453714	5	0.08365
7	479	0.76923077	479	0.752725995	6	0.239544
8	148	0.75	44	0.732563193	7	0
9	438	0.73076923	165	0.688615071	8	0.13308
10	440	0.67307692	148	0.65906618	9	0.007605

Figure 15

3.3.2 Combine factors

To combine the three factors, we need to decide the weight of each factor. Use Analytic

Hierarchy Process[5] to get the determine weights.

First, layers are divided as follows:

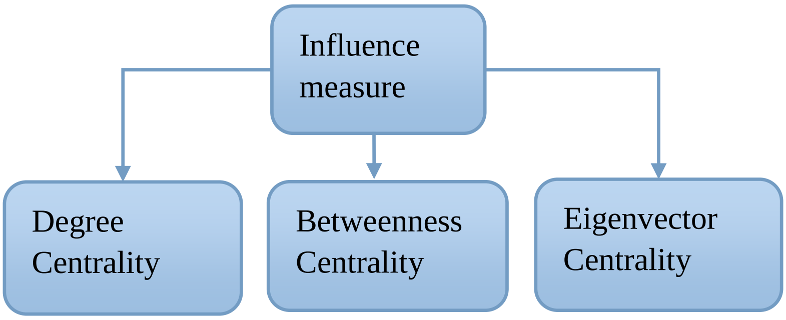

Figure 16

Second, we combine the effect of every two factors in the second layer. We calculate the variance of each factor after normalization. If the variance of a factor is higher than others, this factor appears to be more decisive in influence measure, and should have higher weight.

The variance of each factor is illustrated in Table 1:

Factor	Degree centrality	Betweeness centrality	Eigenvector centrality
Variance	0.02232913	0.018341907	0.02185123

Table 1

So, we can roughly know that the importance of degree centrality is similar to eigenvector centrality, and these two are much more important than betweenness centrality. We can construct Conjugated-Comparative Matrix with Saaty’ s Rule[5] as follow:

$$M_1 = \begin{bmatrix} 1 & 4 & 1 \\ 1/4 & 1 & 1/3 \\ 1 & 3 & 1 \end{bmatrix}$$

After the calculation of the matrix using Summation Method (引用), we obtain the weight vectors:

$$W = (0.4579, \quad 0.1260, \quad 0.4161)$$

We get the final influence measure factor:

$$Influnce = 0.4579 \cdot \deg ree + 0.1260 \cdot betweeness + 0.4161 \cdot eigenvector$$

Using the function above, we calculate the result showed in Figure 17 and 18:

1		Degree	Betweenness	Eigenvec	Influence
2	10	1	0.78800053	1	0.973288067
3	44	0.82692308	0.73256319	0.802281369	0.804780317
4	378	0.80769231	0.36976796	0.885931559	0.785069192
5	165	0.82692308	0.68861507	0.768060837	0.78500369
6	148	0.75	0.65906618	0.764258555	0.744475323
7	479	0.76923077	0.75272599	0.711026616	0.74293242
8	187	0.82692308	1	0.452471483	0.692921461
9	438	0.73076923	0.95189308	0.517110266	0.66972734
10	440	0.67307692	0.39627389	0.688212928	0.644497832

Figure 17

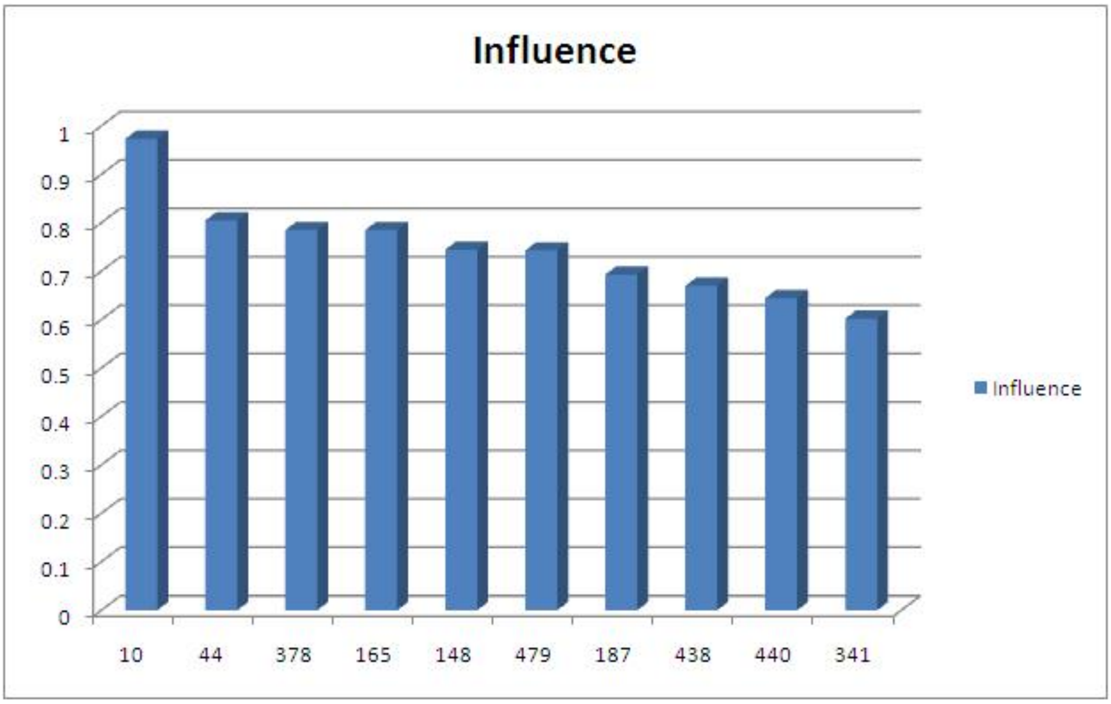

Figure 18

From the result, we can know that author 10 has the most substantial influence in the network. The TOP.10 authors' numbers are shown above and their names can be got in Appenix I. They have the greatest influence in the co-author network of the Erdos1 authors.

3.4 Strengths and Weaknesses

We have analyzed the factors respectively and combined， the results are the same. And from the charts, we can easily see who have more significant influence.

But, the influence we measured can only represent the status of the author within this network, and partly show his influence outside. However, it cannot determine his real influence in the whole academia.

4.Solution of task III

4.1 Description and Analysis

In this task, we develop another type of influence measure to compare the significance of a research paper. So, we will accomplish four tasks as follows:

1.Choose 19 foundational papers in Network Science and number them, the table of which is showed in Appendix II.

2.Build the Adjacency Matrix to demonstrate the relationship between these 19 papers, and draw the network graph of them. Since the papers have two relationships (coauthor and citation) between each other, we build two networks.

3.Use AHP to develop the influence measure, and use it to determine which paper is the most influential in this network.

4.Consider the similar way to determine the relative influence between individual researchers, universities, departments, or journals in science network.

4.2 Build Network

In this part, we build two networks. One depends on the citation relationship, and the other on the co-authorship. It is clear that the first network is a directed graph and the second is a undirected graph.

1.Network of papers dependent on citation relationship:

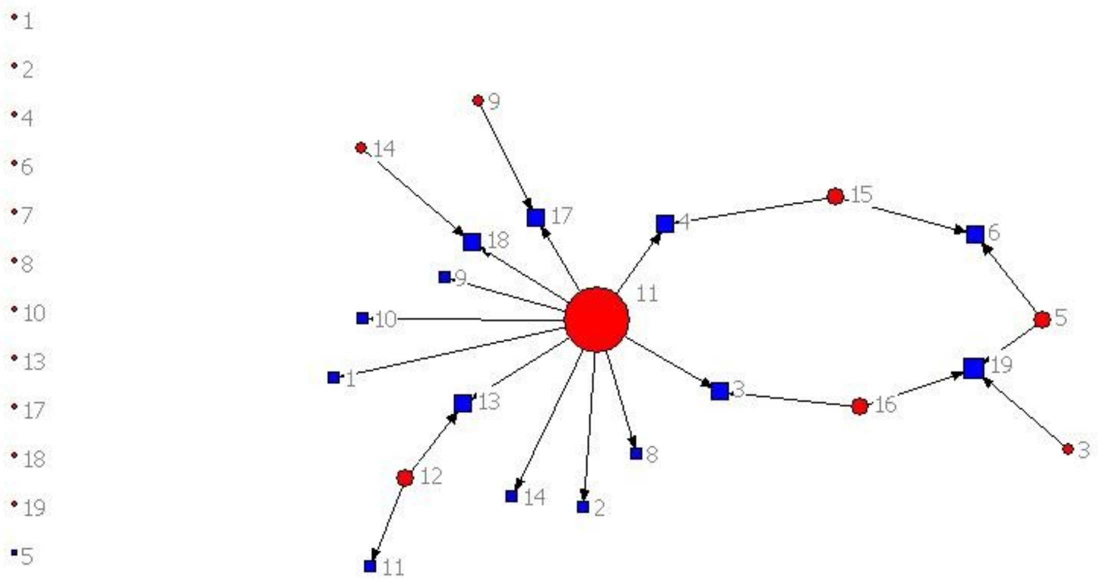

Figure 19

In this graph, the end note of each arrow stands for the paper cited by the starting note of the arrow.

2.Network of papers dependent on co-author relationship:

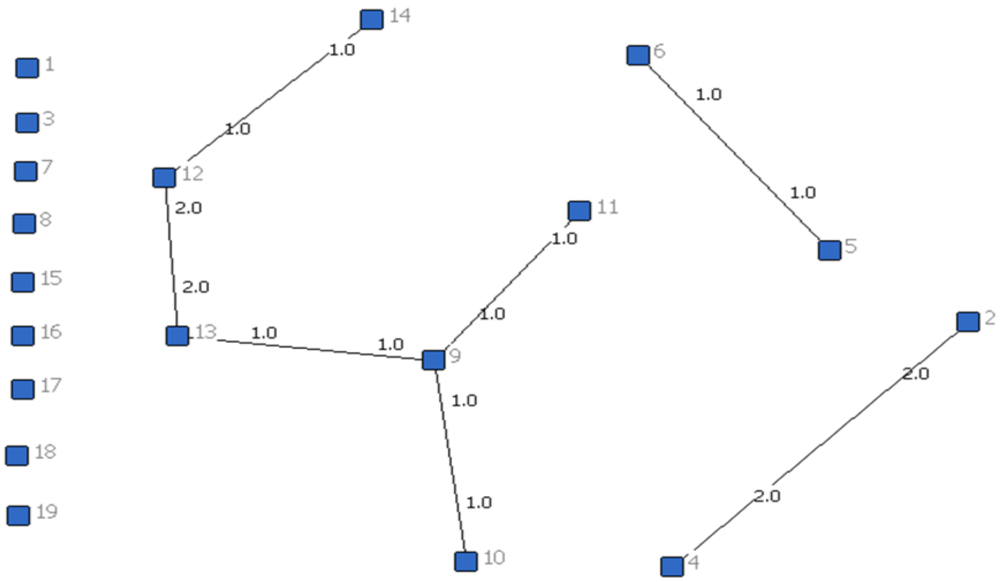

Figure 20

In this network graph each tie between nodes has a weight, indicating the number of co-authors between two papers. For instance, the weight of the link between node 12 and note 13 is 2, because these two papers have two co-authors.

4.3 Analyze citation and co-author network

In this part we redefine degree and closeness centrality, then analyze the two networks.

In the citation network, we choose in-degree and out-degree centrality, betweenness centrality, eigenvector centrality, and "improved closeness centrality" to analyze the relative influence of papers dependent on citation relationship.

In the co-author network, we choose "degree centrality with weight" and "improved closeness centrality" to analyze the relative influence of papers depended on co-author relationship.

4.3.1 Citation network

Using the same method mentioned in Task I, we calculate the in-degree and out-degree centrality, betweenness centrality, and eigenvector centrality by UCINET. However, since this network graph is not a complete graph, we build our method to calculate the closeness centrality.

We use "***a***" to represent the distance between two unlinked notes, which never influence each other. So "***a***" can be regarded as an infinite number. When necessary, we can give "***a***" a specific high value, such as 100, or 1000. Then, we calculate the sum-distance of a note towards all the other ones, and consider it as the "improved closeness centrality" of this note.

The result showed in Figure 21:

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	0	a	a	a	a	a	a	a	a	a	a	a	a	a	a	a	a	a	a
2	a	0	a	a	a	a	a	a	a	a	a	a	a	a	a	a	a	a	a
3	a	a	0	a	a	a	a	a	a	a	a	a	a	a	a	a	a	a	1
4	a	a	a	0	a	a	a	a	a	a	a	a	a	a	a	a	a	a	a
5	a	a	a	a	0	1	a	a	a	a	a	a	a	a	a	a	a	a	1
6	a	a	a	a	a	0	a	a	a	a	a	a	a	a	a	a	a	a	a
7	a	a	a	a	a	a	0	a	a	a	a	a	a	a	a	a	a	a	a
8	a	a	a	a	a	a	a	0	a	a	a	a	a	a	a	a	a	a	a
9	a	a	a	a	a	a	a	a	0	a	a	a	a	a	a	a	1	a	a
10	a	a	a	a	a	a	a	a	a	0	a	a	a	a	a	a	a	a	a
11	1	1	1	1	a	a	a	1	1	1	0	a	1	1	a	a	1	1	2
12	2	2	2	2	a	a	a	2	2	2	1	0	1	2	a	a	2	2	3
13	a	a	a	a	a	a	a	a	a	a	a	a	0	a	a	a	a	a	a
14	a	a	a	a	a	a	a	a	a	a	a	a	a	0	a	a	a	1	a
15	a	a	a	1	a	1	a	a	a	a	a	a	a	a	0	a	a	a	a
16	a	a	1	a	a	a	a	a	a	a	a	a	a	a	a	0	a	a	1
17	a	a	a	a	a	a	a	a	a	a	a	a	a	a	a	a	0	a	a
18	a	a	a	a	a	a	a	a	a	a	a	a	a	a	a	a	a	0	a
19	a	a	a	a	a	a	a	a	a	a	a	a	a	a	a	a	a	a	0
sum-distance	16a+3	16a+3	15a+4	15a+4	18a	16a+2	18a	16a+3	16a+3	16a+3	17a+1	18a	16a+2	16a+3	18a	18a	15a+4	15a+4	13a+8

Figure 21

4.3.2 Co-author network

UCINET cannot process adjacent matrix with quantities out of "0" and "1", so we calculate it by ourselves to obtain the degree centrality.

The result is showed in Figure 22.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
degree	0	2	0	2	1	1	0	0	3	1	1	3	3	1	0	0	0	0	0

Figure 22

We also use the same method mentioned above to calculate the "improved closeness centrality".

4.4 Develop the influence measure

We use similar method in the Task II to develop the influence measure. First, we use Analytic Hierarchy Process to analyze the network and calculate the weights.

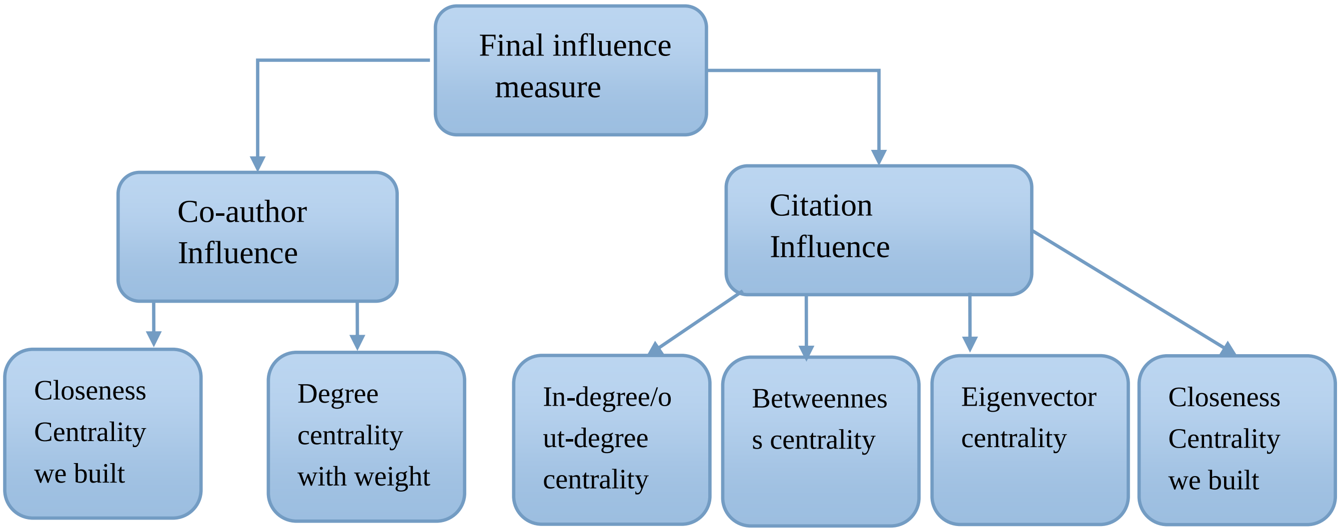

Figure 23

Then calculate the weight in different layers in the same way. We get three functions :

co-author-influence = 0.6·degree + 0.4·closeness

citation-influence = 0.36632·degree + 0.1008·betweenness + 0.33288·eigenvector + 0.2·closeness

$$citation-\inf luence = 0.36632 \cdot \deg ree + 0.1008 \cdot betweenness + 0.33288 \cdot eigenvector + 0.2 \cdot closeness$$

$$Final-\inf luence = 0.2 \cdot co-author-\inf luence + 0.8 \cdot citation-\inf luence$$

The “final influence is” the influence measure we built.

Since the in-degree and out-degree centrality of a note are different, we analyze them respectively.

1.Out-degree analysis, representing the number of a paper’s reference within this network.

The result is showed in Figure 24 and Figure 25:

	co-author			citation					Final
	Degree	Closeness	Influence	OutDegree	Betweeen	eigenvector	Closeness	Influence	Influence
11	0.3333	0.1547296	0.2618919	1	1	1	0.9888459	0.99776919	0.850593719
9	1	0.3267197	0.7306879	0.0909091	0	0.356927711	1	0.35211591	0.427830309
12	1	0	0.6	0.1818182	0	0.356927711	0.9944144	0.38430062	0.427440498
13	1	0.3285832	0.7314333	0	0	0.356927711	1	0.3188141	0.401337931
14	0.3333	0.3267197	0.3306879	0.0909091	0	0.356927711	0.9832944	0.34877479	0.345157406
10	0.3333	0.3267197	0.3306879	0	0	0.262048193	0.9888459	0.28499979	0.294137407
4	0.6667	0.5232226	0.6092891	0	0	0.284638554	0.1547296	0.12569641	0.222414938
2	0.6667	0.3267197	0.5306879	0	0	0.262048193	0.1547296	0.11817653	0.200678802
3	0	0.5232226	0.2092891	0.0909091	0.181818	0.328313253	0	0.16091801	0.170592215
17	0	0.5232226	0.2092891	0	0	0.356927711	0	0.1188141	0.136909087
18	0	0.5232226	0.2092891	0	0	0.356927711	0	0.1188141	0.136909087
5	0.3333	0	0.2	0.1818182	0	0.042168675	0.1547296	0.11158667	0.129269339
19	0	1	0.4	0	0	0.129518072	0	0.04311398	0.114491181
6	0.3333	0.3285832	0.3314333	0	0	0.03313253	0.1547296	0.04197509	0.099866722
1	0	0.3267197	0.1306879	0	0	0.262048193	0	0.0872306	0.095922059
8	0	0.3267197	0.1306879	0	0	0.262048193	0	0.0872306	0.095922059
16	0	0	0	0.1818182	0	0.120481928	0	0.10670966	0.085367728
15	0	0	0	0.1818182	0	0.082831325	0	0.09417653	0.075341222
7	0	0	0	0	0	0	0	0	0

Figure 24

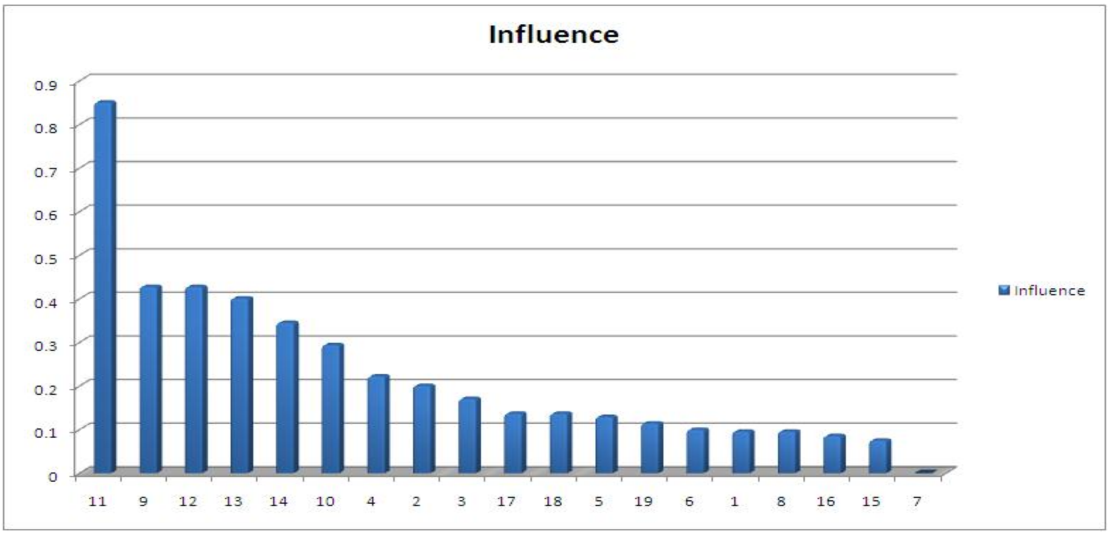

Figure 25

From the result, we can see that the paper 11 has the most significant influence, and its influence is much higher than others. It corresponds to the fact, because the paper 11 cites other papers the most, which means this paper collects widely from other papers. And it also shows this paper contributes little within this network and is controlled by other papers.

2.In-degree analysis, representing the number of a paper being cited by others.

The result is showed in Figure 26 and Figure 27:

	co-author			citation					Final
	Degree	Closeness	Influence	InDegree	Betweeen	eigenvect	Closeness	Influence	Influence
11	0.3333	0.15473	0.2618919	0.333333	1	1	0.988846	0.7535559	0.65522305
13	1	0.328583	0.7314333	0.666667	0	0.356928	1	0.5630274	0.5967086
9	1	0.32672	0.7306879	0.333333	0	0.356928	1	0.4409208	0.49887419
4	0.6667	0.523223	0.6092891	0.666667	0	0.284639	0.15473	0.3699097	0.41778561
14	0.3333	0.32672	0.3306879	0.333333	0	0.356928	0.983294	0.4375796	0.41620128
19	0	1	0.4	1	0	0.129518	0	0.409434	0.40754718
10	0.3333	0.32672	0.3306879	0.333333	0	0.262048	0.988846	0.4071065	0.39182274
12	1	0	0.6	0	0	0.356928	0.994414	0.317697	0.37415759
3	0	0.523223	0.2092891	0.666667	0.181818	0.328313	0	0.3718295	0.33932143
17	0	0.523223	0.2092891	0.666667	0	0.356928	0	0.3630274	0.33227975
18	0	0.523223	0.2092891	0.666667	0	0.356928	0	0.3630274	0.33227975
2	0.6667	0.32672	0.5306879	0.333333	0	0.262048	0.15473	0.2402832	0.29836414
6	0.3333	0.328583	0.3314333	0.666667	0	0.033133	0.15473	0.2861884	0.29523739
1	0	0.32672	0.1306879	0.333333	0	0.262048	0	0.2093373	0.19360739
8	0	0.32672	0.1306879	0.333333	0	0.262048	0	0.2093373	0.19360739
5	0.3333	0	0.2	0	0	0.042169	0.15473	0.044983	0.07598643
16	0	0	0	0	0	0.120482	0	0.040106	0.03208482
15	0	0	0	0	0	0.082831	0	0.0275729	0.02205831
7	0	0	0	0	0	0	0	0	0

Figure 26

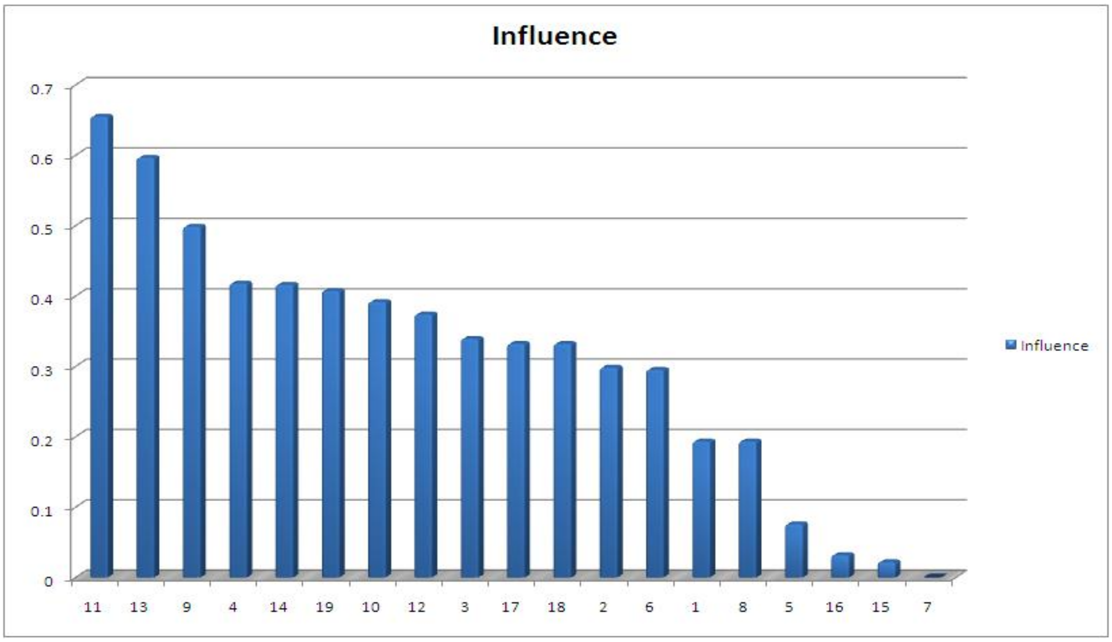

Figure 27

From the result, the paper 11 is the most influential one, because its betweenness centrality and eigenvector centrality are both very high, and it is the most central note. But its influence is not much higher than others, since it is only be cited once.

The paper 13 and 9 also have high influence, because they are widely cited by others. These two papers contribute much within this network and control others papers.

But we cannot say the paper 11 is not important, because it was published in 2003, and the paper 13 and 9 were published in 2002 and 2001.

4.5 Similar analysis in other fields

According to the analysis above, we can use a similar way to determine the role or influence measure of an individual network researcher. We make the notes in the papers' network to represent the researchers of these papers, and build the network of researchers depending on citation relationship.

Similarly, when it comes to measure the role or influence of a university, department, or a journal, we can also build their network depending on their cooperation relationship, and then determine their relative influence in the similar way.

4.6 Strengths and Weaknesses

In this task, we have improved our analysis in Task I&II by building how to calculate the closeness centrality and the degree centrality with weight.

But the result is still limited within this network. And it is also because we have not collected enough data.

5.Solution of task IV

5.1 Description and Analysis

In this task, we implement our algorithm on a completely different set of network influence data. We choose movie actors to analyze. So, we will accomplish three tasks as follow:

1. Choose the TOP.35 most valuable movie actors on website ***vulture***[6], and find their movies in the last 5 years on ***wikipedia***.

2. Build the network between these movie actors dependent on co-star relationship.

3. Use the model in Task II to analyze who is the most influential actor in this network.

5.2 Analyze the co-star network

The co-star network graph is showed in Figure 28:

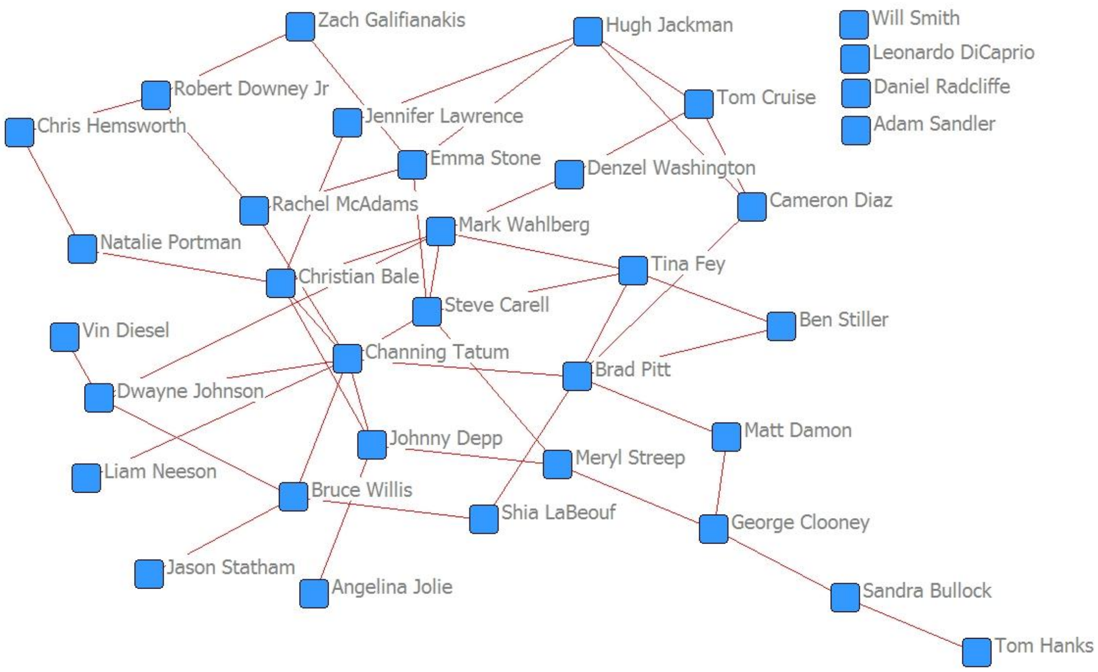

Figure 28

Then we calculate the degree centrality, betweenness centrality, and eigenvector centrality. And use AHP to combine them together to determine who is most influential in this co-star network.

Due to a complete different field, we recalculate the weights of these three determine factors.

$$M_2 = \begin{bmatrix} 1 & 2 & 1/2 \\ 1/2 & 1 & 1/4 \\ 2 & 4 & 1 \end{bmatrix} \qquad W = (0.297 \quad 0.1634 \quad 0.5396)$$

$$Influence = 0.297 \cdot degree + 0.1634 \cdot betweenness + 0.5396 \cdot eigenvector$$

The result is showed in Figure 30:

		Degree	Betweenness	Eigenvector	Influence
24	Channing Tatum	1	1	1	1
5	Brad Pitt	0.75	0.6143953	0.62309368	0.6593635
21	Steve Carell	0.625	0.42630931	0.7037037	0.6350025
16	Christian Bale	0.625	0.45967105	0.61873638	0.5946054
12	Mark Wahlberg	0.625	0.31215077	0.64270153	0.5834322
27	Dwayne Johnson	0.5	0.23408151	0.55119826	0.4841755
3	Johnny Depp	0.5	0.34967326	0.50762527	0.4795512
35	Tina Fey	0.5	0.10244026	0.55119826	0.4626653
23	Bruce Willis	0.5	0.24018689	0.47276688	0.4428515

Figure 30

We can see Channing Tatum is the most influential actor within this network. This is because he has played many movies last five years and cooperated with many actors. Since he is only 34 years old this year, we can tell he is in the rise of his career.

We can see some well-known actors in this list, such as Brad Pitt, Johnny Depp, they are also quite significant in this network.

However, some famous actors do not exist in this list, because they do not shoot a lot of movies these 5 years. This does not mean they are not have great influence, because we have not consider the quality of movies and the audience' opinions.

6.Solution of task V

When modeling, we've use several methods to analyze as above. Centrality indicates the tightness and control ability of resource. Cliques and sub-groups analysis illustrates the communication and circulation of information. Core—Periphery analysis depicts the distribution of core vertices. We should make clear that this model only reflects the influence between each other notes in the network, not towards notes outsides the network.

As social animal, we are living in various social networks. These methods can help us understand the properties of social network around us and make some practical applications. For instance, it is useful for a company to analyze influence of corporations in commercial network, and choose the higher ones to collaborate with.

Procedures of network modeling are as follows:

1. Determine the purpose of network analysis. Then determine what the particular notes means in network, and the definite links between them, which in this example are corporations and the numbers of contracts they have signed(or trading volume) with others. It is necessary to make sure that sufficient notes and links can be collected.

2. Collect the data we need. For example from the annual financial report or corporations' websites. Though a larger amount of data helps to make a more accurate assessment, we should limit our data range depending on collection ability, and consider the exactness of them.

3. Build the relationship matrix. Elements in the matrix represent the links and weight between nodes, then use the methods to analyze and find out the most influential one. Try the best to construct connections with them, and make good use.

7. Reference

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[6] Vulture : Vulture's 100 Most Valuable Stars of 2012:
http://www.vulture.com/2012/07/most-valuable-movie-stars.html

Appendix I

1 ABBOTT, HARVEY LESLIE
2 ACZEL, JANOS D.
3 AGOH, TAKASHI
4 AHARONI, RON
5 AIGNER, MARTIN S.
6 AJTAI, MIKLOS
7 ALAOGLU, LEONIDAS*
8 ALAVI, YOUSEF
9 ALLADI, KRISHNASWAMI
10 ALON, NOGA M.
11 ANDERSON, JAMES MILNE
12 ANDRASFAI, BELA
13 ANKENY, NESMITH CORNETT*
14 ANNING, NORMAN H.*
15 ARKIN, JOSEPH*
16 ARNEY, DAVID CHRISTOPHER
17 ARONOV, BORIS
18 ASH, J. MARSHALL
19 ASHBACHER, CHARLES D.
20 AVIS, DAVID MICHAEL
21 BABAI, LASZLO
22 BABU, GUTTI JOGESH
23 BAGEMIHL, FREDERICK*
24 BALOG, ANTAL
25 BANKOFF, LEON*
26 BARAK, AMNON B.
27 BATEMAN, PAUL TREVIER
28 BAUMGARTNER, JAMES EARL
29 BEASLEY, LEROY B.
30 BECK, ISTVAN
31 BEHZAD, MEHDI^1
32 BEJLEGAARD, NIELS
33 BENKOSKI, STANLEY J.
34 BERGELSON, VITALY
35 BERGER, MARC ARON
36 BERTRAM, EDWARD ARTHUR
37 BIALOSTOCKI, ARIE
38 BLASS, ANDREAS RAPHAEL
39 BLECKSMITH, RICHARD FRED
40 BLEICHER, MICHAEL NATHANIEL
41 BOALS, ALFRED JOHN
257 KOMJATH, PETER
258 KOMLOS, JANOS
259 KOMORNIK, VILMOS
260 KOREN, ISRAEL
261 KOSTOCHKA, ALEXANDR V.
262 KOVARI, TAMAS
263 KRANTZ, STEVEN GEORGE
264 KRATSCH, DIETER
265 KRIVELEVICH, MICHAEL
266 KROO, ANDRAS V.
267 KUBICKA, EWA MARIE
268 KUBICKI, GRZEGORZ
269 KUNEN, KENNETH
270 LACAMPAGNE, CAROLE B.
271 LAM, PETER CHE BOR
272 LARSON, JEAN ANN
273 LASKAR, RENU CHAKRAVARTI
274 LEFMANN, HANNO
275 LEHEL, JENO
276 LEHNER, JOSEPH
277 LENGYEL, BELA A.*
278 LEV, VSEVOLOD F.
279 LEVEQUE, WILLIAM JUDSON*
280 LEWIN, MORDECHAI
281 LI, WEN-CH'ING WINNIE
282 LICK, DON RAYMOND
283 LINIAL, NATHAN
284 LIU, JIUQIANG
285 LOEBL, MARTIN
286 LORENTZ, GEORGE GUNTHER*
287 LOVASZ, LASZLO
288 LOXTON, JOHN HAROLD
289 LUCA, FLORIAN
290 LUCZAK, TOMASZ
291 MACINTYRE, ARCHIBALD JAMES*
292 MAGIDOR, MENACHEM
293 MAHLER, KURT*
294 MAIER, HELMUT
295 MAKAI, ENDRE, JR.
296 MAKKAI, MICHAEL
297 MALDE, PARESH J.

42 BOAS, RALPH PHILIP, JR.*
43 BOES, DUANE CHARLES
44 BOLLOBAS, BELA
45 BONAR, DANIEL DONALD
46 BONDY, JOHN ADRIAN
47 BONNET, ROBERT
48 BOROSH, ITSHAK
49 BOSAK, JURAJ
50 BOVEY, JOHN D.
51 BRENNER, JOEL LEE*
52 BRILLHART, JOHN DAVID
53 BRINDZA, BELA*
54 BROWN, THOMAS CRAIG
55 BROWN, WILLIAM G.
56 BUCK, ROBERT CREIGHTON*
57 BUKOR, JOZEF
58 BURR, STEFAN ANDRUS
59 BUSOLINI, DONALD TERENCE
60 CACCETTA, LOUIS
61 CALKIN, NEIL JAMES
62 CAMERON, PETER J.
63 CANFIELD, EARL RODNEY
64 CARROLL, FRANCIS WIILLIAM
65 CATER, FRANK SYDNEY
66 CATES, MARSHALL L.
67 CATLIN, PAUL ALLEN*
68 CHALK, JOHN H. H.*
69 CHARTRAND, GARY THEODORE
70 CHEN, CHUAN CHONG
71 CHEN, GUANTAO
72 CHEN, HANG
73 CHEN, ROBERT W.
74 CHINN, PHYLLIS ZWEIG
75 CHOI, SHIU LUN GODFREY
76 CHOWLA, SARVADAMAN D. S.*
77 CHUI, CHARLES KAM-TAI
78 CHUNG, FAN RONG KING (GRAHAM)
79 CHUNG, KAI-LAI*
80 CHVATAL, VACLAV (VASEK)
81 CLARK, BRENT N.
82 CLARK, LANE HENRY
83 CLARKSON, JAMES ANDREW*
84 CLUNIE, JAMES G.
85 COHEN, STEPHEN D.
86 COLBOURN, CHARLES JOSEPH
298 MALOUF, JANICE L.
299 MARCUS, SOLOMON
300 MATE, ATTILA
301 MAUDUIT, CHRISTIAN
302 MAULDIN, RICHARD DANIEL
303 MAXSEIN, THOMAS*
304 MAYS, MICHAEL ELLSWORTH
305 MCCANNA, JOSEPH E.
306 MCELIECE, ROBERT JAMES
307 MCKAY, BRENDAN DAMIEN
308 MEIR, AMRAM
309 MELFI, GIUSEPPE
310 MEYER, ANJA GABRIELE
311 MILLS, GEORGE HARLOW
312 MILNER, ERIC CHARLES*
313 MINC, HENRYK
314 MIRSKY, LEONID*
315 MOLLOY, MICHAEL SEAN O'BRIEN
316 MONTGOMERY, HUGH LOWELL
317 MONTGOMERY, PETER L.
318 MOON, JOHN W.
319 MORAN, SHLOMO
320 MORTON, PATRICK
321 MOSER, LEO*
322 MULLIN, RONALD C.
323 MURTY, MARUTI RAM PEDAPROLU
324 MURTY, VIJAYA KUMAR
325 NATHANSON, MELVYN BERNARD
326 NESETRIL, JAROSLAV
327 NETANYAHU, ELISHA*
328 NEVEU, JACQUES
329 NEWMAN, DONALD JOSEPH*
330 NEY, PETER E.
331 NICOLAS, JEAN-LOUIS
332 NIVEN, IVAN MORTON*
333 NORTON, DONALD ALAN
334 O'NEIL, PATRICK EUGENE
335 OBLATH, RICHARD*
336 ODLYZKO, ANDREW MICHAEL
337 OELLERMANN, ORTRUD R.
338 OFFORD, ALBERT CYRIL*
339 ORDMAN, EDWARD THORNE
340 OXTOBY, JOHN C.*
341 PACH, JANOS
342 PALFY, PETER PAL

87 CONWAY, JOHN HORTON
88 COPELAND, ARTHUR HERBERT, SR.*
89 CROFT, HALLARD T.
90 CSAKI, ENDRE
91 CSISZAR, IMRE
92 CZIPSZER, JANOS*
93 DARLING, DONALD A.
94 DARST, RICHARD BRIAN
95 DAVENPORT, HAROLD*
96 DAVIES, ROY O.
97 DAYKIN, DAVID E.
98 DE BRUIJN, NICOLAAS GOVERT
99 DE CAEN, DOMINIQUE*
100 DE KONINCK, JEAN-MARIE
101 DEBOSE, YOLANDA
102 DEHEUVELS, PAUL
103 DELEGLISE, MARC
104 DENES, JOZSEF
105 DESHOUILLERS, JEAN-MARC
106 DEUBER, WALTER A.*
107 DEZA, MICHEL-MARIE
108 DIACONIS, PERSI W.
109 DIAMOND, HAROLD GEORGE
110 DIRAC, GABRIEL ANDREW*
111 DIXMIER, JACQUES
112 DOWKER, YAEL NAIM
113 DRAKE, DAVID ALLYN
114 DUDLEY, UNDERWOOD
115 DUKE, RICHARD ALTER
116 DVORETZKY, ARYEH*
117 ECKLUND, EARL F., JR.
118 EDREI, ALBERT*
119 EGGLETON, ROGER BENJAMIN
120 EL-ZAHAR, MOHAMED H.
121 ELEKES, GYORGY
122 ELLIOTT, PETER D. T. A.
123 ENTRINGER, ROGER CHARLES
124 ERNE, MARCEL
125 EVANS, ANTHONY B.
126 FABER, VANCE
127 FAJTLOWICZ, SIEMION
128 FAUDREE, RALPH JASPER, JR
129 FEJES TOTH, LASZLO*
130 FELDHEIM, ERVIN*
131 FELLER, WILLI K. (WILLIAM)*
343 PALKA, ZBIGNIEW J.
344 PALMER, EDGAR MILAN
345 PAPP, ZOLTAN Z.
346 PARSONS, TORRENCE DOUGLAS*
347 PAYAN, CHARLES
348 PENNEY, DAVID EMROY, II
349 PHELPS, KEVIN THOMAS
350 PINKUS, ALLAN M.
351 PIPPERT, RAYMOND ELMER
352 PIRANIAN, GEORGE*
353 POLLACK, RICHARD M.
354 POLLARD, HARRY STRANGE*
355 POMERANCE, CARL BERNARD
356 POSA, LAJOS
357 PRACHAR, KARL*
358 PREISS, DAVID
359 PRINS, GEERT CALEB ERNST*
360 PUDAITE, PAUL
361 PULLMAN, NORMAN JAY*
362 PURDY, GEORGE BARRY
363 PYBER, LASZLO
364 RADO, RICHARD*
365 RAMACHANDRA, KANAKANAHALLI
366 RAO, SIDDANI BHASKARA
367 RAUZY, GERARD
368 REDDY, A. R.
369 REID, TALMAGE JAMES
370 RENYI, ALFRED A.*
371 REVESZ, PAL
372 REZNICK, BRUCE ARIE
373 RICHARDS, JONATHAN IAN*
374 RICHMOND, LAWRENCE BRUCE
375 RIEGER, GEORG JOHANN
376 RIESEL, HANS
377 ROBINSON, ROBERT WILLIAM
378 RODL, VOJTECH
379 ROGERS, CLAUDE AMBROSE*
380 ROSA, ALEXANDER
381 ROSENBLOOM, PAUL CHARLES*
382 ROSENFELD, MOSHE^1
383 ROTHSCHILD, BRUCE LEE
384 ROUSSEAU, CECIL CLYDE
385 RUBEL, LEE ALBERT*
386 RUBIN, ARTHUR L.
387 RUDIN, MARY ELLEN ESTILL

132 FELZENBAUM, ALEXANDER GERSH
133 FEW, LEONARD
134 FISHBURN, PETER C.
135 FODOR, GEZA*
136 FON-DER-FLAASS, DMITRI G.
137 FOWLER, JOEL CHRISTOPHER
138 FOWLER, THOMAS GEORGE
139 FRAENKEL, AVIEZRI SIEGMUND
140 FRANKL, PETER
141 FREEDMAN, ALLEN R.
142 FREIMAN, GREGORY A.
143 FREUD, GEZA*
144 FREUD, ROBERT
145 FRIED, ERVIN
146 FRIED, HANS*
147 FUCHS, WOLFGANG HEINRICH JOHANNES*
148 FUREDI, ZOLTAN
149 GAAL, STEVEN A. (GAL, ISTVAN SANDOR)
150 GALAMBOS, JANOS
151 GALLAI, TIBOR* (GRUNWALD, TIBOR)
152 GALVIN, FREDERICK WILLIAM
153 GERENCSER, LASZLO
154 GILLIS, JOSEPH E.*
155 GILLMAN, LEONARD*
156 GIMBEL, JOHN GORDON
157 GINZBURG, ABRAHAM
158 GODDARD, WAYNE DEAN
159 GODSIL, CHRISTOPHER DAVID
160 GOLDBERG, MARK K.
161 GOLOMB, MICHAEL*
162 GOODMAN, ADOLPH W.*
163 GORDON, BASIL
164 GOULD, RONALD J.
165 GRAHAM, RONALD LEWIS
166 GRAHAM, SIDNEY WEST
167 GRANVILLE, ANDREW JAMES
168 GRIESER, DANIEL
169 GRILL, KARL
170 GRUBER, PETER MANFRED
171 GRUNBAUM, BRANKO
172 GRUNWALD, GEZA*
173 GUNDERSON, DAVID SHANE
174 GUPTA, HANSRAJ*

388 RUSZINKO, MIKLOS
389 RUZSA, IMRE Z.
390 RYAVEC, CHARLES ALBERT
391 SACHS, HORST
392 SAFFARI, BAHMAN
393 SAIAS, ERIC
394 SAKS, MICHAEL EZRA
395 SALAMON, PETER
396 SALAT, TIBOR*
397 SANDOR, CSABA
398 SARKAR, AMITES
399 SARKOZY, ANDRAS
400 SARKOZY, GABOR N.
401 SAUER, NORBERT W.
402 SCHAER, JONATHAN
403 SCHELP, RICHARD H.
404 SCHERK, PETER*
405 SCHINZEL, ANDRZEJ BOBOLA MARIA
406 SCHMUTZ, ERIC J.
407 SCHNITZER, FRANZ JOSEF*
408 SCHONHEIM, JOHANAN
409 SCHULMAN, LEONARD J. Y.
410 SCHUSTER, SEYMOUR
411 SCHWENK, ALLEN JOHN CARL
412 SEGAL, SANFORD LEONARD*
413 SEIDEL, WLADIMIR P.*
414 SELFRIDGE, JOHN L.
415 SELKOW, STANLEY M.
416 SERESS, AKOS
417 SHALLIT, JEFFREY OUTLAW
418 SHAPIRO, HAROLD NATHANIEL
419 SHAPIRO, HAROLD SEYMOUR
420 SHARMA, AMBIKESHWAR*
421 SHELAH, SAHARON
422 SHENG, TSENG KUO
423 SHIELDS, ALLEN LOWELL*
424 SHISHA, OVED*
425 SHOREY, TARLOK N.
426 SHREVE, WARREN EUGENE
427 SILBERMAN, GABRIEL M.
428 SILVERMAN, RUTH
429 SIMMONS, GUSTAVUS J.
430 SIMONOVITS, MIKLOS

175 GUY, MICHAEL J. T.
176 GUY, RICHARD KENNETH
177 GYARFAS, ANDRAS
178 GYORI, ERVIN
179 GYORY, KALMAN
180 HAJNAL, ANDRAS
181 HALASZ, GABOR
182 HALL, RICHARD ROXBY
183 HAMBURGER, PETER
184 HAMMER, JOSEPH
185 HANANI, HAIM*
186 HANSON, DENIS
187 HARARY, FRANK*
188 HARCOS, GERGELY
189 HARDY, GEORGE EUGENE
190 HARE, WILLIAM RAY, JR.
191 HARNER, CHARLES C.
192 HARTMAN, STANISLAW*
193 HARTTER, ERICH
194 HARZHEIM, EGBERT
195 HATTINGH, JOHANNES HENDRIK
196 HECHLER, STEPHEN HERMAN
197 HEDETNIEMI, STEPHEN TRAVIS
198 HEDRLIN, ZDENEK
199 HEGYVARI, NORBERT
200 HEILBRONN, HANS ARNOLD*
201 HELL, PAVOL
202 HEMMINGER, ROBERT LOUIS
203 HENNING, MICHAEL ANTHONY
204 HENRIKSEN, MELVIN*
205 HERZOG, FRITZ*
206 HERZOG, MARCEL
207 HICKERSON, DEAN ROBERT
208 HIGGS, DENIS ARTHUR
209 HILDEBRAND, ADOLF J.
210 HINDMAN, NEIL B.
211 HOBBS, ARTHUR M.
212 HOFFMAN, ALAN JEROME
213 HOGGATT, VERNER EMIL, JR.*
214 HOLTON, DEREK ALLAN
215 HOLZMAN, RON
216 HORAK, PETER
217 HORVATH, MIKLOS

431 SINGHI, NAVIN MADHAVPRASAD
432 SIRAN, JOZEF
433 SIRAO, TUNEKITI
434 SKILTON, DONALD K.
435 SMITH, BRENT PENDLETON
436 SMITH, PAUL R.
437 SOIFER, ALEXANDER
438 SOS, VERA TURAN
439 SPECKER, ERNST P.
440 SPENCER, JOEL HAROLD
441 SPIRO-SILVERMAN, CLAUDIA A.
442 STATON, WILLIAM A., III
443 STEIN, ALAN H.
444 STEIN, SHERMAN K.
445 STEWART, CAMERON LEIGH
446 STINSON, DOUGLAS ROBERT
447 STONE, ARTHUR HAROLD*
448 STRAIGHT, H. JOSEPH
449 STRAUS, ERNST GABOR*
450 SUBBARAO, MATUKUMALLI VENKATA*
451 SUEN, W.-C. STEPHEN
452 SUN, HUI CHENG
453 SURANYI, JANOS*
454 SWART, HENDRIKA CORNELIA SCOTT (HENDA)
455 SZABADOS, JOZSEF
456 SZALAY, MIHALY
457 SZEGEDY, MARIO
458 SZEGO, GABOR*
459 SZEKELY, LASZLO A.
460 SZEKERES, ESTHER KLEIN*
461 SZEKERES, GEORGE*
462 SZEMEREDI, ENDRE
463 SZUSZ, PETER*
464 TARSKI, ALFRED*
465 TAYLOR, ALAN DANA
466 TAYLOR, HERBERT
467 TAYLOR, SAMUEL JAMES
468 TENENBAUM, GERALD
469 TETALI, PRASAD VENKATA SITARAMA VARA
470 THOMASSEN, CARSTEN
471 TIJDEMAN, ROBERT
472 TOTH, JANOS T.
473 TOTIK, VILMOS

218 HOWORKA, EDWARD R.
219 HSU, DERBIAU FRANK
220 HUNT, GILBERT AGNEW*
221 HWANG, JUN SHUNG*
222 INDLEKOFER, KARL-HEINZ
223 INGHAM, ALBERT EDWARD*
224 IVIC, ALEKSANDAR
225 JABOTINSKY, ERI*
226 JACKSON, STEPHEN CRAIG
227 JACOBSON, MICHAEL SCOTT
228 JAGOTA, ARUN KUMAR
229 JANSON, SVANTE
230 JARNIK, VOJTECH*
231 JIN, GUO PING
232 JONES, FRED B.
233 JOO, ISTVAN*
234 JOO, MIKLOS
235 KAC, MARK*
236 KAINEN, PAUL C.
237 KAKUTANI, SHIZUO*
238 KAPLANSKY, IRVING*
239 KARAMATA, JOVAN*
240 KATAI, IMRE
241 KELLY, PAUL JOSEPH*
242 KENNEDY, JOHN W.
243 KESTELMAN, HYMAN*
244 KHARE, SATGUR PRASAD
245 KIERSTEAD, HENRY A.
246 KISS, PETER*
247 KLAMKIN, MURRAY SEYMOUR*
248 KLAWE, MARIA MARGARET
249 KLEITMAN, DANIEL J.
250 KLUGERMAN, MICHAEL RICHARD
251 KNAPPENBERGER, JONATHAN CHARLES
252 KO, CHAO (KE, ZHAO)*
253 KOCZY, LASZLO A.
254 KOHAYAKAWA, YOSHIHARU
255 KOKSMA, JURJEN FERDINAND^1*
256 KOLESNIK, GRIGORI ABRAMOVICH
474 TOVEY, CRAIG AARON
475 TROTTER, WILLIAM THOMAS, JR.
476 TURAN, PAL*
477 TURK, JOHANNES WILHELMUS MARIA (JAN)
478 TUTTE, WILLIAM THOMAS*
479 TUZA, ZSOLT
480 ULAM, STANISLAW MARCIN*
481 URBANIK, KAZIMIERZ*
482 VAALER, JEFFREY DAVID
483 VALTR, PAVEL
484 VAN KAMPEN, EGBERTUS RUDOLF*
485 VAN LINT, JACOBUS HENDRICUS*
486 VARMA, ARUN KUMAR*
487 VAUGHAN, ROBERT CHARLES
488 VAZSONYI, ANDREW* (WEISZFELD, ENDRE)
489 VERTESI, PETER O. H.
490 VESZTERGOMBI, KATALIN L.
491 VIJAYAN, KAIPILLIL
492 VINCZE, ISTVAN*
493 VOLKMANN, BODO
494 WAGSTAFF, SAMUEL STANDFIELD, JR.
495 WANG, JIAN FANG
496 WEAKLEY, WILLIAM DOUGLAS
497 WEISS, GARY LYNN
498 WEST, DOUGLAS BRENT
499 WILLIAMSON, ALAN GLYNNE
500 WILSON, RICHARD MICHAEL
501 WILSON, ROBIN JAMES
502 WINKLER, PETER MANN
503 WINTNER, AUREL FRIEDRICH*
504 WORMALD, NICHOLAS CHARLES
505 YAO, FOONG FRANCES
506 ZAKS, ABRAHAM
507 ZAKS, SHMUEL
508 ZALCSTEIN, YECHEZKEL
509 ZAREMBA, STANISLAW KRYSTYN*
510 ZHANG, ZHEN XIANG
511 ZIV, ABRAHAM

Appendix II

1	Robert Downey Jr
2	Will Smith
3	Johnny Depp
4	Denzel Washington
5	Brad Pitt
6	Matt Damon
7	Leonardo DiCaprio
8	Angelina Jolie
9	Meryl Streep
10	George Clooney
11	Sandra Bullock
12	Mark Wahlberg
13	Daniel Radcliffe
14	Tom Cruise
15	Jennifer Lawrence
16	Christian Bale
17	Liam Neeson
18	Emma Stone
19	Shia LaBeouf
20	Chris Hemsworth
21	Steve Carell
22	Adam Sandler
23	Bruce Willis
24	Channing Tatum
25	Ben Stiller
26	Rachel McAdams
27	Dwayne Johnson
28	Cameron Diaz
29	Zach Galifianakis
30	Hugh Jackman
31	Vin Diesel
32	Jason Statham
33	Tom Hanks
34	Natalie Portman
35	Tina Fey