Simplex Solver

March 10, 2023

Problem

Given the following linear system and objective function, find the optimal solution.

$$\max 3x_1 + 2x_2$$

$$\begin{cases} x_1 + 3x_2 \leq 5 \\ 2x_1 - x_2 \leq 4 \end{cases}$$

Solution

Add slack variables to turn all inequalities to equalities.

$$\begin{cases} x_1 + 3x_2 + s_1 = 5 \\ 2x_1 - x_2 + s_2 = 4 \end{cases}$$

Create the initial tableau of the new linear system.

$$\begin{array}{cc} \left[\begin{array}{cccc|c} x_1 & x_2 & s_1 & s_2 & b \\ \hline 1 & 3 & 1 & 0 & 5 \\ 2 & -1 & 0 & 1 & 4 \\ \hline -3 & -2 & 0 & 0 & 0 \end{array}\right] & \begin{array}{c} \\ s_1 \\ s_2 \\ \\ \end{array} \end{array}$$

There are negative elements in the bottom row, so the current solution is not optimal. Thus, pivot to improve the current solution. The entering variable is x_1 and the departing variable is s_2.

Perform elementary row operations until the pivot element is 1 and all other elements in the entering column are 0.

$$\begin{array}{cc} \left[\begin{array}{cccc|c} x_1 & x_2 & s_1 & s_2 & b \\ \hline 0 & 7/2 & 1 & -1/2 & 3 \\ 1 & -1/2 & 0 & 1/2 & 2 \\ \hline 0 & -7/2 & 0 & 3/2 & 6 \end{array}\right] & \begin{array}{c} \\ s_1 \\ x_1 \\ \\ \end{array} \end{array}$$

There are negative elements in the bottom row, so the current solution is not optimal. Thus, pivot to improve the current solution. The entering variable is x_2 and the departing variable is s_1.

Perform elementary row operations until the pivot element is 1 and all other elements in the entering column are 0.

$$\begin{array}{c c} \left[\begin{array}{cccc|c} x_1 & x_2 & s_1 & s_2 & b \\ \hline 0 & 1 & 2/7 & -1/7 & 6/7 \\ 1 & 0 & 1/7 & 3/7 & 17/7 \\ \hline 0 & 0 & 1 & 1 & 9 \end{array}\right] & \begin{array}{c} \\ x_2 \\ x_1 \\ \\ \end{array} \end{array}$$

There are no negative elements in the bottom row, so we know the solution is optimal. Thus, the solution is:

$$s_1 = 0, s_2 = 0, x_1 = \frac{17}{7}, x_2 = \frac{6}{7}, z = 9$$