Simplex Solver

April 17, 2023

Problem

Given the following linear system and objective function, find the optimal solution.

$$\max 2x_1 + x_2$$
$$\begin{cases} y_1 + y_2 \geq 6 \\ --2y_1 + y_2 \geq -2 \end{cases}$$

Solution

Add slack variables to turn all inequalities to equalities.

$$\begin{cases} y_1 - -2y_2 + s_1 = 2 \\ y_1 + y_2 + s_2 = 1 \end{cases}$$

Create the initial tableau of the new linear system.

$$\left[\begin{array}{cccc|c} y_1 & y_2 & s_1 & s_2 & b \\ \hline 1 & -2 & 1 & 0 & 2 \\ 1 & 1 & 0 & 1 & 1 \\ \hline -6 & 2 & 0 & 0 & 0 \end{array}\right] \begin{array}{c} \\ s_1 \\ s_2 \\ \\ \end{array}$$

There are negative elements in the bottom row, so the current solution is not optimal. Thus, pivot to improve the current solution. The entering variable is y_1 and the departing variable is s_2.

Perform elementary row operations until the pivot element is 1 and all other elements in the entering column are 0.

$$\left[\begin{array}{cccc|c} y_1 & y_2 & s_1 & s_2 & b \\ \hline 0 & -3 & 1 & -1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ \hline 0 & 8 & 0 & 6 & 6 \end{array}\right] \begin{array}{c} \\ s_1 \\ y_1 \\ \\ \end{array}$$

There are no negative elements in the bottom row, so we know the solution is optimal. Thus, the solution is:

$$s_1 = 1, s_2 = 0, x_1 = 0, x_2 = 6, y_1 = 1, y_2 = 0, z = 6$$